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TECHNICAL NOTE NO. 1

VOLUME II

FEASIBILITY DEMONSTRATION OF LARGE SEGMENTED SOLID PROPELLANT ROCKET MOTOR WITH DUAL TVC SYSTEM AND THERMAL GRAIN STRUCTURAL ANALYSIS

(U)

22 MARCH 1962

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CONTRACT NO. AF 04(611)-8013
(LPC MPO 577 AND MPO 578)

PREPARED FOR **AIR FORCE FLIGHT TEST CENTER**

EDWARDS AIR FORCE BASE, CALIFORNIA

LOCKHEED PROPULSION COMPANY

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LOCKHEED PROPULSION COMPANY
P. O. Box 111
Redlands, California

VOLUME II

THERMAL GRAIN STRUCTURAL ANALYSIS

Contract No. AF 04(611)-8013
LPC MPO 578

Technical Note No. 1

Period Covered

6 December 1961 through 28 February 1962

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ABSTRACT

Technical Note No. 1 is the first quarterly progress report submitted in partial fulfillment of Contract AF 04(611)-8013 (LPC MPO 578-1) in accordance with Paragraph 4.2.1 of AFBM Exhibit 58-1. The reporting period extends from 6 December 1961 to 28 February 1962.

~~The objective of this program is~~ studies were conducted to determine the effects of cyclic temperatures in producing failure in case-bonded solid propellant grains. During this report period an extensive uniaxial experimental program was scheduled for the determination of thermal effects on propellants. A collection of limit elastic solutions for case-bonded grains subjected to thermal inputs was completed and the problem of determining suitably expressed temperature distributions in grains was studied. The problem of transient temperature cycling of uniaxial samples was examined. The tasks subcontracted to New York University and Purdue University have been specified and their effort was begun.

I. INTRODUCTION

Technical Note No. 1 is the first quarterly report submitted in partial fulfillment of Contract AF 04(611)-8013, LPC MPO 578-1. The reporting period extends from 6 December 1961 to 28 February 1962. The general problem and planned approach to this study of transient thermal stress behavior was presented in the proposal. Nevertheless, it is felt that a reiteration of the program objectives are in order. The objectives in brief are then:

(1) To determine if there exists in a cast-bonded solid rocket grain a generalized cyclic thermal input over a given temperature range which produces a greater tendency to failure than a step function input over the same temperature range.

(2) To determine the applicable tendency to failure for the above transient thermal conditions, i.e., a pertinent failure criterion.

(3) To reduce to engineering practice the analytical methods for utilizing the results of (1) and (2) above.

(4) To make recommendations for thermal environmental control and testing of rocket motors based upon (1) and (2) above.

To achieve these objectives, theoretical and experimental investigations have been initiated. The following areas have been treated at LPC:

(1) Theoretical studies for the transient temperature cycling of uniaxial specimens.

(2) Considerations of suitable experiments on uniaxial thermally cycled samples to confirm theory on (1).

(3) Preparation of a collection of limit elastic solutions for steady state and transient thermal inputs in case-bonded grains with material properties which are constant and variable with temperatures.

(4) Studies for the determination of suitable expressed temperature distributions in grains for use in theoretical studies at Purdue University.

(5) Equilibrium testing of uniaxial specimens to provide failure data and pertinent viscoelastic characterization of materials for the theoretical investigations being conducted at Purdue and for the experimental studies at New York University.

The tasks assigned to Purdue University are as follows:

(1) Conduct a bibliographical search and prepare extracts of suitable thermal, thermo-elastic, and thermo-visco-elastic solutions for use on this program. Some items of particular interest are:

(a) Transient heat conduction for hollow cylinders including both axial symmetry and non-axially symmetric cases as well for longitudinal temperature variations.

(b) Transient elastic stress analyses using applicable numerical, analytical or variational techniques for hollow cylinders with

- Symmetry
- Anti-symmetry
- Material properties constant with temperatures
- Material properties variable with temperatures

(2) Set up and program the solution for the axi-symmetric, plane strain, visco-elastic case-bonded grain subject to arbitrary cyclic thermal inputs. Solve certain problems for a range of parameters of interest.

(3) As in (2) above but for non-axial-symmetric thermal inputs.

The tasks assigned to New York University are:

(1) Conduct photo-thermo-visco-elastic tests under the following conditions:

(a) Cold specimen:

- Step function temperature increase
- Cyclic temperature application

(b) Warm specimen:

- Step function temperature decrease
- Cyclic temperature application

(c) Non-symmetrical plane radiation on cylinder.

(d) Check K_e, K_t factors obtained by displacement techniques.

Both of the above institutions have started their respective programs.

II. LOCKHEED PROPULSION COMPANY TASKS

A. Transient Thermal Cycling of Uniaxial Specimens

The problems in this area are as follows:

(a) The development of a suitable analysis for the behavior of uniaxial specimens subjected to cyclic temperatures.

(b) Experiments to confirm theory of item (a) above.

The uniaxial analysis and experimentation will serve the following purposes:

(a) To determine whether there exists a tendency for failure under cyclic temperatures which is greater than that due to a monotonic temperature change.

(b) To determine suitable failure criterion for cyclic thermal inputs.

(c) To correlate experimental data with a finite linear differential operator or an integro-differential stress-strain law.

A preliminary investigation of the effects of uniform cyclic temperatures on uniaxial specimens was performed. The analysis considered materials whose properties varied with temperature, and where response in dilation was assumed to be incompressible and elastic (i.e., Poisson's ratio equal to one-half). Both the finite operator, discrete relaxation spectrum, and the integral treatment for materials whose viscoelastic properties vary with temperature in a completely arbitrary manner may not be possible. However, for materials whose properties vary with temperature according to the "shift phenomena," an analytic treatment should be more tractable.

At this point a definition of the "shift" phenomena is in order. A material is defined as "thermo-rheologically simple" by Schwarzl and Staverman (Ref. 1) if its time dependent relaxation modulus (the stress output for a step function strain input) varies with temperature in a special manner. The requirement is that the relaxation modulus for a given temperature be identical to the relaxation modulus for another temperature if a different time scale is used. This means that if the relaxation modulus is plotted versus time, the curves for different constant temperatures can be shifted an amount depending on the temperature and superimposed along the time scale. Hence, a "reduced time" can be introduced which is a function of real time and the temperature.

The shift phenomena is important since most propellants can be expected to exhibit this property, which simplifies many thermal analysis problems. Before any analysis can be performed in this area, experiments have to be conducted to determine under what temperature ranges the "shift" rule is applicable and to determine quantitative accuracy obtained using reduced variables for analysis. The experimental program necessary to do this is described in Section B of this report.

There are two cases to be considered in connection with thermal cycling of uniaxial specimens: (1) heat input from the ends of a specimen only, and (2) uniform heat input from all sides of the specimen. Case (1), because of the existance of spatial temperature gradients in a specimen, develops a three-dimensional, as opposed to a one-dimensional, stress state (see Appendix A). While it is probably true that the non-axial stresses are small compared to the axial one, the one of fundamental interest, the validity of such an argument needs verification. A set of experiments will be made to develop an insight into the practical aspects of this problem. Demonstration of a negligible magnitude of the non-axial stresses will materially aid in transforming this problem to an analytically tractable one.

Case (2) above represents another uniaxial thermal cycling experiment, one in which a spatially uniform temperature is slowly varied.

Experiments of one of the above types are being considered. The scheme of the experiments will be to cycle a uniaxial tensile specimen fixed at its ends over a wide temperature range. The stress variation with time will be measured. Peak stresses at the limit temperatures for cycles subsequent to the first will be compared to the limit stresses in the first cycle to permit investigation of the possibility of stress accumulation during repeated cycling resulting from viscoelastic stress response retardation.

The experimental data from case (2) experiments will be compared to theoretical predictions for correlation of reduced variable, viscoelastic analyses for the tests. Failure data, where accessible experimentally, will be obtained also. LPC-543A propellant will be used in these tests.

B. Uniaxial Testing at Equilibrium Temperatures

The problem of thermal cycling of case-bonded grains involves theoretical as well as experimental considerations. The present section deals with the testing of uniaxial specimens at varying equilibrium temperatures. Such tests are needed for the following reasons:

(a) To provide suitable failure information. Once the stresses and strains are calculated from a theoretical analysis, failure data are needed to predict failure.

(b) Any theoretical analysis will depend on the material properties. Hence, suitable experimentation is required for the determination of the behavior of the propellants.

The first set of tests will of necessity be uniaxial tests. Once the uniaxial behavior is understood, the testing program will be extended to biaxial and triaxial tests to ascertain whether uniaxial data can in fact be used for problems involving multiple stress fields.

The uniaxial testing program will be separated into the following two areas:

(a) Failure tests which will be used to obtain suitable failure criteria and for specifying safe limits of strain, strain rate, and stresses for the second series of test mentioned in (b) below.

(b) Ramp type strain inputs to determine operator coefficients, which will later be used in any theoretical analysis.

Both of the above series of tests will be performed for various temperatures so that thermal effects on the material properties and failure criteria can be determined. The discussion of these tests is presented below.

1. Failure testing. Failure tests will include constant strain rate tests, ramp type strain inputs, and constant load test for various equilibrium temperatures.

a. Constant strain rate to failure. Uniaxial test samples will be subjected to constant strain rates until failure occurs. The values of the strain at failure, ϵ_r, the maximum stress, σ_m, and the values of the strain at maximum stress, ϵ_1, will be determined for varying rates and temperatures (see Figure 1). The temperature will be varied from -60°F to 140°F. For each temperature a series of test will be performed at different rates. The resulting data will reveal the effect of both strain rates and temperature on the failure of propellants.

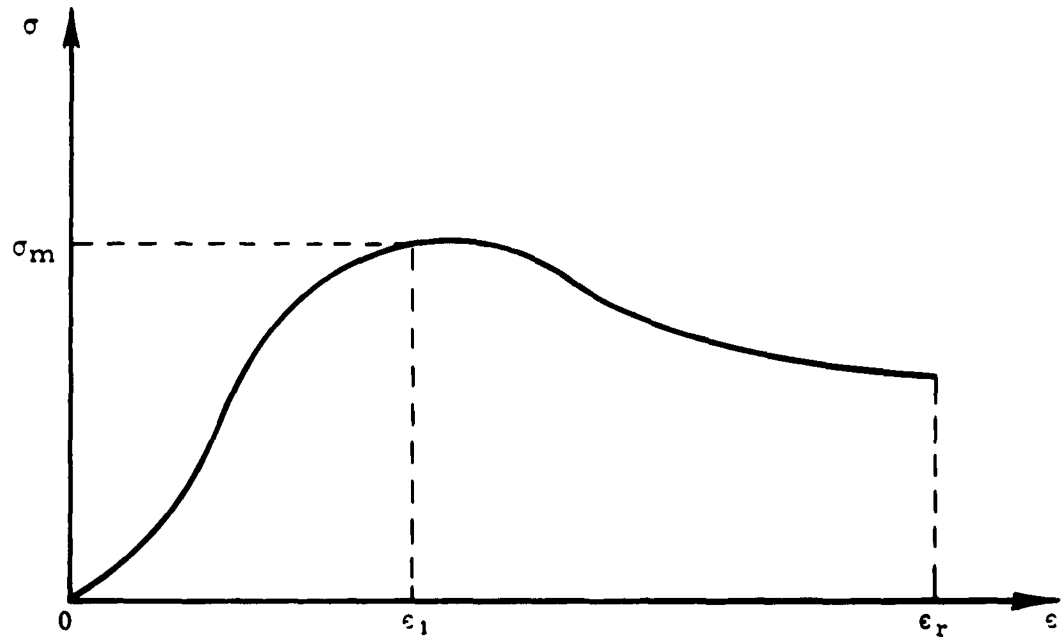

Figure 1 Stress Strain Curve

b. Ramp strain inputs. Uniaxial test samples will be subjected to a constant strain rate until a specified strain level is reached; then the strain will be held fixed (see Figure 2). The ramp rate will be varied as well as the final strain. Again the temperature will be varied from 140°F to -60°F.

The time to failure will be determined for each strain level (Figure 2). Figure 2 illustrates the series of strain inputs for a fixed strain rate and a fixed temperature. The value of the strain and the time for failure can then be plotted as shown in Figure 3. The curve shown is for a fixed rate and a fixed temperature.

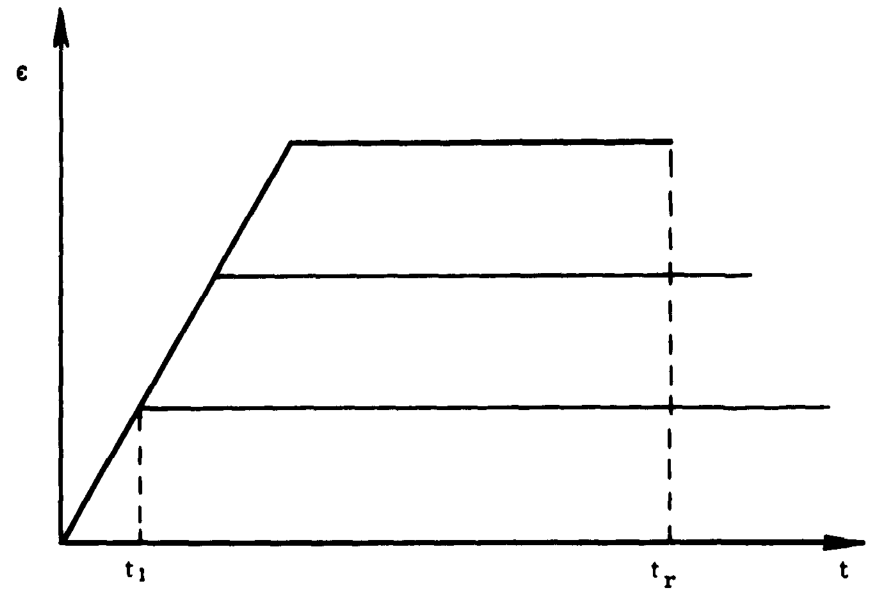

Figure 2 Ramp Strain Input

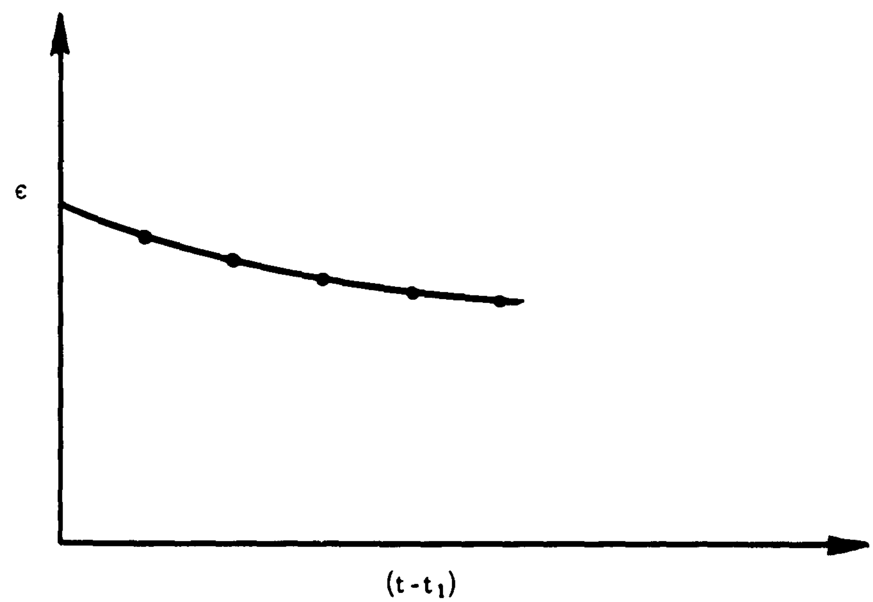

Figure 3 Rupture Strain versus Time

Similar curves would result for each strain rate and temperature. Of special importance is the determination of the effects of strain rates and temperature on the time for failure. Attempts will be made to correlate these data with the results obtained from the constant strain rate tests discussed in subsection a. If such a correlation exists, then failure information can be obtained from constant rate tests. These are less time consuming than the ramp strain tests and hence, more desirable.

c. Constant load or creep tests. Uniaxial test samples will be subjected to a constant strain rate until a certain level of stress is reached; then the load will be maintained at this value. This will be accomplished by lowering a weight on a specimen at a constant rate. The strain at failure and the time to break will be determined for the same series of temperatures as in the tests mentioned above and for different values of the load. Figure 4

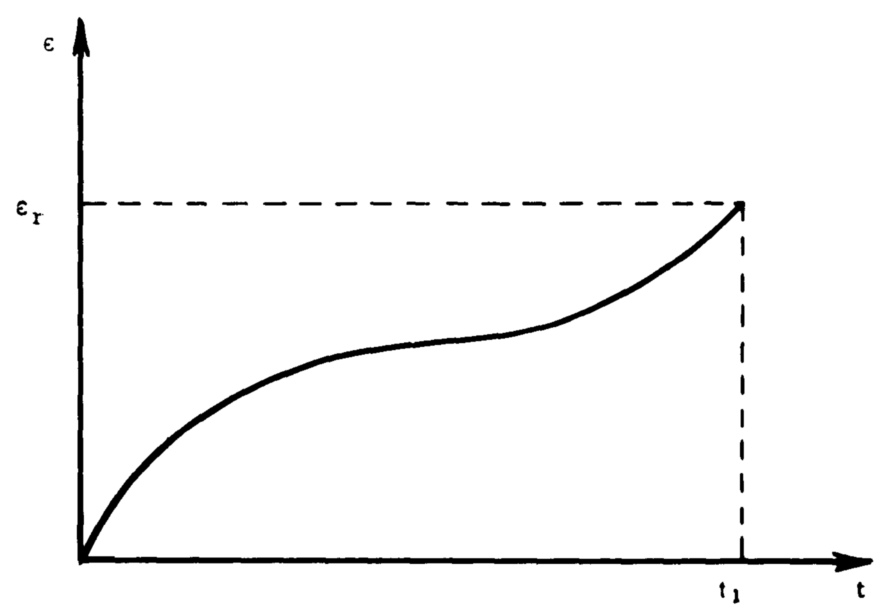

Figure 4 Creep Curve

illustrates a sample strain versus time curve for a fixed load and a fixed temperature. Here again, a correlation of the data obtained from these tests with the data obtained from the constant rate tests will be attempted. This would be desirable since constant load or creep tests are more time consuming than constant rate tests.

2. Operator coefficient determination. The tests which will determine operator coefficients, unlike the aforementioned tests, do not involve failure. Hence, the levels of strain used in these tests will depend on the failure data obtained from the tests described in section A. Uniaxial specimens will be subjected to a ramp type strain input as shown in Figure 5. The resulting

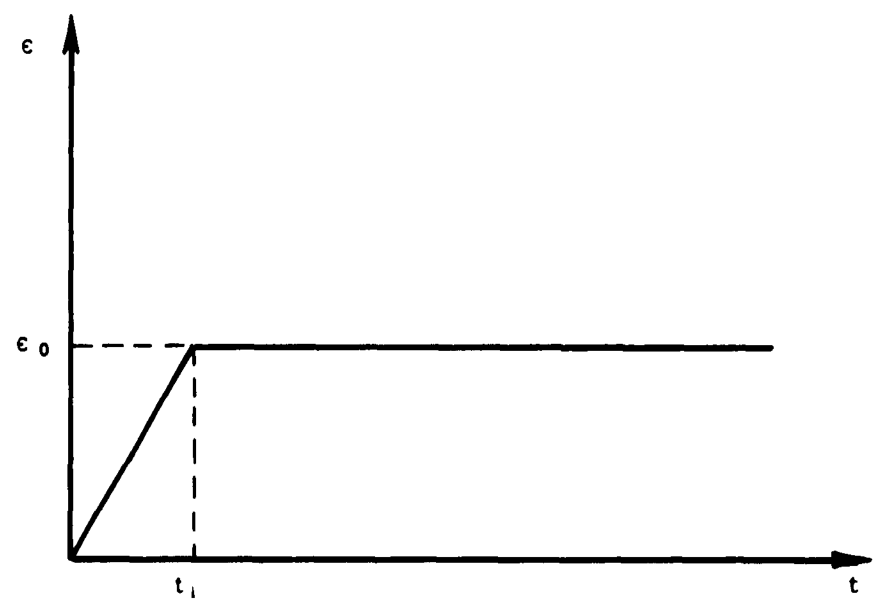

Figure 5 Ramp Strain Input

stress time curve as shown in Figure 6 will be the output. The portion of the stress time curve to the right of t_1 will be used to determine the operator coefficients. The process will be repeated for equilibrium temperatures ranging from -60°F to 140°F. The results should indicate under what temperature ranges the "shift" rule is applicable and should determine the functional relationship between the "shift factor" and temperature. This information is necessary before any theoretical analysis can be performed.

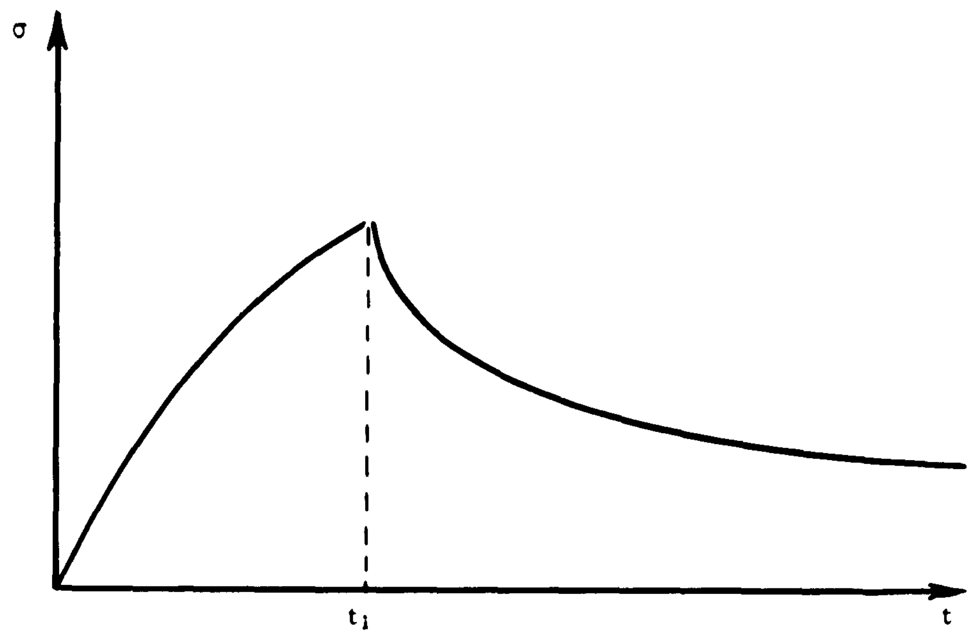

Figure 6 Relaxation Curve

C. Motor Analysis - Cylindrical Case-Bonded Grains

The main objective in this area is to prepare a collection of limit elastic solutions for steady-state and transient thermal inputs in case-bonded grains (circular port) with material properties which are constant and variable with temperatures. These solutions will serve as limit solutions for the viscoelastic analysis, i.e., they could represent the limit solutions at the beginning or end of a thermal cycle. Due to the complexity of a general viscoelastic analysis of case-bonded grains, it is also desirable to investigate these limiting solutions as a preliminary step. Furthermore, there is the possibility that they may lead directly to viscoelastic solutions in special instances.

A search of the literature and investigations performed at LPC have resulted in tabulation of limiting elastic solutions for the following cases:

(a) Motor case and grain at same temperature.

(b) Motor case and grain at different constant temperatures.

(c) Steady and transient thermal stress analysis for constant material properties.

(d) Steady and transient thermal stress analysis with Young's modulus and the coefficient of thermal expansion considered as arbitrary functions of temperature. However, the analytical solution is only valid for Poisson's ratio equal to one-half.

In all of the above instances the temperature field considered is axially symmetrical. The cylinders are considered to have a length much greater than the outside radius (i.e., plan strain). At present, solutions exist only for long cylinders. Study of problems dealing with short cylinders with consideration of end effects has not been initiated. A list of the solutions to (a), (b), (c), and (d) above are listed in Appendix B.

D. Temperature Fields in Cylindrical Grains

The necessary methods for determining temperature fields in hollow circular cylinders are known. However, the analytical expressions for such temperatures are generally too complex for engineering purposes. Approximate methods have been studied and Biots' Energy Method (Ref. 2) appears to give suitably accurate answers which are simpler than the exact solutions. Before any numbers can be extracted from the above mentioned solutions, however, the thermal constants of the materials of interest will have to be determined. Laboratory measurement of the thermal constants will be initiated in the near future. A heat conduction apparatus was purchased in order to determine the thermal conductivity.

A case-bonded propellant slice was available for temperature measurements. This slice had a 120-inch outer diameter with a 60-inch inner diameter. It was 20 inches thick, bonded to a steel case, and had a circular inner port. This test assembly was fabricated in connection with other LPC activities.

Thermocouples cast into the propellant enabled actual temperature field measurements for various thermal conditions to be made. The results of this test are being evaluated. It is hoped that the results will provide a suitable check on any approximate theoretical temperature analysis and will also verify the values of the heat conductivity as determined from other tests.

III. PURDUE UNIVERSITY TASK

This section deals with the progress made at Purdue University during the last report period. A biographical search was initiated which will assist in the present program.

Four general methods were developed for the axi-symmetrical viscoelastic problem with a non-steady thermal input. These methods are not included in this report, since their practicality has not yet been determined. During the next report period, Purdue will continue to investigate the feasibility of these methods and a detailed report on them will be presented.

IV. NEW YORK UNIVERSITY TASK

The effort at New York University was begun and progress during the last report period was of a preliminary nature. Nothing concrete can be reported at this time.

REFERENCES

1. Schwarwl, F. and Staverman, A. J., Journal Appl. Phys. 23, 838, 1952.

2. Boley, B. A., Weiner, J. H., Theory of Thermal Stresses, John Wiley and Sons, 1960.

3. William, M. L., and Blatz, P. J., "Fundamental Studies Relating to Systems Analysis of Solid Propellants," Galcit Final Report No. 101, February 1961.

APPENDIX A

CYCLIC THERMOELASTIC ANALYSIS OF A THIN BAR

It was noted in the text of this report that a cyclic thermal viscoelastic analysis for a thin bar, constrained at the ends and subjected to a cyclic thermal gradient along its axis, was extremely difficult. To gain insight into this problem an elastic material will be considered. If an elastic analysis proves difficult, then the viscoelastic problem will certainly be untractable.

The governing equations for this problem must result from a general three-dimensional treatment (see Figure A-1). This is true since the problem is not a uniaxial one. The three-dimensional treatment will clearly reveal the errors arising from the assumption that the stress field is uniaxial and will indicate the difference between stress fields arising from steady temperatures and those arising from non-steady thermal fields.

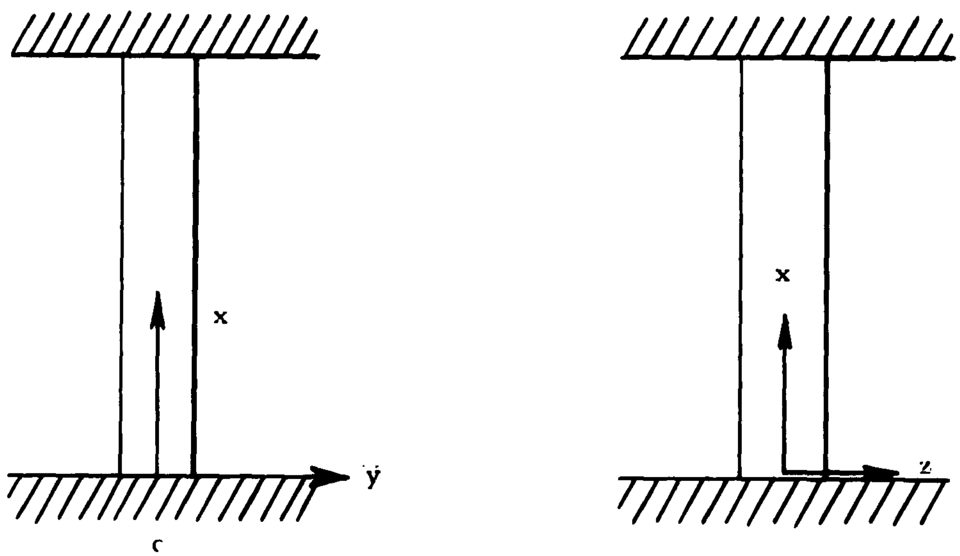

Figure A-1 Geometry and Related Coordinates

In the absence of body forces, the equations of equilibrium are:

$$\frac{\partial \sigma_x}{\partial_x} + \frac{\partial \tau_{xy}}{\partial_y} + \frac{\partial \tau_{xz}}{\partial_z} = 0 \qquad (1.a)$$

$$\frac{\partial \tau_{xy}}{\partial_x} + \frac{\partial \sigma_y}{\partial_y} + \frac{\partial \tau_{yz}}{\partial_z} = 0 \qquad (1.b)$$

$$\frac{\partial \tau_{xz}}{\partial_x} + \frac{\partial \tau_{yz}}{\partial_y} + \frac{\partial \sigma_z}{\partial_z} = 0 \qquad (1.c)$$

where σ_x, σ_y, σ_z are normal stresses in the x, y, and z directions respectively and τ_{xy}, τ_{xz}, τ_{yz} are the shear stresses.

The stress-strain law including thermal expansions is

$$\epsilon_x = \frac{\sigma_x}{E} - \frac{\nu(\sigma_y + \sigma_z)}{E} + \alpha T \qquad (2.a)$$

$$\epsilon_y = \frac{\sigma_y}{E} - \frac{\nu(\sigma_x + \sigma_z)}{E} + \alpha T \qquad (2.b)$$

$$\epsilon_z = \frac{\sigma_z}{E} - \frac{\nu(\sigma_x + \sigma_y)}{E} + \alpha T \qquad (2.c)$$

$$\epsilon_{xy} = \frac{\tau_{xy}(1+\nu)}{E} \qquad (2.d)$$

$$\epsilon_{xz} = \frac{\tau_{xy}(1+\nu)}{E} \qquad (2.e)$$

$$\epsilon_{yz} = \frac{\tau_{yz}(1+\nu)}{E} \qquad (2.f)$$

where ϵ_x, ϵ_y, ϵ_z are normal strains and ϵ_{xy}, ϵ_{xz}, ϵ_{yz} the shear strains.

The complete equations of compatibility are too lengthy to write here, but for our purpose it suffices to include the following two:

$$\frac{\partial^2 \epsilon_x}{\partial y^2} + \frac{\partial^2 \epsilon_y}{\partial x^2} = 2 \frac{\partial^2 \epsilon_{xy}}{\partial x \partial y} \tag{3.a}$$

$$\frac{\partial^2 \epsilon_x}{\partial z^2} + \frac{\partial^2 \epsilon_z}{\partial x^2} = 2 \frac{\partial^2 \epsilon_{xz}}{\partial x \partial z} \tag{3.b}$$

The final equation is the heat conduction equation which for our problem is as follows:

$$\frac{\partial^2 T(x,t)}{\partial x^2} = \frac{1}{k} \frac{\partial T(x,t)}{\partial t} \tag{4}$$

where k = the thermal diffusivity

T = the temperature above a reference temperature.

Suppose now that one assumes the following to be true:

$$\sigma_y = \sigma_z = \tau_{xy} = \tau_{yz} = \tau_{xz} = 0 \tag{5}$$

i.e., the only non-zero stress is σ_x. Through the use of the stress-strain law, equation 2, the expressions for the strains are obtained as follows:

$$\epsilon_x = \frac{\sigma_x}{E} + \alpha T(x,t)$$

$$\epsilon_y = \frac{\nu \sigma_x}{E} + \alpha T(x,t) = \epsilon_z \tag{6}$$

$$\epsilon_{xy} = \epsilon_{yz} = \epsilon_{xz} = 0$$

Inserting these equations into the compatibility expressions, Equation (3.a, b) we have

$$\frac{\partial^2}{\partial x^2} \left[- \frac{\nu \sigma_x}{E} + \alpha T \right] = 0 \tag{7}$$

This is possible if and only if

$$\frac{-\nu \sigma_x}{E} + \alpha T = a_1 + a_2 x \tag{8}$$

The equilibrium equations, based on the relations given in Equation (5) reduce to

$$\frac{\partial \sigma_x}{\partial x} = 0 \qquad (9)$$

Therefore

$$\sigma_x = c(t) \qquad (10)$$

The expression derived for σ_x from Equation 10 is incompatible with that expressed by Equation 10 unless, 'T' is a linear function of x. Such is not the case for non-steady temperatures as can be seen from an examination of the heat conduction equation, Equation (4). Hence, a uniaxial stress field is possible only if the temperature is a linear function of x, which means that the temperature must be steady (i.e., $\partial T/\partial t = 0$). The assumption of a uniaxial stress field becomes more inaccurate as the time gradient of temperature increases. An exact treatment of this problem is quite difficult once the existence of other stresses is realized.

APPENDIX B

LIMIT THERMOELASTIC SOLUTIONS

The solutions presented in this section are for long case-bonded grains, (plain strain), subjected to thermal inputs which are axially-symmetrical (i.e., $T = T(r, t)$).

Plain Strain with Temperature Independent Properties

The geometry and related coordinates are illustrated in Figure B-1. The solution for the stresses and strains are listed below for an arbitrary axially symmetrical temperature field.

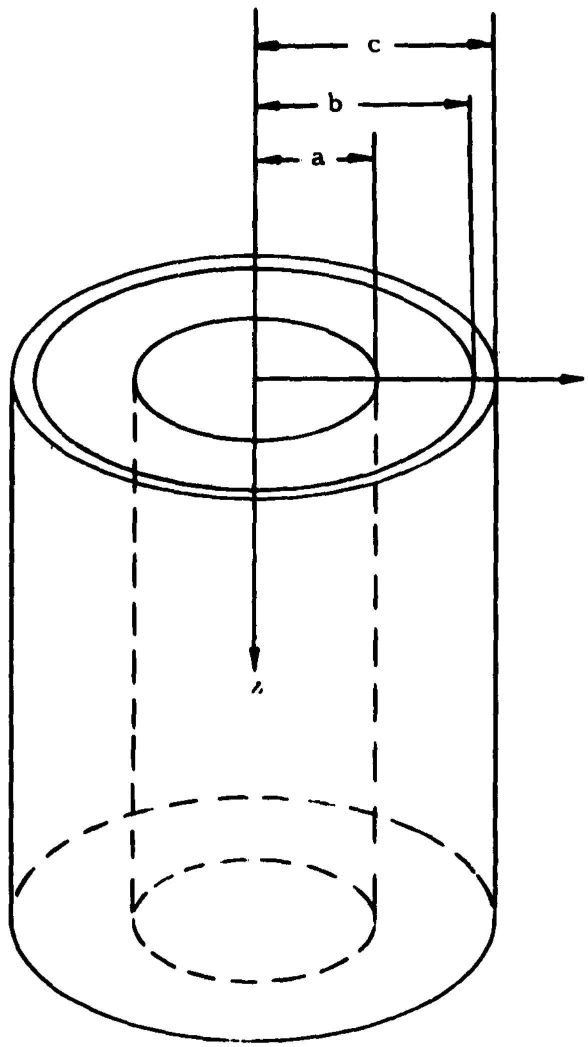

Figure B-1 Geometry and Related Coordinates

Stresses in the Propellant:

$$\sigma_r = \frac{-B}{\rho^2}\int_1^{\rho} T\rho\, d\rho + \frac{1}{(1-\lambda^2)}\left\{-P_i\left(1-\frac{\lambda^2}{\rho^2}\right) + P\ \lambda^2\left(1-\frac{1}{\rho^2}\right) - BI\left(1-\frac{1}{\rho^2}\right)\right\} \tag{B.1}$$

$$\sigma_{\theta} = \frac{B}{\rho^2}\int_1^{\rho} T\rho\, d\rho + \frac{1}{(1-\lambda^2)}\left\{P\ \lambda^2\left(1+\frac{1}{\rho^2}\right) - P_i\left(1+\lambda^2/\rho^2\right) - BI\left(1+\frac{1}{\rho^2}\right)\right\} - BT \tag{B.2}$$

$$\sigma_z = \nu(\sigma_\theta + \sigma_r) - \alpha T E \tag{B.3}$$

where

$$\rho = r/a$$

$$\lambda = b/a \tag{B.4:1}$$

$$I = \int_1^{\lambda} T(\rho, t)\rho\, d\rho \tag{B.4:2}$$

(B.4:3)

P_i = internal pressure

$$B = \frac{\alpha E}{1-\nu} \tag{B.4:4}$$

α = linear thermal coefficient of expansion for the propellant

ν = Poisson's ratio for the propellant

E = Youngs' modulus for the propellant

P' = pressure between case and propellant

$$P' = \frac{P_i[2(1+\nu)\lambda(1-\nu)]}{\lambda D} + \frac{2(1-\nu^2)\beta I}{D} - \frac{2\beta' I'(1-\nu'^2)E(1-\lambda^2)}{D\,E'(1-\lambda'^2)} \tag{B.5:1}$$

Here

$$D \cdot [1+\nu'] \left\{(1-2\nu') + \lambda'^2\right\} \frac{E(1-\lambda^2)}{E'(1-\lambda'^2)} + (1+\nu)\left[\lambda^2(1-2\nu)+1\right] \tag{B.5:2}$$

where

$$I' = \int_1^{\lambda'} T\ \rho' d\rho' \tag{B.6:1}$$

$\rho' = r/b$ (B.6:2)

$\lambda' = c/b$ (B.6:3)

E' = Young's modulus for the case

ν' = Poisson's ratio for the case

$B' = \frac{\alpha' E'}{1-\nu'}$

α' = linear coefficient of expansion for the case.

The Strains in the Propellant:

$$\epsilon_r = \frac{B(1+\nu)}{E\rho^2}\int_1^{\rho} T\rho\, d\rho + \frac{(1+\nu)}{E(1-\lambda^2)}\Big\{P'\lambda^2\left[1-2\nu-\frac{1}{\rho^2}\right] - P_i\left[(1-2\nu)-\lambda^2/\rho^2\right]$$

$$-BI\left[(1-2\nu)-\frac{1}{\rho^2}\right] + \frac{(1+\nu)}{(1-\nu)}\alpha T \qquad \text{(B.7:1)}$$

$$\epsilon_\theta = \frac{B(1+\nu)}{E\rho^2}\int_1^{\rho} T\rho\, d\rho + \frac{(1+\nu)}{E(1-\lambda^2)}\Big\{P'\lambda^2\left[(1-2\nu)+\frac{1}{\rho^2}\right] - P_i\left[(1-2\nu)+\frac{\lambda^2}{\rho^2}\right]$$

$$-BI\left[(1-2\nu)+\frac{1}{\rho^2}\right]\Big\} \qquad \text{(B.7:2)}$$

The Radial Deflection:

$$u' = \frac{(1+\nu)B}{E\rho}\int_1^{\rho} T\rho\, d\rho + \frac{(1+\nu)a}{E(1-\lambda^2)}\Big\{P'\lambda^2\left[(1-2\nu)\rho+\frac{1}{\rho}\right] - P_i\left[(1-2\nu)\rho+\frac{\lambda^2}{\rho}\right]$$

$$-BI\left[(1-2\nu)\rho+\frac{1}{\rho}\right]\Big\} \qquad \text{(B.8)}$$

The Stresses in the Case:

$$\sigma'_r = -\frac{B'}{\rho'^2}\int_1^{\rho'} T\rho' d\rho' - \frac{1}{(1-\lambda'^2)}\Big\{P'\Big(1-\frac{\lambda'^2}{\rho'^2}\Big) + B'I'\Big(1-\frac{1}{\rho'^2}\Big)\Big\} \qquad \text{(B.9:1)}$$

$$\sigma'_\theta = \frac{B'}{\rho'^2}\int_1^{\rho'} T\rho' d\rho' - B'T - \frac{1}{(1-\lambda'^2)}\Big\{P'\Big(1+\frac{\lambda'^2}{\rho'^2}\Big) + B'I'\Big(1+\frac{1}{\rho'^2}\Big)\Big\} \qquad \text{(B.9:2)}$$

$$\sigma'_z = \frac{2P'\nu}{(1-\lambda'^2)} - \frac{2B'I'\nu'}{(1-\lambda'^2)} - \frac{T\alpha' E'}{(1-\nu')} \qquad \text{(B.9:3)}$$

<u>The Strains in the Case:</u>

$$\epsilon'_r = -\frac{B'(1+\nu')}{E'\rho'^2}\int_1^{\rho'} T\rho' d\rho' - \frac{(1+\nu')}{(1-\lambda'^2)E'}\left\{P'\left[(1-2\nu') - \frac{\lambda'^2}{\rho'^2}\right]\right.$$

$$\left. + B'I'\left[(1-2\nu') - \frac{1}{\rho'^2}\right]\right\} + \frac{T\alpha'(1+\nu')}{(1-\nu')} \qquad \text{(B.10:1)}$$

$$\epsilon'_\theta = \frac{B'(1+\nu')}{E'\rho'^2}\int_1^{\rho'} T\rho' d\rho' - \frac{(1+\nu')}{E'(1-\lambda'^2)}\left\{P'\left[(1-2\nu') + \frac{\lambda'^2}{\rho'^2}\right]\right.$$

$$\left. + B'I'\left[(1-2\nu') + \frac{1}{\rho'^2}\right]\right\} \qquad \text{(B.10:2)}$$

<u>The Radial Deflection in the Case:</u>

$$u' = \frac{B'b(1+\nu')}{E'\rho'}\int_1^{\rho'} T\rho' d\rho' - \frac{b(1+\nu')}{E'(1-\lambda'^2)}\left\{P'\left[(1-2\nu')\rho' + \frac{\lambda'^2}{\rho'}\right]\right.$$

$$\left. + B'I\left[\rho'(1-2\nu) + \frac{1}{\rho'}\right]\right\} \qquad \text{(B.11)}$$

The above solutions can be simplified for the following two cases:

(a) Case and grain at the same constant temperature. This simulates a case-bonded grain which has been subjected to a temperature change for a long time. The modulus, E, for the grain would be the equilibrium modulus.

(b) Case and grain at different temperatures. That is, the grain is at one constant temperature, while the case is at another temperature. This simulates a case-bonded grain subjected to a sudden change in temperature. The modulus, E, for this case would be the glassy modulus.

CASE AND GRAIN AT A CONSTANT TEMPERATURE

Stresses in the Propellant:

$$\sigma_r = \frac{1}{(1-\lambda^2)}\left\{P'\lambda^2\left(1-\frac{1}{\rho^2}\right) - P_i\left(1-\lambda^2/\rho^2\right)\right\} \qquad \text{(B.12:1)}$$

$$\sigma_\theta = \frac{1}{(1-\lambda^2)}\left\{P'\lambda^2\left(1+\frac{1}{\rho^2}\right) - P_i\left(1+\lambda^2/\rho^2\right)\right\} \qquad \text{(B.12:2)}$$

$$\sigma_z = \frac{2\nu\left[P'\lambda^2 - P_i\right]}{(1-\lambda^2)} + \alpha T \qquad \text{(B.12:3)}$$

Strains in the Propellant:

$$\epsilon_r = \frac{(1+\nu)}{E(1-\lambda^2)}\left\{P'\lambda^2\left[(1-2\nu) - \frac{1}{\rho^2}\right] - P_i\left[(1-2\nu) - \lambda^2/\rho^2\right]\right\}$$

$$+ \alpha T(1+\nu) \qquad \text{(B.13:1)}$$

$$\epsilon_\theta = \frac{(1+\nu)}{E(1-\lambda^2)}\left\{P'\lambda^2\left[(1-2\nu) + \frac{1}{\rho^2}\right] - P_i\left[(1-2\nu) + \lambda^2/\rho^2\right]\right\}$$

$$+ \alpha T(1+\nu) \qquad \text{(B.13:2)}$$

Radial Deflection of Grain:

$$u' = \frac{(1+\nu)}{E(1-\lambda^2)}\left\{P'\lambda^2\left[(1-2\nu)\rho + \frac{1}{\rho}\right] - P_i\left[(1-2\nu)\rho + \lambda^2/\rho\right]\right\} \qquad \text{(B.14)}$$

where for a thin case

$$P' = \frac{ET\left[(1+\nu)\alpha - \alpha'(1+\nu)\right]}{\frac{(1+\nu)\left[(1-2\nu)\lambda^2 + 1\right]}{(\lambda^2-1)} + (1-\nu'^2)\frac{bE}{hE'}} \qquad \text{(B.15:1)}$$

Here

$h = (c-b) =$ thickness of case (B.15:2)

Stresses in the Case:

$$\sigma'_r = -\frac{1}{(1-\lambda'^2)}\left\{P'\left(1-\lambda'^2/\rho'^2\right)\right\} \qquad (B.16:1)$$

$$\sigma'_\theta = -\frac{1}{(1-\lambda'^2)}\left\{P'\left(1+\lambda'^2/\rho'^2\right)\right\} \qquad (B.16:2)$$

$$\sigma'_z = -\frac{2\nu}{(1-\lambda'^2)}\left\{P'\right\} + \alpha' T \qquad (B.16:3)$$

The Strains in Case:

$$\epsilon'_r = -\frac{(1+\nu')}{(1-\lambda'^2)E'}\left\{P'\left[(1-2\nu') - \lambda'^2/\rho'^2\right]\right\} + T\,\alpha'(1+\nu') \qquad (B.17:1)$$

$$\epsilon'_\theta = -\frac{(1+\nu')}{E'(1-\lambda'^2)}\left\{P'\left[(1-2\nu') + \lambda'^2/\rho'^2\right]\right\} + T\,\alpha'(1+\nu') \qquad (B.17:2)$$

The Radial Deflection:

$$u' = -\frac{(1+\nu')}{E'(1-\lambda'^2)}\left\{P'\left[(1-2\nu')\rho' + \lambda'^2/\rho'\right]\right\} + T\rho' b\,\alpha'(1+\nu') \qquad (B.18)$$

CASE AT T_c AND PROPELLANT AT T_p

Stresses in the Propellant:

[(Same as Equations (B.12:1) - (B.12:3)) except P' is not given by Equation B.15:1 but by the following expression:] (B.19)

$$P' = P_i\left[2(1+\nu)\lambda \quad (1-\nu)\right] + \frac{E\left[(1+\nu)\alpha T_p - \alpha' T_p(1+\nu')\right]}{\frac{(1+\nu)}{(\lambda^2-1)}\left[(1-2\nu)\lambda^2+1\right] + (1-\nu'^2)\frac{Eb}{hE'}} \qquad (B.20)$$

Strains in the Propellant:

[(Same as Equations (b.13:1) to (B.12:3), except T is replaced by T_p, and P' is given by Equation (B.20) instead of (B.15:1).] (B.21)

Stresses and Strains in the Case:

[(These quantities are the same as those expressed by Equation (B.16:1) through Equation (B.18) providing P' is given by Equation (B.20) and T is replaced by T_c)] (B.22)

The above expressions provide a tabulation of the stresses, strains, and deflections in a long case-bonded grain whose material properties are independent of temperature. Equation (B.1) through Equation (B.11) give the solution to an arbitrary axially symmetrical non-steady temperature. While Equation (B.12) through Equation (B.18) give the solution for a case-bonded grain when the case and grain have the same temperature. Equation (B.19) through (B.22) give the solution for a case-bonded grain when the case is at one temperature with the grain at another.

LIST OF SYMBOLS

E = Young's modulus for the grain

E' = Young's modulus for the case

I = defined by equation

I' = defined by equation

P_i = internal pressure

P' = pressure between grain and case

T = temperature above some reference temperature

a = inner radius of the grain

b = outer radius of the grain

c = outer radius of the case

h = $c-b$ = thickness

r = radial coordinate

z = axial coordinate

α = linear coefficient of thermal expansion for the grain

α' = linear coefficient of thermal expansion for the case

B = $\frac{\alpha E}{1-\nu}$

B' = $\frac{\alpha' E'}{1-\nu'}$

ϵ_r = strain in radial direction

ϵ_θ = strain in circumferential direction

ϵ_z = strain in axial direction

λ = b/a

λ' = c/b

ν = Poisson's ratio for the grain

ν' = Poisson's ratio for the case

ρ = r/a

ρ' = r/b