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FINAL TECHNICAL REPORT

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"APPROXIMATIONS USING ORTHOGONAL FUNCTIONS"

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1. General summary of work under the grant

The principal investigator was fortunate in obtaining an excellent graduate student, Mr. K. F. Klopfenstein, to aid in the research under this grant.

Two papers were prepared and submitted for publication as a result of this research; copies of these papers have been sent previously to AFOSR to be filed. These papers were entitled:

[1] "On Singular Sturm-Liouville Eigenvalue Problems," and

[2] "Sturm-Liouville Systems That Generate Families."

The title of the first of these papers [1] has been changed to

[3] "A Remark on Singular Sturm-Liouville Differential Equations" and has been accepted for publication in the AMERICAN MATHEMATICAL MONTHLY.

As yet no word has been received as to the fate of the second of these papers [2]. It was submitted for publication in the SIAM REVIEW.

2. Review of research pursued under the grant

The main portion of the time under the present grant was spent in examining Sturm-Liouville systems for possible functions to use in approximations using orthogonal functions. It is known [4, 5, 6] that the eigenvalues of a Sturm-Liouville system are orthogonal over some interval [a, b] with respect to some function w(x), so, as a first area of exploration, it was decided to examine in more detail the more general types of Sturm-Liouville

systems that arose in [6].

In approximating the solutions of differential equations and integro-differential equations, it is necessary to approximate the various derivatives of a solution as well as the solution itself. The functions studied in [2] were found to possess precisely the properties desired for approximating functions.

A family of Sturm-Liouville systems is defined to be the collection of Sturm-Liouville systems consisting of an equation of the type

$$py'' + sy' + (q + \lambda_i r)y = 0 \tag{1}$$

with corresponding homogeneous boundary conditions of the form

$$\begin{aligned} Ay(a) + By'(a) &= 0 \\ Cy(b) + Dy'(b) &= 0 \end{aligned} \tag{2}$$

and all Sturm-Liouville systems that can be obtained by repeatedly differentiating and integrating equation 1 and applying a particular set of boundary conditions of the form of equation 2 to the same interval [a, b]. In [2] the necessary and sufficient conditions were determined for the coefficients in equation 1 and 2 so that the system would generate a family.

The singular Sturm-Liouville differential equations in particular were discovered to provide fruitful results. It was found that the only singular Sturm-Liouville differential equations that can generate families and which have singularities at both endpoints of the interval are the differential equations of Hermite, Jacobi, and Laguerre. Furthermore, these are the only Sturm-Liouville differential equations that have polynomial eigenfunctions and generate families.

As a side result of the preceding study, a paper [3] developed in which it was shown that a result of Hahn [7] can be derived by different means than he used. In particular it was shown that by relying on the usual assumption that the weight function w(x) be positive and integrable over the interval of orthogonality (a, b), then equation 1 is equivalent, up to a linear transformation of the independent variable, to the differential equations of Hermite, Jacobi, and Laguerre, under the necessary and sufficient conditions determined for that equation to generate a family, for the case in which equation 1 has singularities at both endpoints of the interval [a, b].

It is hoped that the results of [2] and [3] will be useful in the areas of approximation to be studied under the continuation of the grant on "Approximations Using Orthogonal Functions."

REFERENCES

1. F. Max Stein and Kenneth F. Klopfenstein, On Singular Sturm-Liouville Eigenvalue Problems.

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3. F. Max Stein and K. F. Klopfenstein, A Remark on Singular Sturm-Liouville Differential Equations.

4. R. V. Churchill, Fourier Series and Boundary Value Problems, McGraw-Hill, New York, 1941.

5. R. V. Churchill, Operational Mathematics, McGraw-Hill, New York, 1958, Second Ed.

6. E. Dunn and F. Max Stein, Families of Sturm-Liouville Systems, SIAM Review, 3(1961), pp. 54-65.

7. W. Hahn, Über die Jacobischen Polynome und Zwei Verwandte Polynomklassen, Math. Zeitschrift, 39(1935), pp. 634-638.