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CONVERSION OF STRESS RELAXATION MODULUS TO DYNAMIC MODULUS FOR A VISCOELASTIC MATERIAL

MTO-270-2

November 1962

Prepared by

HERCULES POWDER COMPANY
CHEMICAL PROPULSION DIVISION
Bacchus Works
Magna, Utah

Prepared for

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SPACE SYSTEMS DIVISION
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Los Angeles, California

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CONVERSION OF STRESS RELAXATION MODULUS TO DYNAMIC MODULUS FOR A VISCOELASTIC MATERIAL

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FOREWORD

This document describes techniques used in determining material properties of the Stage III, XM-57 Minuteman Motor. It was prepared by the Structural Dynamics Group, Research & Development Department, Bacchus Works, Chemical Propulsion Division, Explosives Department, Hercules Powder Company.

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ABSTRACT

The relationship between the stress relaxation modulus, $E(t)$, and the dynamic modulus, $E^*(i\omega) = E'(\omega) + i\,E''(\omega)$, can be expressed in the form

$$E'(\omega) = \omega \int_0^\infty E(t) \sin \omega t \; dt$$

and

$$E''(\omega) = \omega \int_0^\infty E(t) \cos \omega t \; dt$$

The evaluation of these integrals must be a numerical process since $E(t)$ as obtained from test data is ordinarily given in tabular form. The basic numerical approach presented in this document is to fit $E(t)$ over each time definition interval with a function of the type

$$E(t) \Big|_{t_r}^{t_{n+1}} = A_n e^{-b_n t}$$

The integral defining $E'(\omega)$ can then be expressed as sums of integrals of the type

$$\int_{t_n}^{t_{n+1}} A_n e^{-b_n t} \sin \omega t \, dt$$

This integral and the companion integral for $E''(\omega)$ can be evaluated analytically, resulting in a series approximation for $E^*(i\omega)$ containing no integrals.

The numerical procedure is carried out using a digital computer. An accuracy investigation is performed using the tensile stress relaxation data for polyisobutelene. The regime in which $E(t)$ must be known in order to accurately compute $E^*(i\omega)$ is determined for a frequency spectrum.

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INTRODUCTION

The stress relaxation modulus, E(t), for a linear viscoelastic material can be converted to the dynamic modulus, $E^*(i\omega)=E'(\omega)+iE''(\omega)$, by evaluating the following integrals:[1]

$$E'(\omega) = \omega \int_0^{\infty} E(t) \sin \omega t \, dt$$

$$E'(\omega) = \int_0^{\infty} E(t) \cos \omega t \, dt$$

It is not possible to obtain E(t) from tests in the time range specified by either the lower or upper limit of these integrals. Therefore, what must the time range be for E(t) in order to obtain a good approximation for $E^*(i\omega)$? The data from stress relaxation tests is ordinarily obtained in tabular form. A numerical procedure must be used to evaluate the integrals for $E'(\omega)$ and $E''(\omega)$.

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[1] See List of References.

SECTION II

METHOD OF SOLUTION

Given the time dependent relaxation modulus, E(t), for a linear viscoelastic material, it is desired to compute the frequency dependent dynamic modulus, $E^*(i\omega) = E'(\omega) + i\,E''(\omega)$. The relationship between $E^*(i\omega)$ and E(t) can be expressed in several forms. Commonly used forms for this relationship are:[1,2]

$$E'(\omega) = \omega \int_0^\infty E(t) \sin \omega t \, dt \tag{1}$$

$$E''(\omega) = \omega \int_0^\infty E(t) \cos \omega t \, dt \tag{2}$$

and

$$E'(\omega) = E_c + \omega \int_0^\infty \psi(t) \sin \omega t \, dt \tag{3}$$

$$E''(\omega) = \omega \int_0^\infty \psi(t) \cos \omega t \, dt \tag{4}$$

where: $\psi(t) = E(t) - E_c$

E_c = constant value approached by E(t)

It can be shown that equations (1) and (3) are equivalent, as are equations (2) and (4). This demonstration is based upon the fact that

$$\int_0^\infty \sin x \, dx = 1 \quad \text{and} \quad \int_0^\infty \cos x \, dx = 0$$

The integral in equation (1) can be written as

$$\int_0^\infty E(t) \sin \omega t \, dt = \int_0^{\frac{2m\pi}{\omega}} E(t) \sin \omega t \, dt + \int_{\frac{2m\pi}{\omega}}^\infty E_c \sin \omega t \, dt$$

where m is an integer chosen such that $E(t) = E_c$ for all $t > \frac{2m\pi}{\omega}$

[1,2] See List of References

Since

$$\omega \int_{\frac{2m\pi}{\omega}}^{\infty} E_c \sin \omega t \, dt = E_c \quad \text{and} \quad \omega \int_{\frac{2m\pi}{\omega}}^{\frac{2m\pi}{\omega}+\frac{\pi}{2\omega}} E_c \sin \omega t \, dt = E_c$$

equation (1) can be written as

$$E'(\omega) = \omega \int_0^{\frac{2m\pi}{\omega}+\frac{\pi}{2\omega}} E(t) \sin \omega t \, dt. \tag{5}$$

A similar treatment of equation (2) yields

$$E''(\omega) = \omega \int_0^{\frac{2m\pi}{\omega}} E(t) \cos \omega t \, dt. \tag{6}$$

Because E(t), as obtained from tests, is in tabular form, a numerical method must be used to evaluate the integrals for $E^*(i\omega)$. A procedure for curve fitting will be necessary in any method chosen. Two distinct types of curve fitting methods are available. One type consists of determining a single function which is good for the full range of the independent variable. This is the type of fit accomplished when a multiparameter Maxwell model is assumed for the material.[3] The other type of curve fit method consists of selecting discrete functions to be used in different ranges of the independent variable. The latter method was chosen for further study because of its simplicity and accuracy potential.

Examination of several sets of tabular data for E(t) has indicated that E(t) can be fit over time intervals with functions of the type

$$E(t)\Big|_{t_n}^{t_{n+1}} = a_n e^{-b_n t} \qquad \text{(as shown in Figure 1)}$$

[3]See List of References

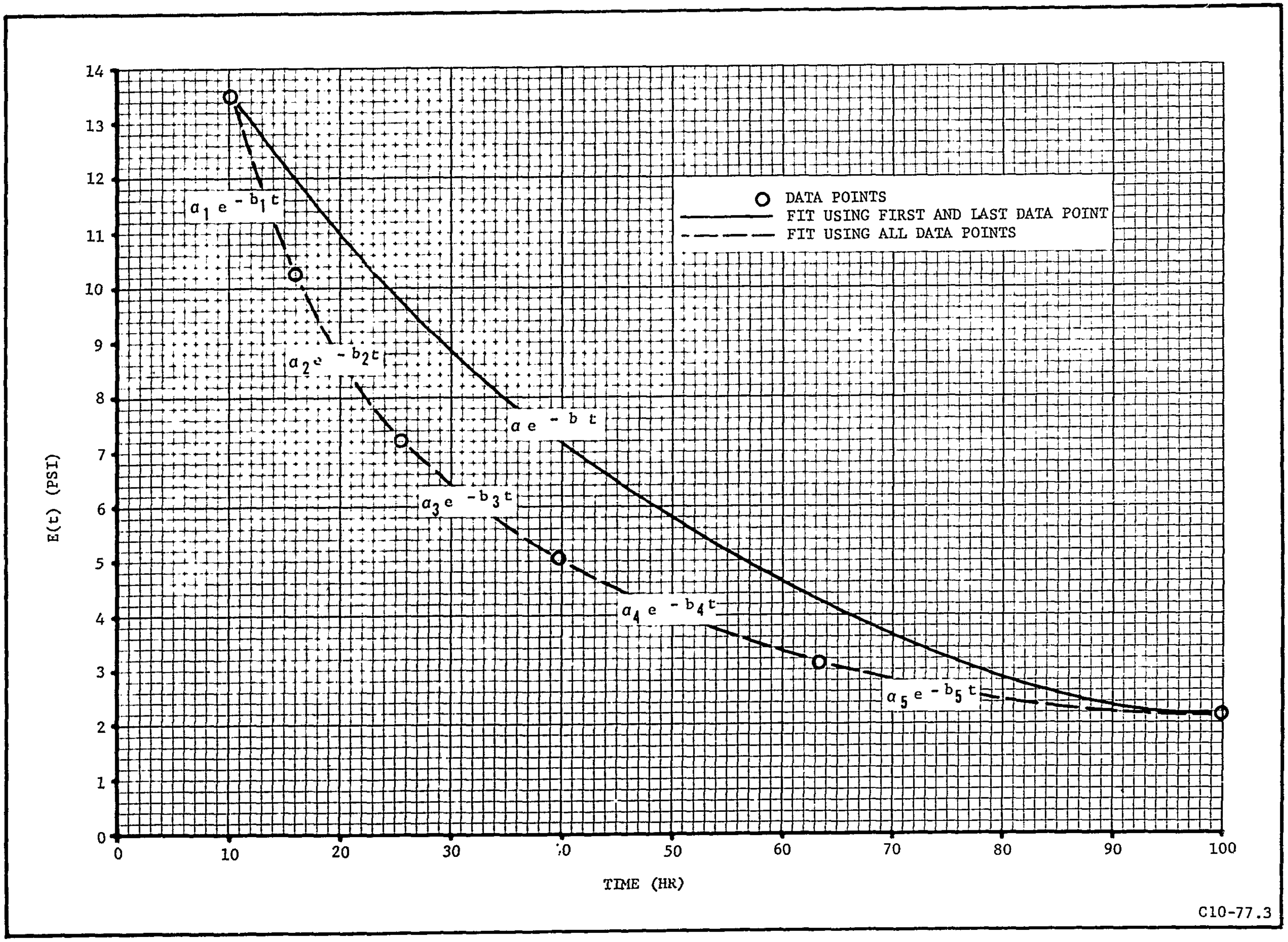


Figure 1. Exponential Fit of Polyisobutelene Stress Relaxation Data for the Time Interval Between 10 and 100 Hours

Substitution of this expression into equations (5) and (6) yields

$$E'(\omega) = \omega \sum_{n=1}^{n_1} \int_{t_n}^{t_{n+1}} a_n e^{-b_n t} \sin \omega t \, dt \tag{7}$$

$$E''(\omega) = \omega \sum_{n=1}^{n_2} \int_{t_n}^{t_{n+1}} a_n e^{-b_n t} \cos \omega t \, dt \tag{8}$$

where n_1 is chosen such that

$$t_{n_1} = \frac{2m\pi}{\omega} + \frac{\pi}{2\omega}$$

and n_2 is chosen such that

$$t_{n_2} = \frac{2m\pi}{\omega}$$

For the time interval between t_n and t_{n+1}, these integrals can be evaluated analytically, the result being

$$E'(\omega) = \omega \sum_{n=1}^{n_1} \frac{a_n}{b_n^2+\omega^2} \left[e^{-b_n t} (b_n \sin \omega t + \omega \cos \omega t) \right]_{t_{n+1}}^{t_n} \tag{9}$$

and

$$E''(\omega) = \omega \sum_{n=1}^{n_2} \frac{a_n}{b_n^2+\omega^2} \left[e^{-b_n t} (b_n \cos \omega t - \omega \sin \omega t) \right]_{t_{n+1}}^{t_n} \tag{10}$$

where: ω = given circular frequency

$$b_n = \frac{\ln \left[\frac{E(t_n)}{E(t_{n+1})}\right]}{t_{n+1} - t_n}$$

$$a_n = E(t_n)\, e^{b_n t_n}$$

Two considerations should be kept in mind when the time increments, t_n, in equations (9) and (10) are chosen. The first consideration is that for small values of t, E(t) changes rapidly with time. The t_n should be chosen in this time regime where the test data points occur. The second consideration concerns convergence.

A method of determining the convergence of the integrals in equations (5) and (6) should be provided. A convergence criterion is easily obtained if t_{n+1} is chosen such that $\sin \omega t_{n+1} = \sin \omega t_n$ and $\cos \omega t_{n+1} = \cos \omega t_n$. If t_n is large compared with $2\pi/\omega$, then the data points thereafter can be assumed to occur at multiples of $2\pi/\omega$ with little error.

Assume $t_{n+1} = t_n + 2\frac{p\pi}{\omega}$ in equation (9), where p is a given positive integer. Further, assume that a_n, b_n, and ω are given for the interval t_n to t_{n+1}. Then the interval contribution, δ_n, is equal to

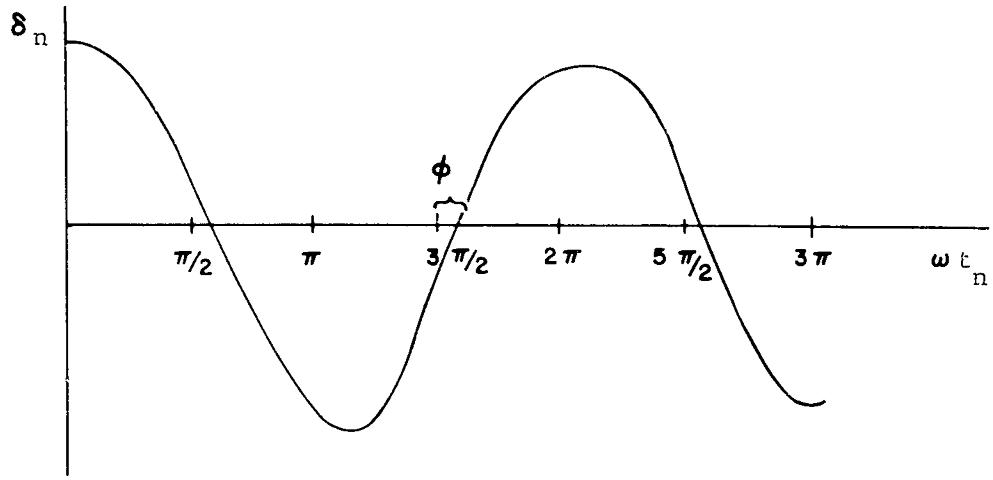
$$\delta_n \Bigg|_{t_n}^{t_n + \frac{2p\pi}{\omega}} = \frac{a_n}{b_n^2 + \omega^2} \left[e^{-b_n t} (b_n \sin \omega t + \omega \cos \omega t) \right]_{t_n + \frac{2p\pi}{\omega}}^{t_n}$$

This equation can be expressed in the form

$$\delta_n \Bigg|_{t_n}^{t_n + \frac{2p\pi}{\omega}} = \frac{a_n}{b_n^2 + \omega^2} \left(1 - e^{-2p\pi \frac{b_n}{\omega}}\right) \left\{ e^{-b_n t_n} \sin (\omega t_n + \phi) \right\} \quad (11)$$

where $\phi = \tan^{-1} \left(\frac{\omega}{b_n}\right)$

A plot of δ_n as function of t_n will appear as



The interval contribution is a maximum when $t_n = q \frac{\pi}{2\omega} - \frac{\phi}{\omega}$, when q is an even integer, and the contribution is zero when q is an odd integer.

As E(t) approaches a constant, b_n and ϕ approach zero. Thus δ_n is small when t_n is an odd multiple of $\pi/2\omega$. This fact, coupled with the fact that t_{n1} must be an odd multiple of $\frac{\pi}{2\omega}$, has the implication that the t_n in equation (9) should be chosen as odd multiples of $\pi/2\omega$ when t_n is sufficiently large to permit the approximation. With the t_n thus chosen, equation (11) reduces to

$$\delta n \Big|_{t_n}^{t_{n+1}} = \frac{b_n}{\omega} \frac{1}{1 + \left(\frac{b_n}{\omega}\right)^2} \left[E(t_n) - E(t_{n+1}) \right] \tag{12}$$

A similar discussion and treatment of $E''(\omega)$ implies that t_n in equation (10) should be chosen as even multiples of $\pi/2\omega$ when t_n is sufficiently large to permit the approximation. With these assumptions concerning t_n, the interval contribution formula for $E''(\omega)$ is identical with equation (12).

A computer program was written for the IBM 1620 digital computer. (See Appendix A.) This program evaluates equations (9), (10), and (12). The time intervals, t_n and t_{n+1}, in equations (9) and (10) are chosen as the intervals at which the data points are given. When a t_n is found such that $t_n > K \left(\frac{2\pi}{\omega}\right)$, where K is an integer given as input, the program assumes that $E(t_n)$ is equal to $E(t_p)$ where $\omega t_p = (\frac{\pi}{2}$ + nearest integral multiple of $2\pi)$ in equation (9) and ωt_p = nearest integral multiple of 2π in equation (10). Thereafter, the program uses equation 12 to compute $E^*(i\omega)$.

SECTION III

DISCUSSION OF RESULTS

To analyze the accuracy of the method, the stress relaxation data for polyisobutelene[4] (see table I) was run using this computer program. The stress relaxation data extends over 16 decades in time, and is acknowledged to be the best data available at the present time. The computer results for $E^* (i\omega)$ are shown in figure 2. The data for the dynamic shear modulus, $G^* (i\omega)$, for polyisobutelene[5] was converted to $E^* (i\omega)$ using the formula $E^* (i\omega) = 3\, G^* (i\omega)$. The results are also plotted in figure 2. The comparison is excellent throughout most of the frequency regime.

The data was further analyzed to determine the sum interval contributions in equations (9), (10) and (12). These results are given in table II for $\omega = 100$ cps. Four different frequencies were run, .01, 1, 100, and 1000 cps, to determine the interval sums up to $t = \frac{10\,(2\pi) + \pi/2}{\omega}$ in equation (9) and $t = 10(\frac{2\pi}{\omega})$ in equation (10). In all cases, the sum contributions added up to results accurate to four significant figures as compared with the final computed answers for $E' (\omega)$ and $E'' (\omega)$ using the full time regime. For $\omega = 1$ cps, $10(\frac{2\pi}{\omega}) = 10$ seconds. Thus an accurate computation for $E^* (i\omega)$ could be obtained from stress relaxation tests run for only 10 seconds.

The minimum time at which the first data point must be known in order to compute $E' (\omega)$ and $E'' (\omega)$ accurately to four significant figures for $\omega = 1$ cps is 1×10^{-3} sec. for $E' (\omega)$ and 1×10^{-5} sec. for $E'' (\omega)$.

The time range over which the stress relaxation modulus for polyisobutelene must be known to compute $E^* (i\omega)$ accurate to four significant figures for a given frequency is approximately 5 decades for $E' (\omega)$ and 10 decades for $E'' (\omega)$. This is true for the frequency range from .01 to 1000 cps, which is presented graphically in figure 3. These limits apply only to polyisobutelene but those for solid propellant should be similar.

[4,5] See List of References

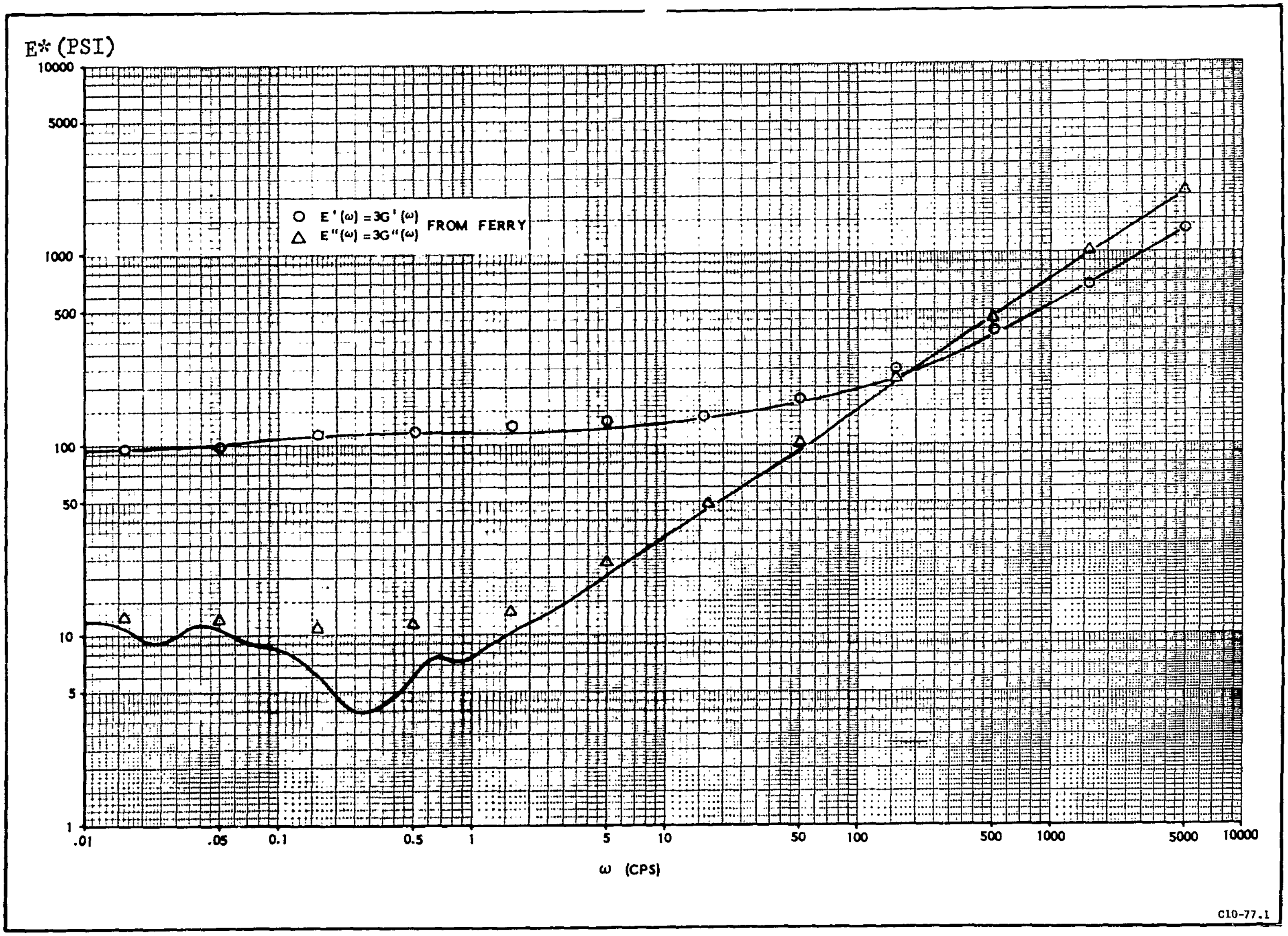


Figure 2. E* (iω) for Polyisobutelene

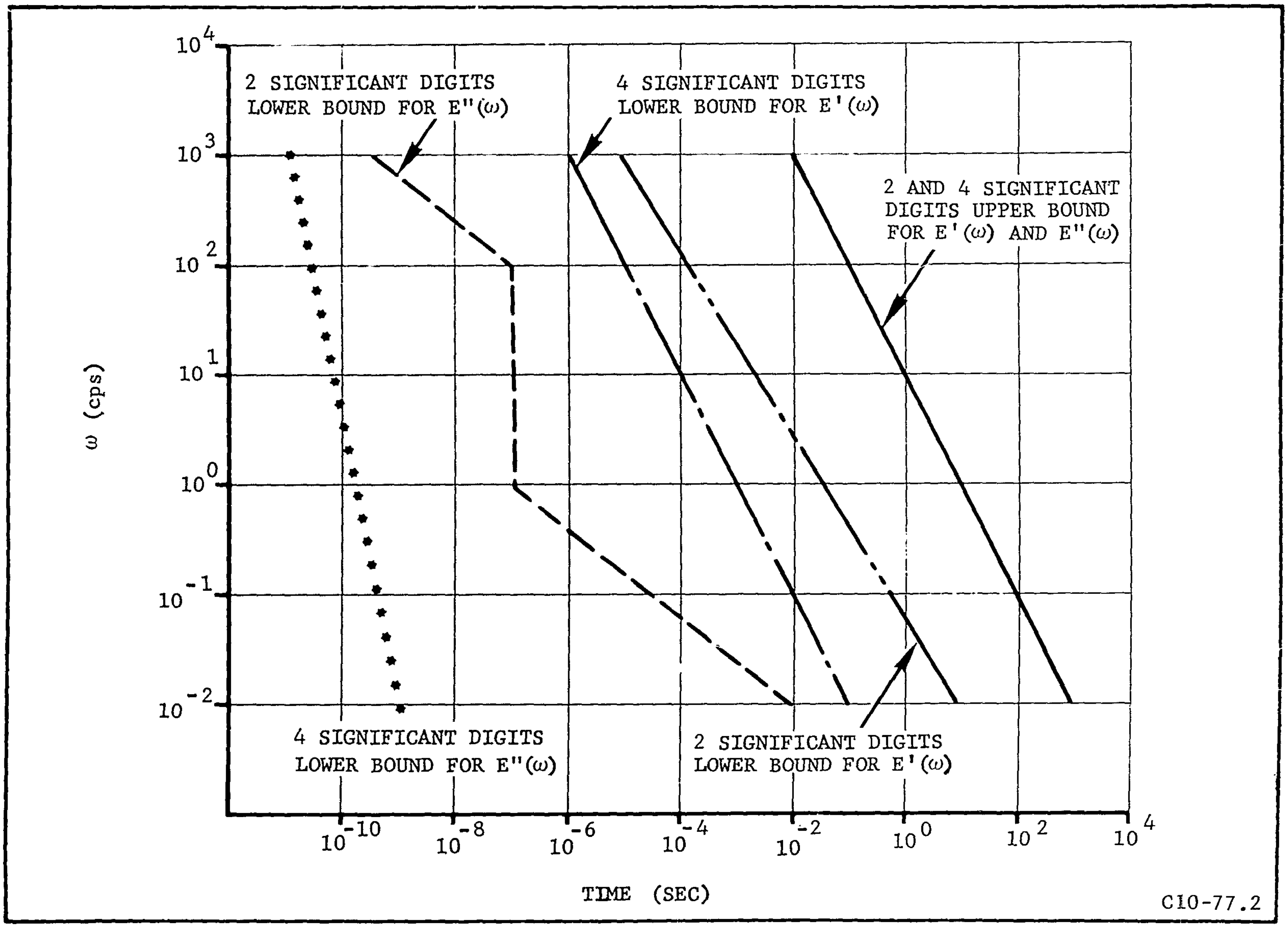


Figure 3. Time Regime in Which E(t) Must Be Known to Compute $E^*(i\omega)$ for Polyisobutelene

TABLE I

STRESS RELAXATION DATA FOR POLYSIOBUTELENE

$\log_{10} t$ (HRS)	$\log_{10} E(t)$ (PSI)
-.45000E+02	.56500E+01
-.14400E+02	.56460E+01
-.14200E+02	.56410E+01
-.14000E+02	.56260E+01
-.13800E+02	.56160E+01
-.13600E+02	.56060E+01
-.13400E+02	.55910E+01
-.13200E+02	.55710E+01
-.13000E+02	.55510E+01
-.12800E+02	.55310E+01
-.12600E+02	.55010E+01
-.12400E+02	.54610E+01
-.12200E+02	.54160E+01
-.12000E+02	.53660E+01
-.11800E+02	.53110E+01
-.11600E+02	.52360E+01
-.11400E+02	.51410E+01
-.11200E+02	.50460E+01
-.11000E+02	.49360E+01
-.10800E+02	.48160E+01
-.10600E+02	.46860E+01
-.10400E+02	.45560E+01
-.10200E+02	.44210E+01
-.10000E+02	.42860E+01
-.98000E+01	.41510E+01
-.96000E+01	.40260E+01
-.94000E+01	.38960E+01
-.92000E+01	.37660E+01
-.90000E+01	.36310E+01
-.88000E+01	.34910E+01
-.86000E+01	.33560E+01
-.84000E+01	.32160E+01
-.82000E+01	.30860E+01
-.80000E+01	.29660E+01
-.78000E+01	.28560E+01
-.76000E+01	.27460E+01
-.74000E+01	.26460E+01
-.72000E+01	.25460E+01
-.70000E+01	.24560E+01
-.68000E+01	.23710E+01
-.66000E+01	.23010E+01
-.64000E+01	.22460E+01
-.62000E+01	.22010E+01
-.60000E+01	.21660E+01
-.58000E+01	.21410E+01
-.56000E+01	.21210E+01
-.54000E+01	.21010E+01
-.52000E+01	.20860E+01
-.50000E+01	.20760E+01
-.48000E+01	.20660E+01
-.46000E+01	.20610E+01
-.44000E+01	.20560E+01
-.42000E+01	.20510E+01
-.40000E+01	.20460E+01
-.38000E+01	.20410E+01
-.36000E+01	.20360E+01
-.34000E+01	.20310E+01
-.32000E+01	.20160E+01
-.30000E+01	.20010E+01
-.28000E+01	.19860E+01
-.26000E+01	.19710E+01
-.24000E+01	.19560E+01
-.22000E+01	.19410E+01
-.20000E+01	.19160E+01
-.18000E+01	.18960E+01
-.16000E+01	.18710E+01
-.14000E+01	.18460E+01
-.12000E+01	.18160E+01
-.10000E+01	.17860E+01
-.80000E-00	.17510E+01
-.60000E-00	.17110E+01
-.40000E-00	.16660E+01
-.20000E-00	.16160E+01
.00000E-99	.15560E+01
.20000E-00	.14910E+01
.40000E-00	.14210E+01
.60000E-00	.13360E+01
.80000E-00	.12410E+01
.10000E+01	.11310E+01
.12000E+01	.10110E+01
.14000E+01	.86100E-00
.16000E+01	.70100E-00
.18000E+01	.50100E-00
.20000E+01	.34100E-00
.22000E+01	.61000E-01
.24000E+01	-.33900E-00
.26000E+01	-.83900E-00

NOTE:

PRINTOUTS MAY BE PUT INTO A MORE READABLE FORM. FOR EXAMPLE THE PRINTOUT -.45000E + 02 BECOMES -- 45.0, BECAUSE THE E + 02 OF THE PRINTOUT DESIGNATES THE NUMBER AND DIRECTION OF THE DECIMAL POINT. NOTE THE FOLLOWING EXAMPLES:

$\pm$ 0.10600 E + 02 = $\pm$ 10.6
$\pm$ 0.82000 E + 01 = $\pm$ 8.20
$\pm$ 0.40000 E - 00 = $\pm$ 0.40
$\pm$ 0.00000 E - 99 = 0
$\pm$ 0.12000 E - 01 = $\pm$ 0.012
$\pm$ 0.23000 E - 02 = $\pm$ 0.0023

C10-92.1

TABLE II

INTERVAL CONTRIBUTIONS FOR E*(iω)

T_n	E'_{n+1}	E''_{n+1}	$E'_{n+1}-E'_n$	$E''_{n+1}-E''_n$
.99999E-45	.44877E-01	.40039E-02	.18029E-10	.40039E-02
.39810E-14	.44877E-01	.63216E-02	.26963E-10	.23177E-02
.63095E-14	.44877E-01	.99115E-02	.66131E-10	.35898E-02
.10000E-13	.44877E-01	.15439E-01	.16146E-09	.55280E-02
.15848E-13	.44877E-01	.24001E-01	.39639E-09	.85619E-02
.25118E-13	.44877E-01	.37186E-01	.96695E-09	.13185E-01
.39810E-13	.44877E-01	.57258E-01	.23320E-08	.20072E-01
.63095E-13	.44877E-01	.87639E-01	.55941E-08	.30380E-01
.10000E-12	.44877E-01	.13362E-00	.13419E-07	.45983E-01
.15848E-12	.44877E-01	.20243E-00	.31798E-07	.68809E-01
.25118E-12	.44877E-01	.30305E-00	.73637E-07	.10062E-00
.39810E-12	.44877E-01	.44768E-00	.16766E-06	.14462E-00
.63095E-12	.44877E-01	.65318E-00	.37740E-06	.20549E-00
.10000E-11	.44878E-01	.94181E-00	.83978E-06	.28863E-00
.15848E-11	.44880E-01	.13359E+01	.18140E-05	.39409E-00
.25118E-11	.44884E-01	.18498E+01	.37430E-05	.51395E-00
.39810E-11	.44891E-01	.25043E+01	.75548E-05	.65452E-00
.63095E-11	.44906E-01	.33242E+01	.14977E-04	.81982E-00
.10000E-10	.44935E-01	.43217E+01	.28859E-04	.99756E-00
.15848E-10	.44989E-01	.55080E+01	.54344E-04	.11862E+01
.25118E-10	.45091E-01	.69017E+01	.10119E-03	.13937E+01
.39810E-10	.45278E-01	.85303E+01	.18732E-03	.16285E+01
.63095E-10	.45623E-01	.10421E+02	.34481E-03	.18914E+01
.10000E-09	.46258E-01	.12618E+02	.63472E-03	.21968E+01
.15848E-09	.47440E-01	.15196E+02	.11822E-02	.25796E+01
.25118E-09	.49654E-01	.18247E+02	.22138E-02	.30491E+01
.39810E-09	.53776E-01	.21829E+02	.41224E-02	.35824E+01
.63095E-09	.61407E-01	.26015E+02	.76311E-02	.41860E+01
.10000E-08	.75372E-01	.30851E+02	.13964E-01	.48353E+01
.15848E-08	.10093E-00	.36433E+02	.25561E-01	.55820E+01
.25118E-08	.14770E-00	.42881E+02	.46775E-01	.64478E+01
.39810E-08	.23383E-00	.50365E+02	.86124E-01	.74840E+01
.63095E-08	.39610E-00	.59254E+02	.16227E-00	.88889E+01
.10000E-07	.70897E-00	.79556E+02	.31287E-00	.10802E+02
.15848E-07	.13188E+01	.83337E+02	.60990E-00	.13280E+02
.25118E-07	.25216E+01	.97830E+02	.12027E+01	.16493E+02
.39810E-07	.49181E+01	.12050E+03	.23964E+01	.20678E+02
.63095E-07	.97399E+01	.14656E+03	.48218E+01	.26051E+02
.10000E-06	.19555E+02	.17943E+03	.98158E+01	.32871E+02
.15848E-06	.39736E+02	.22011E+03	.20180E+02	.40678E+02
.25118E-06	.81201E+02	.26636E+03	.41465E+02	.46257E+02
.39810E-06	.16104E+03	.30121E+03	.79845E+02	.34847E+02
.63095E-06	.28032E+03	.26844E+03	.11928E+03	-.32768E+02
.10000E-05	.31929E+03	.97923E+02	.38971E+02	-.17052E+03
.15848E-05	.85903E+02	.77392E+02	-.23339E+03	-.20530E+02
.25118E-05	.30820E+03	.20661E+03	.22230E+03	.12922E+03
.39810E-05	.20894E+03	.27530E+03	-.99262E+02	.68688E+02
.63095E-05	.28942E+03	.84783E+02	.80486E+02	-.19052E+03
.10000E-04	.22519E+03	.42794E+02	-.64235E+02	-.41988E+02
.15848E-04	.82191E+02	.18515E+03	-.14300E+03	.14236E+03

C10-92.2

TABLE II

INTERVAL CONTRIBUTIONS FOR E*(iω) (Cont)

T_n	E'_{n+1}	E''_{n+1}	$E'_{n+1}-E'_n$	$E''_{n+1}-E''_n$
.25118E-04	.19306E+03	.15458E+03	.11087E+03	-.30570E+02
.39810E-04	.19306E+03	.15458E+03	.28476E-03	.28476E-03
.63095E-04	.19306E+03	.15458E+03	.17761E-03	.17761E-03
.10000E-03	.19306E+03	.15458E+03	.11078E-03	.11078E-03
.15848E-03	.19306E+03	.15459E+03	.69100E-04	.69100E-04
.25118E-03	.19306E+03	.15459E+03	.43102E-04	.43102E-04
.39810E-03	.19306E+03	.15459E+03	.23919E-03	.23919E-03
.63095E-03	.19306E+03	.15459E+03	.14580E-03	.14580E-03
.10000E-02	.19306E+03	.15459E+03	.88874E-04	.88874E-04
.15848E-02	.19306E+03	.15459E+03	.54172E-04	.54172E-04
.25118E-02	.19306E+03	.15459E+03	.33020E-04	.33020E-04
.39810E-02	.19306E+03	.15459E+03	.20127E-04	.20127E-04
.63095E-02	.19306E+03	.15459E+03	.33693E-04	.33693E-04
.10000E-01	.19306E+03	.15459E+03	.12918E-04	.12918E-04
.15848E-01	.19306E+03	.15459E+03	.12093E-04	.12093E-04
.25118E-01	.19306E+03	.15459E+03	.72041E-05	.72041E-05
.39810E-01	.19306E+03	.15459E+03	.61445E-05	.61445E-05
.63095E-01	.19306E+03	.15459E+03	.36183E-05	.36183E-05
.10000E-00	.19306E+03	.15459E+03	.28837E-05	.28837E-05
.15848E-00	.19306E+03	.15459E+03	.21802E-05	.21802E-05
.25118E-00	.19306E+03	.15459E+03	.15790E-05	.15790E-05
.39810E-00	.19306E+03	.15459E+03	.11027E-05	.11027E-05
.63095E-00	.19306E+03	.15459E+03	.88313E-06	.88313E-06
.10000E+01	.19306E+03	.15459E+03	.56648E-06	.56648E-06
.15848E+01	.19306E+03	.15459E+03	.35499E-06	.35499E-06
.25118E+01	.19306E+03	.15459E+03	.27651E-06	.27651E-06
.39810E+01	.19306E+03	.15459E+03	.17729E-06	.17729E-06
.63095E+01	.19306E+03	.15459E+03	.11860E-06	.11860E-06
.99999E+01	.19306E+03	.15459E+03	.68435E-07	.68435E-07
.15848E+02	.19306E+03	.15459E+03	.49600E-07	.49600E-07
.25118E+02	.19306E+03	.15459E+03	.24993E-07	.24993E-07
.39810E+02	.19306E+03	.15459E+03	.16395E-07	.16395E-07
.63095E+02	.19306E+03	.15459E+03	.43859E-08	.43859E-08
.99999E+02	.19306E+03	.15459E+03	.52283E-08	.52283E-08
.15848E+03	.19306E+03	.15459E+03	.32516E-08	.32516E-08
.25118E+03	.19306E+03	.15459E+03	.13375E-08	.13375E-08

NOTE:

PRINTOUTS MAY BE PUT INTO A MORE READABLE FORM. FOR EXAMPLE THE PRINTOUT - 0.045000 E + 02 BECAMES - 45.0, BECAUSE THE E + 02 OF THE PRINTOUT DESIGNATES THE NUMBER AND DIRECTION OF THE DECIMAL POINT. NOTE THE FOLLOWING EXAMPLES:

± 0.10600 E + 02 = ± 10.6
± 0.82000 E + 01 = ± 8.20
± 0.40000 E - 00 = ± 0.40
± 0.00000 E - 99 = 0
± 0.12000 E - 01 = ± 0.012
± 0.23000 E - 02 = ± 0.0023

C10-92.3

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3. Ferry, J. D., "Viscoelastic Properties of Polymers," John Wiley and Sons, Inc., New York City (1961) P. 44

4. Tobolsky, A. V., "Properties and Structure of Polymers," John Wiley and Sons, Inc., New York City (1960) P. 152

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LIST OF SYMBOLS

ω	...	circular frequency
t	...	time
E(t)	...	stress relaxation modulus
$E^*(i\omega)$	...	dynamic modulus
$E'(\omega)$	...	real part of dynamic modulus
$E''(\omega)$	...	imaginary part of dynamic modulus
E_c	...	constant value approach by stress relaxation modulus for large t
ψ (t)	...	$E(t)-E_c$
m	...	see equation (5)
a_n	...	see equation (10)
b_n	...	sec equation (10)
t_n	...	see equation (10)
p	...	see equation (11)
q	...	see equation (11)
n_1	...	see equation (7)
n_2	...	see equation (8)

APPENDIX A

1620 COMPUTER PROGRAM

1. OPERATING INSTRUCTIONS AND INPUT DATA FORMAT

Card. No.		
1	Title card	- Any Hollerith Information in first 45 columns of card
2	Parameter card	- NT, NW, SCL, FORMAT (2I5,E 14.8) NT = number of time increments in stress relaxation table. NW = multiple of $2\frac{\pi}{\omega}$ at which the program will begin using equation (12). SCL = scale factor to multiply input frequency by in order to make the frequency and stress relaxation data in compatable units; SCL is assumed to be 2π if columns 11-25 are left blank.
3	Stress Relaxation	- $\text{Log}_{10}(t_1)$, $\log_{10}(E(t_1))$
4	Table	$\text{Log}_{10}(t_2)$, $\log_{10}(E(t_2))$
5		$\text{Log}_{10}(t_3)$, $\log_{10}(E(t_3))$
.		.
.		.
.		.
.		.
M+2		- $\text{Log}_{10}(t_M)$, $\text{Log}_{10}(E(t_M))$
M+3	Frequencies	$\omega 1$
M+4		$\omega 2$
M+5		$\omega 3$
.		.
.		.
.		.
.		.
M+N+2		ωn
M+N+3		blank card (if new stress relaxation table is to be read in)
M+N+4	Title card	
M+N+5	Parameter card	
.	etc.	
.	.	
.	.	

Format for Frequencies and Stress Relaxation Table (2E10.5)

2. PROGRAM LISTING

```
      E STAR PROGRAM
      DIMENSION TN(90), EN(90), AN(90), BN(90)
      TPI = 6.2831853
      TEN = 10.
  1   READ 10
      PRINT 10
      PUNCH 10
 10   FORMAT (45H
      READ 20, NT, NW, SCL
      PUNCH 20, NT, NW, SCL
 20   FORMAT (2I5,E14.8)
      TRA = NW
      TRA = TRA*TPI
      IF (SCL) 28, 24, 28
 24   SCL = TPI
 28   DO 30 I = 1, NT
      READ 40, TN (I), EN (I)
 30   PUNCH 50, TN (I), EN (I)
 40   FORMAT (6E10.5)
 50   FORMAT (6E11.5)
      TN (1) = TEN**TN (1)
      T1 = TN (1)
      EN (1) = TEN**EN (1)
      E1 = EN (1)
      DO 60 I = 2, NT
      TN (I) = TEN**TN(I)
      T2 = TN (I)
      EN (I) = TEN**EN (I)
      E2 = EN (I)
      BN (I) = LOGF (E1/E2)/(T2-T1)
      AN (I) = E1* EXPF (BN(I)*T1)
      T1 = T2
 60   E1 = E2
 70   READ 40, WC
      IF (WC) 80, 1, 80
 80   PUNCH 50, WC
100   W = WC*SCL
110   EP = 0.
      EPP = 0.
      KSW = 0
      TRAW = TRA/W
      W2 = W*W
      T1 = TN (1)
      DO 170 I = 2, NT
      T2 = TN (I)
      IF (T2-TRAW) 120, 120, 130
```

```
120   WT = W*T2
      CW = COSF (WT)
      CX = CW
      SW = SINF (WT)
      SX = SW
125   WT = W*T1
      CWT = COSF (WT)
      SWT = SINF (WT)
      TE = AN (I)*W/(BN(I)*BN(I) + W2)
      TT = EXPF (BN(I)*T1)
      FN = (BN(I)*SWT+W*CWT)/TT
      FNP = (BN(I)*CWT-W*SWT)/TT
      TT = EXPF (BN(I)*T2)
      FN = TE*(FN-(BN(I)*SW+W*CW)/TT)
      FNP = TE*(FNP-(BN(I)*CX-W*SX)/TT)
      GO TO 160
130   IF (KSW) 150, 140, 150
140   CW = 0.
      SW = 1.
      CX = 1.
      SX = 0.
      KSW = 1
      GO TO 125
150   BNW = BN(I)/W
      FN = BNW*(EN(I-1)-EN(I))/(1.+BNW**2)
      FNP = FN
160   EP = EP+FN
      EPP = EPP+FNP
170   T1 = T2
      PRINT 180, WC, EP, FN
      PUNCH 180, WC, EP, FN
180   FORMAT (5H  EP(F8.2,2H) = E11.5, 12H   LAST TERM = E11.5
      PRINT 190, WC, EPP, FNP
      PUNCH 190, WC, EPP, FNP
190   FORMAT (5H EPP (F8.2,2H) = E11.5, 12H  LAST TERM = E11.5
      GO TO 70
      END
```

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