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WADD TECHNICAL REPORT 61-66

A STUDY TO CORRELATE FLIGHT MEASURED HELICOPTER VIBRATION DATA AND PILOT COMMENTS

WILLIAM J GRANT

VERTOL DIVISION
BOEING AIRPLANE COMPANY

AUGUST 1961

AERONAUTICAL SYSTEMS DIVISION

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A STUDY TO CORRELATE FLIGHT MEASURED HELICOPTER VIBRATION DATA AND PILOT COMMENTS, W. J. Grant, Dec. 1960. 55 p. incl. illus., WADD TR61-66; Contract No. AF33(616)-5240; Project No. 1370; Task No. 13749.

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II. Grant, W. J.

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Lissajous' patterns of resultant displacement, velocity, and acceleration are constructed and evaluated to define those characteristics which best correlate with the pertinent pilot comments. A new measure of comfort level, Equivalent

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Vibration Level (Veq) is defined. These quantities are calculated for all Lissajous' figures, and resultant acceleration is seen to be the most meaningful parameter. An improvement in the degree of correlation between measured vibration and pilot comment is shown through the use of Veq for the patterns of resultant acceleration in lieu of the standard vibration criteria.

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WILLIAM J. GRANT

VERTOL DIVISION
BOEING AIRPLANE COMPANY

AUGUST 1961

FLIGHT DYNAMICS LABORATORY
CONTRACT No. AF 33(616)-5840
PROJECT No. 1370
TASK No. 13740

AERONAUTICAL SYSTEMS DIVISION
AIR FORCE SYSTEMS COMMAND
UNITED STATES AIR FORCE
WRIGHT-PATTERSON AIR FORCE BASE, OHIO

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FOREWORD

The study reported herein was initiated by the Flight Dynamics Laboratory, Aeronautical Systems Division, Wright-Patterson Air Force Base, Ohio, at the request of the U. S. Army Transportation Corps under MIPR-TRECOM-57-34, Department of the Army, Project No. 9-38-01-000, Task No. 302. The work was accomplished by the Vertol Division, Boeing Airplane Company, Morton, Pennsylvania, under Air Force Contract No. AF33(616)-5240, Project No. 1370, "Dynamic Problems in Flight Vehicles," and Task No. 13749, "Methods for Predicting Rotor Induced Helicopter Vibrations." Mr. Otto F. Maurer of the Vehicle-Kinetics Section, Dynamics Branch, Flight Dynamics Laboratory is task engineer on Task No. 13749. The study was initiated 7 April 1957 and is continuing. This report is Phase VII of the subject contract.

The valuable contributions of the Messrs. D. J. Smyers and H. Sternfeld to this project were especially helpful and greatly appreciated.

ABSTRACT

This report presents the results of a study aimed at improving the correlation between recorded helicopter vibration data and pilot comments.

Lissajous' patterns of resultant displacement, velocity, and acceleration are constructed and evaluated to define those characteristics which best correlate with the pertinent pilot comments. A new measure of comfort level, Equivalent Vibration Level (V_{eq}) is defined. These quantities are calculated for all Lissajous' figures, and resultant acceleration is seen to be the most meaningful parameter. An improvement in the degree of correlation between measured vibration and pilot comment is shown through the use of V_{eq} for the patterns of resultant acceleration, in lieu of the standard vibration criteria.

PUBLICATION REVIEW

This report has been reviewed and is published for the exchange and stimulation of ideas.

FOR THE COMMANDER

WILLIAM C. NIELSEN
Colonel, USAF
Chief, Flight Dynamics Laboratory

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I. INTRODUCTION

At the present writing there exists no substantiated vibration comfort criteria which includes the variables of harmonic content, phasing, and motion along more than one axis. Present procedures for determining the acceptability of helicopter vibration data consist of comparing measured harmonic amplitudes individually with the prescribed acceptance limits, and for each direction of motion separately.

Even the most experienced pilot sometimes finds difficulty in trying to distinguish in detail the direction and frequency of the highest amplitude components, and indeed cases have been observed where the pilot comment was "unsatisfactory" yet the measured data met the prescribed limits. The converse of this has also been observed, as have cases where two different flights displayed the same measured vibration characteristics yet the comment was "satisfactory" in one case and "unsatisfactory" in the other.

If such poor correlation exists between pilot comment and measured vibration where the data are considered highly reliable, then it follows that either the pilots' tolerance varies appreciably from flight to flight or indeed the measured data are not being analysed comprehensively enough with respect to all the variables which comprise the physiological impression sensed by the pilot. While the variation of pilot tolerance cannot be lightly dismissed, it is considered essential to first make a more comprehensive analysis and correlation of: (1) combinations of harmonics, (2) direction of motion, and (3) their respective phasing to pilot comment.

The study reported under this contract is believed to be the first specific attempt to evaluate such combinations. To accomplish this, existing recorded data and pilot comment were reviewed, certain cases selected and the data further analysed. The vertical and lateral components from each selected flight were then recombined to form patterns of resultant motion in the vertical-lateral plane. The objective of this study, of course, was to examine, (1)- the degree of correlation between these patterns, which contain all harmonics and phase relationships, and (2)- the corresponding pilot comment - with a view towards establishing a new comfort criteria that would bear a more realistic relationship between vibration measurements and the pilot's reaction.

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II. DATA ACQUISITION

During the course of production flight testing of the H-21 series helicopter, a large volume of cockpit floor vertical and lateral vibration levels was recorded. Approximately 350 aircraft of this type were involved and the data covers the entire airspeed range at normal rotor speed.

MB velocity pickups, located as shown in Figure 1, were used to record all vibration data. The output signals of these pickups were fed through integrating amplifiers and the resulting displacement traces recorded on the oscillograph. Figure 2 is an oscillograph record of the type used in this study. The analysed data was determined to be acceptable by comparing amplitudes of individual harmonics for each direction of motion with the limits prescribed in Reference 1.

Furthermore, it was required that the pilots accept or reject each aircraft on the basis of the vibrations encountered at the time these records were taken, irrespective of the measured data.

III. DATA SELECTION

The "cross-checking" of measured amplitudes and pilot comment on vibration acceptability over the complete speed range resulted in many combinations of pilot and data agreement and disagreement. For this study, several individual flights for which measured data and definite pilot comments existed, were selected at random and divided into five groups. These groups are:

1. The measured vibration level is acceptable and the pilot's comment is "acceptable."

2. The measured vibration level is unacceptable and the pilot's comment is "unacceptable."

3. The measured vibration level is acceptable and the pilot's comment is "unacceptable."

4. The measured vibration level is unacceptable and the pilot's comment is "acceptable."

5. Two flights, on the same helicopter, of apparently similar vibration level evoking conflicting pilot comment.

For this study, one example of group No. 5 is considered while three cases of groups 1 to 4 are included. In all cases, measured vertical and lateral data at the cockpit floor were analysed since it is felt that these modes would have the greatest influence on a pilot's opinion of aircraft vibration level.

IV. HARMONIC ANALYSES

At the time these vibrations were recorded, i.e. during production flight tests, a graphic harmonic analysis (Reference 2, pages 120 - 135) was performed and the individual harmonic amplitudes for both the vertical and lateral vibrations were calculated. No particular attempt was made to establish phase relationships.

The amplitudes were limited to the first three harmonics of rotor speed since in a three-bladed helicopter a pilot's comment is based largely on these frequencies, and indeed the human body is most sensitive in this frequency range (References 3 and 4). The entire data analysis operation was performed manually and the resulting amplitudes are those from which the helicopter's acceptability (by data) was determined. This graphical analysis was performed always using the most typically repetitive rotor cycle from the oscillograph records.

The re-analysis for this study used a more exact 24 ordinate digital analysis as described in Reference 5. Here both phase and a more exact amplitude determination were the objectives. It was performed on the same rotor cycle as was originally chosen and made use of an automatic digital computing machine. The data reading equipment consisted of a Benson-Lehner Model Oscar J data reader, Figure 3, incorporating an IBM Model 026 keypunch for automatic preparation of input cards. These cards were fed into an IBM 650 computer for the harmonic analysis.

This program consisted essentially of a curve-fitting process, matching the curve at a fixed number of points by the classical Fourier expansion:

$$f(t) = \sum_{n=1}^{K} a_n \sin n\omega t + \sum_{n=0}^{K} b_n \cos n\omega t \qquad (1)$$

A simultaneous equation solution is performed for the coefficients of the sine and cosine terms utilizing a minimum of (2K + 1) ordinates in order to provide (2K + 1) equations in (2K + 1) unknowns, where K is the highest harmonic number component sought. Since harmonics as high as 10 and 11 were observed in some cases, a 24 ordinate analysis was selected. This procedure prevents unaccounted-for higher harmonics from being attributed, erroneously to the lower orders which are of primary interest to the program

The sine and cosine coefficients were then restated in terms of resultant and phase angle and since, as in the production check for acceptance, harmonics only up to the third are considered in this investigation, Equation 1 is rewritten for the correlation effort as:

$$f(t) = a_1\cos(\omega t + \Phi_1) + a_2\cos(2\omega t + \Phi_2) + a_3\cos(3\omega t + \Phi_3) \quad (2)$$

Where $a_n \triangleq$ amplitude of n^{th} harmonic

$\phi_n \triangleq$ phase angle of n^{th} harmonic

V. CALIBRATION

The ideal transducer for vibration measurement would have an output whose amplitude and phasing is constant with frequency as shown in Figure 4. The standard velocity pickup circuit as employed by VERTOL has the nominal frequency characteristics shown in Figure 5. The fact that amplitude and phase of pick-up output are a function of frequency results in the recording of a complex waveform which differs considerably from the actual motion sensed by the transducer. This must not be overlooked or a misleading interpretation will follow. The recorded waveform, therefore, must be corrected for frequency effects on phase and amplitude.

A. Phase Calibration

In order to determine the amount of the instrumentation phase shift, three identical recording systems consisting of MB velocity type pickups, integrating amplifiers, galvanometers, and recording oscillograph of the same type used in acquiring the original data were phase calibrated. This required that histories of the system output and actual shaker table motion on the same time base be obtained The record of table motion was obtained by recording a signal from the shaker control oscillator to the shaker armature on an oscillograph. (Figure 6).

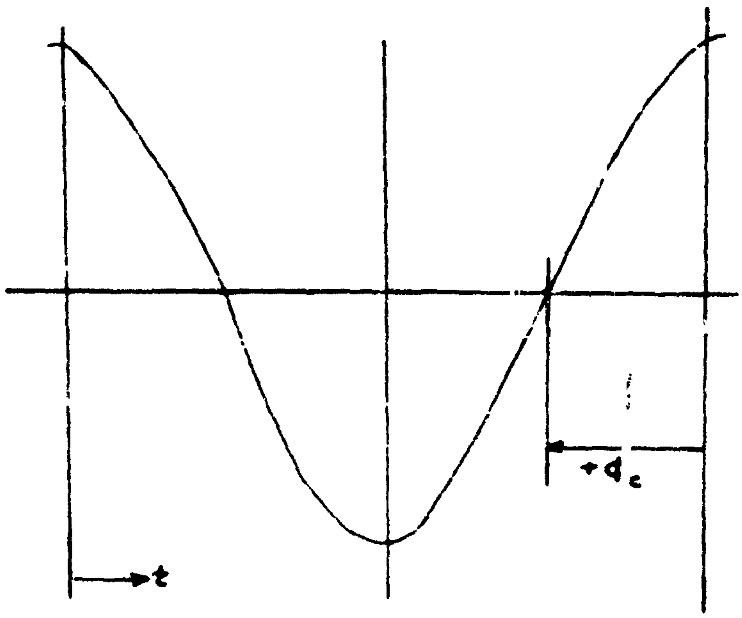
Since the oscillator signal was to be used as the reference for phase calibration it was first necessary to determine the change in phase between the shaker table motion and this signal. To do this a strain gage was mounted on the table support and analog recordings of the voltage from the oscillator and strain gage bridge were obtained as shown in Figure 6.

The MB model C-1 shaker table used for this calibration is an electro-magnetic type in which the table is attached to the armature which is subjected to a constant field. The velocity of the shaker armature and, therefore, the table is proportional to the armature current and, therefore, to the input voltage from the oscillator. The signal from the oscillator thus should lead, by 90°, the displacement signal from the strain gage.

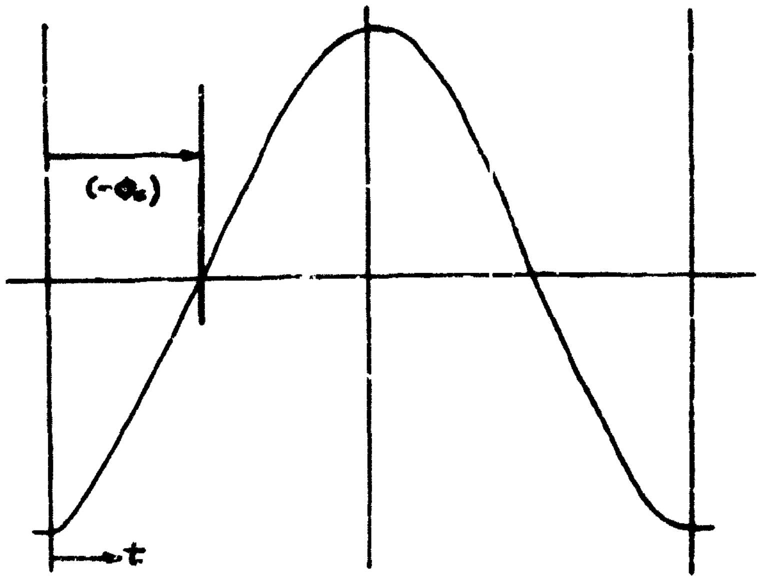
The phase difference of the two signals was checked over the low frequency range of prime interest in this study, and was, in fact, found to be 90°. By this means, the validity of the oscillator signal as a reference for the MB pick-up velocity was proven.

The recorded phase difference between the table motion and the system output, as determined in Figure 7, is thus the total phase shift of the recording systems. Test 2 was performed for all three recording systems; the results for the three systems are given in Figures 8, 9, and 10. The calibration as averaged and used in this study is presented in Figure 11.

The sign convention used in this study for phasing was one in which positive ϕc was read as that angular increment from the zero time reference to the first positive point of inflection of the sign wave to the left of the zero time reference, i.e., in the direction contrary to increasing time.



If Φc is negative, it must be read as that increment of time from the zero reference to the first positive point of inflection to the right of the zero time reference, i.e., in the direction of increasing time.



With this convention in use, the appropriate sign for rephasing of the actual motion to the recorded motion (applying the Φc) would be plus (+) when the actual motion is lagging or occurring after the trace motion, and minus (-) when the actual motion is leading or occurring before the trace in the direction of increasing time.

B. Amplitude Calibration

In order to convert the recorded oscillograph traces to actual amplitudes, it was necessary to determine the sensitivity of output amplitude to frequency. To do this, MB's from each of the recording systems were mounted on the C-1 Shaker Table. The table was then excited at known amplitudes over a frequency range from 2 to 30 cps. Oscillograph records of MB output were taken at various points in the frequency range. These records were then compared to the table displacements, as measured by an MB Type OC-1 calibrated microscope, thus establishing the amplitude sensitivity of the various pickups.

A total of 6 pickups were calibrated in this manner with the resulting scatter band in sensitivity being less than 5% (Figure 12). This close agreement from MB to MB allowed a mean or average curve to be drawn and applied to all pickups. Figure 13 is such a curve and was used for this study.

VI. REANALYSIS OF DATA

A. Correction and Resynthesis of Recorded Motion

As mentioned in the preceding section certain amplitude and phase corrections must be made to Equation (2). Section V describes the determination of these correction factors. Letting k_n = the amplitude conversion factor for the n^{th} harmonic and Φ_{cn} = the phase angle correction for the n^{th} harmonic, the program proceeded to apply these corrections to equation (2) as follows:

$$f(t) = k_1 a_1 \cos(\omega t + \Phi_1 + \Phi c_1) + k_2 a_2 \cos(2\omega t + \Phi_2 + \Phi c_2)$$
$$+ k_3 a_3 \cos(3\omega t + \Phi_3 + \Phi c_3) \qquad (3)$$

The resulting amplitudes and phase angles are then those of the actual motion as sensed by the pilot. Special care was taken to assure consistency in the sign convention for phase angle corrections as described in Section (V-A.)

Having determined actual harmonic amplitudes and phasing, equations (4) and (5) represent the trace excursions which would have been recorded had it been possible to measure vibratory motion with no amplitude attenuation or phase shifts due to the instrumentation.

For vertical motion:

$$x = A_{1x}\cos(\omega t + \Phi_{1x}) + A_{2x}\cos(2\omega t + \Phi_{2x}) + A_{3x}\cos(3\omega t + \Phi_{3x}) \qquad (4)$$

For lateral motion:

$$y = A_{1y}\cos(\omega t + \Phi_{1y}) + A_{2y}\cos(2\omega t + \Phi_{2y}) + A_{3y}\cos(3\omega t + \Phi_{3y}) \qquad (5)$$

Where $A_{ni} \triangleq$ Actual amplitudes in the nth harmonic, ith direction

$\Phi_{ni} \triangleq$ Actual phase angles for the nth harmonic, ith direction

A vectored combination of Equations (4) and (5) would then give the resultant planar motion of a point in space. In order to investigate the resultant velocity and acceleration of such a point, the program included conversion of the above expressions to velocities and accelerations as indicated in Equations (6) and (7).

For velocity:

$$\overset{\circ}{x}_i = \omega A_{1_i} \cos(\omega t + \phi_1 + \pi/2) + 2\omega A_{2_i} \cos(2\omega t + \phi_2 + \pi/2) + 3\omega A_{3_i} \cos(3\omega t + \phi_3 + \pi/2) \qquad (6)$$

and for acceleration:

$$\overset{\circ\circ}{x}_i = \omega^2 A_{1_i} \cos(\omega t + \phi_1 + \pi) + 4\omega^2 A_{2_i} \cos(2\omega t + \phi_2 + \pi) + 9\omega^2 A_{3_i} \cos(3\omega t + \phi_3 + \pi) \qquad (7)$$

B. Evaluation and Plotting of Corrected Wave Forms

It was necessary to numerically evaluate Equations (4) through (7) preliminary to plotting the desired Lissajous' figures. This was accomplished, as a subroutine to the harmonic analysis program, using 10^o increments of the azimuth angle, ωt, between the limits of 0^o and 360^o. The resulting values for x, y, $\overset{\circ}{x}$, $\overset{\circ}{y}$, $\overset{\circ\circ}{x}$, and $\overset{\circ\circ}{y}$ are then, indeed, the ordinates of the desired wave forms. The ordinates are then plotted in sequential time order by use of an Electronic Associates Model 3033-A-2 automatic data plotter.

This plotter accepts digital input from IBM cards and, after converting these data to the analog equivalent, it produces an x-y graphical representation of the digital information.

By definition, a Lissajous' figure is produced by the motion of a point whose plane Cartesian co-ordinates both vary periodically. Since the output of the data plotter is in this form it may be and is herein referred to as Lissajous' patterns of motion of a point in space.

Considering motion in a transverse plane, the reference axis is shown in Figure 14, where x is taken as vertical and y as horizontal. If the cockpit floor was subjected to a vertical vibration only, at a frequency of one per rotor revolution the motion versus time and the projection on the x axis would be as shown in Figure 15. If the cockpit was subjected to simultaneous vertical and lateral vibrations which were in phase with each other, as shown versus time in Figure 16, the resultant or Lissajous' pattern of the two would be the straight line constructed by their projections in the same figure. Presume further that a 90° phase difference be added to the combined vibrations, then Figure 16 would be altered to a circle as in Figure 17.

These cases are, of course, of a basic nature in that they are all of equal frequency and amplitude and involve nothing more than a simple shift in phasing of the input wave forms for Figure 17. Noting the resulting change in the Lissajous' pattern for this shift, the effects of more complex phase shifts and combinations of frequencies on the figures can be visualized.

The preceding discussion describes the entire IBM program from analysis of the vibration records to the graphical representation of resultant displacements, velocities, and accelerations. Figure 18 is a flow diagram of this work as performed by the 650 computer.

VII. PRESENTATION AND DISCUSSION OF RESULTS

In order to seek out the best possible correlation between pilot comment and resultant displacement, velocity, and acceleration, the Lissajous' plots were divided into two groups with respect to pilot comment, i.e., acceptable and unacceptable. The number in the upper right hand corner of each plot identifies it with respect to the groups of data agreement with these comments (Figure 19).

A review of these plots was made in order to determine those characteristics which best define their acceptability and unacceptability to the pilot.

The first parameter investigated was the overall acceleration or amplitude of the figure. This was measured along the major axis of the indicated ellipse. As expected, the unacceptable cases display larger amplitudes but there are cases of acceptable comment which have amplitudes just as high, Plot No. 74 (Figure 37), or higher, Plot No. 70 (Figure 25), than any of the unacceptable data. The conclusion must then be that this parameter is not in itself the only definition of acceptability.

Examining the inclination of the Lissajous' figures of resultant acceleration and acceptable and unacceptable comments, indicates that a majority of the acceptable plots have slopes close to vertical or, in some cases, plots No. 64, 65, 66, and 70 (Figure 25), the figures are completely vertically oriented. This indicates that the apparent slope of the Lissajous' figure has to be included in any attempt to define the acceptability of a given Plot However, this parameter in itself would once again not be sufficient since there are exceptions, Plots No. 81 and 82 (Figure 31).

Eccentricity, which is a function of the ratio of the minor axis of the indicated ellipse to the major axis was also investigated. A majority of the unacceptable figures appear wider than the acceptable cases along their minor axes, Plots No 72, 73 75, 76, and 78 (Figures 26, 38, and 32 respectively) versus 64, 65, 68, 71, 74, and 84 (Figures 25, 37, and 31 respectively). This indicates the need for including eccentricity in determining the acceptability or unacceptability of a given Lissajous' pattern. The eccentricity is determined by

$$e = \sqrt{1 - \frac{m^2}{M^2}}$$

Where $M \triangleq$ length of semi-major axis

$m \triangleq$ length of semi-minor axis

An equivalent vibration level V_{eq} which would best define these figures would then contain these three parameters. A simplified means of arriving at a V_{eq} of this nature would be to combine eccentricity and slope into one parameter. Such a combination is given by the ratio of the peak lateral to the peak vertical acceleration for a given plot. If Lissajous patterns similar to Figure 20 were obtained, a measurement of the peak lateral and vertical components, clearly includes both eccentricity and slope.

Figure 20 represents three ellipses, two of equal eccentricity but different slopes, and two of equal slope but different eccentricity. The ratio of peak accelerations obviously cannot indicate which of the two parameters, eccentricity or slope, is the prevailing influence, but does show the combined effect.

By comparing accelerations which resulted from flights of acceptable pilot comment to those resulting from a condition of unacceptable comment, the presence of this characteristic, and indeed the importance of it in these patterns, can easily be seen. With the exception of Plots No. 80, 81, and 82 (Figure 31), which are of small amplitude, only one of the acceptable data displays a lateral to vertical ratio greater than 0.53 while 11 of the 12 unacceptable cases produce ratios which are higher. This shows that the ratio is important: the question is then raised how can this ratio be combined with the magnitude of acceleration also assumed to be important?

Since vibration level is commonly expressed in g, the proposed measure, "Equivalent Vibration Level," will be in these units. Therefore, a dimensionalizing factor must be applied to the y/x term to make all units compatable. The desired criteria would then be expressed as

$$V_{eq} = A + K\ y/x \qquad (8)$$

where $A \triangleq$ Resultant Acceleration

$Y \triangleq$ Peak Lateral Acceleration

$X \triangleq$ Peak Vertical Acceleration

$K \triangleq$ Dimensionalizing Constant

The value of the dimensionalizing constant was arrived at by considering the average values for the resultant acceleration and slope of the Lissajous' patterns, $\bar{A}$ and $(\overline{y/x})$ respectively.

$$K = \frac{\bar{A}}{(\overline{y/x})} \tag{9}$$

Since the values for unacceptable acceleration resultants and ratios have a great deal of scatter compared to acceptable values, the numerical value of K was determined using the average of the pilot acceptable resultant accelerations and lateral to vertical ratios. It was found to be 2.25 g/in/in for this study.

In addition, as a matter of interest similar averages were obtained using both acceptable and unacceptable data by pilot comment. Both these values for K were used in Equation 8 to calculate "Equivalent Vibration Levels" for all Lissajous' patterns.

Figure 21 shows the data obtained using the K factor derived from all the data. A lack of separation exists between a pilot acceptable and a pilot unacceptable V_{eq} in this plot. By comparing these data to those of Figure 22, where K was determined by using the acceptable data only, it is concluded that the large spread of the unacceptable accelerations and ratios is responsible for the lack of definition between a pilot acceptable and unacceptable V_{eq} in Figure 21, corroborating the original assumption.

A second limit on V_{eq} is suggested by comparisons of cases where there are resultant accelerations with equal or nearly equal amounts of vertical and lateral acceleration. An example of this condition is Plot No. 80 (Figure 31), where both the pilot comment and the original data analysis were acceptable. If the vertical acceleration (x) of Plot No. 80 was to become very small and indeed go to zero while the lateral acceleration was also small but finite, the y/x term would then go to infinity. The resulting V_{eq} would then be of a very unacceptable nature, whereas, the environment itself would be even more acceptable, to the pilot, than the original condition of Plot No. 80.

Therefore, from examination of Plots No. 80, 81, and 82, it appears that when the resultant acceleration is less than 0.4 g's the pilot will accept the vibration encountered regardless of the eccentricity or slope of the resulting Lissajous pattern. Equivalent vibration level is therefore best redefined as being limited:

$$V_{eq} = A \neq K\ y/x \text{ for } A > 0.4g$$

where $A \hat{=}$ Resultant Acceleration

$K \hat{=}$ Dimensionalizing Factor

$Y \hat{=}$ Peak Lateral Acceleration

$X \hat{=}$ Peak Vertical Acceleration

Equivalent vibration levels with this modification are presented in Figure 22. A grey area or overlap is present in this plot. This area of uncertainty must be accepted as that factor which is present in any analysis of human opinion. Figure 22 indicates that an "Equivalent Vibration Level" equal to or less than 1.5 is unquestionably acceptable, whereas a V_{eq} equal to or greater than 2 is seen to be unquestionably unacceptable. Those cases in between these two figures must then be termed "marginal." Comparing the V_{eq} to those cases where pilot comment and the standard vibration criteria were in disagreement, two cases of unacceptable comment but acceptable data, Plots No. 72 and 73 (Figures 26 and 38 respectively), are now shown to be unacceptable by data, and one case which was originally acceptable by pilot comment and unacceptable by data, Plot No. 69 (Figure 25), is now shown to be acceptable by use of the V_{eq}. Further, the four remaining cases of disagreement, Plots No. 70, 71, 66, and 67 (Figures 25 and 26 respectively), are all seen to be in the marginal area, whereas the original analysis termed the data either definitely acceptable or unacceptable, definitions which were in complete disagreement with the pilot's report, thus demonstrating the significant improvement in correlation achieved through the use of the V_{eq}.

Resultant displacement and velocity were also considered and "Equivalent Vibration Levels" were calculated in a similar fashion. Figures 23 and 24 present these data which show no clear definition between an acceptable pilot comment and an unacceptable comment. Thus the choice of resultant acceleration as the most significant parameter is further substantiated.

VIII. CONCLUSIONS AND RECOMMENDATIONS

The principle result of this analysis is the definition of an Equivalent Vibration Level (V_{eq}) for determining pilot acceptance or rejection of helicopter vibration.

Before any strict interpretation or application of this index can be made, certain qualifications must be considered. With the instrumentation located as shown in Figure 1, the resulting motions, correct as they are, do not necessarily represent the exact motion that prompts a pilot's comment. Further, the aircraft used for this study, the H-21C, incorporates a pilot's seat mounted such that is isolates 3rd rotor order in the vertical direction, thus the pilot himself experiences a slightly different motion. The probable influence of longitudinal modes is another factor not to be overlooked in a more exact determination of a three dimensional V_{eq}.

In this work, as in any analysis of human opinion, the amount of data considered is most important. While the flights used for this analysis represent clear cut examples of pilot comment on the vibrations encountered, their total number is indeed a minority However, a distinction between an acceptable and an unacceptable V_{eq}, is becoming apparent, even from this relatively small sampling. It is conceivable that a larger sampling would result in a much clearer distinction.

A fifth and final consideration is that this criteria was determined using data from one type of aircraft only It is probable that a large sampling of various aircraft would then result in a universal Equivalent Vibration Level.

To eliminate qualifications which must now be placed on the V_{eq}, because of the limitations discussed above, three avenues of investigation are open.

The first proposed study would be one in which the pilot's seat itself was instrumented. An analysis of the data resulting from such instrumentation would result in Lissajous' patterns of motion more closely related to the resulting pilot comment than those of Figures 25 to 38

Another possibility is a test program utilizing a vibrating seat with three degrees of freedom (vertical, lateral and longitudinal), and correlating forced motion data with the resulting comments. This program could allow for: (a) changing various parameters individually, e g. amplitude, phase: (b) determining whether vertical, lateral or longitudinal modes were most objectionable In this program accelerometers could be used in lieu of vibration pickups so that direct measurements may be used to calculate V_{eq}, rather than going through the harmonic analysis and resynthesis necessary with MB vibration data. An even more expedient method would use an electronic device producing, on a scope, actual Lissajous patterns which could be photographed and then measured for calculations of V_{eq}.

The final and, it seems, the most logical program would evaluate various types of aircraft and correlate the vibration data taken with pilot comments. This program would evaluate the influence of different seat configurations and other items, such as the reactions of pilots to reciprocating engine aircraft and turbine powered aircraft. Using the instrumentation recommended in the preceding paragraph, in conjunction with the analysis presented in this report, an evaluation of this type could, hopefully, result in a reliable, universally applicable, Equivalent Vibration Level.

REFERENCES

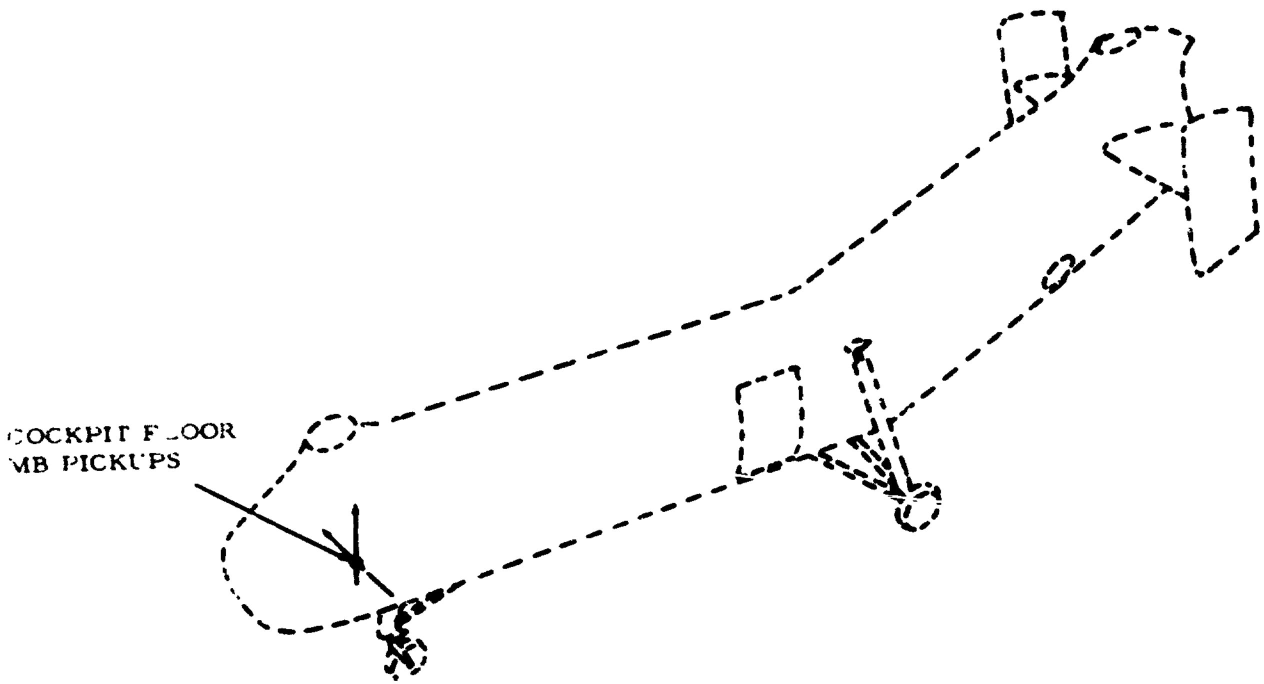
1. Vibration Acceptance Criteria, CCNR(5820)49 on Contract Number AF33(600)-26112

2. Manley, R. G.; "Waveform Analysis" John Wiley & Sons, Inc., New York, N. Y.: Chapman & Hall, Ltd., London, England 1945

3. Coermann, R. R., Ziegenruecker, G. H., Wittwer, A. L., Von Gierke, H. E.; "The Passive Dynamic Mechanical Properties of the Human Thorax-Abdomen System and of the Whole Body System." 30th annual meeting of the Aerospace Medical Association, April 27, 1959, Los Angeles, California.

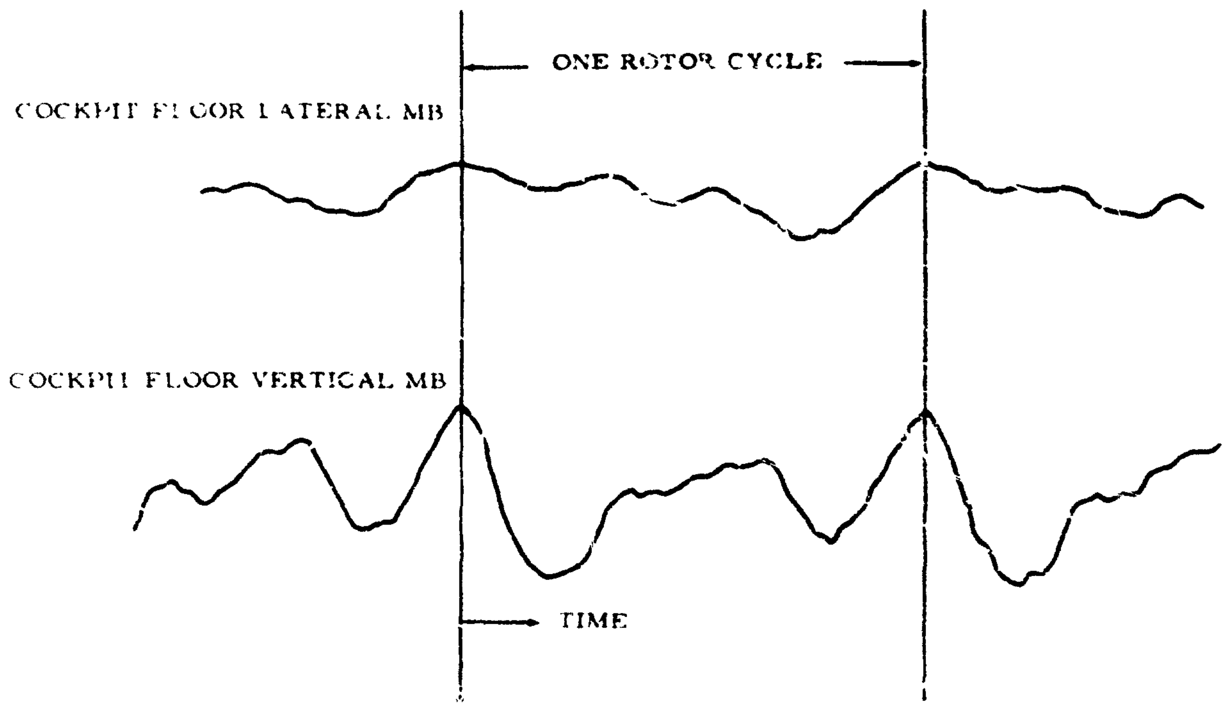
4. Schmitz, M. A., Simons, A. K.; "Man's Response to Low-Frequency Vibration" A.S.M.E. Paper No. 59-A-200 presented at the Annual Meeting, Atlantic City, N. J., November 29 - December 4, 1959

5. Wylie, C. R., Jr.; "Advanced Engineering Mathematics" McGraw-Hill Book Co., Inc., New York, N. Y. 1951



INSTRUMENTATION - H-21 PRODUCTION AIRCRAFT

FIGURE 1



TYPICAL OSCILLOGRAPH RECORD OF MB OUTPUT SIGNALS

FIGURE 2

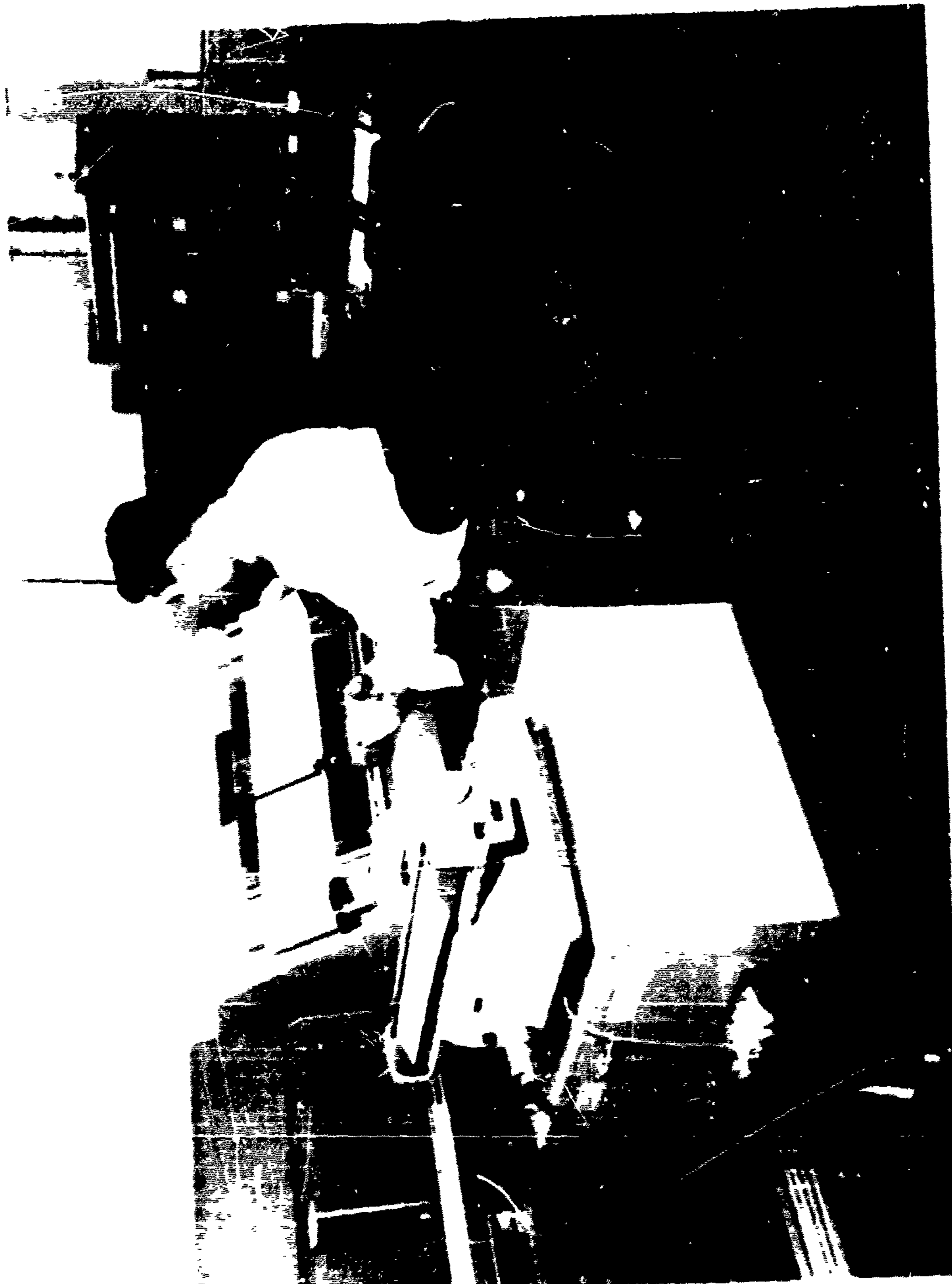
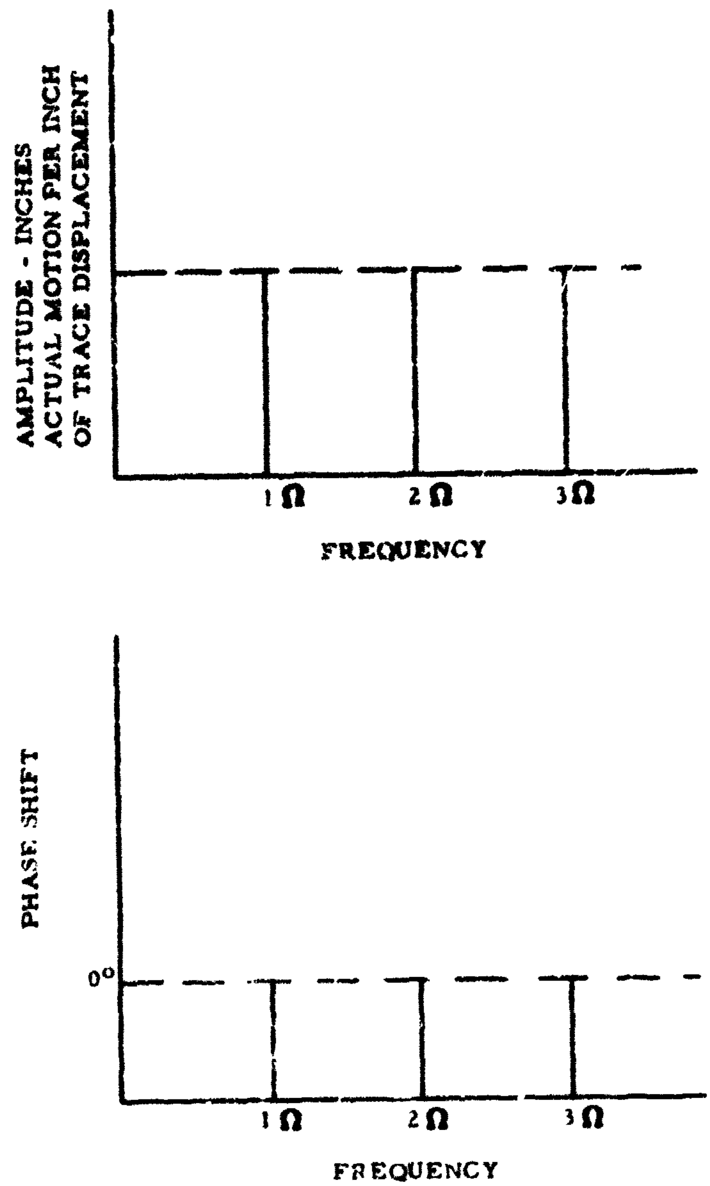
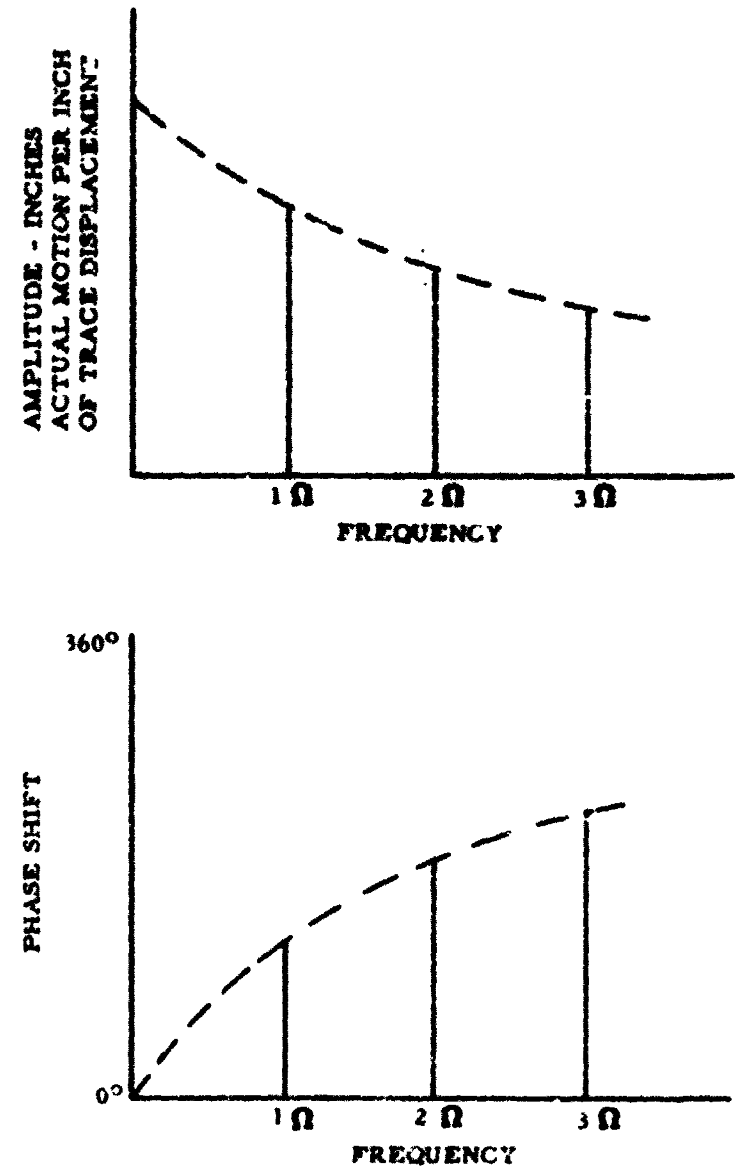


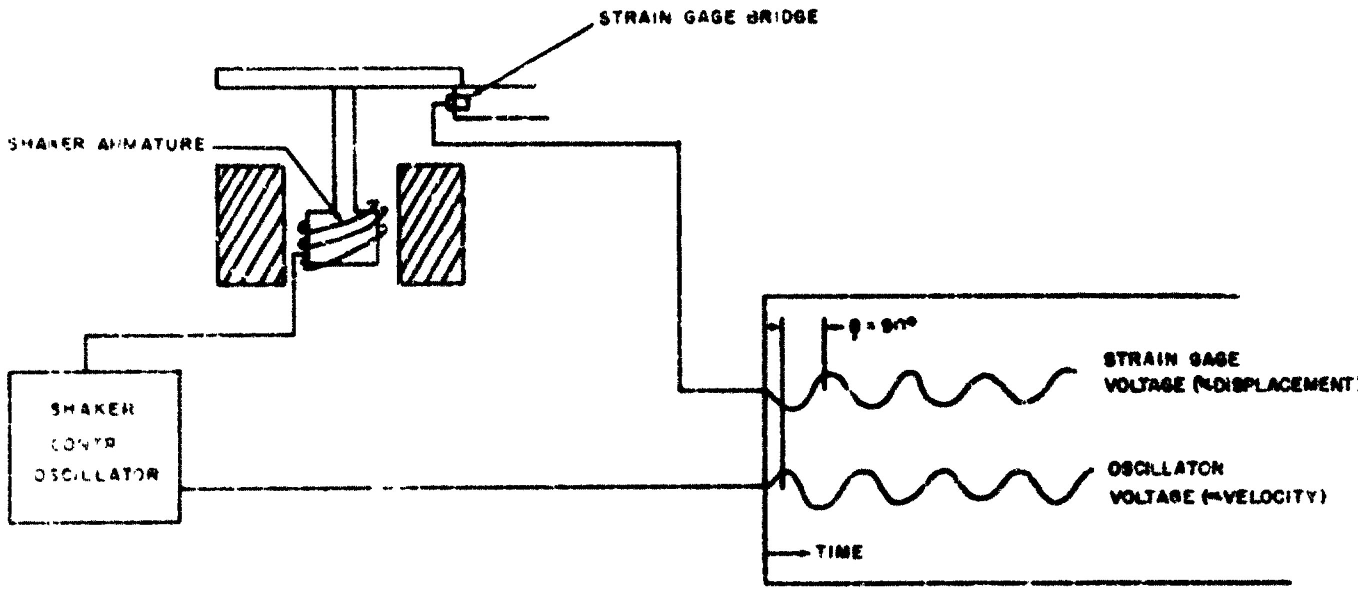
FIGURE 3 AUTOMATIC DATA READING EQUIPMENT



. MPLITUDE AND PHASE CHARACTERISTICS OF AN
IDEAL VIBRATION TRANSDUCER (CONSTANT INPUT)
FIGURE 4

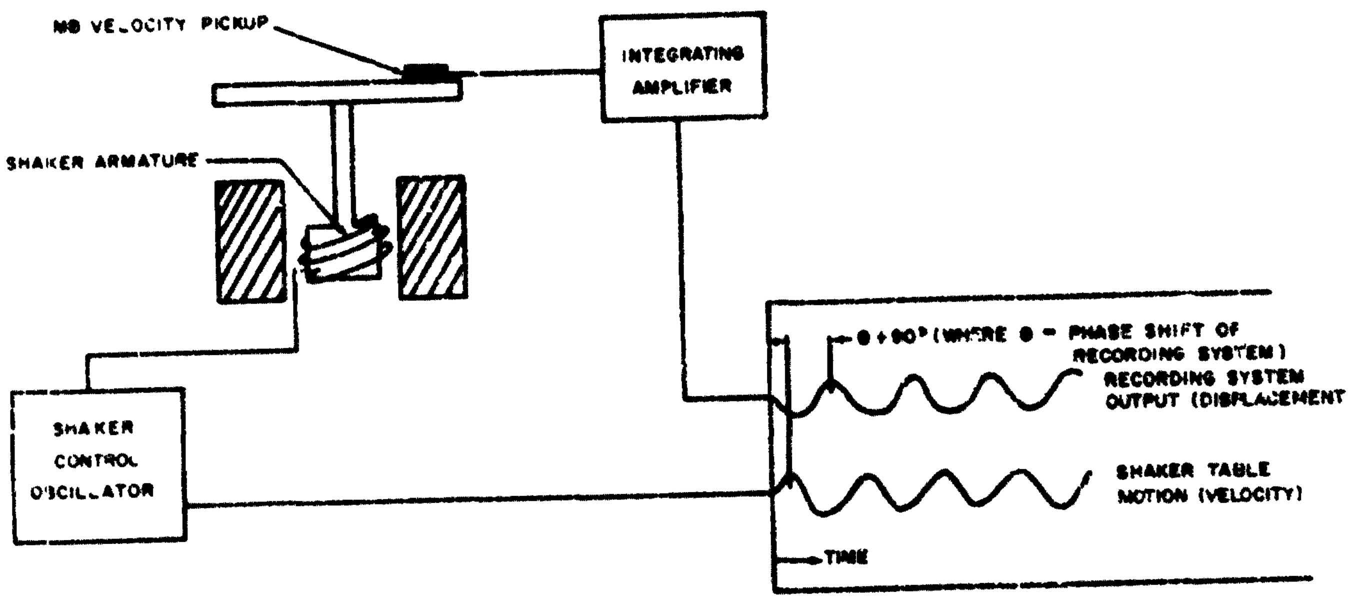


NOMINAL CHARACTERISTICS OF TYPICAL MB
PICKUP - INTEGRATING AMPLIFIER SYSTEM (CONSTANT INPUT)
FIGURE 5



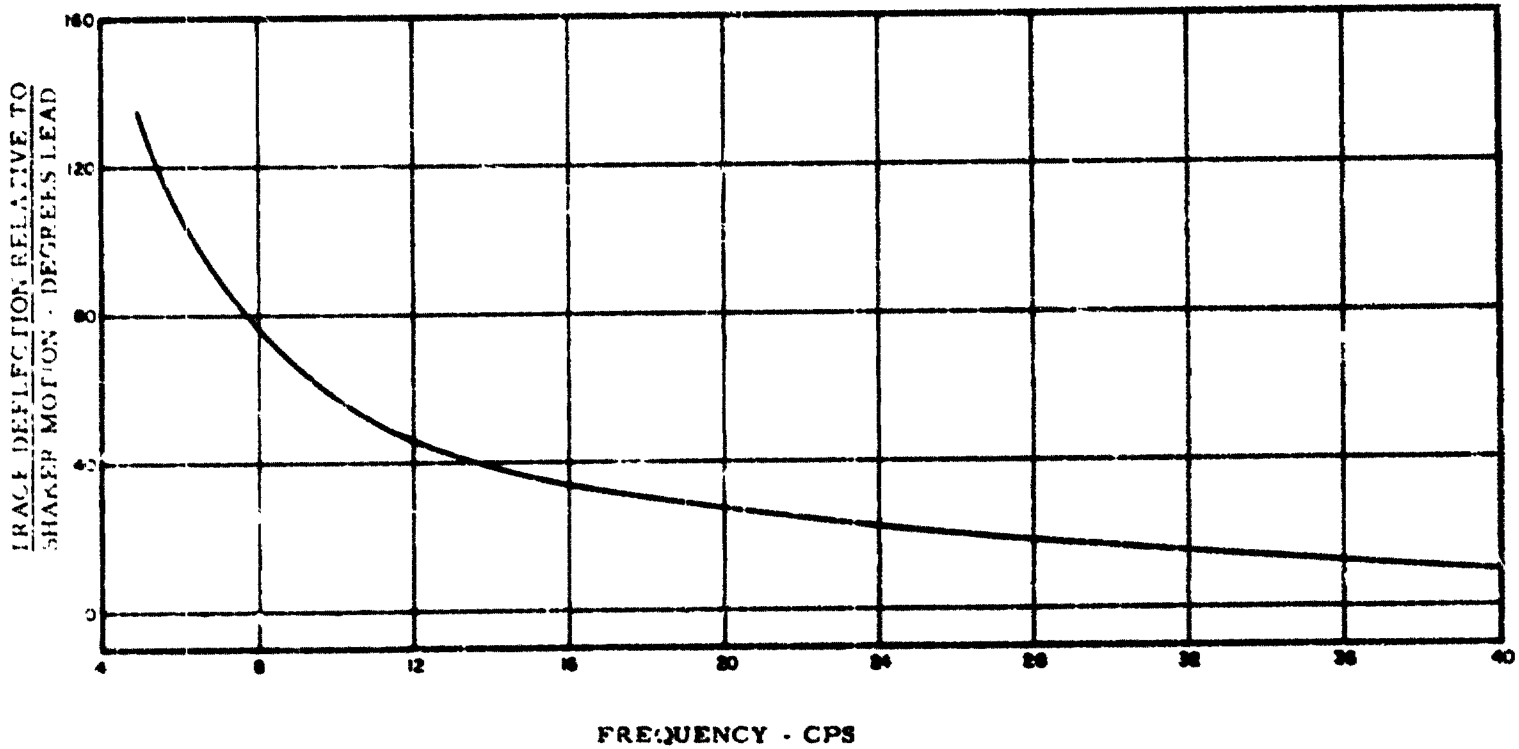
TEST 1 - PHASE CHECK OF SIGNAL GENERATOR

FIGURE 6

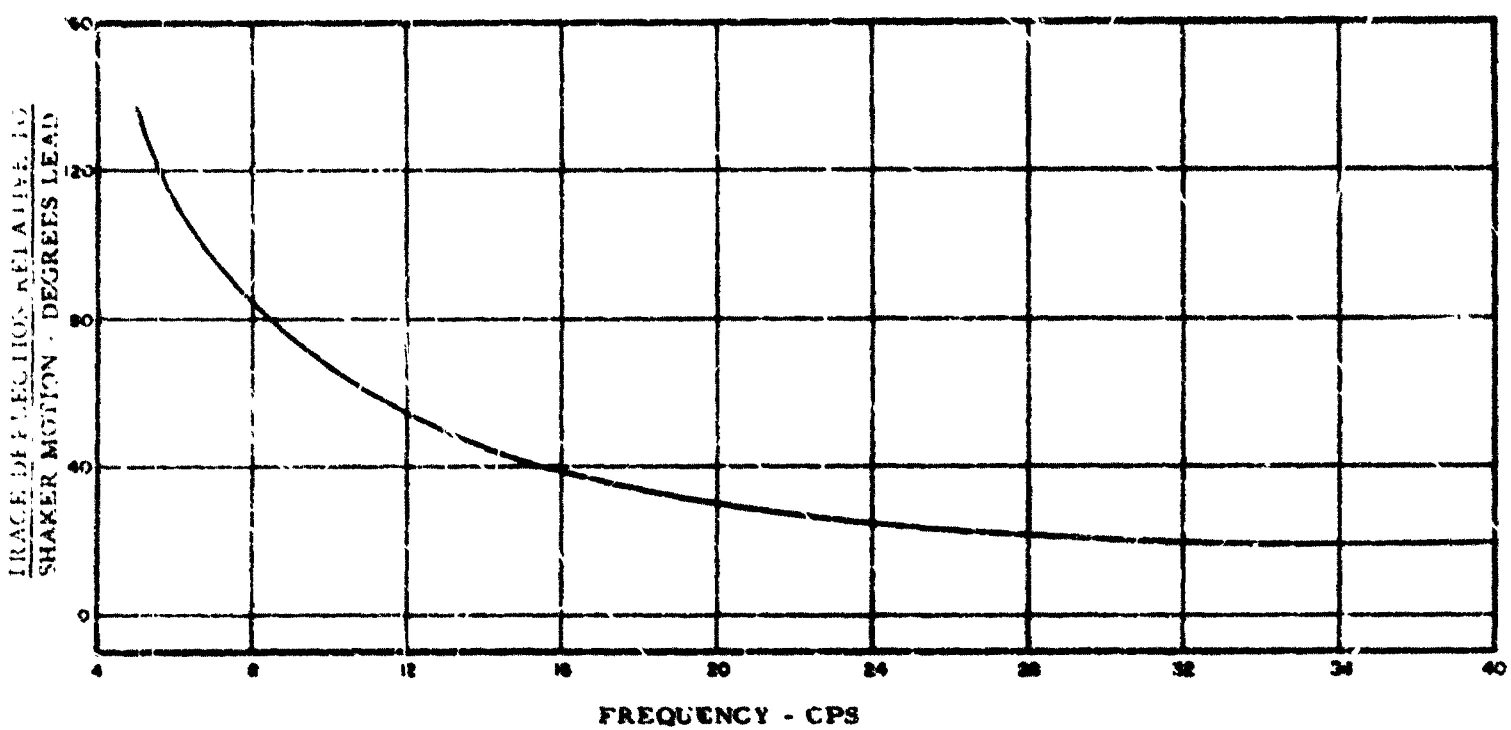


TEST 2 - PHASE CALIBRATION OF MB VELOCITY TYPE PICKUP

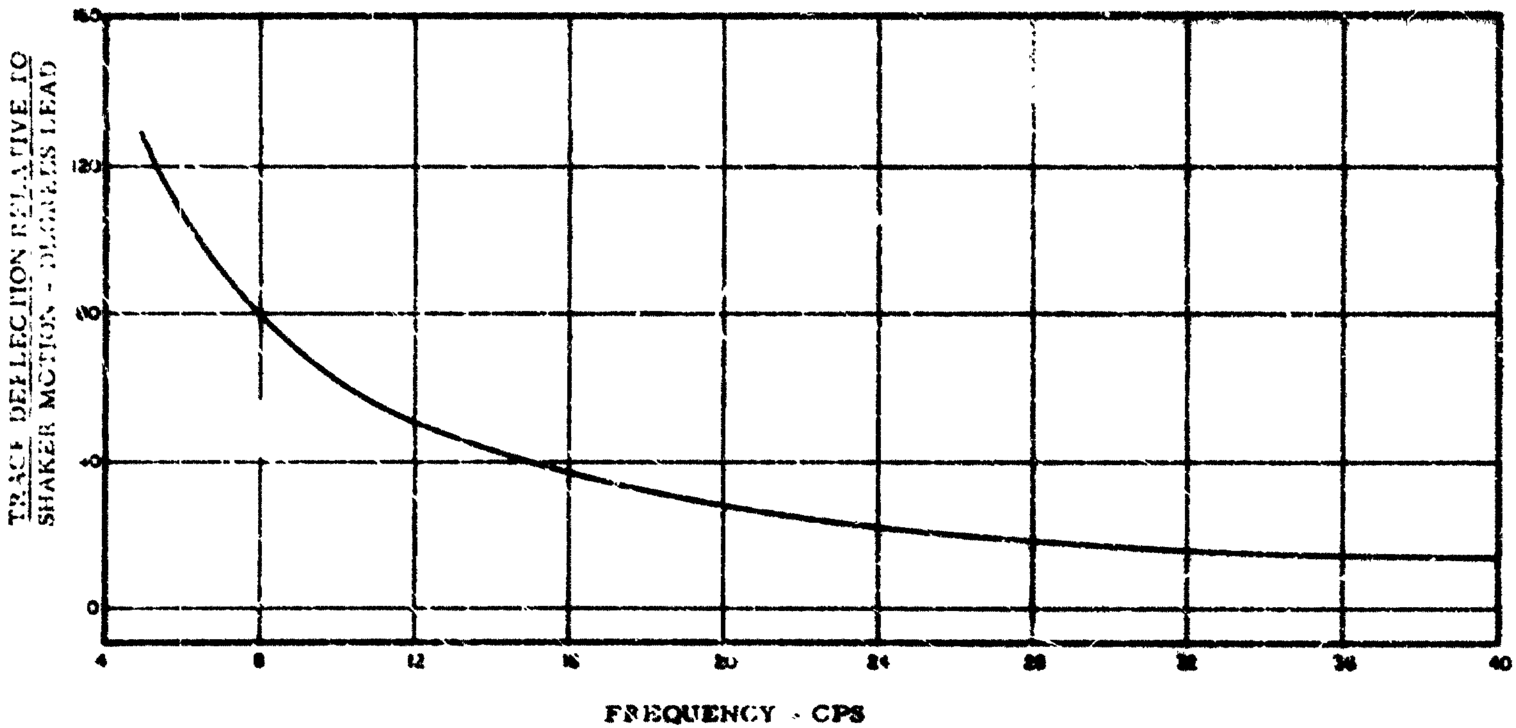
FIGURE 7



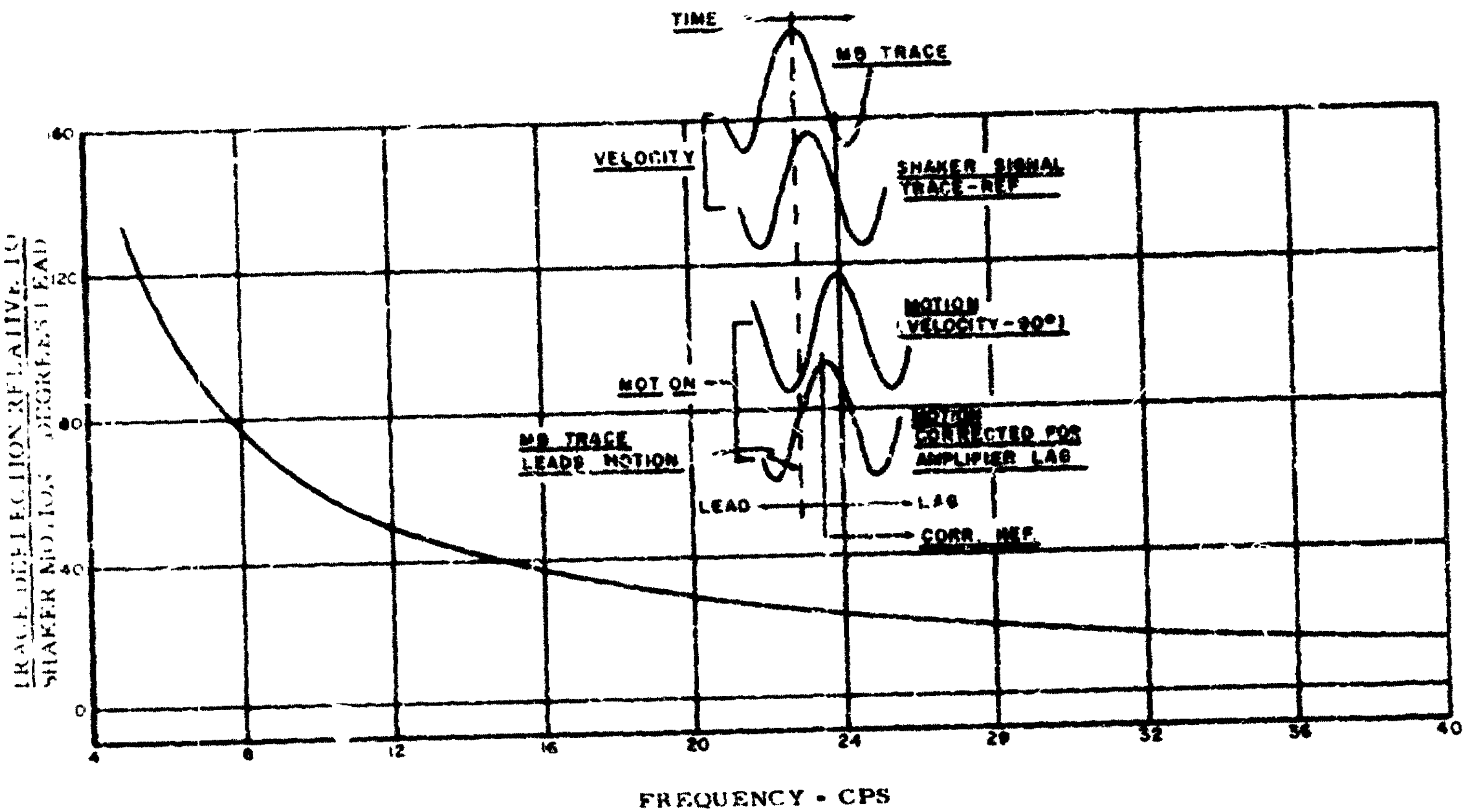
RECORDING SYSTEM PHASE CALIBRATION
"BREADBOARD A"
FIGURE 8



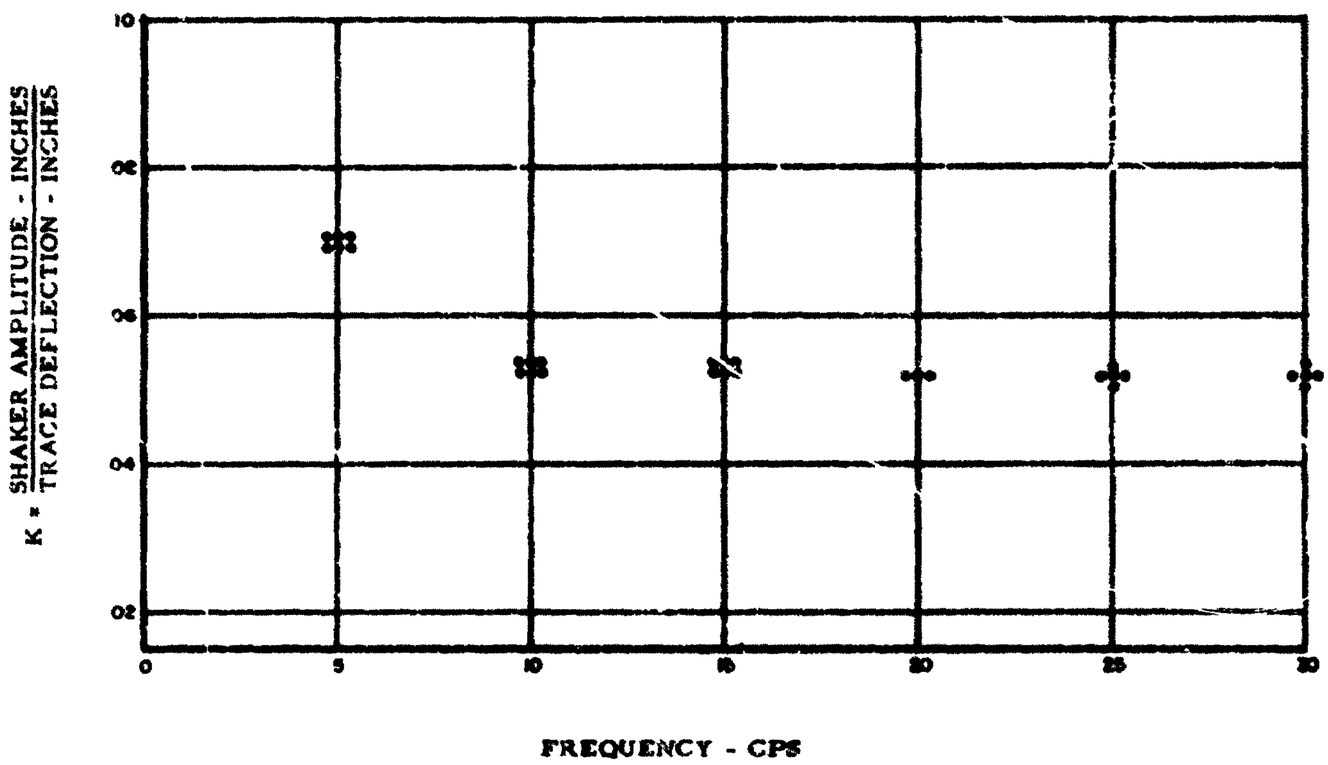
RECORDING SYSTEM PHASE CALIBRATION
"BREADBOARD B"
FIGURE 9



RECORDING SYSTEM PHASE CALIBRATION
"BREADBOARD F"
FIGURE 10

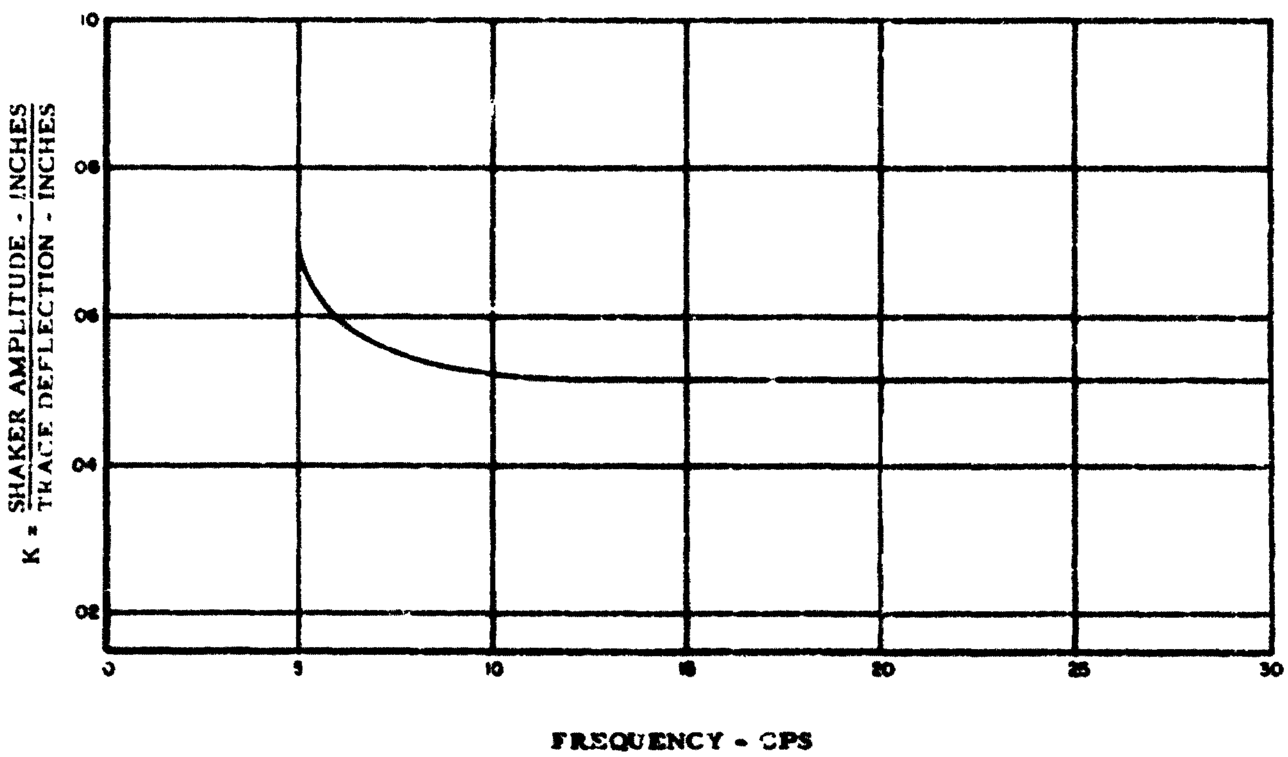


RECORDING SYSTEM PHASE CALIBRATION
(AVERAGE OF 3 RECORDING SYSTEMS)



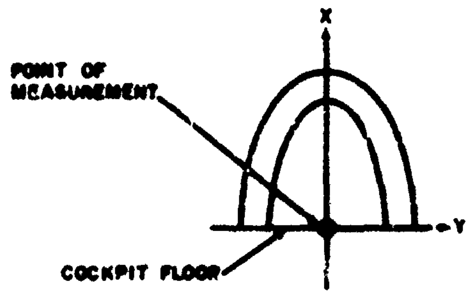
MB TYPE 124 VELOCITY PICKUPS - AMPLITUDE CALIBRATION
SCATTER OF RESULTS FOR SIX PICKUPS

FIGURE 12

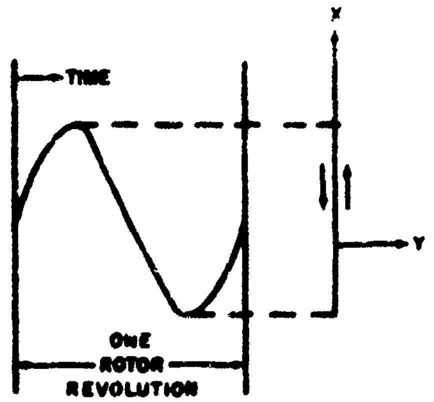


MB TYPE 124 VELOCITY PICKUPS - AMPLITUDE CALIBRATION
AVERAGE OF SIX PICKUPS

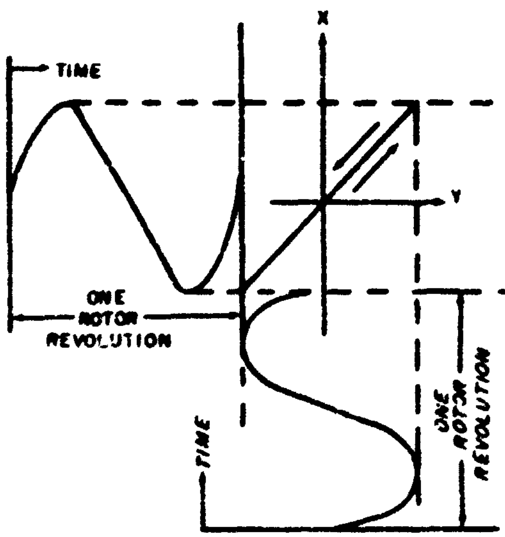
FIGURE 13



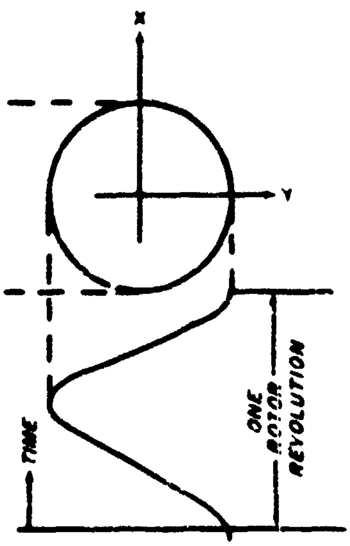
REFERENCE AXIS FOR VIBRATION MEASUREMENTS
FIGURE 14



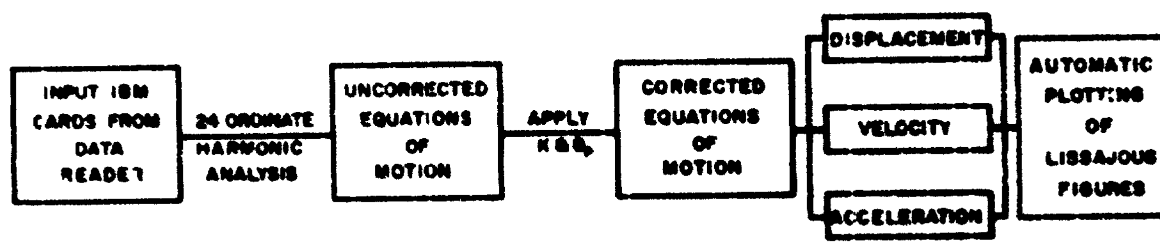
SAMPLE LISSAJOUS FIGURE
FIGURE 15



SAMPLE LISSAJOUS FIGURE
FIGURE 16



SAMPLE LISSAJOUS FIGURE
FIGURE 17



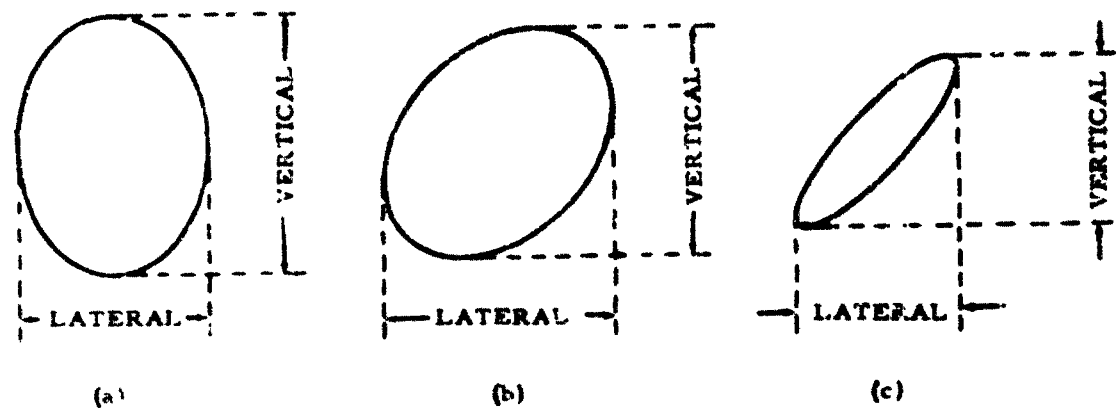
FLOW DIAGRAM - AUTOMATIC DATA ANALYSIS PROGRAM

FIGURE 18

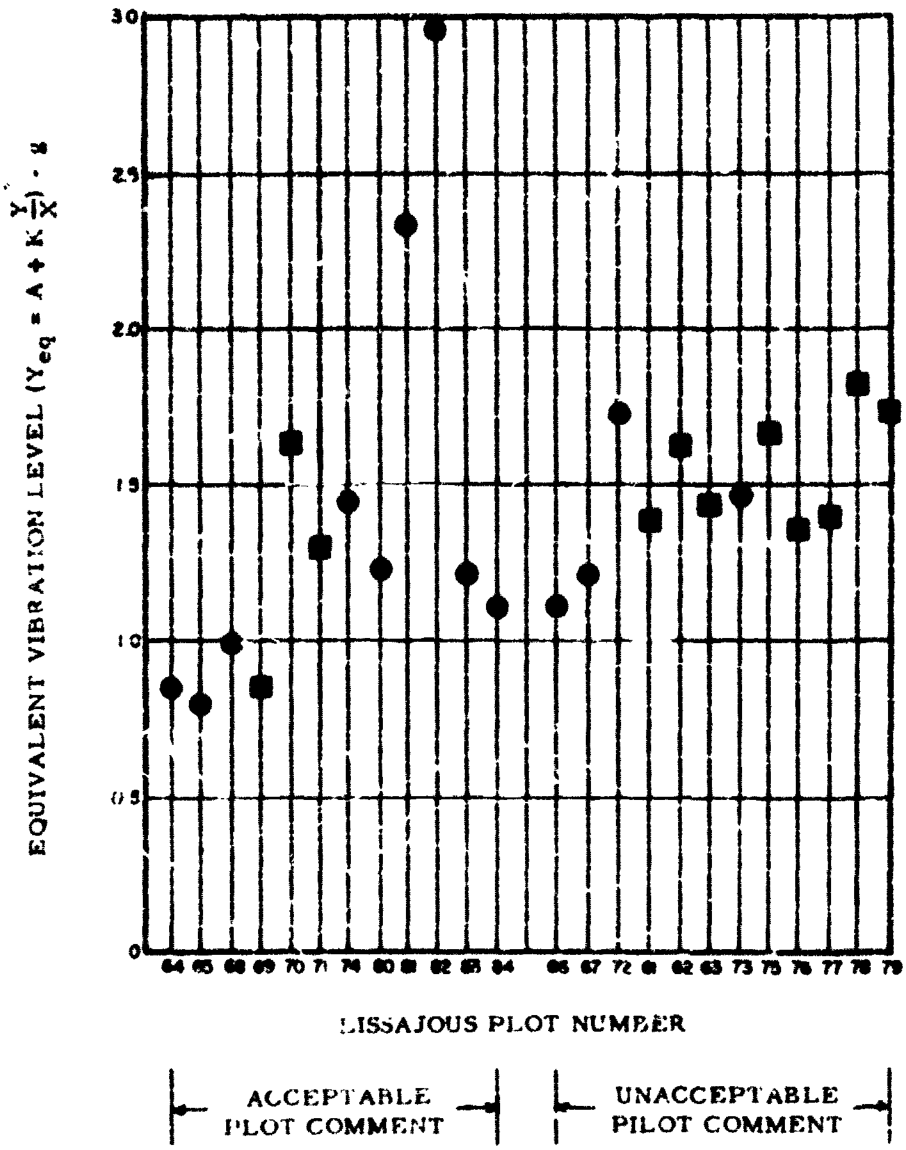
AIRCRAFT	PILOT COMMENT	DATA	DISPLACEMENT	VELOCITY	ACCELERATION
H-21C #177	Unacceptable	Unacceptable	1	31	61
H-21C #274	"	"	2	32	62
H-21C #274	"	"	3	33	63
H-21C #176	Acceptable	Acceptable	4	34	64
H-21C #177	"	"	5	35	65
H-21C #151	Unacceptable	Acceptable	6	36	66
H-21C #155	"	"	7	37	67
H-21C #305	Acceptable	Acceptable	8	38	68
H-21C #267	Acceptable	Unacceptable	9	39	69
H-21C #286	"	"	10	40	70
H-21C #142	"	"	11	41	71
H-21C #155	Unacceptable	Acceptable	12	42	72
H-21C #271	Unacceptable	Acceptable	13	43	73
H-21C #271	Acceptable	Acceptable	14	44	74
H-21C #274	Unacceptable	Unacceptable	15	45	75
			16	46	76
			17	47	77
			18	48	78
H-21C #274	Unacceptable	Unacceptable	19	49	79
H-21C #176	Acceptable	Acceptable	20	50	80
			21	51	81
			22	52	82
			23	53	83
H-21C #176	Acceptable	Acceptable	24	54	84

IDENTIFICATION OF LISSAJOUS FIGURES

FIGURE 19



SAMPLE LISSAJOUS FIGURES
FIGURE 20

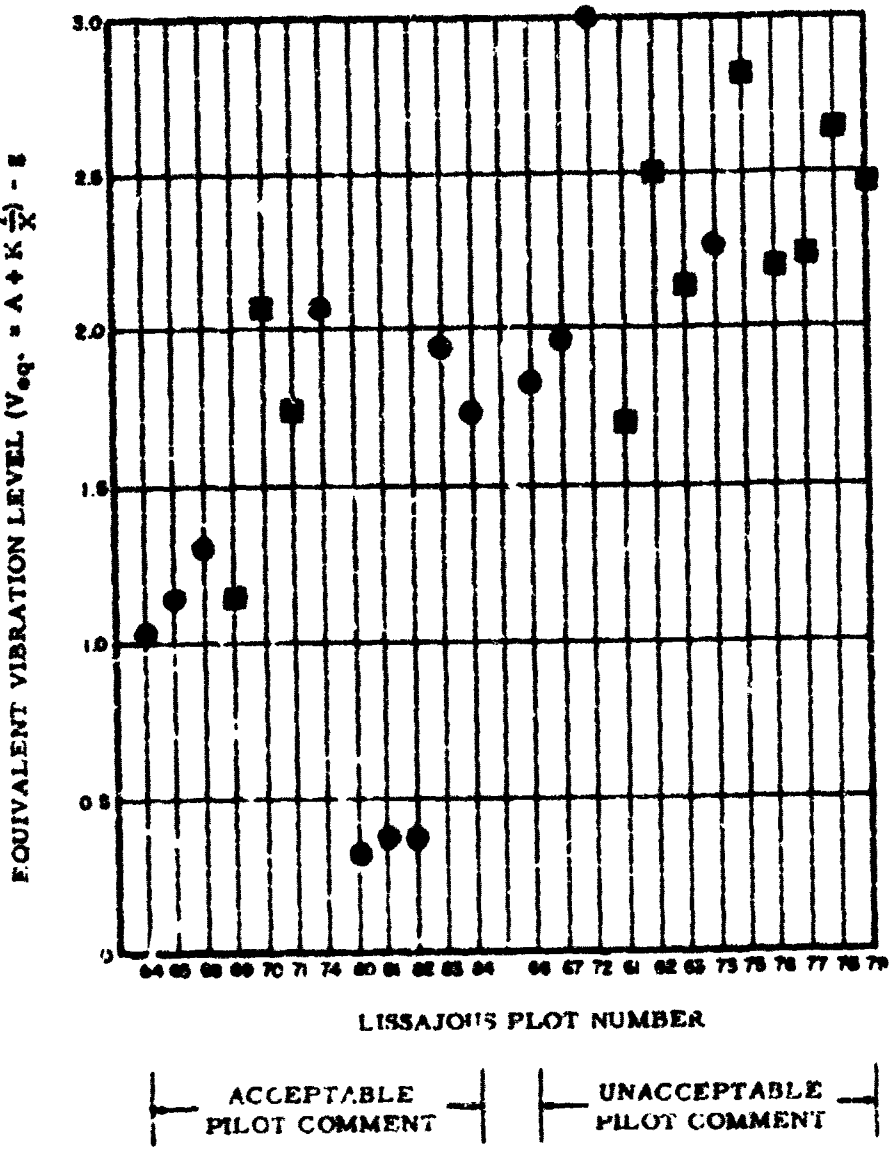


LEGEND ● = MEASURED DATA ACCEPTABLE BY STANDARD CRITERIA
■ = MEASURED DATA UNACCEPTABLE BY STANDARD CRITERIA

WHERE $\bar{A}$ & $\overline{(\frac{Y}{X})}$ = AVERAGE VALUES FOR ALL LISSAJOUS PLOTS

EQUIVALENT VIBRATION LEVELS - RESULTANT ACCELERATION $K = \frac{\bar{A}}{\overline{(\frac{Y}{X})}}$

FIGURE 21

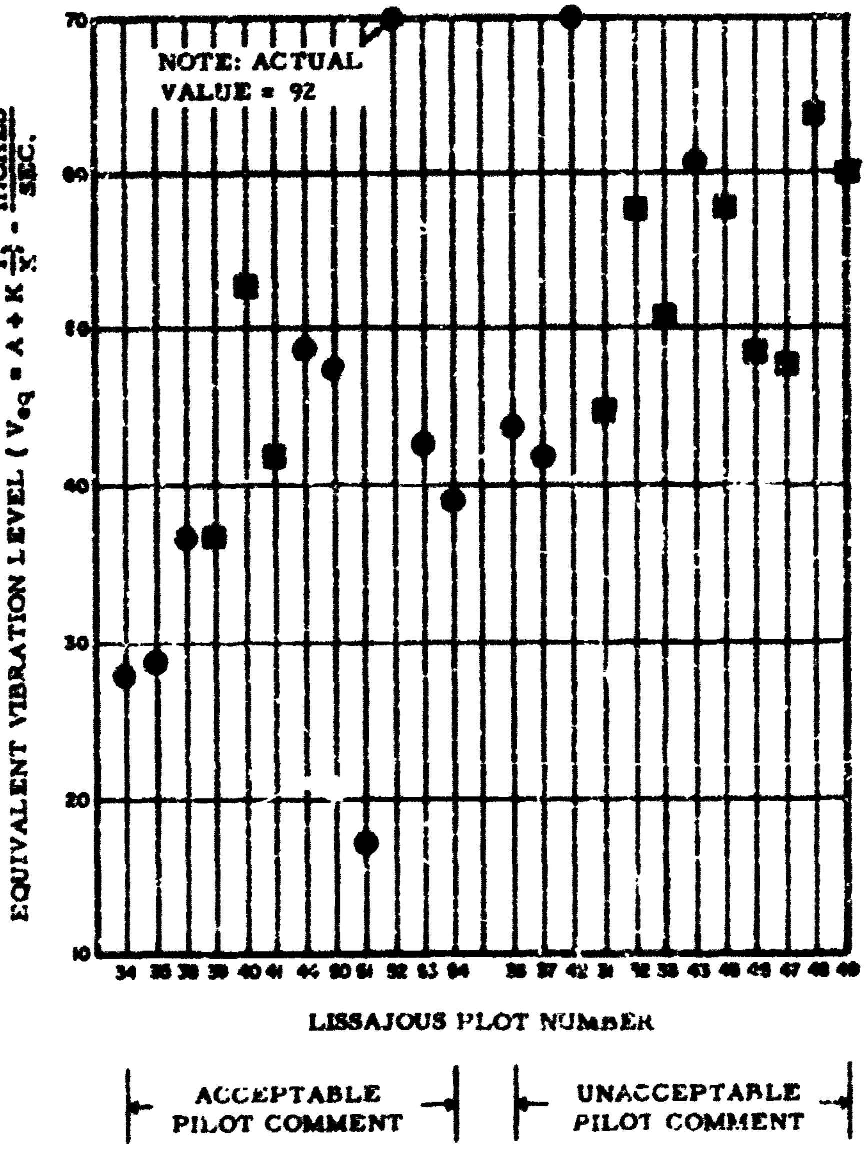


LEGEND: ● = MEASURED DATA ACCEPTABLE BY STANDARD CRITERIA
■ = MEASURED DATA UNACCEPTABLE BY STANDARD CRITERIA

WHERE $\bar{A}$ & $(\frac{\bar{Y}}{X})$ = AVERAGE FOR ACCEPTABLE LISSAJOUS PLOTS ONLY

EQUIVALENT VIBRATION LEVELS - RESULTANT ACCELERATION $K = \frac{\bar{A}}{(\frac{\bar{Y}}{X})}$

FIGURE 22

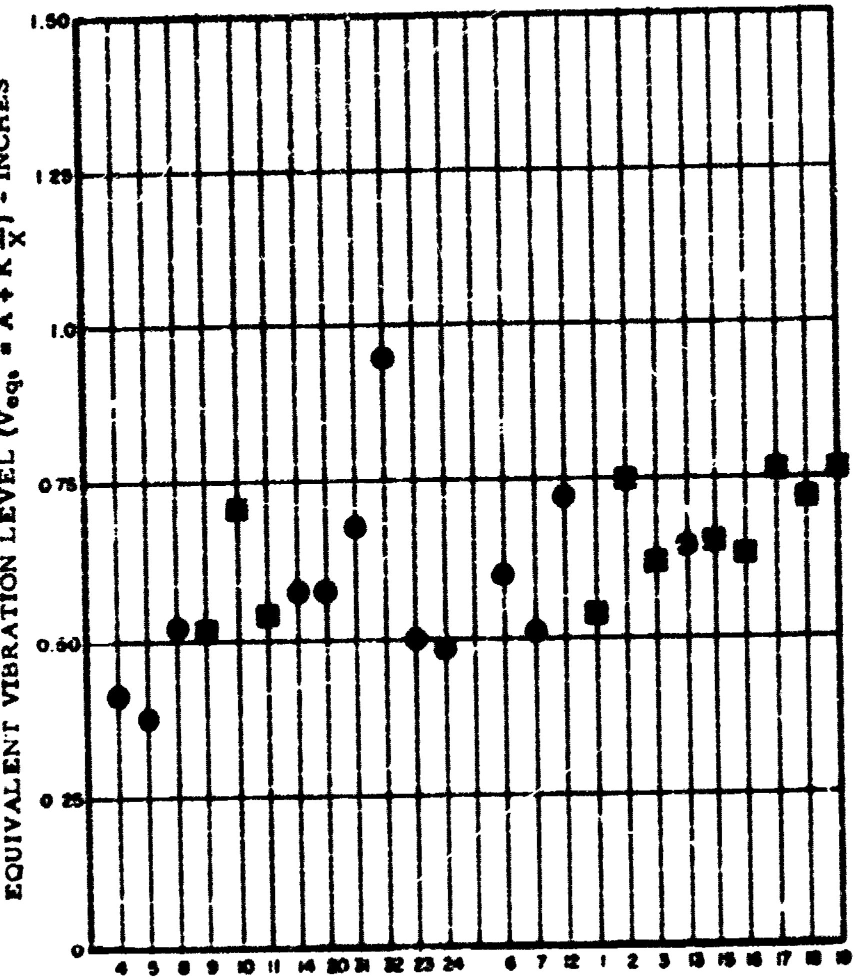


LEGEND: ● = MEASURED DATA ACCEPTABLE BY STANDARD CRITERIA
■ = MEASURED DATA UNACCEPTABLE BY STANDARD CRITERIA

WHERE $\bar{A}$ & $(\frac{\bar{Y}}{X})$ = AVERAGE FOR ACCEPTABLE LISSAJOUS PLOTS ONLY

EQUIVALENT VIBRATION LEVELS - RESULTANT VELOCITY $\quad K = \frac{\bar{A}}{(\frac{\bar{Y}}{X})}$

FIGURE 23



LISSAJOUS PLOT NUMBER

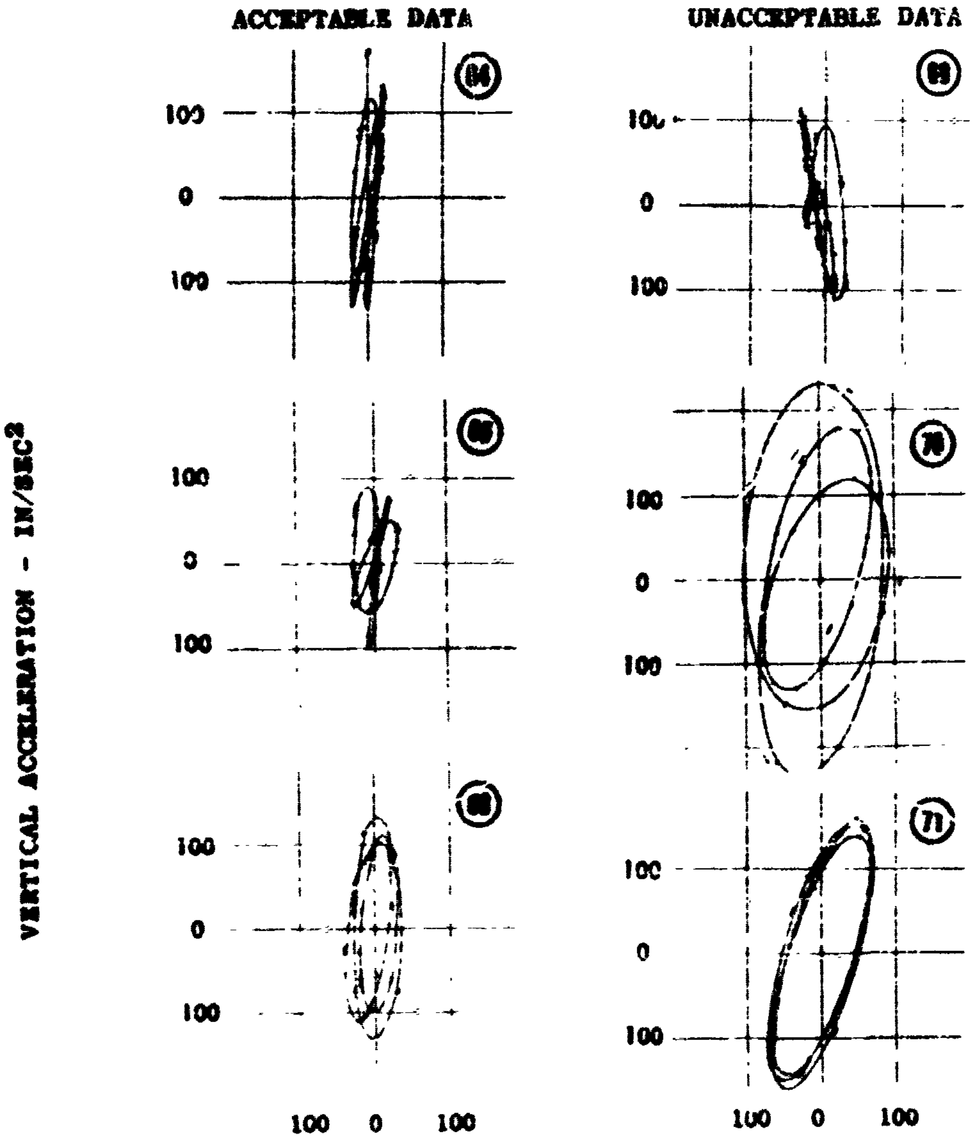
|← ACCEPTABLE PILOT COMMENT →| |← UNACCEPTABLE PILOT COMMENT →|

LEGEND ● – MEASURED DATA ACCEPTABLE BY STANDARD CRITERIA
■ = MEASURED DATA UNACCEPTABLE BY STANDARD CRITERIA

WHERE $\bar{A}$ & $\overline{(\frac{Y}{X})}$ = AVERAGE FOR ACCEPTABLE LISSAJOUS PLOTS ONLY

EQUIVALENT VIBRATION LEVELS - RESULTANT DISPLACEMENT $K = \frac{\bar{A}}{\overline{(\frac{Y}{X})}}$

FIGURE 24

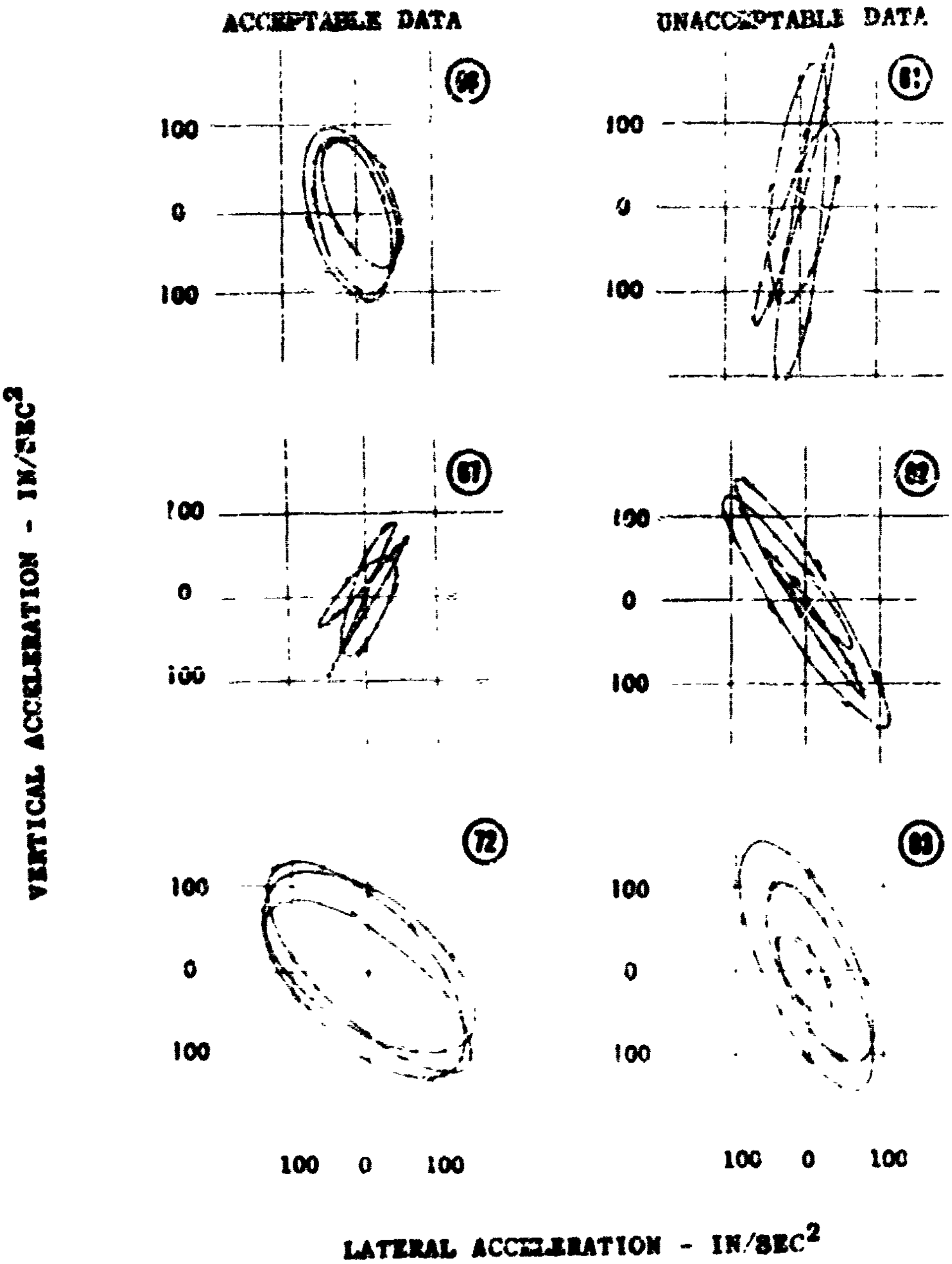


LATERAL ACCELERATION - IN/SEC^2

INDICATED AIRSPEED = 96 KNOTS ROTOR SPEED = 258 RPM

RESULTANT ACCELERATION ACCEPTABLE PILOT COMMENT

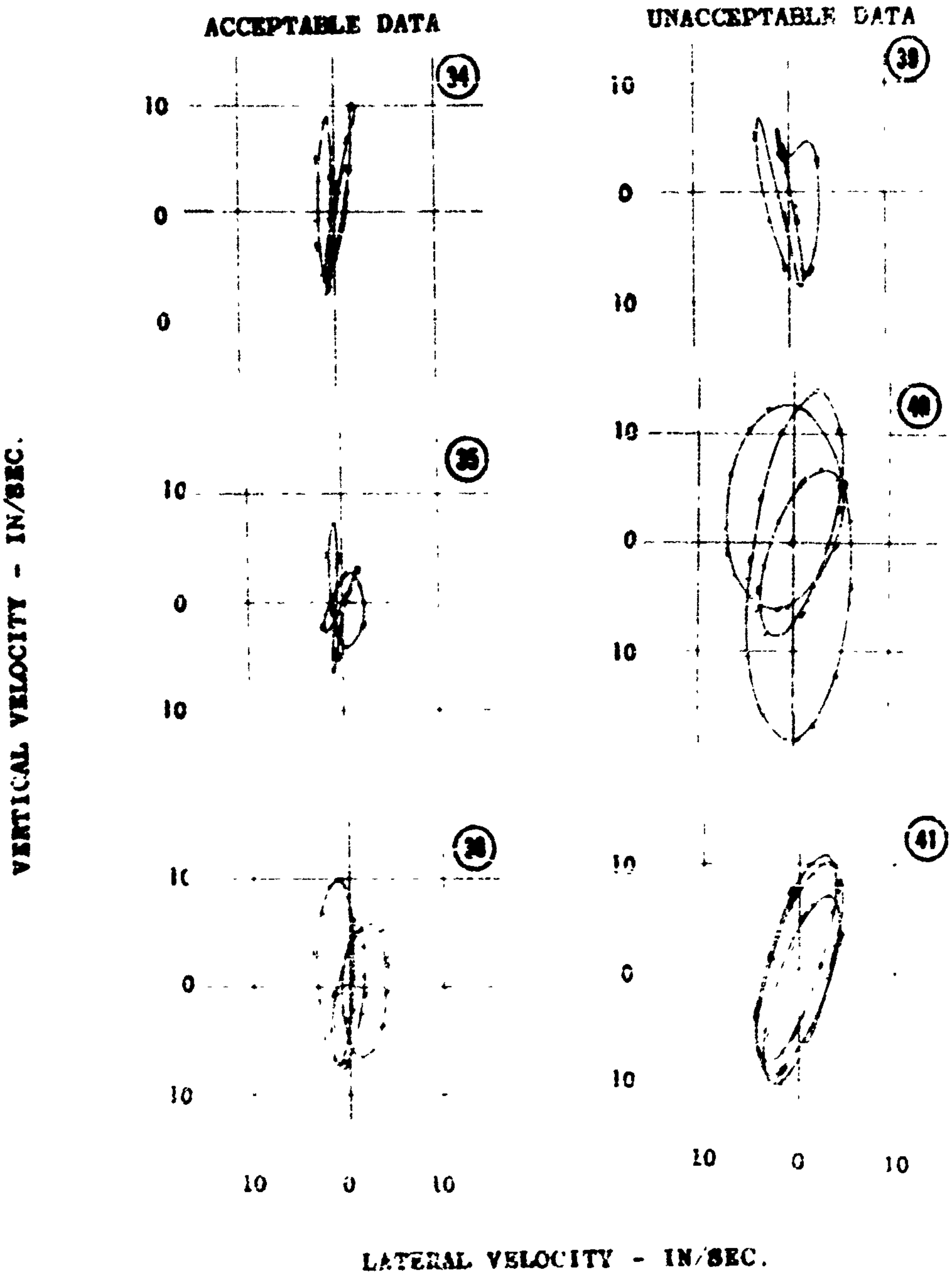
FIGURE 25



INDICATED AIRSPEED = 96 KNOTS ROTOR SPEED = 258 RPM

RESULTANT ACCELERATION UNACCEPTABLE PILOT COMMENT

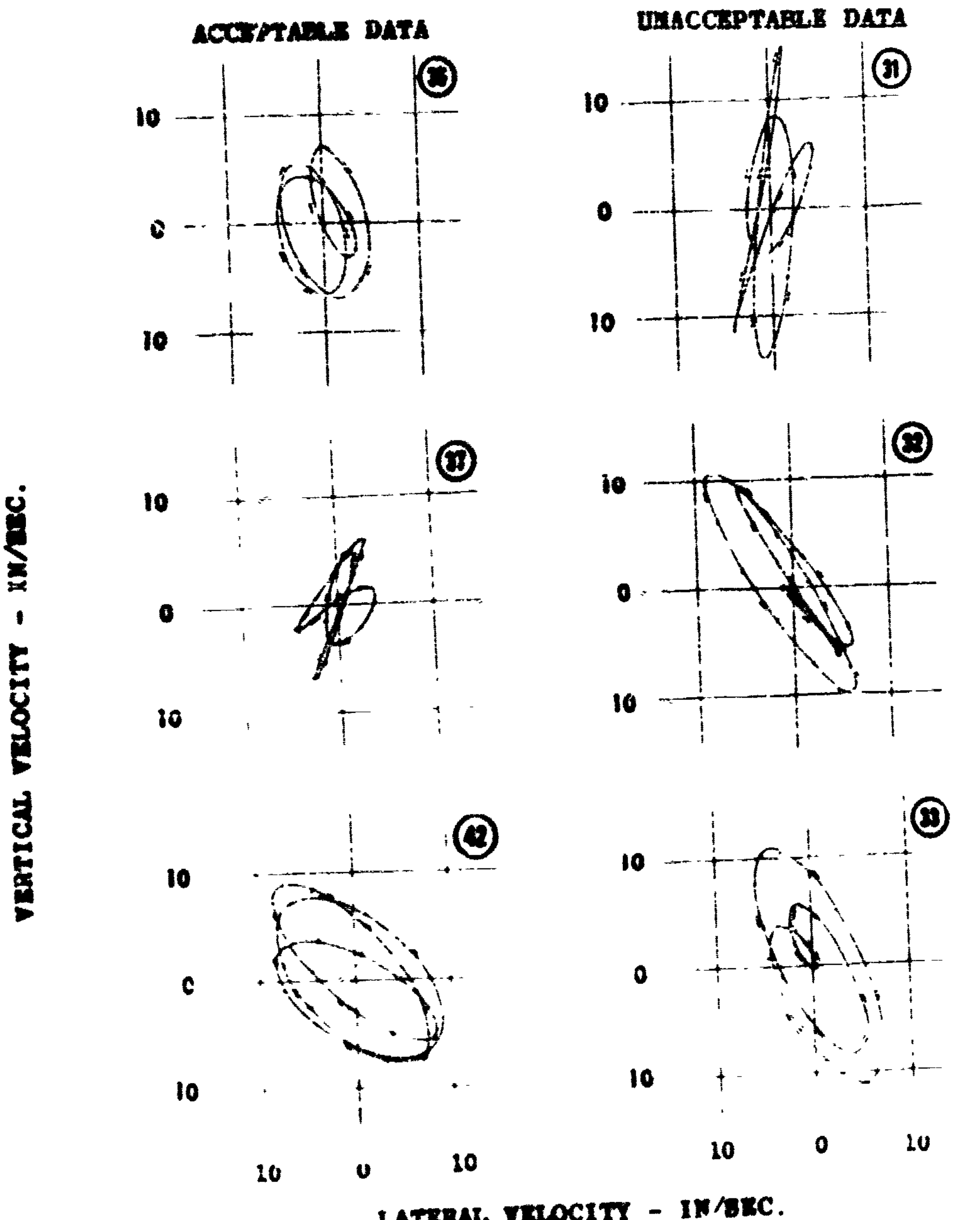
FIGURE 26



INDICATED AIRSPEED - 96 KNOTS ROTOR SPEED - 258 RPM

RESULTANT VELOCITY ACCEPTABLE PILOT COMMENT

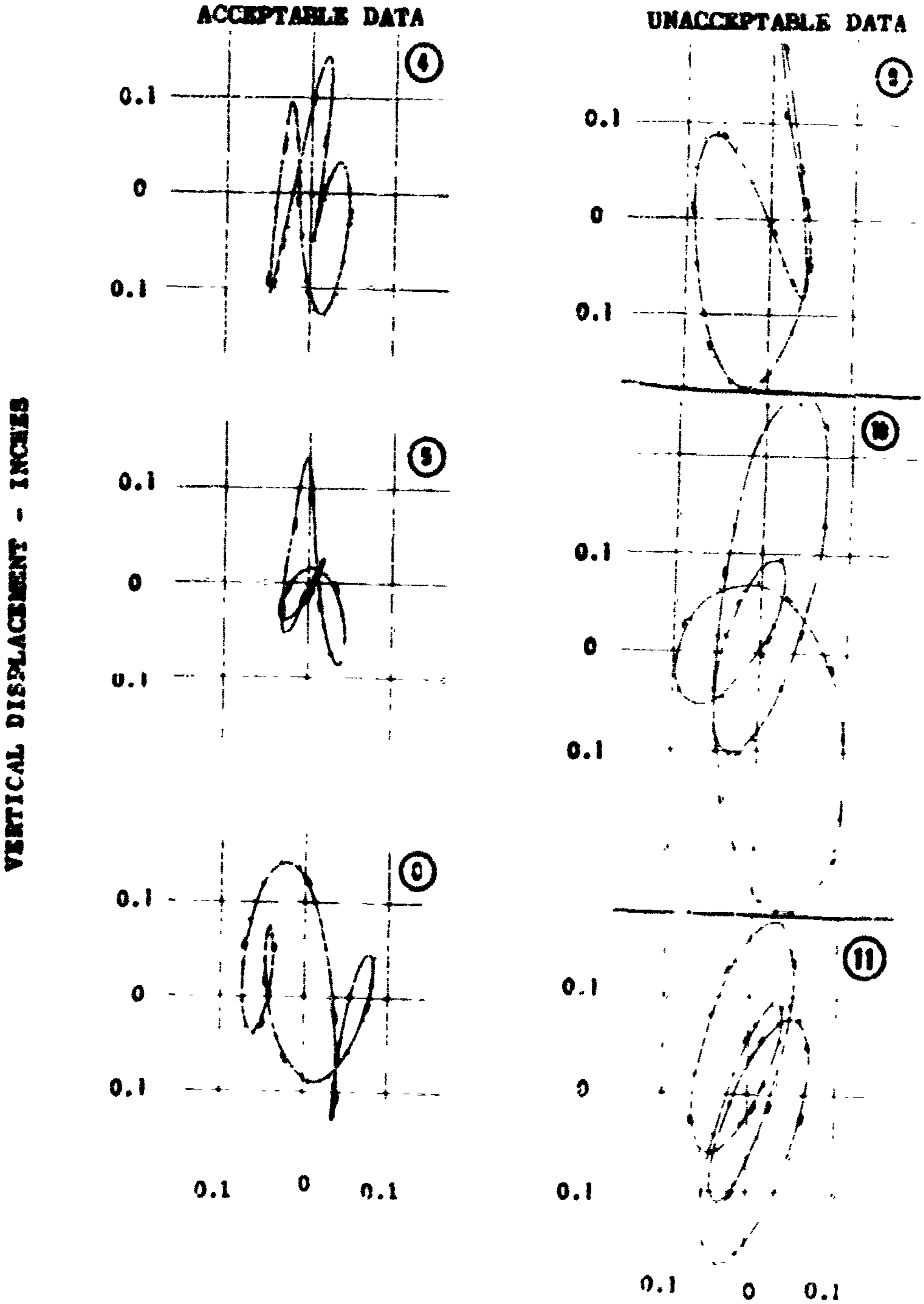
FIGURE 27



INDICATED AIRSPEED = 96 KNOTS   ROTOR SPEED = 258 RPM

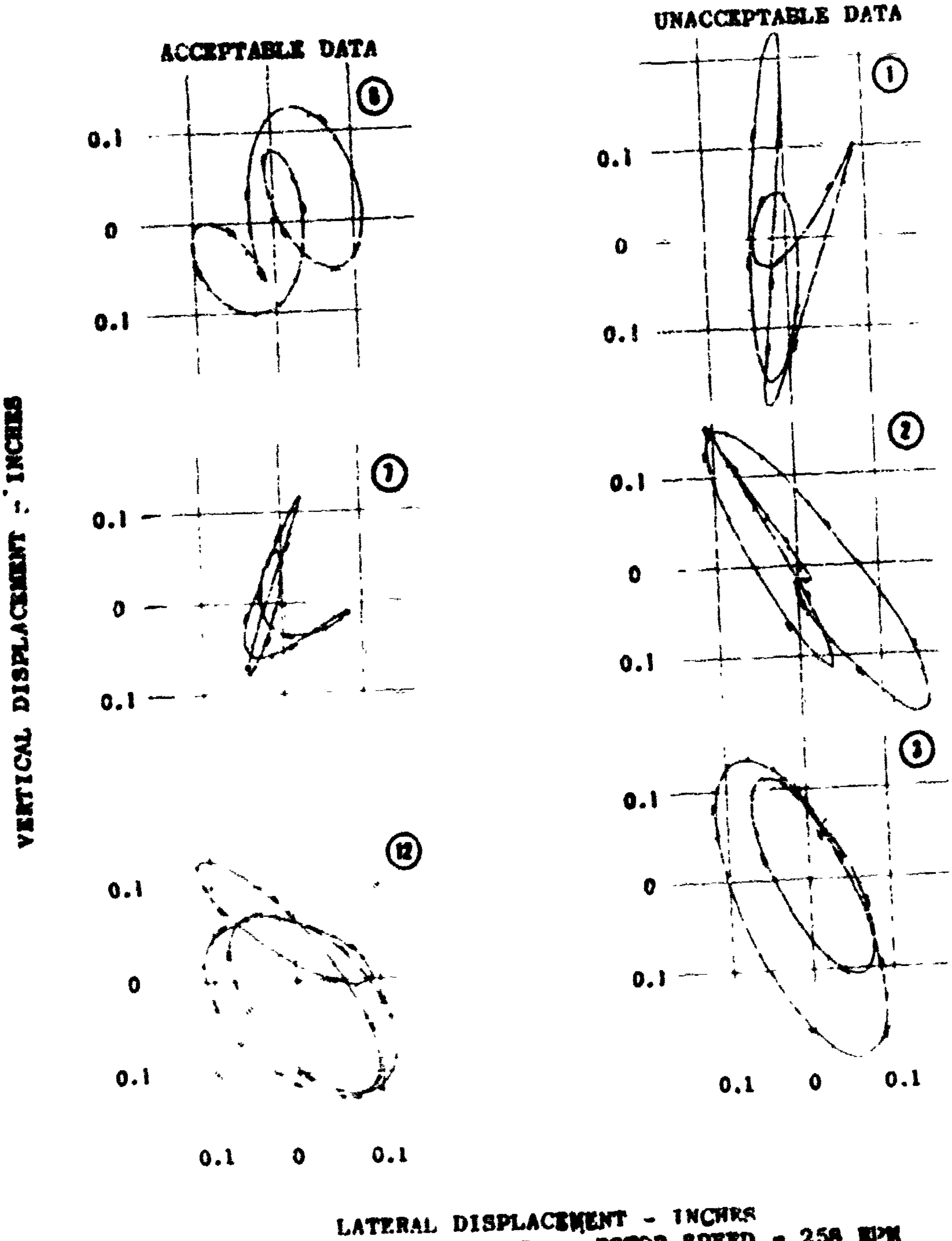
RESULTANT VELOCITY   UNACCEPTABLE PILOT COMMENT

FIGURE 28



INDICATED AIRSPEED = 96 KNOTS ROTOR SPEED = 258 RPM

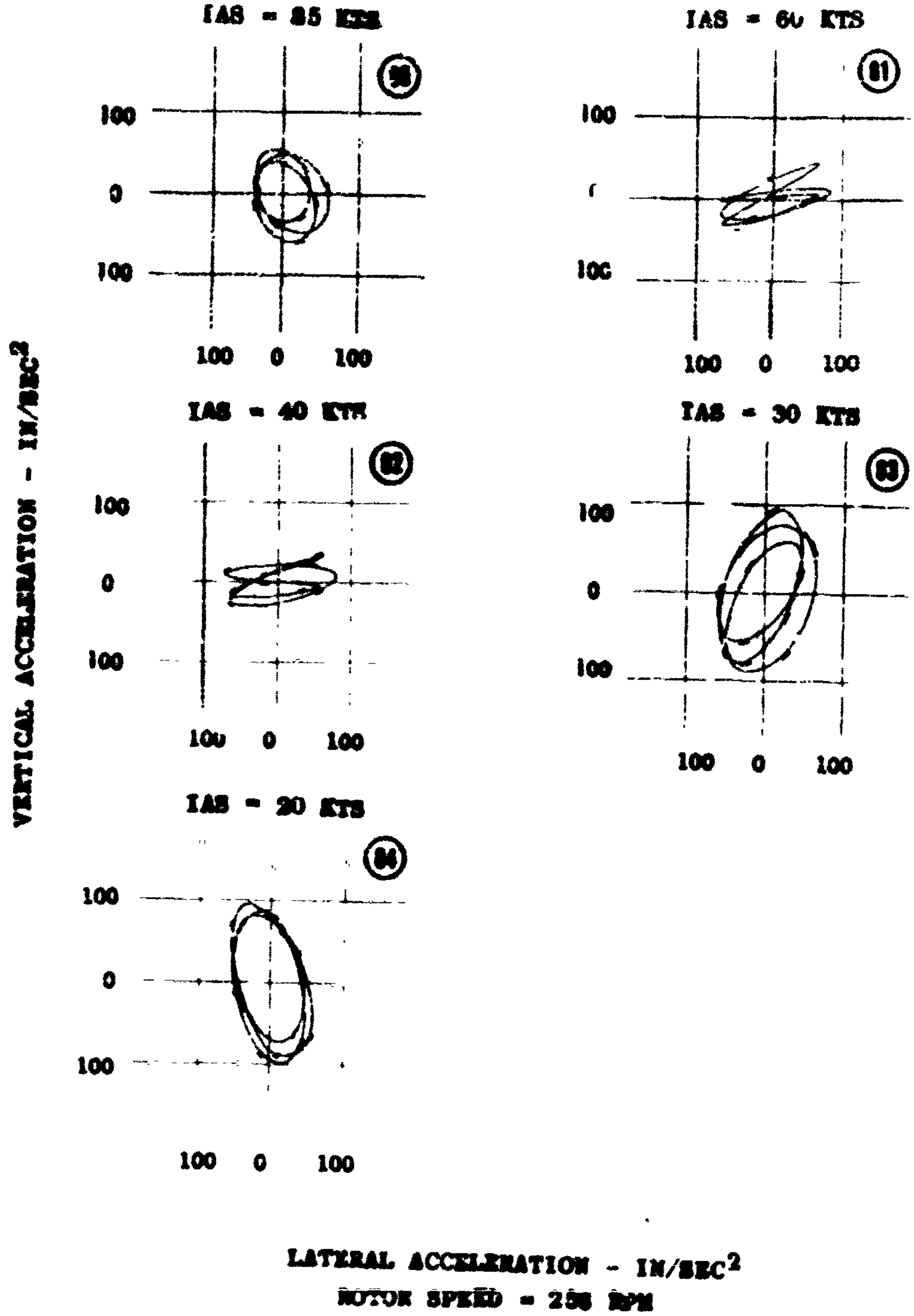
RESULTANT DISPLACEMENT ACCEPTABLE PILOT COMMENT
FIGURE 29



INDICATED AIRSPEED - 96 KNOTS   ROTOR SPEED = 258 RPM

RESULTANT DISPLACEMENT UNACCEPTABLE PILOT COMMENT

FIGURE 30

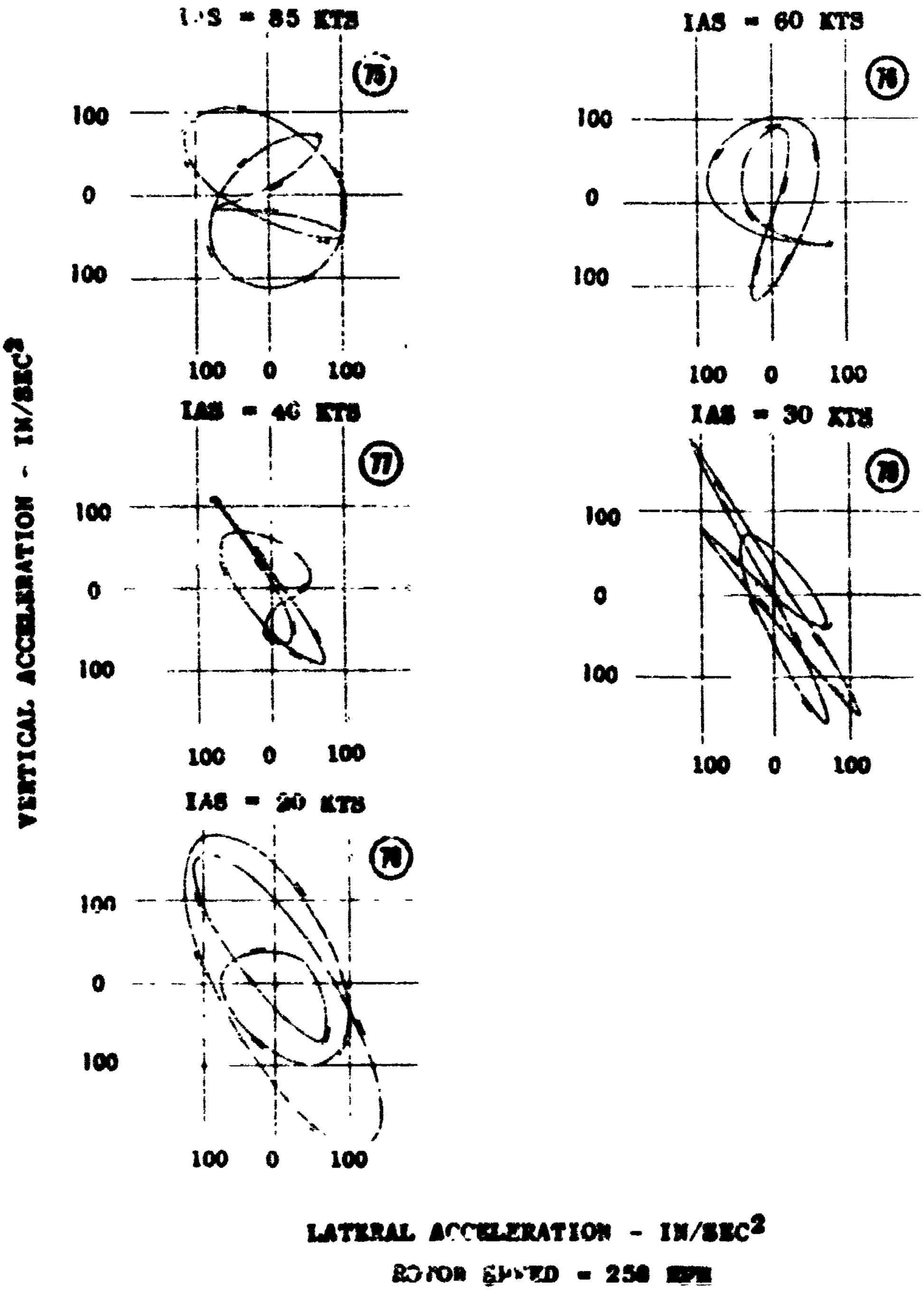


ACCEPTABLE PILOT COMMENT

ACCEPTABLE DATA

RESULTANT ACCELERATION

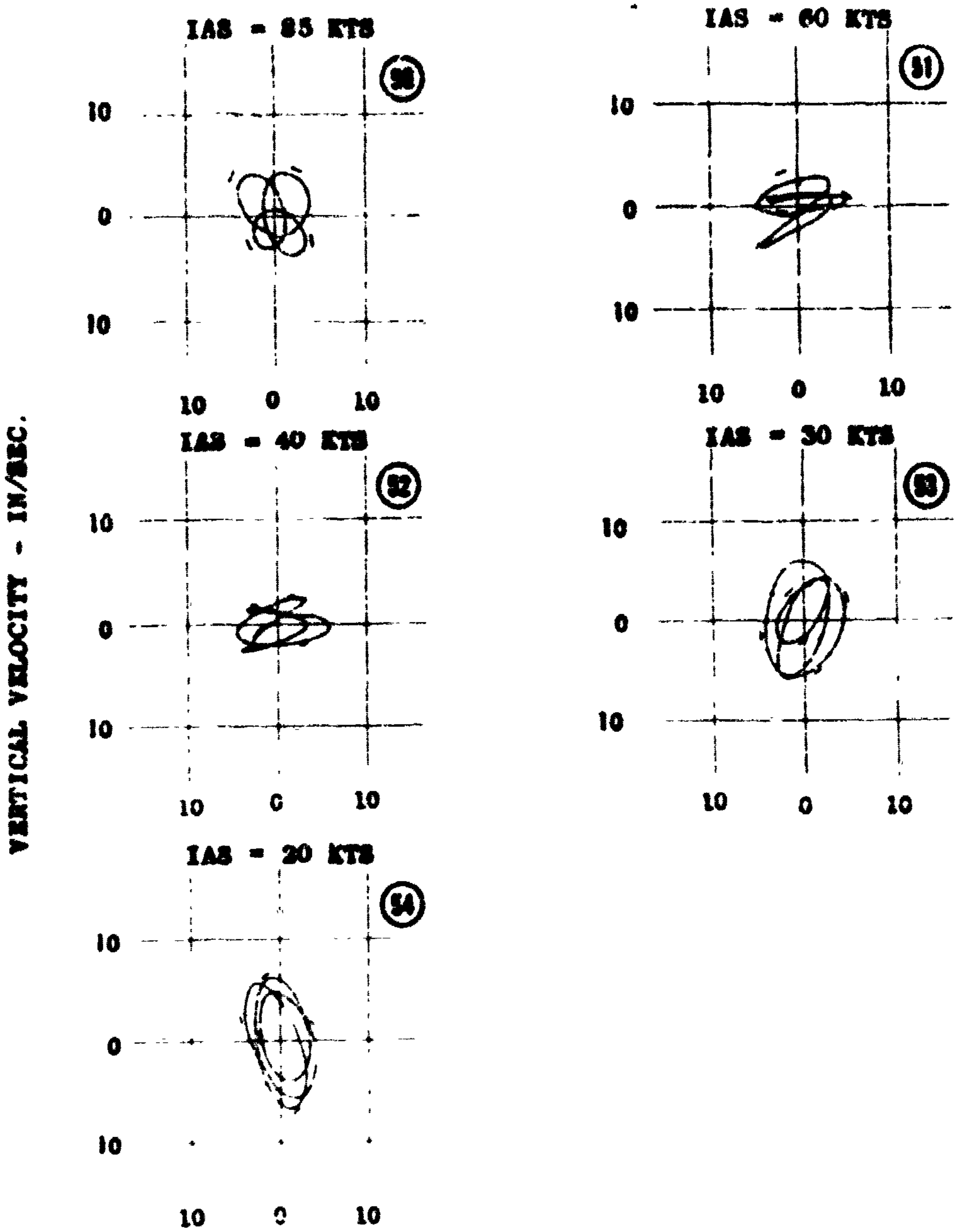
FIGURE 31



UNACCEPTABLE PILOT COMMENT UNACCEPTABLE DATA

RESULTANT ACCELERATION

FIGURE 32



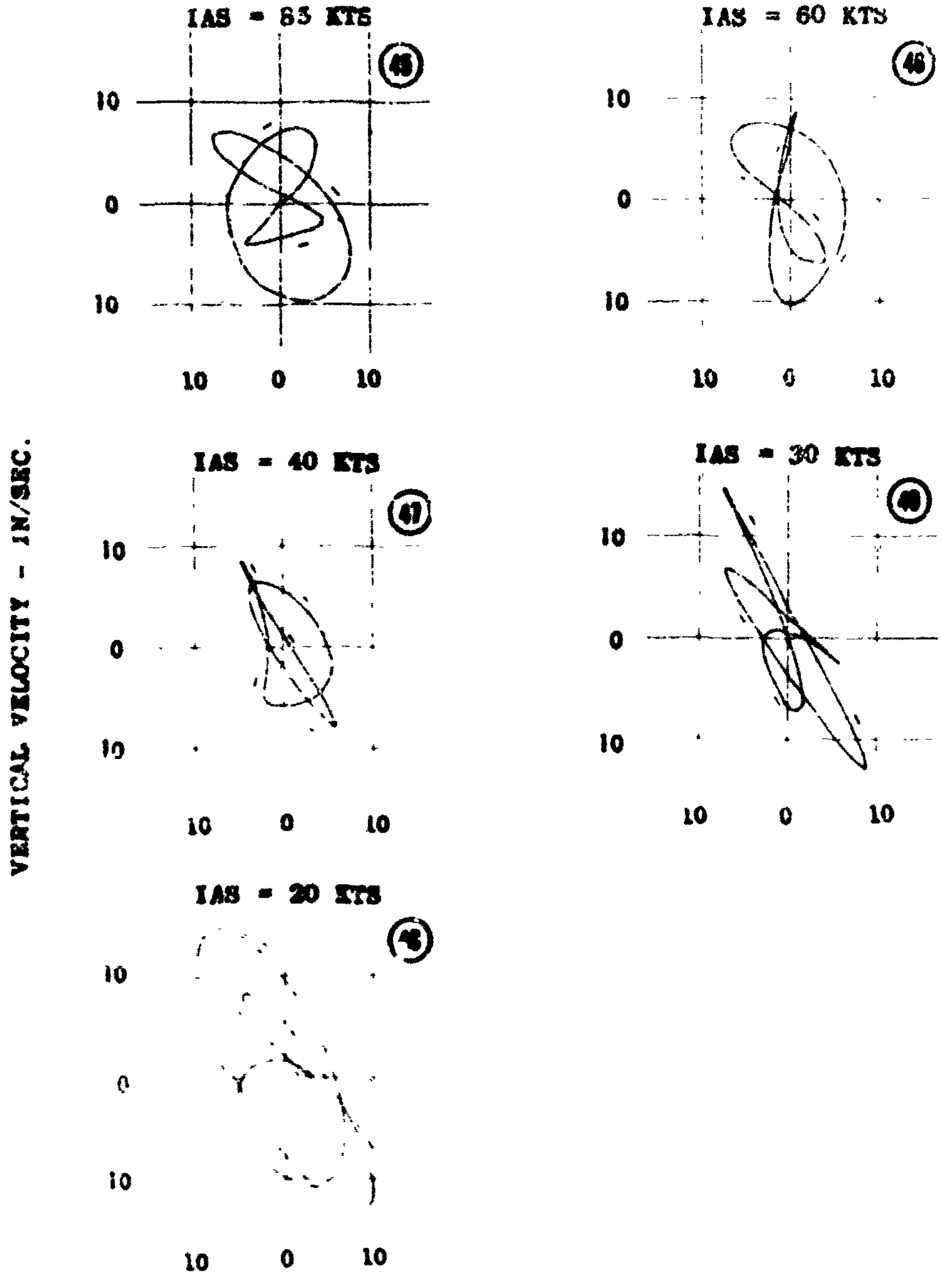
LATERAL VELOCITY - IN/SEC.

ROTOR SPEED = 258 RPM

ACCEPTABLE PILOT COMMENT　　ACCEPTABLE DATA

RESULTANT VELOCITY

FIGURE 33

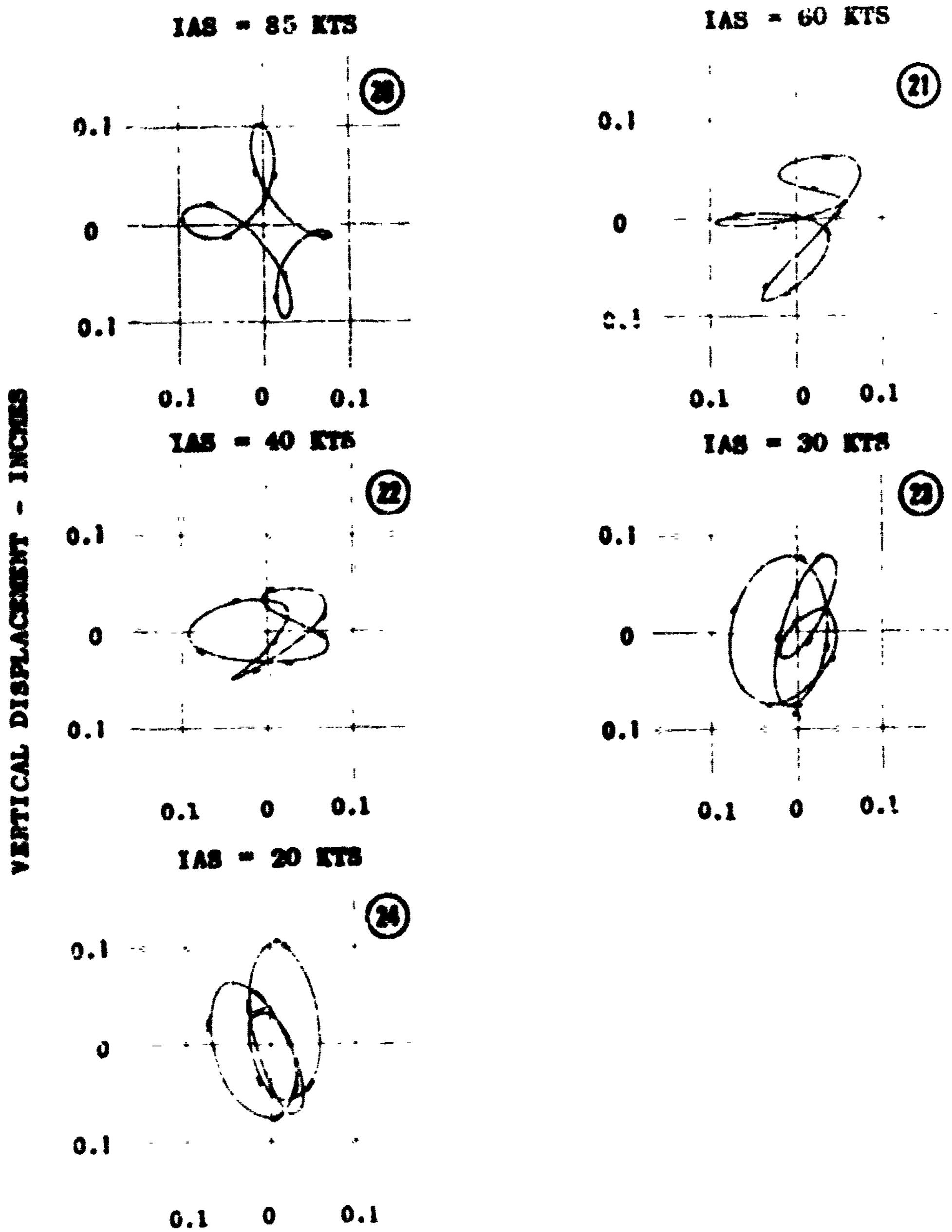


ROTOR SPEED = 258 RPM

UNACCEPTABLE PILOT COMMENT UNACCEPTABLE DATA

RESULTANT VELOCITY

FIGURE 34



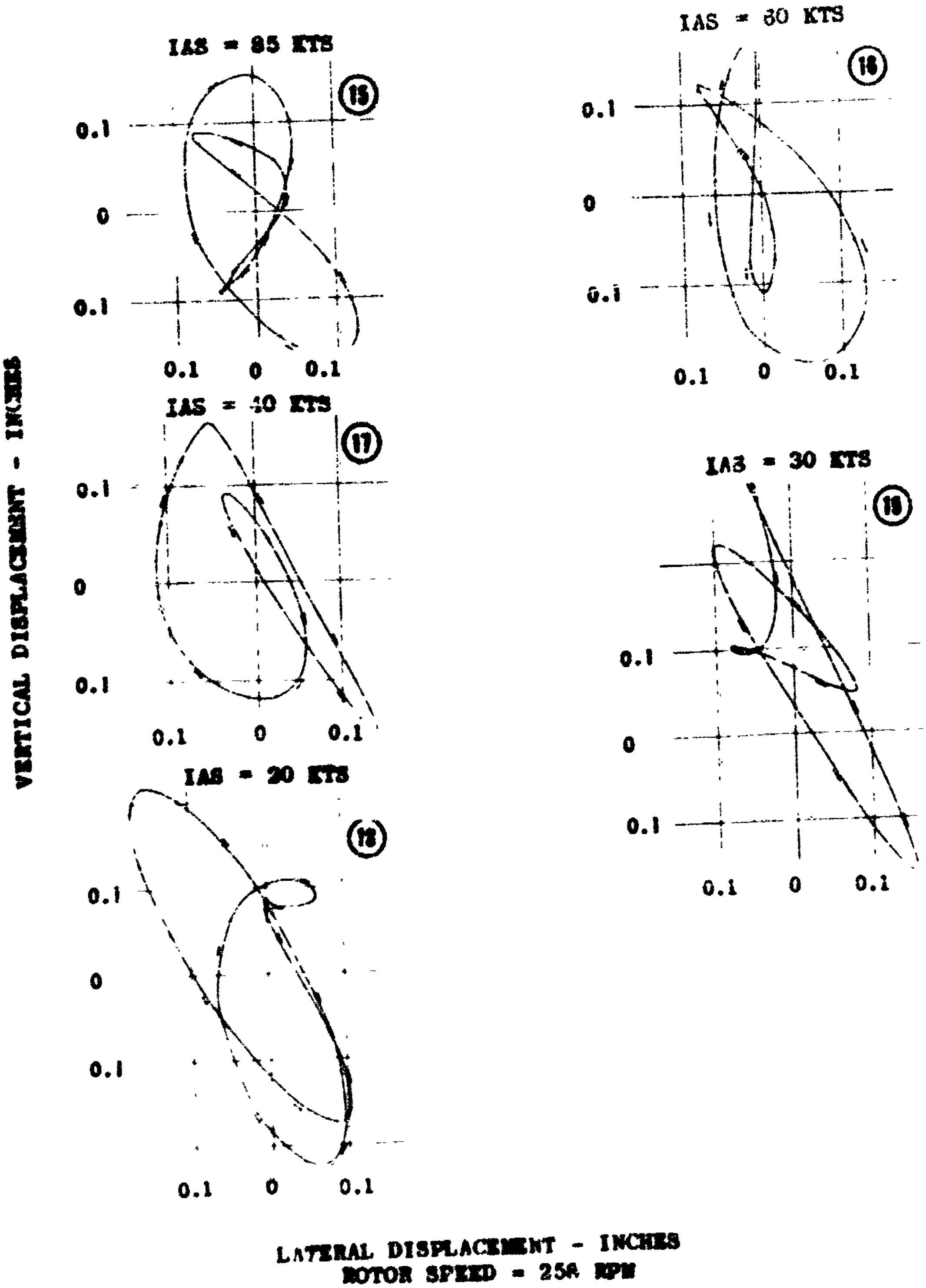
LATERAL DISPLACEMENT - INCHES

ROTOR SPEED = 256 RPM

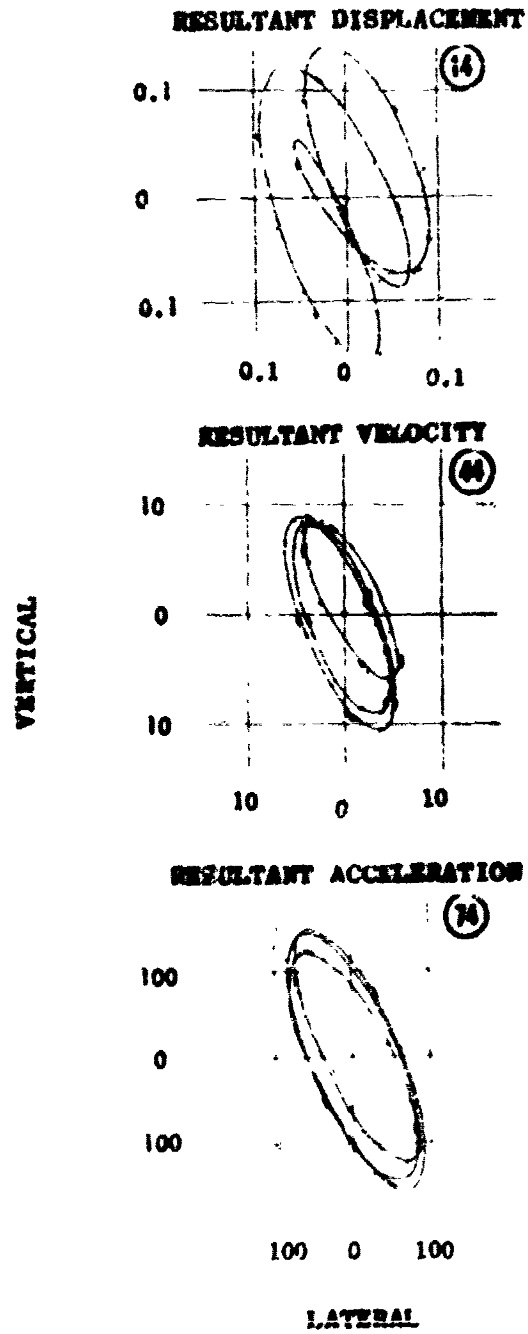
ACCEPTABLE PILOT COMMENT ACCEPTABLE DATA

RESULTANT DISPLACEMENT

FIGURE 35

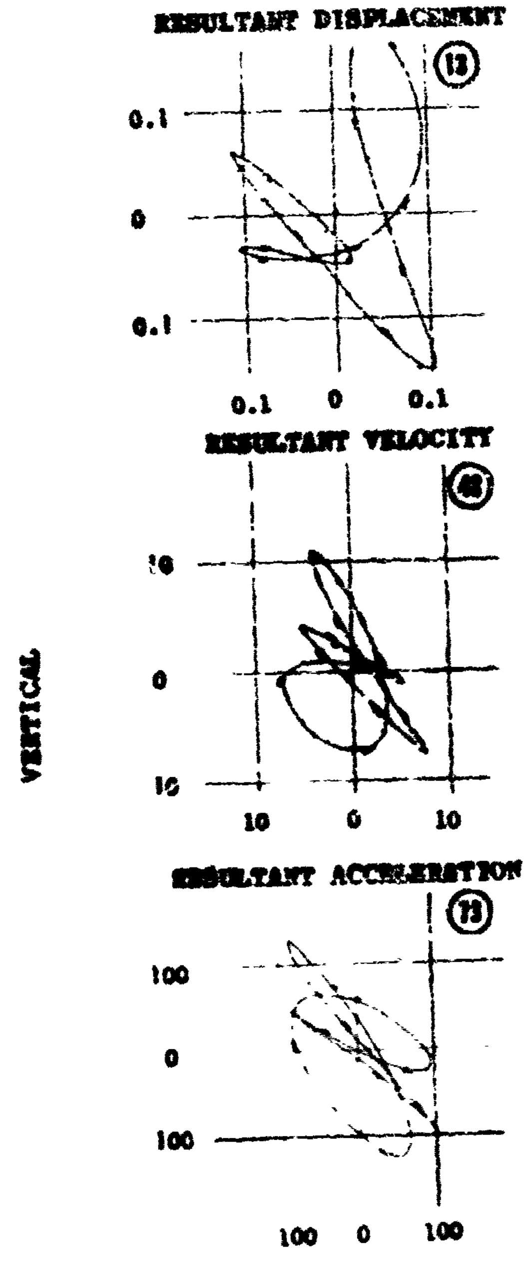


UNACCEPTABLE PILOT COMMENT UNACCEPTABLE DATA
RESULTANT DISPLACEMENT
FIGURE 36



INDICATED AIRSPEED = 36 KNOTS ROTOR SPEED = 258 RPM

ACCEPTABLE PILOT COMMENT ACCEPTABLE DATA
FIGURE 37



LATERAL

INDICATED AIRSPEED = 96 KNOTS ROTOR SPEED = 350 RPM

UNACCEPTABLE PILOT COMMENT ACCEPTABLE DATA
FIGURE 38