63-3-2

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INFRARED TRANSMISSION AND BACKGROUND RADIATION BY CLEAR ATMOSPHERES

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NOTICE : Especially notice corrections # 42, 43, and 44!!!

ERRATA # 1 for Document No. 61SD199
" Infrared Transmission and Background Radiation
by Clear Atmospheres "

by Thomas L. Altshuler

February 6, 1962

1. Sheet FN-610-1 which appears as the first page. In the Abstract in lines 3 and 4, change " path " to read " line of sight "

2. Before page (i) insert the following:

" ACKNOWLEDGEMENT

The author is especially indebted to G.J. Rayl for his support, suggestions, and encouragement during the preparation of the manuscript, and for his effort in publishing and proof reading this document. The author also wishes to thank A.H. Canada and Dr. J.I.F. King for their interest and support in producing this report. "

Remove the ACKNOWLEDGEMENT that appears after page vii.

3. Remove the transparent TEMPLATE A from the back sheet envelope and insert the semi-transparent TEMPLATE A provided with this ERRATA.

4. Page (i): Change " paths " to read " lines of sight " in the following lines:

SECTION II:
SECTION III: Part 1:
Part 2:
SECTION VI: APPENDIX A:

Change " Path " to read " Line of Sight " in SECTION VI: APPENDIX C:

5. Page (iii): Change lines 3-5 below TABLES AND FIGURES to read as follows:

TABLE 1	Concentration of Infrared Absorbing Gases	V-1
TABLE 2	Conversion Factors	V-1
TABLE 3	Deviation of Model from Actual Transmissions	VI-24
TABLE 4	Sources of Transmission Data	VI-25

Also change " Paths " to read "Lines of Sight" on the lines after:
FIGURE 1 , FIGURES 5-9, FIGURES 10-14, FIGURES 49

6. Page (v): Insert between symbols A_{N_2O} and C_{gas} the following line:

" $B_{r\lambda}$ = spectral radiance of target, watts/cm^2 - ster.- micron "

Cross out the 2nd from the bottom line " N_λ = spectral ---etc. "

7. Page (vi): 13th line from the bottom of the page, change " path " to read " line-of-sight ".

8. Page (vii): In the top line below the " DEFINITION OF SYMBOLS " change the symbol " x " to read " X ".

9. Page (vii): Change "path" to read " line-of-sight " in the following lines; 5th line from the top of the page below DEFINITION OF SYMBOLS, 12th line from the bottom of the page, and 3rd line from the bottom of the page.

Insert the following line between the symbols $z_{95\%}$ and θ:

" ϵ_λ = spectral emissive power at wavelength (λ). "

10. Page I-2: Insert an asterisk in the 5th line from the top of the page after the number (1) so that the last part of the line reads, " Taylor and Yates$^{(1)*}$ "

and underline
Change/the first sentance of the third paragraph to read: " It is suggested that those unfamiliar with the subject matter of this report study the material in Section II, Section III Parts 1-4, and Section IV Part 1. " Please note that commas after Section III and Section IV have been deleted in the above sentande as compared to the sentance printed on page I-2.

11. Page II-1: Starting the third paragraph from the top of the page, change " Type 1 " to read " Type I ". Starting the fourth paragraph, change " Type 2 " to read " Type II ". Starting the fifth paragraph change " Type 3 " to read " Type III ".

12. Page II-3: 3rd line from the top of the page change " The infrared path is that of type 2, see Figure 1." to read " The infrared line-of-sight is that of type II, see Figure 1. 6th line from the top of the page change " h = 41.0 km. " to read " h_t = 41.0 km."

17th line from the top of the page change " The infrared path is that of type 1, see Figure 1." to read " The infrared line-of-sight is that of type I, see Figure I.

Bottom line change " The infrared path is that of type 3, see Figure 1." to read " The infrared line-of-sight is that of type III, see Figure 1."

13. Page III-1: In the middle of the page after the symbol "d", 3rd paragraph from the top, change the first xxxxxx line of the paragraph to read as follows:
" d = equivalent path length of dry air in the infrared line-of-sight, corrected to "

14. Page III-2: 3rd line from the top of the page change to read:

Part 1: R < 150 kilometers, horizontal lines of sight, Use Figures 5 to

Reduce the size of the letters " W " and " Z " in the equations (1) & (
to read:

$$w = w_h \times R \qquad \text{eq. (1)}$$

$$z = z_h \times R \qquad \text{eq. (3)}$$

The line below Given in the EXAMPLE, change to read:

" $h_t = h_o$ = 10 km.

15. Page III-2: In the following lines change the capital letters of W and Z to small letters as:
10th line from page bottom, " From Figure 5, w_h = 1.1 x 10^{-3} pr. cm. "
8th line from page bottom, " From Figure 9, z_h = 1.75 x 10^{-3} atmo-cm."
6th line from page bottom, " w = (1.1 x 10^{-3}) x (15) = 1.65 x 10^{-2}pr.cm.
Change the 4th line from the bottom of the page to read,
" z = (1.75 x 10^{-3}) x (15) = 2.62 x 10^{-2} atmo-cm ".
Change the word " paths" in the 3rd line from the bottom of the page to read " lines-of-sight ".

16. Page III-3: 5th line from the top change capital W and W_v to small w and w_v respectively. 6th line from the top change W to w. 7th line from the top change "Z" and "W" to "z" and "w". In the EXAMPLE below <u>Required</u> change " W " and "Z" to "w" and "z". Under <u>Solution</u> change " W_v " to "w_v", change "Z_v" to "z_v". In the " Ans " change "W" to "w", "Z" to "z" and change the formula after "d" to read

$$" d = \frac{1.35 - 0.33}{10 - 5} 20 = \underline{4.16} \text{ km.} "$$

17. Page III-4: Change " path " to " line-of-sight " 13th line from bottom.

18. Page III-5: 2nd line from top of page change "W" to "w".
2nd line under <u>Solution</u> change "type 2" to read " type II ".
After "<u>Ans</u>" change the 3rd line from the bottom of the page to read,
" w = 2 x 1.30 x 10^{-1} - 7.7 x 10^{-3} - 1.23 x 10^{-3} = $\underline{\underline{2.51 \times 10^{-1}}}$ pr. cm. "

19. Page III-6: Change "x" to read "X" in the 7th and 5th lines from th bottom of the page.

20. Page III-9: 5th line from the bottom of the page change " $x_{corrected}$" to read " $z_{corrected}$ ".

21. Page III-10: Underline " Given " in the first line under <u>EXAMPLE</u> . In the 5th ~~and 4th~~ line from the bottom of the page change "x" to "X". In the 4th line from the bottom of the page change " x = 1.6 x 10^{-5}" to " X = 1.6 x 10^{-5} "

? 22. Page III-11: After " Ans " in the middle of the page, change " d = etc." to read " d = 0.0208 x 18 = 3.75 x 10-1 "

23. Page III-13: 10th line above the bottom of the page change " Type 2 " to read " Type II ".

24. Page III-14: 3rd line from the top of the page change " From equation (6) " to read " From equation (6), page III-5 "

Change the line under <u>Step 3</u> in the middle of the page from " To correct for weather changes: " to read " To correct for weather changes; see equation (7), page III-7; ".

3rd line from the bottom of the page change " From Equation 7: " to read " From Equation 7; page III-7: "

25. Page III-16: 3rd line from the top of the page change " Type 2 " to read " Type II "
13th line from the top of the page change " From equation (6) ", to read " From equation (6), page III-5.

ERRATA #1 continued, page 4

26. Page III-16: 7th line from the bottom of the page change " To correct for weather changes:" to read " To correct for weather changes; see equation (9), page III-7.

27. Page III-17: 7th line from the top of the page change " From equation 9: " to read " From equation 9; page III-7: "

28. Page IV-1: In the first paragraph second sentance change " Figures 25-46 " to read " Figures 24-47 "
In the second paragraph first sentance place a comma after " scattering"

Change the third paragraph " This section is divided etc. " to read,

" This section is divided into four parts. Part (1) gives methods for determining the transmission of infrared by the atmosphere. Part (2) gives methods for determining the apparant target radiance. Part (3) gives methods for determining the molecular absorption of infrared by small quantities of gases and vapors. Part (4) presents methods for obtaining the background radiance of the atmosphere."

29. Page IV-2: In the 2nd line from the top, insert the word " Thus, " before the word " Large " at the beginning of the second sentance, and change the word " Large " to " large ".
In the 6th line from the top of the page change " Table 3 " to " Table 4 ".

30. Page IV-3: Change " Step 3: etc." at the top of the page to read
" Step 3: The transmission is obtained by displacing the transmission curve by the same amount and direction that the entry value, obtained in Step 1, differs from the curve, obtained in Step 2. "

Below the EXAMPLE under " Given " change the sentance " From the EXAMPLE etc. " to read " From the EXAMPLE in Case a) Part 4, Section III, page III-4: "

31. Page IV-4: In the 8th line from the top of the page change the formula " $Q_\lambda = \Delta_\lambda N = 0.023_\lambda N_\lambda$ " to read

$$" \quad Q = (\Delta\lambda)\, B_{r\lambda} = 0.023\,\lambda B_{r\lambda} \qquad \text{eq (19)} \quad "$$

At the end of the 4th line from the top of the page after the word " radiance " insert " , Q'_λ , "

Change the formula in the 6th line from the top of the page " $Q'_\lambda = \tau_A \cdot Q_\lambda$" to read " $Q'_\lambda = [\tau_A(\lambda)]\, Q_\lambda$ "

The 11th line from the top of the page after " Let: " change " N_λ " to read " $B_{r\lambda}$ "

8th line from the bottom of the page change " $\log_{10}$." to read " $\log_{10}$(wavelenth - microns). "

32. Page IV-5: Change the 10th line from the bottom of the page from " Appendix F and etc. " to read " Appendix F and are summarized in Table 3., These assumed models fail to apply "

33. Page IV-5: Delete the last three lines of the page, all after " gas in question is below that etc. ". Insert the following statement in place of those last three lines:

" gas in question is below that value given in the curve then the following calculation procedure should be used :

Let: A_{H_2O} = absorption by water vapor
A_{CO_2} = absorption by carbon dioxide
A_{O_3} = absorption by ozone
A_{N_2O} = absorption by nitrous oxide "

34. Page IV-6: Delete everything above " Step 1 ", i.e. the first ten lines. In its place insert the following:

" Let: $d_{95\%}$ = quantity of carbon dioxide or nitrous oxide, obtained from Figures 32-38 and 42-44, needed to make the transmission = 95%.

$w_{95\%}$ = quantity of water vapor, obtained from Figures 24-31, needed to make the transmission = 95%.

$z_{95\%}$ = quantity of ozone, obtained from Figures 39-41, needed to make the transmission = 95%.

u_{H_2O} = actual quantity of water vapor, pr. cm.
u_{CO_2} = actual quantity of carbon dioxide, atmo-cm.
u_{O_3} = actual quantity of ozone, atmo-cm.
u_{N_2O} = actual quantity of nitrous oxide, atmo-cm. "

35. Page IV-6: In the paragraph below " <u>Step 1</u> " delete the 3rd line " transmission line etc. " and replace with the following line, " transmission line at a given wavelength. By using the "Index", the " The quotations around the word " Index " should appear in the text.

In the last line of <u>Step 1</u> after the words " under consideration " add the following words " can be determined. "

Below equation (24) and above " <u>EXAMPLE</u> " add the following statement: " Appendix F part (6) pres~~ents the~~ theory upon which steps 1 and 2 are based. "

36. Page IV-8: Change the sixth symbol, which is defined from the top of the page, from " $T_{A,i}(\lambda)$ " to " $\tau_{A,i}(\lambda)$ "

In the 2nd line below " <u>Step 1</u> " after the words " be isothermal " insert the words " and contains the infrared ray "

Change the word " path " to " line-of-sight " in the following places,

36. continued.

in the paragraph below " Step 2 " in the 4th and 5th lines, in the first line below " Step 3 ", and in the first line below below " Step 4 "

Change the 3rd and 4th lines below " Step 5 " from " Section IV. The (i th) layer etc. " to " Section IV. The (i th) layer is that part of the infrared ~~line-of-sight~~ ray between the (i th) point and the (i th - 1) point.

37. Page IV-9: In the first line below " Step 2 " near the bottom of the page change the word " path " to " line-of-sight ".

In the top left corner of the table below " Step 2 " change the word " Path " to " Line-of-sight ".

38. Page IV-10: In the top left corners of the tables below " Step 3 " and " Step 4 " change the word " Path " to " Line of sight ".

In the table below " Step 4 " in column number "4" and line "$\tau_{O_3}(\lambda)$" change the number " .955 " to " .953 "

39. Page IV-11: In the table in column number "6" and the last line of the column " $Q_i\left({}_{A,i-1} - {}_{a,i} \right)$ " change the number

" 6.85×10^{-7} " to " 6.85×10^{-9} "

40. Page V-1: Below the last line of the page, insert the following:

"

TABLE 2

CONVERSION FACTORS

DISTANCE: 1 kilometer = 3281 feet
= 0.6214 statute miles
= 0.5400 nautical miles

PRESSURE: 1 atmosphere = 1013.3 millibars
= 760 mm Hg at 0° C.
= 14.70 pounds per sq. inch

To find pressure vs. altitude, see Figure 19.

TEMPERATURE: $^{\circ}K = 273.16 + {}^{\circ}C$

$^{\circ}C = \frac{5}{9} ({}^{\circ}F - 32)$ "

41. Page V-2: In the title change the word " PATHS " to read " LINES-OF-SIGHT "

In the first figure " Type I " change the word " PATH " to " RAY "

42. Page V-10: Change the ordinate of Figure 9 to read " 10^{-1} " at the top of the figure and " 10^{-6} " at the bottom of the figure. Also change all the intermediate values ~~accordingly~~ so that they will be consistent.

Also change the title of the ordinant from " Equivalent centimeters, etc. " to " EQUIVALENT ATMO-CM. OF OZONE CORRECTED TO SEA LEVEL "

43. Page V-15: Change the ordinate of Figure 14 to read " 10 " at the top of the figure and " 10^{-4} " at the bottom of the figure. Also change all the intermediate values so that they will be consistent.

Also change the title of the ordinant from " $z_{h-\infty}$, EQUIVALENT, etc." to " $z_{h-\infty}$, EQUIVALENT ATMO-CM. OF OZONE CORRECTED TO SEA LEVEL "

44. Page V-19: In the ordinate of Figure 18 should have the following title " ALTITUDE, KILOMETERS "

45. Page V-50: In Figure 49 in the title change the word " PATHS " to " LINES-OF-SIGHT "

46. Page II-1: At the end of the second paragraph change the words " line of sights " to " lines-of-sight "

Corrections to the Appendix will appear in a later ERRATA.

ERRATA No. 2 for Document No. 61SD199
" Infrared Transmission and Background Radiation
by Clear Atmospheres "

by Thomas L. Altshuler October 13, 1962

1. Page VI-1: The first sentance at the top of the page change " infrared path " to " infrared line of sight "

2. Page VI-1: Change the word " path " to " ray " in the following sentances: R = etc., X = etc., y = etc.

3. Page VI-3: 4th line from the top of the page change " When Θ etc. " to read: " When $\Theta < 8^{\circ}$, and assuming $n \approx 1.0$ this reduces to

4. Page VI-4: First and 2nd lines from the top of the page change " path " to read " line of sight ".

5. Page VI-4: 6th and 7th lines from the top of the page change " $\frac{D}{2}$ " to read " $\frac{D}{2}$ ".

6. Page VI-4: 16th line from the top of the page change " ϕ_{50} = angle between OCY, see Figure 51 " to read " ϕ = angle between ^{50}OCY, see Figure 51. "

7. Page VI-6: Change the formula on the 2nd line from the top of the page from " $\frac{\cos \Theta_{50}}{\sin (\phi - \phi_{50})} = \frac{\frac{D}{2} + h}{r}$ " to read " $\frac{\cos \Theta}{\sin (\phi - \phi_{50})} = \frac{\frac{D}{2} + h}{r}$ "

8. Page VI-9: 2nd and 7th lines from the bottom of the page change " Shaw, reference 15 " to read " Shaw, reference 16 ".

9. Page VI-11: 3rd line from the top of the page change " For intermediate regions, etc. " to read " In order to simplify calculations for intermediate regions, it will be assumed that the transmission can be approximated by a function such as : "

10. Page VI-11: Between 6th line from the top of the page " where n varies from 0 to 1 " and the 7th line from the top of the page " It can be seen, etc. " insert the following sentance : " The maximum errors in making this assumption are given on Table 3, page VI-24 ".

11. Page VI-11: 13th line from the top of the page change " the half widths (α) for carbon dioxide and water vapor are: " to read " the half widths (α) for carbon dioxide and water vapor at STP are approximately:

12. Page VI-11: 21st line from the top of the page change the formula " $\frac{Su}{2\pi d}$ " to read " $\frac{Su}{2\pi\alpha}$ ".

13. Page VI-11: change equation (48) to read: " $L = \frac{\pi S \alpha_o^n}{d^{n+1}}$ "

14. Page VI-12: Change equation (51) to read : "

$$q = \int_0^R u_h \left(\frac{P}{P_o}\sqrt{\frac{T_o}{T}} \right) dR$$

15. Page VI-12: 9th line from the top of the page change " $\frac{1}{2}n = \frac{1}{2}$ " to read " $n = \frac{1}{2}$ ".

16. Page VI-15: The 2nd line of the first paragraph change " are made for, etc. " to read " are made for deviations of existing weather from the model at the altitude (h) of "

17. Page VI-15: In the definitions after " P_m " change to read:

" P_m = pressure (model), see Figure 19 "

18. Page VI-15: In the definitions after " q_h " change to read:

" q_h = sea level equivalent quantity of gas per kilometer horizontal path length at altitude h . "

19. Page VI-15: In the definitions after " T_m " change to read as follows:

" T_m = temperature (model) °K, see Figure 20. "

20. Page VI-15: In the definitions change " u_h etc. " to read:

" u_h = actual quantity of a gas or vapor in a kilometer horizontal path length at altitude h . "

21. Page VI-16: Change equation (54) to read as follows:

$$q \text{ corrected} = q_{model} \frac{f}{f_{model}} \left(\frac{P}{P_m}\right)^{n+1} \left(\frac{T_m}{T}\right)^{\frac{n}{2}+1}$$

22. Page VI-16: Change equation 53 to read as follows:

$$q_h = y\, f \left(\frac{P}{P_o}\right)^{n+1} \left(\frac{T_m}{T}\right)^{\frac{n}{2}+1}$$

23. Page VI-16: In the following lines from the bottom of the page "x" should be changed to " X " : Lines 1,3,4, 6, 11, 14.

24. Page VI-17: In equations (56) and (57) change "x" to " X ".

25. Page VI-17: Change equation (57) to read as follows:

$$C_{H_2O} = \left(\frac{X}{X_{model}}\right)\left(\frac{T_m}{T}\right)^{3/2}$$

26. Page VI-17: The first sentance after "Part 2) To Correct (d)", correct " kilimeters " to " kilometers ".

27. Page VI-21: Correct equation (68) to read:

$$k_{O_3} = e^{0.0460(h' - h)} = e^{0.0460\left[23.0 - h_{max}(O_3)\right]}$$

28. Page VI-22: Correct equation (48) to read:

$$L_o = \frac{\pi S \alpha_o^n}{\delta^{n+1}} .$$

29. Page VI-22: Correct equation (50) to read:

$$\tau(\lambda) = f\left[L_o q\right].$$

30. Page VI-23: Correct equation (69) to read:

$$L = \frac{\pi S \alpha^n}{\delta^{n+1}}$$

31. Page VI-23: Correct equation (48) to read:

$$L_o = \frac{\pi S \alpha_o^n}{\delta^{n+1}}$$

32. Page VI-23: 11th line from the bottom of the page which reads " gas, and that (λ_o) and (δ) remain constant etc. " change to read " gas, and that (α_o^o) and (δ) remain constant over the bands of absorption of a "

33. Page VI-23: 6th line from the bottom of the page which reads " presented in Table 2. etc. " change to read " presented in Table 3. etc. "

34. Page VI-23: 3rd line from the bottom of the page which reads " transmission as a function of (β) and (τ) as well as variations between the " change to read " transmission as a function of (β) and ($\beta^2 x$) as well as variations between the ".

35. Page VI-23: At the bottom of the page change equation (73) to read equation (73A). Then insert the following equation beneath equation (73A): "

$$x = \frac{Su}{2\pi\alpha} \qquad \text{eq (73B)}$$

36. Page VI-24: Change Table 2 to read " TABLE 3 ".

37. Page VI-25: The 4th and 5th lines from the top of the page which reads (the sources described in Table 3. The transmission functions, described as " type of model " in Table 2, will be covered etc.) change to read (the sources described in Table 4. The transmission functions, described ~~as~~ " type of model " in Table 3, etc. ".

38. Page VI-25: Change " TABLE 3 " to read " TABLE 4 "

39. Page VI-26: Change " TABLE 3 " to read " TABLE 4 "

40. Page VI-27: Change " TABLE 3 " to read " TABLE 4 ".

41. Page VI-27: Change equation (49) to read as follows:

$$q = u\left[\frac{P_e}{P_o}\sqrt{\frac{T_o}{T}}\right]^n$$

42. Page VI-11: Change equation (47) to read as follows:

$$\tau(\lambda) = f\left[L_o u\left(\frac{P}{P_o}\sqrt{\frac{T_o}{T}}\right)^n\right]$$

43. Page VI-12: Change equation (49) to read as follows:

$$q = u\left(\frac{P}{P_o}\sqrt{\frac{T_o}{T}}\right)^n$$

44. Page VI-29: Change the first line at the top of the page to read:

" The data at the 4.3µ band was fitted to an Elsasser model (equation 75) with n = 1.0, see ".

45. Page VI-29: Change equation (74) to read as follows:

$$\tau(\lambda) = e^{-\left(\frac{2}{\sqrt{\pi}}\right)\left(L_o q\right)^{1/2}}$$

46. Page VI-29: Change the 6th and 5th lines from the bottom of the page to read as follows:

" 6.3µ rotation band of water vapor. Burch, Howard and Williams,(35) applied the Goody model to their water vapor data at the 2.7µ and 3.2µ bands. Table 3 "

47. Page VI-33: After the legend " Elsasser model " add the words, " eq. (75) where $\beta = 0$. " After the legend " Goody model " add the words, " eq. (74) where $\beta = 0$. "

48. Page VI-33: In the label of the abscissa delete the letters " Lu ".

49. Page VI-34: In the second equation from the top of the page, i.e.

" $\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}}\int_0^x e^{-t^2}\,dt$ ", lable the equation as eq (76)

50. Page VI-34: In the 14th and 20th lines from the bottom of the page, change " Table 2 " to read " Table 3 ".

51. Page VI-35: In the 7th line from the top of the page, change " Where: $A(\lambda)$ = absorption " to read:

" where: $A(\lambda)$ = absorption of gas at wavelength (λ).

52. Page VI-35: Delete the last paragraph of the page and add the following:

" Figure 58 shows the absorption of infrared by various gases at sea level as a function of βx^2. The actual curve shown for H_2O represents one of the family of curves in which the Goody model applies as presented by Plass$^{(20)}$ where $\beta = 0.35$ as given in Table 3. Similarly, the actual curves for O_3 and N_2O come from a family of Elsasser model curves in which $\beta = 6.9$ and 1.1 respectively. The curves shown as the Elsasser model and Goody model where $\beta = 0$ represent the absorption curves used in Figures 24 - 31 and 39 - 44. If one were to have a small quantity of H_2O vapor, or O_3, or N_2O, the expected absorption curves at sea level would more likely follow the actual curves shown in Figure 58 rather than those in which $\beta = 0$. Thus if $\beta^2 x = 0.0035$, the absorption of infrared by water vapor would be 0.012 rather than 0.05 as predicted by the Goody model with $\beta = 0$. By going into the water vapor curves, Figures 24 - 31, and looking for $w_{95\%}$, the actual absorption for that amount of water vapor would be 0.012.

To find the absorption of small quantities of vapor or gas,

Let: $u_{95\%}$ = amount of gas or vapor necessary to obtain 95% transmission from the curves in Figures 24-44 in the same units as u_{H_2O}, u_{CO_2}, u_{O_3}, u_{N_2O}

For H_2O vapor

$A_{H_2O} = (\frac{S}{\delta})\, u_{H_2O}$ from equation (43)

For $u_{95\%}$; $A(\lambda) = 0.012$ from Figure 58

And $w_{95\%} = u_{95\%}$

$$\text{Thus, } A_{H_2O} = 0.012 \frac{u_{H_2O}}{u_{95\%}} = 0.012 \frac{u_{H_2O}}{w_{95\%}} \qquad \text{eq (21)}$$

For O_3 Equation (23) was found in the same way as equation (21)

For N_2O

$A(\lambda) = 0.006$ for $u_{95\%}$ from Figure 58

And $u_{95\%} = 0.027\, d_{95\%}$ from Table 1

$$\text{Thus, } A_{N_2O} = 0.006\,(u_{N_2O})/u_{95\%} = 0.22(u_{N_2O})/d_{95\%} \qquad \text{eq (24)}$$

For CO_2

For carbon dioxide, the curves used in Figures 32 through 38 follow the actual absorption behaviour of carbon dioxide under sea level conditions. Thus,

$$A(\lambda) = 0.05 \text{ for } u_{95\%}$$

$$\text{And} \quad u_{95\%} = 32\, d_{95\%}$$

$$\text{Thus} \quad A_{CO_2} = 0.05 \frac{u_{CO_2}}{u_{95\%}} = 0.0016 \frac{u_{CO_2}}{d_{95\%}} \qquad \text{eq (22)} \quad "$$

53. Page VI-37: Change equation (79) to read as follows:

$$Q_{\lambda_{max}} = 0.0305 \left(\frac{T}{1000}\right)^4. \qquad \text{eq (79)}$$

54. Page VI-38: change the definition of J_λ to read as follows:

" J_λ = spectral irradiance at the observer at (λ), watts/$_{cm^2}$ - micron "

55. Page VII-2: Change reference 37 to read as follows:

" 37. " Optical Engineering Handbook " Section 11, p 44-45,
Dr. J.A. Mauro, Editor, published by General Electric Co.,
Schenectady (1957)

FOR USE OF G-E EMPLOYEES ONLY

GENERAL ELECTRIC

TECHNICAL INFORMATION SERIES

Title Page

AUTHOR	SUBJECT CLASSIFICATION	NO.
T. L. Altshuler	RADIATION, Infrared	61SD199
		DATE Dec. 1, 1961

TITLE

INFRARED TRANSMISSION AND BACKGROUND RADIATION BY CLEAR ATMOSPHERES

ABSTRACT

Procedures are presented for determination of the atmospheric transmission of infrared and sky background for clear weather. Methods for determining the geometry of an infrared path with respect to the Earth, for obtaining the amount of infrared absorbing gases within the path, and for calculating the transmission and background radiation of those gases are covered in detail. This report is intended to supersede TIS R57ELC15, and replaces TIS R60ELC18 which was not published.

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CONCLUSIONS

Atmospheric transmission and background radiation data is presented in handbook form to facilitate the use of this data by those persons interested in "slide-rule" accuracy in the calculation of these quantities. The 1959 ARDC Model Atmosphere is used. Absorption tube measurements form the basis of the atmospheric transmission data wherever possible.

By cutting out this rectangle and folding on the center line, the above information can be fitted into a standard card file.

For list of contents—drawings, photos, etc., and for distribution see next page (FN-610-2).

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DEFINITION OF SYMBOLS

A_{H_2O} = absorption by water vapor.

A_{CO_2} = absorption by carbon dioxide.

A_{O_3} = absorption by ozone.

A_{N_2O} = absorption by nitrous oxide.

C_{gas} = temperature correction for CO_2, N_2O, CH_4, CO, Figure 16.

C_{H_2O} = temperature correction for water vapor, Figure 15.

d = equivalent path length of dry air in the infrared line-of-sight, corrected to sea level conditions, km. This is used to specify the amounts of CO_2, N_2O, CH_4, CO, and haze which have approximately constant mixing ratios with air from 0 km. to 50 km. altitude.

d_h = equivalent path length of dry air in 1 km. actual horizontal path length at an altitude (h), corrected to sea level conditions, km.

d_v = equivalent path length of dry air in a vertical path from altitude h to ∞, corrected to sea level conditions, km.

$d_{h-\infty}$ = equivalent path length of dry air in a slant path from altitude h to ∞, corrected to sea level conditions, km.

$d_{95\%}$ = quantity of carbon dioxide or nitrous oxide, obtained from Figures 32-44, needed to make the transmission = 95%.

f = fractional concentration of gas.

H = irradiance at the observer, watts/cm^2

H_λ = spectral irradiance at the observer over the wavelength interval ($\Delta\lambda$) at wavelength (λ), watts/cm^2 - $\Delta\lambda$

h_o = observer altitude, km.

h_t = target altitude, km.

h_{min} = minimum altitude of infrared ray, km.

h_{low} = lowest altitude of infrared line-of-sight between observer and target, km.

h_{trop} = altitude of tropopause.

$h_{max(O_3)}$ = altitude of maximum ozone concentration.

h' = fictitious altitude, km.

k_{H_2O} = correction factor for water vapor when h' is used, Figure 22.

N_λ = spectral radiance of target, watts/cm^2 - ster.- micron

k_{O_3} = correction factor for ozone when h' is used, Figure 23.

DEFINITION OF SYMBOLS (Con't)

P = pressure (of actual atmosphere when used in Section III)

P_o = sea level pressure = 760 mm Hg.

ΔP = pressure difference between actual atmosphere and ARDC model atmosphere, i.e. $(P_{actual})-(P_{model})$

Q_λ = spectral radiance of target over the wavelength increment $(\Delta\lambda)$, watts/cm^2 - ster. -$\Delta\lambda$

$Q_{\lambda\ max}$ = maximum spectral radiance of target over the wavelength increment $(\Delta\lambda)$, watts/cm^2 - ster. - $\Delta\lambda$

$Q_{\lambda,i}$ = spectral radiance of a black body radiator at the temperature of the gas in the (i th) layer and over the wavelength increment $(\Delta\lambda)$ at wavelength (λ), watts/cm^2 - ster. - $\Delta\lambda$

R = line-of-sight range, km.

R. H. = relative humidity

T = temperature degrees Kelvin (of actual atmosphere when used in Section III)

T_o = sea level temperature = 288°K

ΔT = temperature difference between actual atmosphere and ARDC model atmosphere, i.e. $(T_{actual})-(T_{model})$

u = actual quantity of a gas or vapor, atmo-cm or pr. cm.

u_{H_2O} = actual quantity of water vapor, pr. cm.

u_{CO_2} = actual quantity of carbon dioxide, atmo-cm.

u_{O_3} = actual quantity of ozone, atmo-cm.

u_{N_2O} = actual quantity of nitrous oxide, atmo-cm.

w = equivalent precipitable centimeters (pr.cm.) of water vapor in the infrared path, corrected to sea level conditions, pr. cm.

w_h = equivalent precipitable centimeters of water vapor in 1 km. actual horizontal path length at altitude (h), corrected to sea level conditions, pr. cm.

w_v = equivalent precipitable centimeters of water vapor in a vertical path from altitude h to ∞, corrected to sea level conditions, pr. cm.

$w_{h-\infty}$ = equivalent precipitable centimeters of water vapor in a slant path from altitude h to ∞, corrected to sea level conditions, pr. cm.

w' = fictitious value of w_h, w_v, $w_{h-\infty}$, and w

$w_{95\%}$ = quantity of water vapor, obtained from Figures 24-31, needed to make the transmission = 95%.

DEFINITION OF SYMBOLS (Con't)

x = fractional concentration of water vapor at 100% relative humidity to dry air at 760 mm Hg pressure and at temperature (T). Use a psychrometric chart to determine the value of (x).

z = equivalent atmospheric centimeters (atmo-cm) of ozone in the infrared path, corrected to sea level conditions, atmo-cm.

z_h = equivalent atmospheric centimeters of ozone in 1 km. actual horizontal path length at altitude (h), corrected to sea level conditions, atmo-cm.

z_v = equivalent atmospheric centimeters of ozone in a vertical path from altitude h to ∞, corrected to sea level conditions, atmo-cm.

$z_{h-\infty}$ = equivalent atmospheric centimeters of ozone in a slant path from altitude h to ∞, corrected to sea level conditions, atmo-cm.

z' = ficititious value of z_h, z_v, $z_{h-\infty}$, and z

$z_{95\%}$ = quantity of ozone, obtained from Figures 39-41, needed to make the transmission = 95%.

θ = elevation angle, degrees, i.e. angle with respect to the local horizontal (up + and down -, when looking along the line-of-sight.

θ_o = elevation angle of the infrared path at the observer, degrees.

θ_t = elevation angle of the infrared path at the target, degrees

θ_{min} = elevation angle at h_{min}, degrees.

λ = wavelength in microns.

λ_{max} = wavelength at $Q_{\lambda_{max}}$, microns.

$\Delta\lambda$ = wavelength increment such that:

$$\log_{10}(\lambda + \Delta\lambda) - \log_{10}(\lambda) = 0.01$$

$\tau_A(\lambda)$ = atmospheric transmission through the infrared path of infrared radiation at wavelength (λ)

$\tau_{H_2O}(\lambda)$ = transmission of H_2O vapor at wavelength (λ)

$\tau_{CO_2}(\lambda)$ = transmission of CO_2 at wavelength (λ)

$\tau_{O_3}(\lambda)$ = transmission of O_3 at wavelength (λ)

$\tau_{N_2O}(\lambda)$ = transmission of N_2O at wavelength (λ)

$\tau_{scat}(\lambda)$ = transmission through atmospheric scattering particles (haze) at wavelength (λ)

$\tau_{A,i}(\lambda)$ =atmospheric transmission at wavelength (λ) from the (i th) point on the infrared path to the observer, see Figure 49. There are (n) such points.

ω = solid angle measured from the observer, steradians.

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SECTION I
INTRODUCTION

Life on Earth could not exist without the atmosphere. This life-giving envelope includes the gases that are breathed, is responsible for the weather that brings the rains, and is a warm blanket that protects living creatures from harmful solar radiation. The atmosphere can do these things because of its ability to absorb electromagnetic radiation ranging from X-rays to infrared (heat rays).

In recent years there has been considerable interest in predicting the amount of infrared attenuation by the atmosphere. Scientists and engineers responsible for designing infrared equipment have required the answers to such questions as: How much infrared radiation will be transmitted by the atmosphere which exists between a heat source and a detector, or, How much sky background radiation will a detector "see"? This report attempts to provide the answers to these questions.

In order to provide answers quickly and easily, this report has been arranged such that the methods of solution are covered in sections II through IV. Such solutions involve three steps.

Step 1, covered in Section II, outlines a procedure for determining the geometry, with respect to the Earth, of an infrared ray as it travels between a heat source and a detector. This part of the infrared ray will be called the infrared line-of-sight.

Step 2, covered in Section III, enables one to determine the amounts of various atmospheric gases, vapors, and haze present within the infrared line-of-sight.

Step 3, covered in Section IV, presents methods for obtaining the transmission and radiation of infrared in an atmospheric path.

Numerical examples will be used in Sections II through IV in order to clarify the methods of solution. The figures and tables needed in steps 1 through 3 are grouped in Section V.

Section VI, the appendix, covers the theory upon which the calculation procedure is based. However, it is not necessary to delve into this theory in order to correctly solve atmospheric transmission problems.

This report covers only methods for determining the atmospheric transmission of infrared radiation in clear weather. During conditions of fog or reduced visibility, infrared radiation is scattered in much the same manner as visible light. Thus, except for certain types of monodispersed fogs, one cannot "see" much better with infrared than with visible light. Taylor and Yates(1) have made measurements of infrared penetration in fog, rain and snow.

The situation with clouds, except for cirrus clouds, is much the same as with fog. Elliott and Altshuler(2) discuss methods for determining the atmospheric transmission through cloudy atmospheres.

It is suggested that those unfamiliar with the subject matter of this report study the material in Section II, Section III, Parts 1 - 4, and Section IV, Part 1. This material will enable one to solve most infrared transmission problems encountered in practice. The other parts of Sections III and IV cover methods for determining the atmospheric transmission under varying weather conditions, the infrared absorption of very small quantities of gases, and the background radiation of a clear atmosphere.

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[*] Numbers refer to references listed in back of report.

SECTION II

DETERMINATION OF INFRARED LINE OF SIGHT

This section will cover the methods for determining the geometry of the infrared line-of-sight with respect to the Earth. For calculation purposes an infrared ray is considered to be infinite in extent. The line-of-sight is that part of an infrared ray that exists between a detector and a heat source. The detector will be referred to as the "observer" while the heat source will be referred to as the "target".

All line-of-sight conditions can be grouped into three types according to their geometry with respect to the Earth. Figure 1 shows the geometry of various infrared line of sights.

Type 1 is that geometry in which the infrared ray does not intersect the Earth and the line-of-sight does not include the minimum altitude of the infrared ray.

Type 2 is that geometry in which the infrared ray does not intersect the Earth and the line-of-sight includes the minimum altitude of the infrared ray.

Type 3 is that geometry in which the infrared ray intersects the Earth.

The geometry of an infrared ray can be determined by the use of either Figure 2, 3, or 4 with the plastic Template A provided. These figures will provide the following quantities describing the infrared ray.

h_o = observer altitude, km.
h_t = target altitude, km.
h_{min} = minimum altitude of infrared ray, km.
h_{low} = lowest altitude of the line-of-sight, km.
R = range, length of the line-of-sight, km.
θ_o = elevation angle of the infrared ray at the observer, degrees, i.e. angle the infrared ray makes with the local horizontal at the observer, up + and down -.
θ_t = elevation angle of the infrared ray at the target, degrees.
θ_{min} = elevation angle at h_{min}, degrees.

If any four of the above independent quantities are known, the others can be found. θ_{min} and h_{min} are the only quantities not independent of each other, thus they cannot be used together as two independent quantities. If the range, R, and at least one altitude is known, then the number of independent qualtities may be reduced to three in order to determine the other quantities.

When the range, R, is one of three known quantities and the other two quantities describe conditions at both observer and target positions, use the plastic Template A with Figure 2, 3, or 4. The template, which consists of abscissa and ordinate lines, should be laid on Figure 2, 3, or 4 so that the abscissa of the template coincides with the abscissa on the figure. Mark the altitude of the observer and target positions on the template as horizontal lines (parallel to the abscissa). Mark off the line-of-sight range as vertical lines (parallel to the ordinate). The intersection of the appropriate horizontal and vertical lines represent observer and target positions. Now slide the template across the figure keeping both abscissas coincident, until the observer and target positions on the template are connected by a common curve (or are intermediate to two of the curves) on the figure. The curves of Figures 2, 3, and 4 are the parabolic type, and as such take into account the Earth's curvature and the refraction of infrared in the Earth's atmosphere. The curves were computed for infrared radiation at 2 microns wavelength in which the ray is the average of those at different useful wavelengths. The Appendix A gives the theory upon which Figures 2, 3, and 4 are based. If Figure 4 does not go out to a sufficiently high altitude, then use equations (37) to (41) in Appendix A.

EXAMPLES

1. Given

h_o = 24.0 km.
h_t = 39.5 km.
R = 910 km.

Required

Values for the other geometric variables.

Solution: Use the Template A with Figure 2.

$h_{min} = h_{low}$ = 15.0 km. ($h_o < h_{low} < h_t$)
θ_o = -3.0°

$\theta_t = -5.0^\circ$

$\theta_{min} = 0$

The infrared path is that of type 2, see Figure 1.

2. Given:

$h_o = 8.0$ km.

$h = 41.0$ km.

$R = 500$ km.

Required:

Values for the other geometric variables.

Solution

Use the template A with Figure 2.

$h_{min} = 5.0$ km.

$h_{low} = 8.0$ km.

$\theta_o = +1.6^\circ$

$\theta_t = -6.0^\circ$

$\theta_{min} = 0^\circ$

The infrared path is that of type 1, see Figure 1.

3. Given

$h_o = 17.5$ km.

$h_t = 55.0$ km.

$R = 340$ km.

Required

Obtain values for the other geometric variables.

Solution:

Use the template A with Figure 2.

$h_{min} = 0$ km.

$h_{low} = 17.5$ km.

$\theta_o = +5.0^\circ$

$\theta_t = -8.0^\circ$

$\theta_{min} = 3.0^\circ$

The infrared path is that of type 3, see Figure 1.

SECTION III

DETERMINATION OF AMOUNT OF GASES, VAPORS, AND HAZE IN THE LINE OF SIGHT

This section will cover methods for determining the amounts of infrared absorbing atmospheric constituents in the line of sight, namely, $C0_2$, 0_3, N_20, CH_4, C0, water vapor, and haze. A model atmosphere is assumed in parts 1 through 4 of this section. The pressure and temperature variation with altitude is based upon the 1959 ARDC model atmosphere, see Figures 19 and 20. The amounts of the various infrared absorbing gases and vapors in the atmosphere versus altitude are given in Table 1 and Figures 17 and 18. For most problems, the use of these models is adequate, and the calculation methods in parts 1 through 4 suffice. If, however, weather conditions are known at the observer position, and/or the average latitude of the line of sight is known, then the model atmosphere can be corrected to agree with the weather. Methods for making such corrections are covered in parts 5 and 6 of this section.

Because of the distribution of various gases, water vapor, and haze as a function of altitude, it is convenient to calculate three basic quantities which represent the amounts of such atmospheric constituents. These are:

d = equivalent path length of dry air in the infrared path, corrected to sea level conditions, km. This is used to specify the amounts of $C0_2$, N_20, CH_4, C0, and haze which have approximately constant mixing ratious with air from 0 km. to 50 km. altitude.

w = equivalent precipitable centimeters (pr. cm.) of water vapor in the line of sight, corrected to sea level conditions, pr. cm.

z = equivalent atmospheric centimeters of ozone in the line of sight, corrected to sea level conditions, atmo-cm.

Gases and water vapor absorb infrared in regions of their vibration-rotation bands. Temperature and pressure will affect the width of the spectral lines which will then alter the degree of absorption of infrared by the gas. Consequently, with respect to its infrared absorption, the amount of a gas in the line of sight can be equated to another amount under sea level conditions, 288°K temperature and 760 mm Hg pressure. This sea level amount is termed the "equivalent amount of gas in the line of sight, corrected to sea level conditions," and may be represented by the symbols (d), (w), and (z). The actual amount of the gas in the line of sight is represented by the symbol (u). The Appendices

B to E, give the theory upon which the calculation procedures of this section are based.

Part 1: R 150 kilometers, horizontal paths, Use Figures 5 to 9

Let: d_h = equivalent path length of dry air in 1 km. actual horizontal path length at an altitude (h), corrected to sea level conditions, km.

w_h = equivalent precipitable centimeters of water vapor in 1 km. actual horizontal path length at altitude (h), corrected to sea level conditions, pr. cm.

z_h = equivalent atmospheric centimeters of ozone in 1 km. actual horizontal path length at altitude (h), corrected to sea level conditions, atmo-cm.

$$W = W_h \times R \qquad \text{eq. (1)}$$

$$d = d_h \times R \qquad \text{eq. (2)}$$

$$Z = Z_h \times R \qquad \text{eq. (3)}$$

EXAMPLE

Given

h_o = 10 km.

R = 15 km.

Required

Find w, d, and z

Solution

From Figure 5, $W_h = 1.1 \times 10^{-3}$ pr. cm.

From Figure 7, $d_h = 10^{-1}$ km.

From Figure 9, $Z_h = 1.75 \times 10^{-3}$ atmo-cm.

From equations (1), (2), and (3)

$$\text{Ans.} \begin{cases} W = (1.1 \times 10^{-3}) \times (15) = \underline{\underline{1.65 \times 10^{-2}}} \text{ pr. cm.} \\ d = (\ 10^{-1}\ ) \times (15) = \underline{\underline{1.5}} \text{ km.} \\ Z = (1.75 \times 10^{-3}) \times (15) = \underline{\underline{2.72 \times 10^{-2}}} \text{ atmo.-cm.} \end{cases}$$

Part 2: R < 150 kilometers, slant paths, Use Figures 5 - 9

Let: d_v = equivalent path length of dry air in a vertical path from altitude h to ∞, corrected to sea level conditions, km.

w_v = equivalent precipitable centimeters of water vapor in a vertical path from altitude h to ∞, corrected to sea level conditions, pr. cm.

z_v = equivalent atmospheric centimeters of ozone in a vertical path from altitude h to ∞, corrected to sea level conditions, atmo-cm.

$$W = \frac{W_v\,(\text{observer}) - W_v\,(\text{target})}{h_t - h_o} \times R \qquad \text{eq. (4)}$$

d is similar to W

Z is similar to W

EXAMPLE

Given

h_o = 5 km

h_t = 10 km

R = 20 km

Required

Find d, W, and Z

Solution

From Fig 5, W_v (observer) = 7.1×10^{-2} pr. cm.

W_v (target) = 1.9×10^{-3} pr. cm.

From Fig 7, d_v (observer) = 1.35 km.

d_v (target) = 0.33 km.

From Fig 9, Z_v (observer) = 8.8×10^{-2} atmo-cm.

Z_v (target) = 8.0×10^{-2} atmo-cm.

From equation (4)

$$\text{Ans}\begin{cases} W = \dfrac{7.1 \times 10^{-2} - 1.9 \times 10^{-3}}{10 - 5}\; 20 = \underline{\underline{2.76 \times 10^{-2}}} \text{ pr. cm.} \\[2ex] d = \dfrac{1.35 - 0.33}{10 - 5}\; 20 = \underline{\underline{4.16}} \text{ km.} \\[2ex] Z = \dfrac{8.8 \times 10^{-2} - 8.0 \times 10^{-2}}{10 - 5}\; 20 = \underline{\underline{3.2 \times 10^{-2}}} \text{ atmo-cm.} \end{cases}$$

Part 3: $\theta_{min} > 7^\circ$

Use the same procedure outlined in part 2 above.

Part 4: R > 150 kilometers and $\theta_{min} < 7^\circ$, Use figures 10 - 14

Let: $d_{h-\infty}$ = equivalent path length of dry air in a slant path from altitude h to ∞, corrected to sea level conditions, km.

$w_{h-\infty}$ = equivalent precipitable centimeters of water vapor in a slant path from altitude h to ∞, corrected to sea level conditions, pr. cm.

$z_{h-\infty}$ = equivalent atmospheric centimeters of ozone in a slant path from altitude h to ∞, corrected to sea level conditions, atmo-cm.

Case a) If h_o or h_t is at h_{low}

$$w = \left| w_{h-\infty}(\text{observer})' - w_{h-\infty}(\text{target}) \right| \qquad \text{eq. (5)}$$

d is similar to w

z is similar to w

EXAMPLE

Given

h_o = 15.0 km.

h_t = 00.0 km.

$\theta_{min} = 3^\circ$

Required

Find w, d, and z

Solution

From the methods described in Section 2 of this report, the infrared path is of type 3, Figure 1.

$h_{low} = h_t$

R = 220 km > 150 km, $\theta_{min} < 7^\circ$

From Figure 11, $W_{h-\infty}$ (observer) = 4.1×10^{-3} pr. cm.

From Figure 10, $W_{h-\infty}$ (target) = 38.5 pr. cm.

From Figure 13, $d_{h-\infty}$ (observer) = 7.7×10^{-1} km.

From Figure 12, $d_{h-\infty}$ (target) = 71.5 km.

From Figure 14, $z_{h-\infty}$ (observer) = 7.0×10^{-1} atmo-cm.

From Figure 14, $z_{h-\infty}$ (target) = 1.15 atmo-cm.

From Equation (5),

$$\text{Ans} \begin{cases} w = \left| 4.1 \times 10^{-3} - 38.5 \right| = \underline{\underline{38.5}} \text{ pr. cm.} \\ d = \left| 7.7 \times 10^{-1} - 71.5 \right| = \underline{\underline{70.7}} \text{ km.} \\ z = \left| 7.0 \times 10^{-1} - 1.15 \right| = \underline{\underline{0.45}} \text{ atmo-cm.} \end{cases}$$

Case b) If $h_{low} < h_o$, and $h_{low} < h_t$

$$W = 2\, w_{h - \infty} \text{ (at } h_{min}) - w_{h - \infty} \text{ (observer)} - w_{h - \infty} \text{ (target)} \qquad \text{eq (6)}$$

d is similar to w

z is similar to w

EXAMPLE

Given

$h_o = 15.0$ km.

$h_t = 20.0$ km.

$R = 630$ km.

Required

Find w, d, h

Solution

From the methods described in Section 2 of this report, the infrared path is of type 2, Figure 1.

$h_{low} = 10$ km.; thus $h_{low} < h_o$ and $h_{low} < h_t$

$h_{min} = 10$ km.

From Figure 10, $w_{h - \infty}$ (at h_{min}) = 1.30×10^{-1} pr. cm.

From Figure 11, $w_{h - \infty}$ (observer) = 7.7×10^{-3} pr. cm.

$w_{h - \infty}$ (target) = $1{,}23 \times 10^{-3}$ pr. cm..

From Figure 12, $d_{h - \infty}$ (at h_{min}) = 19.0 km.

From Figure 13, $d_{h - \infty}$ (observer) = 1.43 km.

$d_{h - \infty}$ (target) = 0.23 km.

From Figure 14, $z_{h - \infty}$ (at h_{min}) = 1.70 atmo-cm.

$z_{h - \infty}$ (observer) = 1.11 atmo-cm.

$z_{h - \infty}$ (target) = 0.66 atmo-cm.

From equation (6)

$$\underline{\text{Ans}} \left\{ \begin{array}{lll} w & = 2 \times 1.30 \times 10^{-1} - 7.7 \times 10^{-3} - 1.23 \times 10^{-3} & = \underline{2.21 \times 10^{-1}} \text{ pr. cm.} \\ d & = 2 \times 19.0 - 1.43 - 0.23 & = \underline{36.3} \text{ km.} \\ z & = 2 \times 1.70 - 1.11 - 0.66 & = \underline{1.63} \text{ atmo-cm.} \end{array} \right.$$

Part 5: Weather Change Corrections, Use Table 1 and Figures 15 through 21

If the weather conditions at the observer, and/or the average latitude of the line-of-sight is known, then the model atmosphere can be corrected to agree with the weather. The weather conditions which must be known are the pressure, the temperature, and the concentration of gases and vapors. The temperature can be obtained from Figure 21, taken from D. N. Vachon,(38) or the model atmosphere can be assumed as given in Figure 20. The pressure and atmospheric composition can be assumed to correspond to the model atmosphere, see Table 1 and figures 17 - 19. Under average conditions, the relative humidity may be assumed to be 80% in the troposphere. When these weather conditions do not vary too greatly from the model atmosphere, the correction methods presented in Case a) will suffice. Otherwise, one must employ the methods of Case b) which is much more laborious.

Let: C_{gas} = temperature correction factor for CO_2, N_2O, CH_4, and CO, Figure 16.

C_{H_2O} = temperature correction factor for water vapor, Figure 15.

f = fractional concentration of gas.

P = pressure (of actual atmosphere when used in Section III)

P_o = sea level pressure = 760 mm Hg.

ΔP = pressure difference between actual atmosphere and ARDC model atmosphere, i.e. $(P_{actual}) - (P_{model})$

T = temperature, degrees Kelvin (of actual atmosphere when used in Section III)

T_o = sea level temperature = 288°K

ΔT = temperature difference between actual atmosphere and ARDC model atmosphere, i.e. $(T_{actual}) - (T_{model})$

R.H. = relative humidity

u = actual quantity of a gas or vapor, atmo-cm or pr. cm.

x = fractional concentration of water vapor at 100% relative humidity to dry air at 760 mm Hg pressure and at temperature (T). Use a psychrometric chart to determine the value of (x).

A subscript (corrected) refers to the quantity specified, corrected for weather conditions.

A subscript (model) refers to the quantity specified, as given by the model atmosphere.

Case a) If $\frac{\Delta P}{P} < 0.1$ and $\frac{\Delta T}{T} < 0.2$

This method will give apprcximately the correct amount of gas or water vapor existing under actual weather conditions. It is assumed that $\frac{\Delta P}{P}$ and $\frac{\Delta T}{T}$ remain constant for altitudes between the minimum altitude of the line of sight and 10 km. above that minimum altitude. Thus, errors will be minimized if the weather conditions at the minimum altitude of the line of sight are used, except for ozone. For ozone, use the weather conditions existing at the altitude of maximum value for z_h (corrected) where it is found along the line of sight. To find z_h for the model atmosphere, refer to Figure 9.

$$w_{corrected} = (w_{model}) \times \frac{R.H.}{R.H._{model}} \left[1 + \left(\frac{\Delta P}{P} \right)_{h_{low}} \right] \times C_{H_2O} \qquad \text{eq (7)}$$

$$d_{corrected} = (d_{model}) \times \frac{f}{f_{model}} \left[1 + 2 \left(\frac{\Delta P}{P} \right)_{h_{low}} \right] \times C_{gas} \qquad \text{eq (8)}$$

$$z_{corrected} = (z_{model}) \times \frac{u}{u_{model}} \left[1 + 0.3 \left(\frac{\Delta P}{P} \right)_{h_{max(O_3)}} \right] \times \left[1 - 0.15 \left(\frac{\Delta T}{T} \right)_{h_{max(O_3)}} \right] \qquad \text{eq (9)}$$

EXAMPLE

Given

h_o = 5 km.

h_t = 10 km.

R = 20 km.

R.H. = 80% at h_o

P = 0.525 atmospheres at h_o

P = 0.55 atmospheres at h_t

T = 250°K at h_o

T = 210°K at h_t

Required

Find $w_{corrected}$, $d_{corrected}$, $z_{corrected}$

Solution

This problem is a refinement of the **EXAMPLE** in Part 2 (P. 3) of Section III.

To find w_{model}, d_{model}, and z_{model}

$h_{low} = h_o$

$w_{model} = 2.76 \times 10^{-2}$ pr. cm.

$d_{model} = 4.16$ km.

$z_{model} = 3.20 \times 10^{-2}$ atmo-cm.

From Figure 9, the maximum value of z_h within the infrared path is at h_t = 10 km.

To find $\left[\frac{R.H.}{R.H._{model}}\right]_{H_2O\,vapor}$, $\left[\frac{f}{f_{model}}\right]_{O_2,\,N_2O,\,CH_4,\,CO}$, and $\left[\frac{u}{u_{model}}\right]_{O_3}$

$$\frac{R.H.}{R.H._{model}} = \frac{80\%}{100\%} = 0.80$$, see Figure 17.

Assume:

$$\frac{f}{f_{model}} = 1.0$$

$$\frac{u}{u_{model}} = 1.0$$

To find $\frac{\Delta P}{P}$

From Figure 19, P = 0.530 atmospheres at h_o

P = 0.260 atmospheres at h_t

$$\left[\frac{\Delta P}{P}\right]_{h_o} = \frac{0.525 - 0.530}{0.525} = -0.00953$$

$$\left[\frac{\Delta P}{P}\right]_{h_t} = \frac{0.255 - 0.260}{0.255} = -0.0196$$

To find C_{H_2O}, C_{gas}, and $\frac{\Delta T}{T}$:

From Figure 20 for model atmosphere:

$T = 255.0^\circ K$ at $h_o = 5$ km.

$T = 223.0^\circ K$ at $h_t = 10$ km.

$$\frac{\Delta T}{T} = \frac{250 - 255}{250} = -0.02 \text{ at } h_o = 5 \text{ km.}$$

From Figure 15, with $\left[\frac{\Delta T}{T}\right]_{h_o} = -0.02$

$C_{H_2O} = 0.65$ and,

From Figure 16,

$C_{gas} = 1.032$

$$\left[\frac{\Delta T}{T}\right]_{h_t} = \frac{210 - 223.0}{210} = -0.062 \text{ at } h_t = 10 \text{ km.}$$

From equations (7), (8), and (9)

$$W_{corrected} = (2.76 \times 10^{-2})\,(0.80)\,(1 - 0.0095)\,(0.65)$$

$$d_{corrected} = (4.16)\,(1.00)\,(1 - 2 \times 0.0095)\,(1.032)$$

$$z_{corrected} = (3.20 \times 10^{-2})\,(1.00)\,(1 - 0.3 \times 0.0196)\,(1 + 0.15 \times 0.062)$$

$$\text{Ans} \begin{cases} w_{corrected} = \underline{1.42 \times 10^{-2}} \text{ pr. cm.} \\ d_{corrected} = \underline{4.22} \text{ km.} \\ x_{corrected} = \underline{3.21 \times 10^{-2}} \text{ atmo cm.} \end{cases}$$

Case b) Determination of w, d, and z for Known Weather Conditions

When the weather conditions are known over the path, the following procedure can be used:

$$w_h = (122.56)\,(R.H.)\,(X)\left[\frac{P}{P_o}\right]\left[\frac{T_o}{T}\right]^{3/2} \qquad \text{eq (10)}$$

$$d_h = \left[\frac{f}{f_{model}}\right] \left[\frac{P}{P_o}\right]^2 \left[\frac{T_o}{T}\right]^{3/2} \qquad \text{eq (11)}$$

$$z_h = (u) \left[\frac{P}{P_o}\right]^{0.3} \left[\frac{T_o}{T}\right]^{0.15} \qquad \text{eq (12)}$$

Use Figure 2 to determine the geometry of the line of sight with respect to the Earth. Values for w, d, and z can be obtained as follows:

Let: ΔR_i = incremental distance along the infrared path.

$$w = \sum_{i=0}^{R} w_h \, \Delta R_i \qquad \text{eq (13)}$$

(d) can be obtained in a similar manner to (w).

(z) can be obtained in a similar manner to (w).

EXAMPLE

Given

h_o = h_t = 15 km.

R = 18 km.

P = 91.00 mm Hg

T = 220°K

R.H. = 60%

f = 0.031% for CO_2

Required

Find w, d, and z

Solution

To find x, refer to a psychrometric chart at temperature 220°K.

$x = 1.6 \times 10^{-5}$

To find $\left[\frac{f}{f_{model}}\right]_{CO_2}$:

From Table 1, f_{model} = 0.032%

$$\left[\frac{f}{f_{model}}\right]_{CO_2} = \frac{0.031}{0.032} = 0.97$$

To find u , see Figure 18 assuming the model atmosphere applies.

$$u = 6.3 \times 10^{-3} \text{ atmo-cm}/_{\text{km}} \quad \text{at} \quad h_{\max(O_3)} = 15 \text{ km.}$$

From equations (10), (11), and (12)

$$w_h = (122.56)(0.60)(1.6 \times 10^{-5}) \left[\frac{91.0}{760}\right]\left[\frac{288}{220}\right]^{3/2}$$

$$d_h = (0.97) \left[\frac{91.0}{760}\right]^2 \left[\frac{288}{220}\right]^{3/2}$$

$$z_h = (6.3 \times 10^{-3}) \left[\frac{91.0}{760}\right]^{0.3} \times \left[\frac{288}{220}\right]^{0.15}$$

$$w_h = 2.1 \times 10^{-4} \text{ pr. cm.}$$

$$d_h = 0.0208 \text{ km.}$$

$$z_h = 3.5 \times 10^{-3} \text{ atmo-cm.}$$

From equation (13)

$$\text{Ans} \begin{cases} w = w_h \times R = 2.1 \times 10^{-4} \times 18 = \underline{\underline{3.78 \times 10^{-3}}} \text{ pr. cm.} \\ d = 0.0208 \times 18 = \underline{\underline{3.75}} \, (10^{-1}) \text{ km.} \\ z = 3.5 \times 10^{-3} \times 18 = \underline{\underline{6.3 \times 10^{-2}}} \text{ atmo-cm.} \end{cases}$$

Part 6: Changes in the Altitude of the Tropopause and the Altitude of Maximum Ozone Concentration, Use Figures 21 - 23

This part will cover methods to obtain correct values for (w) and (z) if the altitude of the tropopause and/or the altitude of maximum ozone concentration differs from the model. Part a) will describe the methods used to obtain the correct value for (w). Part b) will describe the methods used to obtain the correct value for (z). The correct values for (d) can be obtained as in Parts 1-5.

Let:

h' = fictitious altitude, km.

h_{trop} = altitude of tropopause

$h_{\max(O_3)}$ = altitude of maximum ozone concentration

k_{H_2O} = correction factor for water vapor when h' is used, Figure 22

k_{0_3} = correction factor for ozone when h' is used, Figure 23

w' = fictitious value of w_h, w_v, $w_{h-\infty}$, and w

z' = fictitious value of z_h, z_v, $z_{h-\infty}$, and z

Case a) Water Vapor - Changes in Altitude of Tropopause

The tropopause is defined as the minimum altitude at which the temperature gradient abruptly changes, see Figure 20. This is assumed to be at 11 km. in the ARDC model atmosphere. It can be seen from Figure 17 that the water vapor mixing ratio also changes slope abruptly at the tropopause. In the troposphere, the water vapor model assumes constant relative humidity. In the stratosphere, the mixing ratio remains constant.

If the altitude of the tropopause changes, it will be assumed that the relative humidity will remain constant at 100% below the tropopause and the mixing ratio will remain constant above the tropopause. It will be assumed further that the temperature is at 288°K at sea level, that the temperature gradient will remain constant and equal to that of the model below the tropopause (see dashed line in Figure 20), and the temperature is constant above the tropopause up to an altitude of 25 km. These models of relative humidity - mixing ratio and temperature versus altitude model will be referred to as the "model atmosphere corrected for tropopause altitude changes." The following method can be used to correct the model atmosphere for changes in tropopause altitude.

If: $h_{low} < h_{trop} - 3$ km., or $h_{low} > h_{trop} + 10$ km.

Use the model

If: $h_{low} \geq h_{trop} - 3$ km. and $h_{low} \leq h_{trop} + 10$ km.

Use the following procedure:

Step 1

Let: $h' = h + 11.0 - h_{trop}$ eq (14)

Go into figures 5, 6, 10, or 11 with the values of (h'_o), (h'_t), and (h'_{min}) as computed from equation (14). Determine the appropriate values of (w') as was done similarly for (w) in Parts 1-4 of Section III.

Step 2

Correct (w') to the proper value of (w_{model}), where:

$$w_{model} = k_{H_2O} \cdot w' \qquad \text{eq (15)}$$

k_{H_2O} can be obtained from Figure 22

Step 3

The above value of (w_{model}) and the "model atmosphere for tropopause altitude changes" should be used as the reference model for correcting for existing weather conditions. Use the same procedure as outlined in Part 5 of Section III in order to make such corrections.

EXAMPLE

Given

h_o = 15.0 km.
h_t = 20.0 km.
R = 475 km.
R.H. = 80% and constant in troposphere
Latitude = 38° North
Date = January 1960

Required

Find w

Solution

From Figure 2, $h_{min} = h_{low} = 13.0$ km. This is a Type 2 infrared path, see Figure 1.

Step 1

To find h_{trop}, refer to Figure 21.

$h_{trop} = 14.0$ km.

From equation (14)

$h'_o = 15.0 + 11.0 - 14.0 = 12.0$ km.
$h'_t = 20.0 + 11.0 - 14.0 = 17.0$ km.
$h'_{min} = h'_{low} = 13.0 + 11.0 - 14.0 = 10.0$ km.

As in Case b) Part 4 Section III (P. 5)

$w'_{h-\infty}$ (at h'_{min}) = 0.130 pr. cm.

$w'_{h-\infty}$ (at h'_o) = 0.027 pr. cm.

$w'_{h-\infty}$ (at h'_t) = 0.0036 pr. cm.

From equation (6)

$w' = 2 \times 0.130 - 0.027 - 0.0036 = 0.229$ pr. cm.

Step 2

$h_{low} - h_{trop} = 13.0 - 14.0 = -1.0$ km.

From Figure 22

$k_{H_2O} = 2.8 \times 10^{-2}$

From Equation (15)

$W_{model} = 2.8 \times 10^{-2} \times 0.0229 = 6.4 \times 10^{-3}$ pr. cm.

Step 3

To correct for weather changes:

$$\frac{R.H.}{R.H._{model}} = \frac{80\%}{100\%} = 0.80$$

Assume the pressure versus attitude corresponds to the model, i.e.

$$\frac{\Delta P}{P} = 0$$

To find C_{H_2O}:

From Figure 21, $T = 210^\circ K$ at $h_{low} = 13$ km.

From Figure 20 assuming $h_{trop} = 14$ km.

$T = 203^\circ K$ at $h_{low} = 13$ km.

$$\left[\frac{\Delta T}{T}\right]_{h_{low}} = \frac{210 - 203}{210} = 0.033$$

From Figure 15

$C_{H_2O} = 2.9$ at $h_{low} = 13$ km.

From Equation 7:

$$\text{Ans}\begin{cases} w_{corrected} = (6.4 \times 10^{-3})\,(0.80)\,(1 + 0)\,(2.9) \\ w_{corrected} = \underline{\underline{1.5 \times 10^{-2}}} \text{ pr. cm.} \end{cases}$$

Case b) Ozone - Changes in Altitude of Maximum Ozone Concentration

It can be seen from Figure 18 that the maximum concentration of ozone occurs at an altitude of 23 km. in the model. The altitude of maximum ozone concentration may be changed by displacing the ozone curve, Figure 18, in altitude without altering its shape. This model will be referred to as the "model atmosphere corrected for altitude of maximum ozone concentration." The following method can be used to correct the model atmosphere for changes in the altitude of maximum ozone concentration.

Step 1

Let: $h' = h + 23.0 - h_{max(0_3)}$ eq (16)

Go into figures 9 or 14 with the values of (h'_o), (h'_t), and (h'_{min}) as computed from equation (16). Determine the appropriate values of (z') as was done similarly for (z) in Parts 1-4 of Section III.

Step 2

Correct (z') to the proper value of (z_{model}), where:

$$z_{model} = k_{0_3} \cdot z' \qquad \text{eq (17)}$$

k_{0_3} can be obtained from Figure 23

Step 3

The above value of (z_{model}) and the "model atmosphere corrected for altitude of maximum ozone concentration" should be used as the reference model for correcting for existing weather conditions. Use the same procedure as outlined in Part 5 of Section III in order to make such corrections.

EXAMPLE

Given

h_o = 15.0 km.
h_t = 20.0 km.
$h_{max(0_3)}$ = 21.0 km.
R = 475 km.
u_v = 0.350 atmo-cm. = amount ozone in a vertical column
Latitude = 38° North
Date = January 1960

Required

Find z

Solution

From Figure 2, $h_{min} = h_{low} = 13.0$ km.

This is a Type 2 infrared path, see Figure 1.

Step 1

From equation (16)

$h'_o = 15.0 + 23.0 - 21.0 = 17.0$ km.

$h'_t = 20.0 + 23.0 - 21.0 = 22.0$ km.

$h'_{min} = h'_{low}, = 13.0 + 23.0 - 21.0 = 15.0$ km.

As in Case b) Part 4 Section III (P. 5)

$z'_{h-\infty}$ (at h'_{min}) $= 1.86$

$z'_{h-\infty}$ (at h'_o) $= 1.25$

$z'_{h-\infty}$ (at h'_t) $= 0.62$

From equation (6)

$z' = 2 \times 1.86 - 1.25 - 0.62 = 1.85$ atmo-cm.

Step 2

From Figure 23

$k_{O_3} = 1.10$

From equation (17)

$z_{model} = 1.10 \times 1.85 = 2.04$ atmo-cm.

Step 3

To correct for weather changes:

To find $\frac{u}{u_{model}}$:

From Figure 18, $u_{model} = 0.229$ atmo-cm.

$u = 0.350$ atmo-cm is given

$$\frac{u}{u_{model}} = \frac{0.350}{0.229} = 1.53$$

Assume the pressure versus altitude corresponds to the model, i.e.

$\frac{\Delta P}{P} = 0$

To find $\frac{\Delta T}{T}$:

From Figure 21:

$T = 210^{o}K$ at $h_t = 20$ km.

From Figure 20, for model atmosphere

$T = 216.6^{o}K$ at $h_t = 20$ km.

$$\frac{\Delta T}{T} = \frac{210 - 216.6}{210} = -0.031$$

From equation 9:

$$z_{corrected} = (2.04)\ (1.53)\ (1 + 0)\ (1 + 0.15 \times 0.031)$$

Ans

$z_{corrected} = \underline{3.13}$ atmo-cm.

SECTION IV

DETERMINATION OF ATMOSPHERIC TRANSMISSION AND RADIATION OF INFRARED

This section will cover methods for obtaining the atmospheric transmission and spectral background radiance of infrared. Once values for (w), (d), and (z) are known, one can go to Figures 25 - 46 and obtain the atmospheric transmission and also calculate the spectral background radiance of the sky.

Infrared is attenuated in the atmosphere by molecular absorption, aerosols "haze" scattering and by absorption by solid particles such as dust. Only molecular absorption and scattering by haze will be considered here, since solid particle absorption is considered to be negligible for most practical applications.

This section is divided into three parts. Part (1) gives methods for determining the transmission of infrared by the atmosphere. Part (2) gives methods for determining the molecular absorption of infrared by small quantities of gases and vapors. Part (3) presents methods for obtaining the background radiance of the atmosphere.

Part (1): Transmission of Infrared by the Atmosphere, Use Figures 24-45

Molecular absorption is caused by the vibration of atoms within a gas molecule and the rotation of the molecule. The vibration of atoms within the gas molecule is responsible for the infrared band absorption spectra while the rotation of the molecules are responsible for the fine line spectra within the absorption bands. If the absorption behavior of an infinitesimal wavelength segment of a spectral line were observed, Beer's absorption law would be obeyed. Thus, the absorption of a spectral band could be obtained by integrating the absorption of each such wavelength increment over the thousands of lines that make up the spectral band. The work involved would be enormous. For this reason, the average absorption of many lines over larger wavelength increments is used, and the resulting spectral band will appear as a smooth curve. The absorption behavior of these larger wavelength increments do not obey Beer's Law, but follow instead much more complicated laws, viz. the Elsasser model, the Goody model, the King model, and the Plass model. Such models are described by Altshuler, ref. 3.

Figures 25-44 are transmission curves on graph paper designed to preserve the shape of the curves as the amount of absorbing gas varies. The abscissa ($\log_{10}\lambda$) was designed with increments equal to 0.01 units. Large errors will not be introduced when calculating the total atmospheric transmission versus wavelength. The data from which these figures were obtained came from numerous sources. A complete description of this is given in the Appendix F and in Table 3. The data for water vapor and carbon dioxide is considered to be the most reliable due to the extensive experimental work done by Howard, Burch, and Williams, reference 4, and by Palmer, reference 5. The data for the 4.5 micron band of N_20 by Burch, reference 6, is excellent also.

The absorption of infrared due to haze is given on Figure 45. It is assumed that the amount of haze will be distributed with a uniform mixing ratio versus altitude, thus values of (d), equivalent path length, are used. Such an assumption agrees with data obtained from Taylor and Yates, reference 1.

The absorption bands for CH_4 and C0 are not included in this report. They are so weak in comparison to those of other gases that they are not considered to affect the atmospheric transmission sufficiently to warrant their inclusion.

Let: λ = wavelength in microns

$\tau_{H_20}(\lambda)$ = transmission of H_20 vapor at wavelength (λ)

$\tau_{C0_2}(\lambda)$ = transmission of $C0_2$ at wavelength (λ)

$\tau_{0_3}(\lambda)$ = transmission of 0_3 at wavelength (λ)

$\tau_{N_20}(\lambda)$ = transmission of N_20 at wavelength (λ)

$\tau_{scat.}(\lambda)$ = transmission through atmospheric scattering particles (haze) at wavelength (λ)

$\tau_A(\lambda)$ = atmospheric transmission through the infrared path of infrared radiation at wavelength (λ)

To obtain the atmospheric transmission:

Step 1: With the use of dividers, mark off on the "index" the amount of absorbing gas present in the infrared path, i.e. (w), (d), and (z) as found in Section III of this report. Use the fine scale between 1 and 10 on the "Index" for interpolating between the major divisions on the "Index."

Step 2: Locate the major division on the "Index" corresponding to the values of (w), (d), or (z) identifying the curve on the figure.

Step 3: The transmission is obtained by displacing the transmission curve by the same amount and direction that the entry value differs from the curve.

Step 4: Read the transmission (τ) for each gas, water vapor, and haze at the same wavelength and calculate:

$$\tau_A(\lambda) = \left[\tau_{H_2O}(\lambda)\right] \times \left[\tau_{CO_2}(\lambda)\right] \times \left[\tau_{O_3}(\lambda)\right] \times \left[\tau_{N_2O}(\lambda)\right] \times \left[\tau_{scat.}(\lambda)\right] \qquad \text{eq (18)}$$

Step 5: Repeat Steps 1 - 4 for each ($\log_{10} \lambda$) division (0.01 units apart) over the wavelength region desired in order to obtain the atmospheric transmission if not indicated otherwise in Figures 24-45.

EXAMPLE

Given

From the EXAMPLE in Case a, Part 4, Section III

w = 38.5 pr. cm.

d = 70.7 km.

z = 0.45 atmo-cm.

Assume a very clear day.

Required

Find the atmospheric transmission at 4.8 μ.

Solution

From Figure 27, $\tau_{H_2O} = 0.18$

From Figure 35, $\tau_{CO_2} = 0.37$

From Figure 39, $\tau_{O_3} = 0.735$

From Figure 42, $\tau_{N_2O} = 1.00$

From Figure 45, $\tau_{scat.} = 0.28$

From equation (18)

$\tau_A(4.8) = 0.18 \times 0.37 \times 0.735 \times 1.00 \times 0.28$

Ans $\tau_A(4.8) = \underline{0.0137 = 1.4\%}$

Part 2. Apparent Target Radiance, Use Figures 46, 47, and Template B.

The results of the atmospheric transmission can be plotted as a function of wavelength. These results can then be used for calculating the amount of radiant energy that will be transmitted by the line-of-sight. This apparent spectral radiance at the target as a function of wavelength per unit wavelength interval can be found by taking the product of the spectral target radiance and the line-of-sight transmission in that interval. If target data is presented in terms of spectral target radiance per unit interval, Q_λ, the apparent target radiance is given by

$$Q'_\lambda = \tau_A \cdot Q_\lambda$$

where

$$Q_\lambda = \Delta_\lambda N = 0.023\ \lambda\ N_\lambda$$

If, on the other hand, the target spectral radiance is known to be that of a Planckian radiator, the following procedure can be used to find Q_λ.

Let: N_λ = spectral radiance of target, watts/cm^2 - ster. - μ

Q_λ = spectral radiance of target over the wavelength increment ($\Delta\lambda$), watts/cm^2 - ster. - $\Delta\lambda$

$Q_{\lambda_{max}}$ = maximum spectral radiance of target over the wavelength increment ($\Delta\lambda$), watts/cm^2 - ster. - $\Delta\lambda$

T = temperature of target, degrees Kelvin

λ = wavelength, microns

λ_{max} = wavelength at $Q_{\lambda_{max}}$, microns

$\Delta\lambda$ = wavelength increment such that:

$$\log_{10}(\lambda + \Delta\lambda) - \log_{10}(\lambda) = 0.01 \qquad \text{eq (19)}$$

For black body type of radiation, refer to Figures 46 and 47 together with Template B. Template B should lay over Figure 46 so that the horizontal line of the template coincides with the abscissa (the line $\dfrac{Q_\lambda}{Q_{\lambda_{max}}} = 10^{-4}$). Slide the template back and forth keeping these lines coincident until the vertical (λ_{max}) line of the template coincides with the appropriate target temperature indicated in the bottom scale of Figure 46. Now read off $\dfrac{Q_\lambda}{Q_{\lambda_{max}}}$ directly as a function of $\log_{10}$.

From Figure 47, ($Q_{\lambda_{max}}$) is given as a function of target temperature. Therefore,

$$Q_\lambda = \epsilon_\lambda \left[\frac{Q_\lambda}{Q_{\lambda_{max}}} \right] Q_{\lambda_{max}} \qquad \text{eq (20)}$$

where ϵ_λ is the spectral emissive power ($\epsilon_\lambda = \epsilon = 1$ for a black body, or total radiator.)

Appendix G covers the theory upon which Figures 46 and 47 and Template B are based.

EXAMPLE

Given

$T = 800^{o}K$

Required

Find Q_{λ} at $\lambda = 10$

Solution

From Figure 46 and Template B

$$\frac{Q_{\lambda}}{Q_{\lambda_{max}}} = 0.45$$

From Figure 47

$$Q_{\lambda_{max}} = 1.2 \times 10^{-2} \text{ watts/cm}^{2} \text{ - ster - } \Delta\lambda$$

From equation (20)

Ans $Q_{\lambda} = (0.45)(1.2 \times 10^{-2}) = \underline{\underline{5.4 \times 10^{-3}}} \text{ watts/cm}^{2} \text{ - ster - } \Delta\lambda$

Part (3): Molecular Absorption of Infrared by Small Quantities of Gases and and Vapors, Use Figure 48 and Figures 24-45

When the quantities of gases and vapors are sufficiently small, the absorption of infrared is proportional to the quantity of gas. Normally, such absorption behavior is known as the "linear region" of the absorption curve, and occurs at low values of absorption (A gas $<$ 0.05) irrespective of which absorption model applies; viz, the Elsasser model, etc. In actual fact the absorption function is dependent on the spectral line half width. The latter in turn is a function of pressure. A specific absorption model that best represents the behavior of the gas has been used with each gas in question. The Figures 24 through 44 are the results of these assumptions. The arguments for model selection are presented in Appendix F and are summarized in Table 2. These assumed models fail to hold when dealing with small quantities of gas (Figure 58). Calculation based on the linear region of the model absorption curve then have to be corrected to bring these values into agreement with the actual absorption.

Figure 48 shows the linear absorption regions for various gases and vapors. The absorption refers to the absorption of a gas in the line-of-sight. The altitude refers to the lowest altitude of the line-of-sight (h_{low}). If the absorption of the gas in question is below that value given in the curve then the relationship $A = 0.05 \frac{u}{u_{95\%}}$ holds as follows. For CO_2: $A = \frac{0.05}{32} \frac{u_{CO_2}}{u_{95\%}} = 0.0016 \frac{u_{CO_2}}{d_{95\%}}$

where 1 km @ SL = 32 atmo-cm for CO_2

No other corrections need be made since the data plotted in Figure 32 through Figure 38 are based on experimental data (Figure 58).

H_20:

$$A = 0.012\ \frac{u_{H_20}}{w_{H_20}} \quad \text{from Figure 58}$$

for 0_3:

$$A = 0.07\ \frac{u_{0_3}}{z_{95\%}} \quad \text{from Figure 58}$$

for N_20:

$$A = 0.06\ \frac{u_{N_20}}{u_{95\%}} \quad \text{or} \quad A = \frac{0.06}{0.027}\ \frac{u_{N_20}}{d_{95\%}} = 0.22\ \frac{u_{N_20}}{d_{95\%}}$$

where 1 km. @ SL = 0.027 Atmo-cm. for N_20

Calculation Procedure

Step 1:

Use Figures 24-44 to determine $w_{95\%}$, $d_{95\%}$, and $z_{95\%}$. This is done by measuring the distance between the absorption curve and the 95% transmission line for increments of log λ = 0.01. This will give the amount of gas needed to give a transmission of 95% at the wavelength under consideration.

Step 2:

Solve for A_{H_20}, A_{C0_2}, A_{0_3}, and A_{N_20} from:

$$A_{H_20} = 0.012\ \frac{u_{H_20}}{w_{95\%}} \qquad \text{eq (21)}$$

$$A_{C0_2} = 0.0016\ \frac{u_{C0_2}}{d_{95\%}} \qquad \text{eq (22)}$$

$$A_{0_3} = 0.007\ \frac{u_{0_3}}{z_{95\%}} \qquad \text{eq (23)}$$

$$A_{N_20} = 0.22\ \frac{u_{N_20}}{d_{95\%}} \qquad \text{eq (24)}$$

EXAMPLE

Given

$u_{H_20} = 1.9 \times 10^{-4}$ pr. cm.

$u_{C0_2} = 1.07 \times 10^{2}$ atmo-cm.

$u_{0_3} = 2.5 \times 10^{-2}$ atmo-cm.

$u_{N_20} = 9.05 \times 10^{-2}$ atmo-cm.

$h_{low} = 10$ km.

Required

Find A_{H_20}, A_{C0_2}, A_{0_3}, and A_{N_20} at $\lambda = 4.0\mu$

Solution

At 4.0μ

From Figure 27, $w_{95\%} = 20$ pr. cm.

From Figure 35, $d_{95\%} = 32$ km for $C0_2$

From Figure 39, $z_{95\%} = \infty$

From Figure 42, $d_{95\%} = 4.2$ km. for N_20

From equations (21 - 24)

$$\underline{\text{Ans}}\begin{cases} A_{H_20} = 0.012\ \dfrac{1.9 \times 10^{-4}}{20} = \underline{1.14 \times 10^{-7}} \\[2ex] A_{C0_2} = 0.0016\ \dfrac{1.07 \times 10^{2}}{32} = \underline{\underline{5.3 \times 10^{-3}}} \\[2ex] A_{0_3} = 0.007\ \dfrac{2.5 \times 10^{-2}}{\infty} = \underline{\underline{0}} \\[2ex] A_{N_20} = 0.22\ \dfrac{9.05 \times 10^{-2}}{4.2} = \underline{\underline{4.7 \times 10^{-3}}} \end{cases}$$

From Figure 48, these absorptions are below the curves at h = 10 km, so the calculated values of low absorption are correct.

Part (4): Background Radiation of the Atmosphere, Use Figures 1 - 49.

When gases or vapors absorb infrared radiation, they become hotter unless they radiate energy. If the amount of energy radiated equals that absorbed, then the temperature of the gas will remain at equilibrium.

The same mechanism governs the infrared radiation and absorption behavior of gases. Thus, the amount and spectral distribution of the radiated energy is a function of the infrared transmission characteristics of the gas as well as its temperature. The following procedure may be used to determine the background radiation:

Let: H = irradiance at the observer, watts/cm^2

H_λ = spectral irradiance at the observer over the wavelength interval ($\Delta\lambda$) at wavelength (λ), watts/cm^2 - $\Delta\lambda$

$Q_{\lambda,i}$ = spectral radiance of a black body radiator at the temperature of the gas in the (i th) layer and over the wavelength increment ($\Delta\lambda$) at wavelength (λ), watts/cm^2 - ster. - $\Delta\lambda$

T_i = temperature of the gas, degrees Kelvin, (i th) layer.

λ = wavelength, microns.

$\Delta\lambda$ = wavelength increment such that:

$$\log_{10}(\lambda + \Delta\lambda) - \log_{10}(\lambda) = 0.01$$

$T_{A,i}(\lambda)$ = atmospheric transmission at wavelength (λ) from the (i th) point on the infrared path to the observer, see Figure 49. There are (n) such points.

ω = solid angle measured from the observer, steradians.

Step 1

Divide the atmosphere into horizontal layers which may be considered to be isothermal, see Figure 49. Find the temperature of these layers from Figures 20 or 21 unless actually known. Above the tropopause, the atmosphere can be considered to be isothermal.

Step 2

Determine the intersection of the infrared ray with the boundaries of the atmospheric layers, see Figure 49. The portion of the infrared ray between its intersection with a boundary at the (i th) location and the observer is known as the " (i th) infrared path ". Determine the geometry of that path with respect to the Earth as explained in Section II.

Step 3

Determine the values of w, d, and z in the " (i th) infrared path " using the methods described in Section III.

Step 4

Determine the atmospheric transmission in the " (i th) infrared path " at wavelength (λ) as described in Parts (1) and (2) of Section IV.

Step 5

Determine the values of $Q_{\lambda,i}$ for each layer of atmosphere using the methods described under " Thermal Energy Calculations " Part (1) of Section IV. The (i th) layer is within the bounds the infrared path at the (i th) point.

Step 6

Determine H_λ from:

$$H_\lambda = \omega \sum_{i=1}^{n} Q_{\lambda,i} \left[\tau_{A,i-1}(\lambda) - \tau_{A,i}(\lambda) \right] \qquad \text{eq (25)}$$

If the atmosphere is isothermal for the entire infrared ray extending from the observer, then equation (25) can be simplified to:

$$H_{\lambda} = Q_{\lambda}\left[1 - r_A(\lambda)\right] \qquad \text{eq (26)}$$

Step 7

Repeat steps 4 through 6 for each wavelength increment ($\Delta\lambda$) over the wavelength region of interest (λ_1 to λ_2), and determine H as follows:

$$H = \sum_{\lambda=\lambda_1}^{\lambda_2} H_{\lambda} \qquad \text{eq (27)}$$

Appendix H covers the theory upon which equations (25) through (27) are based.

EXAMPLE

Given

h_o = 17.3 km
θ_o = -3°
ω = 10^{-4} steradians
Haziness - average

Required

Find H_{λ} at $\lambda = 5.0\,\mu$, i.e. $\log_{10}\lambda = 0.70$

Solution

Step 1

Divide the atmosphere into isothermal layers, see Figure 49, and determine their temperature from Figure 20.

Layer	1	2	3	4	5	6	7
Altitude of layer bottom	11 km	10	9	8	9	10	11 km
T	217°K	223°K	229°K	236°K	229°K	223°K	217°K

Step 2

Determine the geometry of the infrared path.

Path	1	2	3	4	5	6	7
h_t	11 km	10	9	9	10	11	∞
h_{low}	11 km	10	9	8	8	8	8

h_o = 17.3 km.
h_{min} = 8.0 km.

Step 3

To find w, d, and z. From Figures 10 - 14.

Path	1	2	3	4	5	6	7
w, pr. cm	3.0×10^{-2}	5.3×10^{-2}	1.27×10^{-1}	1.01	1.08	1.10	1.14
d, km.	5.4	8.6	14.6	52	58	61	67
z, atmo-cm	0.40	0.50	0.60	1.10	1.20	1.30	2.60

Step 4

To find $\tau_{A,i}(\lambda)$ at $\log_{10}\lambda = 0.70$:

Read transmission versus w, d, or z at $\log_{10}\lambda = 0.70$ in Figures 27, 35, 39, 42, and 45.

Path	1	2	3	4	5	6	7
$\tau_{H_2O}(\lambda)$	.89	.86	.78	.50	.49	.49	.49
$\tau_{CO_2}(\lambda)$	1.00	1.00	.99	.958	.954	.953	.950
$\tau_{O_3}(\lambda)$	.971	.968	.965	.955	.950	.947	.928
$\tau_{N_2O}(\lambda)$	1.00	1.00	1.00	1.00	1.00	1.00	1.00
$\tau_{scat}(\lambda)$	.83	.73	.60	.16	.13	.12	.09
$\tau_{A,i}(\lambda)$	.727	.608	.448	.073	.058	.053	.039

Step 5

To find $Q_{\lambda,i}$, from Figures 46, 47.

Layer	1	2	3	4	5	6	7
$Q_\lambda / Q_{\lambda_{max}}$	1.3×10^{-2}	1.8×10^{-2}	2.3×10^{-2}	2.8×10^{-2}	2.3×10^{-2}	1.8×10^{-2}	1.3×10^{-2}
$Q_{\lambda_{max}}$	6.9×10^{-5}	7.6×10^{-5}	8.4×10^{-5}	9.4×10^{-5}	8.4×10^{-5}	7.6×10^{-5}	6.9×10^{-5}
$Q_{\lambda i}$	9.0×10^{-7}	1.37×10^{-6}	1.93×10^{-6}	2.63×10^{-6}	1.93×10^{-6}	1.37×10^{-6}	9.0×10^{-7}

Step 6

To find H_λ from equation (25),

i	1	2	3	4	5	6	7
$\tau_{A,i-1}$	1	.727	.608	.448	.073	.058	.053
$\tau_{A,i}$	.727	.608	.448	.073	.058	.053	.039
$(\tau_{A,i-1} - \tau_{A,i})$	.273	.119	.160	.375	.015	.005	.014
$Q_{\lambda,i}$	9.0×10^{-7}	1.37×10^{-6}	1.93×10^{-6}	2.63×10^{-6}	1.93×10^{-6}	1.37×10^{-6}	9.0×10^{-7}
$Q_{\lambda i}(\tau_{A,i-1} - \tau_{A,i})$	2.46×10^{-7}	1.63×10^{-7}	3.09×10^{-7}	9.86×10^{-7}	2.89×10^{-8}	6.85×10^{-7}	1.26×10^{-8}

$$\sum_{i=1}^{7} Q_{\lambda i}\,(\tau_{A,i-1} - \tau_{Ai}) = 1.75 \times 10^{-6}$$

$$H_\lambda = \omega \sum_{i=1}^{7} Q_{\lambda_i}\,(\tau_{A,i-1} - \tau_{Ai}) = 10^{-4} \times 1.75 \times 10^{-6}$$

Ans $H_\lambda = \underline{\underline{1.8 \times 10^{-10}}}$ watts/cm^2 -$\Delta\lambda$

TABLE 1

CONCENTRATION OF INFRARED ABSORBING GASES

Gas	Concentration, parts per million	Amount in 1 km path at sea level, atmo-cm.
CO_2	320	32.0
N_2O	0.27	0.027
CH_4	2.4	0.24
CO	1.1	0.11

These gases all have approximately constant mixing ratios with air versus altitude between 0 km. and 50 km.

GEOMETRIC RELATIONSHIPS FOR VARIOUS TYPES OF INFRARED PATHS

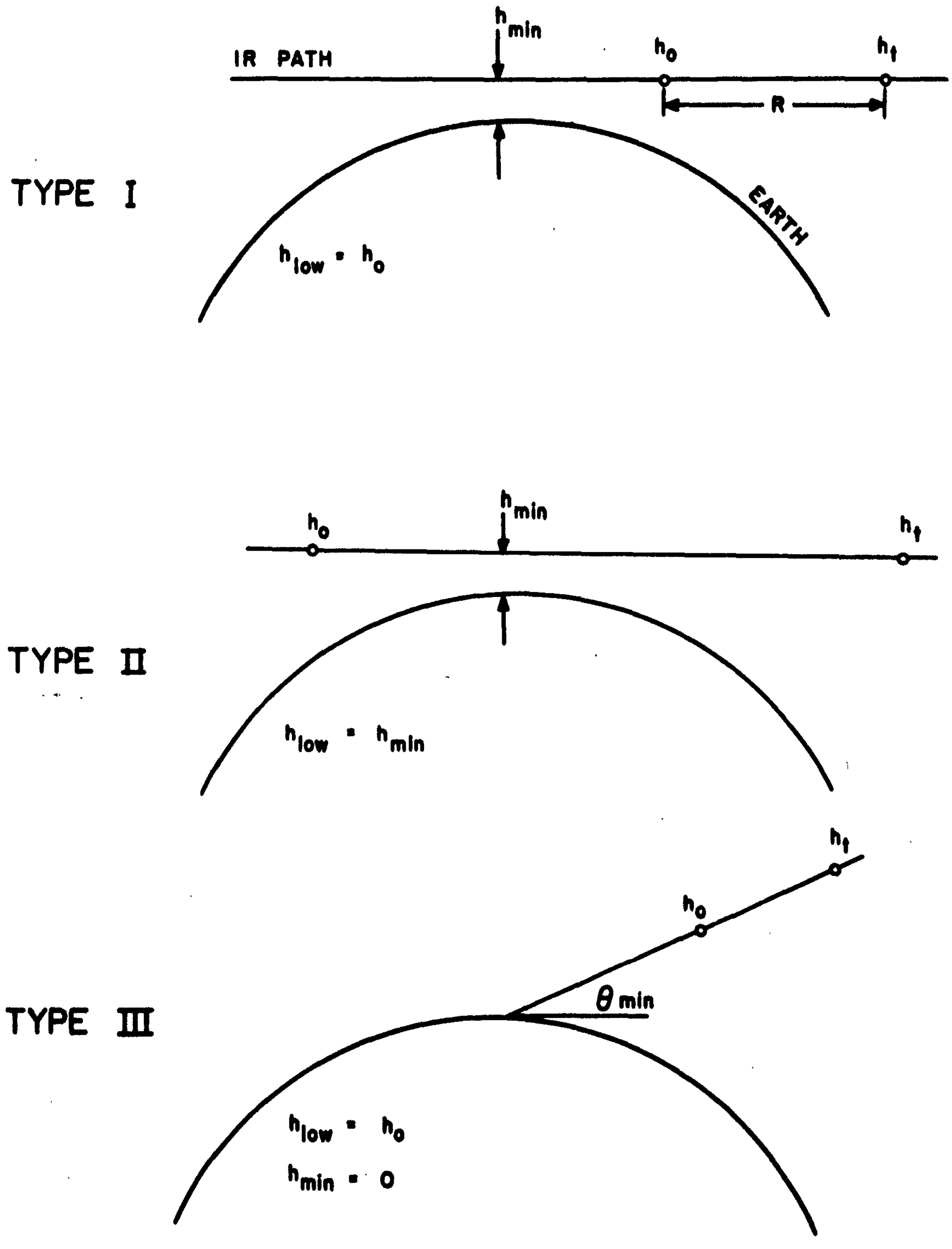

Figure 1

TABLE 1

CONCENTRATION OF INFRARED ABSORBING GASES

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GEOMETRIC RELATIONSHIPS FOR VARIOUS TYPES OF INFRARED PATHS

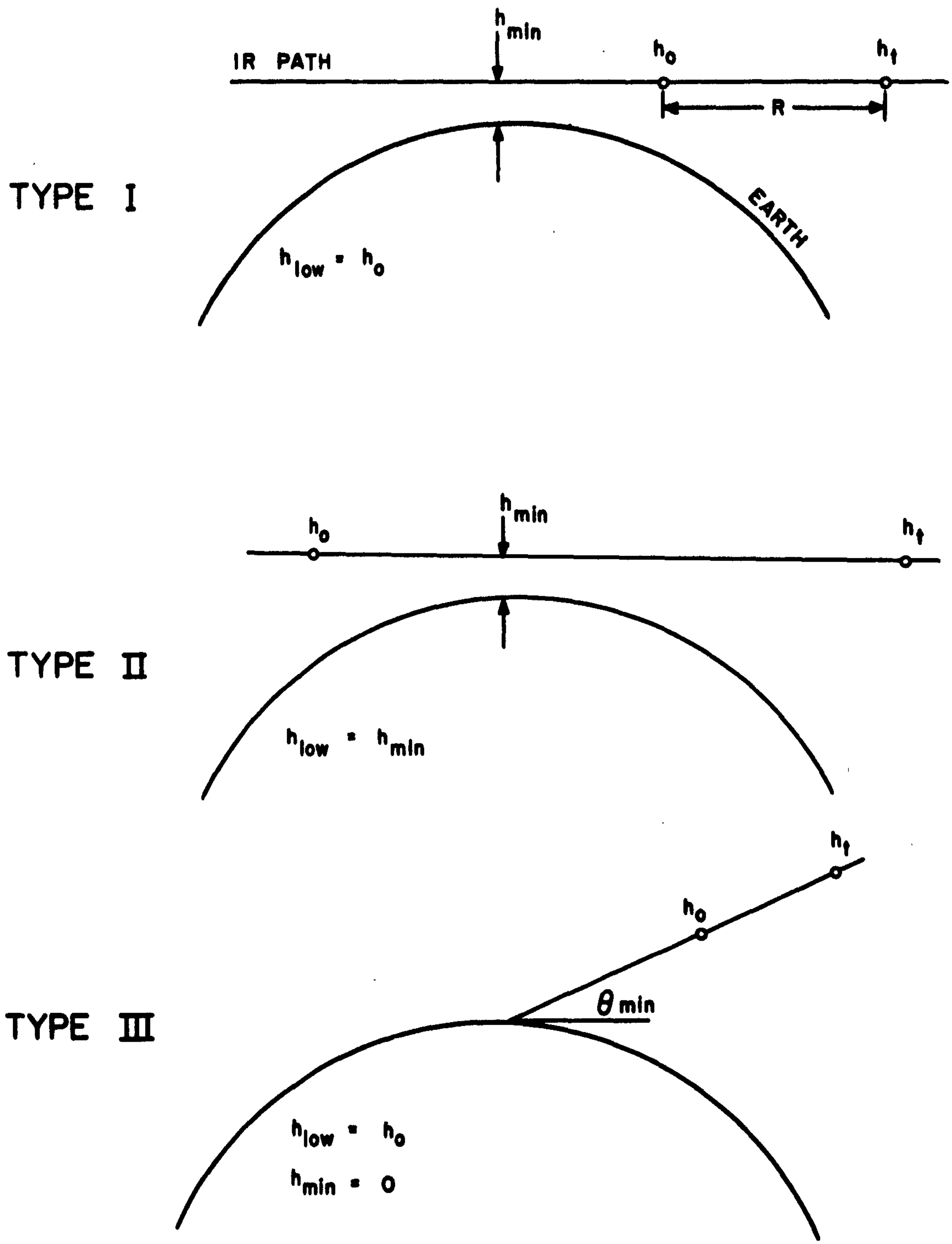


Figure 1

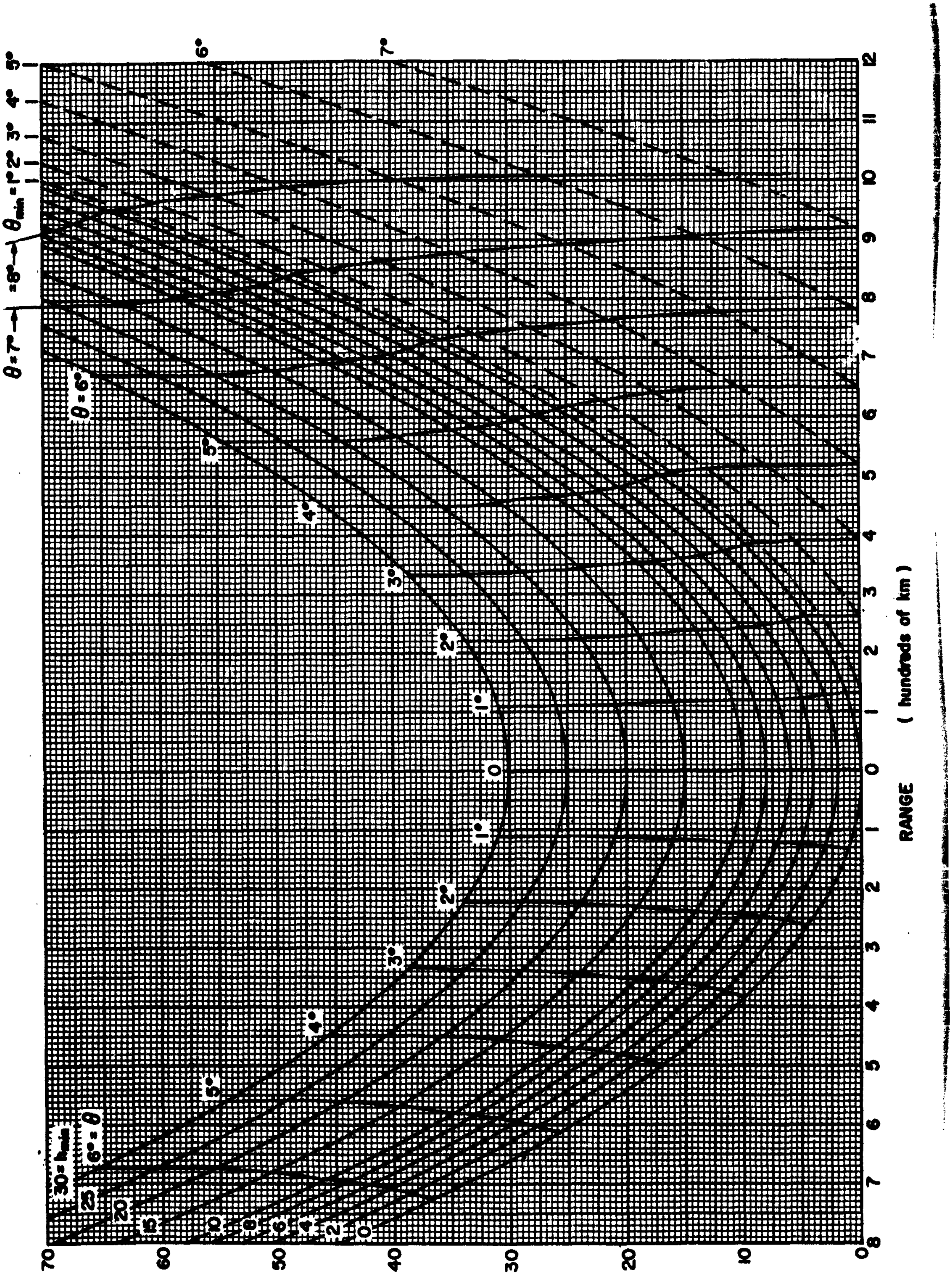

ALTITUDE (km)
0
10
20
30
40
50
60
70
RANGE (hundreds of km)
8
7
6
5
4
3
2
1
0
1
2
3
4
5
6
7
8
9
10
11
12
30 = h_{min}
25
20
15
10
8
6
4
2
0
6° = θ
5°
4°
3°
2°
1°
0
1°
2°
3°
4°
5°
θ = 6°
θ = 7°
= 8°
θ_{min} = 1° 2° 3° 4° 5°
6°
7°

Figure 2

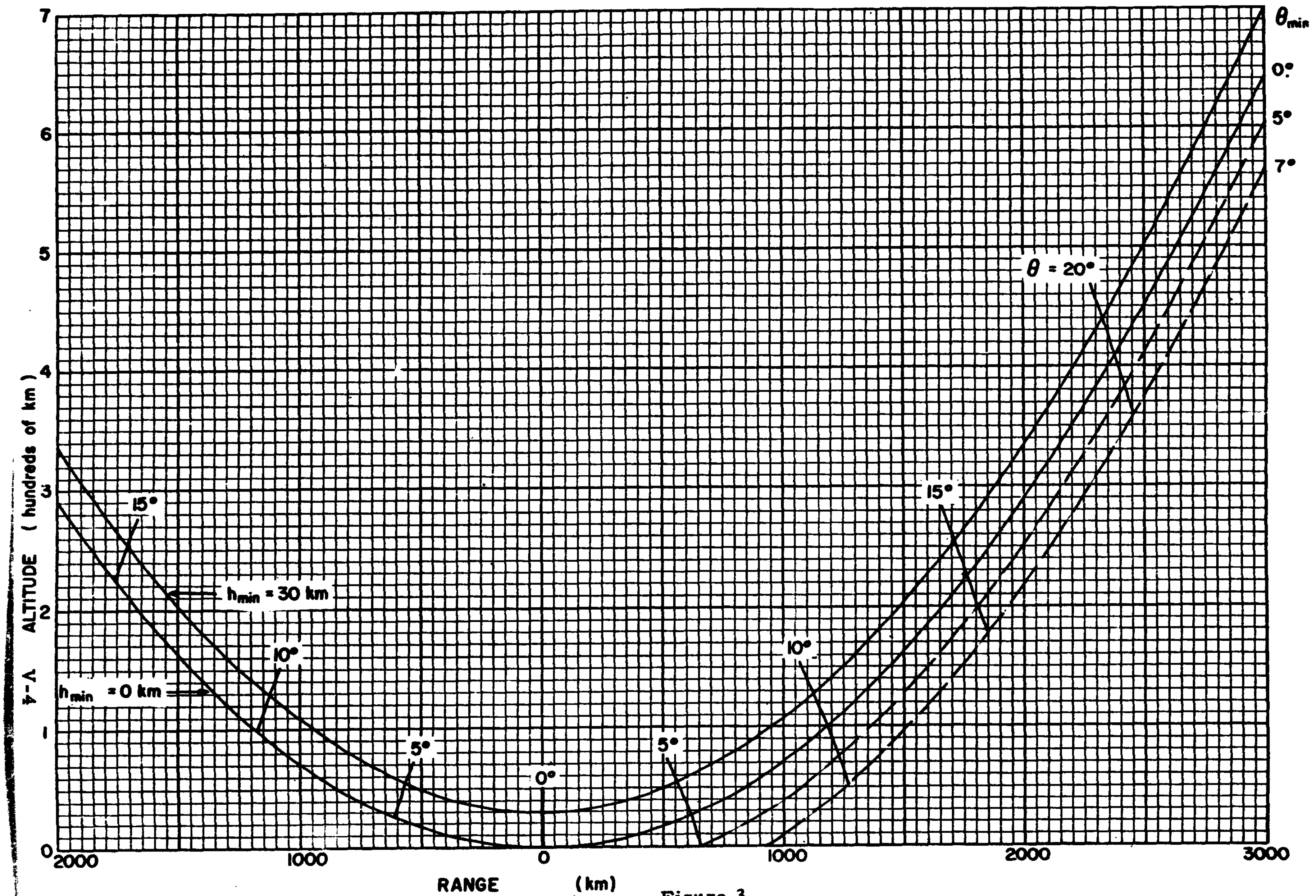

7
6
5
4
3
2
1
0
ALTITUDE (hundreds of km)
2000
1000
0
1000
2000
3000
RANGE (km)
θ_{min}
0°
5°
7°
θ = 20°
15°
10°
5°
0°
5°
10°
15°
h_{min} = 30 km
h_{min} = 0 km

Figure 3

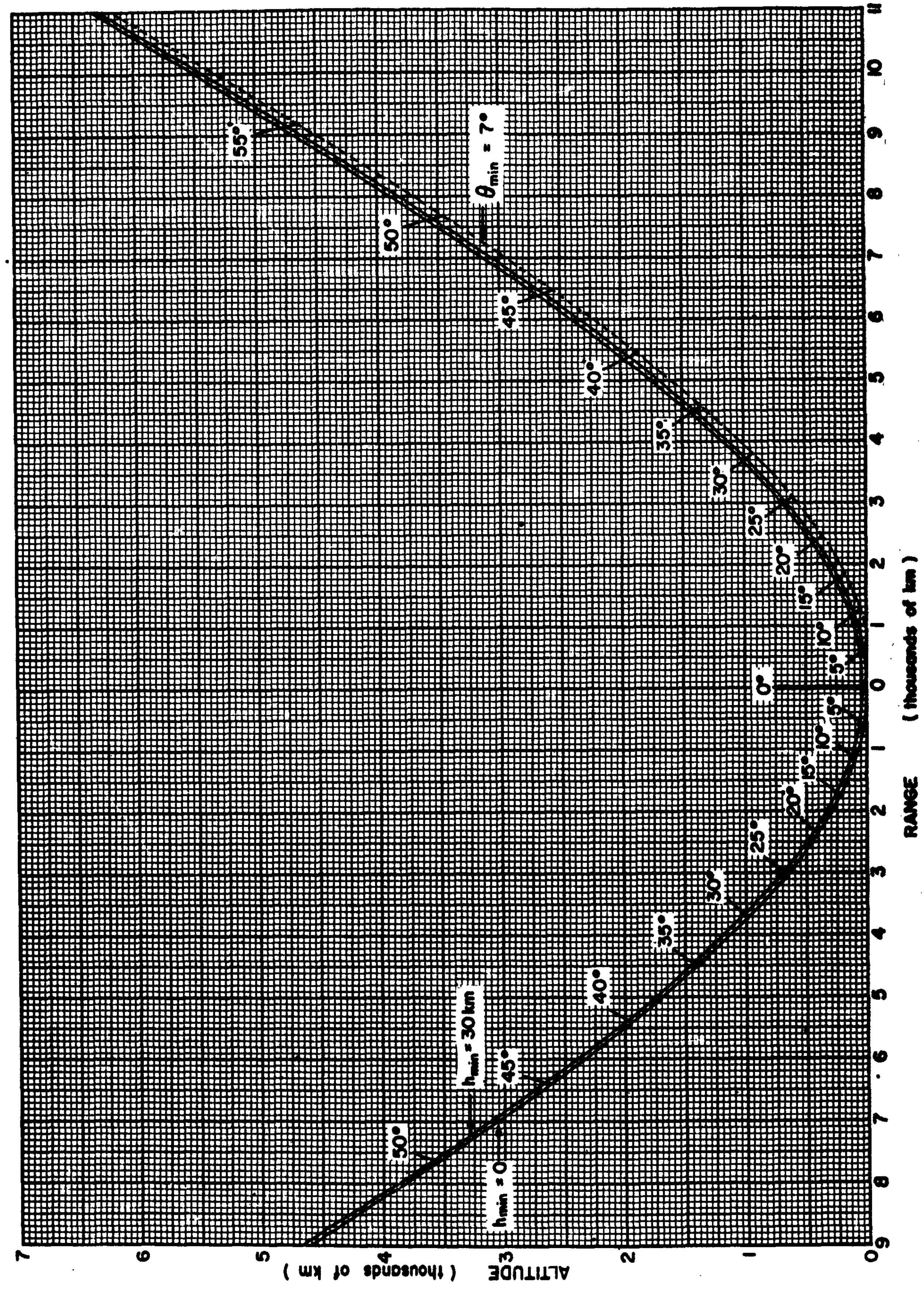

ALTITUDE (thousands of km)
0
1
2
3
4
5
6
7
RANGE (thousands of km)
9
8
7
6
5
4
3
2
1
0
1
2
3
4
5
6
7
8
9
10
11
0°
5°
10°
15°
20°
25°
30°
35°
40°
45°
50°
55°
h_{min} = 30 km
h_{min} = 0
θ_{min} = 7°

Figure 4

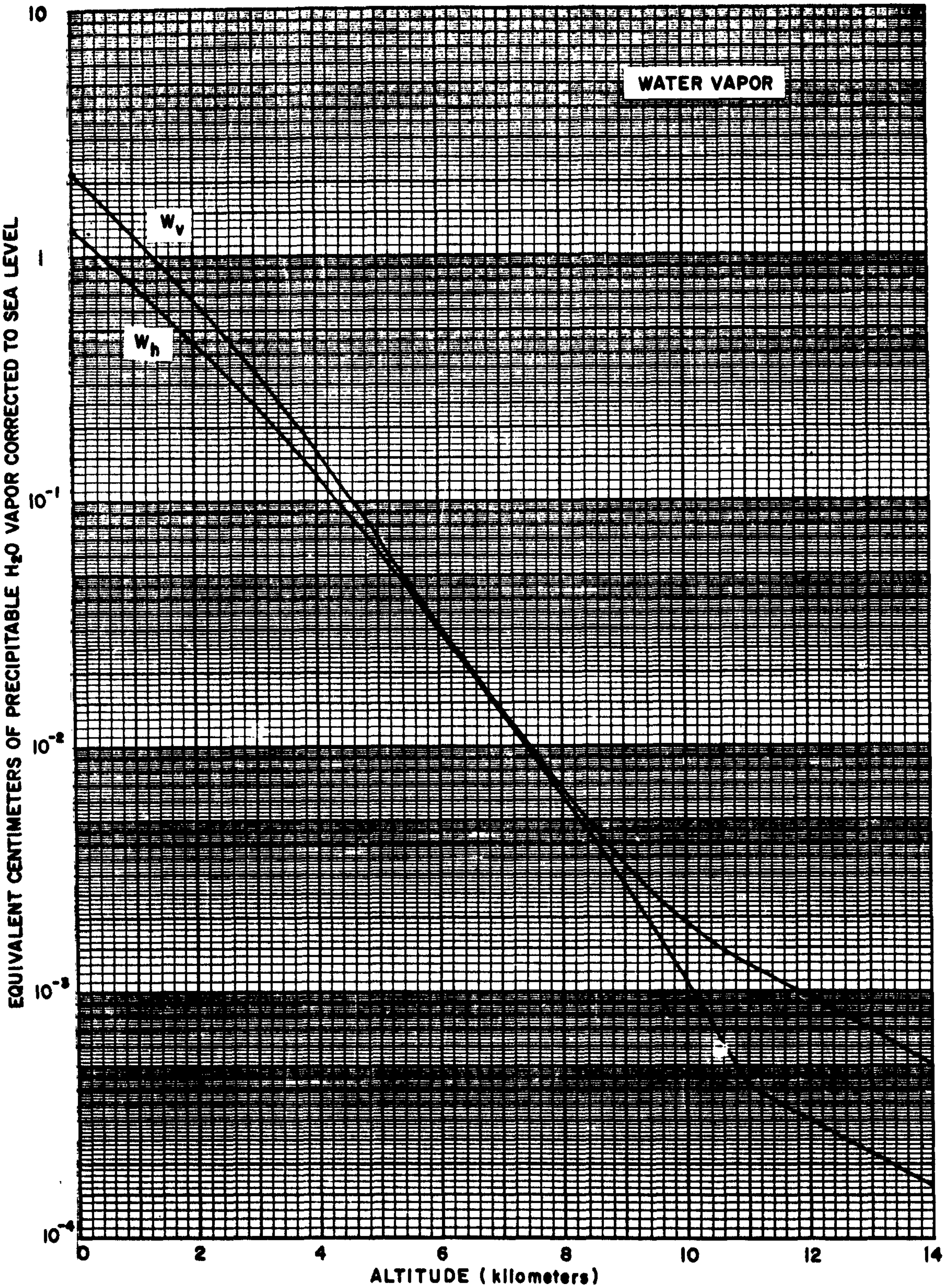

WATER VAPOR
W_v
W_h
EQUIVALENT CENTIMETERS OF PRECIPITABLE H_2O VAPOR CORRECTED TO SEA LEVEL
10
1
10^{-1}
10^{-2}
10^{-3}
10^{-4}
0
2
4
6
8
10
12
14
ALTITUDE (kilometers)

Figure 5

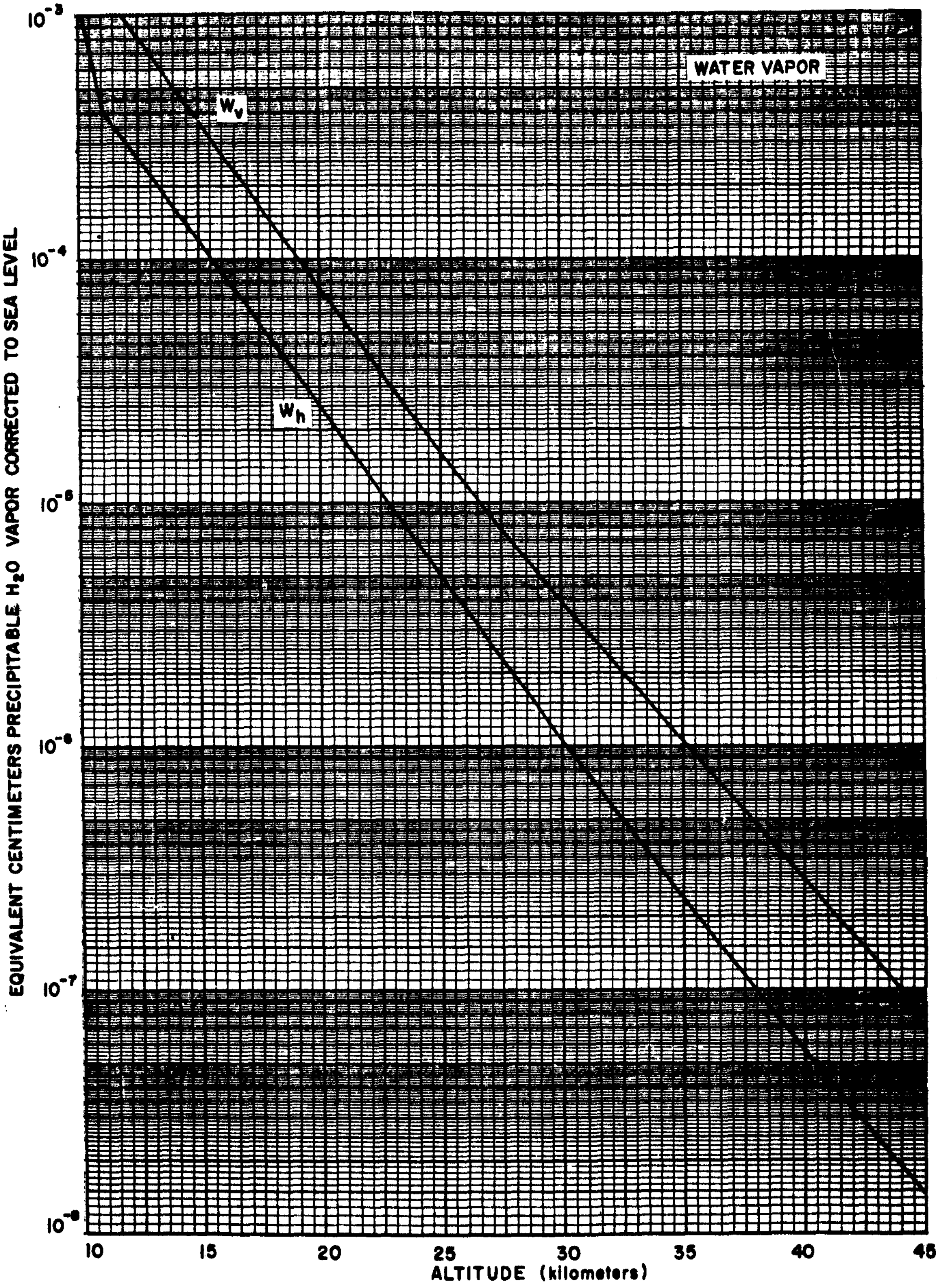
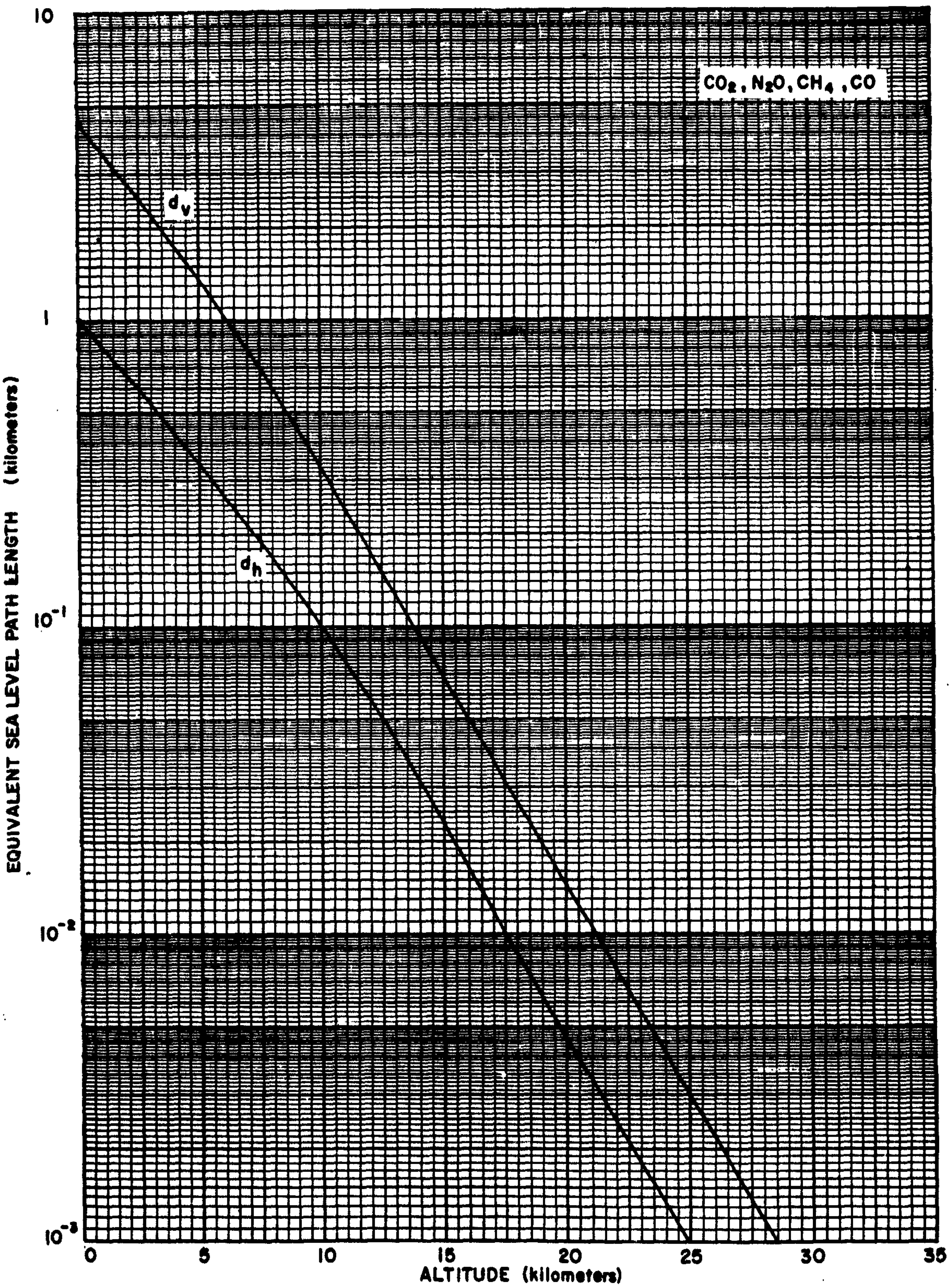

WATER VAPOR
W_v
W_h
EQUIVALENT CENTIMETERS PRECIPITABLE H_2O VAPOR CORRECTED TO SEA LEVEL
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
10^{-8}
10
15
20
25
30
35
40
45
ALTITUDE (kilometers)

Figure 6



10
1
10^{-1}
10^{-2}
10^{-3}
EQUIVALENT SEA LEVEL PATH LENGTH (kilometers)
CO_2, N_2O, CH_4, CO
d_v
d_h
0
5
10
15
20
25
30
35
ALTITUDE (kilometers)

Figure 7

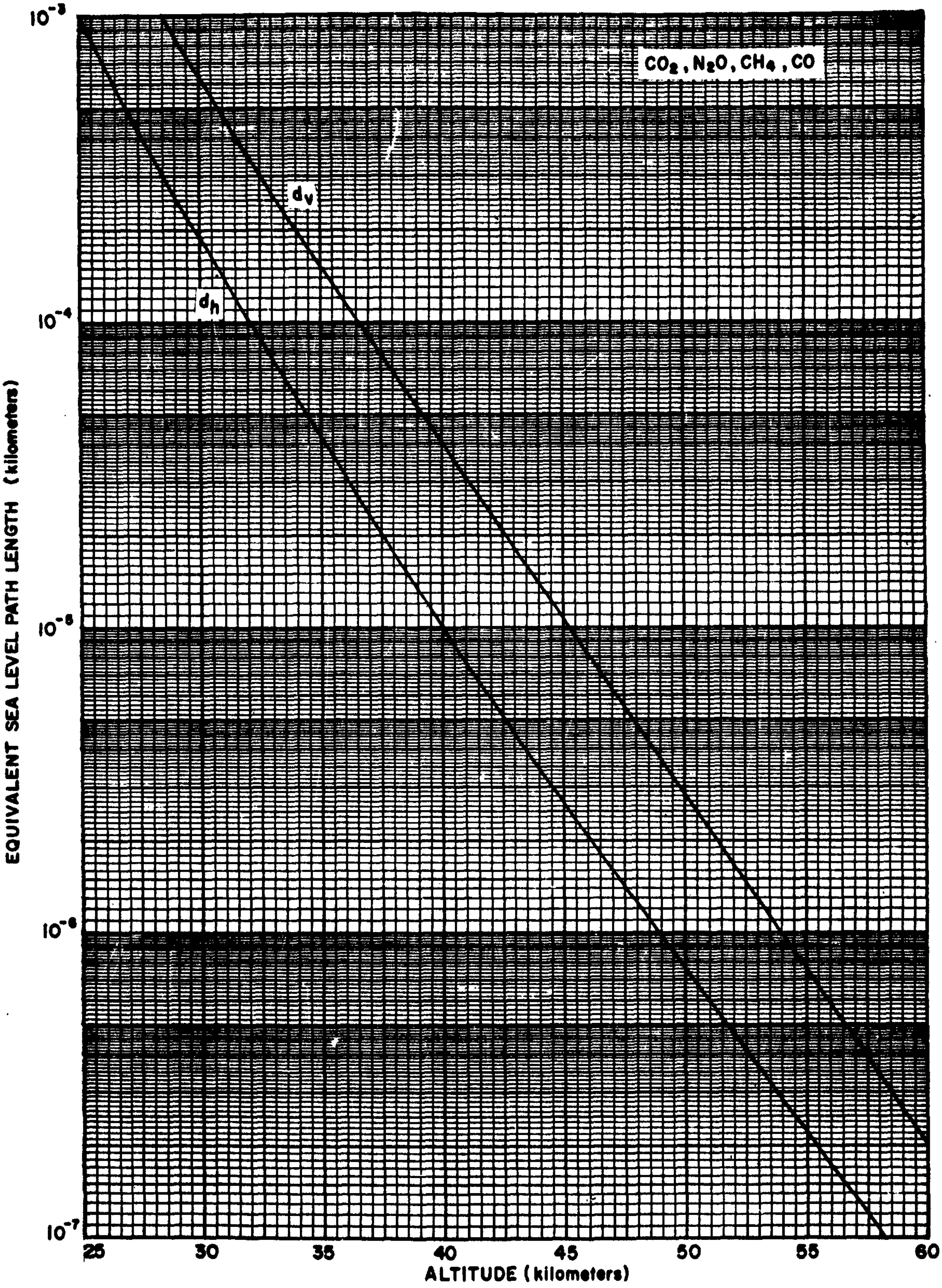

CO$_2$, N$_2$O, CH$_4$, CO
d_v
d_h
EQUIVALENT SEA LEVEL PATH LENGTH (kilometers)
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
25
30
35
40
45
50
55
60
ALTITUDE (kilometers)

Figure 8

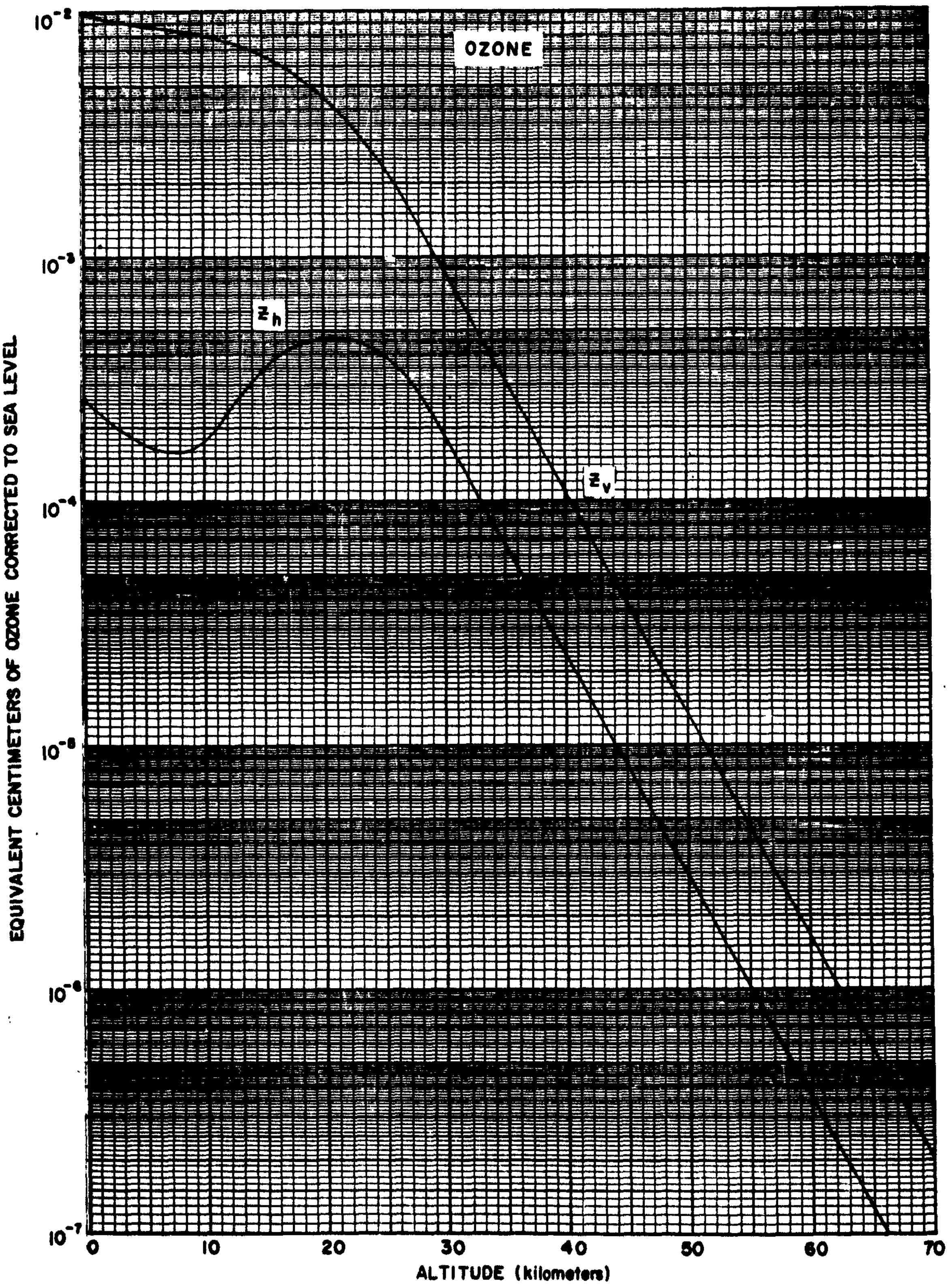
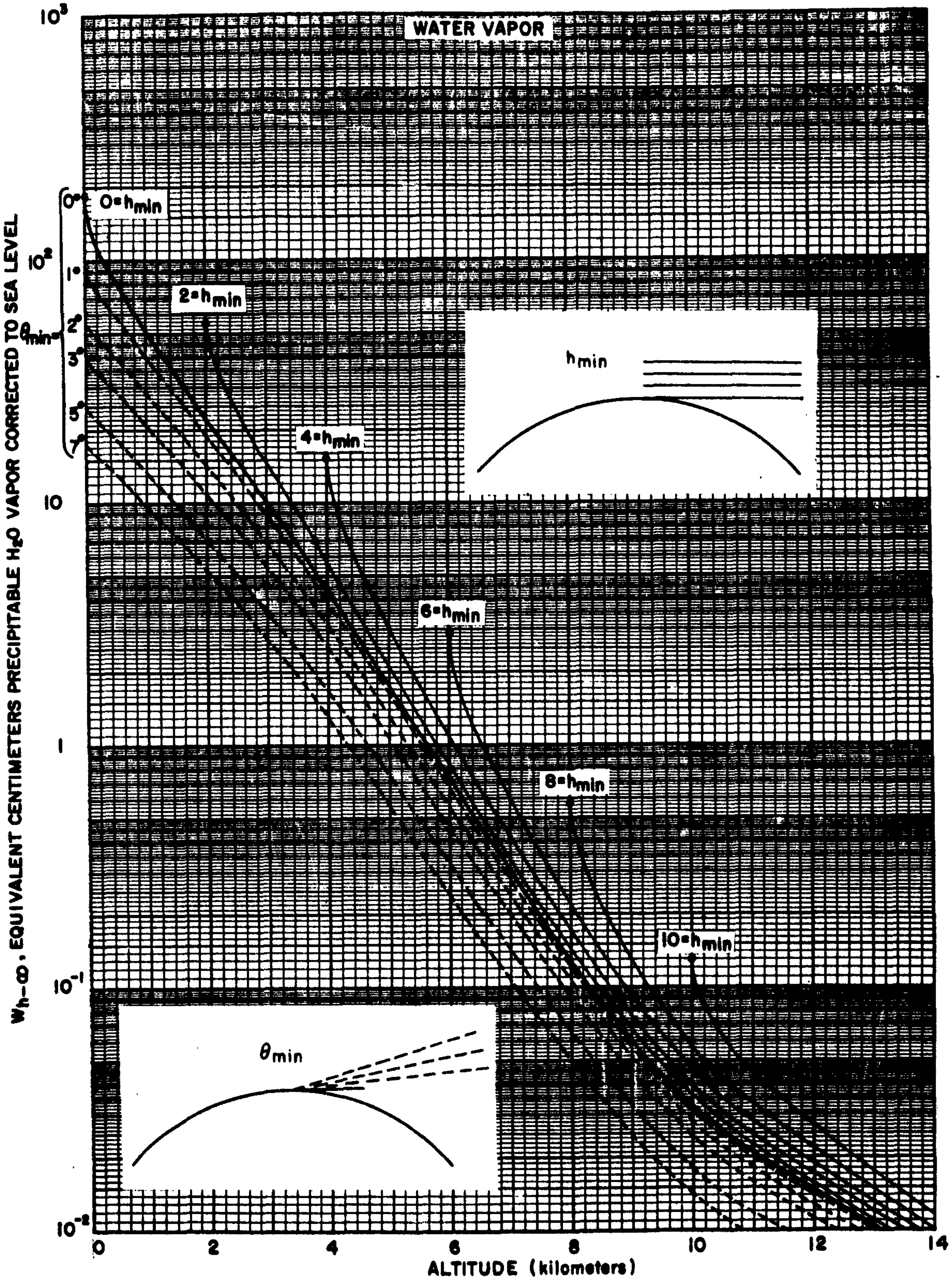

OZONE
Z_h
Z_v
EQUIVALENT CENTIMETERS OF OZONE CORRECTED TO SEA LEVEL
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
0
10
20
30
40
50
60
70
ALTITUDE (kilometers)

Figure 9



WATER VAPOR
$W_{h-\infty}$, EQUIVALENT CENTIMETERS PRECIPITABLE H_2O VAPOR CORRECTED TO SEA LEVEL
10^3
10^2
10
1
10^{-1}
10^{-2}
0 = h_min
2 = h_min
4 = h_min
6 = h_min
8 = h_min
10 = h_min
θ_min =
0°
1°
2°
3°
5°
7°
h_min
θ_min
0
2
4
6
8
10
12
14
ALTITUDE (kilometers)

Figure 10

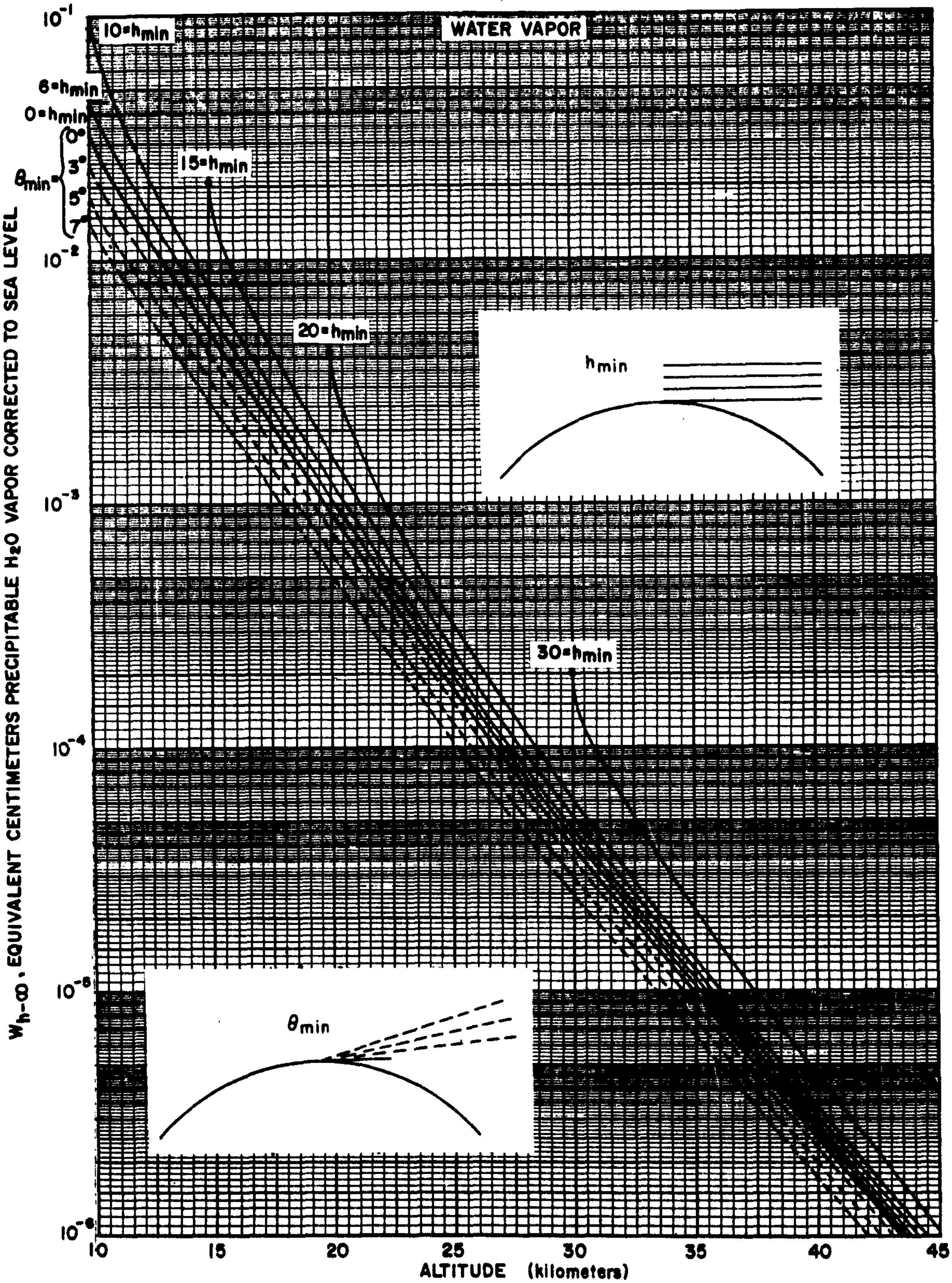

WATER VAPOR
10=hmin
6=hmin
0=hmin
θmin=
0°
3°
5°
7°
15=hmin
20=hmin
30=hmin
hmin
θmin
10⁻¹
10⁻²
10⁻³
10⁻⁴
10⁻⁵
10⁻⁶
W h-∞, EQUIVALENT CENTIMETERS PRECIPITABLE H₂O VAPOR CORRECTED TO SEA LEVEL
10
15
20
25
30
35
40
45
ALTITUDE (kilometers)

Figure 11

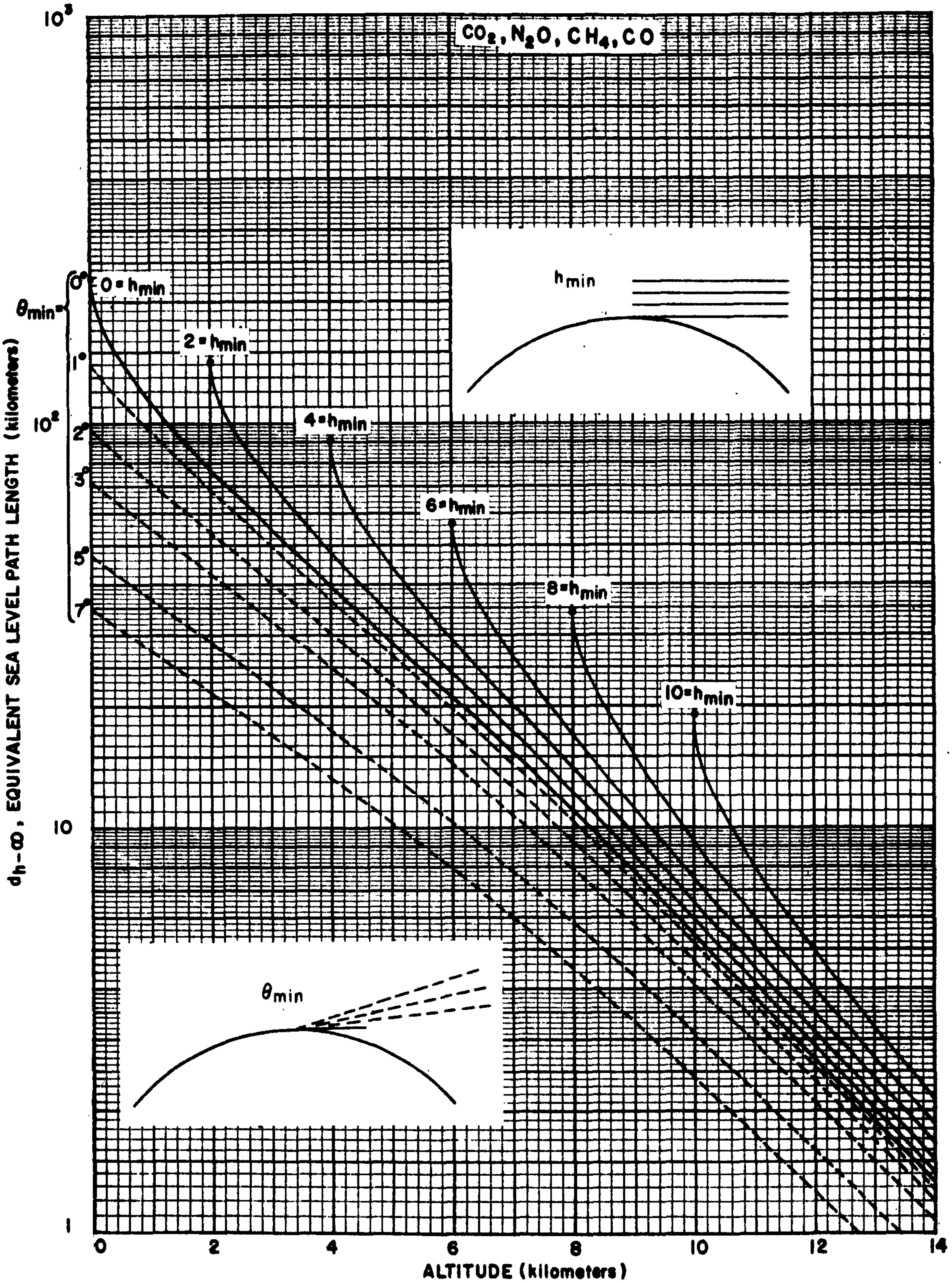
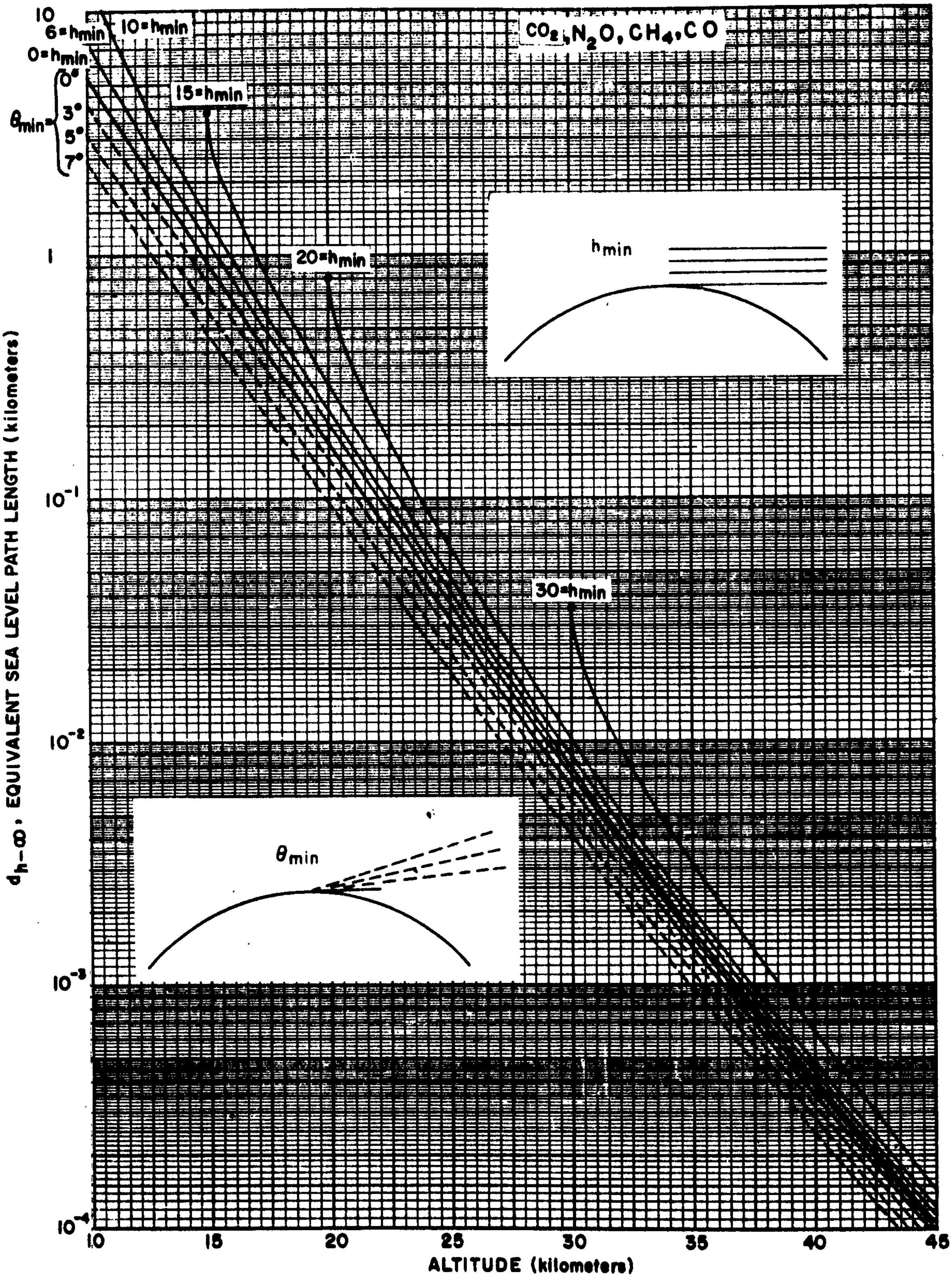

CO_2, N_2O, CH_4, CO
h_{min}
θ_{min}
θ_{min}=
0°
1°
2°
3°
5°
7°
0 = h_{min}
2 = h_{min}
4 = h_{min}
6 = h_{min}
8 = h_{min}
10 = h_{min}
10^3
10^2
10
1
0
2
4
6
8
10
12
14
ALTITUDE (kilometers)
$d_{h-\infty}$, EQUIVALENT SEA LEVEL PATH LENGTH (kilometers)

Figure 12



10
6=hmin
10=hmin
0=hmin
θmin=
0°
3°
5°
7°
15=hmin
CO2, N2O, CH4, CO
hmin
20=hmin
1
10⁻¹
30=hmin
10⁻²
θmin
10⁻³
10⁻⁴
10
15
20
25
30
35
40
45
ALTITUDE (kilometers)
dh-∞, EQUIVALENT SEA LEVEL PATH LENGTH (kilometers)

Figure 13

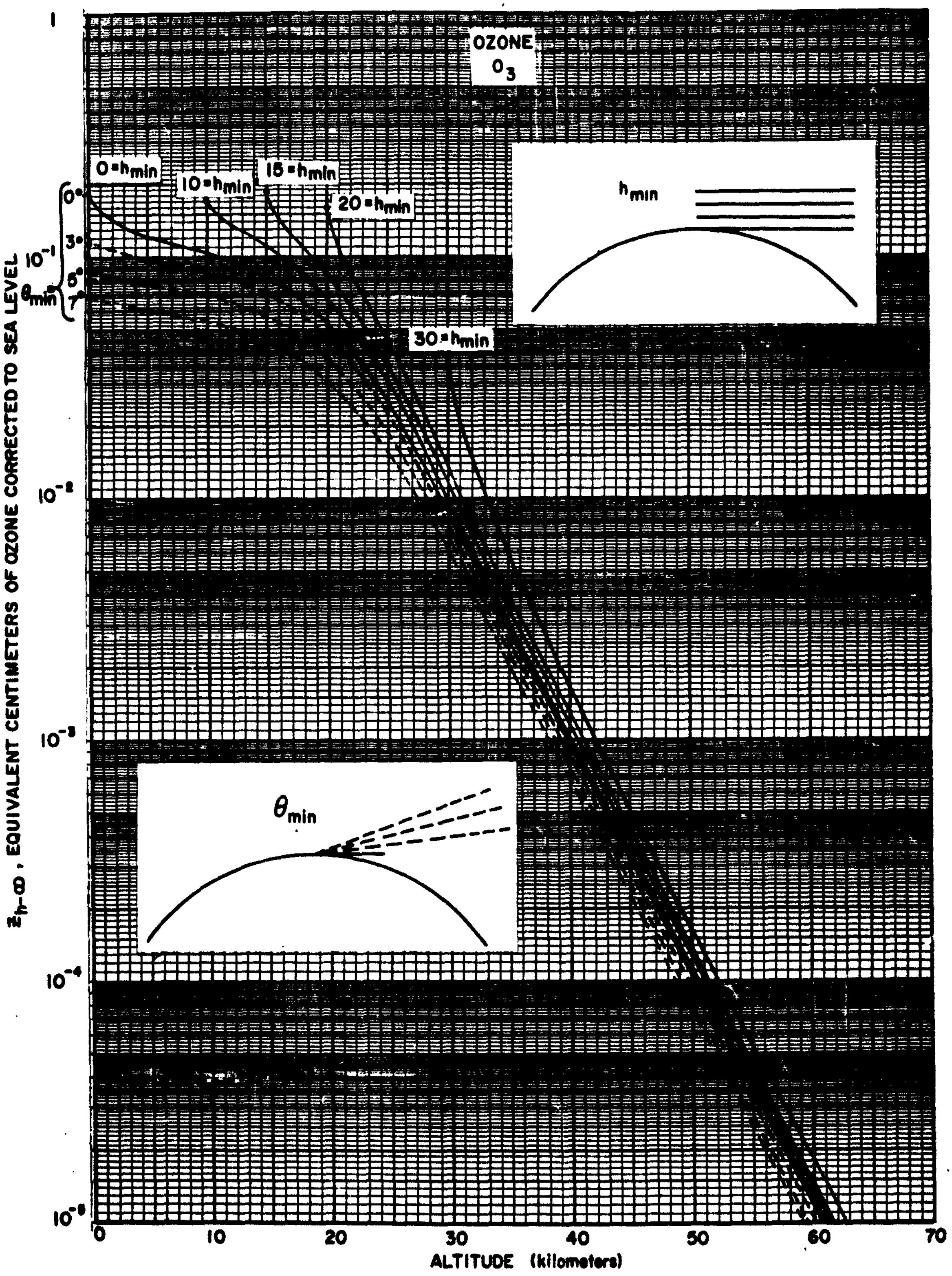

OZONE
O_3
$0 = h_{min}$
$10 = h_{min}$
$15 = h_{min}$
$20 = h_{min}$
$30 = h_{min}$
h_{min}
θ_{min}
0°
3°
5°
7°
θ_{min}
1
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
$z_{h-\infty}$, EQUIVALENT CENTIMETERS OF OZONE CORRECTED TO SEA LEVEL
0
10
20
30
40
50
60
70
ALTITUDE (kilometers)

Figure 14

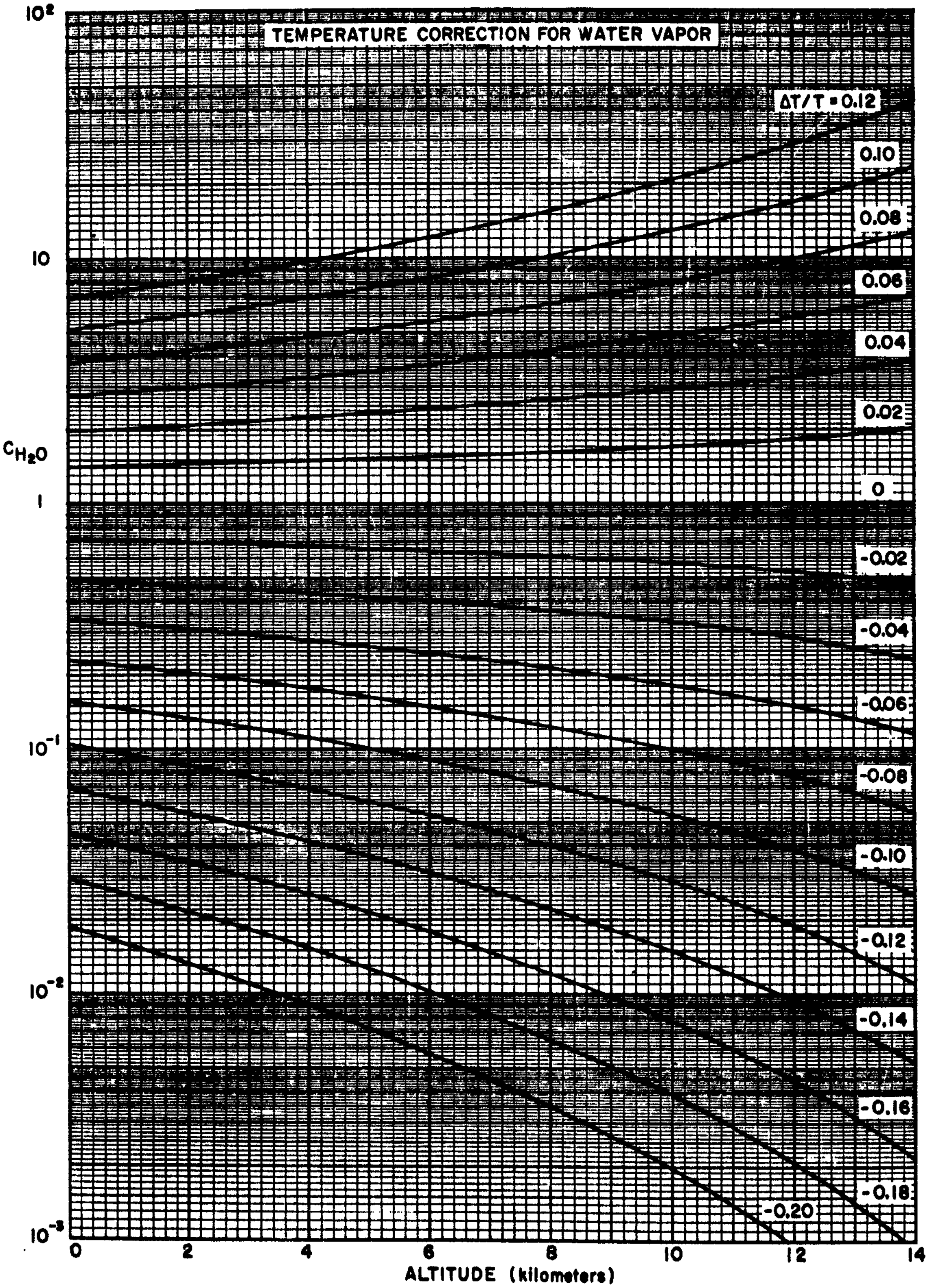

TEMPERATURE CORRECTION FOR WATER VAPOR
C_{H_2O}
10^2
10
1
10^{-1}
10^{-2}
10^{-3}
ΔT/T = 0.12
0.10
0.08
0.06
0.04
0.02
0
-0.02
-0.04
-0.06
-0.08
-0.10
-0.12
-0.14
-0.16
-0.18
-0.20
0
2
4
6
8
10
12
14
ALTITUDE (kilometers)

Figure 15

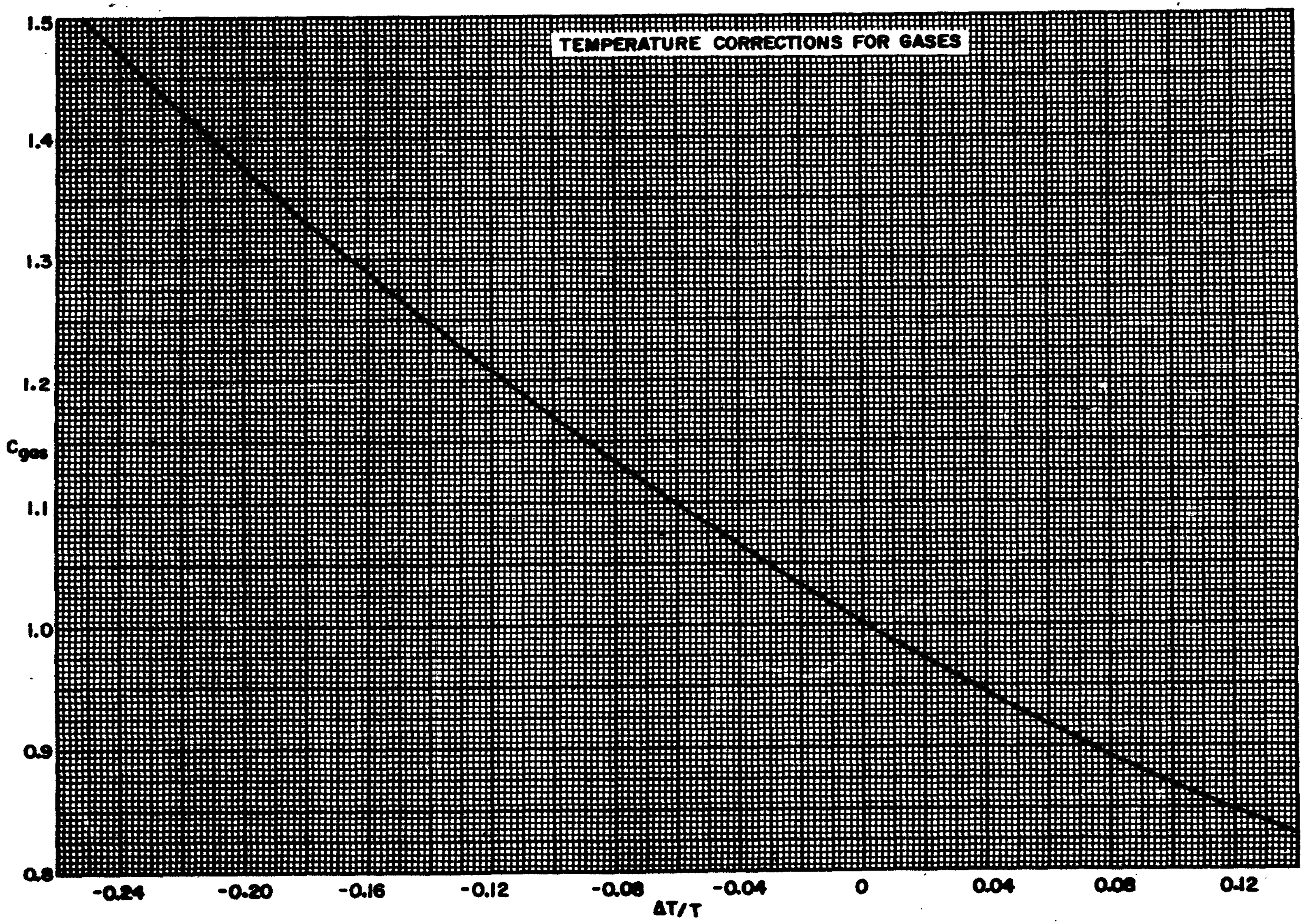

TEMPERATURE CORRECTIONS FOR GASES
C_{gas}
1.5
1.4
1.3
1.2
1.1
1.0
0.9
0.8
-0.24
-0.20
-0.16
-0.12
-0.08
-0.04
0
0.04
0.08
0.12
ΔT/T

Figure 16

MODEL ATMOSPHERE, WATER VAPOR VERSUS ALTITUDE

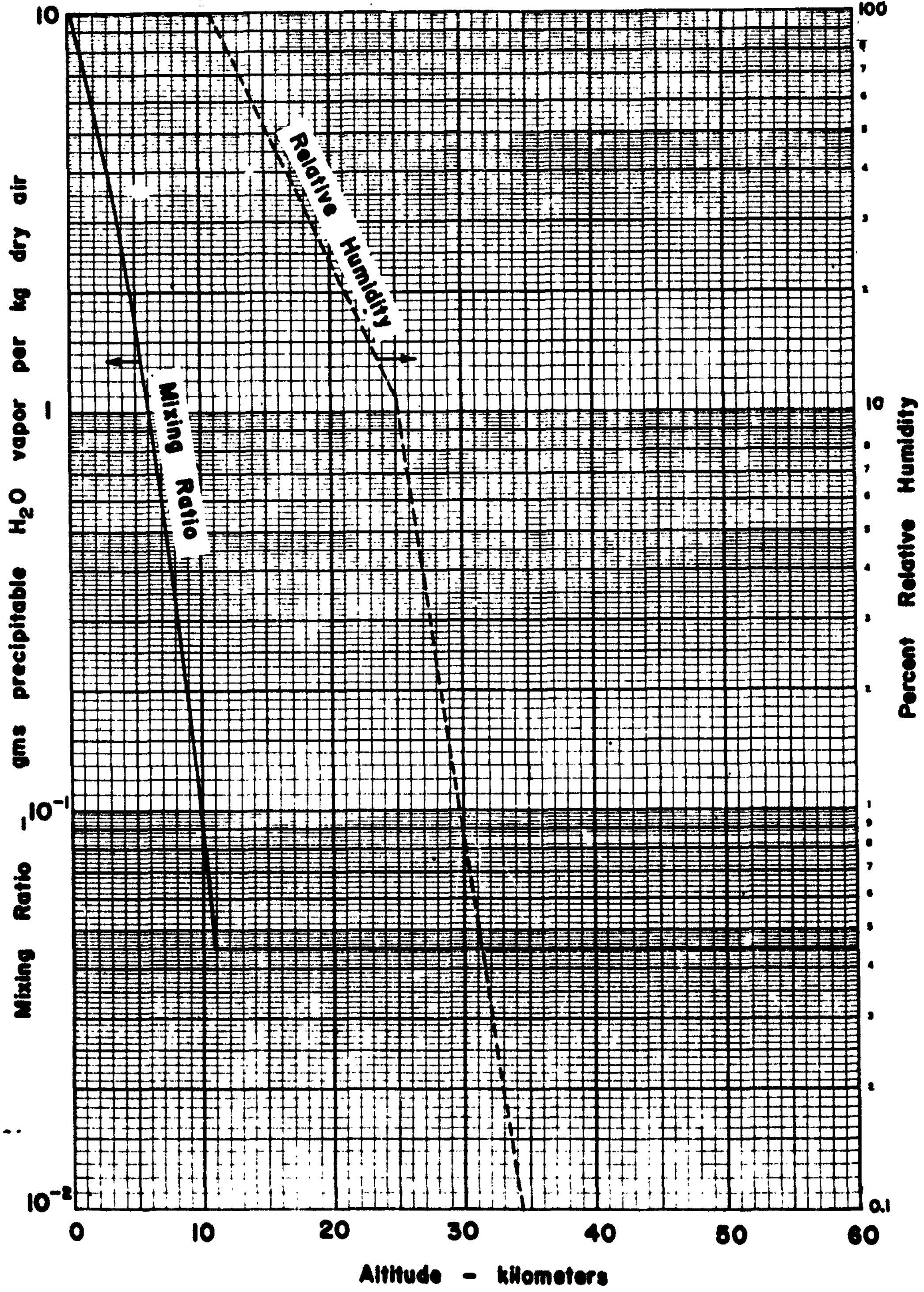


Figure 17

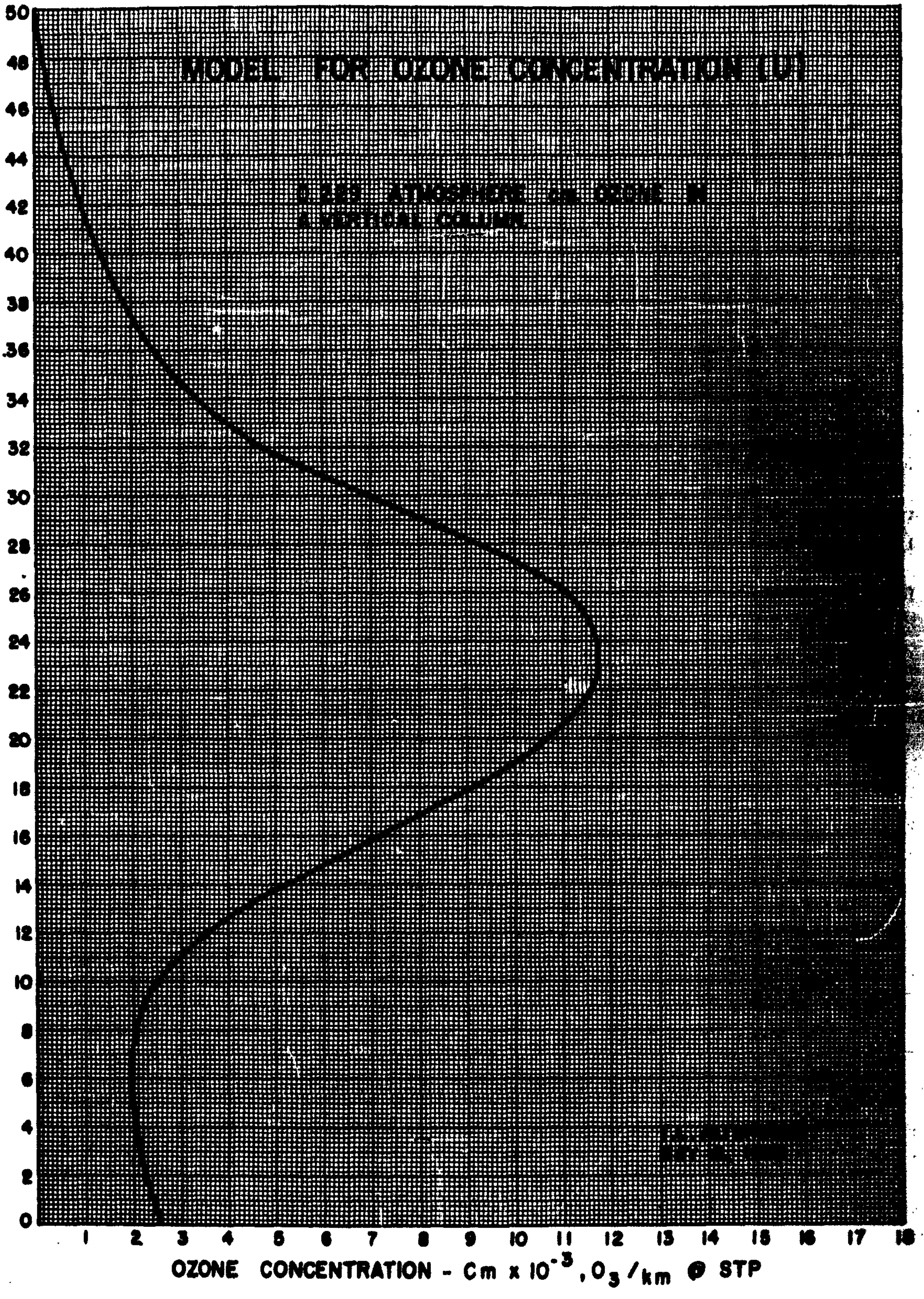

MODEL FOR OZONE CONCENTRATION (U)
0.223 ATMOSPHERE cm OZONE IN A VERTICAL COLUMN
50
48
46
44
42
40
38
36
34
32
30
28
26
24
22
20
18
16
14
12
10
8
6
4
2
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
OZONE CONCENTRATION - Cm x 10^{-3}, O$_3$/km @ STP

Figure 18

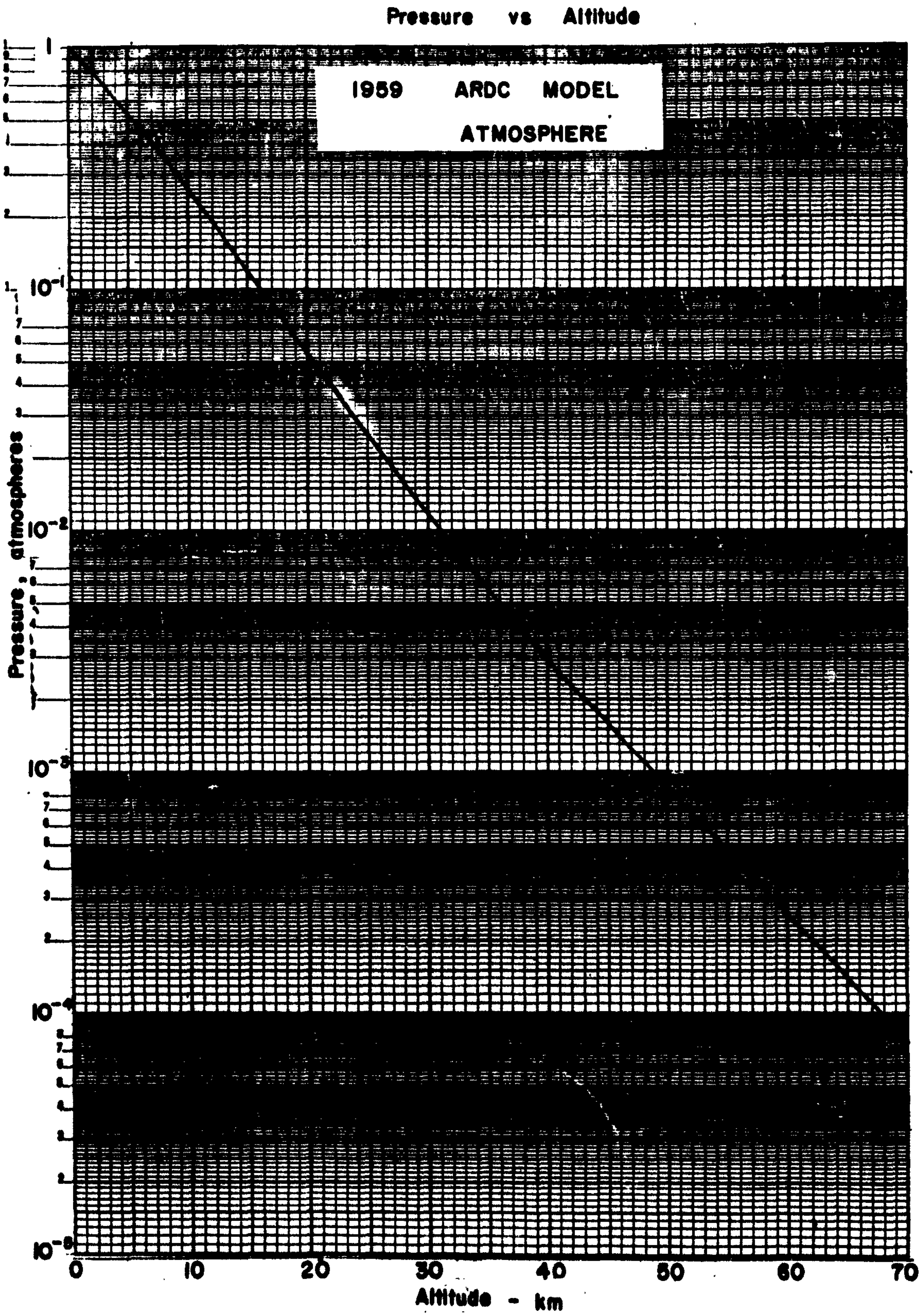

Pressure vs Altitude
1959 ARDC MODEL
ATMOSPHERE
Pressure, atmospheres
1
10^-1
10^-2
10^-3
10^-4
10^-5
0
10
20
30
40
50
60
70
Altitude - km

Figure 19

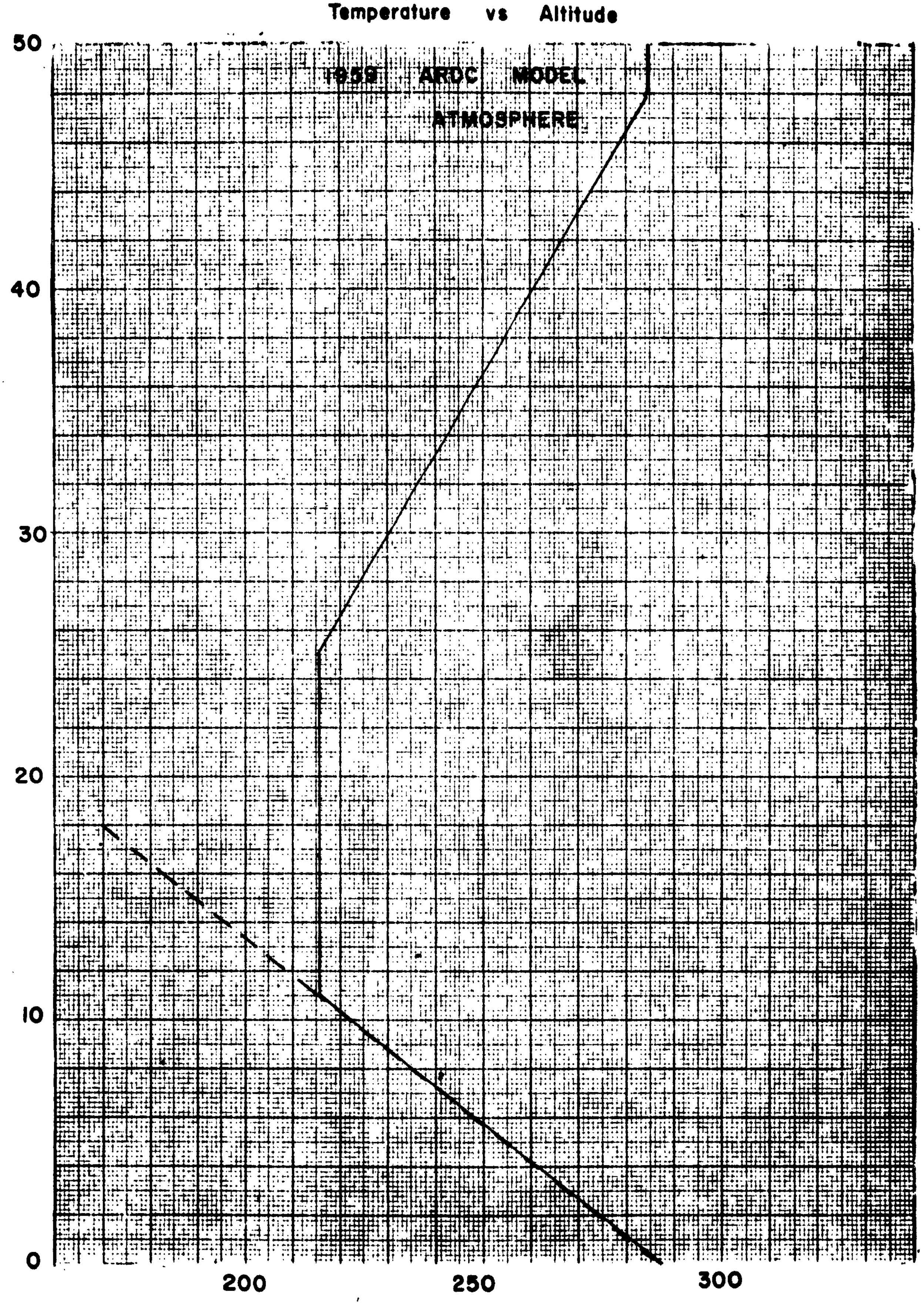
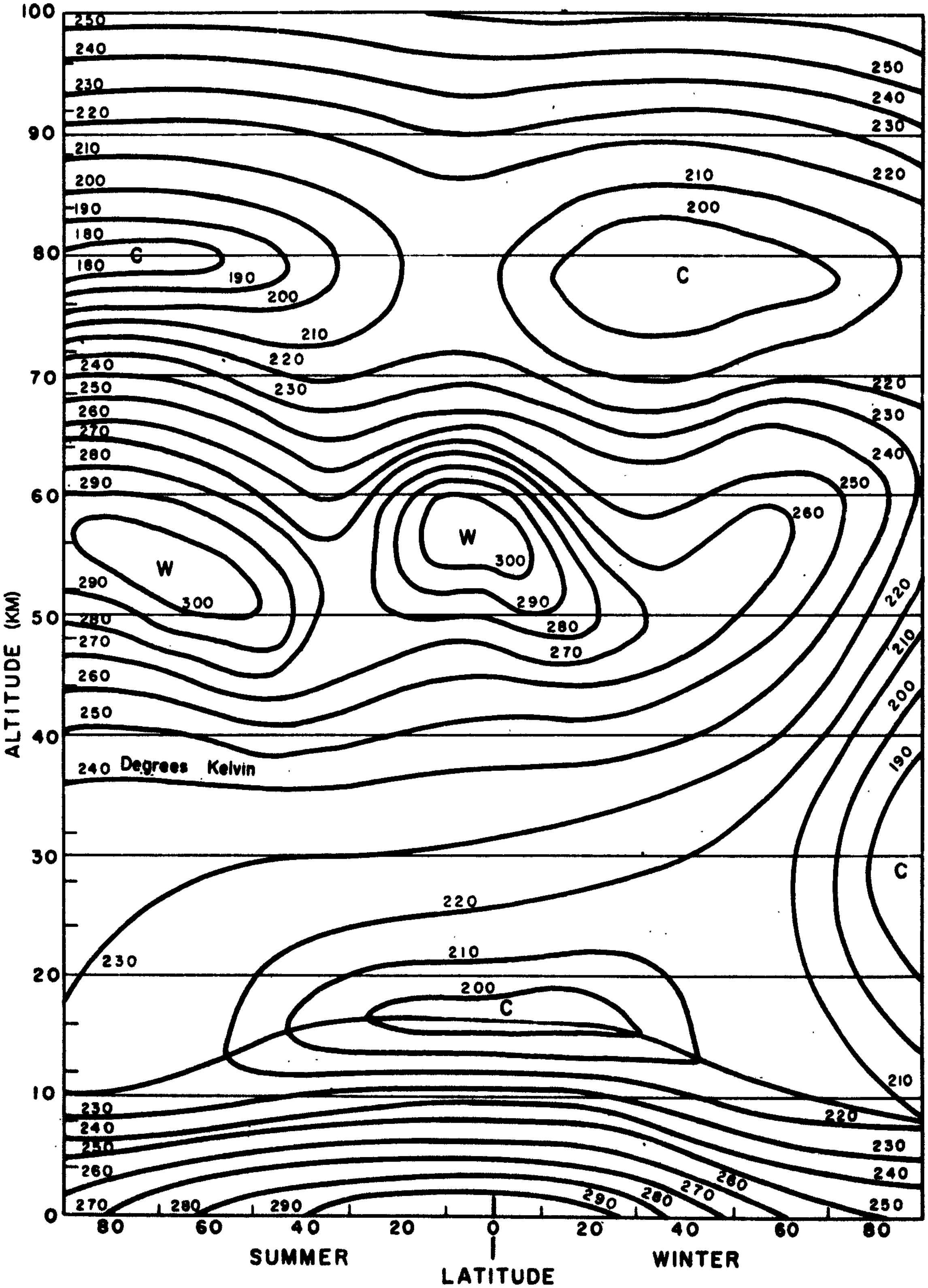

Temperature vs Altitude
1959 ARDC MODEL
ATMOSPHERE
Altitude - km
50
40
30
20
10
0
200
250
300
Temperature, °K

Figure 20



VERTICAL DISTRIBUTION OF ATMOSPHERIC TEMPERATURE AS A FUNCTION OF LATITUDE AND SEASONS

Figure 21

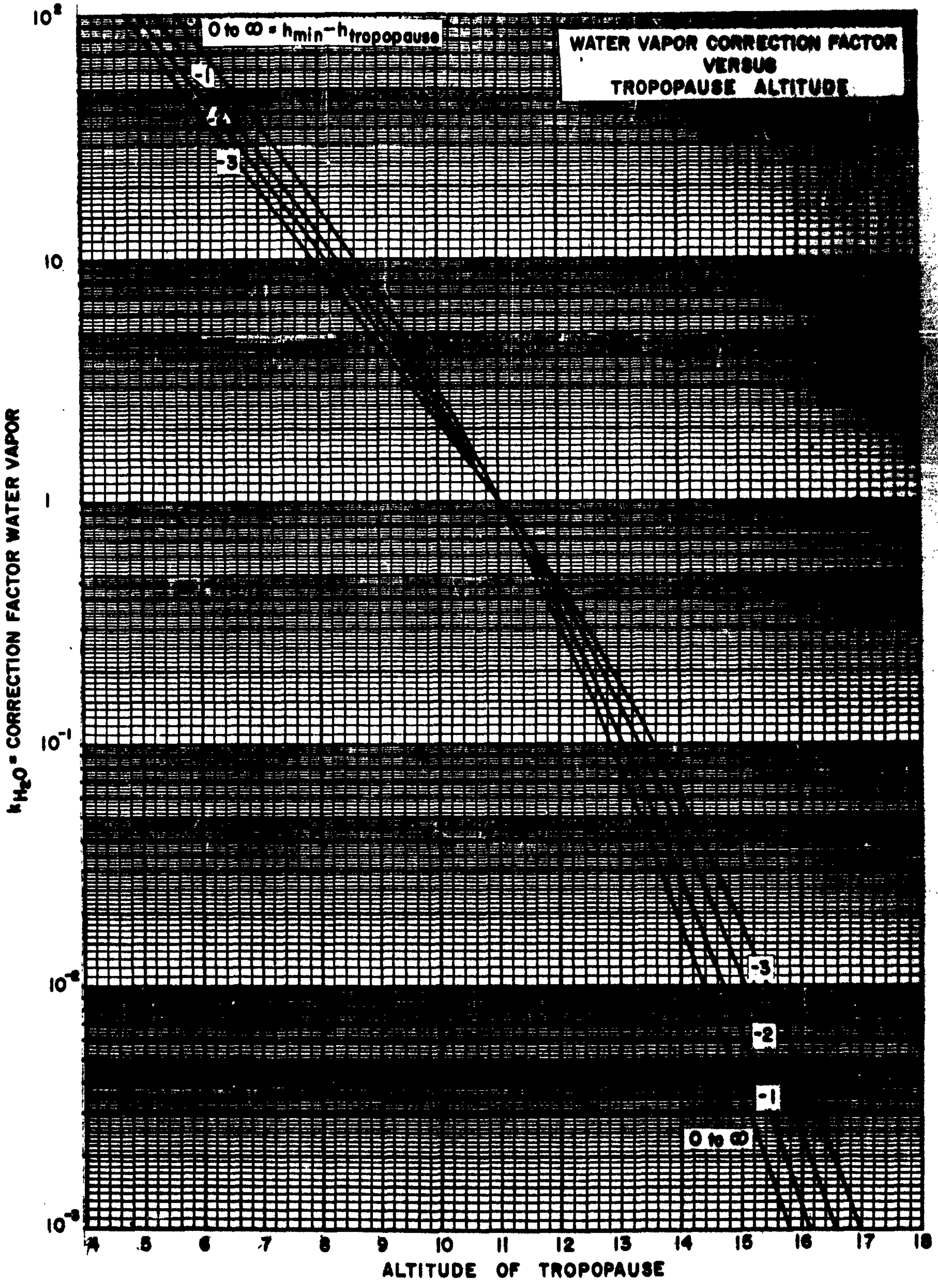

WATER VAPOR CORRECTION FACTOR VERSUS TROPOPAUSE ALTITUDE
0 to ∞ = h min − h tropopause
-1
-2
-3
-3
-2
-1
0 to ∞
k H2O = CORRECTION FACTOR WATER VAPOR
10²
10
1
10⁻¹
10⁻²
10⁻³
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
ALTITUDE OF TROPOPAUSE

Figure 22

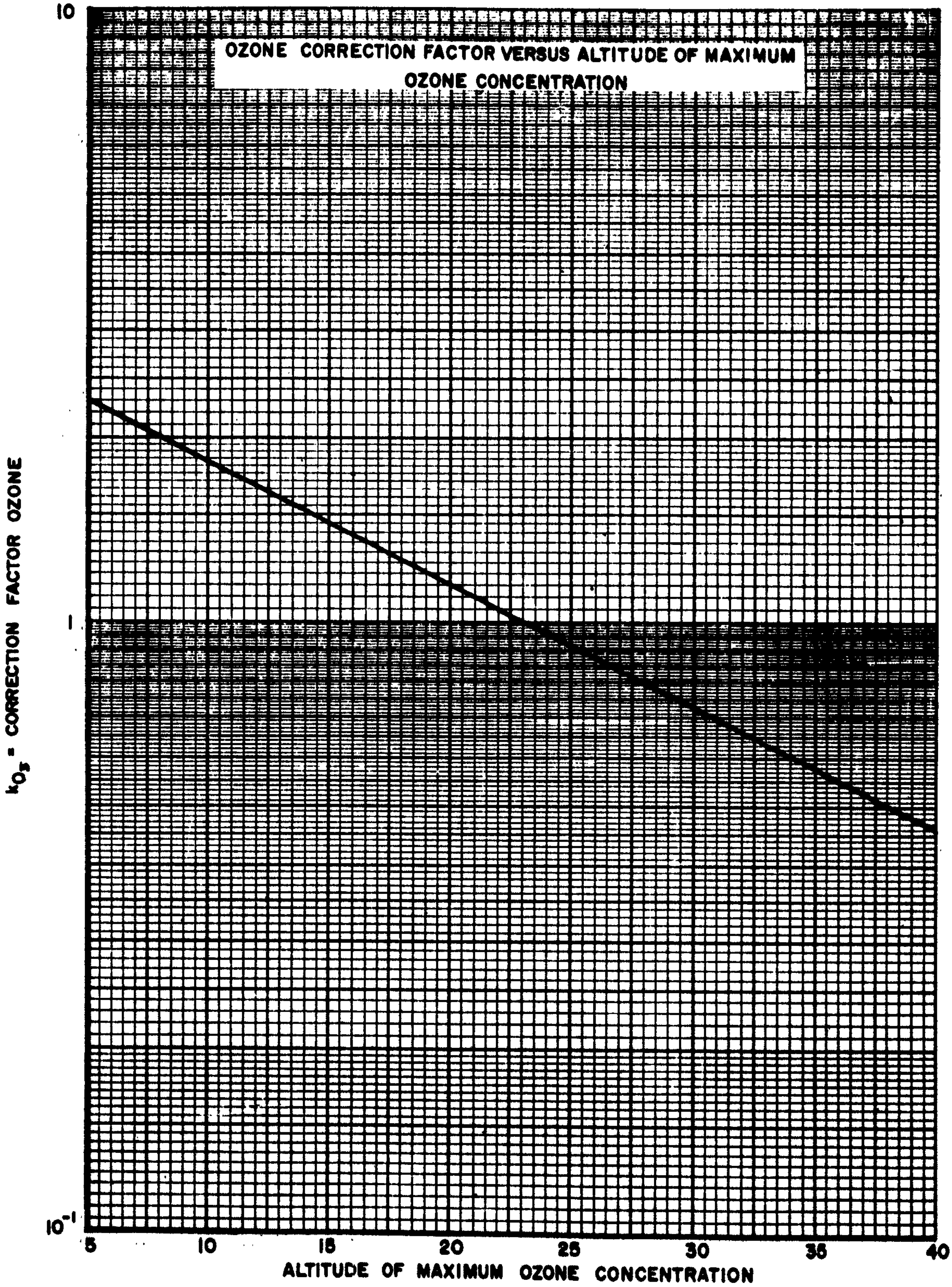

OZONE CORRECTION FACTOR VERSUS ALTITUDE OF MAXIMUM OZONE CONCENTRATION
10
1
10⁻¹
k_{O_3} = CORRECTION FACTOR OZONE
5
10
15
20
25
30
35
40
ALTITUDE OF MAXIMUM OZONE CONCENTRATION

Figure 23

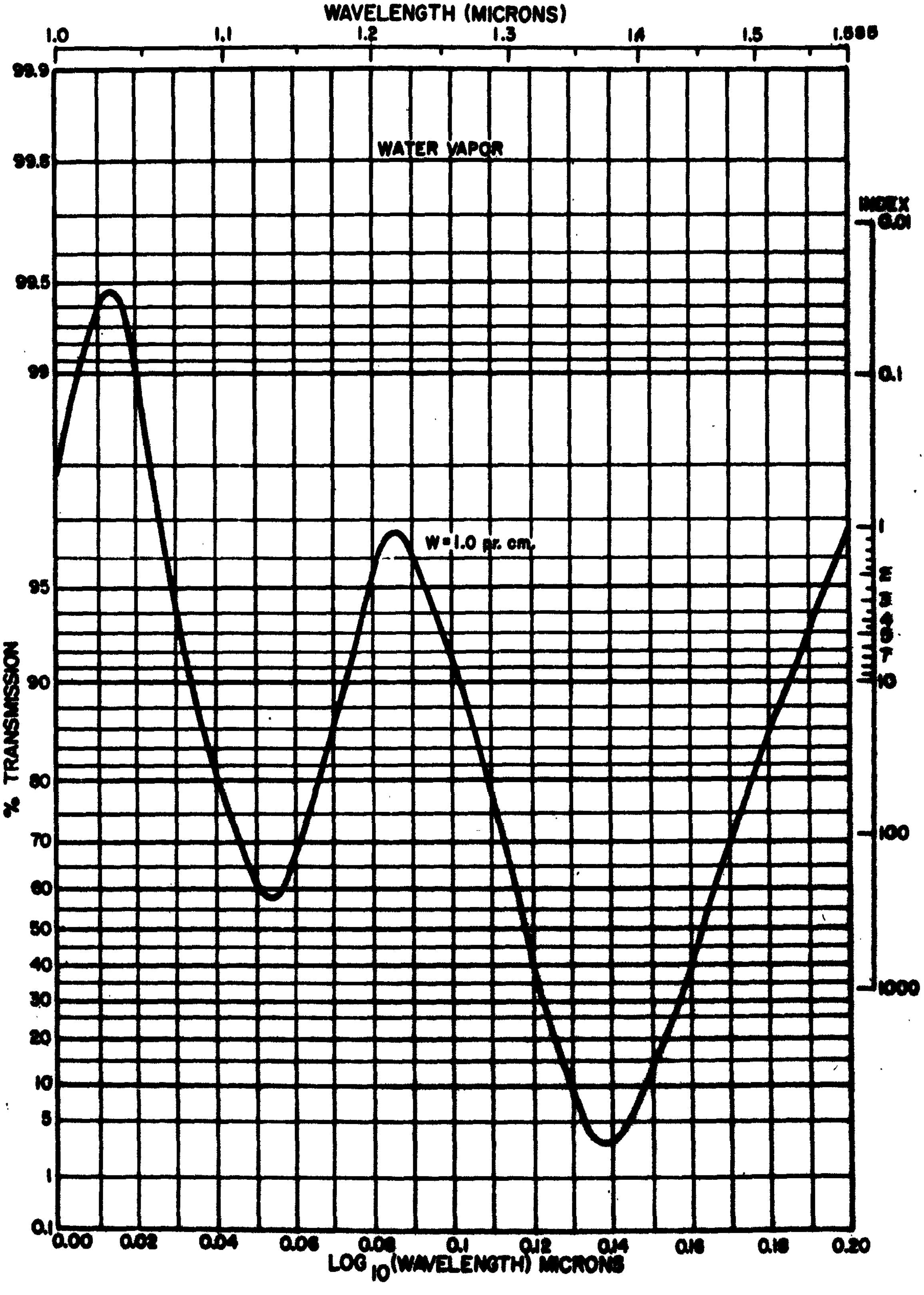

WAVELENGTH (MICRONS)
1.0
1.1
1.2
1.3
1.4
1.5
1.585
WATER VAPOR
W=1.0 pr. cm.
% TRANSMISSION
99.9
99.8
99.5
99
95
90
80
70
60
50
40
30
20
10
5
1
0.1
INDEX
0.01
0.1
1
2
3
4
5
7
10
100
1000
0.00
0.02
0.04
0.06
0.08
0.1
0.12
0.14
0.16
0.18
0.20
LOG 10 (WAVELENGTH) MICRONS

Figure 24

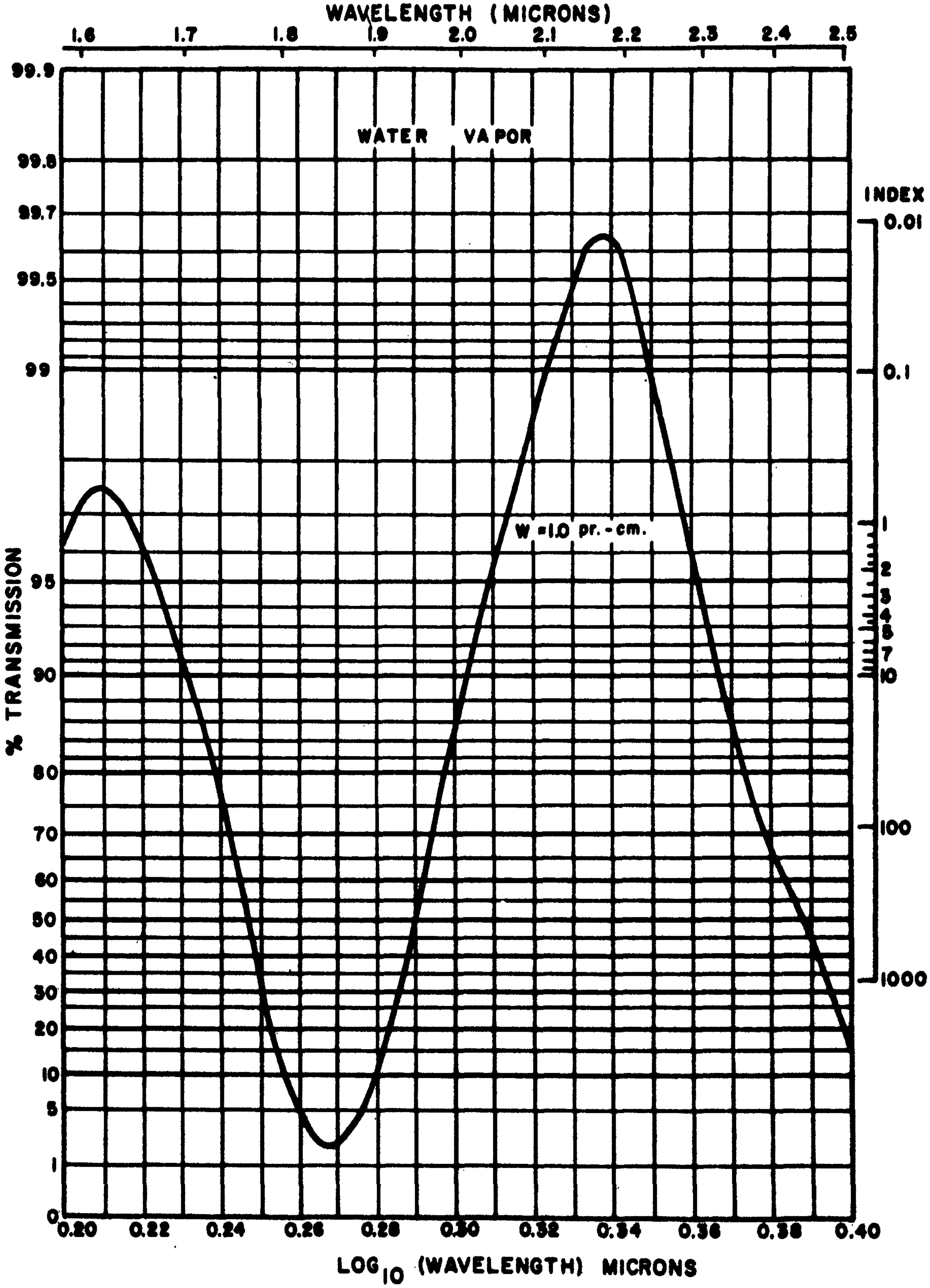

WAVELENGTH (MICRONS)
1.6
1.7
1.8
1.9
2.0
2.1
2.2
2.3
2.4
2.5
WATER VAPOR
W = 1.0 pr.-cm.
% TRANSMISSION
99.9
99.8
99.7
99.5
99
95
90
80
70
60
50
40
30
20
10
5
1
0
INDEX
0.01
0.1
1
2
3
4
5
7
10
100
1000
0.20
0.22
0.24
0.26
0.28
0.30
0.32
0.34
0.36
0.38
0.40
LOG$_{10}$ (WAVELENGTH) MICRONS

Figure 25

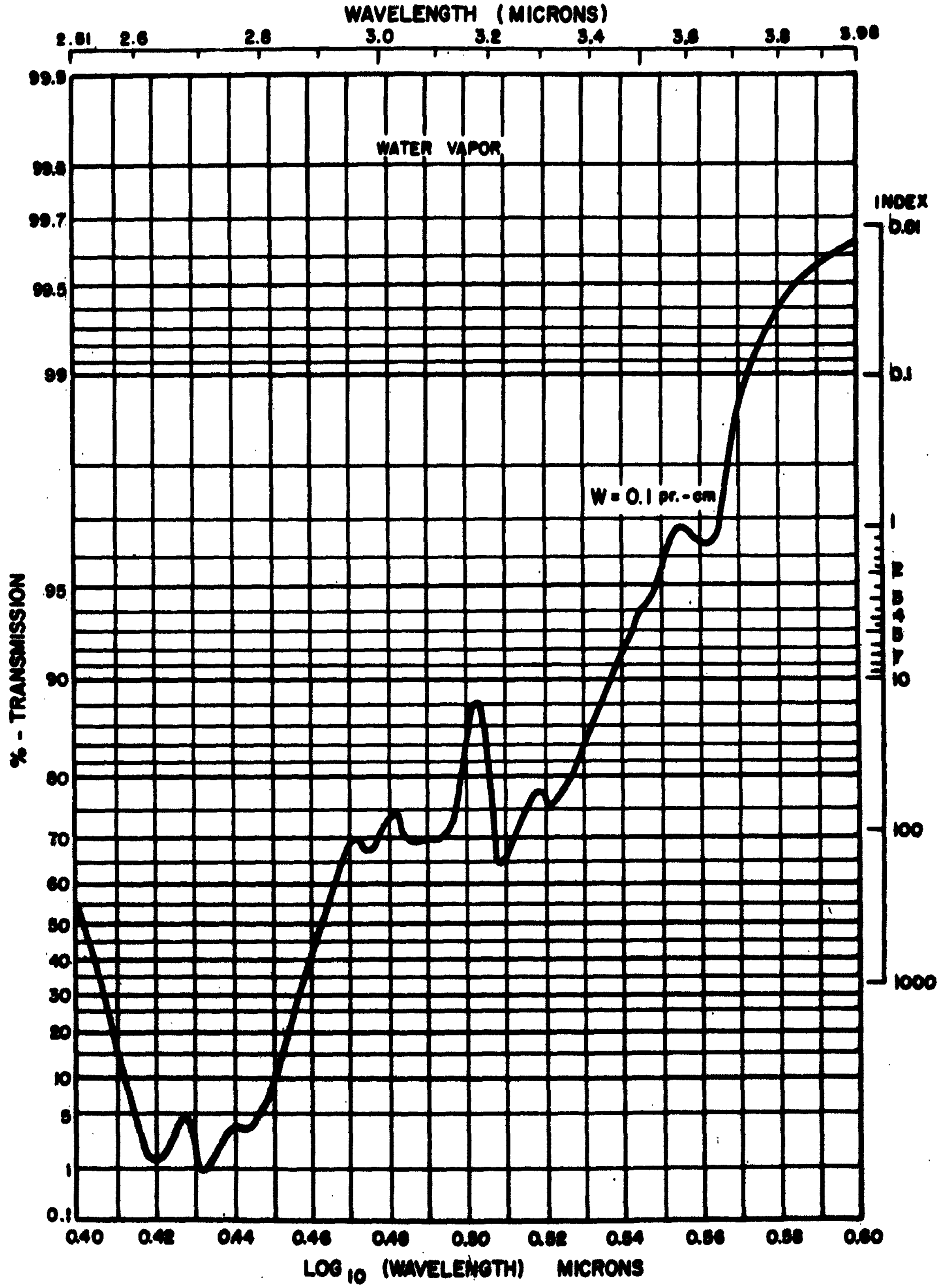

WAVELENGTH (MICRONS)
2.51
2.6
2.8
3.0
3.2
3.4
3.6
3.8
3.98
WATER VAPOR
W = 0.1 pr.-cm
INDEX
0.01
0.1
1
2
3
4
5
7
10
100
1000
% - TRANSMISSION
99.9
99.8
99.7
99.5
99
95
90
80
70
60
50
40
30
20
10
5
1
0.1
0.40
0.42
0.44
0.46
0.48
0.50
0.52
0.54
0.56
0.58
0.60
LOG 10 (WAVELENGTH) MICRONS

Figure 26

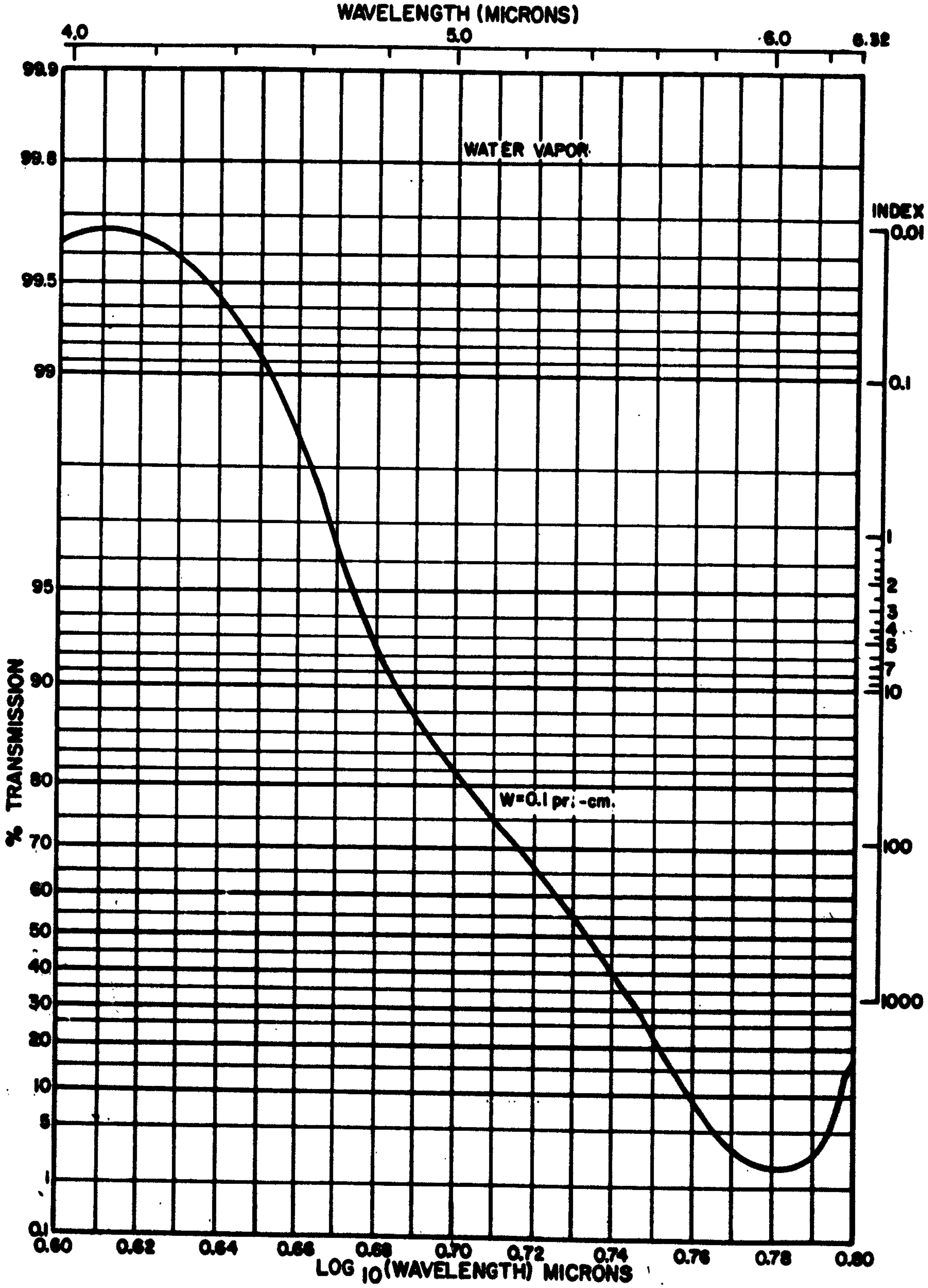

WAVELENGTH (MICRONS)
4.0
5.0
6.0
6.32
WATER VAPOR
INDEX
0.01
0.1
1
2
3
4
5
7
10
100
1000
W=0.1 pr. -cm.
% TRANSMISSION
99.9
99.8
99.5
99
95
90
80
70
60
50
40
30
20
10
5
1
0.1
0.60
0.62
0.64
0.66
0.68
0.70
0.72
0.74
0.76
0.78
0.80
LOG 10 (WAVELENGTH) MICRONS

Figure 27

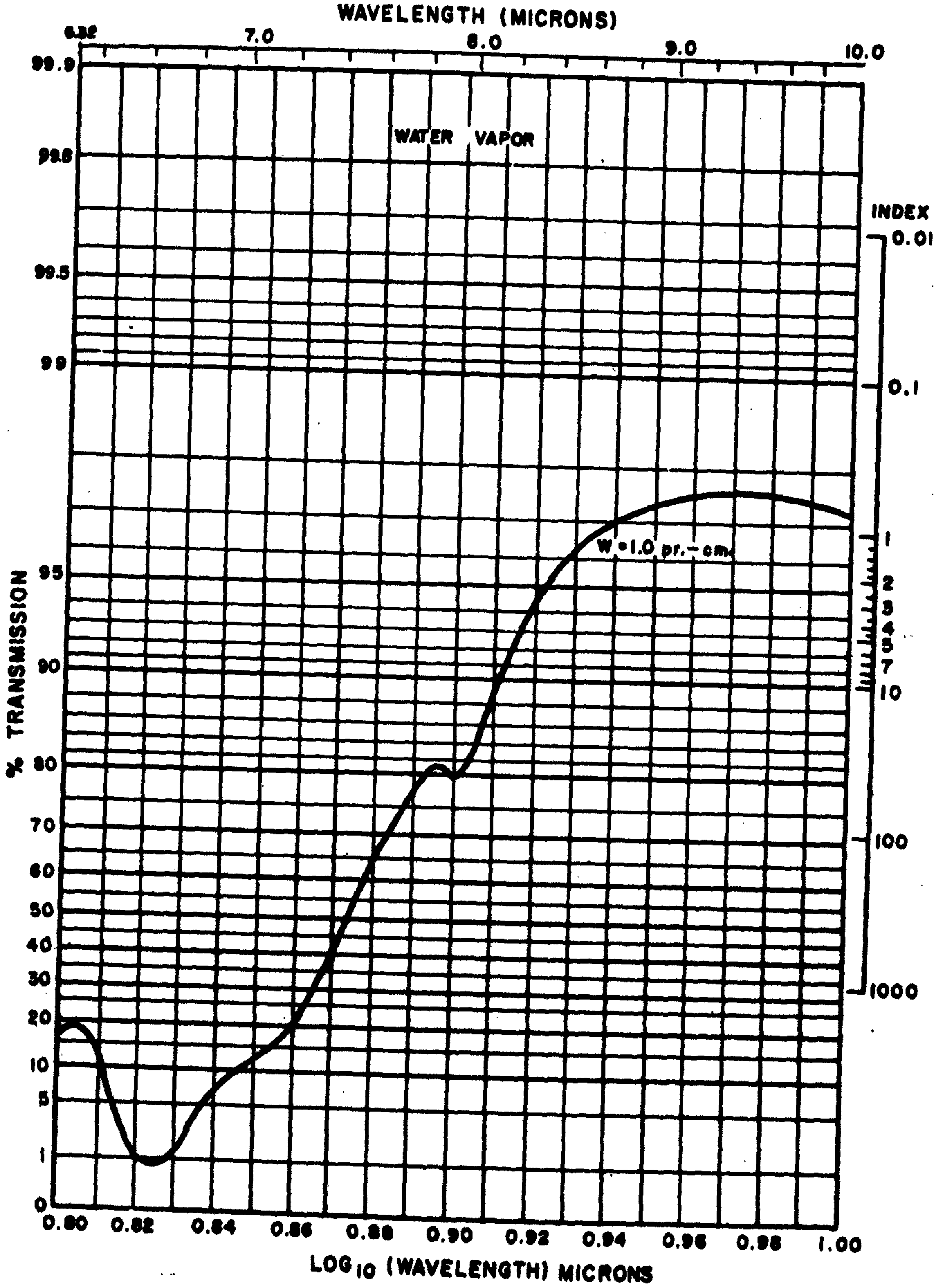

WAVELENGTH (MICRONS)
6.32
7.0
8.0
9.0
10.0
WATER VAPOR
INDEX
0.01
0.1
1
2
3
4
5
7
10
100
1000
W = 1.0 pr.-cm.
% TRANSMISSION
99.9
99.8
99.5
99
95
90
80
70
60
50
40
30
20
10
5
1
0
0.80
0.82
0.84
0.86
0.88
0.90
0.92
0.94
0.96
0.98
1.00
LOG10 (WAVELENGTH) MICRONS

Figure 28

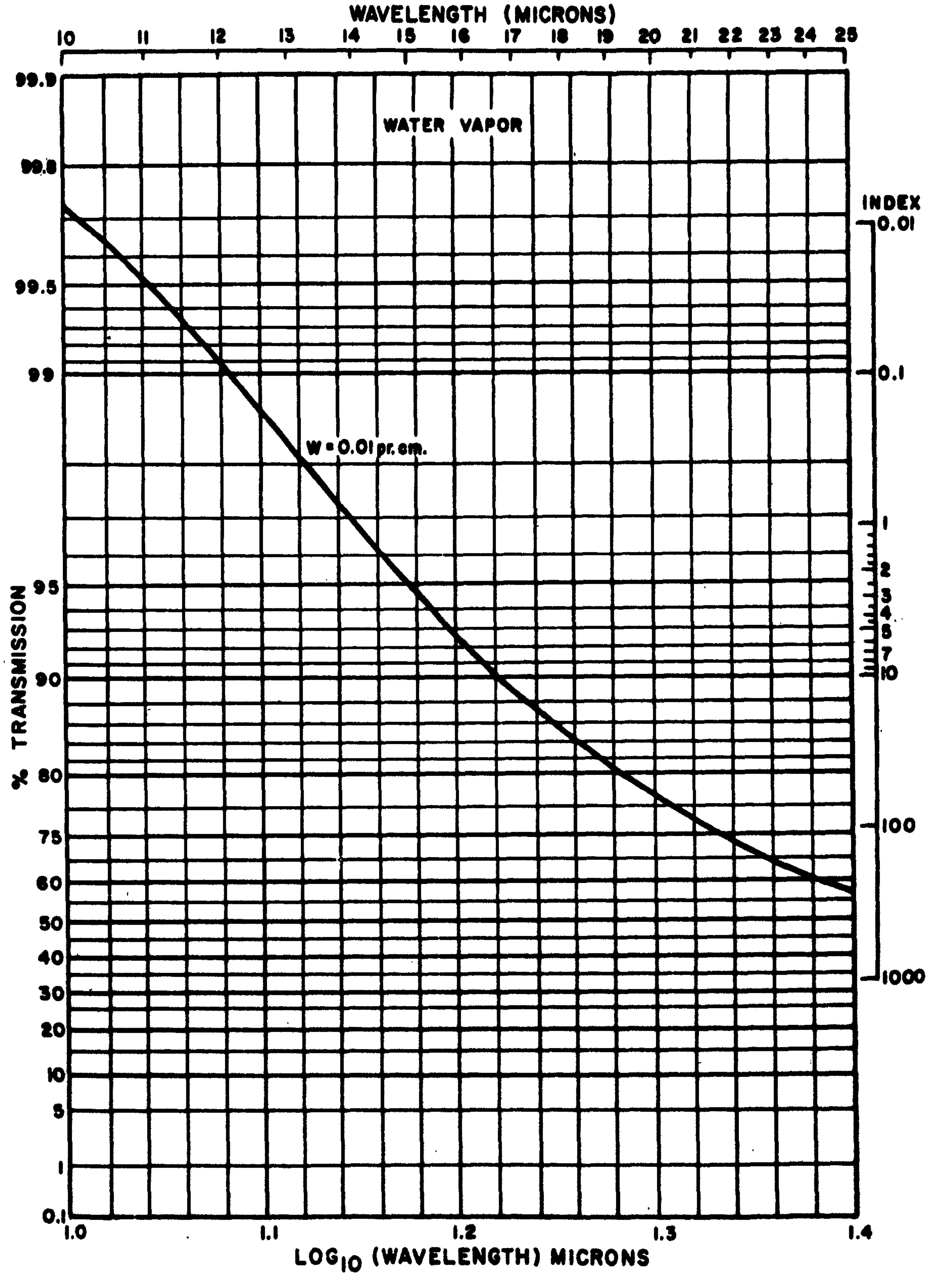

WAVELENGTH (MICRONS)
10
11
12
13
14
15
16
17
18
19
20
21
22
23
24
25
WATER VAPOR
W = 0.01 pr. cm.
INDEX
0.01
0.1
1
2
3
4
5
7
10
100
1000
% TRANSMISSION
99.9
99.8
99.5
99
95
90
80
75
60
50
40
30
20
10
5
1
0.1
1.0
1.1
1.2
1.3
1.4
LOG$_{10}$ (WAVELENGTH) MICRONS

Figure 29

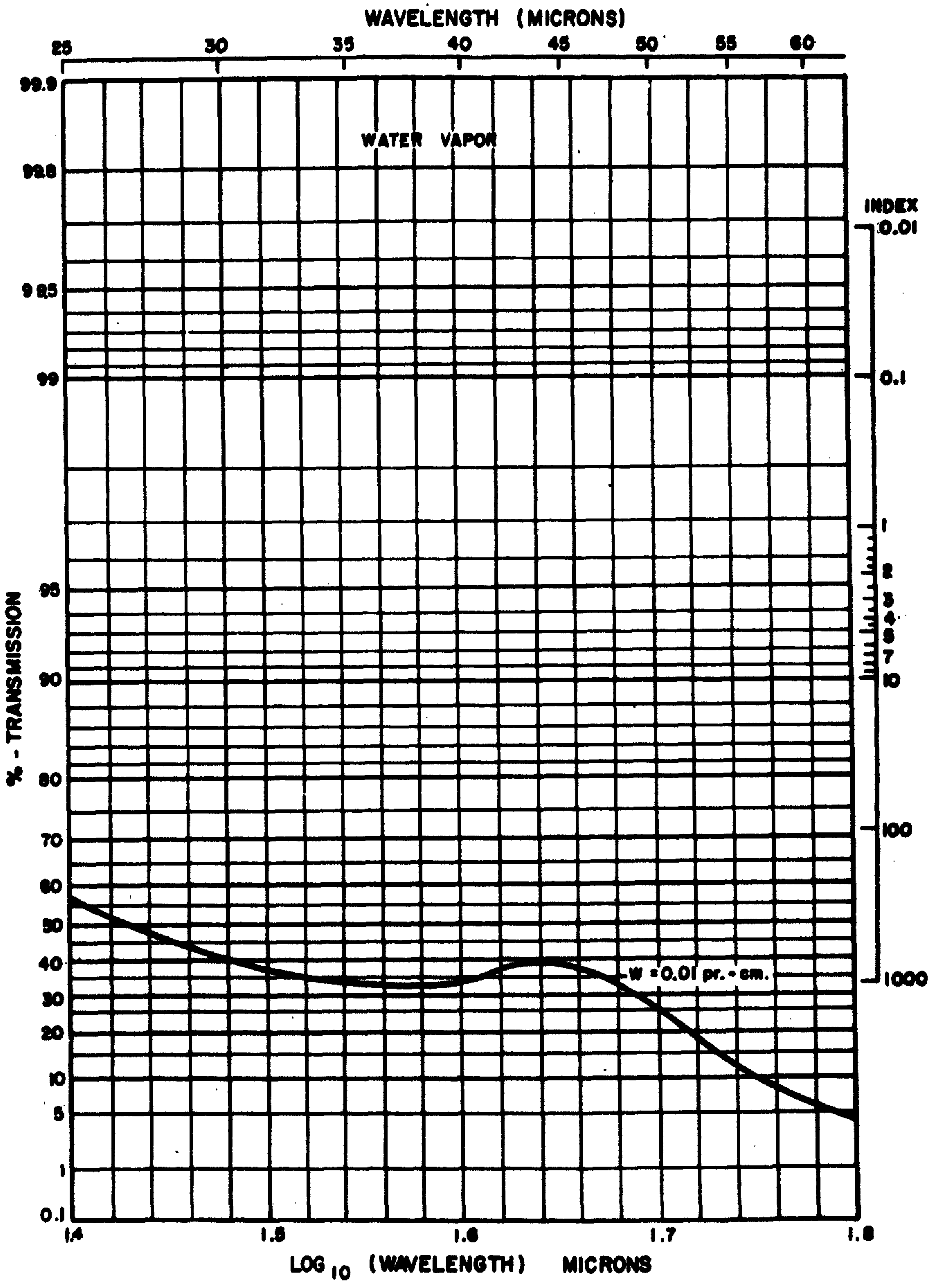

WAVELENGTH (MICRONS)
25
30
35
40
45
50
55
60
WATER VAPOR
99.9
99.8
99.5
99
95
90
80
70
60
50
40
30
20
10
5
1
0.1
% - TRANSMISSION
INDEX
0.01
0.1
1
2
3
4
5
7
10
100
1000
W = 0.01 pr.-cm.
1.4
1.5
1.6
1.7
1.8
LOG$_{10}$ (WAVELENGTH) MICRONS

Figure 30

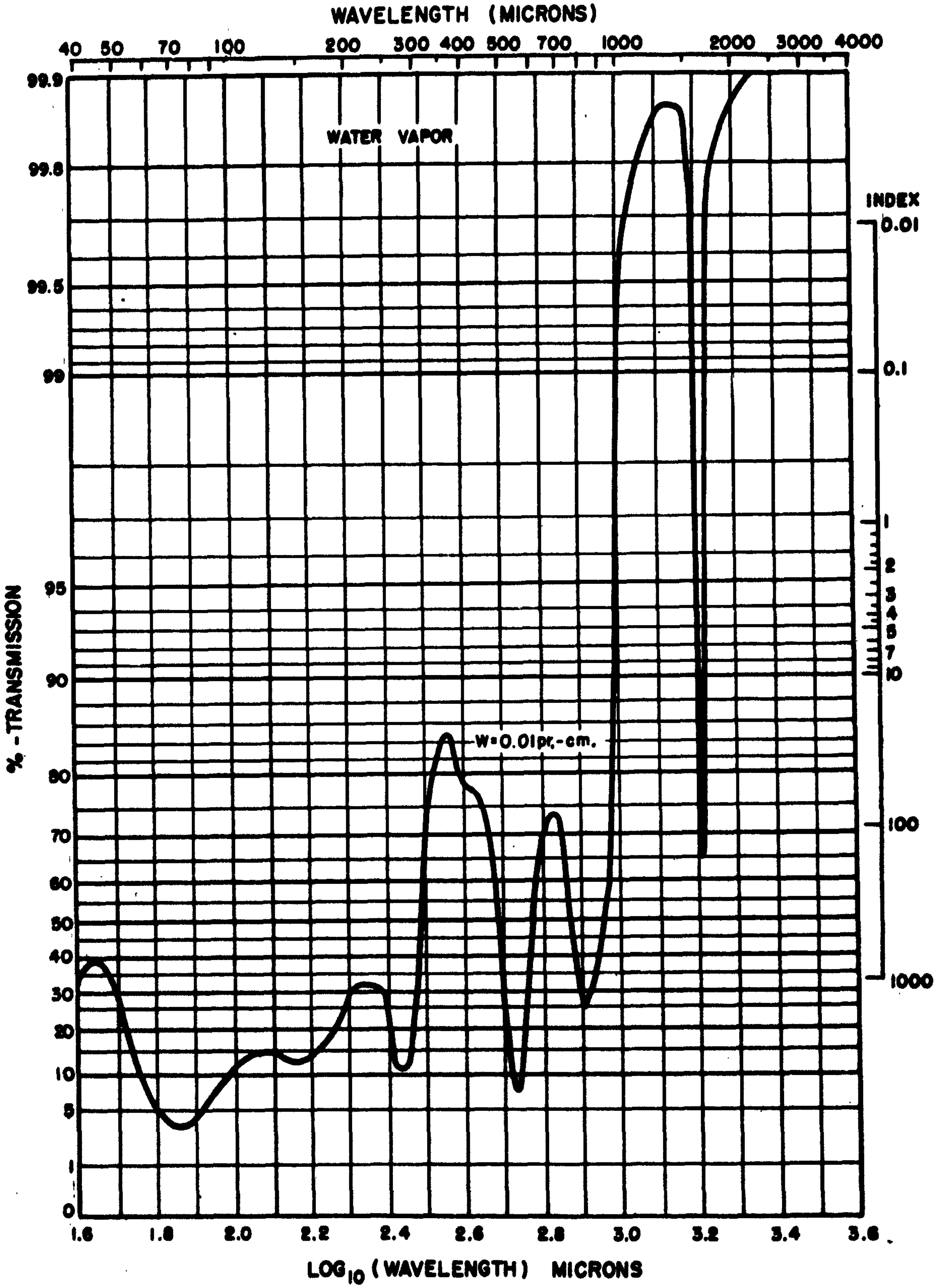

WAVELENGTH (MICRONS)
40
50
70
100
200
300
400
500
700
1000
2000
3000
4000
WATER VAPOR
99.9
99.8
99.5
99
95
90
80
70
60
50
40
30
20
10
5
1
0
% - TRANSMISSION
INDEX
0.01
0.1
1
2
3
4
5
7
10
100
1000
W=0.01pr.-cm.
1.6
1.8
2.0
2.2
2.4
2.6
2.8
3.0
3.2
3.4
3.6
LOG10 (WAVELENGTH) MICRONS

Figure 31

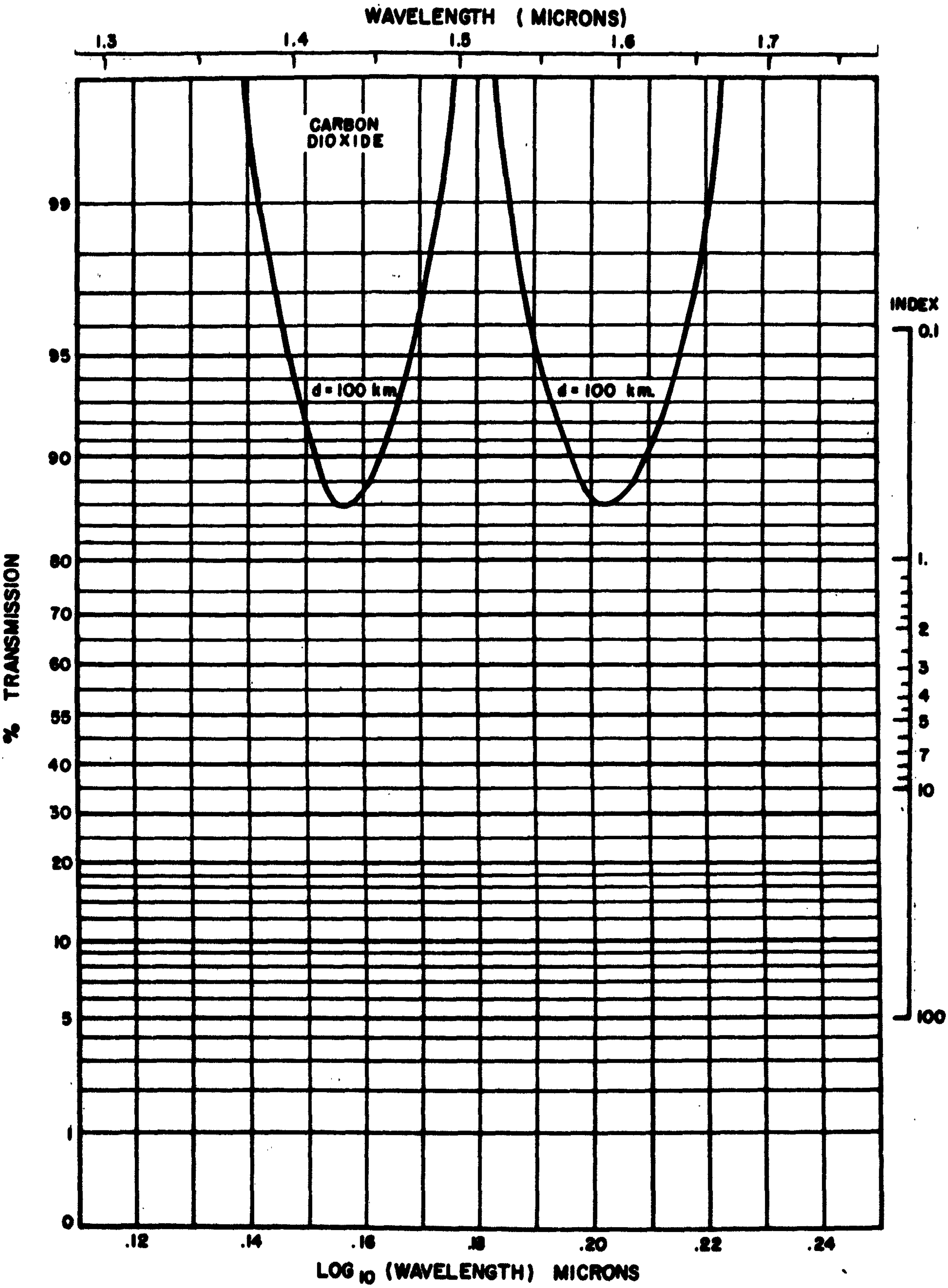

WAVELENGTH (MICRONS)
1.3
1.4
1.5
1.6
1.7
CARBON DIOXIDE
d = 100 km.
d = 100 km.
% TRANSMISSION
99
95
90
80
70
60
55
40
30
20
10
5
1
0
INDEX
0.1
1.
2
3
4
5
7
10
100
.12
.14
.16
.18
.20
.22
.24
LOG$_{10}$ (WAVELENGTH) MICRONS

Figure 32

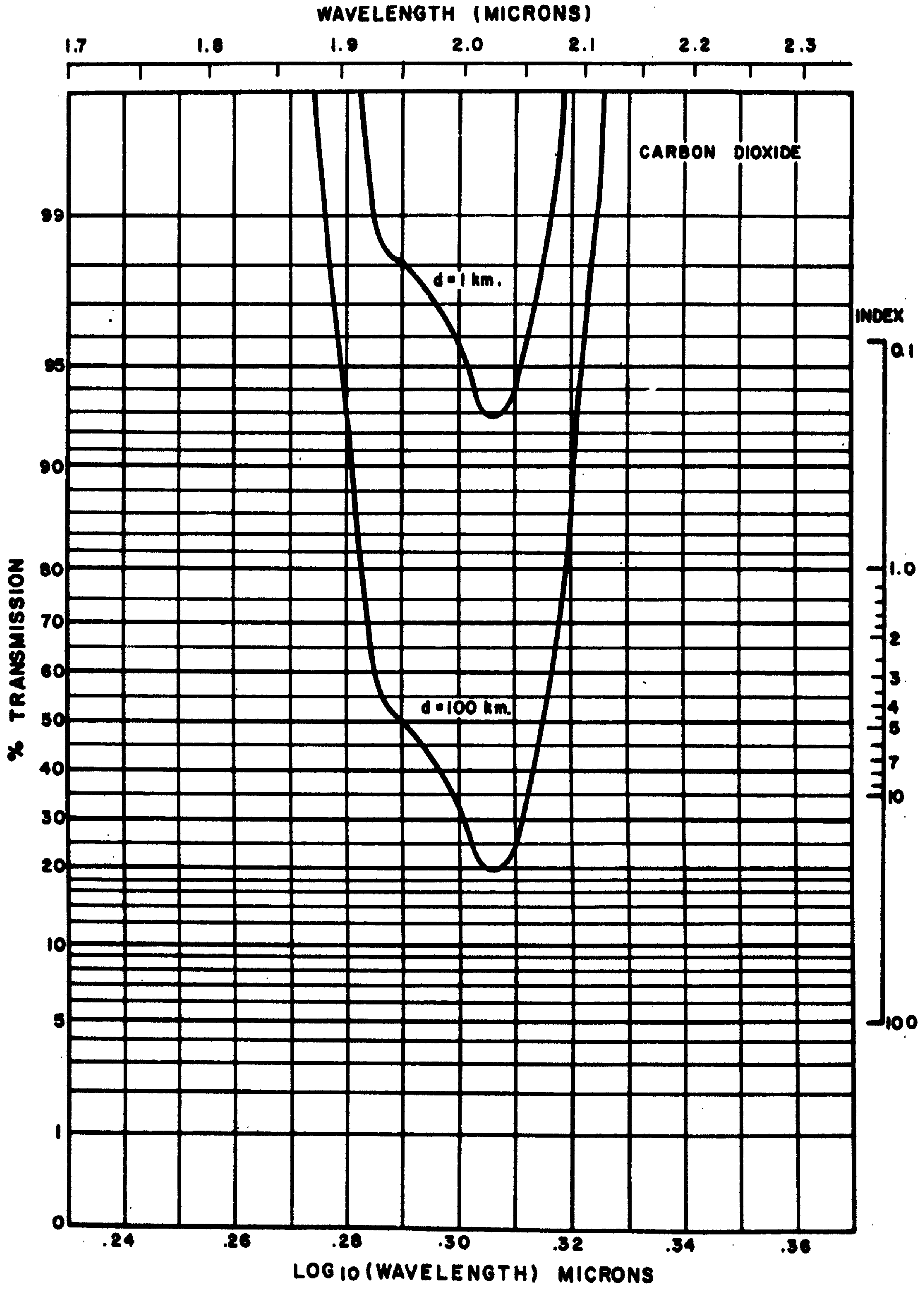

WAVELENGTH (MICRONS)
1.7
1.8
1.9
2.0
2.1
2.2
2.3
CARBON DIOXIDE
d = 1 km.
d = 100 km.
INDEX
0.1
1.0
2
3
4
5
7
10
100
% TRANSMISSION
99
95
90
80
70
60
50
40
30
20
10
5
1
0
.24
.26
.28
.30
.32
.34
.36
LOG 10 (WAVELENGTH) MICRONS

Figure 33

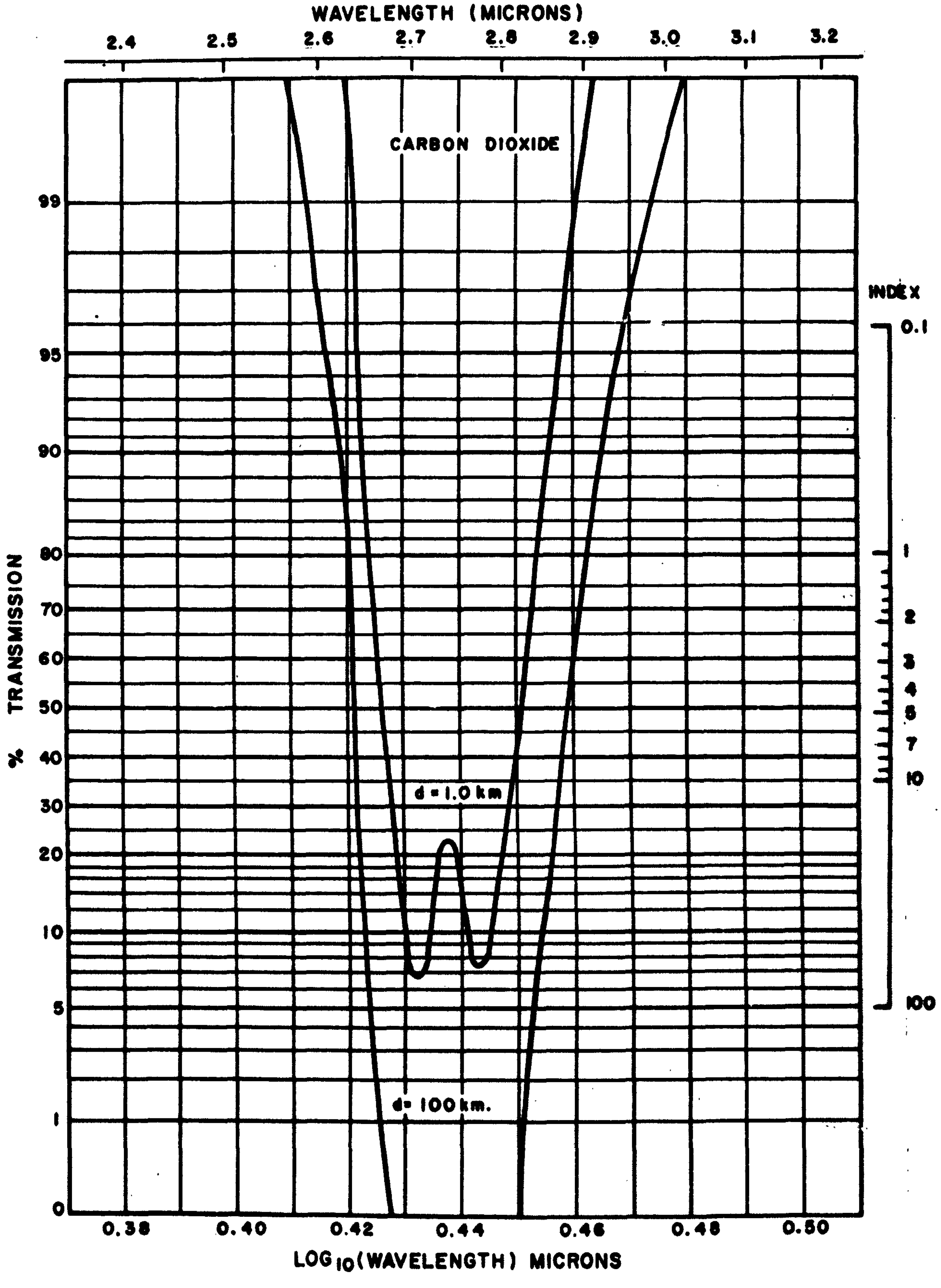

WAVELENGTH (MICRONS)
2.4
2.5
2.6
2.7
2.8
2.9
3.0
3.1
3.2
CARBON DIOXIDE
% TRANSMISSION
99
95
90
80
70
60
50
40
30
20
10
5
1
0
INDEX
0.1
1
2
3
4
5
7
10
100
d = 1.0 km
d= 100 km.
0.38
0.40
0.42
0.44
0.46
0.48
0.50
LOG10 (WAVELENGTH) MICRONS

Figure 34

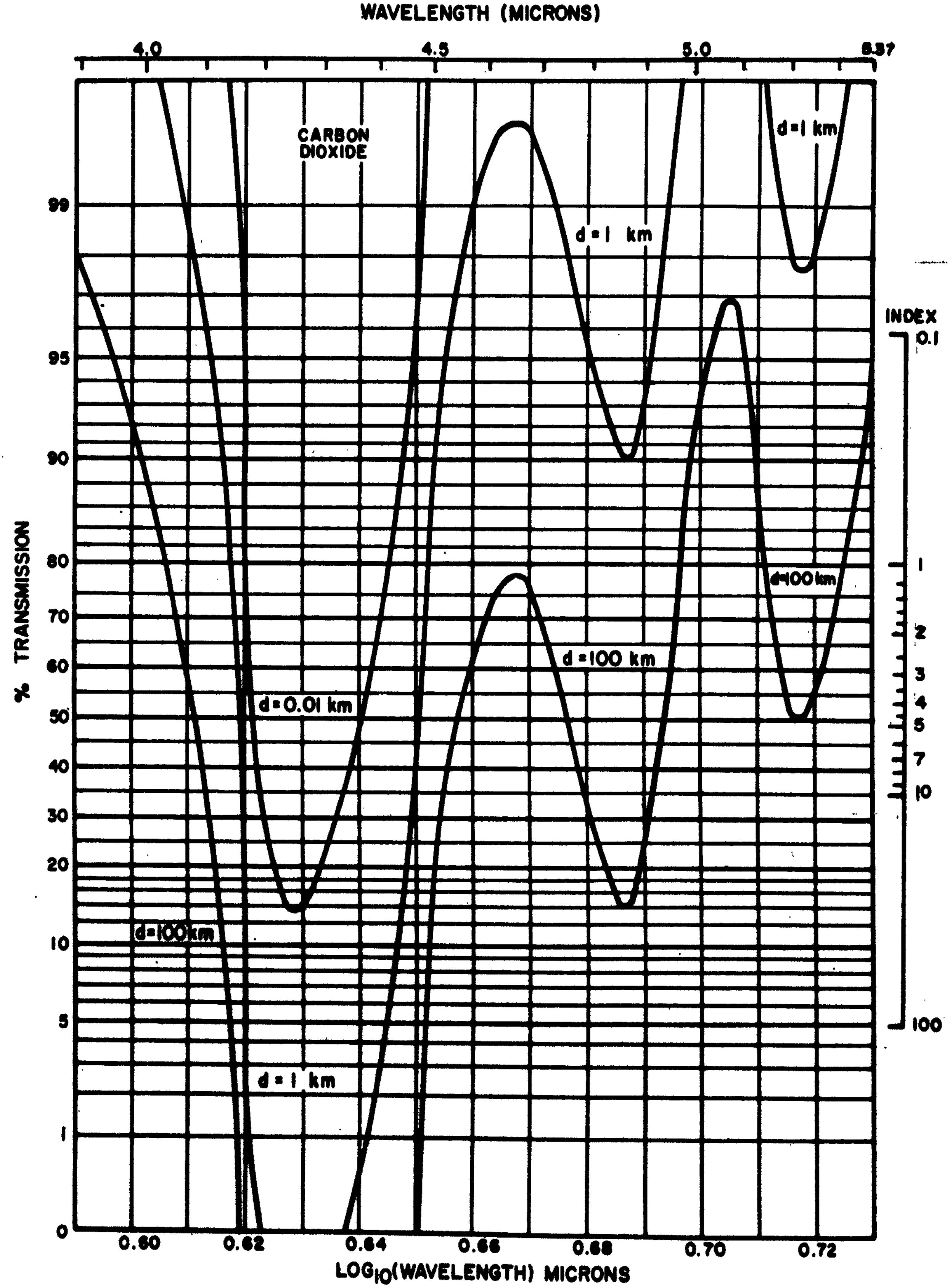

WAVELENGTH (MICRONS)
4.0
4.5
5.0
5.37
CARBON DIOXIDE
d = 1 km
d = 1 km
d = 100 km
d = 100 km
d = 0.01 km
d = 100 km
d = 1 km
INDEX
0.1
1
2
3
4
5
7
10
100
% TRANSMISSION
99
95
90
80
70
60
50
40
30
20
10
5
1
0
0.60
0.62
0.64
0.66
0.68
0.70
0.72
LOG$_{10}$(WAVELENGTH) MICRONS

Figure 35

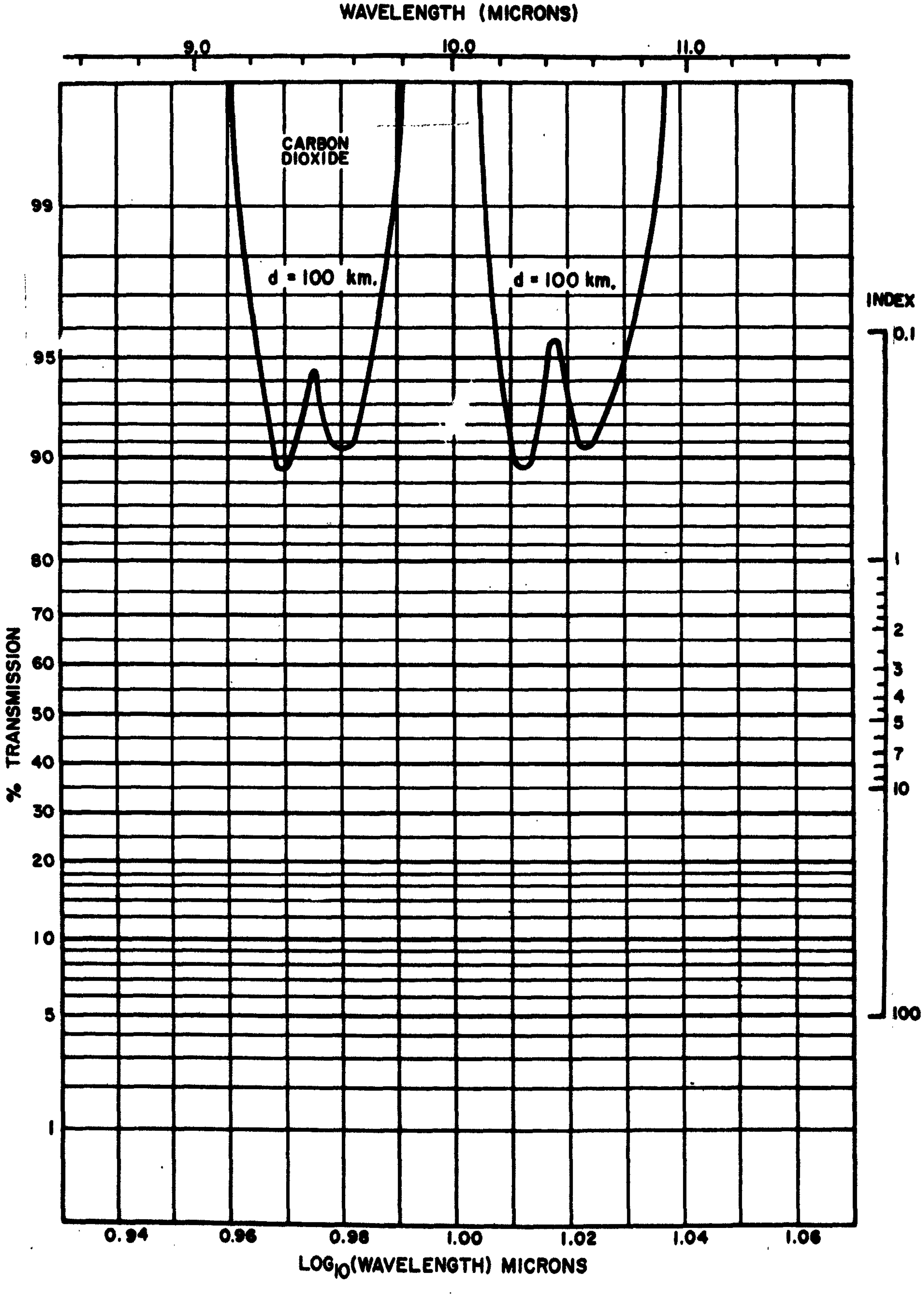

WAVELENGTH (MICRONS)
9.0
10.0
11.0
CARBON DIOXIDE
d = 100 km.
d = 100 km.
99
95
90
80
70
60
50
40
30
20
10
5
1
% TRANSMISSION
INDEX
0.1
1
2
3
4
5
7
10
100
0.94
0.96
0.98
1.00
1.02
1.04
1.06
LOG$_{10}$(WAVELENGTH) MICRONS

Figure 36

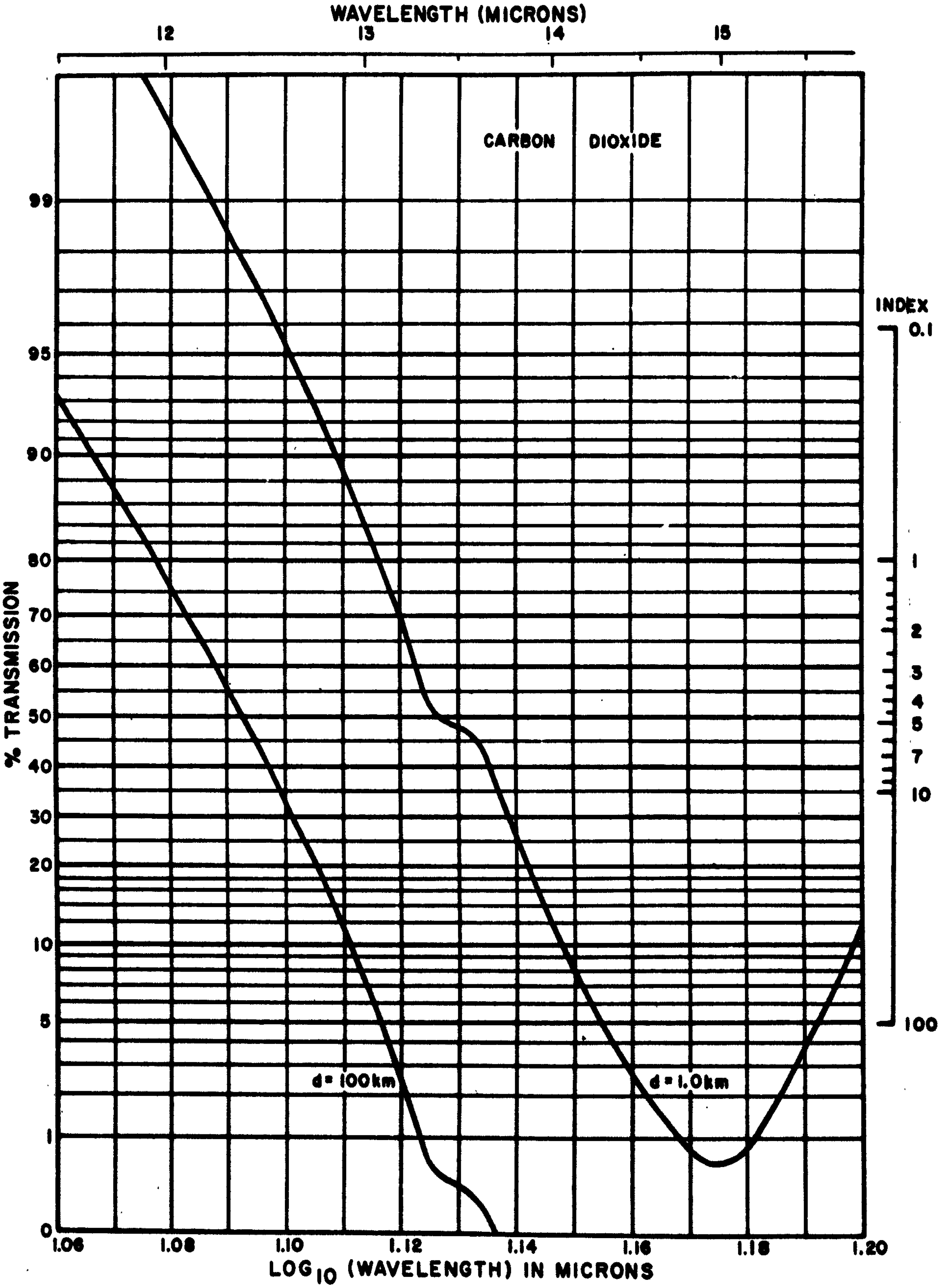

WAVELENGTH (MICRONS)
12
13
14
15
CARBON DIOXIDE
99
95
90
80
70
60
50
40
30
20
10
5
1
0
% TRANSMISSION
INDEX
0.1
1
2
3
4
5
7
10
100
d= 100km
d= 1.0km
1.06
1.08
1.10
1.12
1.14
1.16
1.18
1.20
LOG10 (WAVELENGTH) IN MICRONS

Figure 37

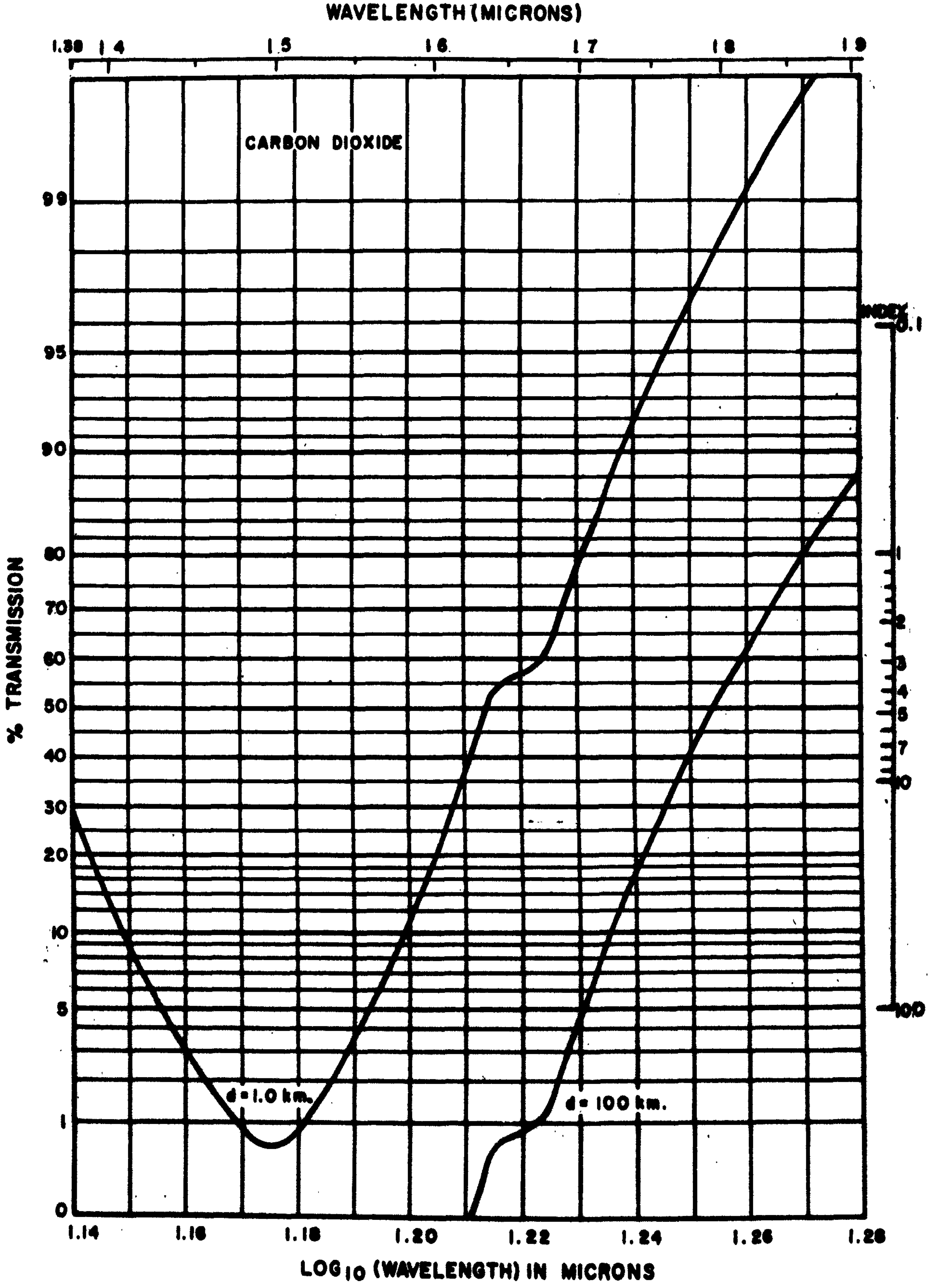

WAVELENGTH (MICRONS)
1.38
1.4
1.5
1.6
1.7
1.8
1.9
CARBON DIOXIDE
% TRANSMISSION
99
95
90
80
70
60
50
40
30
20
10
5
1
0
INDEX
0.1
1
2
3
4
5
7
10
100
d = 1.0 km.
d = 100 km.
1.14
1.16
1.18
1.20
1.22
1.24
1.26
1.28
LOG10 (WAVELENGTH) IN MICRONS

Figure 38

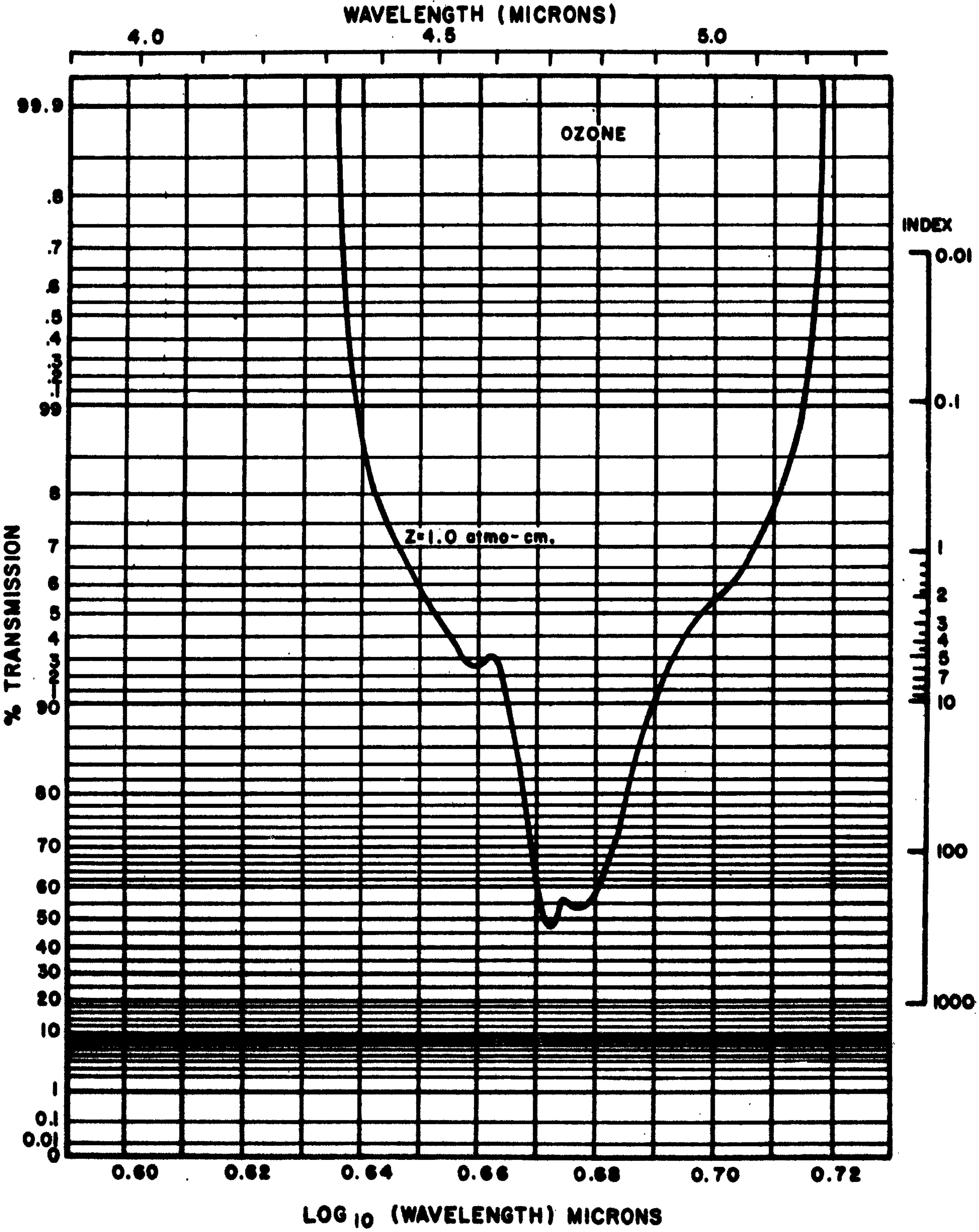

WAVELENGTH (MICRONS)
4.0
4.5
5.0
OZONE
INDEX
0.01
0.1
1
2
3
4
5
7
10
100
1000
Z=1.0 atmo-cm.
% TRANSMISSION
99.9
.8
.7
.6
.5
.4
.3
.2
.1
99
8
7
6
5
4
3
2
1
90
80
70
60
50
40
30
20
10
1
0.1
0.01
0
0.60
0.62
0.64
0.66
0.68
0.70
0.72
LOG 10 (WAVELENGTH) MICRONS

Figure 39

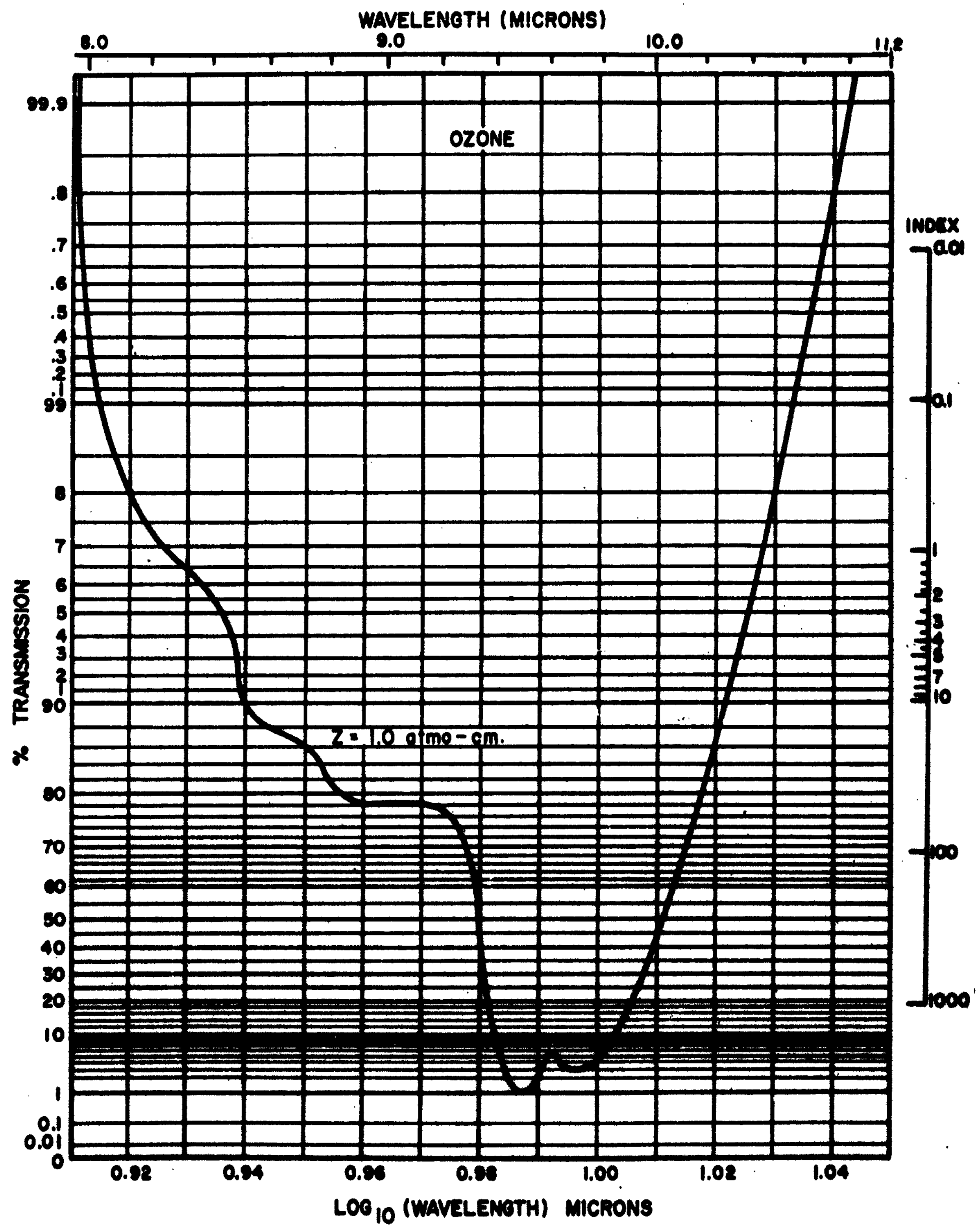

WAVELENGTH (MICRONS)
8.0
9.0
10.0
11.2
OZONE
Z = 1.0 atmo-cm.
% TRANSMISSION
99.9
.8
.7
.6
.5
.4
.3
.2
.1
99
8
7
6
5
4
3
2
1
90
80
70
60
50
40
30
20
10
1
0.1
0.01
0
INDEX
0.01
0.1
1
2
3
4
5
7
10
100
1000
0.92
0.94
0.96
0.98
1.00
1.02
1.04
LOG10 (WAVELENGTH) MICRONS

Figure 40

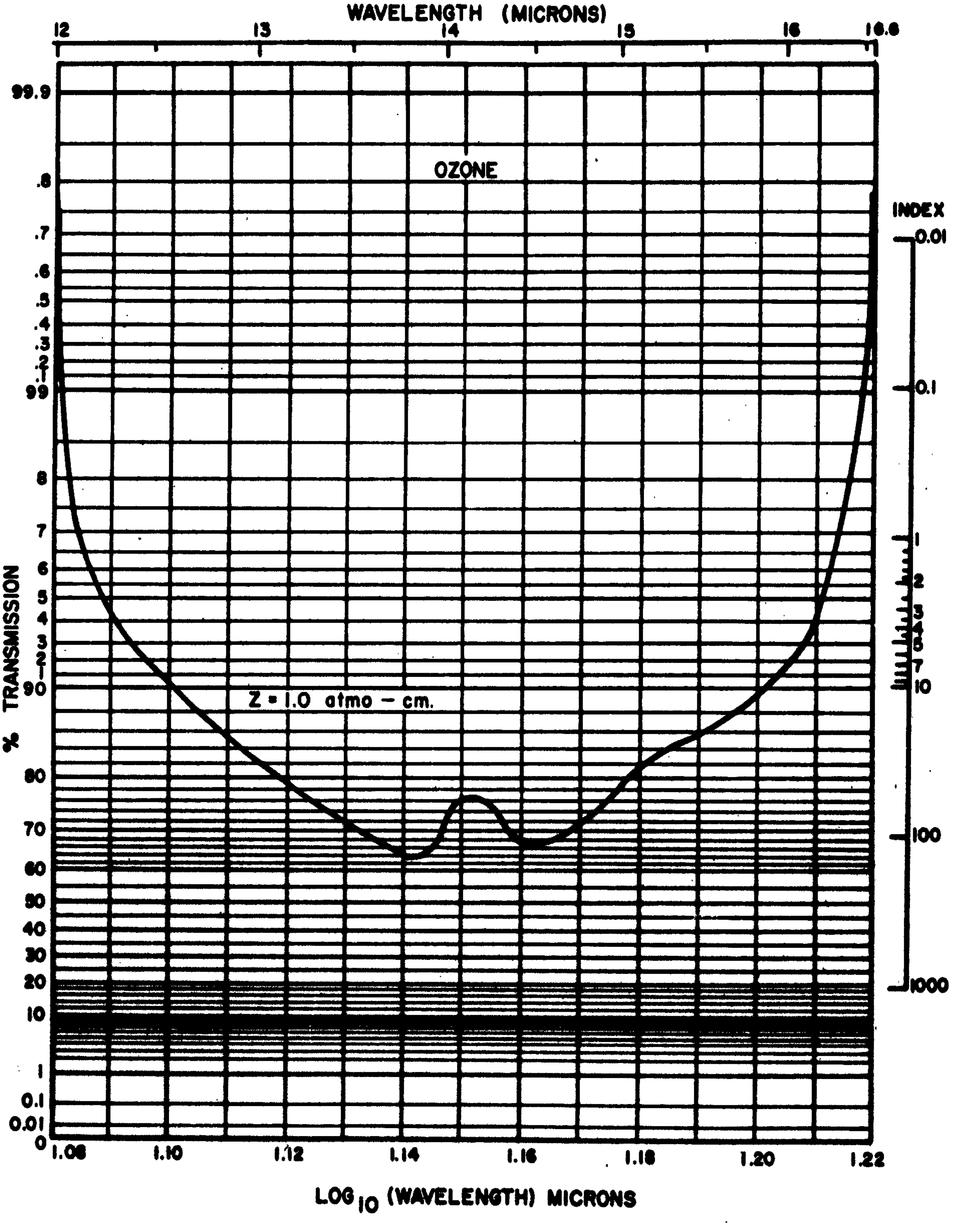

WAVELENGTH (MICRONS)
12
13
14
15
16
16.6
OZONE
INDEX
0.01
0.1
1
2
3
4
5
7
10
100
1000
Z = 1.0 atmo - cm.
% TRANSMISSION
99.9
.8
.7
.6
.5
.4
.3
.2
.1
99
8
7
6
5
4
3
2
1
90
80
70
60
50
40
30
20
10
1
0.1
0.01
0
1.08
1.10
1.12
1.14
1.16
1.18
1.20
1.22
LOG10 (WAVELENGTH) MICRONS

Figure 41

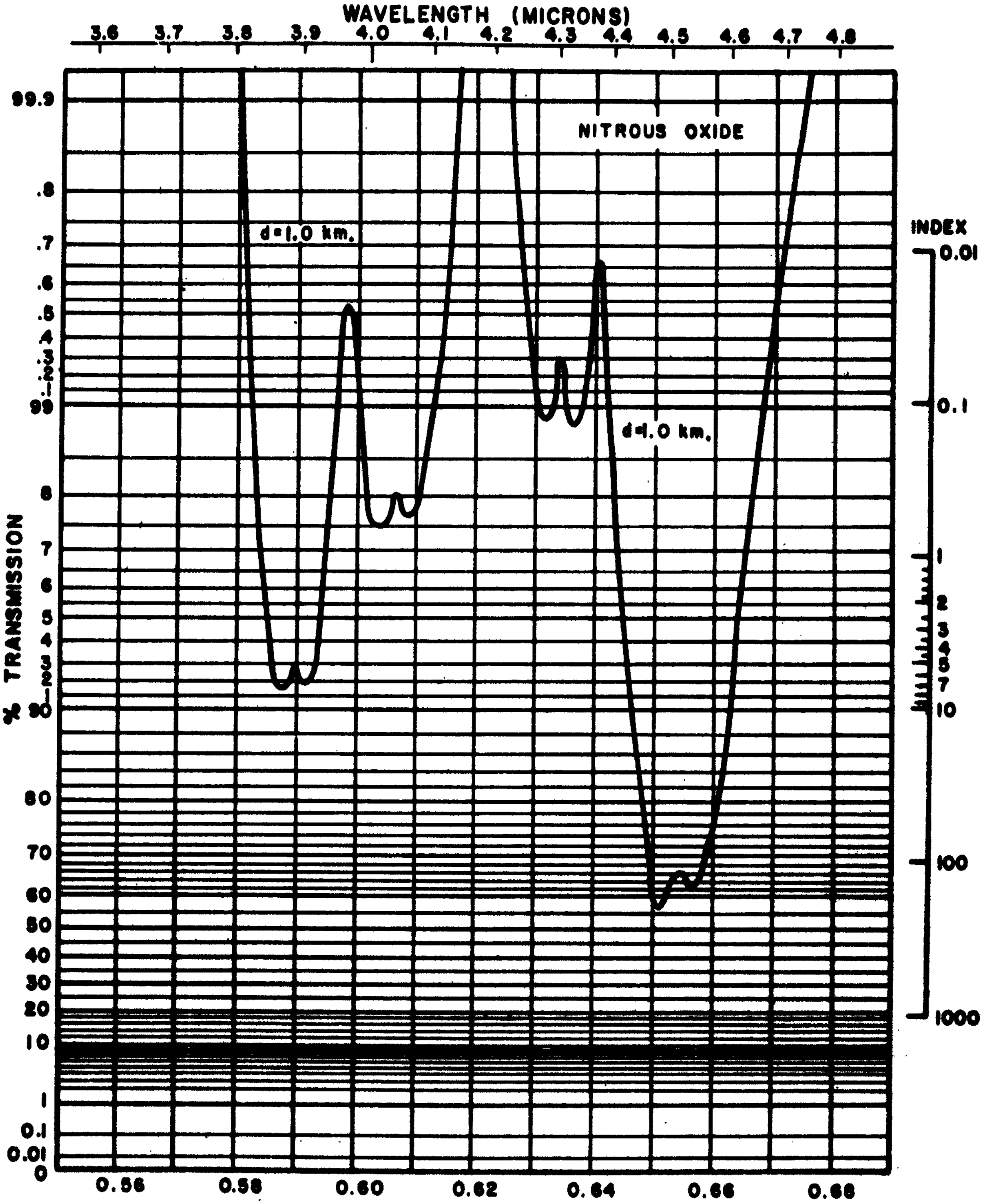

WAVELENGTH (MICRONS)
3.6
3.7
3.8
3.9
4.0
4.1
4.2
4.3
4.4
4.5
4.6
4.7
4.8
NITROUS OXIDE
d=1.0 km.
d=1.0 km.
INDEX
0.01
0.1
1
2
3
4
5
7
10
100
1000
% TRANSMISSION
99.9
.8
.7
.6
.5
.4
.3
.2
.1
99
8
7
6
5
4
3
2
90
80
70
60
50
40
30
20
10
1
0.1
0.01
0
0.56
0.58
0.60
0.62
0.64
0.66
0.68
LOG10 (WAVELENGTH) MICRONS

Figure 42

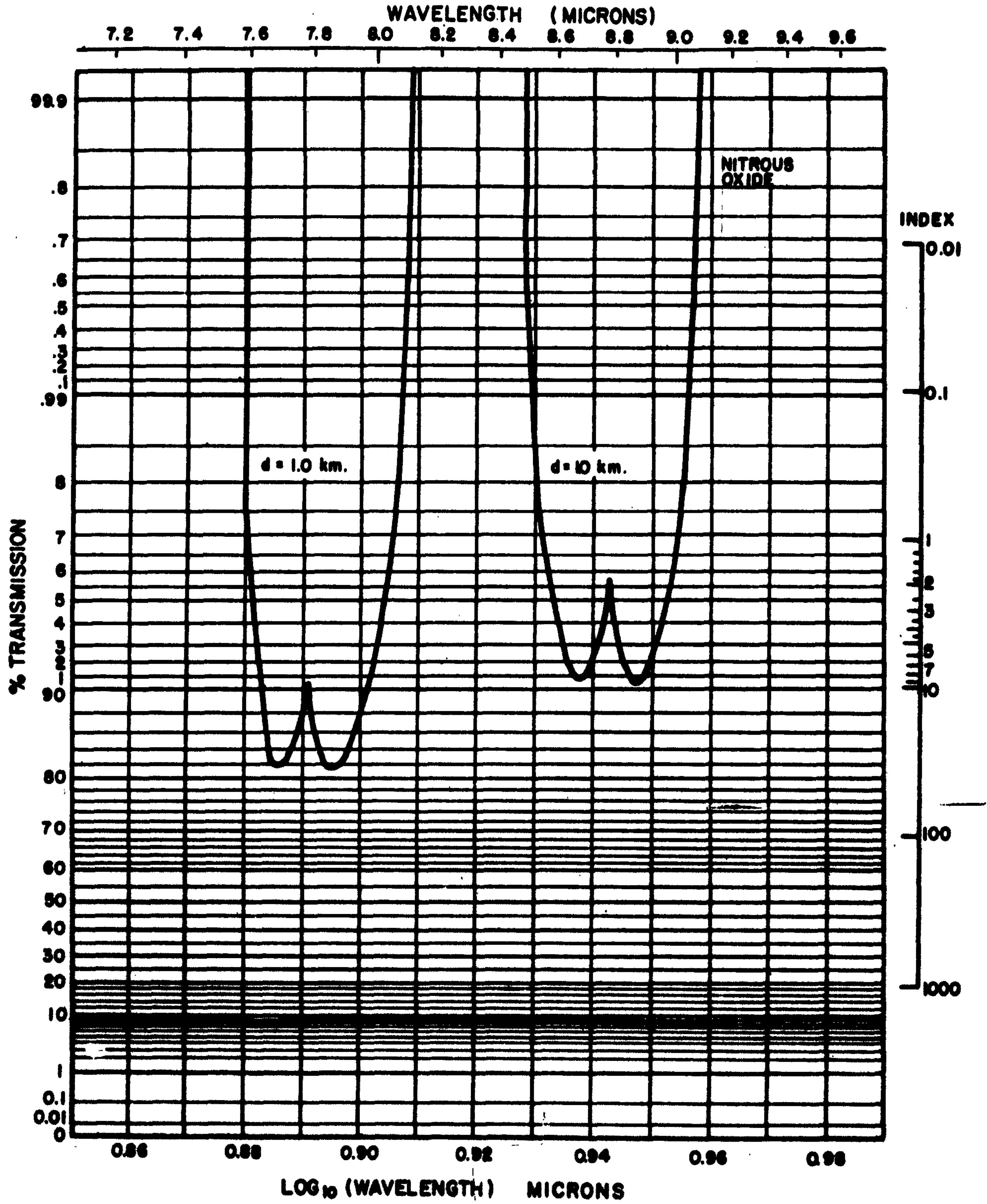

WAVELENGTH (MICRONS)
7.2
7.4
7.6
7.8
8.0
8.2
8.4
8.6
8.8
9.0
9.2
9.4
9.6
% TRANSMISSION
99.9
.8
.7
.6
.5
.4
.3
.2
.1
.99
8
7
6
5
4
3
2
1
90
80
70
60
50
40
30
20
10
1
0.1
0.01
0
NITROUS OXIDE
d = 1.0 km.
d = 10 km.
INDEX
0.01
0.1
1
2
3
5
7
10
100
1000
0.86
0.88
0.90
0.92
0.94
0.96
0.98
LOG₁₀ (WAVELENGTH) MICRONS

Figure 43

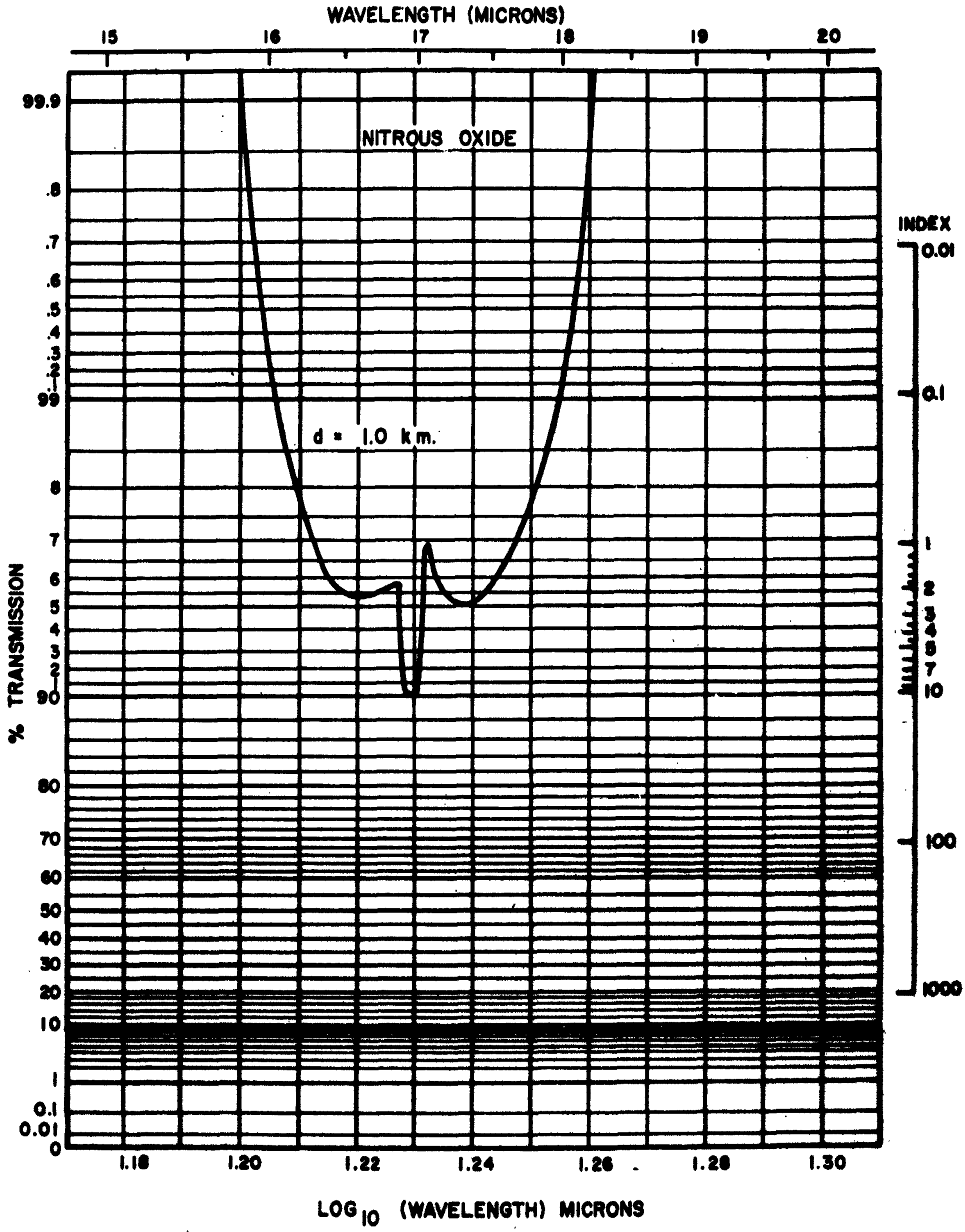
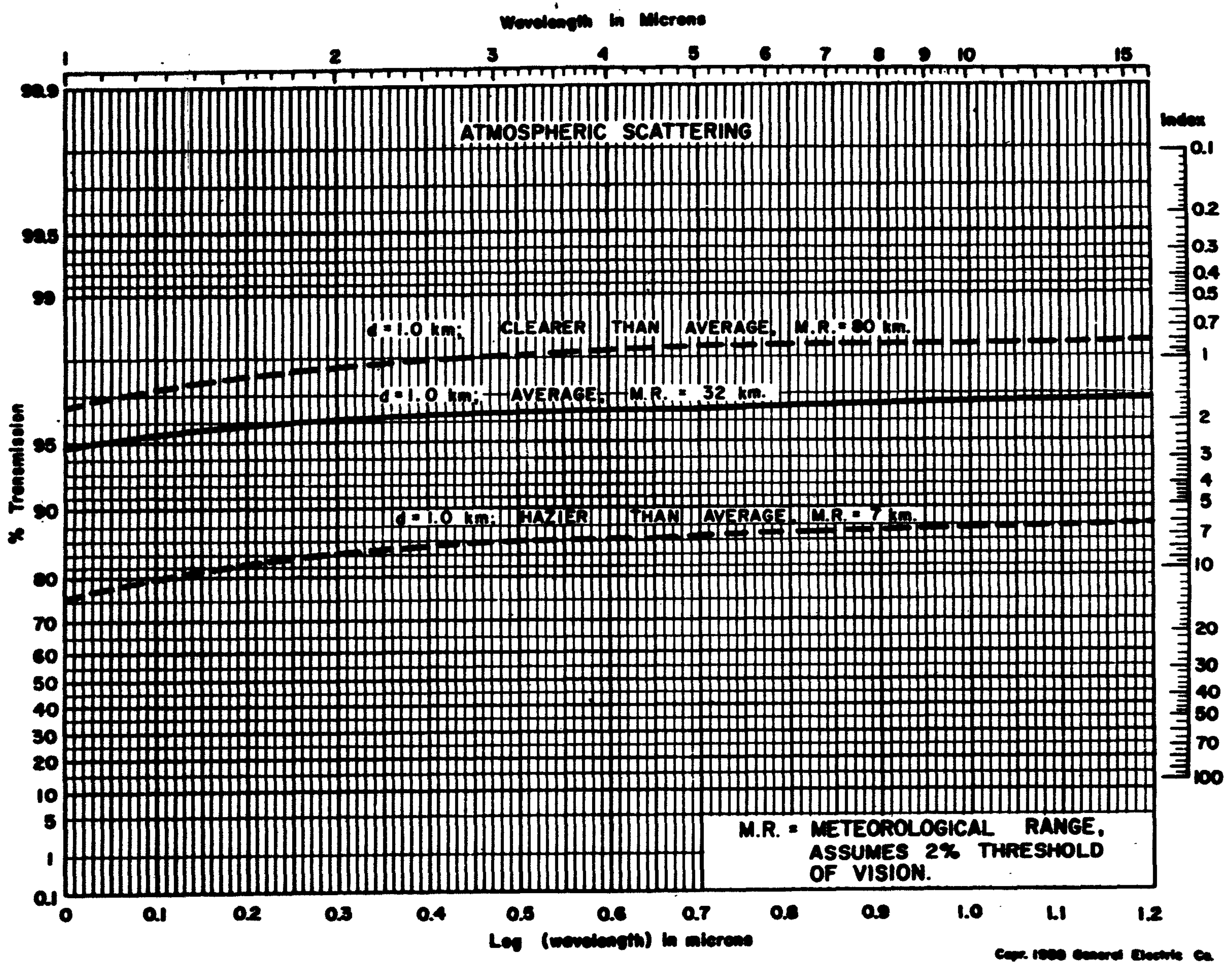

WAVELENGTH (MICRONS)
15
16
17
18
19
20
NITROUS OXIDE
d = 1.0 km.
% TRANSMISSION
99.9
.8
.7
.6
.5
.4
.3
.2
.1
99
8
7
6
5
4
3
2
90
80
70
60
50
40
30
20
10
1
0.1
0.01
0
INDEX
0.01
0.1
1
2
3
4
5
7
10
100
1000
1.18
1.20
1.22
1.24
1.26
1.28
1.30
LOG₁₀ (WAVELENGTH) MICRONS

Figure 44



Copr. 1960 General Electric Co.

Figure 45

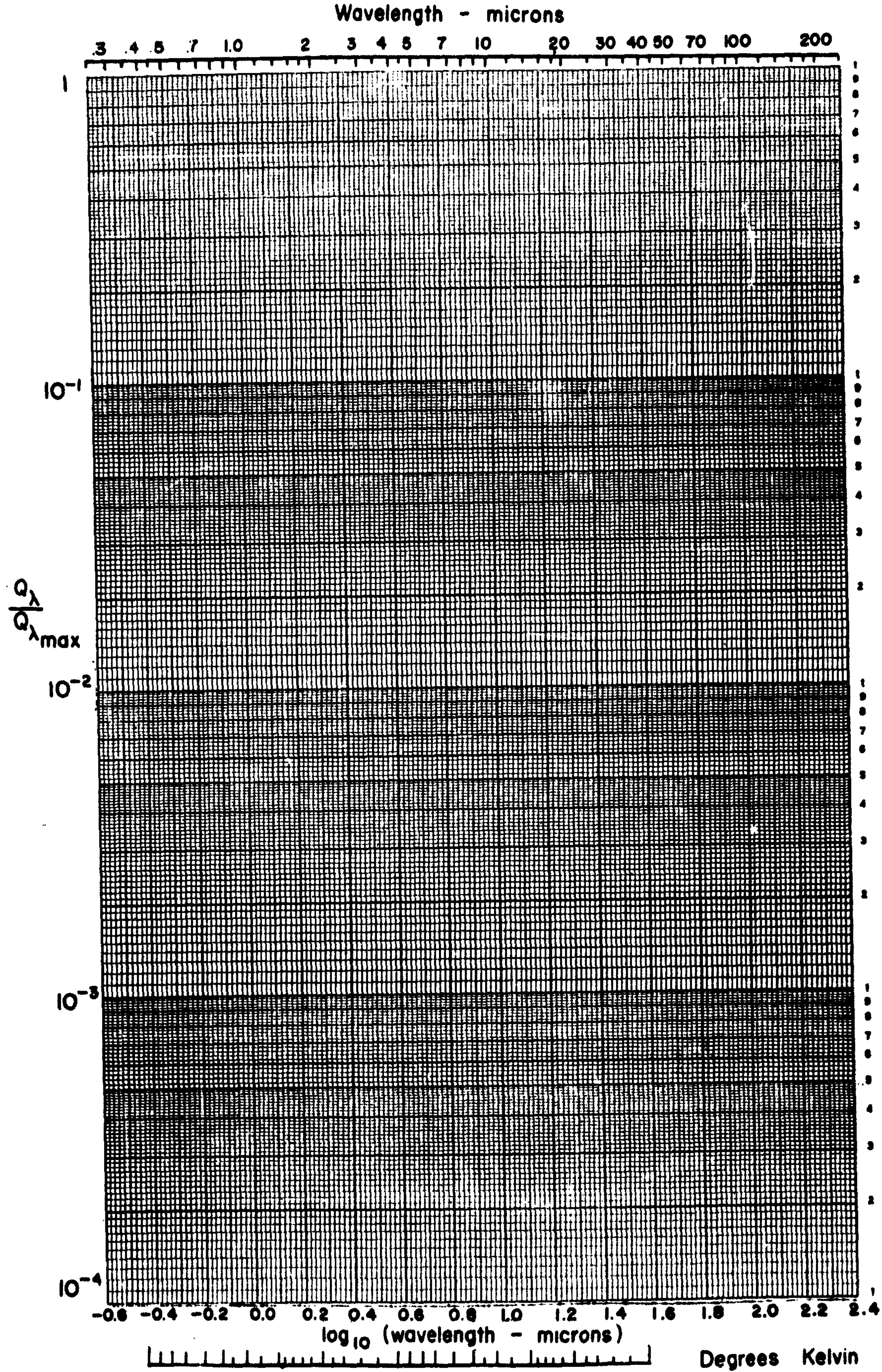

Wavelength - microns
.3 .4 .5 .7 1.0 2 3 4 5 7 10 20 30 40 50 70 100 200
1
10⁻¹
10⁻²
10⁻³
10⁻⁴
Qλ / Qλmax
-0.6 -0.4 -0.2 0.0 0.2 0.4 0.6 0.8 1.0 1.2 1.4 1.6 1.8 2.0 2.2 2.4
log10 (wavelength - microns)
Degrees Kelvin
10,000 7 5 4 3 2 1000 7 5 4 3 2 100

Figure 46

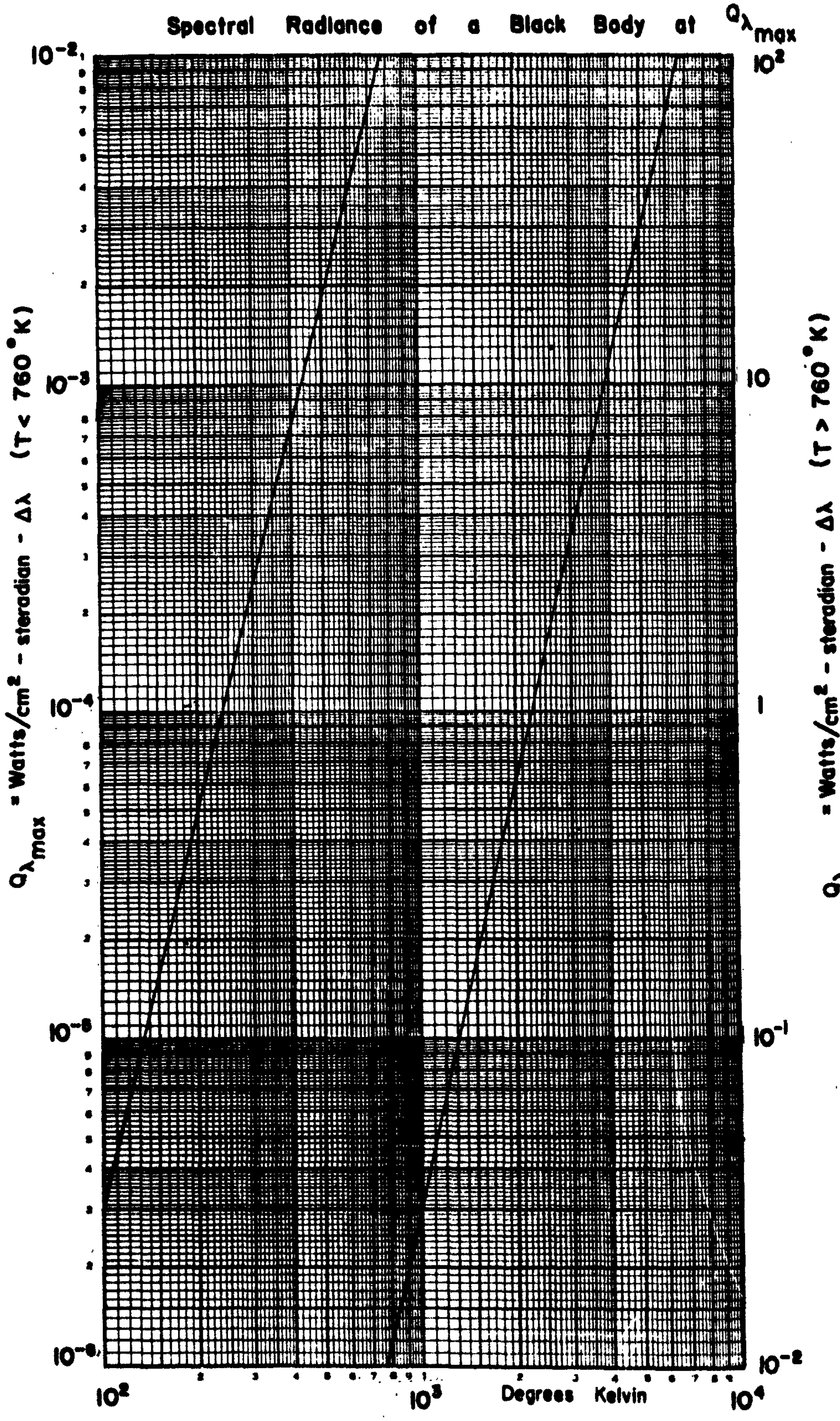

Spectral Radiance of a Black Body at Qλmax
10⁻²
10⁻³
10⁻⁴
10⁻⁵
10⁻⁶
10²
10
1
10⁻¹
10⁻²
Qλmax = Watts/cm² - steradian - Δλ (T < 760°K)
Qλmax = Watts/cm² - steradian - Δλ (T > 760°K)
10²
10³
10⁴
Degrees Kelvin

Figure 47

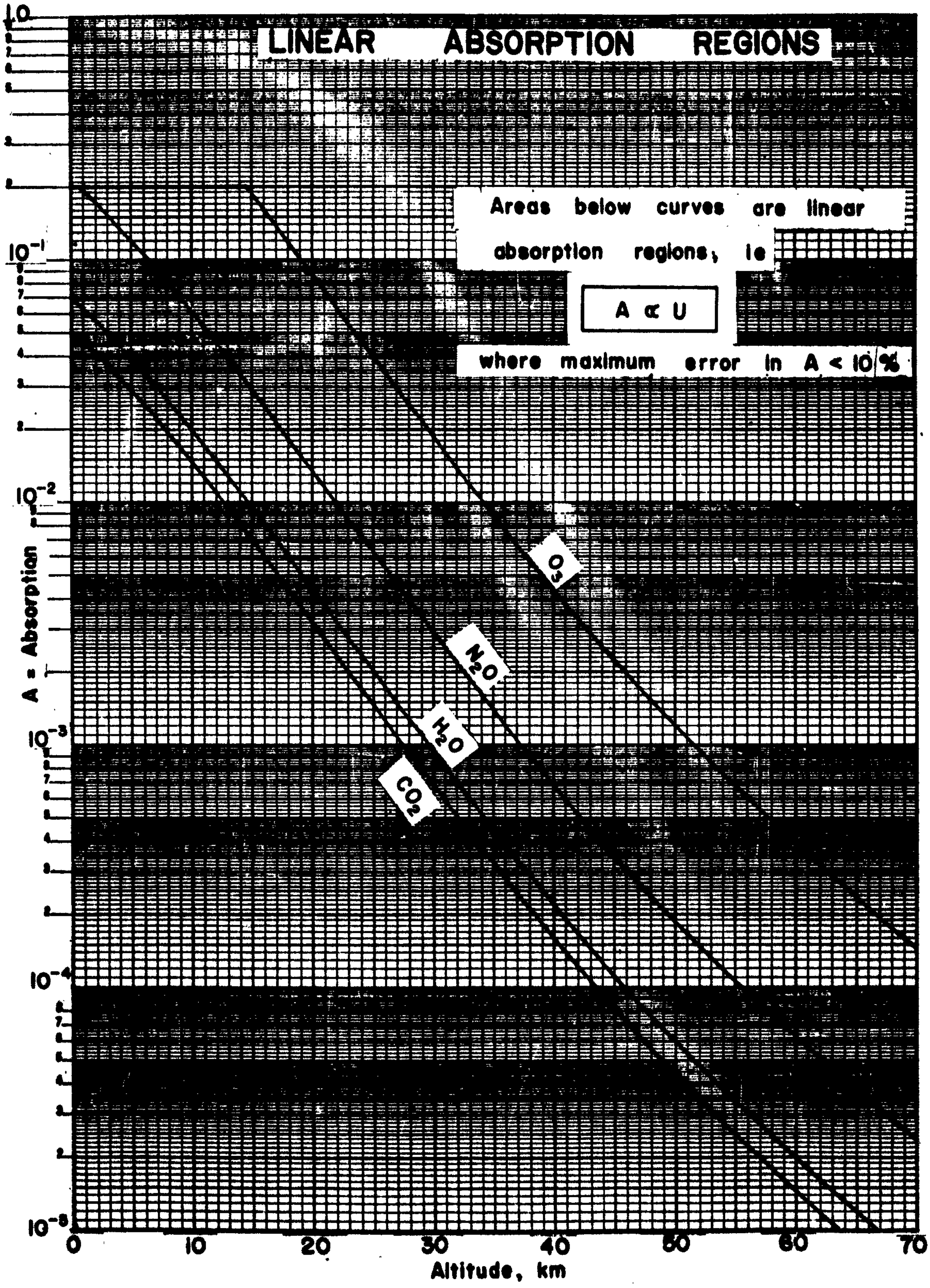

LINEAR ABSORPTION REGIONS
Areas below curves are linear absorption regions, ie
A ∝ U
where maximum error in A < 10 %
O3
N2O
H2O
CO2
A = Absorption
Altitude, km
10
10⁻¹
10⁻²
10⁻³
10⁻⁴
10⁻⁵
0
10
20
30
40
50
60
70

Figure 48

INFRARED PATHS FOR BACKGROUND RADIATION

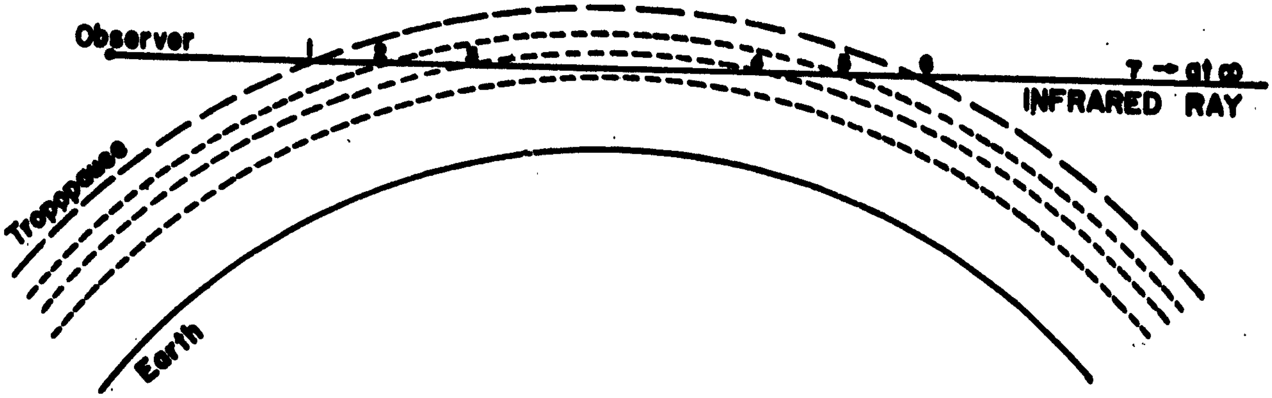


Figure 49

APPENDIX A

INFRARED PATHS

Due to the Earth's curvature and the refraction of infrared rays as they pass through the atmosphere, the geometry of the infrared path with respect to the Earth is not simple. Thus, Figures 2, 3, and 4 were derived from the following theory.

Let: a = height from the Earth's surface to point Y, see Figure 50.

D = diameter of the Earth = 12,713.53 km. (1959 ARDC Model Atmosphere)

h = height from the Earth's surface to point X, km.

n = index of refraction of the atmosphere with respect to infrared radiation.

P = air pressure at point X, mm Hg

P_s = sea level pressure = 760 mm Hg

R = range from point 0 to point X along the infrared path, km.

r = distance between the center of the Earth and point X, km.

T = air temperature at point X, °Kelvin

$T_o = 273.17^\circ K$

X = point on infrared path assuming refraction, see Figure 50.

y = point directly above Point X and laying on a straight line tangent to the infrared path at point 0, see Figure 50.

α = angle between a tangent to the infrared beam at point X and a line parallel to a tangent to the Earth at point 0.

θ = elevation angle

θ_o = elevation angle at point 0

ϕ = angle subtended at the Earth's center and points 0 and X.

λ = wavelength

$$\Delta h = r\,\Delta\phi\,\tan\theta + r\,(\sec\Delta\phi - 1)$$

$$\Delta R = r\,\Delta\phi\,\sec\theta$$

$$r = \frac{D}{2} + h$$

$$\theta = \phi + \alpha$$

By series expansion, $(\sec\Delta\phi - 1) = \frac{\Delta\phi^2}{2}$

Combining terms,

$$\Delta h = \left(\frac{D}{2} + h\right)\left[\tan(\phi + \alpha) + \frac{\Delta\phi}{2}\right]\Delta\phi \qquad \text{eq (28)}$$

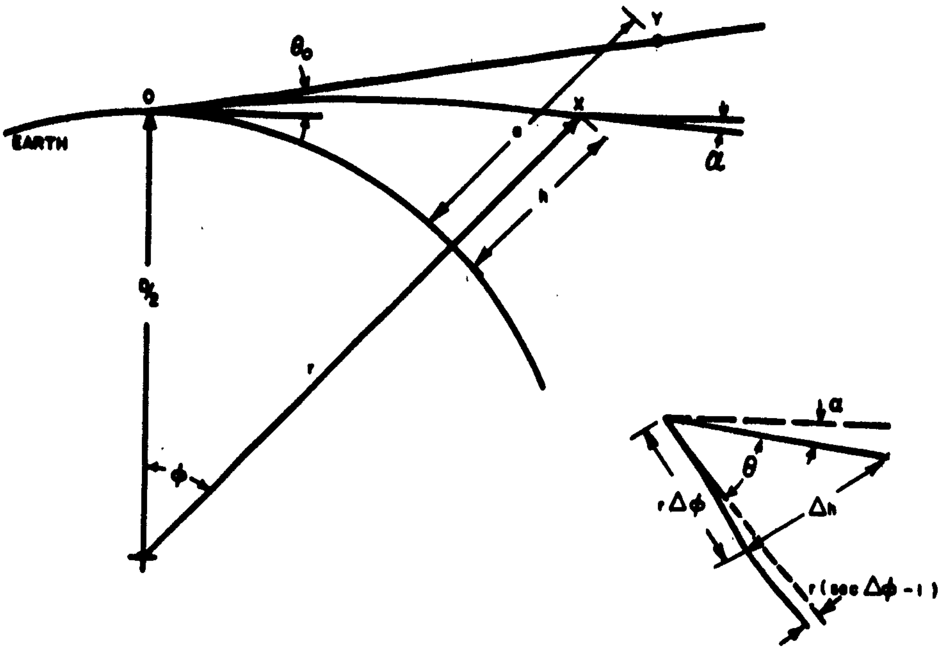

θ_0
O
Y
X
EARTH
α
a
h
a/2
r
ϕ
α
θ
$r\Delta\phi$
Δh
$r(\sec\Delta\phi - 1)$

Figure 50

$$\Delta R = (\frac{D}{2} + h) \sec (\phi + \alpha) \Delta\phi \qquad \text{eq (29)}$$

From Snell's Law

$$\frac{n + \Delta n}{n} = \frac{\sec (\theta + \Delta\alpha)}{\sec \theta}$$

When $\theta < 8^{\circ}$, this reduces to

$$\Delta\alpha = \frac{\Delta n}{\theta} = \frac{\Delta n}{(\phi + \alpha)}$$

From Penndorf, reference 7

$n = 1 + 28797 \times 10^{-8} \left(\frac{T_s}{T}\right) \left(\frac{P}{P_s}\right)$ at $\lambda = 2.0\ \mu$ which is a good average value for infrared.

From the 1959 ARDC model atmosphere to determine T and P vs. h

$$\left.\begin{aligned} n &= 1 + 0.00027299\ e^{-0.10611 \times h} \\ \Delta n &= 0.000028967 e^{-0.10611 \times h}\ \Delta h \end{aligned}\right\} \quad 0 \leq h \leq 9.5 \text{ km.}$$

$$\left.\begin{aligned} n &= 1 + 0.00027299 e^{(-0.15351 \times h + 0.44076)} \\ \Delta n &= -0.000041906 e^{(-0.15351 \times h + 0.44076)}\ \Delta h \end{aligned}\right\} \quad h > 9.5 \text{ km.}$$

Summarizing the important equations,

$$\Delta h = (6,356.77 + h) \left[\tan (\phi + \alpha) + \frac{\Delta\phi}{2}\right] \qquad \text{eq (28)}$$

$$\Delta R = (6,356.77 + h) \sec (\phi + \alpha) \Delta\phi \qquad \text{eq (29)}$$

$$\Delta\alpha = \frac{\Delta n}{\phi + \alpha} \qquad \text{eq (30)}$$

$$\Delta n = -0.000028967 e^{-0.10611 \times h}\ \Delta h\ ;\ 0 \leq h \leq 9.5 \text{ km.} \qquad \text{eq (31)}$$

$$\Delta n = -0.000041906 e^{(-0.15351 \times h + 0.44076)} \Delta h\ ;\ h > 9.5 \text{ km} \qquad \text{eq (32)}$$

$h = \Sigma\ \Delta h$, over the infrared path — eq (33)

$R = \Sigma\ \Delta R$, " " " " — eq (34)

$\alpha = \Sigma\ \Delta\alpha$. " " " " — eq (35)

$\theta = \phi + \alpha$ — eq (36)

This summation was done on a Librascope LGP 30 computer to give the date for Figures 2, 3, and 4.

To Extend the Infrared Path to Any Range

If none of the infrared path is below an altitude of 50 km., or if $\theta_{min} > 7^\circ$, assume there is no refraction of the infrared ray and that the Earth is a sphere. The problem reduces to a simple one of plane geometry and trigonometry.

Otherwise,

Let: $b = \frac{D}{2} + 50$ km, see Figure 51

$\frac{D}{2}$ = radius of Earth = 6356.77 km.

h = altitude, km.

R = range from (h_{min}) to point y, km, see Figure 51.

R_{50}= range from (h_{min}) to point X, km, see Figure 51.

r = range from point X to point y, km, see Figure 51.

$z = h - 50$ km, see Figure 51.

θ_{50}= elevation angle at point X, degrees, see Figure 51.

θ = elevation angle at point y, degrees, see Figure 51.

ϕ_{50}= angle between OCX, see Figure 51.

ϕ_{50}= angle between OCY, see Figure 51.

$$\cos(90^\circ + \theta_{50}) = \frac{b^2 + r^2 - (b+z)^2}{2br}$$

$$z^2 + 2bz - r^2 - 2br \sin\theta_{50} = 0$$

$$z = -b + \sqrt{b^2 + r^2 + 2br \sin\theta_{50}}$$

$$r = -b(\sin\theta_{50}) + \sqrt{b^2(\sin\theta_{50})^2 + z^2 + 2bz}$$

$$\theta_{50} = \sin^{-1}\left(\frac{z^2 + 2bz - r^2}{2br}\right)$$

$$h = z + 50$$

$$R = R_{50} + r$$

$$b = \frac{D}{2} + 50 = 6406.77$$

$$b^2 = 41046701.83$$

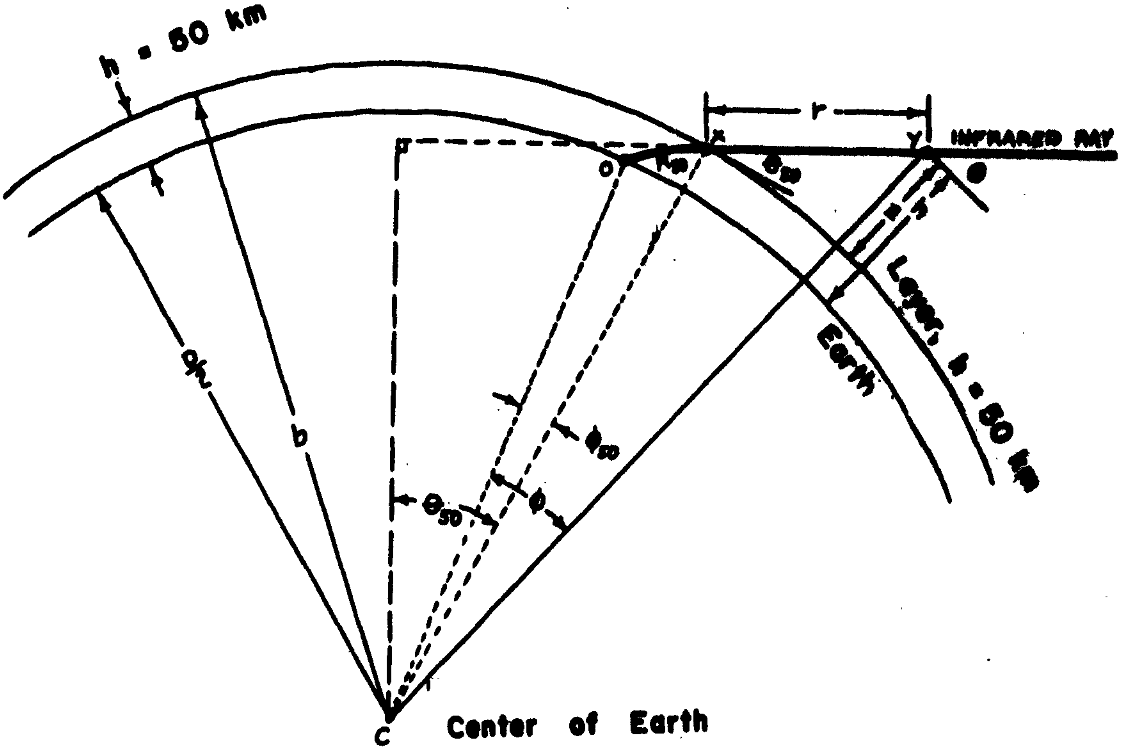

h = 50 km
r
X
Y
INFRARED RAY
O
r_{50}
θ_{50}
θ
h
Layer, h = 50 km
Earth
a
b
ϕ_{50}
θ_{50}
ϕ
C
Center of Earth

Figure 51

$$\theta = \theta_{50} + \phi - \phi_{50}$$

$$\frac{\cos \theta_{50}}{\sin (\phi - \phi_{50})} = \frac{\frac{D}{2} + h}{r}$$

$$\phi - \phi_{50} = \sin^{-1}\left[\frac{r \cos \theta_{50}}{D/_2 + h}\right]$$

Combining the above equations,

$$h = -6356.77 + \sqrt{b^2 + (R - R_{50})^2 + 2b\,(R - R_{50}) \sin \theta_{50}} \qquad \text{eq (37)}$$

$$R = R_{50} - b(\sin \theta_{50}) + \sqrt{b^2 (\sin \theta_{50})^2 + (h - 50)^2 + 2b\,(h - 50)} \qquad \text{eq (38)}$$

$$\theta_{50} = \sin^{-1}\left[\frac{(h - 50)^2 + 2b\,(h - 50) - (R - R_{50})^2}{2b\,(R - R_{50})}\right] \qquad \text{eq (39)}$$

$$\theta = \theta_{50} + \sin^{-1}\left[\frac{(R - R_{50}) \cos \theta_{50}}{h + 6356.77}\right] \qquad \text{eq (40)}$$

$$b = 6406.77 \text{ km.} \qquad \text{eq (41)}$$

These equations (37) - (41) can be used with Figure 2 to determine the geometry of any infrared path, regardless of range, with respect to the Earth.

APPENDIX B

AMOUNTS OF GASES AND VAPORS VS. ALTITUDE

Part 1) H_2O Vapor

Figure 17 shows the mixing ratio and relative humidity of water vapor as a function of altitude. Constant relative humidity (100%) is assumed below the tropopause and constant mixing ratio is assumed above the tropopause. The mixing ratio in the troposphere was determined from assuming the temperature and pressure is that of the 1959 ARDC model atmosphere. A psychrometric chart was used to obtain the amount of water vapor per unit volume as a function of temperature.

The assumption of constant relative humidity in the troposphere is considered justified for the following reasons. Data giving averages of water vapor versus altitude for Columbia, Pittsburgh, Washington, D.C., and Cape Hatteras were compiled by Martin, reference 8. Since he assumes square root pressure broadening while this report assumes linear pressure broadening, some of the data was converted to H_2O vapor mixing ratio versus altitude. Figure 52 shows plots of such data for Washington D.C. Martin's data was converted to water vapor mixing ratio by assuming that the average weather conditions coincided with the 1959 ARDC model and atmosphere corrected for temperature. The temperature correction assumes that the lapse rate remains the same as in the ARDC model, but the sea level temperature is that value indicated in Figure 52. These sea level temperatures were obtained from the Weather Almanac, reference 9. From Figure 52, it can be seen that the relative humidity remains at about 80% in the troposphere for altitudes above 1.5 km. For simplicity, a model of 100% relative humidity is assumed in the tropopause. The corrections can be easily made for existing weather conditions. Otherwise, assume 80% relative humidity to correct the model, see Part 5) of Section III.

The assumption of constant mixing ratio in the stratosphere is made due to data obtained from reference 10.

Part 2) CO_2

Carbon dioxide appears to be uniformly distributed in the atmosphere with a concentration of 320 parts per million of air, see Table 1. This data was obtained from Goldberg, reference 11.

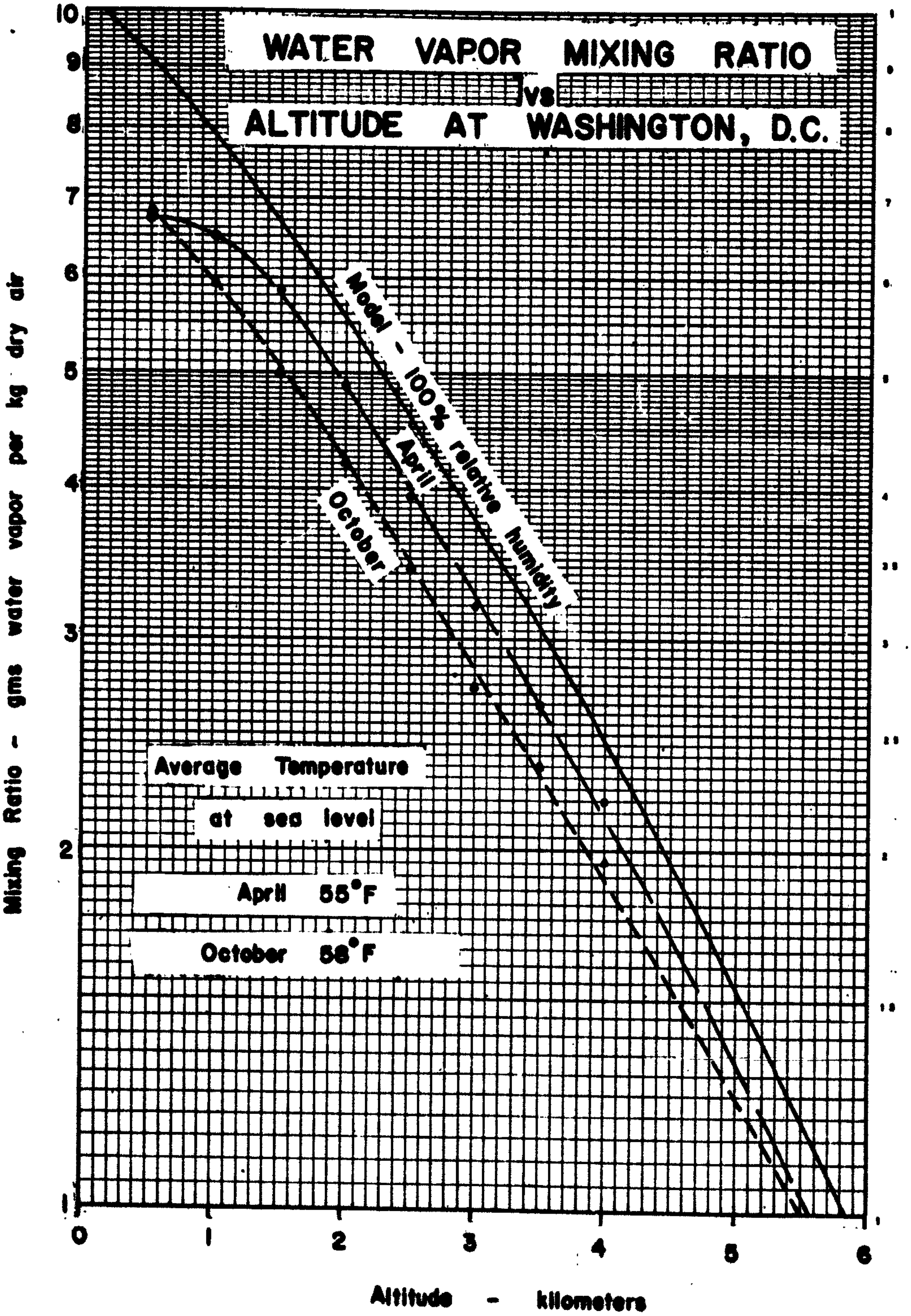

WATER VAPOR MIXING RATIO vs ALTITUDE AT WASHINGTON, D.C.
Model - 100% relative humidity
April
October
Average Temperature at sea level
April 55°F
October 58°F
Mixing Ratio - gms water vapor per kg dry air
Altitude - kilometers
10
9
8
7
6
5
4
3
2
1
0
1
2
3
4
5
6

Figure 52

Part 3) 0_3

The distribution of ozone in the atmosphere is assumed to be that as shown in Figure 18. The maximum ozone concentration is considered to exist at an altitude of 23 km.

This Figure, 23 was obtained by averaging values of ozone concentration at Flagstaff, Arizona, as given by Miller, reference 12. For sea level, the value of ozone concentration was taken to be 2.6 parts per million from Taylor and Yates, reference 13.

Part 4) N_20

Nitrous oxide appears to be uniformly distributed in the atmosphere with a concentration of 0.27 parts per million, see Table 1. Birkeland and Shaw, reference 14, have measured the abundance of nitrous oxide in ground level air. Goldberg and Müller, reference 15, state that the mixing ratio is uniform for N_20 versus altitude.

Part 5) CH_4

Methane appears to be uniformly distributed in the atmosphere with a concentration of 2.4 parts per million, see Table 1. Shaw, reference 15, measured the concentration of methane. Goldberg and Müller, reference 15, state that the mixing ratio of methane is uniform versus altitude.

Part 6) C0

Carbon monoxide appears to be uniformly distributed in the atmosphere with a concentration of 1.1 parts per million according to Shaw, reference 15. See Table 1.

APPENDIX C

DETERMINATION OF AMOUNTS OF GASES AND VAPORS IN THE INFRARED PATH

Part 1) Determination of w, d, and z

The transmission of infrared through the atmosphere is a complex function of the quantity of gas, its pressure and temperature, and the wavelength region under consideration. Section III covered methods for determining the quantity of gas present in the infrared path reduced to the common denominator of sea level conditions, i.e. 760 mm Hg pressure and 288°K temperature. Figures 5 through 14 were derived in the following manner.

Let: L_o = generalized absorption coefficient, see eq (48)
n = exponent, varies from 0 to 1.0.
P = pressure
P_o = pressure = 760 mm Hg
q = sea level equivalent quantity of a gas, i.e. w, d, or z.
R = distance (range) of the infrared path, km.
S = total intensity of a spectral line.
T = temperature, degrees Kelvin
T_o = temperature = 288°K
u = actual quantity of a gas or vapor
u_h = actual quantity of a gas or vapor per kilometer horizontally at altitude (h).
α = half width of a spectral line
α_o = half width of spectral line at sea level conditions
δ = average distance between lines in wavelength region (λ).
λ = wavelength, microns.
$\tau(\lambda)$ = transmission of a gas at wavelength region (λ) covering at least 15 spectral lines.

Elsasser, reference 17, Goody, reference 18, King, reference 19, and Pass, reference 20, have developed theories for predicting the transmission of infrared through various gases. These theories are represented by equations (42), (43), and (44) as:

$$\tau(\lambda) = f(u, \alpha, \delta, S) \qquad \text{eq (42)}$$

For small quantities of gases, the transmission reduces to:

$$\tau(\lambda) = 1 - \frac{Su}{\delta} \qquad \text{eq (43)}$$

For large quantities of gases when the atmospheric pressure is sufficiently low,

$$\tau(\lambda) = f\left(\frac{Su\alpha}{\delta^2}\right) \qquad \text{eq (44)}$$

For intermediate regions, the transmission can be approximated by a function such as:

$$\tau(\lambda) = f\left[\left(\frac{Su}{\delta}\right)\left(\frac{\alpha}{\delta}\right)^{n}\right] \qquad \text{eq (45)}$$

where n varies from 0 to 1.

It can be seen that equation (45) reduces to equations (43) or (44) if n = 0 or n = 1.

The spectral line intensity (S), the average line spacing (δ), and the half width (α) is a function of the type of gas and the wavelength (λ) under consideration. The spectral lines are broadened due to natural broadening. Doppler brordening, and Lorentz broadening. According to Howard, Burch, and Williams$^{(4)}$, the half widths (α) for carbon dioxide and water vapor are:

$\alpha_N \approx 10^{-4}$ cm^{-1} due to natural broadening.

$\alpha_D \approx 10^{-3}$ cm^{-1} due to Doppler broadening.

$\alpha_L \approx 10^{-1}$ cm^{-1} due to Lorentz broadening.

Conditions are similar for ozone and for nitrous oxide. Kinetic theory predicts that the Lorentz half width is linearly proportional to pressure and inversely proportional to the square root of the absolute temperature. Plass,$^{(20)}$ predicts that even though the Doppler width is considerably larger than the Lorentz width, the infrared absorption for large values of $\frac{Su}{2\pi d}$ is determined essentially only by the Lorentz width. This means that the transmission values of less than 90%, the Lorentz broadening predominates for altitudes less than 50 km. Therefore,

$$\alpha = \alpha_o \frac{P}{P_o}\sqrt{\frac{T_o}{T}} \qquad \text{eq (46)}$$

When this equation is combined with equation (45)

$$\tau(\lambda) = f\left[L_o u\left(\frac{P}{P_o}\sqrt{\frac{T_o}{T}}\right)^{n}\right] \qquad \text{eq (47)}$$

where: $$L_o = \frac{\pi S \alpha_o^{\,n}}{d^{\,n+1}} \qquad \text{eq (48)}$$

If, $$q = u\left(\frac{P}{P_o}\sqrt{\frac{T_o}{T}}\right)^n \qquad \text{eq (49)}$$

Then, $$\tau(\lambda) = f\left[L_o q\right] \qquad \text{eq (50)}$$

The problem is therefore to find the value of (q) within the infrared path.

$$q = \int_o^R u_h \left(\frac{P}{P_o}\sqrt{\frac{T_o}{T}}\right)^n dR \qquad \text{eq (51)}$$

The pressure and temperature are assumed to be those of the ARDC model atmosphere. The values of (u) are taken from Table 1 and Figures 17 and 18. Figures 5 through 14 were obtained by solution of equation (51) over the infrared path. Altshuler,(21) has done this previously, but square root pressure broadening, $(n = \frac{1}{2})$, was used. This report supersedes reference (21) and uses better values of (n). Carpenter,(22) uses linear pressure broadening (n = 1.0), and solves equation (51) mathematically for water vapor and carbon dioxide.

Part 2) Determination of n.

Figure 53 shows the effect of α/δ upon absorption. For carbon dioxide under conditions normally found in the atmosphere, the α/δ ratio, varies from 0.04 at sea level to 0.001 at an altitude of 15 kilometers. It can be seen that there is a slight variation in the absorption curves below 20-percent absorption. These curves are Elsasser model curves; the information was taken from Plass,(20) The region below the shaded area gives an absorption A_2 which follows the linear absorption law. In this region the line half width has no effect upon absorption, i.e. n = 0. The region above the shaded area gives an absorption A, which follows the error function law. Note that the power of (α) is the same as the power of (u), i.e. n = 1.0. The shaded area is the intermediate region, and the Howard, Burch, and Williams data,(4) fall partially within this region. However, for most infrared system applications, fairly long paths are involved and (α) has the same power of (u). For this reason, it will be assumed that n = 1.0 for carbon dioxide. Actually, the infrared absorption of carbon dioxide deviates from the Elsasser model above 80% absorption, probably due to clustering of spectral lines, see Appendix F. However, this does not affect the foregoing discussion.

For water vapor, conditions are similar to carbon dioxide, where the α/δ ratio is approximately 0.055 at sea level. Thus n = 1.0 for most applications.

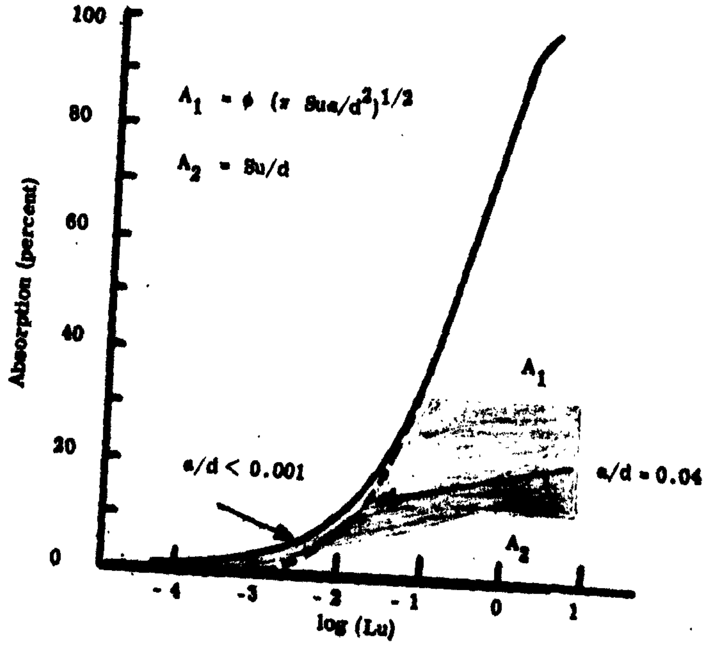

100
80
60
40
20
0
Absorption (percent)
$A_1 = \phi\ (\pi\ Sua/d^2)^{1/2}$
$A_2 = Su/d$
A_1
$a/d < 0.001$
$a/d = 0.04$
A_2
- 4
-3
- 2
- 1
0
1
log (Lu)

Figure 53

For ozone, the $^{\alpha}/_{\delta}$ ratio is approximately 1.1 at sea level. Thus, for most applications, the pressure broadening exponent varies from 0.1 to 0.6. By analyzing data from Walshaw,(23) it is considered that the exponent $n = 0.3$ will give the best results for correcting ozone at the altitudes of maximum concentration to its sea level equivalent.

For nitrous oxide, the $^{\alpha}/_{\delta}$ ratio is approximately 0.17 at sea level. This means that for many applications the exponent of $n < 1.0$. Burch,(6) suggests that $n = 0.8$ for his data. However, serious errors are not introduced if $n = 1.0$ and such a value will be assumed for the sake of simplicity in calculations.

Summarizing:

$n = 1.0$ for CO_2
$n = 1.0$ for H_2O vapor
$n = 0.3$ for O_3
$n = 1.0$ for N_2O

APPENDIX D

CORRECTION FOR WEATHER CONDITIONS

To correct for existing weather conditions, corrections for w, d, and z are made for deviations of existing weather from the model at the altitude of maximum gas concentration in the infrared path. This procedure reduces errors based on the assumption that:

$$\frac{q\ \text{(corrected)}}{q_h\ \text{(corrected)}} = \frac{q\ \text{(model)}}{q_h\ \text{(model)}} \qquad \text{eq (52)}$$

Where (q) is the sea level equivalent quantity of a gas, i.e. w, d, or z. Equation (52) is approximately true if $(\frac{\Delta P}{P})$ and $(\frac{\Delta T}{T})$ remain constant for altitudes near those of the maximum gas concentration in the infrared path. The theory for weather corrections follows:

Let: C_{gas} = temperature correction factor for (d)
C_{H_2O} = temperature correction factor for (w)
d = sea level equivalent amount of CO_2, N_2O, CH_4, and CO.
f = fractional concentration of gas
n = exponent, varies from 0 to 1, see Appendix C
P = pressure (actual)
P_m = pressure (model)
P_o = pressure = 760 mm Hg
ΔP = $P - P_m$
q = sea level equivalent quantity of gas in the infrared path, i.e. w, d, or z.
q_h = sea level equivalent quantity of gas per kilometer horizontal path length.
T = temperature (actual) °K
T_m = temperature (model) °K
T_o = temperature = 228°K
ΔT = $T - T_m$
R.H. = relative humidity
u = actual quantity of a gas or vapor.
u_h = actual quantity of a gas or vapor in a kilometer horizontal path length.
x = fractional concentration of water vapor at 100% relative humidity to dry air at 760 mm Hg and at temperature (T)
y = conversion factor - converts units of fractional concentration at 760 mm Hg and 288°K to units of w, d, or z.

$$u_h = y\,f\,\frac{P}{P_o}\left(\frac{T_o}{T}\right)$$

$$q_h = u_h\left(\frac{P}{P_o}\sqrt{\frac{T_o}{T}}\right)^{n} \qquad \text{eq (44)}$$

$$q_h = y\,f\left(\frac{P}{P_o}\right)^{n+1}\left(\frac{T_o}{T}\right)^{\frac{n}{2}+1} \qquad \text{eq (53)}$$

From equation (52)

$$q \text{ corrected} = q \text{ model}\ \frac{q_h\ (\text{corrected})}{q_h\ (\text{model})}$$

Combining with equation (53)

$$q \text{ corrected} = q \text{ model}\ \frac{f}{f_{\text{model}}}\left(\frac{P}{P_m}\right)^{n+1}\left(\frac{T_m}{T}\right)^{\frac{n}{2}+1} \qquad \text{eq (54)}$$

Part 1) to Correct (w)

Since (x) for water vapor is measured at 760 mm Hg pressure and the amount of water vapor per unit volume depends only upon temperature and relative humidity and not upon external pressure, then,

$$f = (\text{R.H.})\,(x)\,\frac{P_o}{P}$$

To convert the fractional concentration compared to dry air at 760 mm Hg and 288°K to precipitable centimeters.

$$y = 122.56$$

From Appendix C, $n = 1.0$ Thus, from equation (53)

$$w_h = (122.56)\,(\text{R.H.})\,(x)\left(\frac{P}{P_o}\right)\left(\frac{T_o}{T}\right)^{3/2} \qquad \text{eq (10)}$$

This is equation (10), Case (b), Part 5, Section III.

By obtaining values of (x) from a psychrometric chart, a formula was devised which will give values of x within 10% accuracy between -60°C and +55°C.

$$x = 3.80 \times 10^{-3}\, e^{(T-273)\left(\frac{40.2}{T}\right)^{1.345}} \qquad \text{eq (55)}$$

Since n = 1.0, see Appendix C, equation (54) becomes:

$$w_{corrected} = w_{model} \frac{R.H.}{R.H._{model}} \left(\frac{P}{P_m}\right)\left[\frac{x}{x_{model}} \left(\frac{T_m}{T}\right)^{3/2}\right] \qquad \text{eq (56)}$$

The item in the bracket is only temperature dependent

Let:

$$C_{H_20} = \frac{x}{x_{model}} \left(\frac{T_o}{T}\right)^{3/2} \qquad \text{eq (57)}$$

$$\frac{P}{P_m} = 1 + \frac{\Delta P}{P_m} \approx 1 + \frac{\Delta P}{P}$$

Then, equation (56) becomes

$$w_{corrected} = (w_{model}) \times \frac{R.H.}{R.H._{model}} \times \left(1 + \frac{\Delta P}{P}\right) \times C_{H_20} \qquad \text{eq (7)}$$

This is equation (7), see Case (a) part 5, Section III. Values for C_{H_20} from equations (55) and (57) were obtained by a Librascope LGP 30 computer and plotted in Figure 15.

Part 2) To Correct (d)

To convert fractional concentration to equivalent sea level kilimeters,

$$y = \frac{1}{f_{model}}$$

From Appendix C, n = 1.0 Thus, from equation (53)

$$d_h = \left(\frac{f}{f_{model}}\right)\left(\frac{P}{P_o}\right)^2 \left(\frac{T_o}{T}\right)^{3/2} \qquad \text{eq (11)}$$

This is equation (11), Case (b), Part 5, Section III.
Equation (54) also becomes:

$$d_{corrected} = d_{model} \frac{f}{f_{model}}\left(\frac{P}{P_m}\right)^2 \left(\frac{T_m}{T}\right)^{3/2} \qquad \text{eq (58)}$$

$$\text{Let: } C_{gas} = \left(\frac{T_m}{T}\right)^{3/2} \qquad \text{eq (59)}$$

$$\text{and: } \left(\frac{P}{P_m}\right)^2 = \left(1 + \frac{\Delta P}{P_m}\right)^2 \approx 1 + 2\frac{\Delta P}{P_m} \approx 1 + 2\frac{\Delta P}{P} \qquad \text{eq (60)}$$

by neglecting higher orders of $\frac{\Delta P}{P_m}$ and letting $\frac{\Delta P}{P_m} = \frac{\Delta P}{P}$

Combining equations (58), (59), and (60)

$$d_{corrected} = d_{model} \frac{f}{f_{model}} \left[1 + 2 \frac{\Delta P}{P} \right] \times C_{gas} \qquad \text{eq (8)}$$

This is equation (8), see Case (a), Part 5 Section III. Values for C_{gas} are plotted in Figure 16.

Part 3) To Correct (z)

From Appendix C, n = 0.3 Thus, from equation (53), and letting $u_h = u$

$$z_h = u \left(\frac{P}{P_o} \right)^{0.3} \left(\frac{T_o}{T} \right)^{0.15} \qquad \text{eq (12)}$$

This is equation (12), Case (b), Part 5 Section III.

Equation (54) also becomes

$$z_{corrected} = z_{model} \frac{u}{u_{model}} \left(\frac{P}{P_m} \right)^{0.3} \left(\frac{T_m}{T} \right)^{0.15} \qquad \text{eq (61)}$$

$$\left(\frac{P}{P_m} \right)^{0.3} = \left(1 + \frac{\Delta P}{P_m} \right)^{0.3} = \left(1 + \frac{\Delta P}{P} \right)^{0.3} = 1 + 0.3 \frac{\Delta P}{P} \qquad \text{eq (62)}$$

$$\left(\frac{T_m}{T} \right)^{0.15} = \left(1 - \frac{\Delta T}{T} \right)^{0.15} = 1 - 0.15 \frac{\Delta T}{T} \qquad \text{eq (63)}$$

By neglecting higher orders of $\frac{\Delta P}{P}$ and $\frac{\Delta T}{T}$

Combining equations (61), (62), and (63)

$$z_{corrected} = z_{model} \frac{u}{u_{model}} \left[1 + 0.3 \frac{\Delta P}{P} \right] \left[1 - 0.15 \frac{\Delta T}{T} \right] \qquad \text{eq (9)}$$

This is equation (9), see Case (a), Part 5 Section III. Values for u are normally considered the same as for u_h.

APPENDIX E

CHANGES IN THE ALTITUDE OF TROPOPAUSE AND THE ALTITUDE OF MAXIMUM OZONE CONCENTRATION

In correcting for the changes in altitude of the tropopause and the maximum ozone concentration, fictitious altitudes are used to place the infrared path in the correct relative position to the "knee" of the concentration versus altitude curves, see Figures 17 and 18. Then corrections must be made due to changes in concentration and pressure broadening of the gases and vapors. The principles of such corrections are as follows:

Let: h = altitude, real
h' = fictitious altitude
h_{trop} = altitude of tropopause
$h_{max(O_3)}$ = altitude of maximum ozone concentration.
k_{H_2O} = correction factor for water vapor when h' is used.
k_{O_3} = correction factor for ozone when h' is used.
P = pressure
P_o = 760 mm Hg
T = temperature
T_o = 288°K
w' = fictitious value of w_h, w_v, $w_{h-\infty}$, and w.
z' = fictitious value of z_h, z_r, $z_{h-\infty}$, and z.

Part (1) Changes in Altitude of Tropopause

A model whose tropopause altitude is different than 11 km. has the following characteristics.

1. The relative humidity is 100% below the tropopause.
2. The mixing ratio is constant above the tropopause.
3. The temperature at sea level = 288°K
4. The temperature in the stratosphere remains constant.
5. The lapse rate is -6.5°C per kilometer altitude (increasing altitude sign is +) in the troposphere.

If: $h' = h + 11.0 - h_{trop}$ eq (14)

and h' is used with figures 5 and 6 to get w_h, then

$$w_h \text{ (model)} = k_{H_2O} \cdot w'_h \quad \text{eq (64)}$$

Values for (k_{H_2O}) were found by determining (w_h) analytically and comparing those values with $h_{trop} = 11.0$ km. to those with $h_{trop} \neq 11.0$ km. This was done for various altitudes of (h) and (h') where $h' = h + 11.0 - h_{trop}$.

Furthermore, it is assumed that

$$\frac{W_v \text{ (model}}{W'_v} = \frac{W_h \text{ (model)}}{W'_h} = k_{H_2O} \qquad \text{eq (65)}$$

for the same values of (h) and (h_{trop}). The errors involved is less than 10%. Similarly, for any infrared path,

$$W \text{ (model)} = k_{H_2O} \cdot W' \qquad \text{eq (15)}$$

The values for k_{H_2O} are plotted in Figure 22 and equation (15) is given in Case a) Part 6, Section III.

Part (2) Changes in Altitude of Maximum Ozone Concentration

It is assumed that the same ozone concentration as in Figure 18 exists except the altitude scale represents (h') instead of (h), where

$$h' = h + 23.0 - h_{max(O_3)} \qquad \text{eq (16)}$$

From equation (12)

$$z_h = u\left[\frac{P}{P_o}\right]^{0.3}\left[\frac{T_o}{T}\right]^{0.15} \qquad \text{eq (12)}$$

where u = concentration of ozone in a horizontal path.

For 12 km $\leqslant$ h $\leqslant$ 40 km.

$$\left(\frac{P}{P_o}\right)^{0.3}\left(\frac{T_o}{T}\right)^{0.15} = e^{(-0.0460 \cdot h + 0.0921)} \qquad \text{eq (65a)}$$

Equation (65a) is an empirical equation based on the 1959 ARDC model atmosphere, see Figures 19 and 20.

$$z_h\text{(model)} = u_{model}\left[e^{(-0.0460 \cdot h + 0.0921)}\right]$$

$$z_h' = u'\left[e^{(-0.0460 \cdot h' + 00921)}\right]$$

Let: $z_h(\text{model}) = k_{0_3} \cdot z'_h$ eq (66)

$$k_{0_3} = \frac{z_h(\text{model})}{z'_h} = \frac{u_{\text{model}}}{u'}\left[e^{0.0460(h' - h)}\right] \quad \text{eq (67)}$$

Since: $u_{\text{model}} = u'$

From equations (67) and (51), it is apparent that:

$$k_{0_3} = e^{0.0460(h' - h)} = \frac{z_h\,(\text{model})}{z'_h} = \frac{z\,(\text{model})}{z'}$$

since $(h' - h) = 23.0 - h_{\max(0_3)}$ = constant, equation (16)

Therefore:

$$z_{\text{model}} = z' \cdot k_{0_3} \quad \text{eq (17)}$$

And:

$$k_{0_3} = e^{0.0460(h' - h)} = e^{0.0460(23.0 - h_{\max(0_3)})} \quad \text{eq (68)}$$

The values of k_{0_3} are plotted in Figure 23 and equation (17) is given in Case b) Part 6, Section III.

APPENDIX F

TRANSMISSION OF INFRARED THROUGH GASES AND VAPORS

The transmission of infrared through various gases, as a function of wavelength, is a complex function of spectral line intensity, line half width, average spacing between lines, and the distribution of the lines with respect to each other. Elsasser,(17) Goody,(18) King,(19) and Plass,(20) have developed theories to determine transmission functions for various spectral line models. The graph paper in Figures 24-44 employs transmission functions, and were developed as follows:

Let: L_o = generalized absorption coefficient, see equation (48), a function of (λ)
L = generalized absorption coefficient, see equation (69), a function of (λ)
n = exponent, varies from 0 to 1.0
q = sea level equivalent quantity of a gas, i.e. w, d, or z
S = total intensity of a spectral line, a function of (λ)
u = actual quantity of a gas or vapor
α = half width of a spectral line
α_o = half width of a spectral line at sea level conditions, i.e. 760 mm Hg and 288°K
β = defined in equation (73)
δ = average distance between lines in wavelength region (λ)
λ = wavelength, microns
τ = transmission
$\tau(\lambda)$ = transmission in wavelength region (λ).

As was discussed in Appendix C, the transmission function can be approximated by:

$$\tau(\lambda) = f\left[\left(\frac{Su}{\delta}\right)\left(\frac{\alpha}{\delta}\right)^{n}\right] \qquad \text{eq (45)}$$

where n varies from 0 to 1.0

If:

$$L_o = \frac{\pi S \alpha_o}{d^{n+1}} \qquad \text{eq (48)}$$

$$(\lambda) = f\left[L_o q\right] \qquad \text{eq (50)}$$

Similarly, if:

$$L = \frac{\pi S\alpha}{d^{n+1}} \qquad \text{eq (69)}$$

$$\tau(\lambda) = f\left[Lu\right] \qquad \text{eq (70)}$$

Thus, the transmission is a function of (L_oq), and vice versa.

$$L_oq = g\left[\tau(\lambda)\right] = Lu \qquad \text{eq (71)}$$

$$\log_{10} L_o + \log_{10} q = \log_{10}\left\{g\left[\tau(\lambda)\right]\right\} = \log_{10} L + \log_{10} u \qquad \text{eq (72)}$$

Therefore, if the function $g\left[\tau(\lambda)\right]$ is known, a logarithm of that function can be plotted as a graphical scale versus wavelength as the abscissa. It can then be seen that transmission curves of a gas for a given value of (q) has the same shape as curves for any other value of (q) Figures 24 - 45 were developed in such a manner.

In order to determine the function $g\left[\tau(\lambda)\right]$, it can be seen from equations (48) and (50) that:

$$L_o = \frac{\pi S\alpha_o}{\delta^{n+1}} \qquad \text{eq (48)}$$

and

$$\tau(\lambda) = f(L_o q) \qquad \text{eq (50)}$$

It can be assumed that (S) is only a function of wavelength (λ) for a given gas, and that (λ_o) and (δ) remain constant over the bands of absorption of a gas. However, (n) is a function of (S), (q), (α), and (δ). Transmission [illegible] therefore are a function of (λ), (n), and (q) for a given gas. In order to [illegible] the problem, (n) is assumed to remain constant. Then for a given gas, $[\tau($ [illegible] is only a function of (q) and (λ). The errors in making such an assumption [illegible] presented in Table 2. The table shows the values of transmission where the [illegible]ference is maximum between the "model", used in Figures 24 - 44, and the "actual" transmission as predicted by Plass,(20) Plass shows the variation of transmission as a function of (β) and (τ) as well as variations between the Elsasser, the Goody, and the Plass models of a spectral line distribution.

$$\beta = 2\pi\alpha/_\delta \qquad \text{eq (73)}$$

TABLE 2

DESCRIPTION		H_2O	CO_2	O_3		N_2O	
Type of model		Goody	Exp. #1	Elsasser		Elsasser	
δ (cm^{-1} at STP)*		2.0	1.5	0.09		0.84	
α (cm^{-1} at STP)*		0.11	0.06	0.1		0.14	
Band		6.3 μ rotation	4.3 μ	9.6 μ		17 μ 7.8 μ	
$\beta = 2\pi \frac{\alpha}{\delta}$		0.35	0.25	6.9		1.1	
0 km. altitude	τ (model)[†]	84%	93%	83%	20%	83%	20%
	τ (actual)[†]	90%	93%	93%	7%	93%	13%
10 km. altitude	τ (model)[†]	96.3%	93%	83%	20%	83%	20%
	τ (actual)[†]	97.8%	89%	92%	8%	87%	14%
20 km. altitude	τ (model)[†]	99.35%	93%	83%	20%	83%	20%
	τ (actual)[†]	99.60%	88%	86%	17%	84%	19%
30 km. altitude	τ (model)[†]	99.92%	93%	83%	20%	83%	20%
	τ (actual)[†]	99.95%	88%	84%	19%	83%	20%

*Values of α and δ were taken from Curtis and Goody, reference 34 for H_2O, O_3, and N_2O. For CO_2, they were taken from Howard, Burch, and Williams, reference 4.

[†] The maximum difference between the actual transmission and the transmission of the model are at those transmission values indicated in the Table, except for transmissions of less than 20% for H_2O, O_3, and N_2O. The models and the actual transmission values were made to coincide at 50% transmission.

Both the "model" and the "actual" transmission were made to coincide at 50% transmission, which is most representative of the actual methods employed in making Figures 24 - 44.

It has just been seen that (L_0) is only a function of (λ) and the type of gas considered, and that $\left[\tau(\lambda)\right]$ is a function of (L_0q). The methods described in Section III were used to determine (q). The values of (L_0) were found from the sources described in Table 3. The transmission functions, described as "type of model" in Table 2, will be covered for each gas in Parts (1) - (4) as follows. Part (5) will cover the absorption laws for overlapping bands and the justification of equation (18) in Part (1), Section IV. Part (6) covers the molecular absorption of infrared by small quantities of gases and vapors.

TABLE 3

Gas or Vapor	λ	ν	P	p	P_e	T	u	n	q	Ref
CO_2							atmo-cm		atmo-cm	
	1.3-1.7	6000-7000	760	50.0	777	295	8100	0.80	8200	4
	1.8-2.1	4600-5400	760	50.0	777	295	8630	0.78	8700	4
	2.5-3.1	3500-3700	737	1.0	738	295	22	0.86	21	4
	2.5-3.1	3500-3700	755	10.0	758	295	1619	0.86	1614	4
	4.0-5.4	1800-2500	1.0	1.0	1.3	295	18	0.80	0.11	4
	4.0-5.4	1800-2500	734	9.8	737	295	1043	0.80	1018	4
	9.0-11	910-1100	760	608	942	295	5600	0.80	6580	24
	9.0-11	910-1100	6080	6080	7900	295	8000	0.27	15,100	25
	12-19	550-850	750	0.25	750	295	3	0.88	2.9	4
	12-19	550-850	745	4.0	746	295	173	0.88	169	4
H_2O							pr. cm.		pr. cm.	
	* 1.0-1.05	95-10,000								
	1.05-1.2	8200-9500			740	295	3	0.52	2.9	4
	1.05-2.2	4600-9500			740	295	50	0.60	49	4
	1.05-2.2	4600-9500			740	295	0.05	0.60	0.049	4
	1.2 -2.2	4600-8250			740	295	0.71	0.60	0.70	4
	2.3 -3.65	2800-4400	740	16.3	821	295	1.68	0.62	1.75	4
	2.3 -3.65	2800-4400	740	16.3	821	295	0.140	0.62	0.147	4
	2.3 -3.65	2800-4400	740	2	750	295	0.017	0.62	0.017	4
	* 3.65-4.5	2200-2800								
	4.5 -9.1	1100-2200			746	295	1.03	0.60	1.02	4
	4.5 -9.1	1100-2200			746	295	0.047	0.60	0.046	4
	4.5 -9.1	1100-2200	123	2.5	136	295	0.021	0.70	0.006	4
	** 9.1 -13	1100-770			760	291	4.18	1.0	4.18	(run 70)

TABLE 3

Gas or Vapor	λ	ν	P	p	P_e	T	u pr.cm.	n	q pr.cm.	Ref
H_2O**	9.1-13	1100-770			760	285	6.7	1.0	6.7	1(run 61)
**	9.1-13	1100-770			760	298	0.57	1.0	0.57	1(run 60)
*	13-16	625-770								
**	16-20	500-625			760	298	0.57	1.0	0.57	1(run 60)
	20-30	333-500	606	4.0	626	290	0.074	1.0	0.061	5
	20-30	333-500	600	1.6	608	290	0.0325	1.0	0.026	5
	20-30	333-500	590	0.2	591	290	0.0050	1.0	0.0039	5
	30-40	250-333	603	0.15	604	283	0.00318	1.0	0.00255	5
	40-2500	4-250			760	293	0.0104	1.0	0.0103	26
Gas. O_3					†		atmo-cm.	††	atmo-cm.	
	4.4-5.1	1960-2270	230	230	370	295	9.08	0.3	7.38	27, 28
	8.0-9.3	1075-1250	160	160	257	295	6.32	0.3	4.58	27
***	9.3-10.2	980-1075					0.31		0.13	29
	12.0-16.5	606-833	160	160	257	295	6.32	0.3	4.58	27
N_2O							atmo-cm.		atmo-cm.	
****	3.8-4.1	2240-2630							0.062	30, 31
	4.24-4.36	2290-2360	400	400	448	300	12.6	0.8	8.1	32
	4.4 -4.7	2130-2270			750	295	5.8	0.7	5.7	6
	4.4 -4.7	2130-2270			759	295	0.0101	0.7	0.0100	6
	7.6 -8.1	1235-1315	100	100	112	288	0.79	0.8	0.17	33
	8.5 -9.1	1100-1175	240	240	269	288	1.9	0.8	0.83	33
	16.0-18.0	555-625	160	160	179	288	5.5	0.8	1.73	33

The atmospheric scattering was taken from reference 1.

* Curve was estimated to fit the transmission at the extremities of the wavelength interval

** The transmission was determined by correcting for scattering using σ = scattering coefficient = 0.03/km.

*** The same vertical ozone distribution as the model was assumed. Thus, from Figures 9 and 18:

$$\frac{q}{u} = \frac{q_v}{u_v} = \frac{0.098}{0.229} = 0.428$$

**** The value of q = 0.062 atmo-cm. was obtained by matching the transmission of the 4.5 μ band of N_2O from reference 30 with that obtained from reference 6. The magnitude of the transmission of the 3.8 - 4.1 μ band of N_2O was obtained from reference 30 while details of the shape of the transmission curve comes from reference 31.

TABLE 3

† $P_\epsilon = P + 0.3p$ for CO_2 from Burch,(6)

$P_\epsilon \simeq P + 5p$ for H_2O from Burch,(6)

$P_\epsilon = P + 0.61p$ for O_3 from Walshaw,(23)

$P_\epsilon = P + 0.12p$ for N_2O from Burch,(6)

†† The values for (n) are those given with the data if available. Otherwise they are assumed to equal those values summarized in Appendix C.

Let: n = exponent, varies from 0 to 1.0

p = partial pressure, mm Hg

P = total pressure, mm Hg

P_o = 760 mm Hg

P_ϵ = effective pressure, mm Hg

q = equivalent quantity of gas corrected to sea level conditions, i.e. pressure = 760 mm Hg, temperature = 288°K

T = absolute temperature, degrees Kelvin. Room temperature is assumed to be 295°K

T_o = sea level temperature = 288°K

u = actual quantity of gas from which spectral measurements were made.

λ = wavelength - microns

ν = frequency - cm^{-1}

$$q = u\left[\frac{P_\epsilon}{P_o}\sqrt{\frac{T_o}{T}}\right]^n \qquad \text{eq (49)}$$

Part (1) CO_2

In order to obtain the transmission function of carbon dioxide, data from Howard, Burch, and Williams,(4) was analyzed. The absorption within a narrow wavelength interval could be plotted versus the logarithm of various values of (u) of the gas, all at constant pressure, see Figure 54. These data points can then be superimposed upon the curve of an absorption model by adding the logarithm of the absorption coefficient (L) to the logarithm of the quantity of gas (u). This can be done by sliding the data points horizontally until they lie upon the curve with a minimum of scatter, see Figure 54. Equation (72) justifies this procedure.

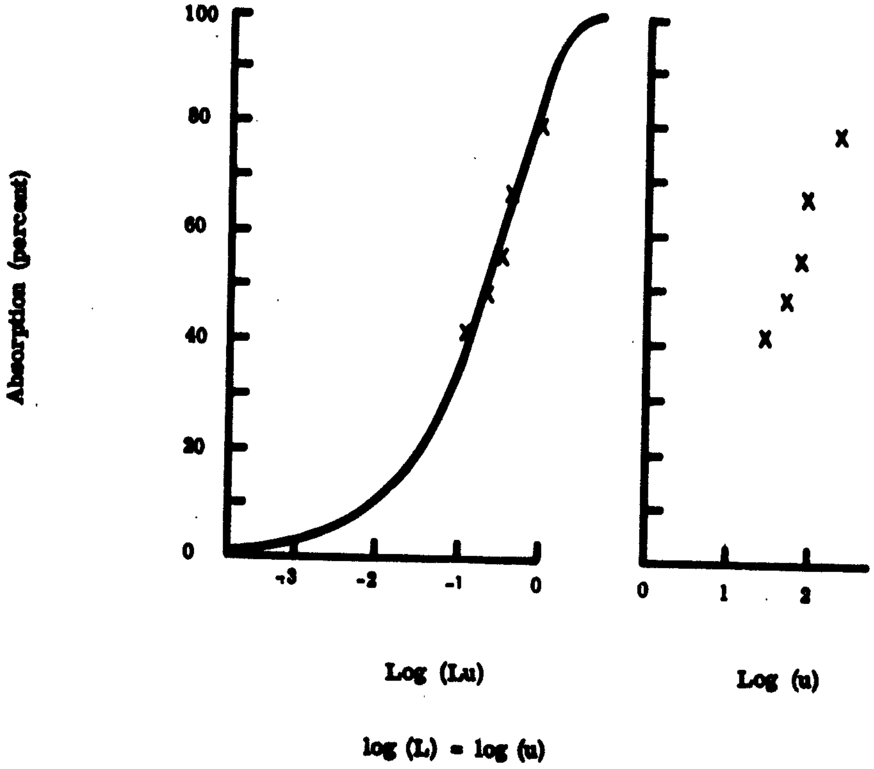

Absorption (percent)
100
80
60
40
20
0
-3
-2
-1
0
0
1
2
Log (Lu)
Log (u)
log (L) = log (u)

Figure 54

The data at the 4.3 μ band was fitted to an Elsasser model with n = 1.0, see Figure 55. The heavy lines bounding most of the data points are limits within which 95% of the data will fall.

The data was then fitted to various empirical curves until one was found giving the least scatter of data points, see Figure 56. The heavy lines bounding most of the data points are limits within which 95% of the data will fall. This procedure was considered necessary because the absorption data which can normally be obtained in a multiple traversal cell spectroscope are limited to values of (u) of less than two orders of magnitude for the gas being measured when other parameters are kept constant. Thus the data could appear to satisfactorily fit a number of different curves.

Figure 57 compares the Elsasser model with the experimental curve #1, see Figure 56. At low absorption values, the experimental curve #1 shows less absorption than the Elsasser model because ($n < 1.0$). At high absorption values, the experimental curve #1 shows less absorption than the Elsasser model probably due to some irregularity or clustering of the spectral lines.

Figures 32 - 38 use the experimental curve #1 for the transmission model.

Part (2) H_20 Vapor

In Figures 24 - 31, the transmission function was assumed to obey the Goody model. This model states:

$$\tau(\lambda) = e^{-}\left[(L_0 q)^{1/2}\right] \qquad \text{eq (74)}$$

and: n = 1.0 is assumed in equation (48)

According to Curtis and Goody,$^{(34)}$ they applied the Goody model to the 6.3 rotation band of water vapor. Burch, Howard and Williams,$^{(35)}$ applied the Goody model to their water vapor data at the 2.7 μ and 3.2 μ bands. Table 2 shows the relative fit of the model with (n = 1) to the "actual" condition where (n) varies with altitude. For low transmission values, the model may be pessimistic if there is clustering of spectral lines. Data has not been analyzed to determine if this is so, as was the case with carbon dioxide.

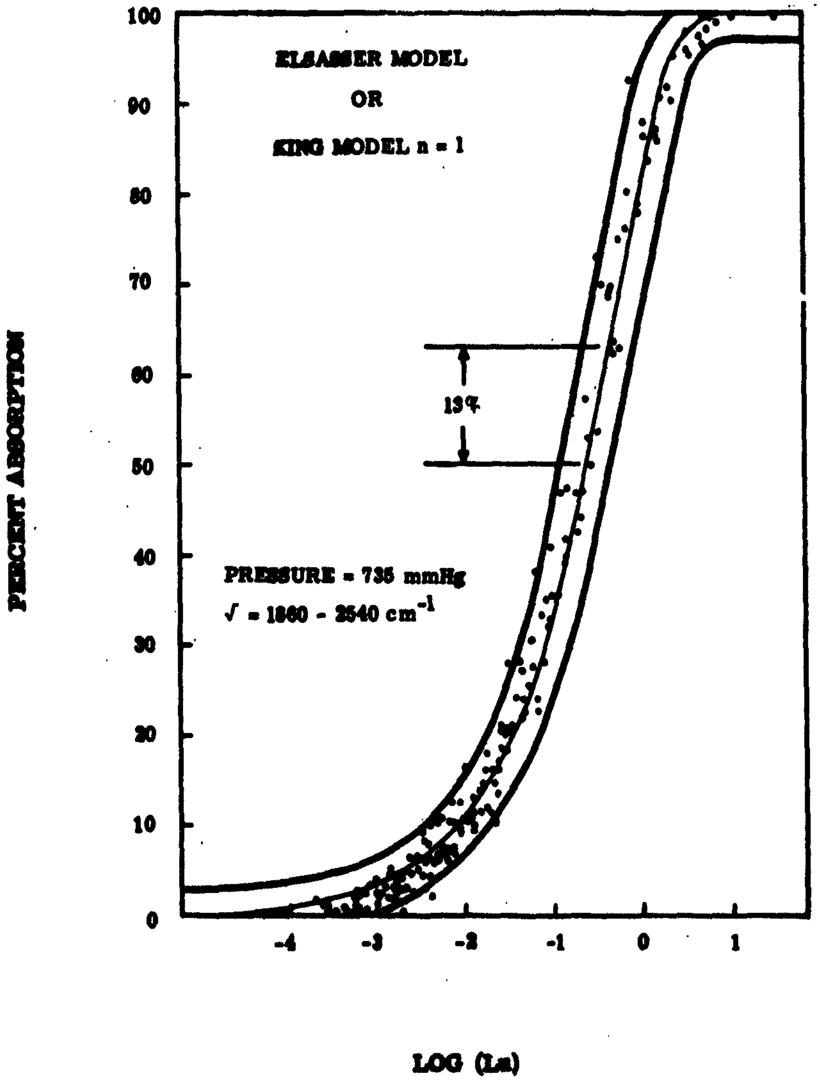

ELSASSER MODEL
OR
KING MODEL n = 1
PRESSURE = 735 mmHg
ν = 1860 - 2540 cm⁻¹
13%
PERCENT ABSORPTION
100
90
80
70
60
50
40
30
20
10
0
-4
-3
-2
-1
0
1
LOG (Lu)

Figure 55

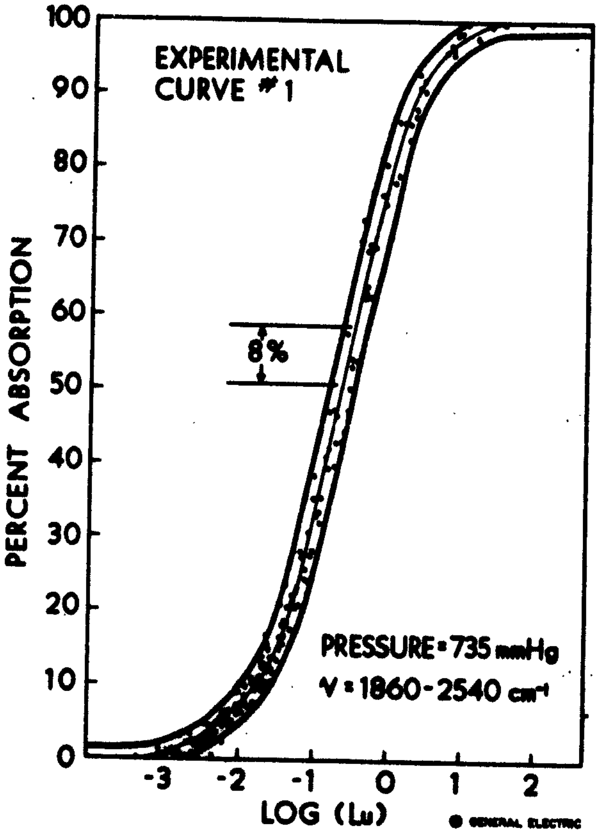

EXPERIMENTAL
CURVE #1
PERCENT ABSORPTION
100
90
80
70
60
50
40
30
20
10
0
8%
PRESSURE = 735 mmHg
ν = 1860-2540 cm⁻¹
-3
-2
-1
0
1
2
LOG (Lu)
● GENERAL ELECTRIC

Figure 56

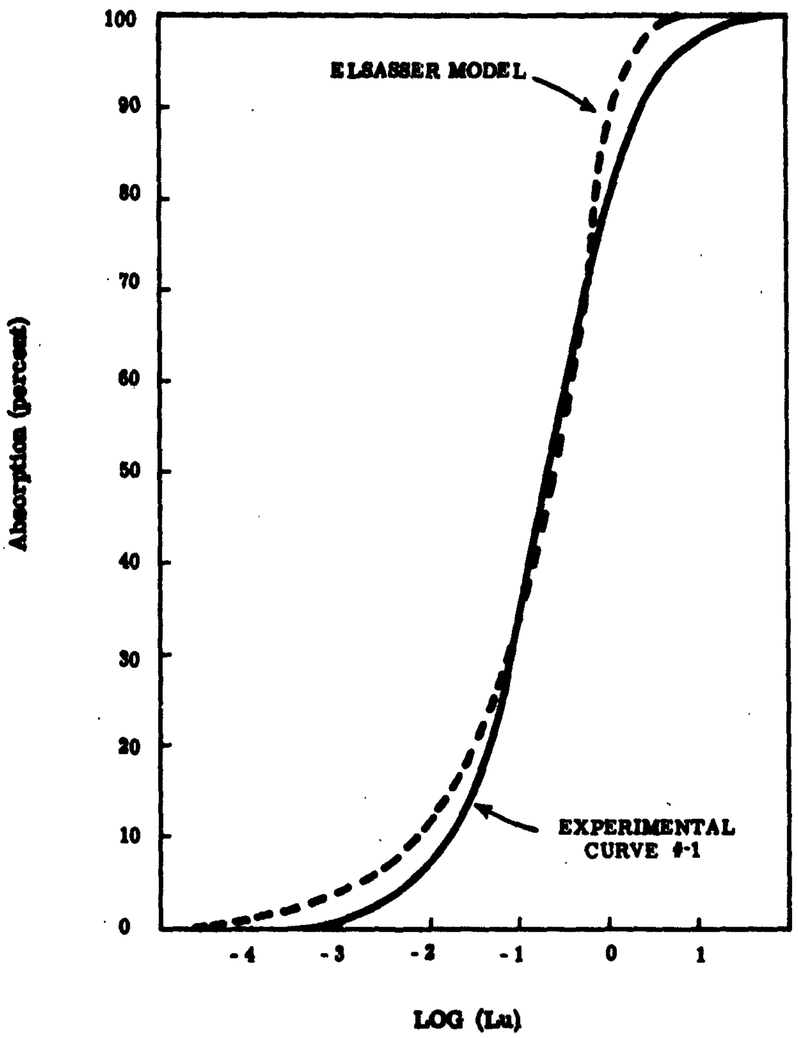

100
90
80
70
60
50
40
30
20
10
0
Absorption (percent)
ELSASSER MODEL
EXPERIMENTAL CURVE #-1
-4
-3
-2
-1
0
1
LOG (Lu)

Figure 57

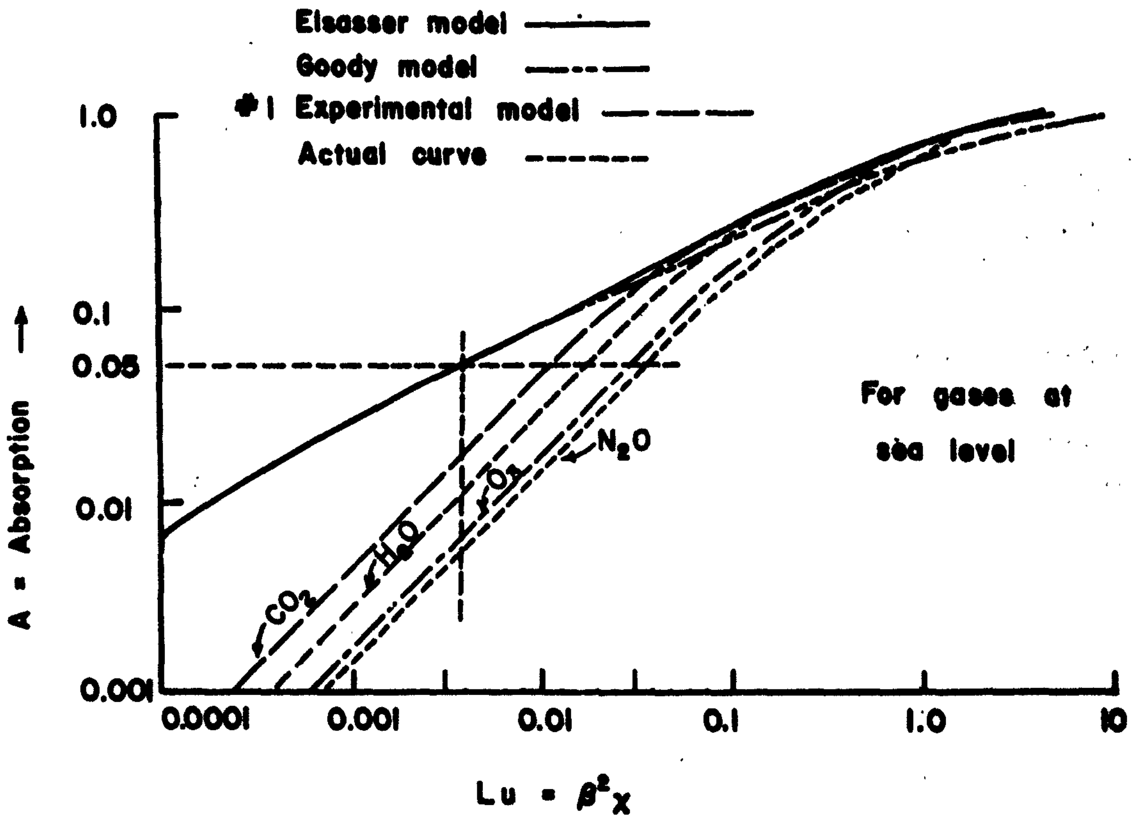

Elsasser model
Goody model
#1 Experimental model
Actual curve
1.0
0.1
0.05
0.01
0.001
A = Absorption
0.0001
0.001
0.01
0.1
1.0
10
Lu = $\beta^2 x$
CO_2
H_2O
O_2
N_2O
For gases at sea level

Figure 58

Part (3) O_3

In Figures 39-41, the transmission function was assumed to obey the Elsasser model. This model states:

$$\tau(\lambda) = 1 - \operatorname{erf}\left[(L_0 q)^{1/2}\right] \qquad \text{eq (75)}$$

where:

$$\operatorname{erf}(x) = \sqrt{\frac{2}{\pi}} \int_0^x e^{-t^2}\, dt$$

According to Curtis and Goody,(34) they suggest the use of the Goody model with the 9.6μ band of ozone. However, since ($\beta \sim 7$) at sea level and ($\beta \sim 0.3$) at ($h_{max(O_3)} \doteq 23$ km), the absorption versus ($L_0 q$) is steeper than that of the Goody model where ($\beta = 0$) for the useful transmission range, see Plass,(20) The Elsasser model is also steeper than the Goody model at ($\beta = 0$). Therefore, it is assumed that the Elsasser model will give a more correct prediction of transmission than the Goody model for the useful transmission range. Table 2 shows how the "actual" transmission varies from that of the "model" for various altitudes.

Part (4) N_2O

In Figures 42 - 44, the transmission function was assumed to obey the Elsasser model, see equations (75) and (76). Curtis and Goody,(34) suggest the use of the Elsasser Model for the 7.8μ and 17μ band of nitrous oxide. Table 2 shows how the "actual" transmission varies from that of the "model" for various altitudes.

Part (5) Absorption Laws for Overlapping Bands

Equation (18), Part (1) of Section IV states:

$$\tau_A(\lambda) = [\tau_{H_2O}(\lambda)] \times [\tau_{CO_2}(\lambda)] \times [\tau_{O_3}(\lambda)] \times [\tau_{N_2O}(\lambda)] \times [\tau_{scat}(\lambda)] \qquad \text{eq (18)}$$

Since the spectral lines of one gas does not overlap the spectral lines of another gas, the lines of one gas may be considered to have a random arrangement with those of the other gas. Therefore, the spectral lines of each gas may be replaced by an average transmission of fifteen or so lines, and the effective total transmission of such average transmissions of each gas in combination is the same as the actual total transmission of the gases in combination. Therefore, equation (18) is justified. Burch, Howard and Williams,(36) performed an experiment which proves the validity of equation (18) for the overlapping bands of carbon dioxide and water vapor in the 2.7μ region.

Part (6) Molecular Absorption of Infrared by Small Quantities of Gases and Vapors.

If the quantities of gases or vapors are small, then equation (43) applies, i.e.

$$A(\lambda) = - \tau(\lambda) + 1 = \frac{Su}{\delta} \qquad \text{eq (43)}$$

where: $A(\lambda)$ = absorption

Since the actual transmission of small quantities of gases at sea level will deviate from the models used in Figures 24-44, corrections must be made to determine the actual value of $(\frac{Su}{\delta})$ when ($\tau(\lambda) = 95\%$) in the Figures. Using these corrections and values in Table 1 to convert (d) to (q), equations (21) - (24) of Part (2) Section IV results from equation (43).

APPENDIX G

THERMAL ENERGY CALCULATIONS

Equation (19) in Part (1) of Section IV and Figures 46 and 47 with TEMPLATE B were developed in the following manner.

Let: $B_{r\lambda}$ = spectral radiance of target, watts/cm^2 - ster. - μ

C_1 = 3.74×10^4 watts/cm^2 - micron

C_2 = 1.439×10^4 micron degrees

Q_λ = spectral radiance of target over the wavelength increment ($\Delta\lambda$), watts/cm^2 - ster. - $\Delta\lambda$

$Q_{\lambda_{max}}$ = maximum spectral radiance of target over the wavelength increment ($\Delta\lambda$), watts/cm^2 - ster. - $\Delta\lambda$

T = temperature of target, °K

λ = wavelength - microns

λ_{max} = wavelength at $Q_{\lambda_{max}}$, microns

$\Delta\lambda$ = wavelength increment such that:

$$\log_{10}(\lambda + \Delta\lambda) - \log_{10}(\lambda) = 0.01$$

$$\log_{10}(\lambda) = \frac{1}{2.30} \log_e \lambda$$

$$d(\log_{10}\lambda) = \frac{d\lambda}{2.30\,\lambda}$$

$$\Delta\lambda = 2.30\,\lambda\Delta(\log_{10}\lambda) = 2.30\,\lambda\left[\log_{10}(\lambda + \Delta\lambda) - \log_{10}(\lambda)\right]$$

$$\Delta\lambda = 2.30\,\lambda \times 0.01 = 0.0230\,\lambda$$

$$Q_\lambda = B_{r\lambda}\Delta\lambda \quad \text{by definition.}$$

$$Q_\lambda = 0.0230\,\lambda\; B_{r\lambda} \qquad \text{eq (19)}$$

In order to find Q_λ, $Q_{\lambda_{max}}$, and λ_{max}:

From reference 37:

$$B_{r\lambda} = \frac{1}{\pi}\;\frac{C_1\,\lambda^{-5}}{e^{C_2/\lambda T} - 1}$$

Combining this with equation (19)

$$Q_\lambda = \frac{0.0230}{\pi} \quad \frac{C_1 \quad \lambda^{-4}}{e^{C_2/\lambda \cdot T}-1} \qquad \text{eq (77)}$$

Differentiating equation (77) so that

$$\frac{dQ\lambda}{d\lambda} = 0$$

$$e^{C_2/\lambda \cdot T}\left(\frac{C_2}{\lambda T} - 4\right) + 4 = 0$$

Solving for $\lambda = \lambda_{max}$

$$T\lambda_{max} = 3670 \text{ micron degrees} \qquad \text{eq (78)}$$

Substituting equation (78) into equation (77)

$$Q_{\lambda_{max}} = 0.0303 \left(\frac{T}{1000}\right)^4 \qquad \text{eq (79)}$$

Figure 46 and Template B were constructed from equations (77) and (78). Figure 47 was constructed from equation (79).

APPENDIX H

BACKGROUND RADIATION OF THE ATMOSPHERE

Equations (25) and (26) were derived as follows:

Let: H_λ = spectral irradiance at the observer over the wavelength interval $(\Delta\lambda)$, at wavelength (λ), watts/cm^2 - $\Delta\lambda$

J_λ = spectral irradiance at the observer at $(\lambda)/$

k = absorption coefficient

$Q_{\lambda,i}$ = spectral radiance of a black body radiator at the temperature of gas in the (ith) layer and over the wavelength increment $(\Delta\lambda)$ at wavelength (λ), watts/cm^2 - ster. - $\Delta\lambda$

T_i = temperature of the gas in the (ith) layer, °K

u = quantity of gas between a wafer in the sky and the observer, see Figure 59.

λ = wavelength, microns.

$\Delta\lambda$ = wavelength increment such that:

$$\log_{10}(\lambda + \Delta\lambda) - \log_{10}(\lambda) = 0.01$$

$\tau_{A,i}(\lambda)$ = atmospheric transmission at wavelength (λ) from the (ith) point on the infrared path to the observer, see Figure 49. There are (n) such points.

ω = solid angle measured from the observer, steradians.

subscripts H_20, $C0_2$, 0_3, and N_20 refer to the gases.

Let $(d\,J_\lambda)$ be the incremental amount of irradiance received by the observer, which was radiated by a wafer of sky at wavelength (λ). The wavelength interval $(d\lambda)$ of (J_λ) is considered to be sufficiently small, much smaller than the half width of a spectral line. Then,

$$d\,J_\lambda = \omega\left(\frac{Q_{\lambda i}}{\Delta\lambda}\right)\left(e^{-ku}\right)\left(k\,du\right)$$

where: $ku = k_{H_20}\,u_{H_20} + k_{C0_2}\,u_{C0_2} + k_{0_3}\,u_{0_3} + k_{N_20}\,u_{N_20}$

And: (e^{-ku}) is the transmission of the atmosphere in wavelength interval $(d\lambda)$

And: $(k du)$ is the emissivity of the wafer of sky, see Figure 58.

$$J_\lambda = \sum_{i=1}^{n} \int_{u_{i-1}}^{u_i} \omega\left(\frac{Q_{\lambda i}}{\Delta\lambda}\right)\left(e^{-ku}\right)\left(k\,du\right)$$

APPENDIX H

BACKGROUND RADIATION OF THE ATMOSPHERE

Equations (25) and (26) were derived as follows:

Let: H_λ = spectral irradiance at the observer over the wavelength interval ($\Delta\lambda$), at wavelength (λ), watts/cm^2 - $\Delta\lambda$

J_λ = spectral irradiance at the observer at (λ)/

k = absorption coefficient

$Q_{\lambda,i}$ = spectral radiance of a black body radiator at the temperature of gas in the (ith) layer and over the wavelength increment ($\Delta\lambda$) at wavelength (λ), watts/cm^2 - ster. - $\Delta\lambda$

T_i = temperature of the gas in the (ith) layer, $^\circ$K

u = quantity of gas between a wafer in the sky and the observer, see Figure 59.

λ = wavelength, microns.

$\Delta\lambda$ = wavelength increment such that:

$$\log_{10}(\lambda + \Delta\lambda) - \log_{10}(\lambda) = 0.01$$

$\tau_{A,i}(\lambda)$ = atmospheric transmission at wavelength (λ) from the (ith) point on the infrared path to the observer, see Figure 49. There are (n) such points.

ω = solid angle measured from the observer, steradians.

subscripts H_2O, CO_2, O_3, and N_2O refer to the gases.

Let (dJ_λ) be the incremental amount of irradiance received by the observer, which was radiated by a wafer of sky at wavelength (λ). The wavelength interval ($d\lambda$) of (J_λ) is considered to be sufficiently small, much smaller than the half width of a spectral line. Then,

$$dJ_\lambda = \omega\left(\frac{Q_{\lambda i}}{\Delta\lambda}\right)\left(e^{-ku}\right)\left(kdu\right)$$

where: $ku = k_{H_2O}\,u_{H_2O} + k_{CO_2}\,u_{CO_2} + k_{O_3}\,u_{O_3} + k_{N_2O}\,u_{N_2O}$

And: (e^{-ku}) is the transmission of the atmosphere in wavelength interval ($d\lambda$)

And: (kdu) is the emissivity of the wafer of sky, see Figure 58.

$$J_\lambda = \sum_{i=1}^{n} \int_{u_{i-1}}^{u_i} \omega\left(\frac{Q_{\lambda i}}{\Delta\lambda}\right)\left(e^{-ku}\right)\left(kdu\right)$$

$$J_\lambda = \sum_{i=1}^{n} \omega \frac{Q_{\lambda i}}{\Delta\lambda}\left(e^{-ku_{i-1}} - e^{-ku_i}\right)$$

$$H_\lambda = \int_{\lambda}^{\lambda+\Delta\lambda} J_\lambda \, d\lambda = \int_{\lambda}^{\lambda+\Delta\lambda} \sum_{i=1}^{n} \omega \frac{Q_{\lambda i}}{\Delta\lambda}\left(e^{-ku_{i-1}} - e^{-ku_i}\right) d\lambda$$

$$H_\lambda = \sum_{i=1}^{n} \omega Q_{\lambda i}\left[\frac{1}{\Delta\lambda}\int_{\lambda}^{\lambda+\Delta\lambda} (e^{-ku_{i-1}}) \, d\lambda - \frac{1}{\Delta\lambda}\int_{\lambda}^{\lambda+\Delta\lambda} (e^{-ku_i}) \, d\lambda \right]$$

But: $\tau_A(\lambda) = \frac{1}{\Delta\lambda}\int_{\lambda}^{\lambda+\Delta\lambda} (e^{-ku}) \, d\lambda$ by definition

Therefore,

$$H_\lambda = \omega \sum_{i=1}^{n} Q_{\lambda i}\left[\tau_{A,i-1}(\lambda) - \tau_{A,i}(\lambda) \right] \qquad \text{eq (25)}$$

If the temperature of the sky being viewed is isothermal, equation (25) reduces to:

$$H_\lambda = \omega Q_\lambda \left[1 - \tau_A(\lambda) \right] \qquad \text{eq (26)}$$

Equations (25) and (26) are found in Part (3) of Section IV.

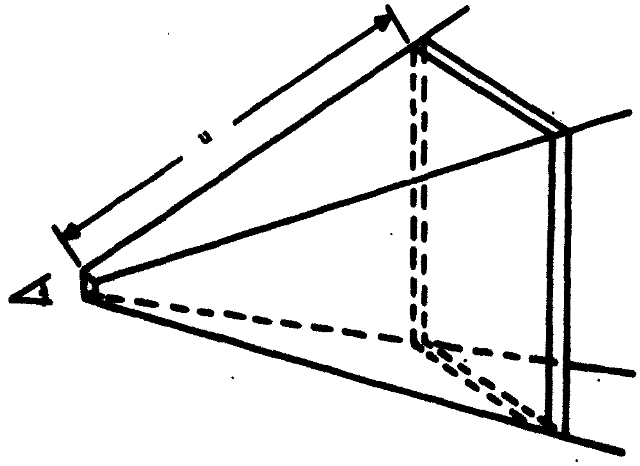


Figure 59

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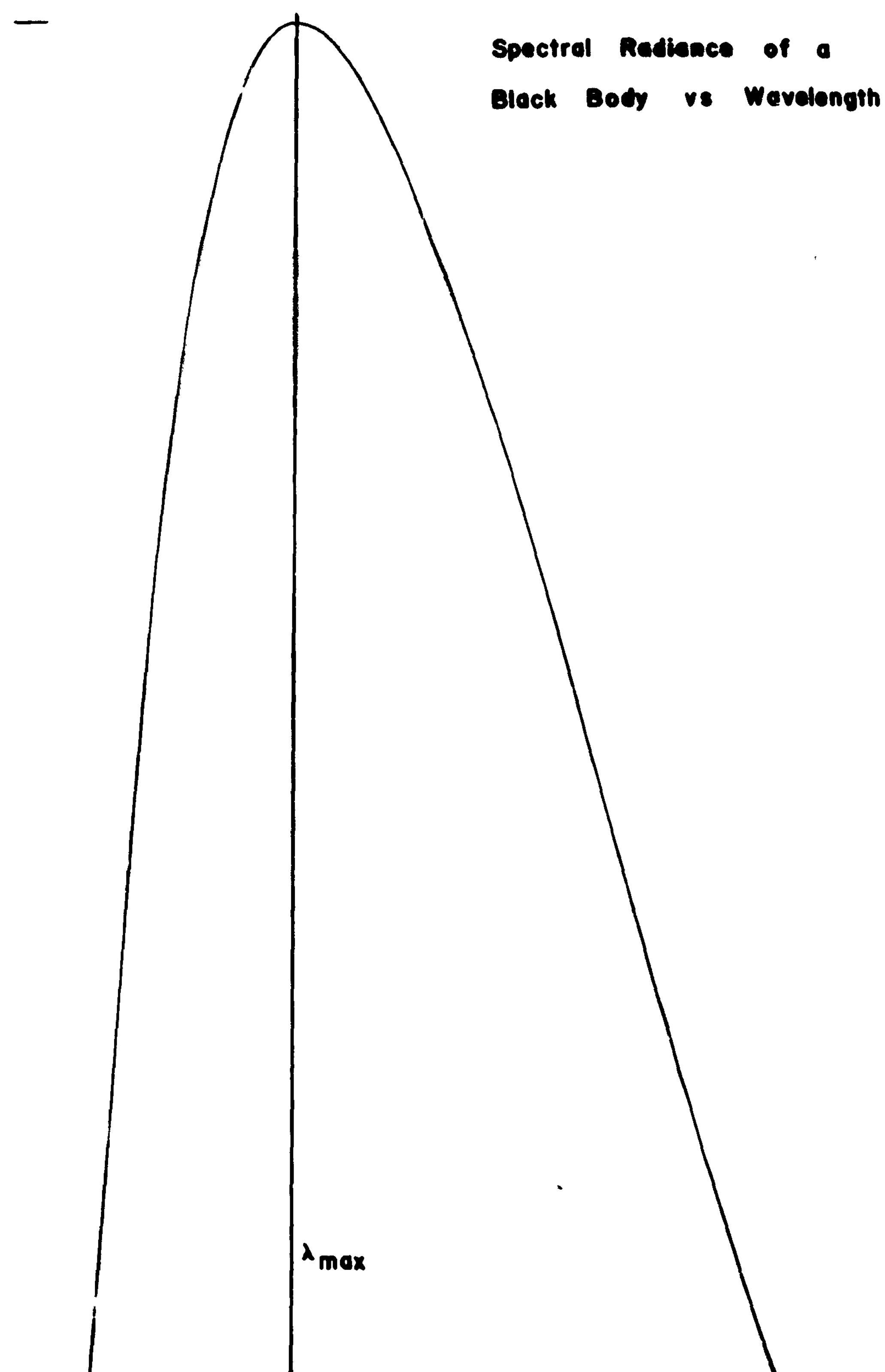
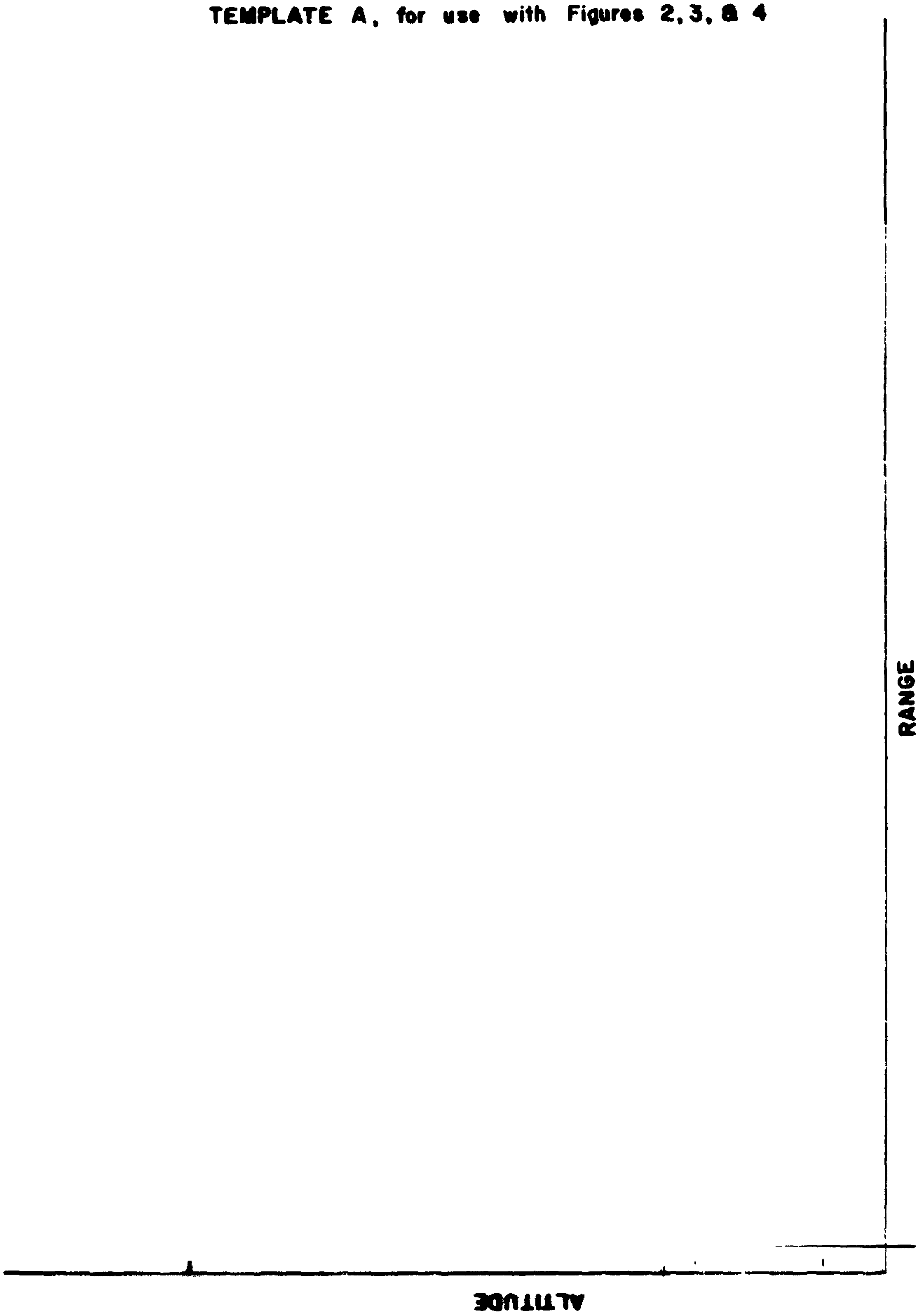
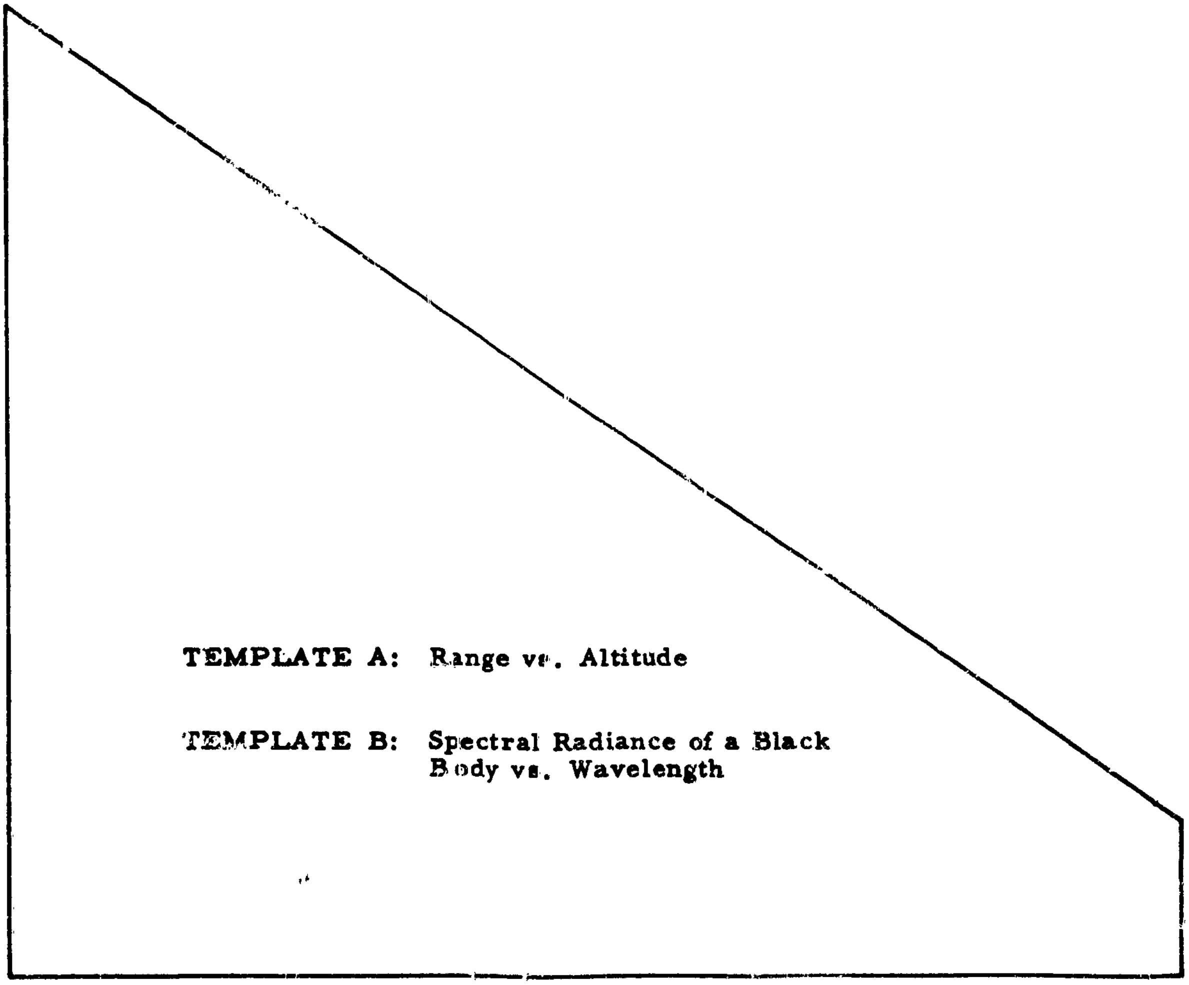
TEMPLATE B, for use with Figure 46


Spectral Radiance of a Black Body vs Wavelength

TEMPLATE A, for use with Figures 2, 3, & 4





TEMPLATE A: Range vs. Altitude

TEMPLATE B: Spectral Radiance of a Black Body vs. Wavelength