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A Study of the Noise Characteristics of Antennas Due to External Thermal Noise Sources

by

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R. Lawrie, D. Brown

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Department of ELECTRICAL ENGINEERING

THE OHIO STATE UNIVERSITY
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REPORT
by
THE OHIO STATE UNIVERSITY RESEARCH FOUNDATION
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Submitted by	H. N. Dawirs, R. Caldecott, R. Lawrie, D. Brown Antenna Laboratory Department of Electrical Engineering
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A STUDY OF THE NOISE CHARACTERISTICS OF ANTENNAS DUE TO EXTERNAL THERMAL NOISE SOURCES

This report describes a study of the antenna noise characteristics due to thermal noise generated by hot objects external to the antenna and fed into the antenna through the electromagnetic field. The objective of the study is to obtain information that can be used to design antennas with improved signal-to-noise ratios.

Unfortunately the noise received by electromagnetic radiation from hot objects external to the antenna is a function of the intensities of these sources, their distribution, the distribution and reflecting properties of any reflecting surfaces in the vicinity, as well as the antenna patterns (both polarizations must be considered) and their orientation. Thus such noise characteristics of an antenna are not unique, but are a complex function of environment and orientation as well as pattern, and so must be investigated for each individual antenna with respect to its particular location and application. This report demonstrates such an investigation for a particular situation, that of a parabolic reflecting antenna over flat terrain with the main beam pointed vertically upwards as shown in Fig. 1. This antenna was

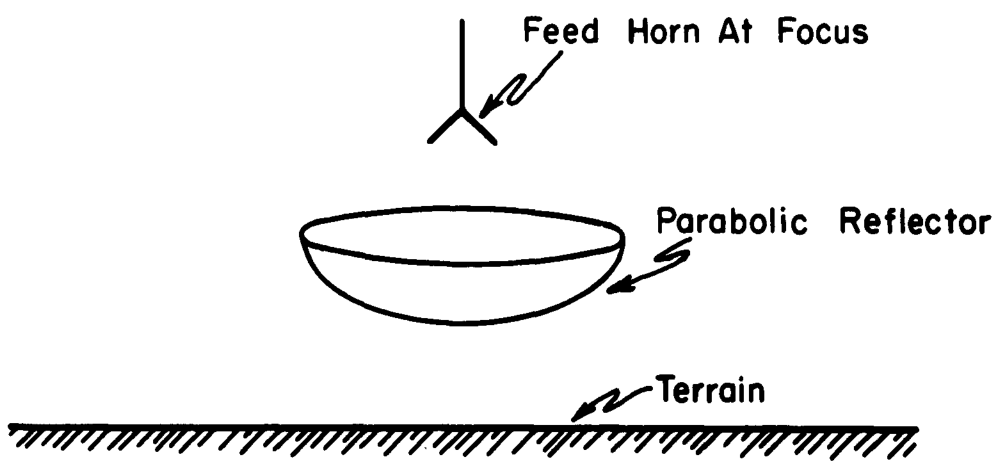

Fig. 1. Horn fed parabolic reflector over flat terrain with main beam directed vertically upward.

chosen as a high-gain configuration commonly used for microwaves. The flat terrain and the orientation of the antenna were chosen as a conveniently specified reference situation that can be treated fairly simply.

PRELIMINARY CONSIDERATION OF PARABOLIC REFLECTOR ANTENNAS

It would seem that, in general, the signal-to-noise ratio of an antenna could be improved, as far as external noise sources are concerned, simply by increasing the gain in the direction of the signal source in order to increase the signal received, while the received noise is reduced by decreasing the gain in all other directions. However, it is not always possible to increase the gain in one direction while decreasing the gain in all other directions, and some compromise is usually necessary.

Since there is no unique signal associated with an antenna, but the signal output is proportional to its gain, which is unique, the noise characteristics of an antenna are more properly specified by gain-to-noise rather than by signal-to-noise ratio. In this report the gain of an antenna will be referred to a uniformly illuminated aperture of the same size and shape as that of the antenna being considered. No loss is considered to exist in either case since internal loss in the antenna is another source of noise that is considered separately.

The noise will be specified as the ratio of the equivalent temperature in degrees Kelvin to a reference temperature. Since this is equivalent to a power ratio it may properly be expressed in decibels.

Figure 2 shows approximations to the gain and the gain-to-noise ratio of an antenna of the type being considered if the focal length is very long.[1] The angle subtended by the reflector at the focus is 2β while 2α is the angle between the nulls in the feed patterns as shown in Fig. 3.

These curves indicate that the null beamwidth, 2α, of the feed pattern should be narrower for a maximum gain-to-noise ratio than for maximum gain. However it must be emphasized that these curves are only approximations based on the following assumptions:

1. The parabolic reflector has an extremely long focal length;

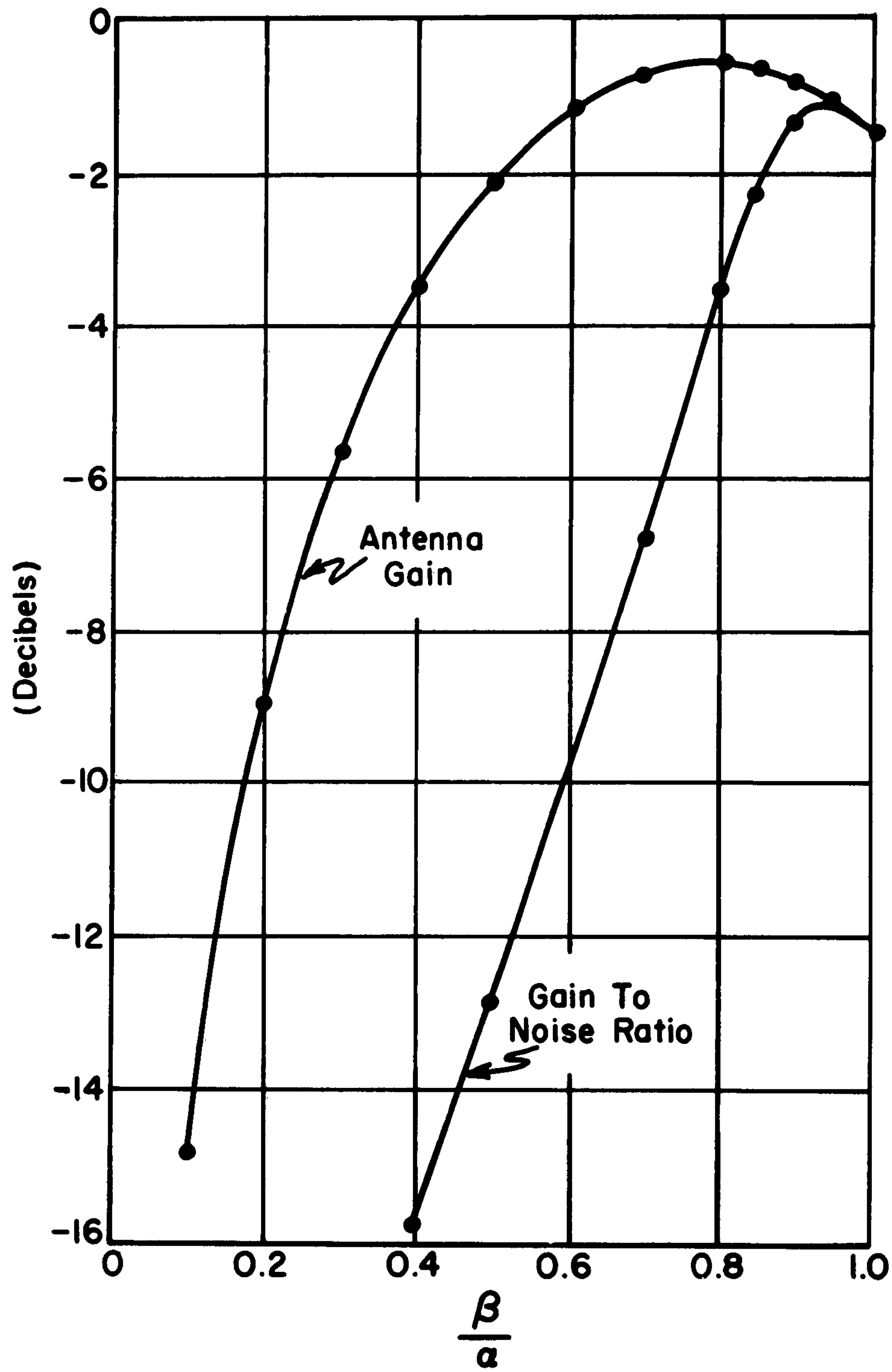

Fig. 2. Gain and gain-to-noise ratio approximations for a very long focal length parabolic reflector antenna.

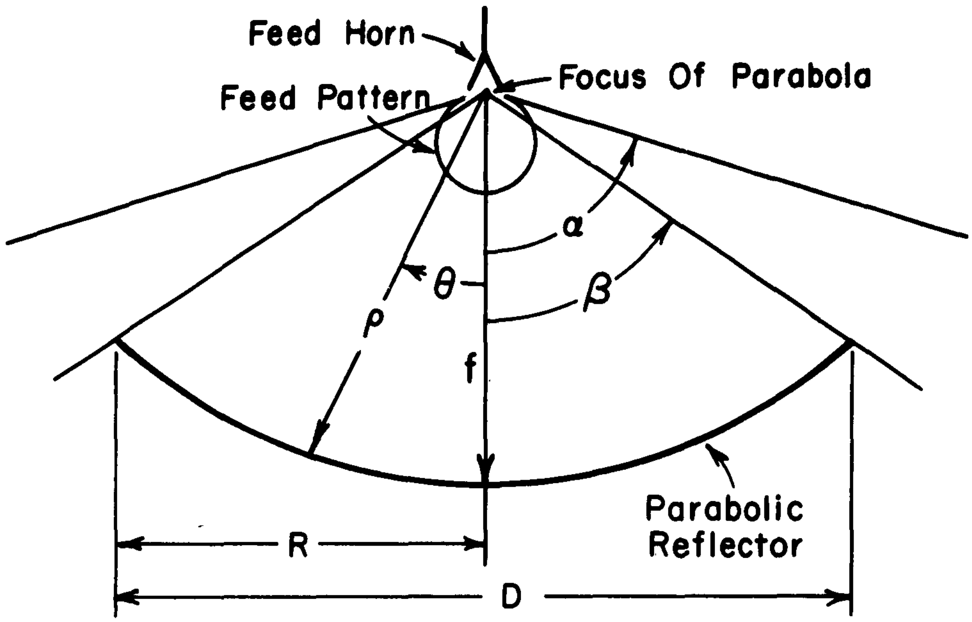

Fig. 3. Coordinates and parameters of the antenna system.

2. The feed-horn pattern is a simple cosine for $-\pi/2 \leq \theta \leq \pi/2$ and zero everywhere else.

3. The main beam is pointed towards a cold part of the sky which has a temperature of $10^{\circ}K$.

4. The spill-over and side lobes above the horizon is neglected.

5. Any spill-over of the feed pattern between the edge of the reflector and the horizon sees a perfect absorber at a temperature of 290°.

These are obviously very idealistic assumptions requiring further study to check the validity of the results.

GAIN CALCULATIONS

The assumption that the focal length of the parabolic reflector is extremely long was made to simplify calculation of the gain in the previous section. Such focal lengths are rarely found in reflectors used for antennas. A more accurate approximation of the gain of practical antennas may be derived as follows: (Still more accurate methods are available[2] but the approximations developed here will be adequate for our purposes.)

The relative gain G_r is defined to be the ratio of the power density at a distant point on the axis, due to the parabolic antenna, to the power density at the same point that would be produced by a uniformly illuminated aperture of the same size, shape and orientation as that of the parabolic antenna and with the same power input. Both antennas are assumed to be lossless.

The on-axis field at a large distance d from the parabolic reflector will be:

(1.1) $$E_p = \int_o^\beta \frac{E(\theta)}{\rho d} (2\pi \ \rho \ \sin\theta)\rho \ d\theta$$

(1.2) $$= \frac{2\pi}{d} \int_o^\beta E(\theta)\rho \ \sin\theta \ d\theta$$

where (see Fig. 3)

$E(\theta)$ is the field pattern of the illuminating antenna referred to a unit distance from the source.

$\frac{E(\theta)}{\rho}$ is the field intensity at the surface of the reflector.

ρ and θ are the polar coordinates of the parabolic surface. (ρ is the length from the iocus to the surface at an angle θ from the axis.)

The equation of the parabolic surface in polar coordinates is:

(2) $$\rho = \frac{2f}{1 + \cos\theta}$$

where f is the focal length of the parabola.

Assuming that the field pattern of the illuminating feed may be approximated by

(3.1) $$E(\theta) = E \cos\left(\frac{\pi}{\alpha}\ \frac{\theta}{2}\right) \qquad 0 \le \theta \le \alpha$$

(3.2) $$E(\theta) = 0 \qquad \theta > \alpha\ .$$

(2) may be substituted into (1.2) to obtain the expression

(4) $$E_p = \frac{4\pi f}{d} \int_o^{\beta} \frac{\cos\left(\frac{\pi}{\alpha}\ \frac{\theta}{2}\right) \sin\frac{\theta}{2}}{\cos\frac{\theta}{2}}\, d\theta$$

for the field at a distance d on the axis, due to the parabola. The corresponding power density at d will be

(5) $$P_p = \frac{E_p^2}{z} = \frac{16\pi^2 f^2}{z\, d^2}\left[\int_o^{\beta} \frac{\cos\left(\frac{\pi}{\alpha}\ \frac{\theta}{2}\right) \sin\frac{\theta}{2}}{\cos\frac{\theta}{2}}\, d\theta\right]^2$$

where z is the intrinsic impedance of space.

The total power delivered by the source can be calculated to be

(6) $$W = \int_o^{\alpha} \frac{1}{z}\left(\frac{E(\theta)}{\rho}\right)^2 (2\pi\ \rho \sin\theta)\rho\ d\theta$$

which becomes

(7) $$W = \frac{2\pi}{z} \int_o^{\alpha} \left[\cos\left(\frac{\pi}{\alpha}\ \frac{\theta}{2}\right)\right]^2 \sin\theta\, d\theta$$

for the cosine approximation (3) of the feed pattern.

If the total power W is uniformly distributed over the aperture of the parabola, the power density will be

(8) $$P_o = \frac{W}{\pi R^2}$$

where R = D/2 is the radius of the aperture. The power density will also be

(9) $$P_o = \frac{E_o^2}{z}$$

where E_o is the field intensity over the aperture. It follows from (8) and (9) that:

(10) $$E_o = \frac{1}{R}\sqrt{\frac{zW}{\pi}} \quad .$$

Now the field on the axis of the aperture at a distance d may be calculated for the uniformly illuminated aperture to be

(11) $$E_a = \int_o^R \left(\frac{E_o}{d}\right) 2\pi r \, dr .$$

Substituting the value of E_o from (10) into (11) and integrating yields

(12) $$E_a = \frac{\pi R}{d}\sqrt{\frac{zW}{\pi}} \quad .$$

The power density at d on the axis due to the uniformly illuminated aperture will be

(13) $$P_a = \frac{E_a^2}{z} \quad .$$

Substituting the value of W from (7) into (12) and the resulting value of E_a into (13) yields

(14) $$P_a = \frac{2\pi^2 R^2}{zd^2} \int_0^\alpha \left[\cos\left(\frac{\pi}{\alpha}\ \frac{\theta}{2}\right)\right]^2 \sin\theta\, d\theta$$

which is the power density at d on the axis due to the uniformly illuminated aperture.

Now the relative gain of the antenna with respect to the uniformly illuminated aperture will be, as defined previously

(15) $$G_r = \frac{P_p}{P_a}$$

which, upon substituting in expressions for P_p and P_a from (5) and (14) respectively becomes

(16) $$G_r = \frac{8f^2}{R^2} \frac{\left[\int_0^\beta \frac{\cos\left(\frac{\pi}{\alpha}\ \frac{\theta}{2}\right) \sin\frac{\theta}{2}}{\cos\frac{\theta}{2}}\, d\theta\right]^2}{\int_0^\alpha \left[\cos\left(\frac{\pi}{\alpha}\ \frac{\theta}{2}\right)\right]^2 \sin\theta\, d\theta} .$$

Now the angle 2β subtended by a parabolic reflector at the focus is related to its diameter, D, and focal length, f, by the expression

(17) $$\frac{4f}{D} = \cot\frac{\beta}{2}$$

or in terms of the radius R of the reflector

(18) $$\frac{2f}{R} = \cot\frac{\beta}{2} .$$

The integral in the denominator of (16) may be evaluated in closed form as

(19) $$\int_0^\alpha \left[\cos\left(\frac{\pi}{\alpha}\ \frac{\theta}{2}\right)\right]^2 \sin\theta\, d\theta = 1 + \frac{\cos^2\frac{\alpha}{2}}{\left(\frac{\alpha}{\pi}\right)^2 - 1} .$$

Using the results of (18) and (19) in (16) yields the expression

$$G_r = 2\left(\cot\frac{\beta}{2}\right)^2 \frac{\left\{\int_0^{\beta}\cos\left(\frac{\pi}{\alpha}\frac{\theta}{2}\right)\tan\frac{\theta}{2}\,d\theta\right\}^2}{1+\dfrac{\left(\cos\frac{\alpha}{2}\right)^2}{\left(\frac{\alpha}{\pi}\right)^2-1}} \tag{20}$$

for the gain of the parabolic reflector referred to the uniformly illuminated aperture.

The integral in the numerator of (20) can be evaluated for specific values of π/α which are rational, but becomes very difficult for any but the most simple ratios of small integers. As a result it is convenient to plot gain as a function of the angle 2β for a few judiciously chosen values of α as shown in Fig. 4. This, however is not the most convenient form for application as one usually would like to determine an optimum feed for a particular reflector. The gain should also be plotted in decibels for convenience.

The curves shown in Fig. 5, where the gain in db is plotted as a function of the null beamwidth 2α for specific values of β are better suited for this purpose. Since the angle β is unique, as determined by (17) for any given reflector, the corresponding curve on Fig. 5 describes the relative gain characteristics of the reflector as a function of the null beamwidth, 2α, of the feed pattern.

Figure 6 shows a plot of the illumination taper of the reflector as a function of the angle β, required for maximum gain which is determined from the curves shown in Fig. 4.

Figure 7 shows the relative gain curves plotted as a function of the ratio β/α. With the exception of the deep dishes (reflectors with a subtended angle of 2β greater than about 150°) these curves coincide very closely in the maximum gain region. An average of all the curves for $\beta \leq 60°$ is shown in Fig. 20 as a universal approximation to all the gain curves. It is to be noted that this approximation is very similar to the approximation developed in an earlier report[1] and shown in Fig. 2, but that it does not represent the curves for $\beta > 60°$.

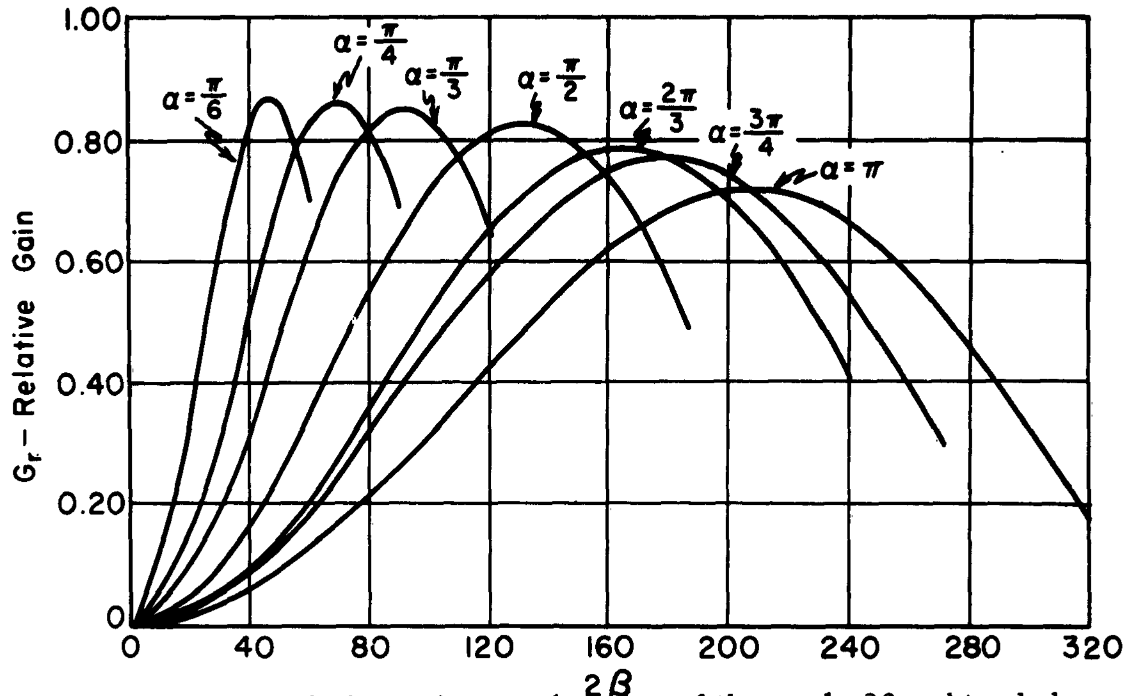

Fig. 4. Plots of relative gain as a function of the angle 2β subtended by the parabolic reflector (2α is the angle between nulls of the cosine approximation of the pattern).

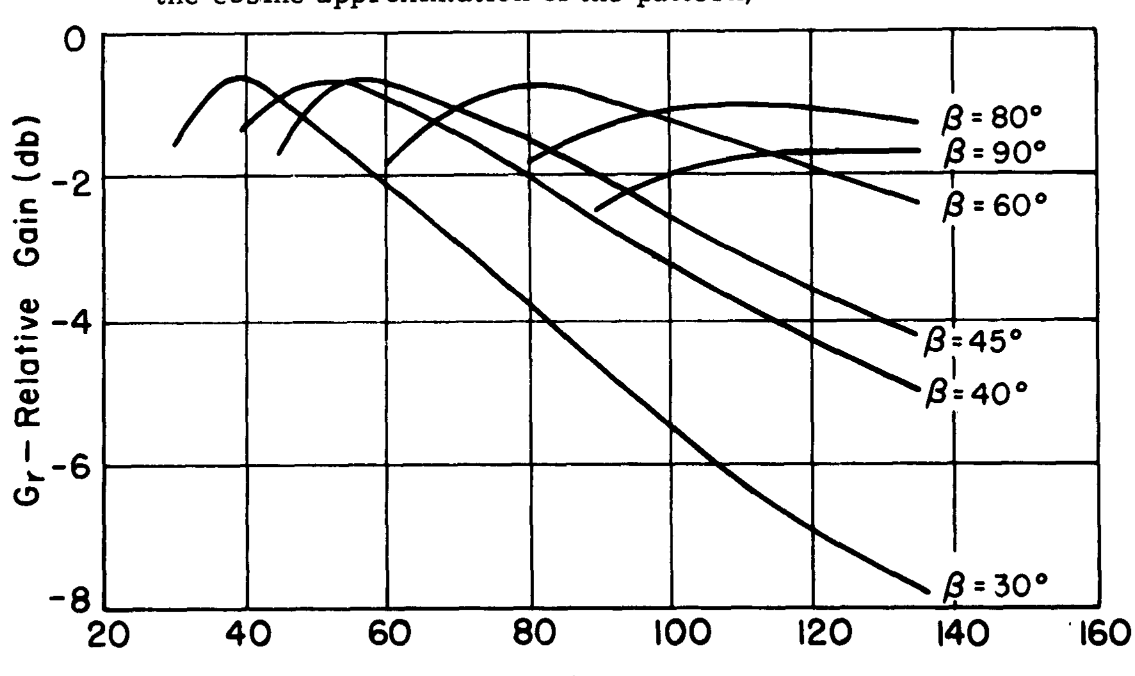

Fig. 5. Plots of relative gain as a function of the null beamwidth 2α of the antenna (2β is the angle subtended at the focus by the reflector).

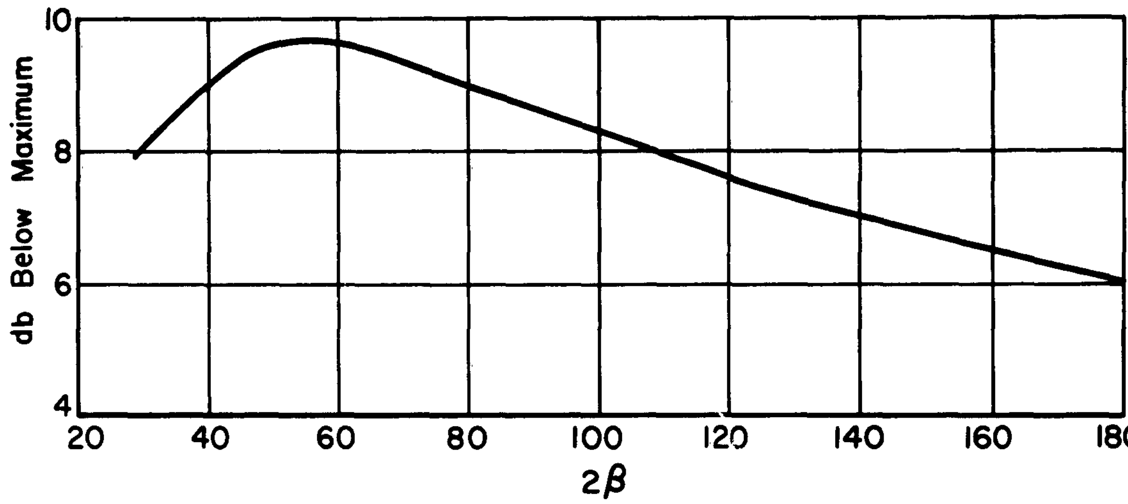

Fig. 6. Edge illumination relative to the maximum illumination required for maximum gain.

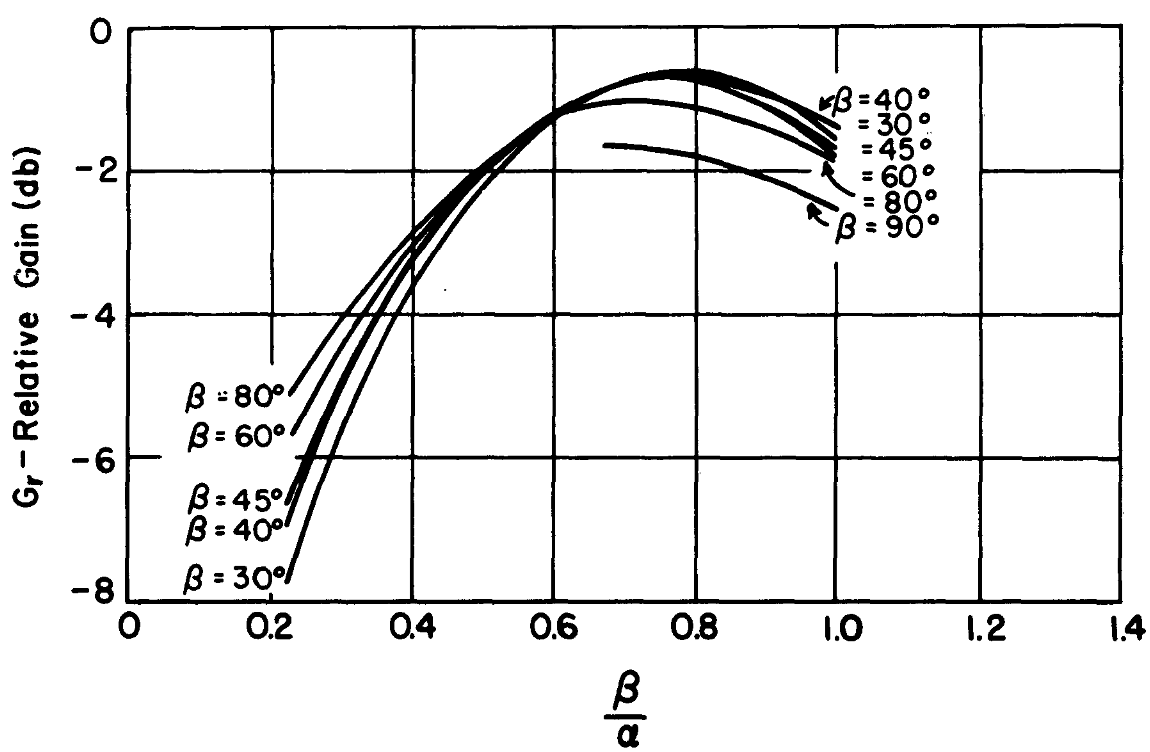

Fig. 7. Relative gain curves plotted as a function of β/α.

APPROXIMATION OF FEED PATTERN

In the gain calculations of the previous section, and in calculations of both gain and gain-to-noise as presented in a previous report[1] it has been assumed that the feed pattern could be expressed by means of a simple cosine approximation. Figures 8 and 9 show a number of typical horn patterns such as might be used to illuminate a parabolic reflector. Figures 10 through 15 show these same patterns in rectangular coordinates with approximating cosine curves drawn through the maximum and half-power points of each curve. The validity of these approximations may be questioned since the nulls of the cosine curves do not coincide with the nulls of the patterns. However, the approximations are quite adequate for most gain calculations since the curves fit the patterns in the central high-gain sections and, in practice, β is usually larger than α, so that most of the regions where the fit is not so good are not involved in the calculations.

Since complete symmetry was assumed in the derivation of the gain expression, some difficulty arises when the E- and H-plane patterns are different; but unless the patterns are greatly different an average value of α may be determined from the two pattern approximations which will usually be satisfactory for the gain calculations.

Thus it would appear that the cosine approximation of the pattern would be satisfactory for most gain calculations. However it is easily seen that the cosine approximation of the pattern is totally inadequate for noise calculations, since the cosine curve bears little or no relation to the pattern at the larger angles where the major thermal noise contributions are received.

NOISE CONTRIBUTION FROM EXTERNAL HOT OBJECTS

The thermal noise received by an antenna from hot bodies in its radiation field may be expressed in terms of temperature as:

$$T = \frac{1}{2\pi} \int G \, T_e \, d\Omega \qquad (21)$$

where:

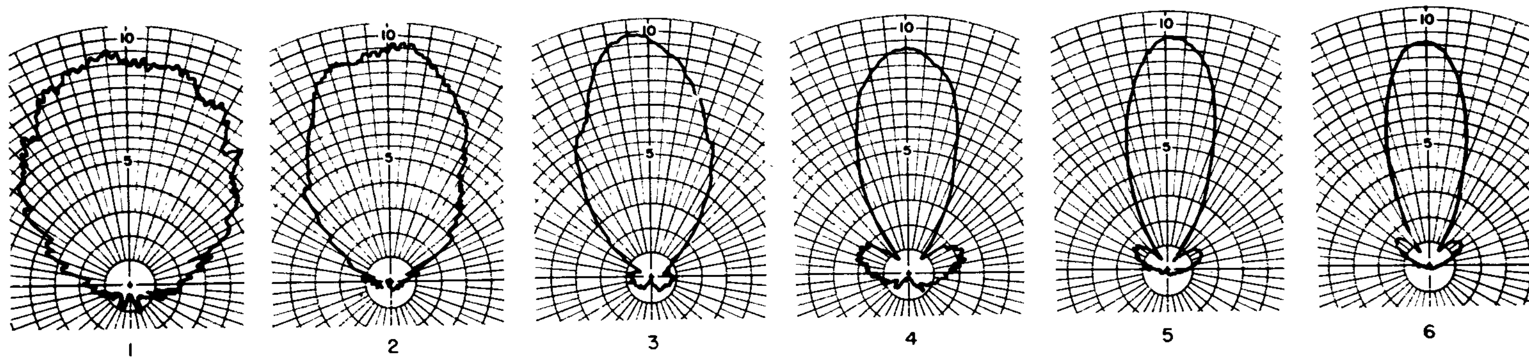

Fig. 8. E-plane patterns of primary feed horn.

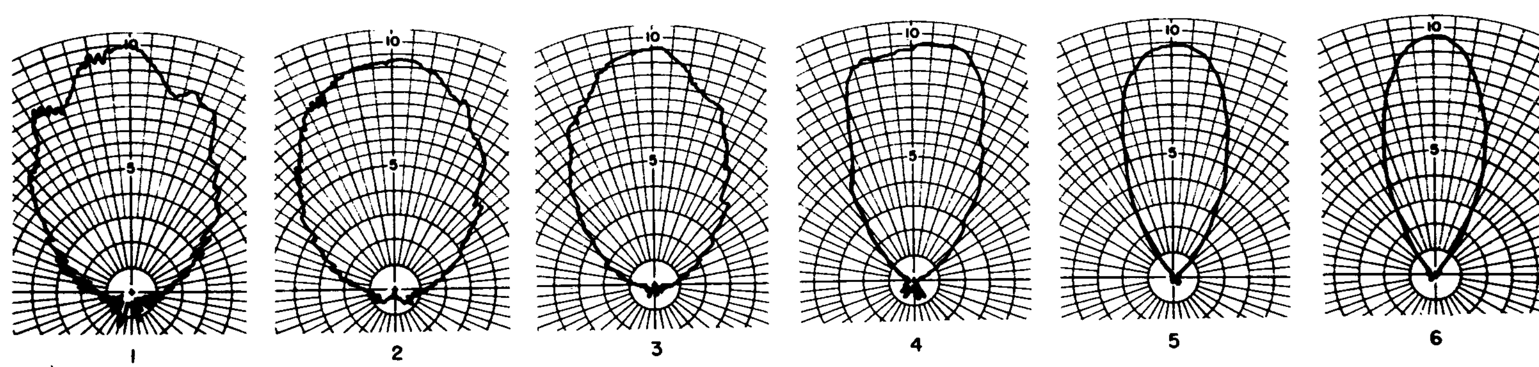

Fig. 9. H-plane patterns of primary feed horn.

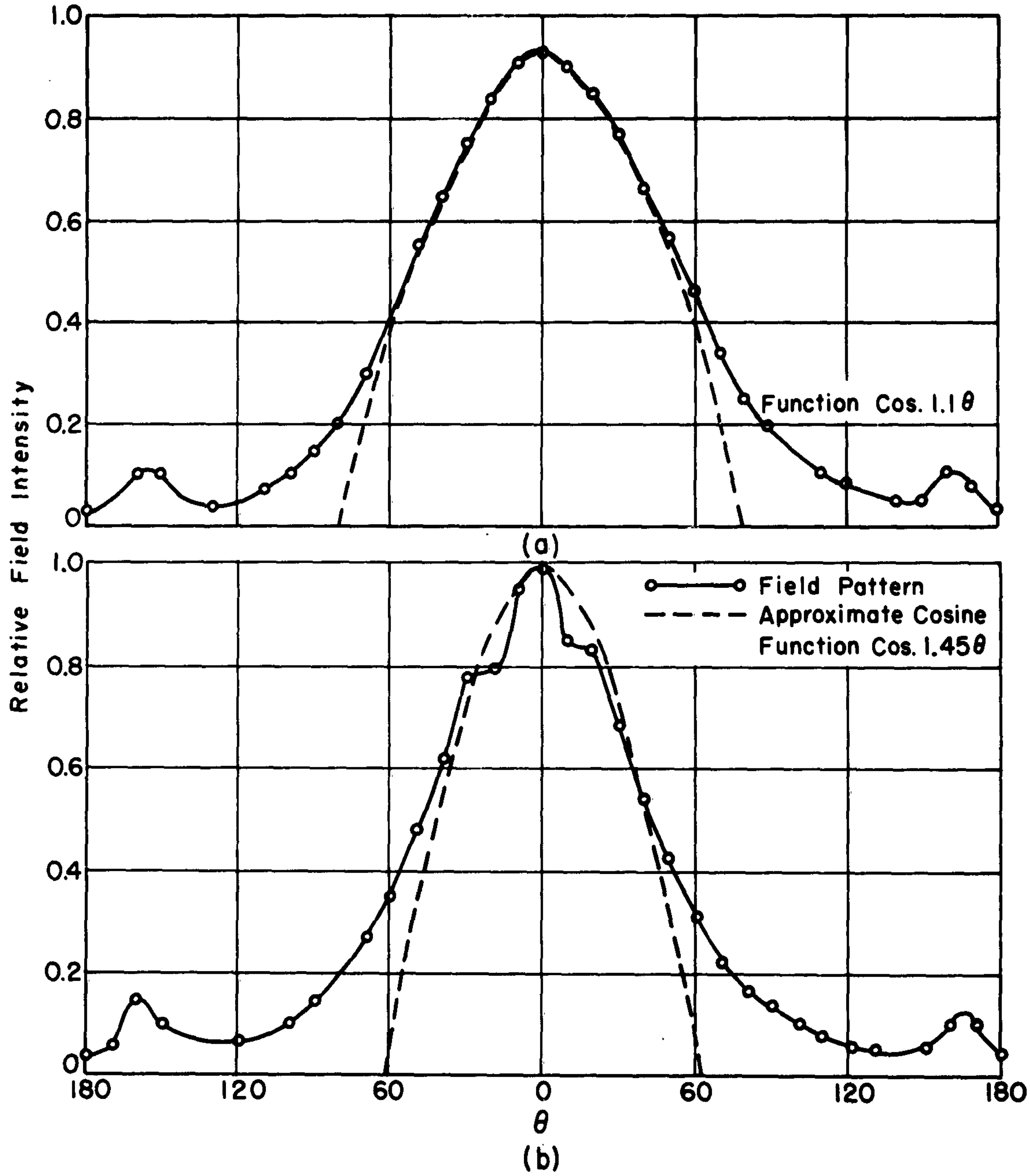

Fig. 10. Field pattern of No. 1 horn.
a. E-plane.
b. H-plane.

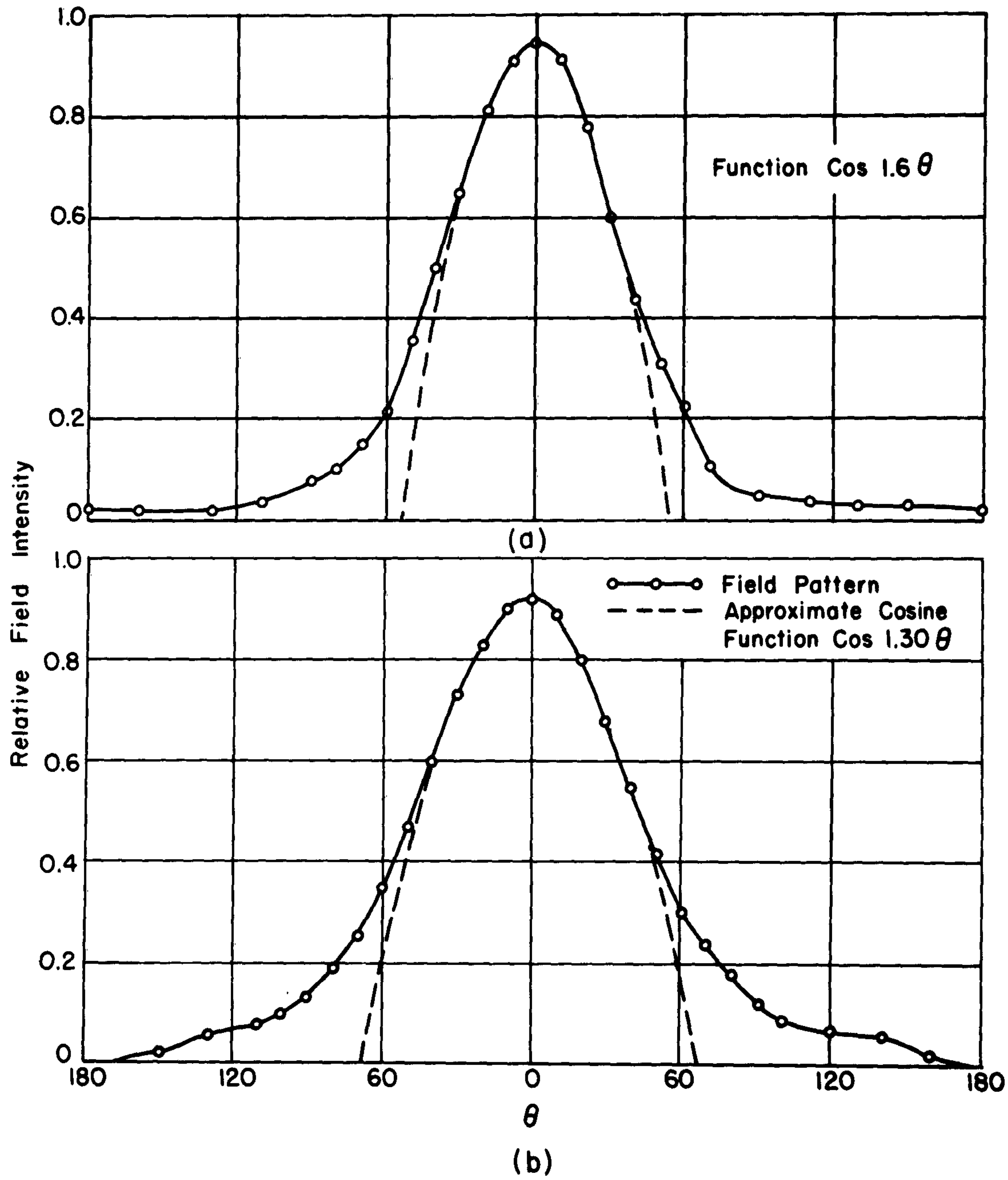

Fig. 11. Field pattern of No. 2 horn.
a. E-plane.
b. H-plane.

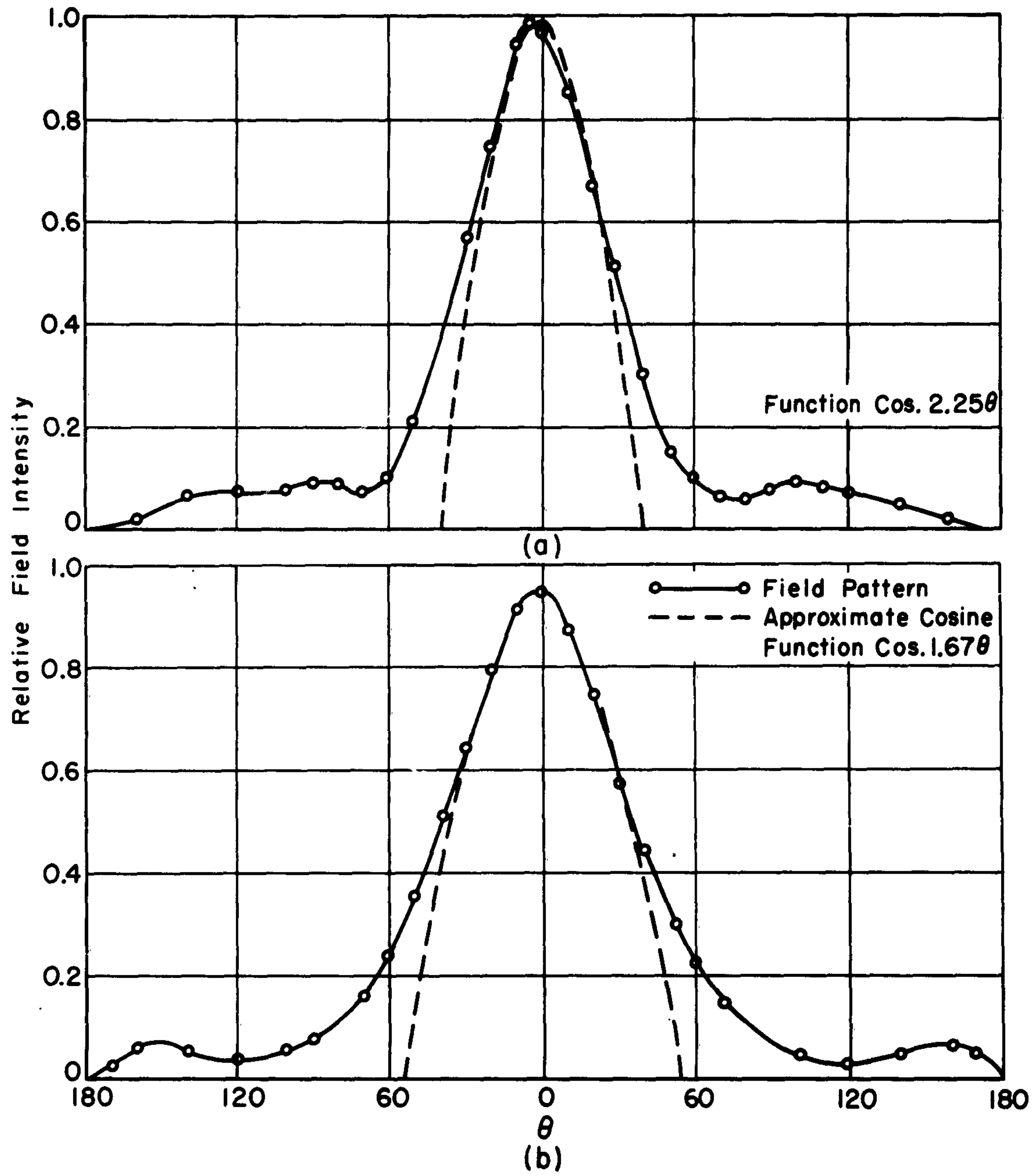

Fig. 12. Field pattern of No. 3 horn.
a. E-plane.
b. H-plane.

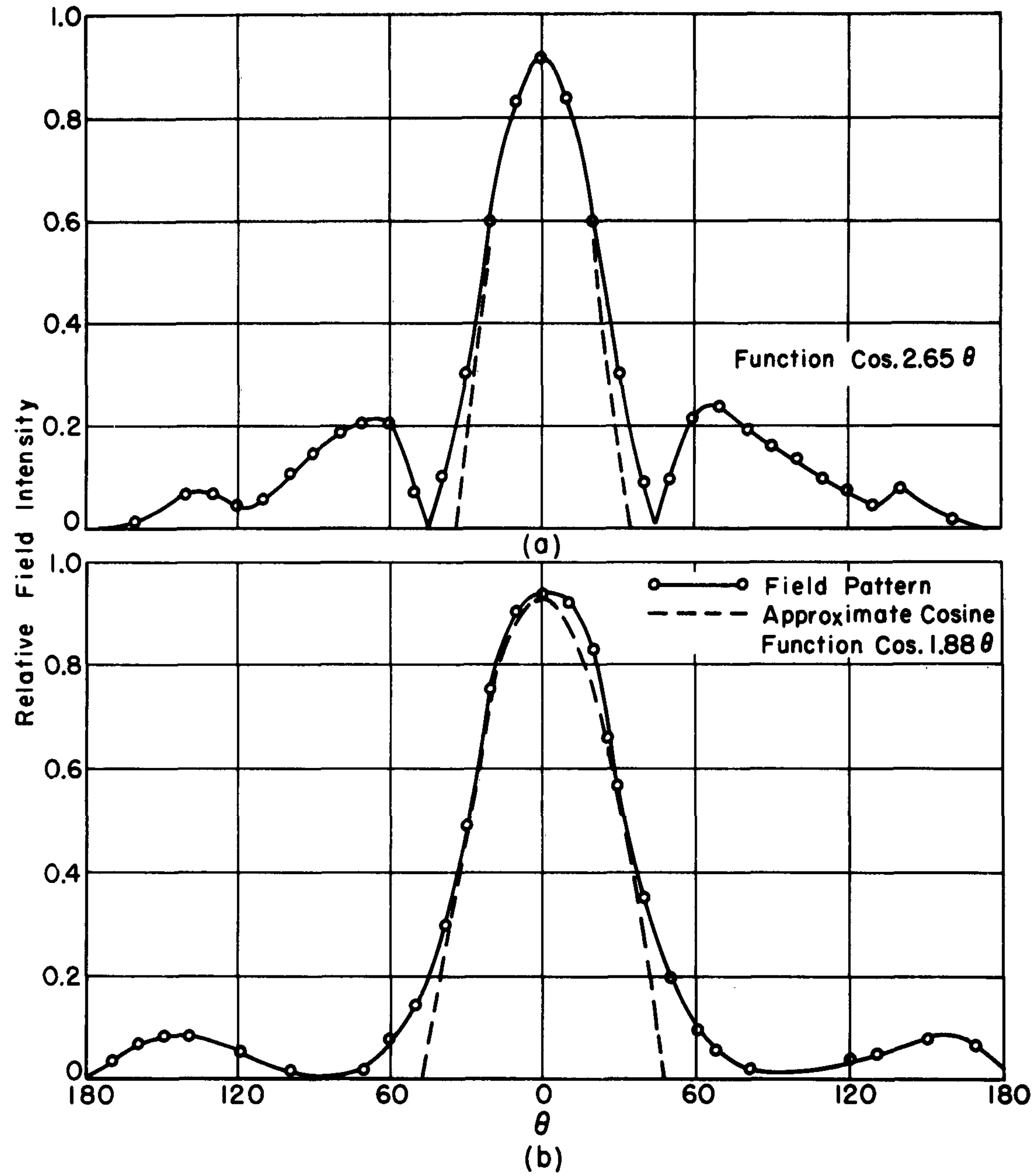

Fig. 13. Field pattern of No. 4 horn.
a. E-field.
b. H-field.

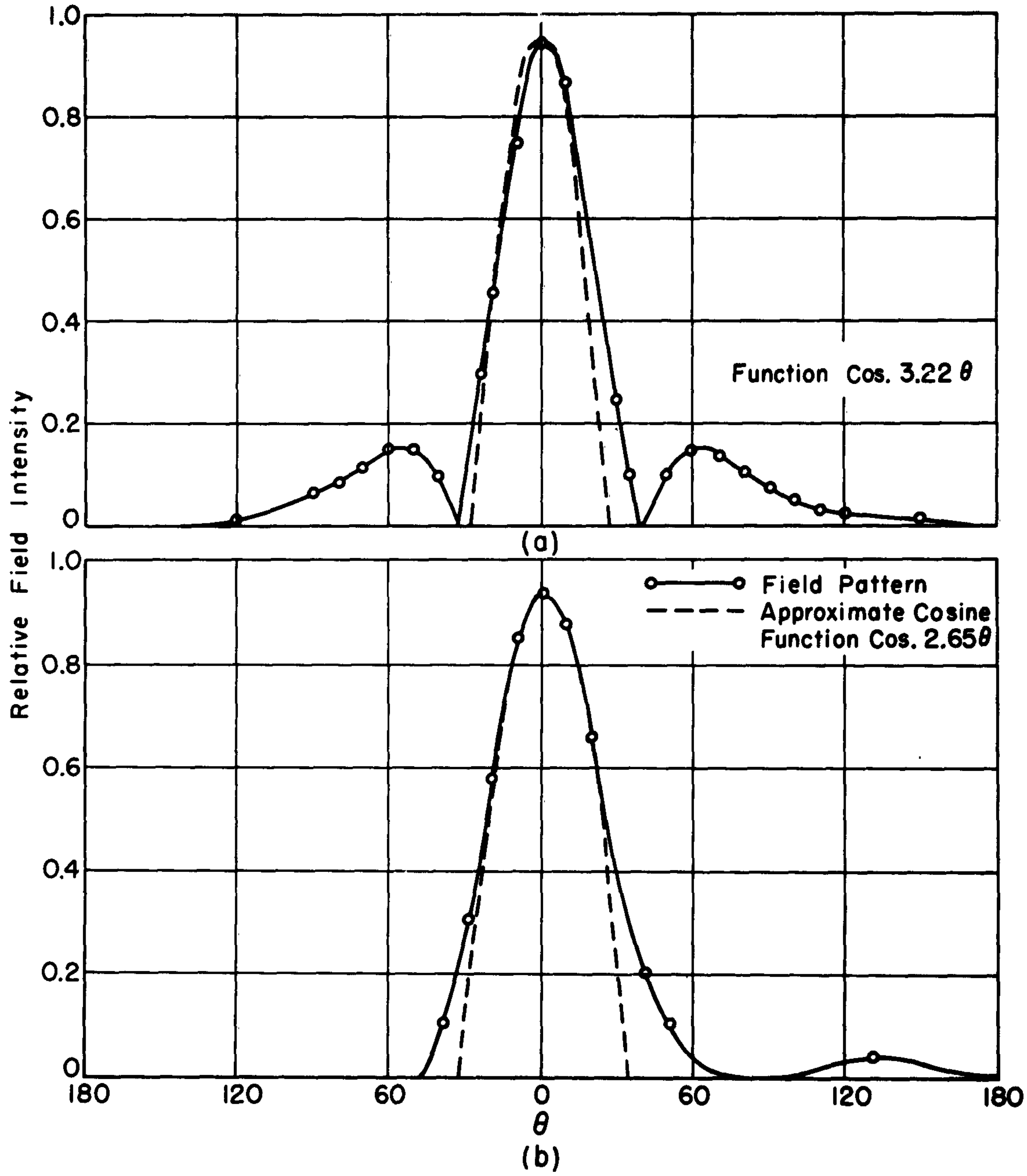

Fig. 14. Field pattern of No. 5 horn.
a. E-field.
b. H-field.

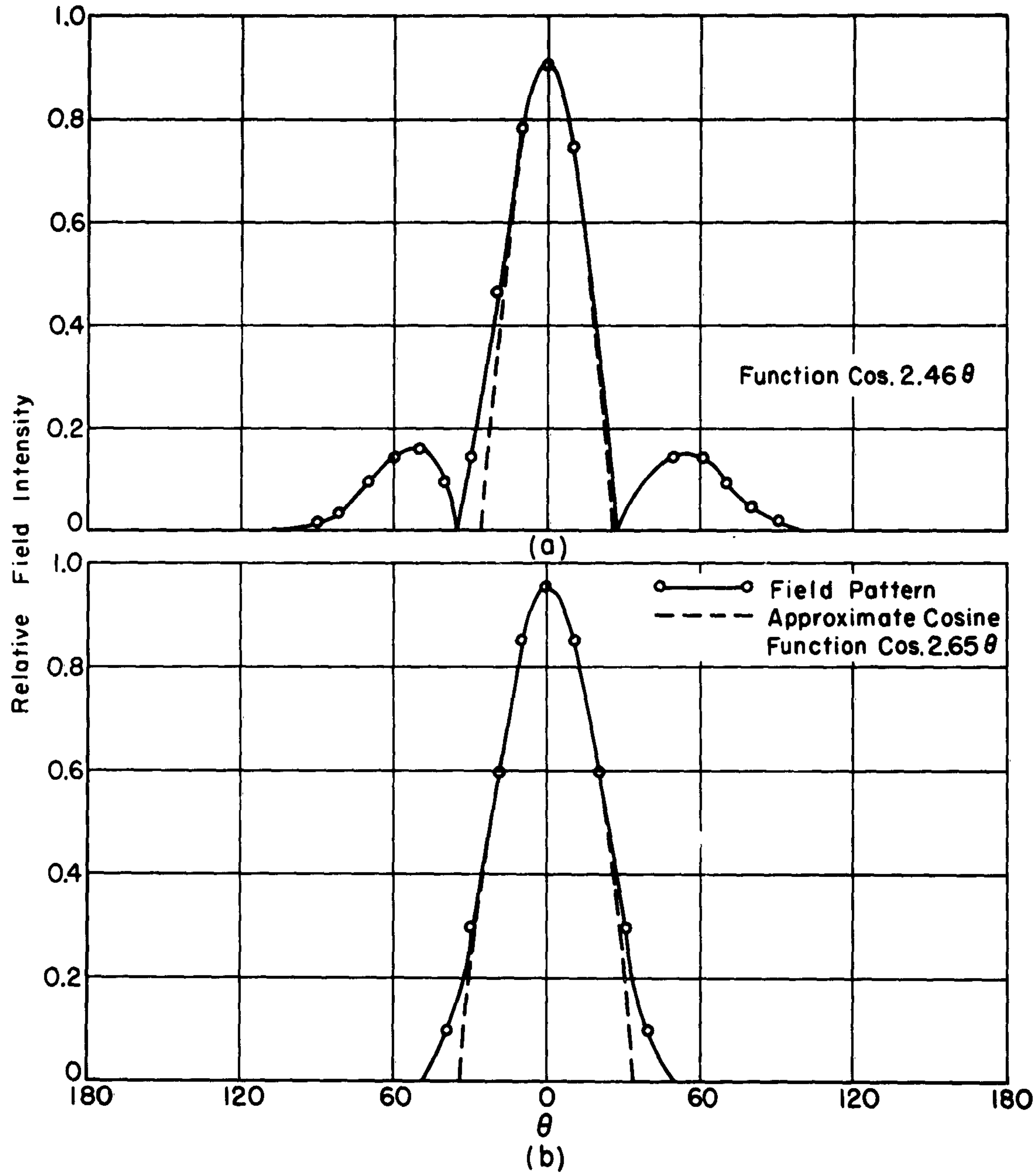

Fig. 15. Field pattern of No. 6 horn.
a. E-field.
b. H-field.

G is the antenna gain in the direction of $d\Omega$,

$d\Omega$ is the increment of solid angle,

T_e is the effective temperature in the direction of $d\Omega$.

The noise contribution of a hot body consists of two parts. $T_a|\rho|^2$ is the noise contribution due to the temperature of the hot body. $T_r(1-|\rho|^2)$ is the noise contribution due to the reflection of incident radiation from other sources by the surface of the hot body. The effective temperature T_e may be written:

$$T_e = T_a|\rho|^2 + T_r(1 - |\rho|^2),$$

where

T_a is the actual temperature of the hot body in the direction of $d\Omega$,

ρ is the transmission coefficient of the hot body,

T_r is the effective temperature of radiation incident on the surface of the hot body.

The integration must be carried out over the entire sphere surrounding the antenna.

This expression does not include noise entering the antenna from discrete sources such as radio stars and electrical interference, which must be treated separately.

It is immediately obvious that the analytic calculation of thermal noise received by an antenna is quite complex since it requires a complete knowledge of the location, temperature, and reflection characteristics of everything visible to the antenna, as well as the three-dimensional field pattern of the antenna itself.

Even in the simple case of the parabolic antenna pointed vertically upwards, the estimation of thermal noise received by the antenna is quite complex. In this case the major part of such noise will be received by the feed horn from the hot earth in the region between the edge of the reflector and the horizon. A knowledge of the complete three-dimensional pattern in this region is necessary in order to be able to calculate the received noise. In addition it is necessary to know the characteristics of the earth as a hot body in this region. This depends upon the surface characteristics and angle of incidence as well as its physical temperature. Figure 16 shows a measurement

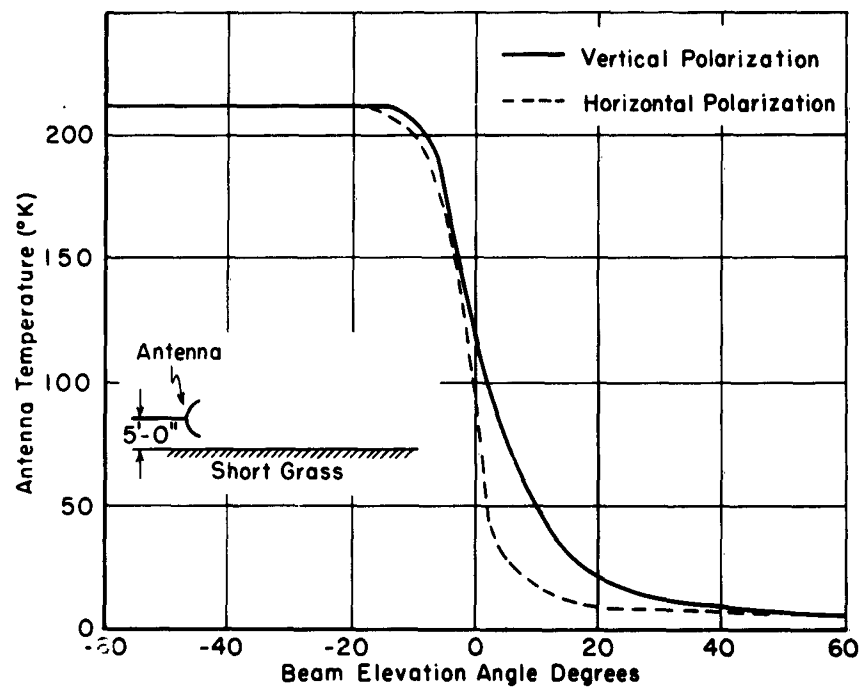

Fig. 16. Effective temperature of flat earth covered with short grass measured at X-band as a function of the angle of incidence.

of the effective temperature of the earth for one particular case, as a function of the angle of incidence. This measurement was made with an X-band radiometer using a four-foot parabolic reflector antenna. It is somewhat approximate as no attempt was made to account for the effect of side lobes and spillover. A more detailed treatment of the apparent temperatures of surfaces may be found in the references.[3,4]

Figure 17 shows approximations of the noise received by the typical feed horns whose patterns are shown in Figs. 8-15. In these approximations each pattern was assumed to be a symmetrical pattern of revolution whose cross section was determined by averaging the E- and H-plane patterns of Figs. 8-15. (It is recognized that this is a rather crude approximation of the three dimensional pattern but is

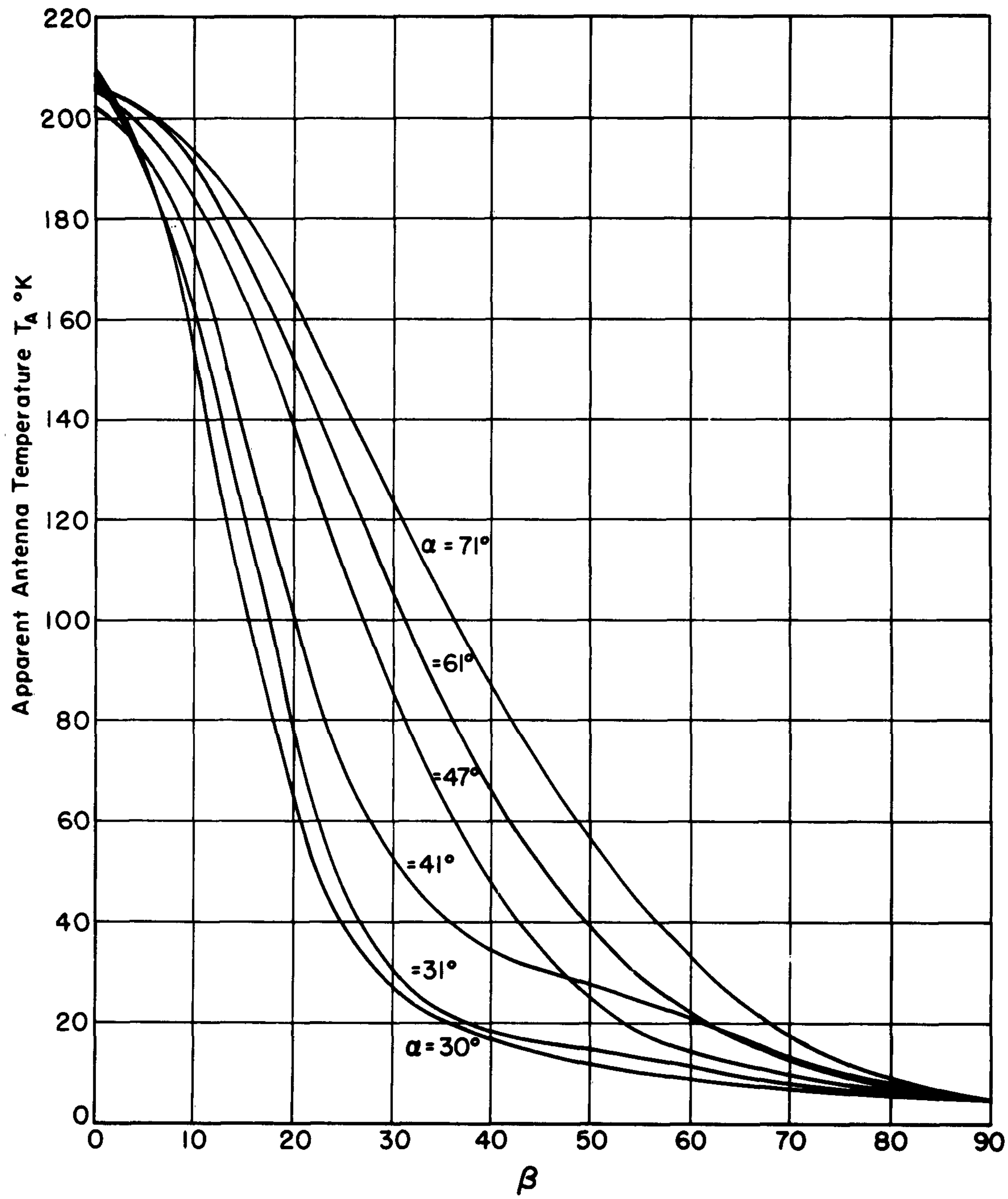

Fig. 17. Approximation of thermal noise received by typical feed horns for parabolic reflectors plotted as a function of angle β.

probably good enough for the example. More accurate results, of course, require a much more detailed knowledge of the three-dimensional pattern.) For these calculations it was assumed that the effective temperature of the earth was 210°K in the sector seen by the feed horn between the edge of the reflector and the horizon. This value was taken from the measured curve of Fig. 16. The fact that the apparent temperature drops off near the horizon was neglected to simplify the calculations. This is partially justified by the fact that the pattern gain in this direction is small for most of the horns considered, but for more precise results, such variations should be taken into account.

The effective temperature in all other directions was assumed to be 5°K for the calculation, which is about the typical X-band sky temperature. It was assumed that there was no loss in the reflecting surfaces and there were no other sources of thermal noise visible by the feed horn.

Since patterns were available for only a limited number of feed horns, a curve of effective temperature was plotted for each horn as a function of β, where 2β is the angle subtended by the reflector. However it is usually more convenient to have curves of the effective temperature plotted as a function of α, where 2α is the null beamwidth of the feed pattern, for typical values of β as shown in Fig. 18. These curves are plotted in db, referred to 5°K, since 5°K is the minimum noise temperature that can be obtained under the assumed circumstances.

It should be noted here that in Fig. 17 the inconsistent behavior of the curve for $\alpha = 41^\circ$ is due to poor feed design or construction which broadened the skirts of the main lobe as shown in Fig. 12. This also accounts for the humps in the curves of Fig. 18 at $\alpha = 41^\circ$. In looking at Fig. 18 it is important to remember that it applies to a particular set of feeds and that it is the trend of the curves and not the point-by-point values that are significant. Figure 18 shows that the apparent antenna temperature is dependent not only on the angle between nulls of the main lobe, but on the overall feed pattern.

Figure 19 shows curves of the gain-to-noise ratio as calculated from the measured horn patterns, for the horn-fed parabolic reflector, plotted as a function of β/α. The range of these curves are limited on one end by the fact that no gain calculations are made for values of α less than β, and on the other end by the number of horns, and hence the number of values of α available. That is the range of α must be

(22) $$\beta < \alpha < \alpha_m$$

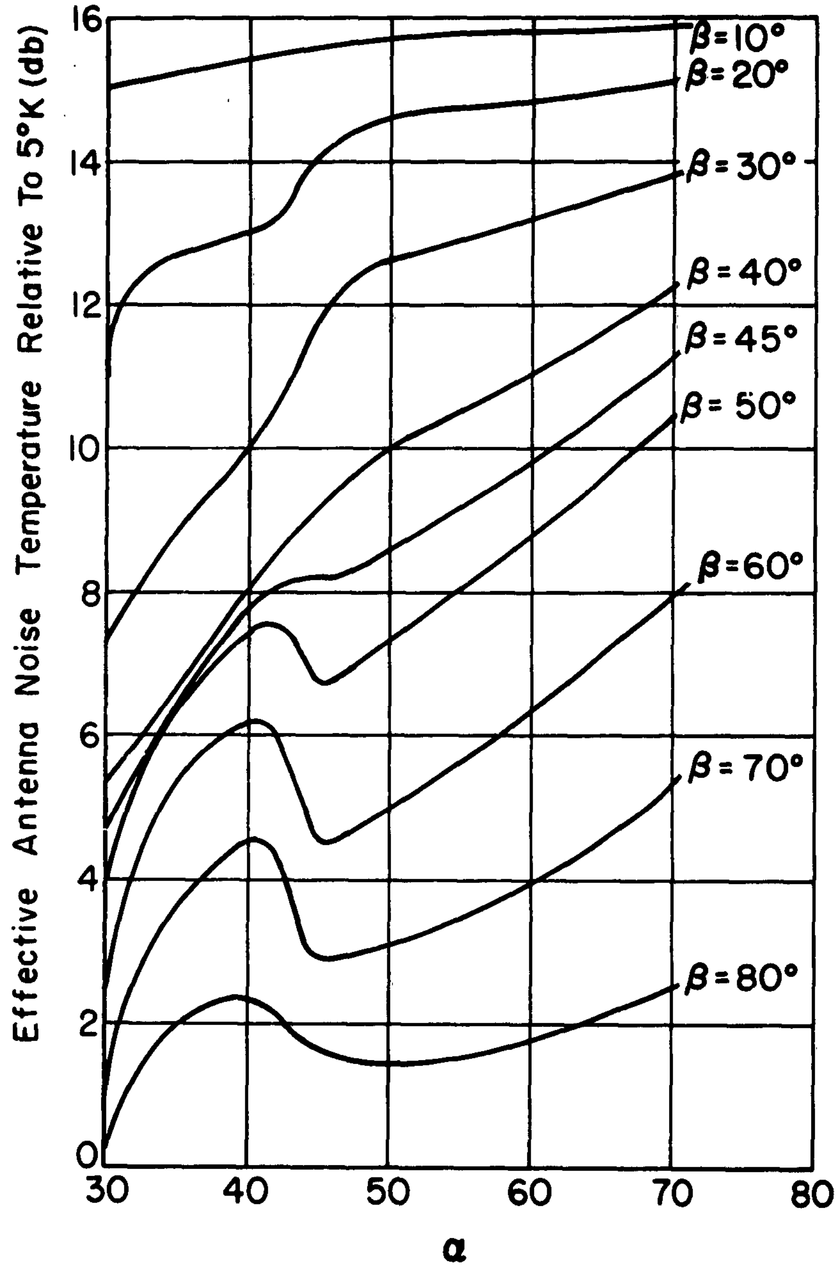

Fig. 18. Approximation of thermal noise received by typical feed horns for parabolic reflectors plotted as a function of angle α.

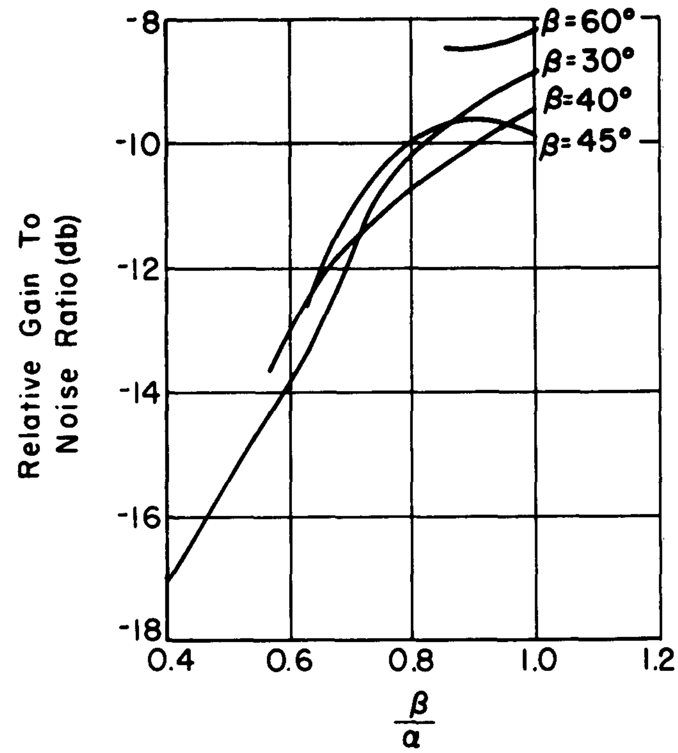

Fig. 19. Ratios of relative gain to apparent thermal noise of a horn illuminated parabolic reflector as a function of β/α.

where α_m is the maximum value of α obtainable with the horns measured. From this it can be seen that

$$(23) \qquad 1 < \frac{\alpha}{\beta} < \frac{\alpha_m}{\beta}$$

or

$$(24) \qquad 1 > \frac{\beta}{\alpha} > \frac{\beta}{\alpha_m}$$

where 1 and β/α_m are the limits of the range of β/α and $\alpha_m = 71°$.

An average of all of the gain-to-noise curves is shown in Fig. 20 along with the averaged gain curves discussed earlier. It can be seen by comparing these curves with the approximations of Fig. 2 (reproduced

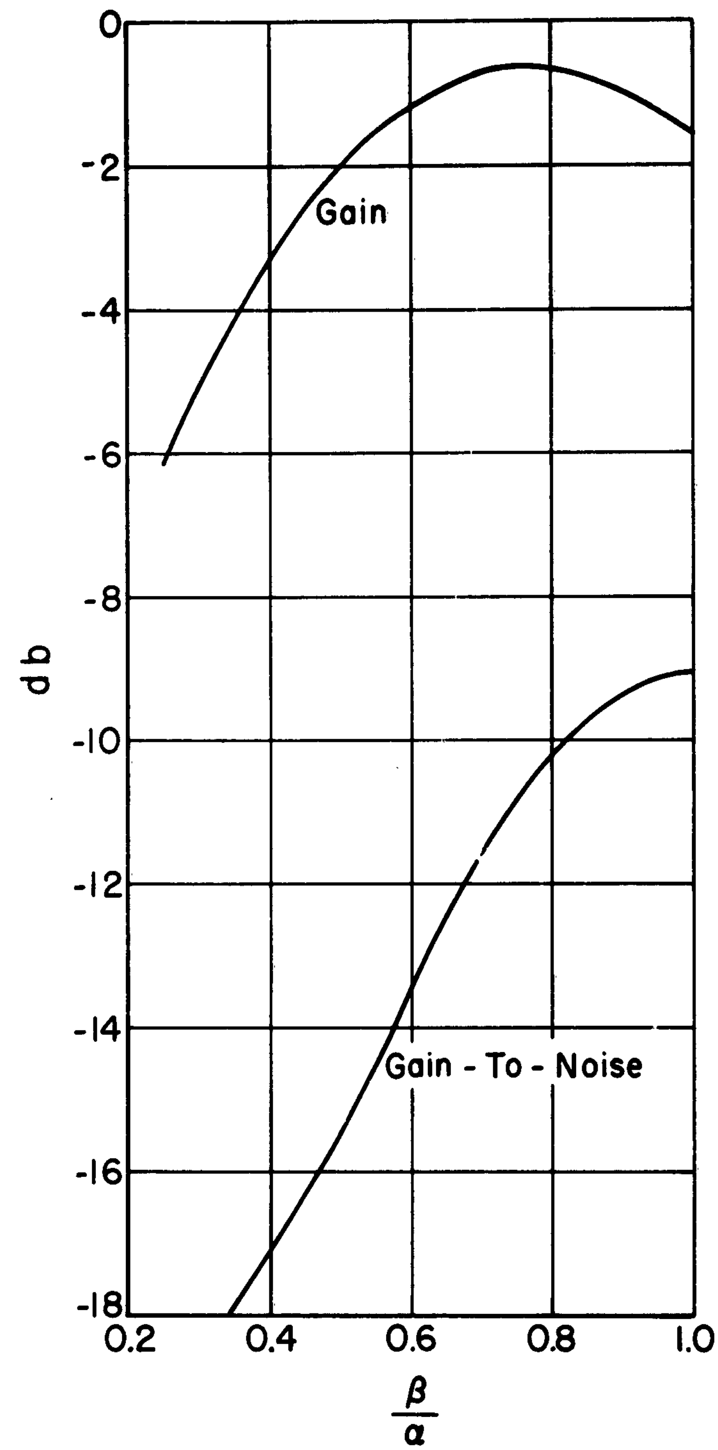

Fig. 20. Average curves of relative gain and gain-to-noise ratio of horn illuminated parabolic reflectors plotted as functions of β / α.

from an earlier report[1]) that, while the gain curves agree quite well, the gain-to-noise ratio curves are quite different.

For one thing, the curve calculated from measured patterns indicate that, in general, a greater illumination taper is required for maximum gain-to-noise ratio than was indicated by the approximate curves. Of even more significance is the fact that the maximum gain-to-noise ratio that can normally be acheived in practice is much less than was indicated by the approximate curves. That is, the calculated curves imply a maximum gain-to-noise ratio of about 9 db less than theoretically possible where the approximate curve indicates that values could be obtained to within about one db of the theoretical limit. It must be emphasized at this point that only thermal noise sources external to the antehna are being considered and that these curves do not include such noise sources as feedline losses, reflection losses, mismatch, etc.

APPLICATIONS: SHIELDED FEEDS AND CASSEGRAIN SYSTEMS

The above developments indicate some ways in which the reception by an antenna of thermal noise from external hot bodies may be reduced.

For the particular type of antenna being considered in detail, that of a parabolic reflector with the main beam pointed vertically upwards, the noise can be reduced considerably by simply shielding the feed horn from the hot earth as shown in Fig. 21. The apparent

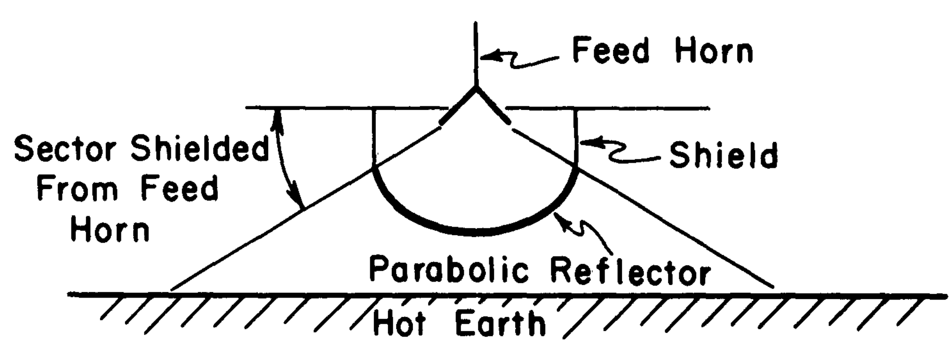

Fig. 21. Shielding the feed horn of the parabolic reflector from the hot earth.

temperature of a four-foot parabolic reflector was reduced from about 27° to about 15° by such shielding as measured by an X-band radiometer. A shield of this type will have little effect upon the main pattern, and hence the gain of the antenna.

Smaller shields may be used to shield against specific hot bodies. For example the smaller cylindrical shield, such as shown in Fig. 22,

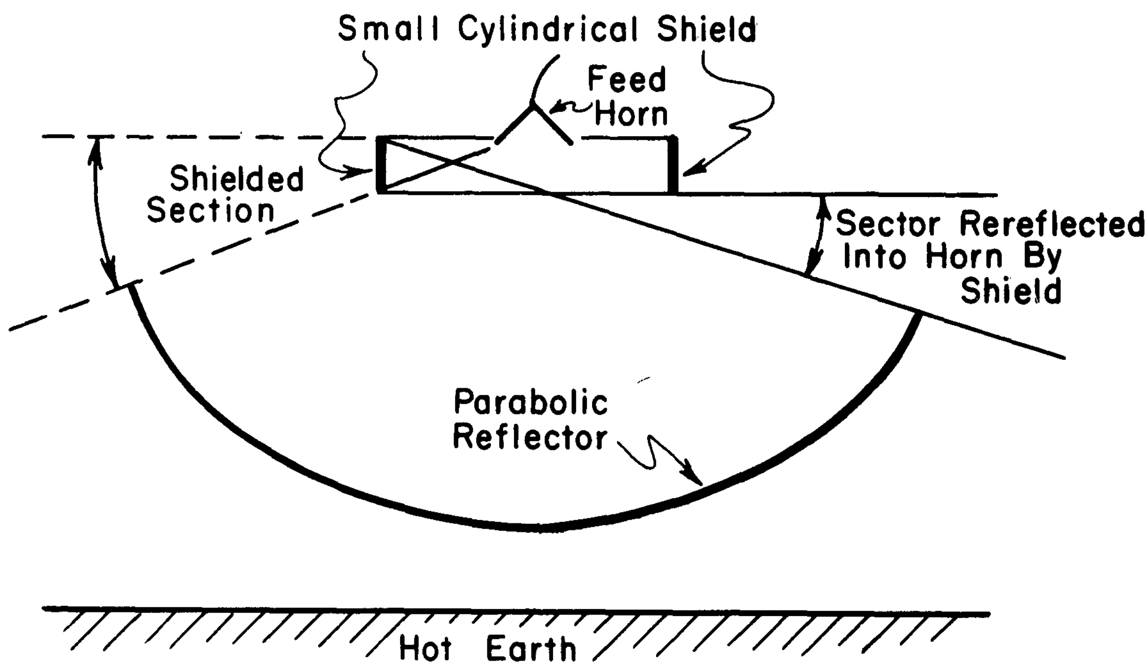

Fig. 22. Small cylindrical shield around the feed horn of a parabolic reflector.

may be used to shield the feed horn against such noise sources as the sun. The effectiveness of such a shield decreases rapidly as the size decreases due to diffraction and the multiple reflection effects indicated in Fig. 22.

The shielding may be accomplished by the reflector itself by increasing the subtended angle 2β as seen by the feed horn at the focal point. For example if $\beta \geq 90°$ the feed horn would be completely shielded from the hot earth when the antenna is pointed vertically upwards. This would result in a decrease in gain as indicated by Figs. 4 and 7, of several db. However the decrease in noise may be much greater, resulting in an overall improvement in the gain-to-noise ratio.

The system of shielding described above becomes increasingly difficult as the size of the reflector increases. The Cassegrain system has received considerable attention in recent years as a possible method of feeding low-noise antennas.[5] Its advantages are that it reduces feed-line loss by eliminating the length of cable or waveguide leading from the back of the reflector to the focal point, and reduces spillover at the edges of the main reflector because of the somewhat greater directivity of the secondary reflector compared with simulated point source feeds.

Disadvantages include additional reflection loss, though this is relatively small, and a second lot of spillover. Spillover at the secondary reflector will be of a similar order to that occurring originally at the main reflector. Its significance will depend on the application of the antenna. For an antenna pointed near the horizon a portion of this spillover will pick up thermal radiation from the ground and will, therefore, be as serious as if it occurred at the main reflector. For an antenna pointed nearer the zenith, this spillover from the secondary reflector will be directed towards the sky and will, therefore, contribute only a small amount of noise. A Cassegrain feed is thus most likely to be beneficial in the case of an antenna directed well above the horizon and when the antenna size is large enough for the feed line length to be significant. In any case it must be remembered that the best condition obtainable is that given by the ideal antenna definition. A simple feed system will usually give a gain-temperature ratio within 9 db of the ideal figure. Thus a simple Cassegrain system is not likely to yield an improvement of much more than 6 db unless the feed line losses for the horn fed case are very high.

The Cassegrain system was originally devised for use on optical telescopes. When the aperture is sufficiently large that diffraction effects may be neglected, then the only losses are at the reflecting surfaces. For a very small antenna the secondary reflector becomes so small that it exhibits a very poor directivity and the Cassegrain system will no longer work. Between these extremes there must exist a minimum aperture for which a Cassegrain system is satisfactory.

One of the problems with a Cassegrain fed antenna is blocking of the main aperture by the secondary reflector. Apart from the loss of power, this produces an increase in side lobe level which may be detrimental, particularly to any type of tracking antenna. The effect of this is illustrated in Fig. 23. If it is assumed that the minimum acceptable level for the first side lobe is 20 db below the main beam then the maximum size of the secondary aperture is about one eighth

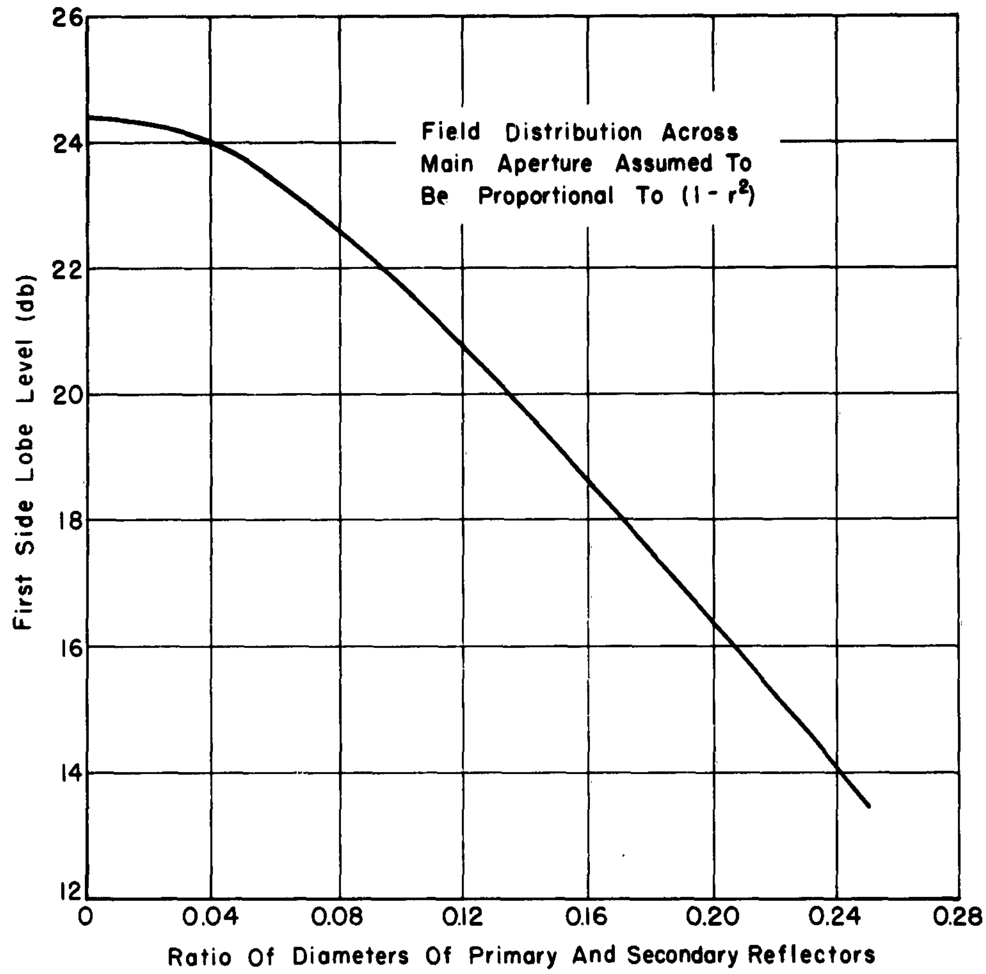

Fig. 23. Effect of aperture blocking on side lobe level in a Cassegrain antenna.

of the main aperture. Since the main aperture must be at least eight times the diameter of the secondary reflector the minimum antenna size is determined by the minimum size of the secondary reflector. For the simple analysis presented above the secondary reflector must be large enough to behave, approximately at least, as an optical reflector. This requires a diameter of several wavelengths, probably five at least. This would imply a minimum main aperture of forty wavelengths for a practical Cassegrain system.

The main use of a simple Cassegrain feed would be to reduce noise by reducing feedline loss and spillover. However, even if the noise can be made to approach the ideal case, the gain of any practical paraboloid still falls short of the ideal because of the illumination taper. This stems from two causes, namely taper of the primary feed pattern and the fact that the edges of the aperture are further from the focal point than the center. A special type of Cassegrain system could possibly invert one of these effects so that the two tend to compensate each other. Such a system is illustrated in Fig. 24.

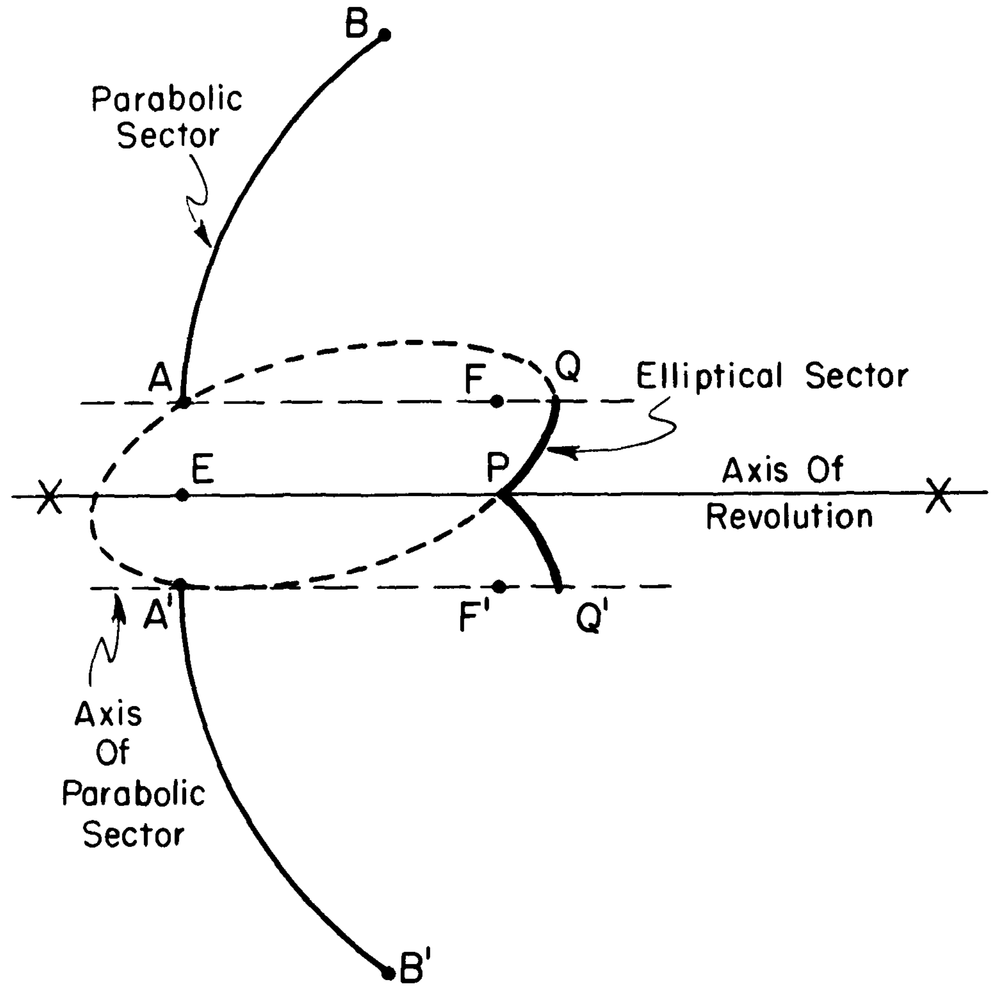

Fig. 24. Inverted Cassegrain feed.

AB represents the arc of a parabola with apex at A and focus at F. The main reflector is formed by rotating the arc about the axis XX, parallel to AF, to form a surface of which the lower half is represented by A'B'. The focal point F generates a ring represented in the figure by the points F and F'. Next consider an ellipse with foci at E and F. A section PQ of this ellipse, if illuminated by a source at E, will reflect the illumination onto AB. If PQ is now also rotated about XX it will form a cusp represented by QPQ'. If this system is now used as a Cassegrain antenna with a source at E having an intensity maximum along the axis XX, the illumination from the center of the feed will be directed to the edge of the main reflector and that from the edge of the feed to the center of the main reflector. By suitable adjustment of the parameters it is possible that a more nearly uniform illumination might be obtained.

CONCLUSIONS

It has been shown that, even though a feed-horn pattern may be approximated by $\cos \frac{\pi}{\alpha} \frac{\theta}{2}$ for calculating the gain of a parabolic reflector, this approximation is totally inadequate for calculating the thermal noise characteristics of such an antenna.

In addition, it has been shown that the gain approximations for a parabolic reflector antenna, presented in a previous report, are quite adequate for most purposes except for cases in which the reflector subtends angles close to 180°.

It has been shown that the maximum gain-to-noise ratio is achieved in practice when the null of the cosine curve which approximates the feed horn pattern nearly coincides with the edge of the reflector.

Calculations of thermal noise received by a practical horn-fed parabolic reflector indicate that the gain-to-noise ratio that can be expected from such an antenna is of the order of 9 db less than ideal.

Methods of decreasing the received thermal noise, and hence increasing the gain-to-noise ratio, by shielding were presented.

Consideration of Cassegrain type antennas for low-noise antenna applications was presented. Specifically it was shown that Cassegrain feeds are advantageous only for large antennas, with diameters of the order of forty wavelengths or more. A novel type of Cassegrain feed was discussed as a possible means of obtaining more desirable illumination of the main aperture.

RECOMMENDATIONS

It is recommended that the low-noise designs of horn-fed parabolic reflectors be checked by means of radiometer measurements on experimental models.

It is recommended that a study of the low-noise capabilities of Cassegrain antennas be studied in much the same manner as that of the horn-fed reflector described in this and the preceding report.[1]

BIBLIOGRAPHY

1. Caldecott, Ross, "Investigation of Low Noise Antenna Systems" Report 1041-1, 1 December 1960, Antenna Laboratory, The Ohio State University Research Foundation; prepared under Contract AF 19(604)-6134, Air Force Command and Control Development Division, Air Research and Development Command, Laurence G. Hanscom Field, Bedford, Massachusetts.

2. Silver, S., Microwave Antenna Theory and Design, McGraw Hill Book Co., Inc., 1949.

3. Chen, S.N.C. and Peake, W.H., "Apparent Temperatures of Smooth and Rough Terrain," I.R.E. Transactions of Antennas and Propagation, Vol. AP-9, pp. 567-572, November 1961.

4. Peake, W.H., "The Interaction of Electromagnetic Radiation with Some Natural Surfaces," Report 898-2, 30 May 1959, Antenna Laboratory, The Ohio State University Research Foundation; prepared under Contract AF 33(616)-6158, Aeronautical Systems Division, Air Force Systems Command, United States Air Force, Wright-Patterson Air Force Base, Ohio.

5. Hannan, Peter W., "Microwave Antennas Derived from the Cassegrain Telescope," I.R.E. Transactions of Antennas and Propagation, Vol. AP-9, pp. 140-153, March 1961.