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AUTOCORRELATION OF PULSE POSITION MODULATED SIGNALS

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Preface

This thesis is submitted by the author, Captain Nicholas G. Gionis, in partial fulfillment of the requirements for the graduate diploma at the United States Air Force Institute of Technology. The thesis concerns the construction of an auto-correlator for pulse position modulated signals and the experimental and theoretical proof that the device does, in fact, accomplish its designed purpose.

The author is indebted to Captain Matthew Kabrisky of the Electrical Engineering Department, the faculty sponsor of the thesis, for his encouraging aid, helpful suggestions, valuable guidance, and untiring patience throughout the project. The author also gratefully acknowledges the helpful assistance given by Professor Gordon M. Russell of the University of Arizona in the form of personal correspondence. In addition valuable assistance was given by the Electrical Engineering Laboratory technicians under the supervision of Mr. Melvin Corbin and Mr. Robert Durham and from the Mathematics Department Computer Laboratory by Mr. Floyd Hild. This problem was established by Major Everette T. Garrett, formerly of the Electrical Engineering Department.

Nicholas G. Gionis

Contents

Contents

List of Figures

List of Figures

Abstract

One method of evaluating a communication receiver is by its ability to accurately discriminate desired information signals from combined signal plus noise. It is the purpose of this thesis to experimentally and theoretically examine a device (autocorrelator) which was designed to operate with decreased signal to noise ratios.

A generator for a particular type of information carrier, pulse position modulation, was constructed, and an appropriate noise source was used.

The combined signal plus noise was sent through the autocorrelator, and a comparison of signal to noise ratios with and without correlation was made in order to ascertain the effectiveness of the device in performing its function.

AUTOCORRELATION IN PULSE POSITION MODULATION

I. Introduction

As more and more sophisticated communication and radar systems are developed, the problem of noise interference becomes increasingly more difficult to solve (the "decibel scrounging" problem). In particular, where communication systems are expected to operate at great distances (i.e., from satellites or in radar systems), the need for more sensitive systems to operate with extremely small signals becomes necessary. Yet as the information signals decrease in amplitude, the ever present noise remains at a comparatively fixed level, and consequently signal to noise ratios become exceedingly small. Where methods of filtering by active or passive networks worked previously for large signal to noise ratios, they will not work satisfactorily for small signal to noise ratios.

A relatively new method of obtaining the desired information from received information plus noise signals has been developed in recent years. It is the purpose of this thesis to build and operate such a device and prove that it is able to select the desired signal from an ensemble of noise. The device I have chosen to build was designed by Professor Gordon M. Russell of the Department of Electrical Engineering of the University of Arizona. In addition it has been necessary to design the desired information carrier (a pulse position modulated signal generator) and to obtain some satisfactory source of noise. The purpose of this thesis is to show that Professor Russell's device can take a signal in which the desired information is partially concealed by noise and produce a satisfactory output signal. The scheme of attack will be to compare the output signal of the device with the original signal before the noise is added and to evaluate the output signal quality in the presence of increasing amounts of additional noise.

The detection method discussed in the paper--functioning to pick out desired information from noise-- is termed correlation. Basically a correlator can be defined as a device which, by knowing something about the desired information signal, can distinguish this signal from its surroundings (i.e., from noise). A mathematical procedure to describe such a process will be defined in this thesis. In Chapter II a brief, semi-qualitative description of mathematical correlation functions will be given with an explanation of how correlation techniques are used in the study of signal and noise functions. Examples of correlation functions for various signals will be given, but it is not intended that a detailed analysis of correlation functions will be presented. This would involve time not available for this study. Several excellent references for non-mathematical treatment of correlation functions are listed in the bibliography (see Ref. 3, 5, and 9). The thesis, "Theory and Application of Correlation Techniques," by 1/Lt Paul H. Hass served as a guide for the basic outline of Chapter II.

Chapter III will describe the autocorrelator used in the laboratory for this thesis and develop proof that the device does operate with decreased signal to noise ratios.

Chapter IV will present the pulse position modulated signal generator, giving detailed description of the principles of operation. In addition, the device used as the noise source will be briefly described.

Appendix A will give the construction details of the pulse position modulated signal generator; Appendix B will show the mathematical design of the delay line component of the correlator; and Appendix C will show the various photographs of oscilloscope waveforms observed in the autocorrelator.

II. Correlation Functions

Definition

If there exists a system with an input x and an output y, it is possible to write a statistical relationship such as p(x,y), the joint distribution function, between x and y. If there are no disturbing influences in the system, then x and y will be uniquely related. If there are disturbing influences in the system, x and y will only be partially related. The product moment

$$m_{11} = \text{av } xy = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} xy\, p(x,y)\,dx\,dy \qquad (1)$$

serves as a measure of correlation between x and y.

If stationary process is assumed, then instead of the ensemble average above (Eq 1), a time average

$$R_{xy} = \lim_{T\to\infty} \frac{1}{T}\int_{0}^{T} x(t)\, y(t)\, dt \qquad (2)$$

may be written. Eqs (1) and (2) are identical if the process is ergodic, that is, one in which the ensemble statistical properties do not vary with time. All ergodic processes are stationary, but not all stationary processes are ergodic. Under the assumption of ergodicity,

$$m_{11} = \text{av } xy = R_{xy} \qquad (3)$$

R_{xy} is called the cross correlation function since it serves as a measure of the correlation of x(t) and y(t). If x(t) and y(t) follow each other closely and are of the same sign (i.e., they correlate positively), the average xy (or R_{xy}) will accumulate positively. If x(t) and y(t) follow closely and are of opposite sign (i.e., they correlate negatively), the average xy (or R_{xy}) will accumulate negatively. On the other hand, if x(t) and y(t) vary independently (i.e., uncorrelated), they tend to cancel each other and the average (or R_{xy}) would approach zero.

If $x_1(t+\tau)$, which is $x_1(t)$ delayed by a factor τ, is substituted for y(t), Eq (2) becomes

$$R_{xx} = \lim_{T\to\infty} \frac{1}{T}\int_0^T x_1(t)\; x_1(t+\tau)\,dt \qquad (4)$$

and is called the time autocorrelation function and denoted by $\phi_{11}(\tau)$. If, in addition, x(t) is written as $x_1(t)$ and $x(t+\tau)$ is written as a different function $x_2(t+\tau)$, then Eq (2) becomes

$$\phi_{12}(\tau) = \lim_{T\to\infty} \frac{1}{T}\int_0^T x_1(t)\; x_2(t+\tau)\,dt \qquad (5)$$

and is called the time cross correlation function.

Autocorrelation Function of Periodic Functions

The autocorrelation function of a periodic function, $f_1(t)$, is defined as

$$\phi_{11}(\tau) = \frac{1}{T}\int_0^T f_1(t)\ f_1(t+\tau)\,dt \qquad (6)$$

where τ is a continuous displacement in the interval from $-\infty$ to $+\infty$. The Fourier expansion of $f_1(t)$ is

$$f_1(t) = \frac{1}{T}\sum_{n=-\infty}^{\infty} F_1(n)\,\epsilon^{j\omega_n t} \qquad (7)$$

where

$$F_1(n) = a_n - jb_n = \int_0^T f_1(t)\,\epsilon^{-j\omega_n t}\,dt \qquad (8)$$

The Fourier expansion of $f_1(t+\tau)$ is

$$f_1(t+\tau) = \frac{1}{T}\sum_{n=-\infty}^{\infty} F_1(n)\,\epsilon^{j\omega_n (t+\tau)} \qquad (9)$$

Substituting Eq (9) into Eq (6)

$$\phi_{11}(\tau) = \frac{1}{T}\int_0^T f_1(t)dt\,\frac{1}{T}\sum_{n=-\infty}^{\infty} F_1(n)\,\epsilon^{j\omega_n (t+\tau)} \qquad (10)$$

or rearranging Eq(10)

$$\phi_{11}(\tau) = \frac{1}{T}\sum_{n=-\infty}^{\infty} F_1(n)\,\epsilon^{j\omega_n\tau}\,\frac{1}{T}\int_0^T f_1(t)\,\epsilon^{j\omega_n t}\,dt \qquad (11)$$

The integral in Eq (11) is the conjugate of Eq (8). Substituting $\overline{F_1(n)}$ into Eq (11)

$$\begin{aligned}\phi_{11}(\tau) &= \frac{1}{T}\sum_{n=-\infty}^{\infty} \frac{F_1(n)\overline{F_1(n)}}{T}\,\epsilon^{j\omega_n\tau} \\ &= \frac{1}{T}\sum_{n=-\infty}^{\infty} \frac{|F_1(n)|^2}{T}\,\epsilon^{j\omega_n\tau}\end{aligned} \qquad (12)$$

The autocorrelation function for an argument of zero ($\tau = 0$) is

$$\phi_{11}(0) = \frac{1}{T}\int_0^T f_1^{\,2}(t)\,dt \qquad (13)$$

which is the mean square value of $f_1(t)$. If it is assumed that there is a load resistor of one ohm and that $f_1(t)$ represents either current or voltage, then the mean square power consumed is the sum of the power contributed by the individual harmonics into which $f_1(t)$ has been resolved (see Eq 12). This is known as Parseval's Theorem for periodic functions.

If $\frac{|F_1(n)|^2}{T}$ is defined as the power spectrum $\Phi_{11}(n)$, then

$$\phi_{11}(\tau) = \frac{1}{T}\sum_{n=-\infty}^{\infty} \Phi_{11}(n)\,\epsilon^{j\omega_n\tau} \qquad (14)$$

and

$$\Phi_{11}(n) = \int_0^T \phi_{11}(\tau)\, \epsilon^{-j\omega_n \tau}\, d\tau \tag{15}$$

Eqs (14) and (15), which are Fourier transforms of each other, constitute the autocorrelation theorem. The autocorrelation function and the power spectrum are Fourier transforms of each other.

An important property is that the autocorrelation function of a periodic function is periodic. This provides the basis of detecting signals obscured in noise if it is known before hand that the signals have periodic properties. This will be evident after reading the section below on the autocorrelation properties of random functions. If the periodic function has the form

$$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos(n\omega t + \theta_n) \tag{16}$$

and is substituted into Eq (6)

$$\phi_{11}(\tau) = \lim_{T\to\infty} \frac{1}{T}\int_0^T \left[\frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos(n\omega t + \theta_n)\right] \left[\frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos(n\omega t + n\omega\tau + \theta_n)\right] dt \tag{17}$$

the following equation is obtained:

$$\phi_{11}(\tau) = \frac{a_0^2}{4} + \frac{1}{2}\sum_{n=1}^{\infty} a_n^2 \cos n\omega\tau \tag{18}$$

The autocorrelation function is periodic, retains the fundamental and harmonic frequencies of the periodic function, but drops all phase angles (see Ref 6:1166).

Autocorrelation Function of Aperiodic Functions

The Fourier integral of an aperiodic function $f_1(t)$ is

$$F_1(\omega) = \int_{-\infty}^{\infty} f_1(t)\, \epsilon^{-j\omega t}\, dt \tag{19}$$

and the inverse Fourier transform is

$$f_1(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F_1(\omega)\, \epsilon^{j\omega t}\, d\omega \tag{20}$$

An aperiodic function has a continuous spectrum and therefore $F_1(\omega)$ is an amplitude density function. The spectrum of a periodic function is not continuous, and therefore $F_1(\omega)$ for a periodic function is an amplitude spectrum.

The autocorrelation function of an aperiodic function

$$\phi_{11}(\tau) = \int_{-\infty}^{\infty} f_1(t)\, f_1(t+\tau)\, dt \tag{21}$$

does not include a specified interval nor is a mean value taken.

The inverse Fourier transform of $f_1(t+\tau)$ is

$$f_1(t+\tau) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F_1(\omega)\, \epsilon^{j\omega(t+\tau)}\, d\omega \tag{22}$$

Substituting Eq (22) into Eq (21) and rearranging

$$\phi_{11}(\tau) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F_1(\omega)\,\epsilon^{j\omega\tau}d\omega \int_{-\infty}^{\infty} f_1(t)\,\epsilon^{j\omega t}dt \qquad (23)$$

The integral in Eq (23) is the conjugate of Eq (19). Making this substitution, Eq (23) reduces to

$$\phi_{11}(\tau) = \frac{1}{2\pi}\int_{-\infty}^{\infty} |F_1(\omega)|^2\,\epsilon^{j\omega\tau}d\omega \qquad (24)$$

If $|F_1(\omega)|^2$ is defined as the energy density spectrum and denoted by $\Phi_{11}(\omega)$, then Eq (24) reduces to

$$\phi_{11}(\tau) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \Phi_{11}(\omega)\,\epsilon^{j\omega\tau}d\omega \qquad (25)$$

and the inverse

$$\Phi_{11}(\omega) = \int_{-\infty}^{\infty} \phi_{11}(\tau)\,\epsilon^{-j\omega\tau}dt \qquad (26)$$

The autocorrelation function with zero argument reduces to

$$\phi_{11}(0) = \int_{-\infty}^{\infty} f_1^{\,2}(t)dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} |F_1(\omega)|^2 d\omega \qquad (27)$$

This is known as Parseval's equality.

Autocorrelation Function of Random Functions

A random function is an ensemble of time functions about which only statistical properties can be given. It is a function the cause and effect of which cannot be uniquely related. The autocorrelation function of a stationary random function is

$$\phi_{11}(\tau) = \lim_{T \to \infty} \frac{1}{T} \int_0^T f_1(t)\, f_1(t+\tau)\, dt \tag{28}$$

It is impossible to write a Fourier transform of a random process, but from the autocorrelation function is it possible to write an expression for a random function in the frequency domain as

$$\phi_{11}(\tau) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \Phi_{11}(\omega)\, \epsilon^{j\omega\tau}\, d\omega \tag{29}$$

where $\Phi_{11}(\omega)$ is defined as it was for an aperiodic function. The inverse of Eq (29) is

$$\Phi_{11}(\omega) = \int_{-\infty}^{\infty} \phi_{11}(\tau)\, \epsilon^{-j\omega\tau}\, d\tau \tag{30}$$

Eqs (29) and (30) are known as the Wiener-Khintchine Theorem.

For zero argument Eq (28) reduces to

$$\phi_{11}(o) = \lim_{T \to \infty} \frac{1}{T} \int_0^T f_1^{\,2}(t)\, dt \tag{31}$$

which is the mean value of $f_1(t)$. Eq (29) for zero argument

$$\phi_{11}(0) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \Phi_{11}(\omega)\, d\omega \tag{32}$$

Equating Eqs (31) and (32)

$$\lim_{T\to\infty} \frac{1}{T}\int_{0}^{T} f_1^2(t)\,dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} \Phi_{11}(\omega)\, d\omega \tag{33}$$

The above expression shows that the mean-square value of $f_1(t)$ is related to $\Phi_{11}(\omega)$. If $f_1(t)$ is a voltage or current and a load resistor of one ohm is assumed, then the mean-square value of $f_1(t)$ is the mean power and equals

$$\frac{1}{2\pi}\int_{-\infty}^{\infty} \Phi_{11}(\omega)\, d\omega \tag{34}$$

It can therefore be said that $\Phi_{11}(\omega)$ represents the power-density spectrum of $f_1(t)$.

The properties of the autocorrelation function of a stationary random process include:

1) $\phi_{11}(\tau) = \phi_{11}(-\tau)$ (an even function)
2) $\phi_{11}(0) = \lim_{T\to\infty} \frac{1}{T}\int_{0}^{T} f_1^2(t)\,dt$
3) Any periodicity that may be hidden in the random function will be present in the autocorrelation function.
4) The autocorrelation function is maximum for a zero argument.

5) The autocorrelation function approaches zero as $\tau \rightarrow \infty$ if there is no periodicity or d.c. component present. If there is a d.c. component present, the autocorrelation function approaches the d.c. power. (see Fig. 1).

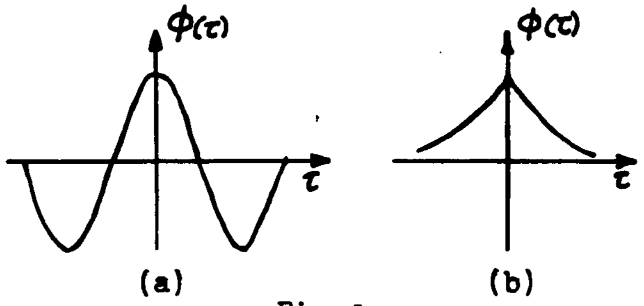

Fig. 1

Autocorrelation Function of a (a) Periodic Function (b) Band-limited White Noise

Cross-Correlation Functions

The cross-correlation functions for periodic, aperiodic, and random functions are listed below:

1) Periodic function

$$\phi_{12}(\tau) = \frac{1}{T}\int_0^T f_1(t)\, f_2(t+\tau)\,dt$$

2) Aperiodic function

$$\phi_{12}(\tau) = \int_{-\infty}^{\infty} f_1(t)\, f_2(t+\tau)\,dt$$

3) Random function

$$\phi_{12}(\tau) = \lim_{T\rightarrow\infty} \frac{1}{T}\int_0^T f_1(t)\, f_2(t+\tau)\,dt$$

The properties of stationary random functions are:

1) $\phi_{12}(\tau)$ is not an even function.

2) $\phi_{12}(\tau)$ may not be maximum at $\tau = 0$.

3) The cross-correlation function of two "uncorrelated" stationary random functions is zero.

4) The cross-correlation function approaches zero as $\tau \to \infty$ if there is no periodicity or d.c. value present.

Illustrative Problem

A hypothetical problem will be presented to illustrate the correlation techniques and their ability to discriminate against noise. The received signal is

$$f_m(t) = S(t) + n(t) \tag{35}$$

where S(t) is the desired information signal and n(t) is the random white noise. The correlation function of this signal, $f_m(t)$, is

$$\phi_{mm}(\tau) = \lim_{T\to\infty} \frac{1}{T}\int_0^T f_m(t)\, f_m(t+\tau)\, dt \tag{36}$$

Substituting in Eq (36) the expression for $f_m(t)$

$$\phi_{mm}(\tau) = \lim_{T\to\infty} \frac{1}{T}\int_0^T [S(t) + n(t)] \tag{37}$$

$$[S(t+\tau) + n(t+\tau)]\, dt$$

and multiplying

$$\phi_{mm}(\tau) = \lim_{T\to\infty} \frac{1}{T}\int_0^T [S(t)S(t+\tau) + n(t)n(t+\tau) \tag{38}$$

$$+ S(t+\tau)n(t) + S(t)n(t+\tau)]\,dt$$

This reduces to

$$\phi_{mm}(\tau) = \lim_{T\to\infty} \frac{1}{T}\int_0^T S(t)S(t+\tau)\,dt \tag{39}$$

$$+\lim_{T\to\infty} \frac{1}{T}\int_0^T n(t)n(t+\tau)\,dt + \lim_{T\to\infty} \frac{1}{T}\int_0^T n(t)S(t+\tau)\,dt$$

$$+\lim_{T\to\infty} \frac{1}{T}\int_0^T S(t)n(t+\tau)\,dt$$

or

$$\phi_{mm}(\tau) = \phi_{ss}(\tau) + \phi_{nn}(\tau) + \phi_{ns}(\tau) + \phi_{sn}(\tau) \tag{40}$$

The first term, $\phi_{ss}(\tau)$, is the autocorrelation function of the desired signal and therefore contains all frequencies present in the original signal. The second term is the autocorrelation function of the random noise and, as stated above in the properties of random functions, the autocorrelation function approaches zero as τ approaches infinity. It is assumed in this illustration that the noise does not have any periodicity or d.c. component. The third and fourth terms are cross-correlation functions which are completely uncorrelated and equal zero.

Therefore, if a sufficiently long delay, τ, is used, the correlation function of the input signal plus noise reduces to the autocorrelation function of the desired information signal. This is illustrated by Fig. 2.

Examples of Correlation Devices

Any device which is capable of successfully performing the correlation equation

$$F(\tau) = \lim_{T \to \infty} \frac{1}{T} \int_0^T f_1(t)\, f_2(t+\tau)\, dt \tag{41}$$

may be termed a correlator. Obviously this device must be capable of a delay τ, of multiplying two functions, and of averaging the product of these two functions over a period of time T. A device capable of performing these functions is a correlator in the mathematical definition of the word. From a practical approach, devices which allow a system to operate with a decreased signal to noise ratio have been called correlators by some.

Two examples of the mathematical definition of a correlator are illustrated in Fig. 3. A qualitative description of an engineering correlator is used to locate satellites. A photograph is made of the area in which it is suspected that the satellite is located. It will most likely be impossible to recognize the satellite because of the star background. A second photograph is taken a short time later and the two photographs are compared. Since there

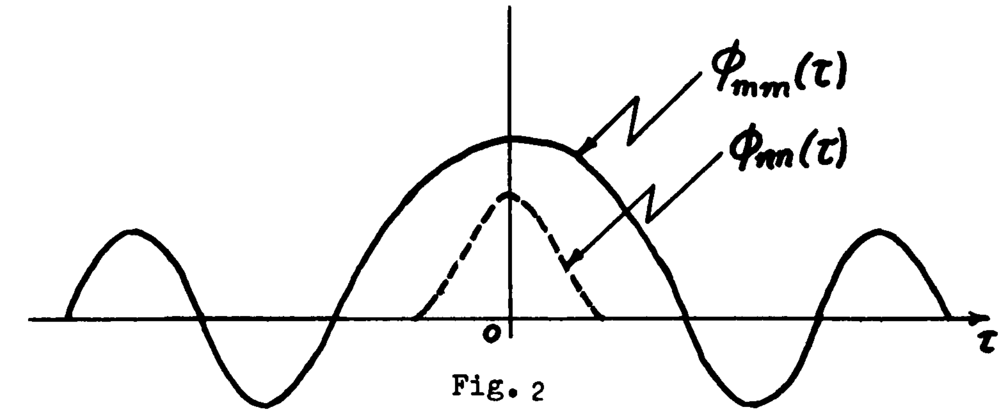

Fig. 2

Autocorrelation Function of a Periodic Signal plus Noise and a Noise

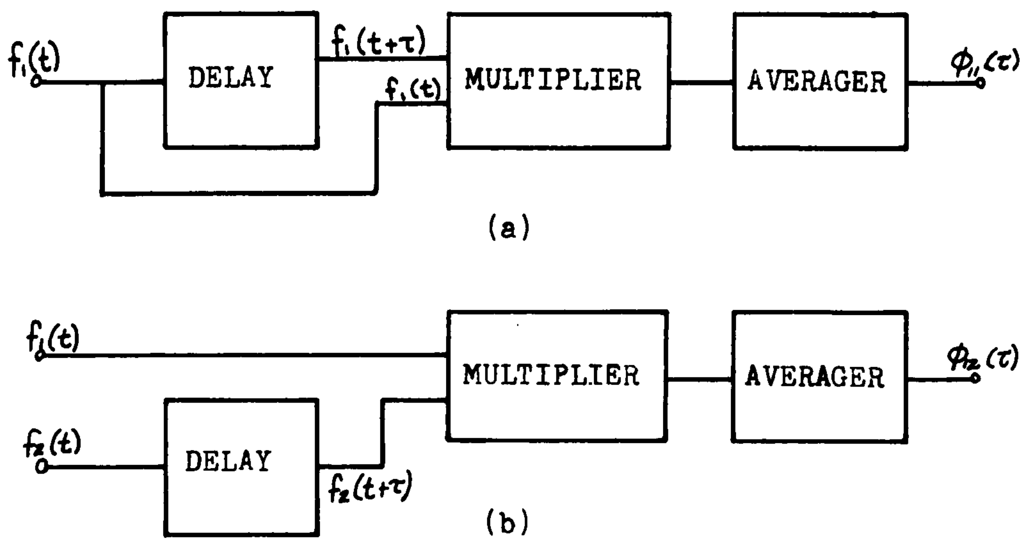

Fig.3

(a) Autocorrelator (b) Crosscorrelator

will be no relative movement of the star background and there will be movement of the satellite, it will be possible to locate the satellite. This is a very general analysis of a correlation system, but it does illustrate an engineering definition of correlation. It is possible, by knowing something about the desired signal, to obtain this signal.

From the analysis in this chapter it is obvious that a correlator functions to separate desired information concealed in noise from the unwanted noise. It accomplishes this task by having a priori knowledge of the desired signal. In essence, the correlator takes an input signal that is concealed in noise (i.e., a small signal to noise ratio) and produces an output which has a larger signal to noise ratio. The mathematical autocorrelation function performs this task. Any other device which performs this task by using a priori knowledge is called a correlator (or autocorrelator) by many.

III. Autocorrelator

In this chapter a device which is capable of performing correlation of a pulse position modulated signal will be analyzed. It will be proved, mathematically and experimentally, that correlation is performed by showing that the signal to noise ratio of the output is larger than the signal to noise ratio of the input to the device. This means that for a given signal to noise ratio at the output, a smaller input signal to noise ratio is necessary. Assuming that the device does perform correlation, it will be possible to locate a signal concealed in noise by increasing the signal to noise ratio. In addition is will be shown that the autocorrelation function serves as a reasonable model of this device.

*"Fig. 4 illustrates in block diagram the device proposed by Professor Russell, and also shows the waveforms associated with the numbered points in the circuit. The input pulse position modulated signal does not contain noise since it is the purpose of the figure and of the following explanation to show only that the device in no way effects the input signal. Waveshape (1) shows the input signal which is delayed, (2), and inverted, (3). The signal is delayed by exactly one pulse width of the input signal. This is the one factor that the correlator must know about the incoming signal before it can identify the desired information.

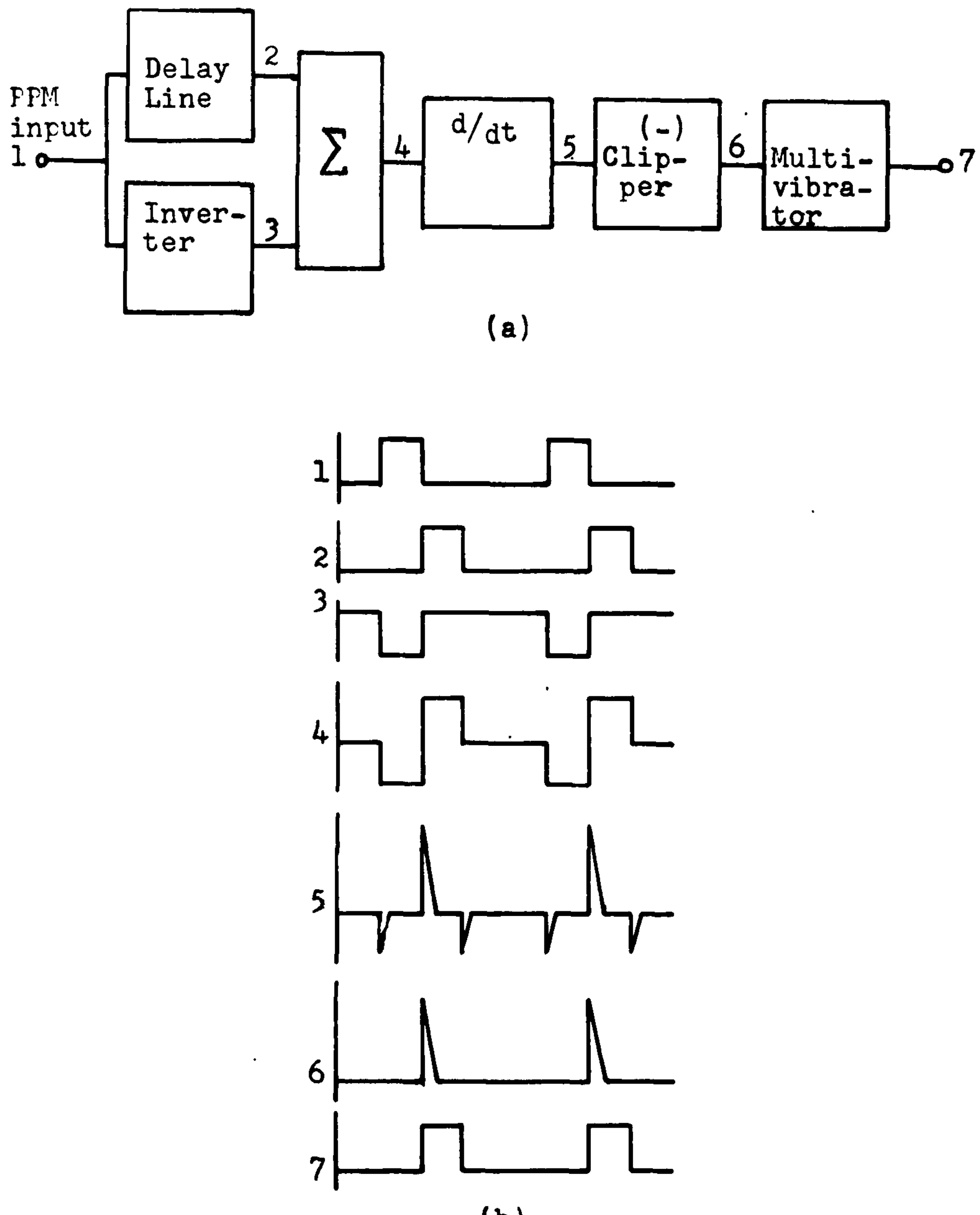

Fig. 4

PPM Autocorrelator and Waveforms

Waveshapes (2) and (3) are added, the sum (4) is differentiated (5), the negative pulses are clipped (6), and the positive pulses are used to trigger the one shot multivibrator. If the multivibrator is adjusted to operate with the same pulse width and amplitude as the input pulse position modulated signal, then the input and output signals will be identical, except that the entire output will be delayed by one pulse width. This delay in no way effects the information content of the pulse position modulated signal." The device I have chosen to construct (see Fig. 5) is similar in idea and operation to Professor Russell's autocorrelator.

The change in position of the pulses is the information carried by a pulse position modulated signal. This means that other characteristics of the signal (i.e., pulse width, pulse amplitude, etc.) are unimportant. It can be proved that the autocorrelation function is a reasonable model for the circuit performance of the device constructed in this thesis.

If two pulse samples of the same signal, with zero delay, are subtracted (the samples are inverted, but otherwise identical), the result will be zero. If the delay is greater than zero but less than one pulse width, the difference will be the amplitude of either pulse or zero. For exactly one pulse width delay, the difference will be twice the amplitude of either pulse. As the delay is further increased, it is evident that the difference will again go from zero

*Gordon M. Russell, MODULATION AND CODING IN INFORMATION SYSTEMS, $\bar{\underline{C}}$, Pg 196, 1962 by Prentice-Hall, Inc., Englewood Cliffs, N. J. Reprinted by permission.

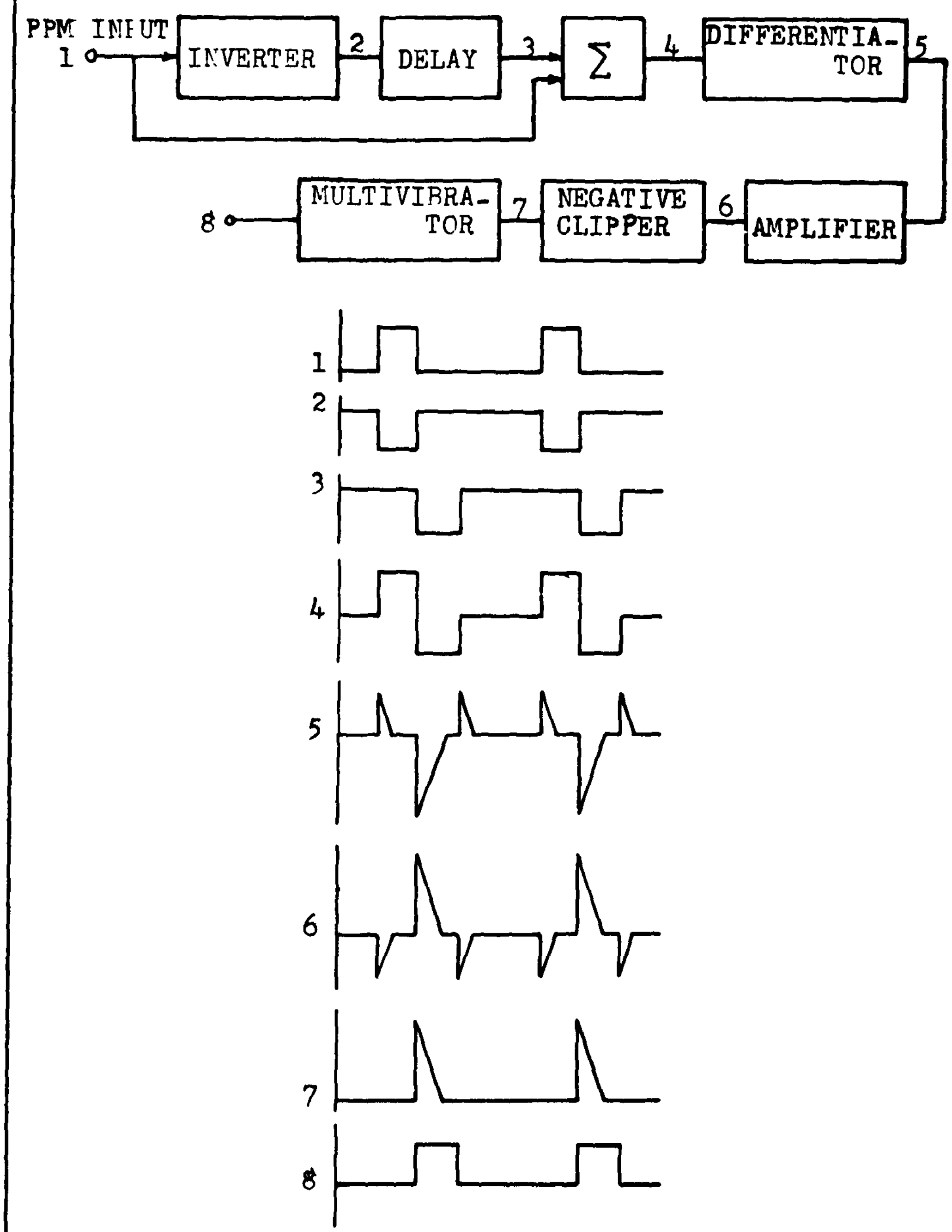

Fig. 5

PPM Autocorrelator and Waveforms

to one times the amplitude to twice the amplitude of either signal. The output of the correlator for a periodic pulse signal is therefore periodic. This is analogous to the mathematical autocorrelation function for a pulse position modulated signal. The difference between the two correlation methods is the amplitude of the output function, but as previously stated, this is unimportant. The important factor, the difference in position of the maximum amplitudes, is the same for the two methods. This analogy is proof that the autocorrelation function is a reasonable model of the device.

In addition, from theory on noise, it is known that when two uncorrelated white noise samples are added, the resultant sum equals the sum of the root mean square voltages of each sample. If two correlated samples are added (delay is equal to zero), the resultant sum is twice the amplitude of each sample. The delay of one pulse width "uncorrelates" the noise. This is analogous to the action of the autocorrelation function for random white noise.

A second method (using a threshold multivibrator) of proving that this device functions as a correlator involves its ability to identify a signal when it is concealed in noise. The bias on the non-conducting tube is adjusted so that the pulse, which results from differentiating the correlated signal, barely triggers the multivibrator. Only when the differentiated noise pulses exceed the signal trigger in magnitude will the multivibrator give a false output. The noise amplitude is increased to the point where any further increase in noise will trigger the multivibrator and give a false output. The signal to noise ratio of the input to the correlator is then measured. Next, the input is applied directly to the grid of the non-conducting multivibrator tube (correlator not in the circuit), and the bias is adjusted so that the signal pulse barely triggers the multivibrator. The noise amplitude is once again increased to the point where any further increase in noise triggers the multivibrator and gives false output. This signal to noise ratio is measured and compared to the signal to noise ratio of the input with the correlator in the circuit. If the signal to noise ratio with the correlator in the circuit is less than the signal to noise ratio without the correlator, then the device successfully functions as a correlator. The signal to noise ratio at the grid of the multivibrator must be the same for a satisfactory output whether the correlator is in the circuit or not. Therefore it is obvious that the correlator has increased the signal to noise ratio.

Mathematical Proof

This proof consists of measuring the input signal to noise ratio of the correlator and comparing it to the output signal to noise ratio. A hypothetical pulse will be used, and the theoretical increase in signal to noise ratio will be the limiting increase that can be achieved in any actual device.

The signal to noise ratio may be defined as

$$S/N = 10 \log \frac{P_s}{P_n} = 10 \log \frac{e_s^2/R}{P_n} \text{ db} \qquad (42)$$

It will be assumed that there is one watt power of white noise at the input, that the pulse position modulated pulse is six volts in amplitude, one microsecond in pulse width, ten microseconds in pulse repetition interval, and that the pulse is taken across an output resistor of one ohm. The rms value of the pulse signal is

$$V_{rms} = \sqrt{\frac{1}{T}\int_0^T V^2 dt} = \sqrt{\frac{1}{10}\int_0^1 6^2 dt} \qquad (43)$$

$$= 1.9 \text{ volts rms}$$

The signal to noise ratio of this signal plus noise is

$$S/N = 10 \log \frac{e_s^2/R}{P_n} = 10 \log \frac{1.9^2/1}{1} \qquad (44)$$

$$= 5.563 \text{ db}$$

The correlator adds (see Fig. 5 waveform 4) the input signal plus noise to the delayed and inverted signal plus noise. This results in a peak to peak value of the signal of twelve volts and a value of white noise of two watts power. When a sample of white noise is added to itself, delayed by some factor as in the above discussion, the power doubles. The new signal to noise ratio for two samples is therefore

$$S/N = 10 \log \frac{e_s^2}{R} \Big/ P_n \tag{45}$$

where

$$V_{rms} = \sqrt{\frac{1}{10}\int_0^1 12^2 dt} = 3.8 \text{ volts rms} \tag{46}$$

Therefore

$$S/N = 10 \log \frac{3.8^2}{1} \Big/ 2 = 8.57 \text{ db} \tag{47}$$

If two more samples are added to the above two samples, making a total of four samples, the signal to noise ratio becomes 11.6 db. For six samples, the signal to noise ratio is 13.4 db; for eight, 14.6 db. A plot of signal to noise ratios versus number of samples added is shown in Fig. 6. This graph shows that the signal to noise ratio increases logarithmically as the number of samples are increased.

This constitutes proof that the correlator increases the signal to noise ratio of the input signal. The most important factor here is that when one sample of signal is added to itself, the peak to peak value of signal voltage doubles, while when one sample of

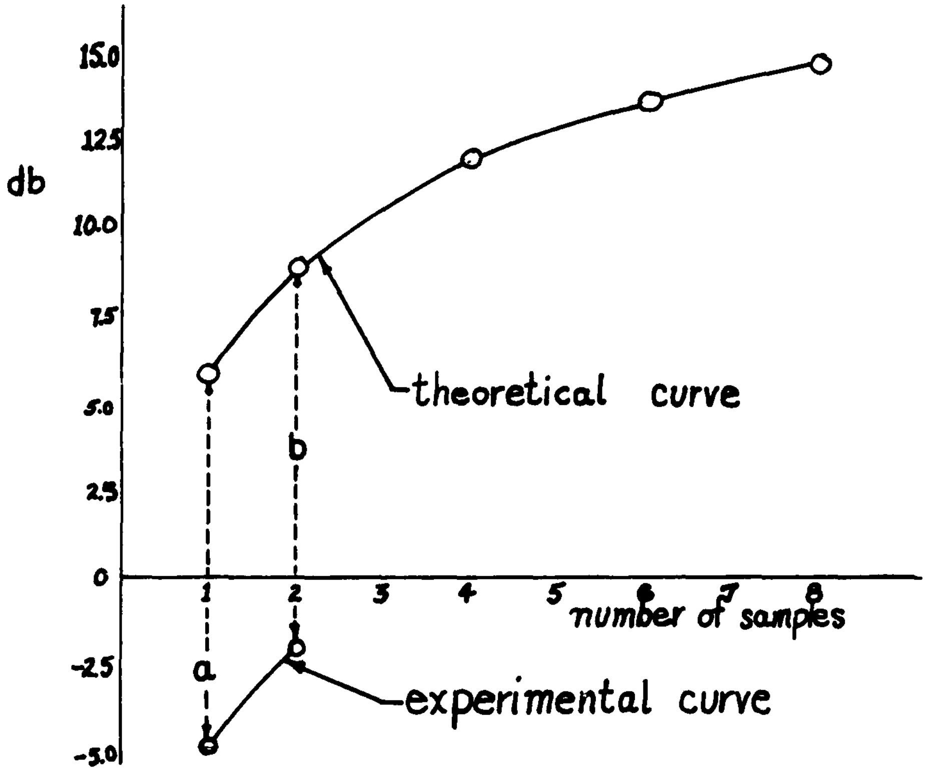

Fig.6

S/N ratio versus number of samples

white noise is added to itself, the value of power doubles.

The method chosen to experimentally prove that the device actually performs correlation (a decrease in signal to noise ratio required for successful operation) consists of triggering a threshold multivibrator with a signal that has been correlated, and comparing this signal to noise ratio with the signal to noise ratio of a signal, used to trigger the multivibrator, that has not been correlated. This was done with the following results. With the correlator in the circuit, the signal to noise ratio necessary to trigger the multivibrator and obtain the desired output was -4.97 db. Without the correlator in the circuit, the signal to noise ratio required was -2.06 db. This illustrates that without correlation a larger signal to noise ratio (-2.06 db) is required than when correlation is present (-4.97 db). If these two points are plotted on the theoretical curve of Fig. 6, they both lie an approximately equal distance below the theoretical curve. Since only the shape and not the location of the curve is important, the fact that these two experimental values of signal to noise ratio lie an equal distance from the curve proves that the theoretical and experimental results agree.

Fig. 7 shows a photograph of the laboratory mock-up for this thesis.

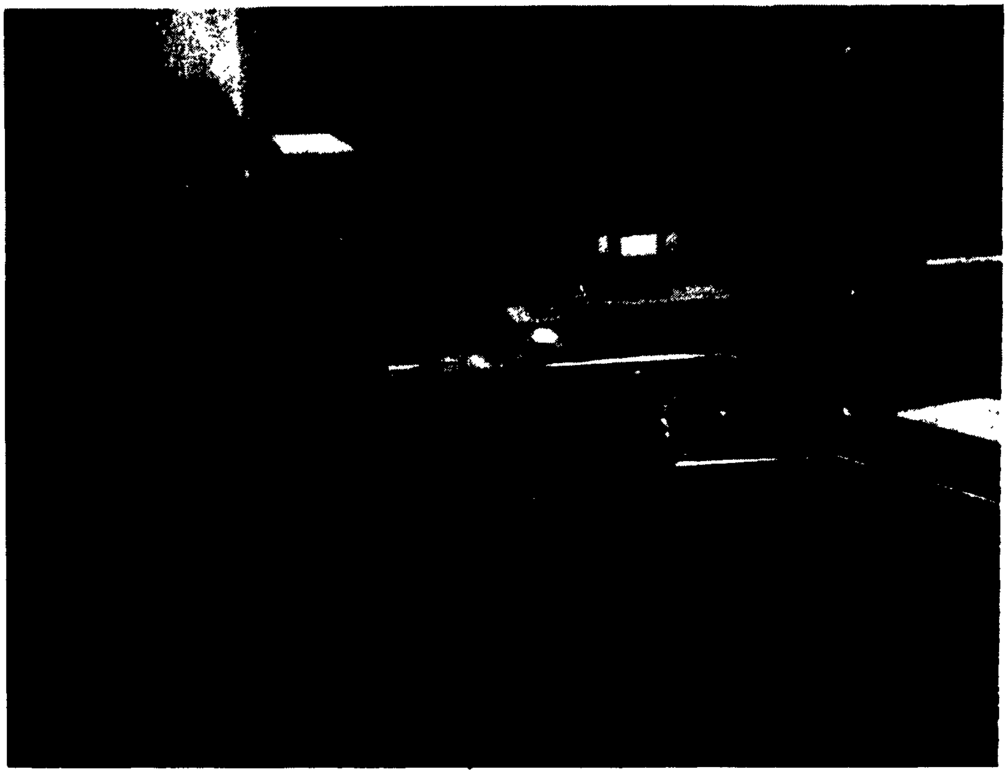

Fig. 7

Laboratory Mock-up
of Thesis Project

IV. Associated Equipment

The purpose of this work is the autocorrelation of a pulse position modulated signal. This signal is added to noise and the combined signal plus noise is sent to the autocorrelator. From these two statements it is obvious that two associated pieces of equipment needed to perform the thesis are a pulse position modulated signal generator and a noise source. In this chapter the two devices will be presented, and in Appendix A, the construction details of the pulse position modulated signal generator will be given.

Pulse Position Modulated Signal Generator

In pulse position modulation, the pulse width and amplitude are unchanged, but the occurrence of the pulse is varied in accordance with a modulating signal. The device constructed to perform pulse position modulation was proposed by Harold S. Black in Modulation Theory (see Ref 2:285).

The output from a square wave generator is differentiated into a triangular shaped pulse. The negative portion of the differentiated pulse is clipped and the positive portion is added to a sine wave. Fig. 8 illustrates the circuit which accomplishes the above.

Following the sine-plus-saw tooth wave is a negative clipper, appropriately set at a reference voltage E_1 (see Fig. 9 waveshape 4).

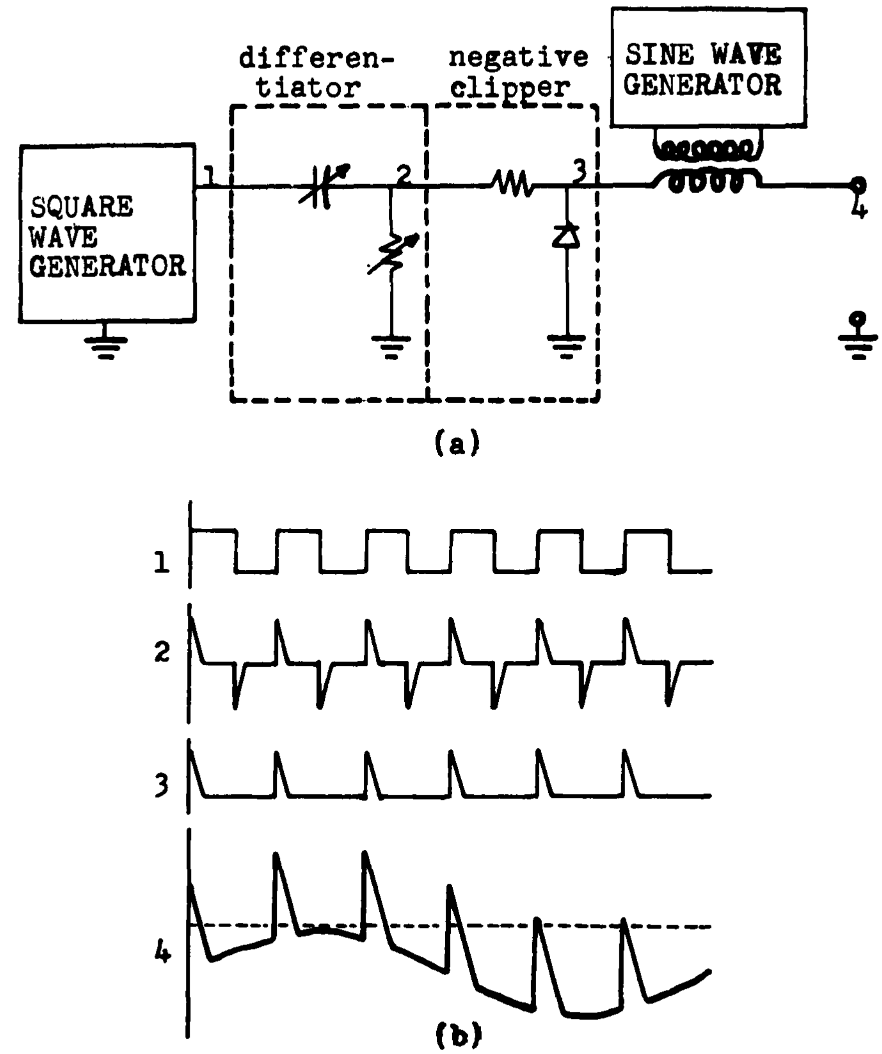

Fig. 8

(a) Portion of PPM Generator (b) Waveshapes

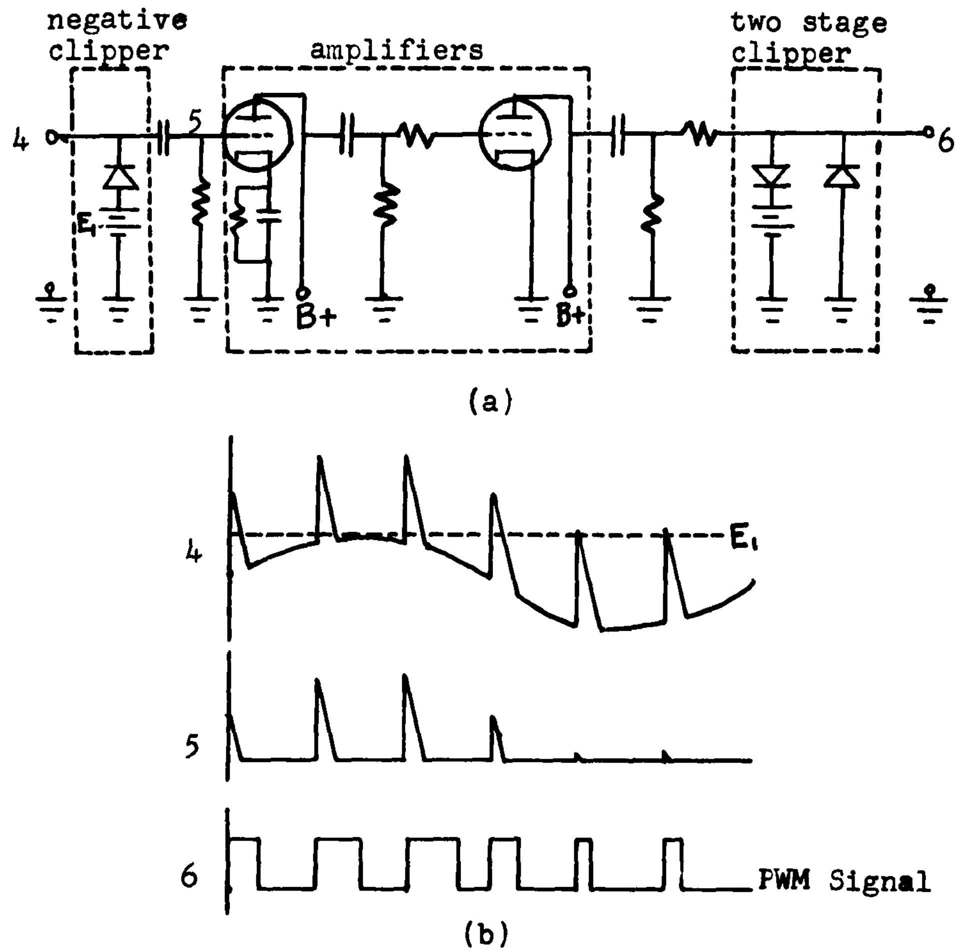

Fig. 9

(a) Portion of PPM Generator (b) Waveshapes

The resulting wave, after two stages of amplification, is sent through a two stage diode limiter (see Fig. 9 waveshape 6). Because of the amplification and the limiting action, the output wave of this section has a very steep trailing edge. This output wave is a pulse width modulated wave with the trailing edge being modulated. The circuit for accomplishing the functions listed in this paragraph is shown in Fig. 9.

The pulse width modulated signal is differentiated. The positive spike resulting from the differentiation of the leading edge of the pulse width modulated signal is not modulated, but the negative spike resulting from the differentiation of the trailing edge is modulated. For this reason the positive spikes are clipped, and the negative spikes, after being amplified, are used to trigger a one-shot multivibrator. The triggering spike is a position modulated spike, and therefore the output of the one-shot multivibrator is a pulse position modulated signal. The sine wave is the modulating signal in this circuit. The circuit that accomplishes the conversion of the pulse width modulated signal into the pulse position modulated signal is shown in Fig. 10.

The pulse position modulated signal obtained from the generator has an unmodulated pulse repetition rate of two hundred and seventy pulses per second, a pulse width of approximately seven hundred

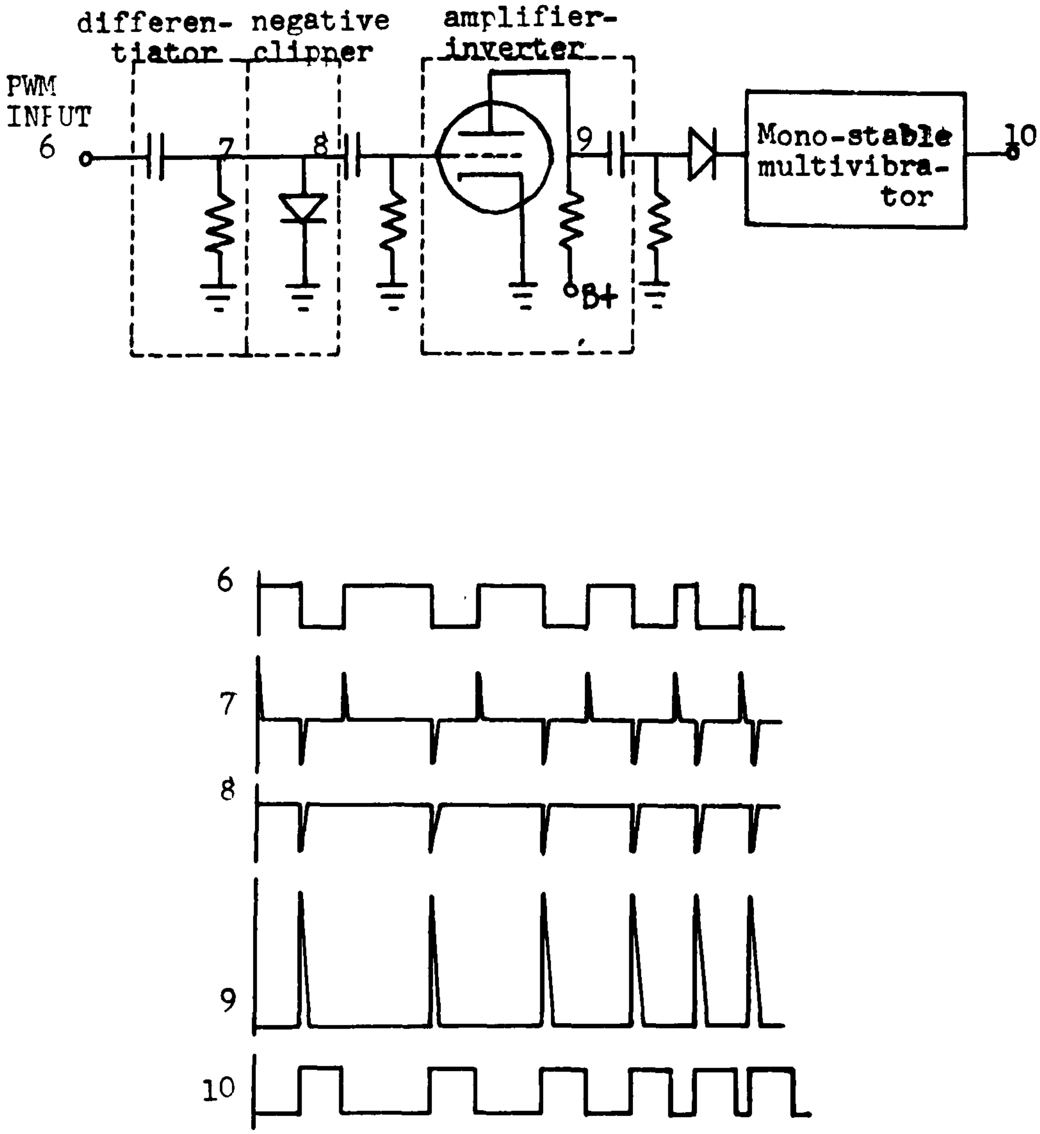

Fig. 10

Final portion of PPM generator

microseconds with rise time of approximately fifty microseconds. This means, essentially, that the highest frequency component of importance present in the signal is twenty kilocycles per second and the lowest frequency component present is one and half kilocycles per second. The noise frequency band, therefore, most effective is obscuring the information of the signal is approximately from one to twenty kilocycles per second.

Fig. 11 illustrates the entire pulse position modulated signal generator, the construction details of which are given in Appendix A.

Noise Source

A radio receiver, R-390A/URR, shown in Fig. 12 was used as the noise source. The receiver bandwidth selector knob was placed in the sixteen kilocycle position, which means that the noise frequency band is approximately as stated above.

Delay Line

Fig. 13 is a photograph of the delay line used in the autocorrelator. For reasons of availability of equipment, it was decided to construct a lumped-parameter delay line. A lumped-parameter delay line gives the delay required with less attenuation but more distortion than distributed-parameter lines. The lumped line is made up of a cascaded series of coils and capacitors. For the delay required in the autocorrelator, it was necessary to

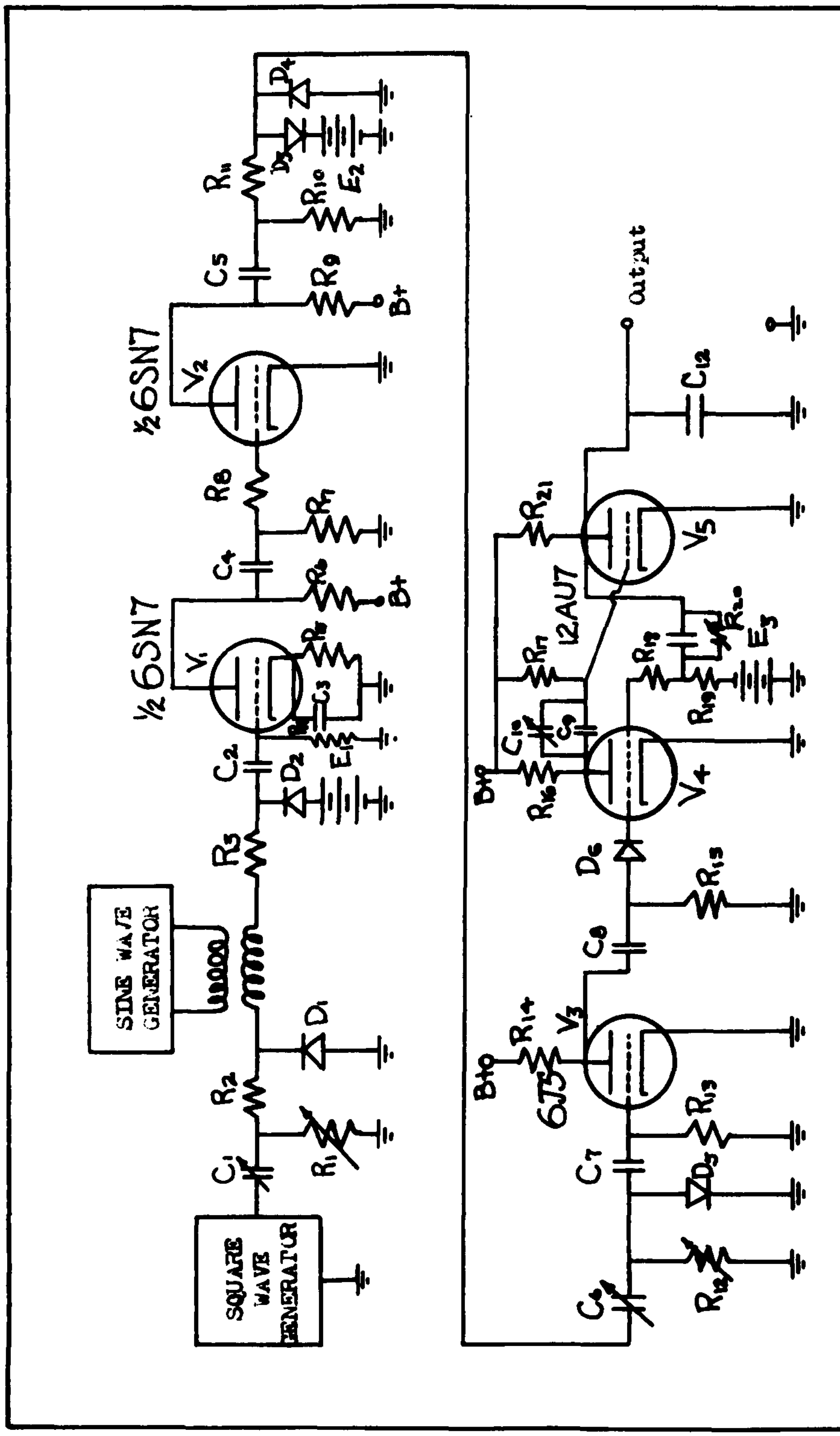

Fig. 11

Pulse Position Modulator

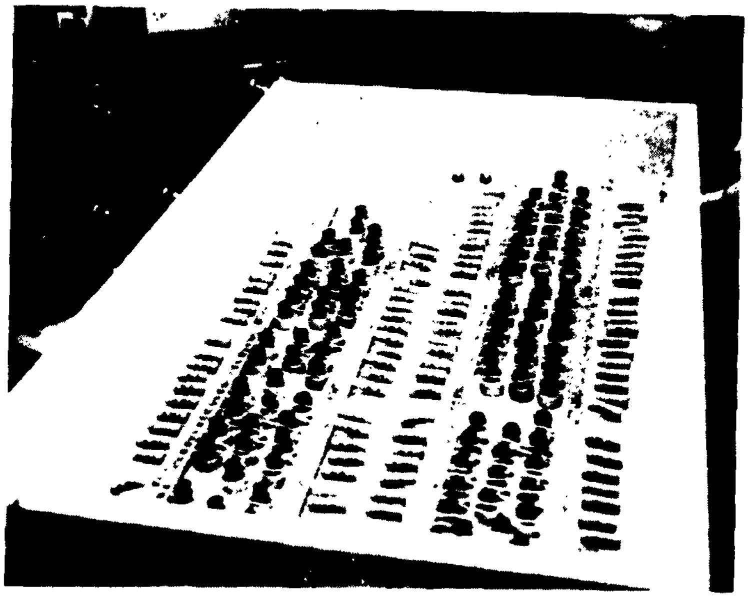

Fig. 12

Delay Line, Bottom View

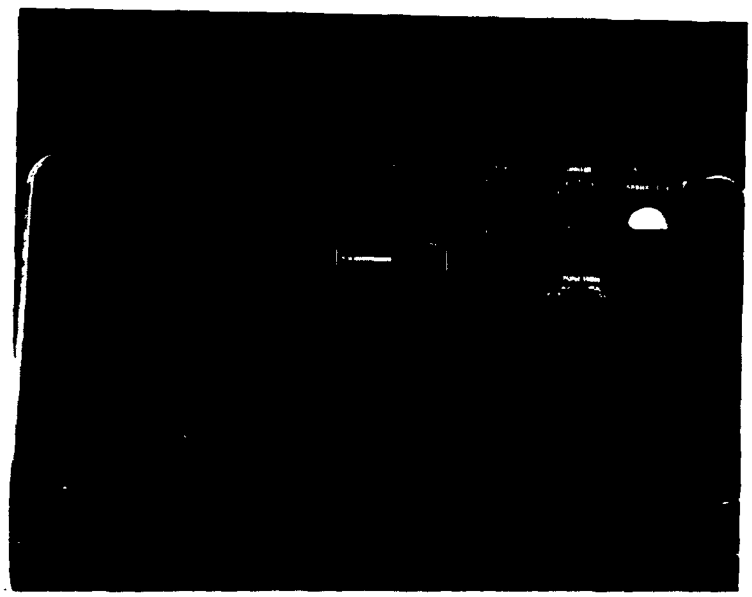

Fig. 13

Radio Receiver, Front View

connect fifty-five sections of T network. The mathematical design of this delay line is presented in Appendix B.

V. Summary

It has been the purpose of this thesis to build a device capable of performing correlation. It has been necessary to present some theoretical matter on correlation functions, and this was done in Chapter II. An illustrative example showing a correlator and its ability to perform its designed function for a restricted form of input signal is presented at the end of Chapter II.

An engineering definition of a correlator was introduced, and a correlator of this type for a particular signal, pulse position modulated signal, was presented in Chapter III. It was proved mathematically and experimentally that the device did indeed enable a receiver to operate with a decreased signal to noise ratio.

Chapter IV presented some of the associated equipment used in this thesis. One device, the pulse position modulated signal generator, was explained in detail, and its construction details are given in Appendix A.

There are many related topics that can be investigated in the future. It would be extremely interesting to construct a correlator that performs the mathematical definition of correlation for arbitrary input signals. This could be accomplished entirely on the analog computer. A second topic for future investigation would be to devise practical correlators for other types of signals (i.e., PWM, PAM, or PCM).

This study has proved that the device proposed by Professor Russell does perform correlation and is a feasible method of increasing signal to noise ratios for satisfactory operation.

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Appendix A

Construction Details of PPM Generator

Parts List

Resistors

R 1 - 2K/pot	R12 - 3K/pot
R 2 - 22K	R13 - 5.6M
R 3 - 22K	R14 - 24K
R 4 - 27K	R15 - 100K
R 5 - 10K	R16 - 15K
R 6 - 10K	R17 - 1M
R 7 - 56K	R18 - 56K
R 8 - 560K	R19 - 100K
R 9 - 10K	R20 - 100K/variable
R10 - 10K	R21 - 15K
R11 - 1K	

Capacitors

C 1 - 0.2 microfarads	C 7 - 0.1 microfarads
C 2 - 0.1 microfarads	C 8 - 0.1 microfarads
C 3 - 0.5 microfarads	C 9 - 0.004 microfarads
C 4 - 0.1 microfarads	C10 - variable
C 5 - 0.5 microfarads	C11 - 0.005 microfarads
C 6 - 0.04 microfarads	C12 - 0.003 microfarads

Diodes

D 1, D 2, D 3, D 4, D 5, D 6. 1N315

Tube Complement

V 1 - ½6SN7
V 2 - ½6SN7
V 3 - 6J5
V 4 - ½12AU7
V 5 - ½12AU7

Terminal Connections

67 Variable capacitor, C 1
68 Variable capacitor, C 1
69 DC clipping voltage, E 1

70 Heater voltage, 6SN7
71 Blank
72 Heater voltage, 6J5
73 Variable capacitor, C 2
74 DC clipping voltage, E 2
75 Plate voltage, 6SN7, pin 5
76 Plate voltage, 6SN7, pin 2
77 Variable capacitor, C 2
78 Blank
79 Heater voltage, 6J5
80 PPM output
81 Heater voltage, 6SN7
82 Sine wave input
83 Sine wave input
84 Square wave input
85 Heater voltage, 12AU7
86 Heater voltage, 12AU7
87 Plate voltage, 12AU7
88 Negative bias, 12AU7
89 100K step resistor
90 100K step resistor
91 Plate voltage, 6J5

<u>Block Diagram of PPM Generator</u>

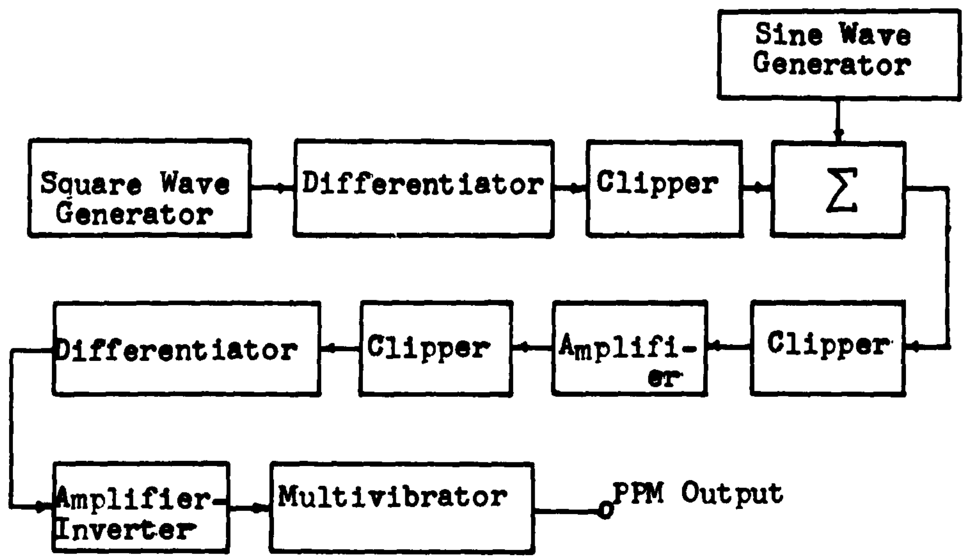

Fig. 14

Block Diagram of PPM Generator

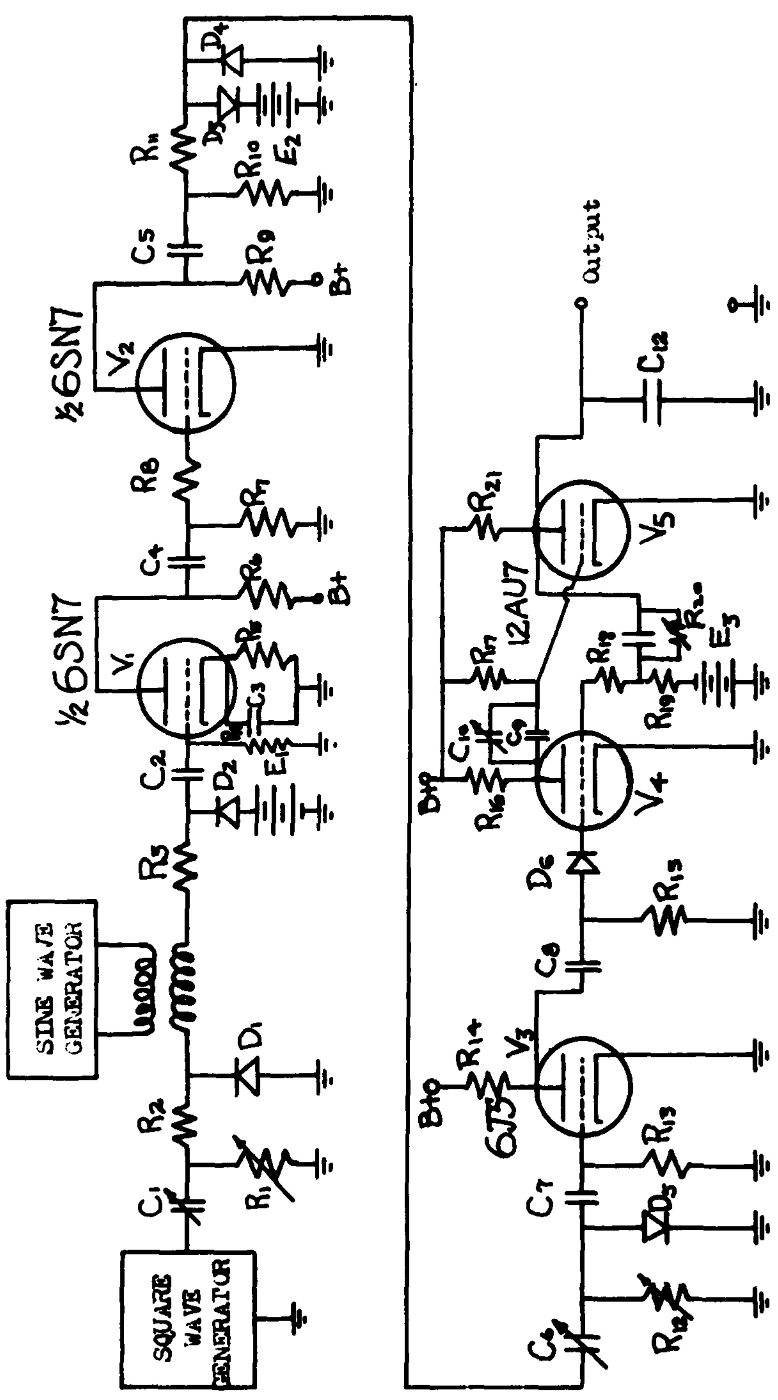

Fig. 15

Pulse Position Modulator

Description of PPM Generator

The basic theory of this device has been explained in Chapter IV. In this section any special circuits not fully described in Chapter IV will be discussed.

The amplitude values of the square wave, of the sine wave, and of the clipping voltage, E_1, are not critical. The important factor is the relative size of these amplitudes. As explained previously, the differentiated square wave is added to the sine wave. This sum is then clipped at a level slightly greater than the positive peak value of the sine wave. Since each differentiated pulse must be present after the clipping stage, then the amplitude of these pulses, and therefore of the input square wave, must be greater than the peak to peak value of the sine wave. Fig. 16b illustrates a case where this is not true, and Fig. 16a shows a satisfactory clipping level. In summary, E_1 must be equal to or greater than the peak value of the sine wave. In addition, the square wave input must be greater than the peak to peak value of the sine wave.

The purpose of the 6SN7 vacuum tube amplifier and the two stage clipper in to convert the variable amplitude pulses into a pulse width modulated signal. This is accomplished by amplifying the variable amplitude triangular pulses and clipping at an appropriate level. The resultant wave is a pulse width modulated signal with all pulses of the same amplitude. The resulting pulse width modulated signal has its trailing edge modulated. These pulses are differentiated,

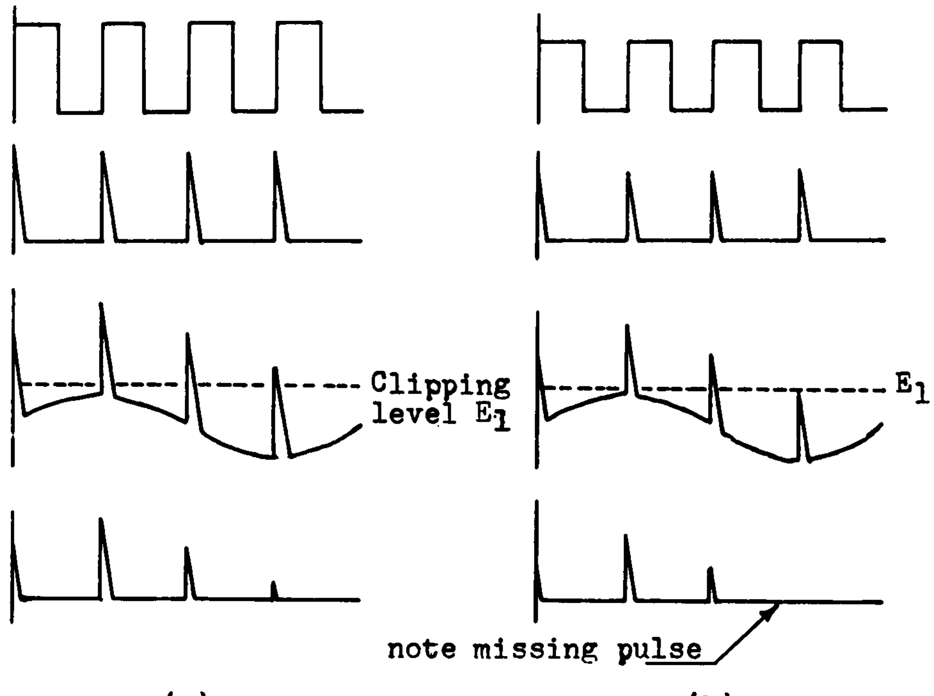

Fig. 16

Relative amplitudes of square wave output,
E_1 clipping level, and sine wave output
(a) Correct square wave amplitude
(b) Incorrect square wave amplitude

clipped, amplified, and used to trigger the one shot multivibrator. Capacitor C 10 serves to vary the pulse width of the pulse position modulated signal, while C 12 is used as a high frequency filter. The negative bias, E_3, is adjusted so that the differentiated pulse is capable of triggering the multivibrator.

The sine wave is added into the circuit in the manner shown so that the transformer can act as an isolation transformer to line frequencies.

The frequency of the sine wave is not critical, and in this thesis was chosen to be 45 cycles per second. The only restriction on the frequency of the square wave is the sampling theorem which states

$$\text{sampling frequency} > \text{twice modulating frequency}$$

In my case, the sampling frequency was 270 cps.

Several blank terminal connections and tube sockets were built into the system in case of future modification or additions. Figs. 17 and 18 are photographs of the top and bottom view of the PPM generator.

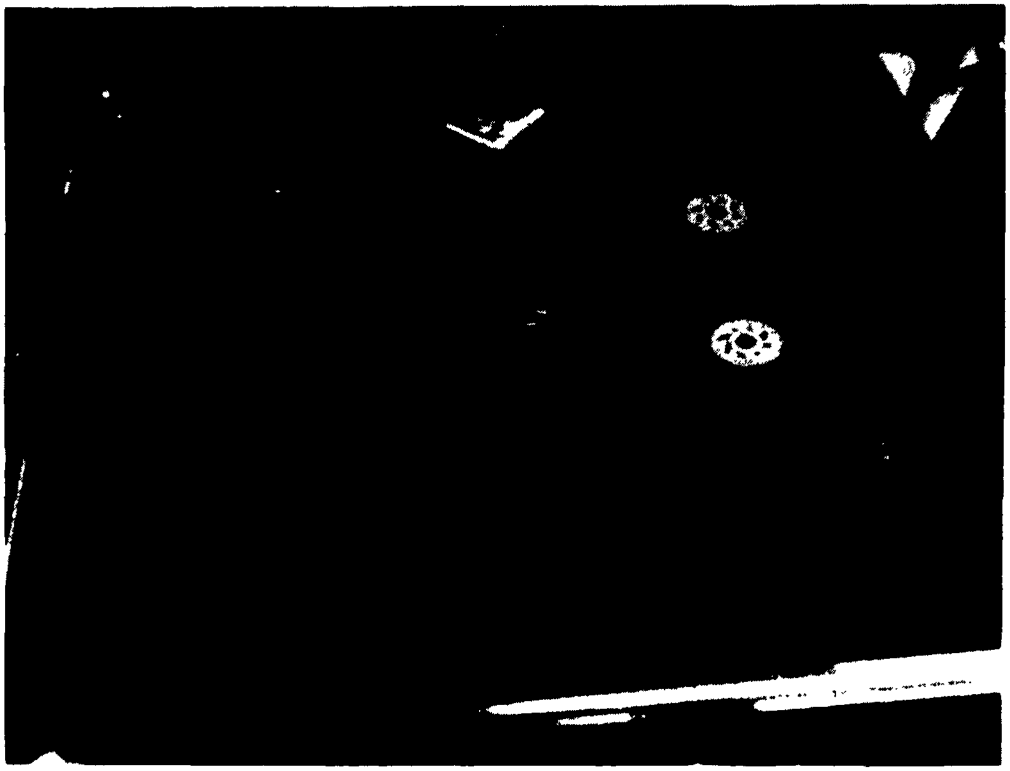

Fig. 17

Pulse Position Modulated Signal
Generator, Top View

Fig.18

Pulse Position Modulated Signal
Generator, Bottom View

Appendix B

Delay Line

The delay line was designed using equations obtained from Pulse and Digital Circuits by Jacob Millman and Herbert Taub (see Ref 7:293). The equations are derived by solving the differential equation for the circuit of Fig. 19 and giving the results in graphical form. The equations are derived assuming a termination of resistance equal to the characteristic impedance of the network. Following are listed the equations used in the design of the delay line:

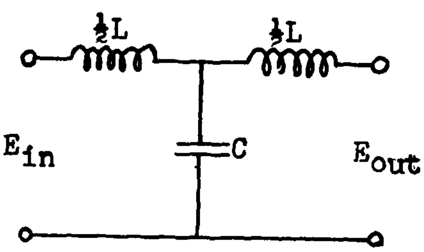

Fig. 19

Lumped-parameter delay line

1) $R_o = \sqrt{L/C}$

2) $t_s = 1.07\sqrt{LC}$ delay per section

3) $t_{r1} = 1.13\sqrt{LC}$ rise time per section

4) $t_d = n t_s$ total delay; n equals number of sections

5) $t_r = n^{1/3} t_{r1}$ total rise time

6) $n = \left(\frac{t_d}{t_r}\right)^{1.5} \left(\frac{t_{r1}}{t_s}\right)^{1.5}$ derived from Eqs (4) and (5)

Dividing Eq (3) by Eq (2)

7) $\frac{t_{r1}}{t_s} = \frac{1.13\sqrt{LC}}{1.07\sqrt{LC}} = 1.06$

Substituting Eq (7) into Eq (6)

8) $n = 1.1\left(\frac{t_d}{t_r}\right)^{1.5}$

The following values of C and L are obtained by solving Eqs (1), (2), and (4).

9) $C = \frac{t_d}{1.07 n R_o}$

10) $L = \frac{t_d R_o}{1.07 n}$

An estimated delay of 700 microseconds and rise time of 50 microseconds are obtained from the oscilloscope presentation of the output of the pulse position modulated signal generator. Substituting these values into Eqs (8)

11) $n = 1.1\left(\frac{700}{50}\right)^{1.5}$ sections

Solving Eqs (1), (9), and (10) simultaneously

12) $CL = 132 \times 10^{-12}$

Assuming a value of 20 millihenries for inductance, the capacitance will equal 6600 microfarads.

This is the general approach used to design the delay line. Fifty five sections of 10000 micro-microfarad capacitors and 20 millihenry inductors were required for the correct delay. Reasons for the discrepancy between calculated and actual values may include improper estimate of t_d and t_r, disregard of mutual coupling effect present in the circuit, estimates used in the derivations of the equations listed above, and improper loading of the line. The mathematical calculations were used to obtain approximate values of L, C, and n. Actual values were obtained by trial and error until the required delay (i.e., the pulse width of the pulse position modulated signal was achieved.

Appendix C

Oscilloscope Photographs

This appendix contains photographs of oscilloscope waveforms for various sections of the pulse position modulated autocorrelator and short narrative descriptions of each photograph.

Fig. 20 shows the output signal of the pulse position modulated signal generator without any modulation present. In Fig. 21, an expanded view of one pulse of the pulse position modulated signal is shown with modulation present. Fig. 22 shows a noise sample from the R-390A radio receiver and Fig. 23 shows the sum of the pulse position modulated signal plus noise.

As explained in Chapter III, it is necessary to invert and delay the pulse position modulated signal and then add the result to the original signal. These functions are illustrated in Figs. 24 and 25. It can be noted that the process of inverting and delaying does distort the original signal somewhat. This is seen most clearly in Fig. 25 where an easy comparison between the original signal and the inverted and delayed signal can be made. The lower pulse is the delayed and inverted signal. Because of the photographic process used, the photographs read from right to left instead of the usual left to right. The lower pulse is spread somewhat and distorted slightly in shape. Nevertheless, the delay is precisely equal to the pulse width as desired. Fig. 26 shows the waveform of Fig. 25 with the random noise added.

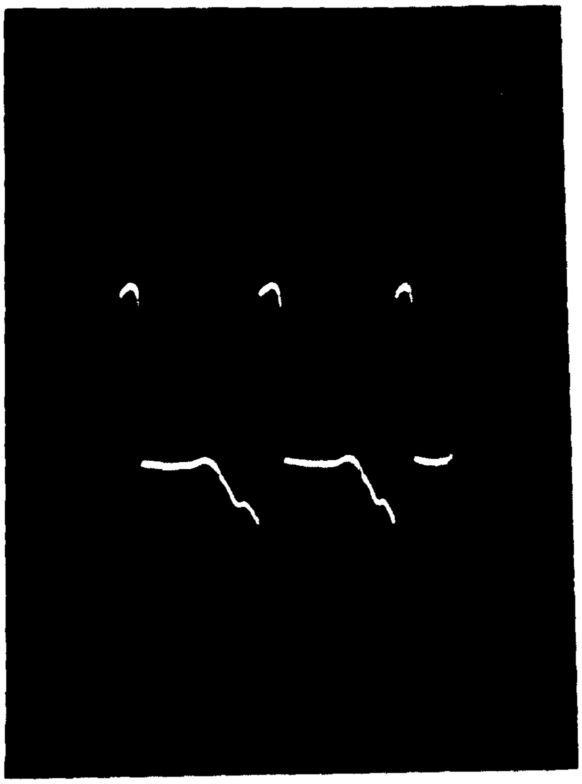

Fig. 20

Pulse Position Modulated Signal

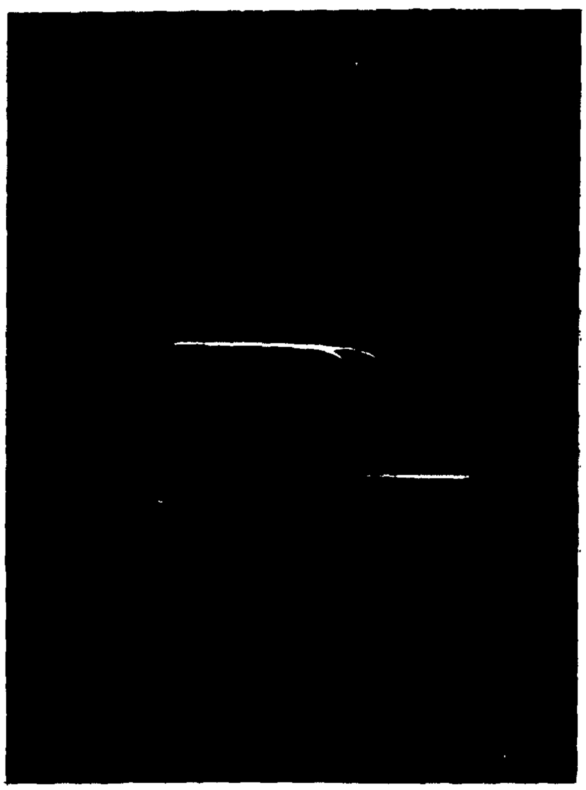

Fig. 21

Expanded Pulse Position Modulated Signal, Showing Modulation

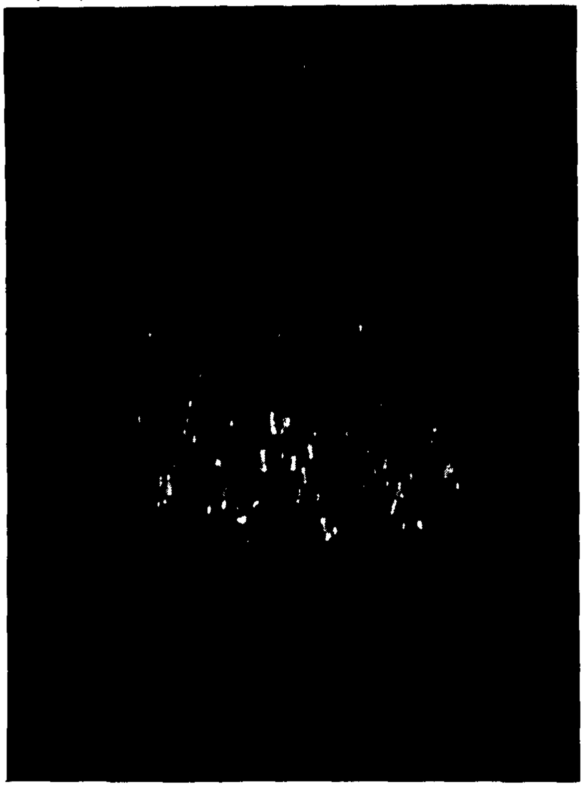

Fig. 22

Random Noise

Fig.23

Pulse Position
Modulated Signal
and Noise

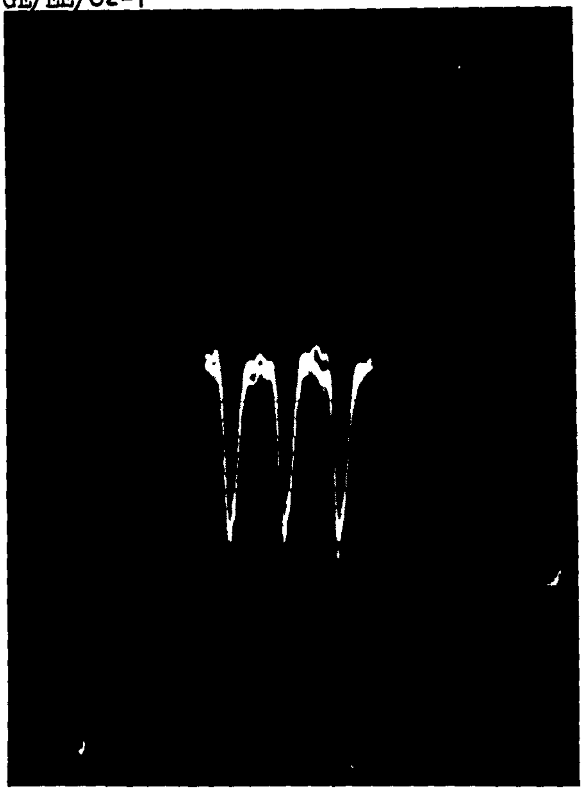

Fig. 24

Pulse Position Modulated Signal, Inverted and Delayed

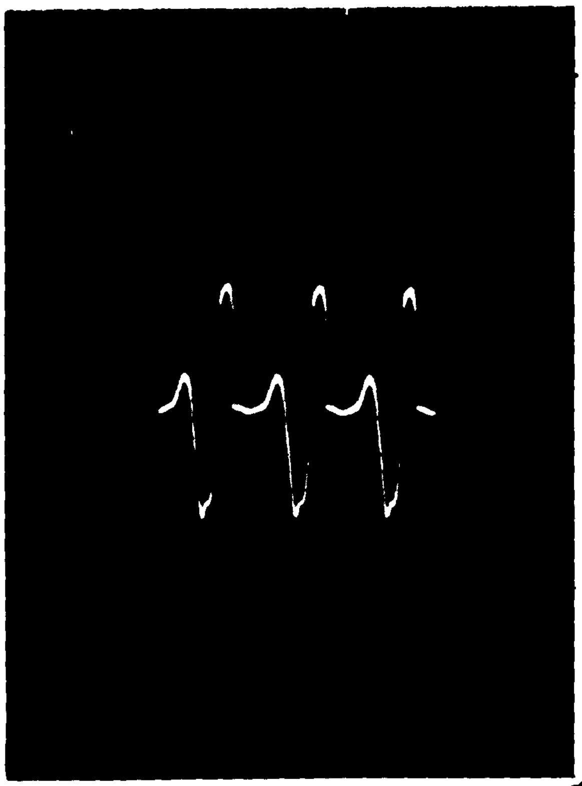

Fig. 25

PPM Signal Plus Inverted and Delayed PPM Signal

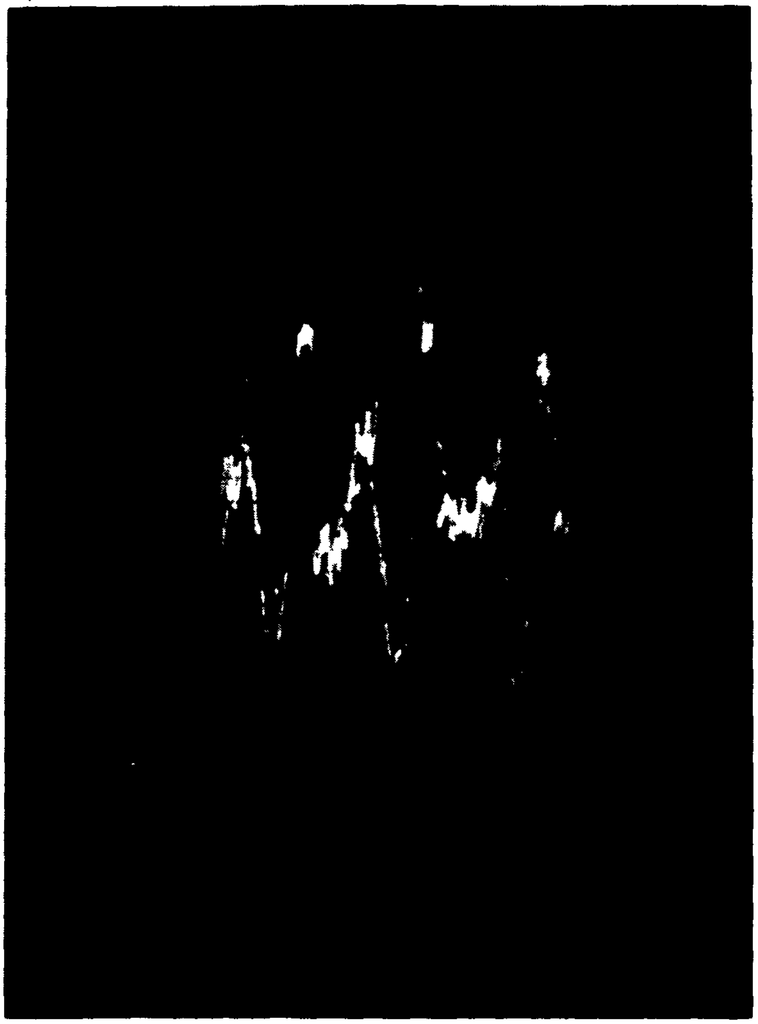

Fig. 26

Pulse Position Modulated Signal
Plus Inverted and Delayed Signal,
and Noise

Fig. 27 shows an expanded view of Fig. 25 after it has been differentiated. These differentiated peaks show the modulation clearly. Fig. 28 is identical to Fig. 27 except that it is not expanded, but has random noise added. By careful observation it is possible to see three negative peaks. These peaks are used, after amplification, to trigger the threshold multivibrator.

Fig. 29 shows in the bottom view the output of the pulse position modulated generator in expanded form, while the top view shows the output of the autocorrelator. Modulation can be seen in both views, and it is noted that, except for amplitude and pulse width, both signals are identical. The modulation present in the bottom view can also be seen in the top view. Fig. 30 is identical to Fig. 29 except that the threshold multivibrator output is adjusted so that the pulse width and amplitude is identical to the input pulse position modulated signal.

The bottom view of Fig. 31 shows the original pulse position modulated signal plus noise and the top view shows the output of the correlator. It can be observed that the desired signal occurs simultaneously in both waveforms. Fig. 32 illustrates what occurs when the amplitude of the noise gets too large and the resultant output of the correlator in not a reproduction of the input as required. Fig. 33, bottom view, shows the pulse position modulated signal plus noise, while the top view shows the output of the

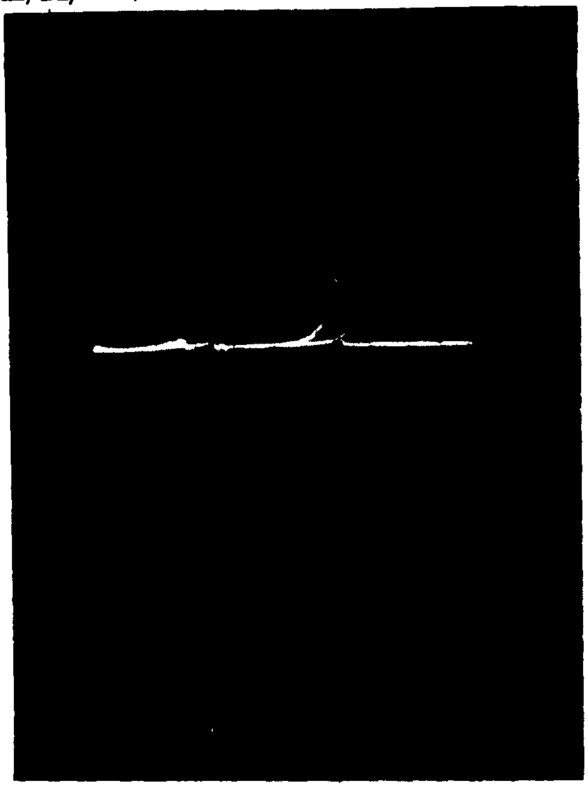

Fig. 27

Differentiated
Correlated Signal,
Expanded View

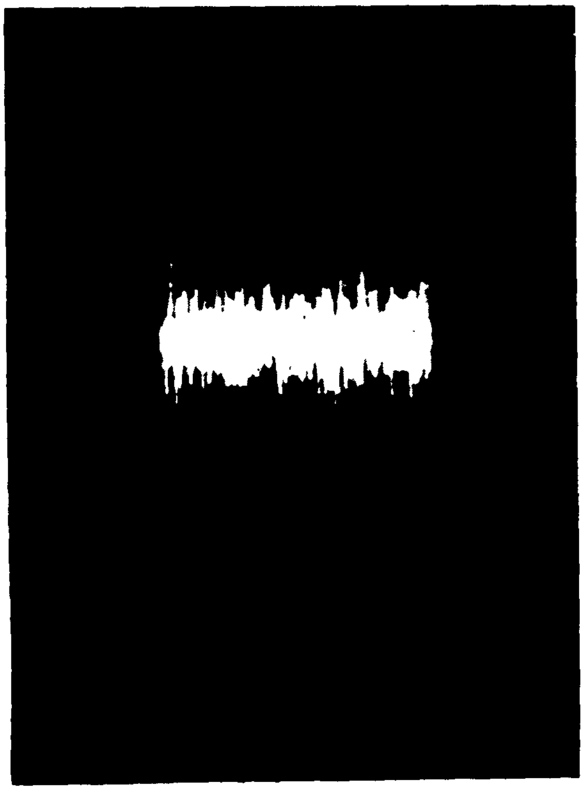

Fig. 28

Differentiated
Correlated Signal
Plus Noise

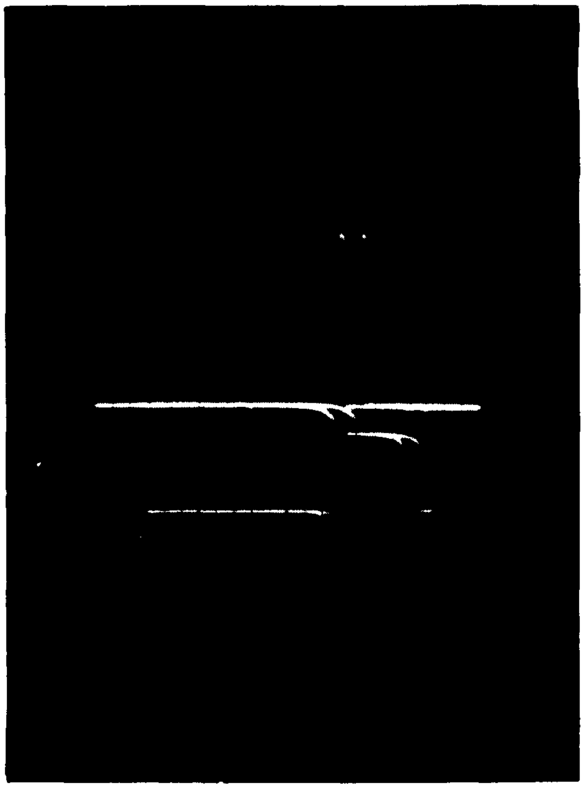

Fig. 29

Top View, Output of Correlator Multivibrator

Bottom View, Original PPM Signal Expanded View

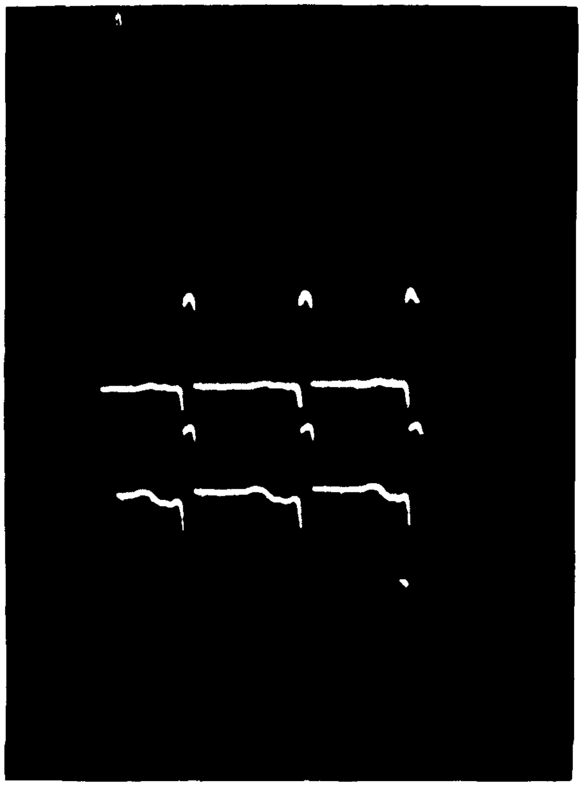

Fig.30

Top View, Output of Correlator Multivibrator

Bottom View, Original PPM Signal

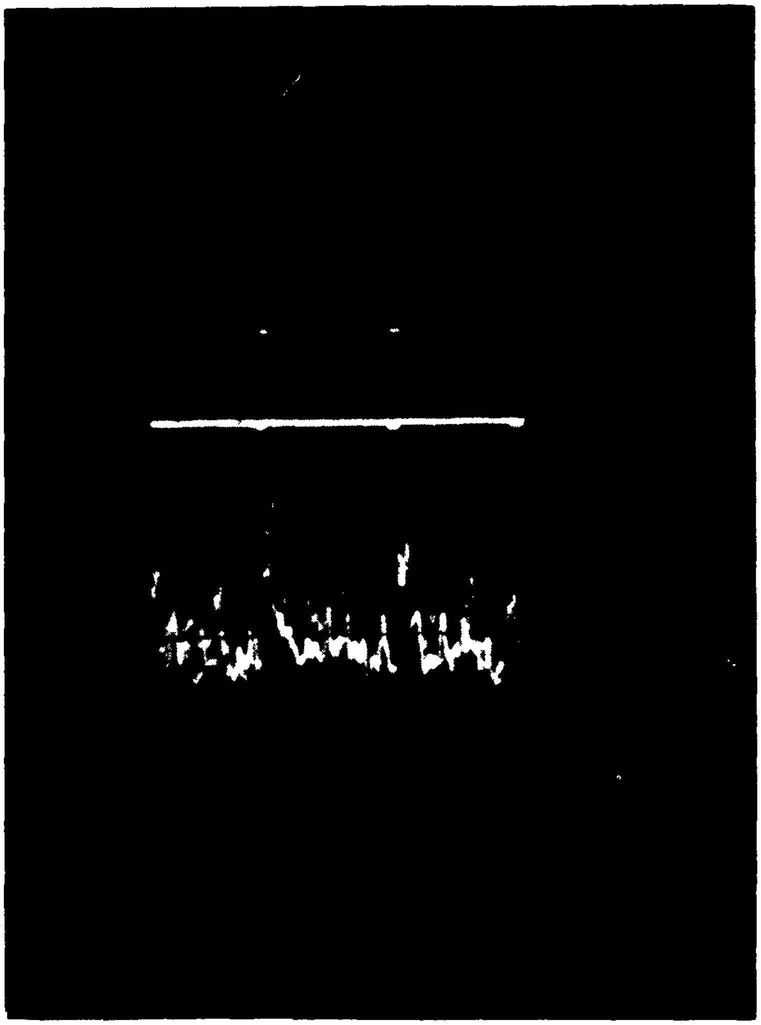

Fig. 31

Top View, Output of Correlator Multivibrator

Bottom View, Original PPM Signal Plus Noise

Fig. 32

Top View, Output of Correlator Multivibrator

Bottom View, Original PDM Signal Plus Noise threshold multivibrator when correlation is not performed (i.e., the pulse position modulated signal plus noise is used to trigger the multivibrator). By comparing Figs. 31 and 33 it is possible to observe the benefits of correlation. It is obvious that the rms value of noise present in Fig. 31 is greater than the rms noise value of Fig. 33. Yet is is possible to get the correct output from the threshold multivibrator with the larger rms value of noise as illustrated in Fig. 31 because of the presence of the autocorrelator in the circuit. In Fig. 33 the output is incorrect, although a smaller rms value of noise is present. The reason is obvious-- there is no correlator present in the circuit from which the waveforms of Fig. 33 were obtained. These two photographs (Figs. 31 and 33) conclusively prove the value and effectiveness of the autocorrelator.

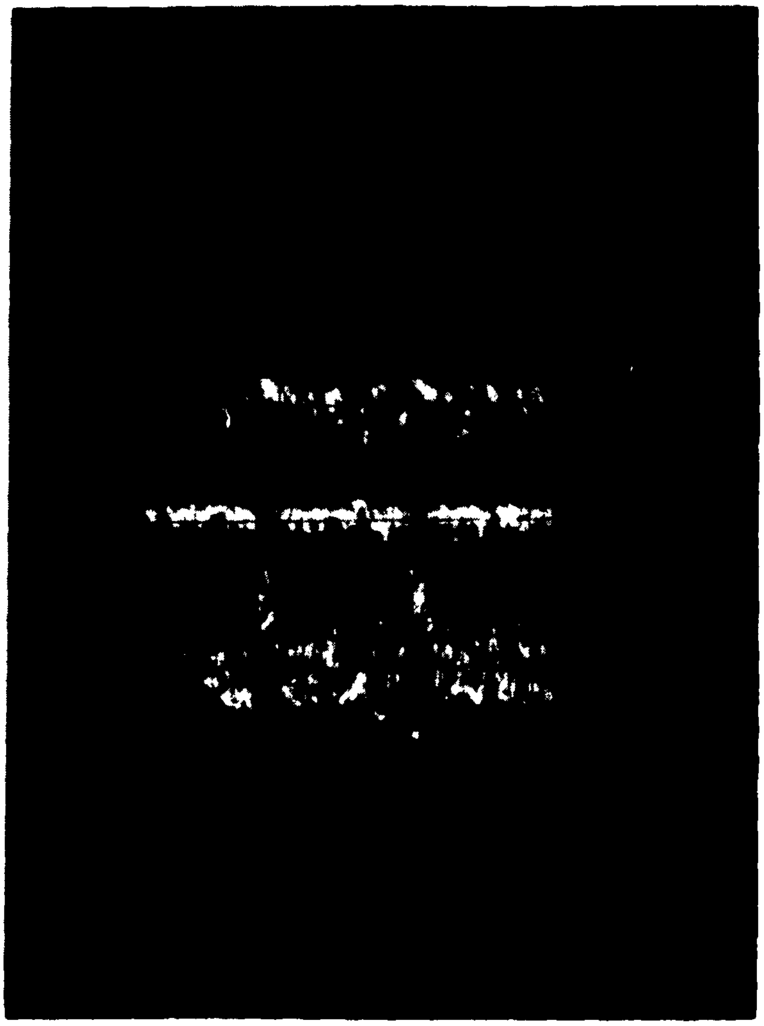

Fig. 33

Top View, Output of Final
Multivibrator without Correlation

Bottom View, Original PPM Signal
Plus Noise