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FINAL REPORT FOR GRANT AFOSR 62-87

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Research Conducted at Yale University, New Haven, Conn.

Summary

This report outlines the research done at Yale University under Grant AFOSR 62-87. The background and results of the research are given in the two listed publications, and that material has not been entirely duplicated here. Some pertinent descriptions and sketches of the experimental apparatus that have not been published are, however, included.

Part B) of the report describes the density measurements as a function of temperature and pressure (PVT relations) made on solid He^4. The isothermal compressability and the volume changes at the phase transitions have been accurately measured in the region of the body-centered-cubic solid phase. An estimate is made of the isobaric thermal expansion coefficient. The data suggest that thermodynamic relations predicting a negative thermal expansion coefficient in the solid might be altered by the fact that the compressability of the solid at the melting curve is greater than at higher pressures. Plans for continuing the research begun under this Grant are described.

Part A) is a summary of the calculations made on the data of several experiments studying the flow of superfluid helium. It has been found that there is ample evidence that a dissipative interaction can exist between the superfluid and the boundaries of the flow system when the superfluid velocity is greater than a certain critical value. Some authors have expressed the opinion that the superfluid component of liquid helium II can only interact with the normal fluid component.

Personnel: The persons who have worked under this Grant are the Principal Investigator, John N. Kidder, and various shop, electronics, and glass technicians.

Publications: "Critical Velocities and Boundary Interactions in the Isothermal Flow of Superfluid Helium." J. N. Kidder and W. M. Fairbank, Phys. Rev. 127, 987 (1962).

"Density Measurements in Solid He^4", Proceedings of the 8th International Conference on Low Temperature Physics, London, England (to be published).

The two publications represent the results of two different phases of the work done under Grant AFO SR 62-87. It will simplify this report to consider these two phases separately.

A) Boundary Interactions in the Flow of Superfluid Helium.

The hydrodynamics of superfluid helium has been the subject of experimental and theoretical investigation for thirty years.[1] One such experiment was conducted at Duke University by W. M. Fairbank and J. N. Kidder, and was the subject of the latter's Ph.D. dissertation. This experiment substantiated previous work by finding clear evidence that, in an isothermal flow scheme where only the superfluid is moving, the superfluid will flow without resistance below a certain critical velocity. The critical velocity is a function of channel geometry and temperature.

The work at Duke was terminated without a successful interpretation of the pressure gradients observed at velocities greater than the critical velocity. Since these data were considered significant, at Yale an extensive effort was made to examine them in greater detail, in hopes of being able to make quantitative statements as to the origin of the observed pressure gradients.

One could describe the data equally well by two equations relating the superfluid velocity v_s, the critical velocity v_c, the pressure gradient ∇p, and two proportionality constants α_1 and α_2:

$$\nabla p = \alpha_1 (v_s - v_c)^{1.7} \quad (1)$$

$$\nabla p = \alpha_2 v_s (v_s - v_c) \quad (2)$$

Equation (1) is almost exactly the relation that describes the turbulent flow of ordinary fluids in circular cross section tubes[2], and Eq. (2) was similar to the experimentally observed relation between the lift on an airfoil and the velocity of superfluid helium[3]. These similarities are discussed in greater detail in the Physical Review article.

There was a dual purpose to the work at Yale and the subsequent publication: The first was to establish the thesis that a superfluid-boundary interaction had in fact been observed, and the second was to show that the introduction of such an interaction could clarify the results of other experiments. It was also possible that examination of other data would reveal which of the two forms (Eq. (1) or Eq. (2)) gave the most suitable description of the superfluid force.

In 1951 Atkins published the results of an experiment on the gravitational flow of liquid helium.[4] Helium was allowed to flow in and out of a glass reservoir through a capillary tube attached to the base. The velocity was measured by the rate of change of the level in the reservoir, and the pressure gradient was calculated from the difference in level inside and outside the reservoir. The experiment was repeated for different capillary lengths and diameters and at different temperatures.

Atkins attempted to explain his data on the assumption that two dissipative or resistive mechanisms were impeding the flow of the helium, which in this case was the flow of both the superfluid and the normal fluid. They were the viscous interaction between the normal fluid component and the wall of the capillary, and the mutual friction interaction between the two fluids. This, however, was not entirely satisfactory, and Atkins noted that the introduction or consideration of some form of turbulence, possibly in the superfluid, might be necessary.

To introduce a superfluid-boundary interaction into the two-fluid hydrodynamical equations to give a pressure gradient of the form of Eqs. (1) or (2) is simple in principle. The analysis of a set of data to determine whether or not such a term is relevant can be much more difficult. It is perhaps for this reason as much as any other that the relatively small effect of the proposed superfluid force has not been considered by most authors.

The details of how the two-fluid equations were analysed, and the results are given in the Physical Review paper. Theoretical curves corresponding to Atkins' experimental values were calculated with and without a superfluid-boundary interaction included in the equations. If the inclusion of this term gave better agreement between theory and experiment than the step was considered valid.

It was found that in some cases there was definite evidence that the superfluid force was a significant factor in Atkins' results. For those sets of data where the analysis was not conclusive or negative, it was shown that experimental uncertainty could have made a definitive analysis impossible.

The authors considered the paper complete when the analysis of, and comparison with, Atkins' results had been made. But then in December, 1961, an important set of papers was published by Staas, Taconis, and Van Alphen at Leiden.[5,6] They had studied the flow of liquid helium and claimed to find no superfluid-boundary interaction. Clearly this work had to be taken into consideration.

Numerical calculations showed that a superfluid force of the type discussed here could have been undetected in some of their experiments, and could have been "masked" by normal fluid turbulence in the others. Furthermore, the results of their experiment designed to detect the two-fluid mutual friction force[6] are more easily understood if one includes a pressure gradient of the form of Eq. (1) or (2).

This problem can be best resolved by further experiments specifically designed to study the superfluid-boundary interaction. Such a program is now under way at Dartmouth College under Grant AFO SR 74-63, which is essentially a continuation of Grant AFO SR 62-87 with the same principal investigator but at a different institution. The first experiments will be a study of the flow of superfluid helium at 0.35°K, where the complicating effects of the normal fluid are not present.

In the meantime, the work at Yale under Grant AFO SR 62-87 served to establish the importance of considering these effects in studying superfluid hydrodynamics. It was shown that some previous data that had not been completely understood might be better explained by considering the superfluid force. While this research was not part of the program originally proposed, it is believed that it made a significant contribution to superfluid hydrodynamics. The one question left unresolved was whether Eq. (1) or Eq. (2) gave the most description of the superfluid force. It is hoped that the experiments at Dartmouth give the answer.

B) Density Measurements in Solid Helium

Measurements of pressure-volume-temperature (PVT) relations in solid He^4, He^3, and isotopic mixtures are of interest both in themselves and because of other thermodynamic properties than can be calculated from them. Such properties as the nuclear magnetic susceptibility can only be measured accurately if the PVT equation of state is known. Two years ago the only measurements of this sort that had been made were on the region near the melting curve. This suggested the advisability of building the system described herein to measure directly changes in density as a function of temperature and pressure.

The density of solid helium contained in a microwave cavity can be calculated from the resonant frequency of the cavity. The relatively low surface to volume ratio of a microwave cavity allows better pressure and density equilibrium within the sample than with other comparable systems (e.g., a parallel plate capacitor).

The resonant frequency of the sample cavity was measured by comparison with a calibrated secondary standard wavemeter, using the simple and straight-forward circuit shown in Fig. 1. Part of the signal from a frequency modulated klystron was reflected from the wavemeter, detected, and displayed on an oscilloscope. If the wavemeter resonance was within the band of frequencies generated by the klystron, a resonant curve would be seen on the oscilloscope. The oscilloscope sweep was synchronized with the "sweep" (frequency modulation) of the klystron.

The remaining klystron signal was doubled in frequency by a crystal multiplier, reflected from the sample cavity, detected, and displayed with the second beam of the same oscilloscope. When the two resonant curves coincided on the oscilloscope screen, the sample cavity frequency was twice that of the standard wavemeter.

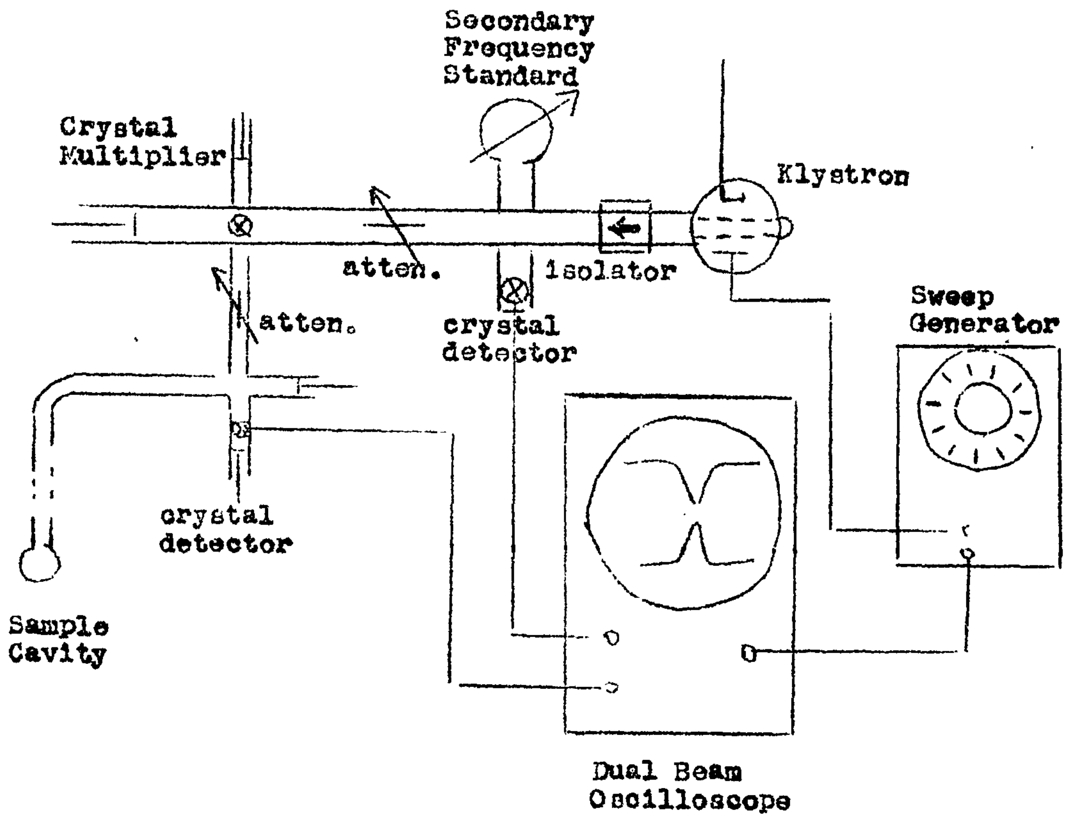

Fig. 1

The microwave circuit used for comparing the resonant frequency of the secondary standard wavemeter with the sample cavity containing solid helium. The circuit impedances were matched so that each resonant curve was symmetrical. The resonances were compared by displaying them simultaneously (one inverted) on a dual beam oscilloscope, then tuning the wavemeter to line up the 'peaks'.

The frequency multiplication of the signal to the sample cavity was not necessary in principle, but considerations of cost, accuracy, and simplicity made it advisable. The sample cavity was made as small as possible, and was therefore in the K-band (18 - 26.5 Kmc). The greatest accuracy and least expense were achieved by having the klystron and secondary standard wavemeter in the X-band (8 - 12.4 Kmc).

The He^4 sample gas was commercial bottled helium purified by passing it through a liquid nitrogen cooled charcoal trap. A glass tubing and Toepler pump system was built for handling and storing the gas samples. To compress the helium gas, and therefore the solid sample, to the required pressures of up to 200 atmospheres, the gas was enclosed in one side of a mercury filled, stainless steel U-tube, and nitrogen gas from a pressurised cylinder was applied to the other side. A 0.027 inch diameter stainless steel capillary tube connected the microwave sample cavity with the helium side of the mercury filled U-tube. This pressurizing system is shown in Fig. 2.

The pressure in the sample cavity was measured externally by first measuring the pressure of the nitrogen gas with a Heise bourdon tube gauge, and then correcting this reading for the difference in the height of the mercury levels in the two arms of the U-tube. The mercury levels were determined by locating two steel ball bearings, floating on the mercury surface, with external magnetic search coils.

The sample was refrigerated with a conventional liquid helium bath in a Dewar flask, connected to a high speed pumping system. The temperature was measured by the vapor pressure of the helium bath. A liquid He^3 pumping system was built for achieving temperatures in the range 0.35°K to 1.1°K. This system consisted of a diffusion pump and a mechanical pump, operating in a closed cycle to preserve the He^3 gas. Only a few

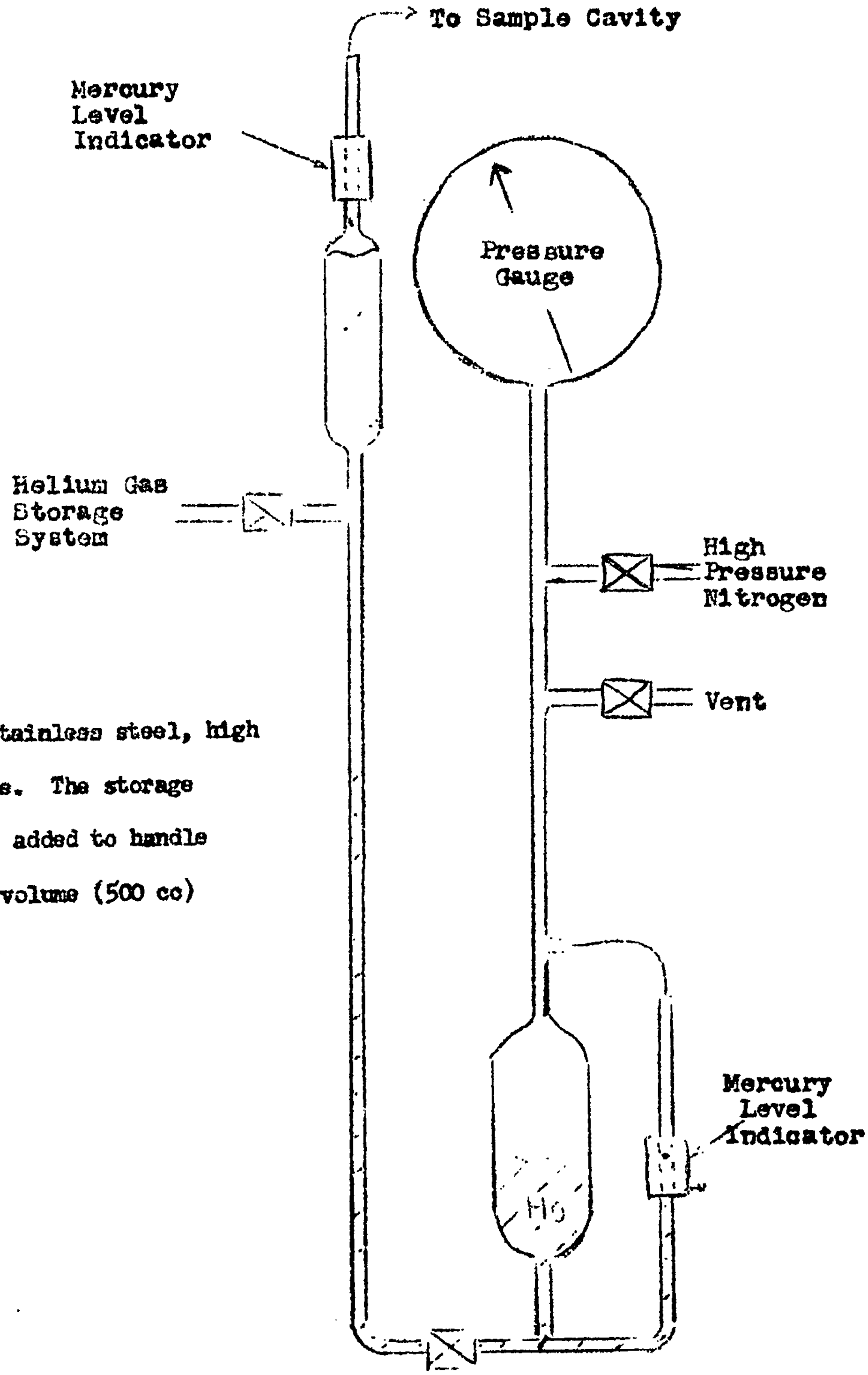

Fig. 2 - The stainless steel, high pressure U-tubes. The storage cylinders were added to handle the necessary volume (500 cc) of sample gas.

cc's of liquid He^3 are required in the cryostat, as the bulk of the refrigeration comes from the He^4 bath at $1.1^{\circ}K$. Again, in such a system, the temperature is measured by the vapor pressure of the bath.

As stated above, the pressure in the solid sample was measured from outside the cryostat, and this method would be always accurate if the sample were a fluid. However, since it is a solid, there is the danger that the small diameter fill line will become plugged and that there will not be pressure and density equilibrium within the cavity. This problem can be solved two ways: By heating the insulated fill line to keep the small diameter section melted, or by measuring the pressure directly at the sample. The first approach was used in the experimental work reported here, and a technique for using the latter has been developed.

The fill line was insulated by a vacuum jacket, and at a point just above the end of the vacuum jacket and where the fill line was connected to the cavity, a small wire heater was wound on the capillary tube. That portion of the fill line that was at the temperature of the helium bath and filled with solid was of a large enough diameter to permit the solid to flow. As described in the London Conference paper, this technique gave reasonable assurance of having equilibrium conditions.

A capacitance-type transducer pressure gauge has been built to measure the pressure of the sample 'in situ', inside the cryostat.[7] Thus far, it has only been tested at room temperature. Some further work will be needed to insure that it will be sufficiently sensitive and single valued for the intended use. Fig. 3 is a sketch of the present experimental sample cavity and the proposed capacitance pressure gauge.

The measurements made were on solid He^4 in the region of the γ (body-centered-cubic) solid phase. Continuous measurements of the density of the solid were made at pressures greater than the melting pressure. The

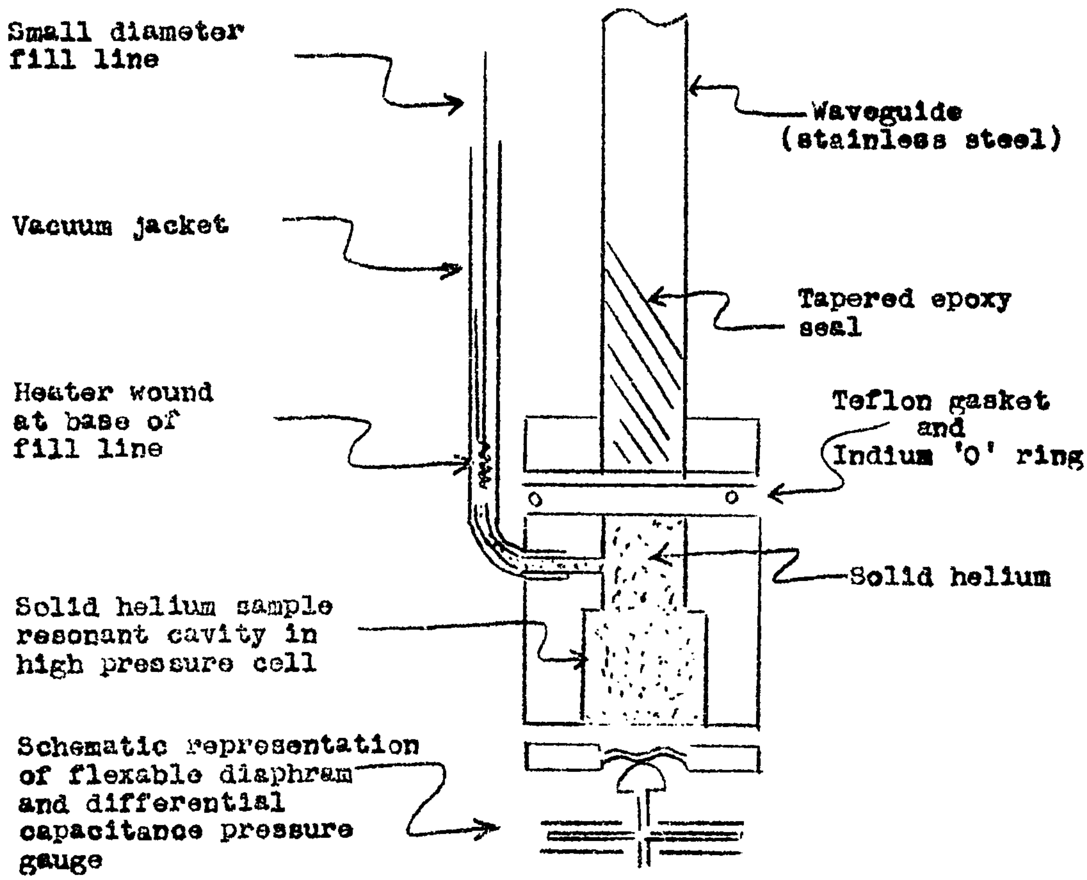

Fig. 3

The sample cavity, high pressure cell, waveguide, and fill line. This part of the apparatus was in a liquid helium bath. The waveguide seal of epoxy resin with a 0.010 inch Teflon gasket and indium 'O' ring contained the solid helium at high pressures but transmitted the microwave signal. The construction of the fill line is described in the text.

isothermal compressability of both the γ (b.c.c.) and the α (hexagonal-close-packed) phases were measured directly. It was also possible to determine accurately the volume change on melting and the volume difference between the two solid phases. The isobaric thermal expansion coefficient was observed to be everywhere positive, and reasonable measurements of its magnitude were made.

From basic thermodynamic considerations, using measured parameters of the solid and the liquid, it had been predicted that the thermal expansion coefficient of solid He^4 would be negative over certain ranges of temperature along the melting curve.[8,9] Our measurements did not confirm this prediction, but an alternate explanation was suggested. It appears that the compressability of the solid may be greater at the melting curve than at higher pressures. If further investigation, now under way, confirms this, it could give a solution to the thermodynamic relations without requiring an anomalous negative thermal expansion coefficient in the solid.

The experiments started at Yale are being continued at Dartmouth College under Grant AFO SR 74-63. As has been discussed in our correspondence with the AFO SR, almost all of the equipment built and purchased at Yale has been moved to Dartmouth. The additional necessary facilities are nearly completed and measurements will be resumed shortly.

The new research program at Dartmouth will begin with an investigation of the magnitude of the isothermal compressability of solid He^3. Swenson and Heltemes[10] have found evidence from specific heat measurements that the compressability of the body-centered-cubic phase is greater than that of the liquid, which would be an anomalous effect. It is also of interest to determine directly the magnitude of the volume change in the (b.c.c.) - (h.c.p.) phase transition in solid He^3.

We would also like to study the PVT relations of solid He^3 - He^4 isotopic mixtures, as little is known of their solid phase diagrams or melting curves. Considering the insufficient data and the interest in solid helium, the density measurements started under Grand AFO SR 62-87 should continue to be fruitful for some time to come.

Notes and References

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