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TRANSLATION

USE OF EXPERIMENTAL DATA IN CALCULATING THE RELIABILITY OF RADIOELECTRONIC EQUIPMENT, BASED ON THE POISSON DISTRIBUTION

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Use of Experimental Data in Calculating the Reliability of Radioelectronic Equipment, Based on the Poisson Distribution.

by

G.B.Linkovskiy

A discussion is held on the use of experimental data on the average time of faultless operation in calculating the reliability of the number of failures during Poisson distribution.

At the IV American national symposium on reliability the reliability administrator of the known RCA company C.M.Ryerson presented an extensive report entitled " Theory of Reliability Testing, Based on Poisson Distribution " [1]. Analysis of this report is contained in the review by B.R.Levin [2] . According to the law of Poisson distribution the probability that exactly k failures will take place during the time t, equals

$$P_k(t)=\frac{1}{k!}\left(\frac{t}{\bar{m}}\right)^k e^{-\frac{t}{\bar{m}}} \quad (1)$$

where $\bar{m}$ - average time of faultless operation.

If in expression (1) is written k = 0, then we will obtain

$$P_0(t)=e^{-\frac{t}{\bar{m}}}. \quad (2)$$

This means that probability of faultless operation follows the exponential law. Formula (2) serves as basis for empirical determination of $\bar{m}$. The fact is, probability of opposite breakdown to the moment of time t, which we will designate by K(t), will have the form of

$$K(t)=1-P_0(t)=1-e^{-\frac{t}{\bar{m}}}. \quad (3)$$

If we consider the moment of time, when the first failure of the apparatus does take place (connected at t = 0) as an accidental value xi, then K(t) appears to be a distribution probability xi:

$$P\{\xi<t\}=K(t). \quad (4)$$

at $t \geq 0$. Density of probability of random value xi is equal

$$p_\xi(t) = \frac{dK(t)}{dt} = \frac{1}{\overline{m}} e^{-\frac{t}{\overline{m}}} \quad \text{при } t \geq 0. \qquad (5)$$

The average time $\overline{m}$ to the first stoppage, as is known, appears to be a mathematical expectation (average) random value xi.

$$M\xi = \int_0^\infty \frac{t}{\overline{m}} e^{-\frac{t}{\overline{m}}} dt = \overline{m}. \qquad (6)$$

Watching the performance of the apparatus, individual magnitudes of random values xi are measured: $t_1, t_2, \ldots, t_n$, i.e. its selective values. For optimum statistical evaluation of value $\overline{m}$, which appears to be an unknown distribution, it is necessary to employ the method of maximum probability [3]. This method is effective only at greater n [4]. The probability function here has the form of

$$L(t_1, t_2, \ldots, t_n; \overline{m}) = \frac{1}{\overline{m}^n} \exp\left(-\frac{\sum_{i=1}^n t_i}{\overline{m}}\right). \qquad (7)$$

Compiling by ordinary laws the probability equation, we find the selective value $\overline{m}^*$

$$\overline{m}^* = \frac{1}{n} \sum_{i=1}^n t_i. \qquad (8)$$

Next is necessary to formulate a confidential interval for the random value $\overline{m}^*$, considering, that all t_i appear to be independent uniformly distributed random values according to (5). But $Mt_i = \overline{m}$ and $Dt_i = D\,xi = \overline{m}^2$. Then

$$M\overline{m}^* = \overline{m}, \quad D\overline{m}^* = \frac{\overline{m}^2}{n}. \qquad (9)$$

For simplicity of calculation we will consider, that we have a discrete selection of greater volume n. In view of this the average value of selection of $\overline{m}^*$ is asymptotically normal with the mathematical expectation $\overline{m}$ and dispersion $D\overline{m}^* = \frac{D\,xi}{n}$ [3, pp. 379-382]. Because of greater n we have approximately

$$M\overline{m}^* = \overline{m} \approx \overline{m}^*; \quad D\overline{m}^* = \frac{D\xi}{n} = \frac{\overline{m}^2}{n} \approx \frac{\overline{m}^{*2}}{n}. \qquad (10)$$

Next, in view of the above mentioned normalcy of the random value $\overline{m}^*$ we can formulate a probability interval for the average $M\overline{m}^* = \overline{m}$. After making easy transforms under the sign of probability P analogous to the report [5], we will obtain

$$P = P\left(\overline{m}^* - u\sqrt{D\overline{m}^*} < \overline{m} < \overline{m}^* + u\sqrt{D\overline{m}^*}\right) = \frac{1}{\sqrt{2\pi}} \int_{-u}^{u} e^{-\frac{z^2}{2}} dz. \qquad (11)$$

In the right side of this equation figures a tabulated function [6]. It can easily be seen, that regardless of how close the given reliability P would be to unity (on account of selecting greater |u|) at sufficiently great n, it is always possible to realize a conveniently greater accuracy in evaluating $\overline{m}$.

Now we will analyse the function of probability distribution showing that the number of failures θ for the fixed time t is smaller than the fixed value c - 1. Here we will use the results obtained by G. Kramer [3] pp. 387-402] in mathematical statistics, developed by V.I. Siforov [5] when investigating one problem of the theory of transmission of announcements. Since the observed values give us only and empirically average $\overline{m}^*$, then we can determine only the empirical probability $P^*\{\theta \leqslant c-1\}$ according to formula

$$P^*\{\theta \leqslant c-1\} = 1 - P^*\{\theta \geqslant c\} = \sum_{k=0}^{c-1} \frac{1}{k!}\left(\frac{t}{\overline{m}^*}\right)^k e^{-\frac{t}{\overline{m}^*}} \qquad (12)$$

In this way, $P^*\{xi \leqslant c-1\}$ - random value, which appears to be a function of the selected average $\overline{m}^*$: $P^*\{\theta \leqslant c-1\} = H(\overline{m}^*)$. We will its distribution. On the basis of G. Kramer [3, pp. 387-402] results at very general assumptions about the functions $H(\overline{m}^*)$, taking place in the given problem, we conclude, that the random value $P^*\{\theta \leqslant c-1\}$ is an asymptotic normal. Its mathematical expectation and dispersia are situated in the following formulas:

$$MP^*\{\theta \leqslant c-1\} \approx P\{\theta \leqslant c-1\} = H(\overline{m}) \approx H(\overline{m}^*) = \widetilde{H}; \qquad (13)$$

$$DP^*\{\theta \leqslant c-1\} \approx \mu_2(\overline{m}^*)H_1^2, \qquad (14)$$

где = where

$$H_1 = \frac{dH(x)}{dx}\bigg|_{x=\overline{m}} \approx \frac{dH(x)}{dx}\bigg|_{x=\overline{m}^*} = \widetilde{H}_1. \qquad (15)$$

Here μ_2 - second central moment

$$\mu_2(\overline{m}^*) = D\overline{m}^*. \qquad (16)$$

We obtain

$$H_1 = e^{-\frac{t}{\overline{m}}} \sum_{k=0}^{c-1}\left[\frac{t^k}{(k-1)!\,\overline{m}^{k+1}}\left(\frac{t}{k\overline{m}} - 1\right)\right]. \qquad (17)$$

Finally we obtain the following formulas:

$$MP^*\{\xi\leqslant c-1\}\approx \exp\left(-\frac{t}{\overline{m}^*}\right)\cdot\sum_{k=1}^{c-1}\frac{1}{k!}\left(\frac{t}{\overline{m}^*}\right)^k. \tag{18}$$

$$DP^*\{\xi\leqslant c-1\}\approx D\overline{m}^*\cdot\exp\left(-\frac{2t}{\overline{m}^*}\right)\cdot\left\{\sum_{k=0}^{c-1}\left[\frac{t^k}{(k-1)!\,\overline{m}^{k+1}}\left(\frac{t}{k\,\overline{m}}-1\right)\right]\right\}^2. \tag{19}$$

Because of the normalcy of the random value $P^*\{xi \angle c-1\}$ the confidental interval for its average $MP^*\{xi \angle c-1\} = P\{xi \angle c-1\}$ is formulated ordinarily:

$$P=P\left\{\widetilde{H}-u\sqrt{D\overline{m}^*}\,\widetilde{H}_1<P\{\xi\leqslant c-1\}<\widetilde{H}+u\sqrt{D\overline{m}^*}\,\widetilde{H}_1\right\}=$$

$$=\frac{1}{\sqrt{2\pi}}\int_{-u}^{u}\exp\left(-\frac{z^2}{2}\right)dz. \tag{20}$$

In this plan it is necessary to use experimental data when calculating the reliability of an apparatus.

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