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TROPOSPHERIC AND IONOSPHERIC RADAR REFRACTION EFFECTS

FINAL REPORT

BY
L. J. Anderson
S. Weisbrod
L. A. Morgan
H. D. Loucks

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ABSTRACT

This report describes simple methods for computing refraction effects, such as radar elevation angle and range errors, total ray bending and Faraday rotation, caused by the troposphere and the ionosphere. These methods are applied to radiosonde and ionogram data for Fairbanks, Thule and Leuchars, Scotland, to obtain the statistical distribution of refraction effects at these locations.

The application of the methods to the computation of refraction effects at a given time and location is described in detail, and numerical examples are given.

An interesting property was discovered of the ionospheric component of elevation angle error. Unlike the tropospheric component, this quantity actually increases with increasing elevation angle, reaching a maximum value at an elevation angle of between 100 and 200 milliradians and then decreases. The tropospheric component decreases rapidly enough to make the combined elevation angle error decrease with increasing elevation angle (at frequencies above 50 mc), but at a slower rate than would otherwise be expected.

The statistical analysis shows that tropospheric refraction is greatest and most variable at Fairbanks. There is a seasonal trend at all stations, with maximum refraction at Thule and Fairbanks in winter, and in Scotland in summer. Ducting is weak at all stations; the lowest trapped radar frequency being 800 megacycles. Maximum terrain clutter ranges for particular terrain features of 350 miles can be expected at Fairbanks, and less than this at the other stations.

Ionospheric effects exhibit a strong diurnal trend in winter at Fairbanks and Scotland, with a maximum in the afternoon. Little diurnal trend exists at Thule. Summertime diurnal variations are small at all stations. The effects during 1954 (sunspot minimum) are similar to those during 1957 (sunspot maximum) except that they are scaled down by an order of magnitude.

INTRODUCTION

The purpose of the work done under Engineering Changes A & B of this contract is to investigate the effects of the troposphere and the ionosphere on ground radar tracking of high altitude targets. The investigation has been divided into the following three phases,

(1) Mathematical analysis of refractive bending, radar elevation angle error, range error, and polarization rotation, as well as the effects of tropospheric ducts.

(2) Development of procedures for computing the above effects using available radiosonde and ionogram data.

(3) Analysis of data for three specified locations, to determine the statistical distributions of the above parameters at these locations.

The details of the mathematical development were presented in SRA Report No. 68, dated September, 1958. In this report, the basic expressions for determining total bending γ, elevation angle error δ, range error Δr, and polarization rotation Ω, were derived. Subsequent SRA Reports (numbers 80 and 87) described the various approximations which are justified in the development of computational procedures for determining γ, δ, Δr and Ω from radiosonde and ionogram data.

In the present report we shall be concerned primarily with presenting computational techniques and with the results of applying these techniques to actual data at three specific locations. Statistical distributions of the parameters, seasonally and diurnally will be presented for each location. Tropospheric effects are treated separately from ionospheric effects, since the latter are frequency sensitive and the former are not. Methods are given for combining the two components for any desired frequency (above 100 mc). Finally, sample curves are presented in which the components have been combined for frequencies of 100 mc and 400 mc, for some typical and extreme refraction conditions. The frequency of occurrence of tropospheric ducts at each location is also presented, together with the resulting maximum ranges for sea and ground clutter echoes.

TROPOSPHERIC REFRACTION EFFECTS

The two tropospheric refraction effects of direct interest in radar tracking are the elevation angle error δ and the range error Δr. In order to obtain δ, it is first necessary to compute the actual ray bending γ. This is done by dividing the tropospheric column into layers whose boundaries are the radiosonde significant levels. Since these levels were originally chosen such that linear interpolation of temperature and/or dew point between levels gives a good recovery of the original radiosonde trace, we are quite justified in assuming a constant gradient of refractive index n, within a given layer. For such a layer the incremental ray bending is

$$\Delta\gamma_{jk} = \frac{2\,(n_j - n_k)}{\tan\beta_j + \tan\beta_k} \tag{1}$$

and the incremental range error is

$$\Delta r = \frac{(n_j + n_k)\;(h_k - h_j)}{\sin\beta_j + \sin\beta_k} \tag{2}$$

In these expressions the j subscript refers to the lower boundary of the layer and the k subscript refers to the upper boundary of the layer. The n's are refractive indices at these heights, the h's the heights of the boundaries, and the β's are the ray inclination angles at the layer boundaries (see Figure 1). The value of β at each layer boundary is determined from Snell's law

$n\rho \cos\beta = n_o\, a \cos\alpha_o$

where a is the earth's radius

and ρ is a + h, the height of the layer boundary

Since one customarily computes the refractivity $N = (n-1) \times 10^6$ from radiosonde data, one can express $\Delta\gamma$ as

$$\Delta\gamma_{jk} \text{ (mr)} = \frac{N_j - N_k}{500 \, (\tan \beta_j + \tan \beta_k)} \tag{3}$$

Figure 2 is a graphical solution of Snell's law giving 500 tan β. To use the figure, enter the left margin at the value of $N_o - N$ appropriate to the boundary level of interest; move horizontally to the curve corresponding to the layer height (the numbers on the curves are in thousands of feet). Now move vertically to the curve corresponding to the desired α_o in milliradians. Finally move horizontally to the right margin and read 500 tan β. Using these curves, it is relatively simple to compute the $\Delta\gamma$'s for any arbitrary set of points defining the N profile. A numerical example of such a computation is given in the last section.

In order to obtain the elevation angle error δ_H for a target at a particular height H, one sums up the $\Delta\gamma$'s from radar height (assumed to be zero) to the target height to obtain γ_H.

then

$$\delta_H = \frac{\gamma_H \tan \beta_H - (N_o - N_H) \times 10^{-6} + \gamma_H^2/2}{\gamma_H + \tan \beta_H - \tan \alpha_o} \tag{4}$$

In the case of range errors, one simply adds the Δr's for each layer between 0 and H to obtain the total range error Δr_H. The range error is such a small percentage of the total range that one is not usually concerned with tropospheric Δr's when tracking aircraft and other relatively low altitude targets. In the case of missile tracking, one is concerned with both the total tropospheric component and at least part of the ionospheric component of range error.

The statistical analysis of range errors has therefore been limited to that caused by the total troposphere. Since this component is small compared to the ionospheric range error component (at 100 mc), it was felt justified to use an approximate method for computing Δr for the troposphere, rather than the more accurate (but more expensive) method of summing individual layer increments.

In the approximate method, the troposphere is assumed to have a constant N gradient of 12 units/1000 ft up to the height where N becomes zero, N_o thus being the only remaining variable. In such an atmosphere

$$H = \frac{N_o}{.012} \quad (\text{ft})$$

then from (2)

$$\Delta r^t = \frac{HN_o}{\text{Sin } \beta_H + \text{Sin } \alpha_o} = \frac{N_o^{\,2} \times 10^{-12}}{.012 \ (\text{Sin } \beta_H + \text{Sin } \alpha_o)}$$

From Snell's law

$$\text{Sin } \beta = \sqrt{\text{Sin}^2 \alpha_o - 2 N_o \times 10^{-6} + 2 H/a}$$

$$= \sqrt{\text{Sin}^2 \alpha_o + 6 N_o \times 10^{-6}}$$

which gives finally

$$\Delta r^t = \frac{N_o^{\,2} \times 10^{-12}}{.012 \left[\text{Sin } \alpha_o + (\text{Sin}^2 \alpha_o + 6 N_o \times 10^{-6})^{1/2} \right]} \qquad (5)$$

Comparisons of this expression with the more accurate, layer by layer method showed that if the approximate results were multiplied by a constant factor of 1.60, they agreed with the more accurate results within 10 ft at all elevation angles. It also turned out that Δr^t is rather insensitive to the actual structure of the N profile, and is a function primarily of N_o at a given elevation angle. One can compute Δr^t with negligible error therefore by the following expression:

$$\Delta r^t \ (\text{ft}) = \frac{.000136 \ N_o^{\,2}}{\sin \alpha_o + (\sin^2 \alpha_o + 6 N_o \times 10^{-6})^{1/2}} \qquad (6)$$

This is the expression used in the statistical computations of the tropospheric component of the range error.

Since the methods which have been developed for computing refractive effects are equally applicable to the troposphere and the ionosphere, superscripts "t" and "i" will be used to distinguish between the two cases whenever necessary.

Statistical Analysis

The methods described above were used to compute δ and Δr^t for various target heights and elevation angles. Three years of radiosonde data (1955 through 1957) were obtained for Leuchars, Scotland; Thule, Greenland; and Fairbanks, Alaska. Four radiosonde ascents a day were available from Thule and two per day from the other two stations. For each ascent, δ was computed for target heights of 10, 50, 100 and 300 thousand feet and for α_o's of 0, 10, 30, 100, 300 and 1000 milliradians. A statistical distribution was run by calendar month on these computations, as well as on the Δr^t computations for the same α_o's.

It was found that all of the distributions were very nearly normally distributed, hence the required statistical information can be well represented by tabulations of the median values and standard deviations. Tables I, II and III list these parameters for the δ distributions for Leuchars, Thule and Fairbanks, respectively. Table IV lists the same parameters for the Δr^t distributions for all stations.

A distinct seasonal change of δ and Δr^t is apparent at Leuchars, with the highest values occurring during the summer and the lowest during the winter. This is caused by higher N's during the summer, because of increased moisture. At Thule, the trend is reversed, with higher values occurring in winter. This is due to thicker surface ducts being built up during the long winter night. Because of low temperatures, the moisture increase during summer is too small to be effective. At Fairbanks, a double maximum is observed, one in winter and one in summer. Since this station is located almost midway (in latitude) between Leuchars and Thule, it is reasonable to expect that the winter night ducts and the increased summer moisture are about equally effective in influencing the δ's.

Extrapolation to Greater Target Heights

In order to combine the δ's at heights above the top of the troposphere (arbitrarily taken to be 300,000 ft), one must first compute the tropospheric δ's at the desired heights, and then combine these values with corresponding ionospheric δ's.

Since the tropospheric computations terminate at a height of 300,000 ft, we require a method of performing the extrapolation to greater heights using the information at 300,000 ft. One can show from the geometry of tropospheric refraction that the value of δ_H^t, the tropospheric component of δ at height H, is given by:

$$\delta_H^t = \delta_{300}^t + (\gamma_{300}^t - \delta_{300}^t) \frac{\tan \alpha_H - \tan \alpha_{300}}{\tan \alpha_H - \tan \alpha_0} \qquad (7a)$$

$$= \delta_{300}^t + (\gamma_{300}^t - \delta_{300}^t) \left[\frac{(\sin^2\alpha_o + 2H/a + H^2/a^2)^{\frac{1}{2}} - (\sin^2\alpha_o + .026906)^{\frac{1}{2}}}{(\sin^2\alpha_o + 2H/a + H^2/a^2)^{\frac{1}{2}} - \sin\alpha_o}\right] \qquad (7b)$$

where subscripts 300 and H refer to the values of the various parameters at the heights of 300,000 feet and some greater height H.

Equation 7 may be derived in the following manner. At heights above 300,000 feet, equations relating γ and δ reduce to (see equation 12 and 14)

$$\varepsilon = \gamma - (N_o - N) \tan \alpha$$

$$\delta = \frac{\varepsilon \tan \alpha}{\tan \alpha - \tan \alpha_o}$$

Above 300,000 feet N is assumed to be zero and we can write

$$\varepsilon^t_{300} = \delta^t_{300} - N_o/\tan \alpha_{300}$$

$$\varepsilon^t_H = \delta^t_{300} - N_o/\tan \alpha_H$$

$$\delta^t_{300} = \varepsilon^t_{300} \tan \alpha_{300} / (\tan \alpha_{300} - \tan \alpha_o)$$

$$\delta^t_H = \varepsilon^t_H \tan \alpha_H / (\tan \alpha_H - \tan \alpha_o)$$

Eliminating N_o, ε^t_{300} and ε^t_H from the above set of four equations yields (7a)

Form (7b) is obtained by noting that

$$(a + h) \cos \alpha = a \cos \alpha_o$$

and using appropriate numerical values for the height of 300,000 feet.

At ionospheric heights or above, Equation 7a may be readily evaluated with the aid of Figure 4.

The value of the square bracketed term is a function only of α_o and H, and has been computed for heights of 100, 300, 1000 and 2500 n.mi. for α_o's of 0, 10, 30, 100, 300 and 1000 milliradians.

In obtaining the values of δ^t_H then, the distributions of δ_{300} and γ_{300} were plotted. Since δ_{300} and γ_{300} can be considered to be well correlated, one obtains the median value of δ^t_H from the median values of δ_{300} and γ_{300}. The standard deviations were nearly the same for both of these quantities in all cases, so the standard deviation of δ^t_H was obtained by interpolation of the standard deviations of δ_{300} and γ_{300}. The values of δ^t_H at the heights and α_o's specified above are tabulated in Tables XVI, XVII and XVIII for Leuchars, Thule and Fairbanks, respectively.

Tropospheric Ducts

Table V shows the percentage of total radiosonde ascents in which strong surface N gradients (>45 N units / 1000 ft) were observed. It can be seen that Leuchars shows a definite increase in occurrence during the summer, while Thule has the maximum in winter. However, the occurrence at both of these stations is small compared to that at Fairbanks. At this station there are frequent trapping gradients during winter months, and a marked decrease during summer.

Analyses of the surface duct characteristics, during the month of maximum occurrence, were made at each station. These analysis showed that the ducts are rather thin (under 300 ft on the average) and that relatively few of them are capable of trapping radar energy. Table VI shows the numerical results for 1955, '56 and '57. The minimum radar frequency which is trapped at each station was determined using the SRA trapping criteria for thin surface ducts shown in Figure 3. These criteria curves were derived by applying S. A. Schelkunoff's approximate solution to the wave equation to exponential surface layers.[1] They are the only such curves which have successfully stood the test of experimental confirmation. In Figure 3, the ordinate is the height h of the top of the layer and the abscissa is the minimum ΔB needed for trapping, where $\Delta B = N_o - N_{Top} - .012h$. To use the curves, plot the point at h and ΔB corresponding to the layer in question, if the point lies to the right of the curve for a given frequency, that frequency will be trapped.

Re-distribution of Energy

One can expect that for the percentages of time listed in Table V, the lowest lobe of the radar interference pattern (below elevation angles of 0.5^o) will be distorted somewhat. This is due to extraction of some of the energy normally contained in this lobe to increase the fields at very low elevation angles. The effect of this re-distribution of energy, will be to reduce the detection probability for targets at elevation angles somewhat below the maximum line of the lowest lobe. The energy which appears at angles just above the horizon line will be stronger than normal in these cases, but it will still be considerably weaker than it is just below the first lobe maximum line, so it will not provide increased detection probability just above the horizon. It is characteristic that under near-trapping conditions, the lobe structure in the lowest 0.5° is quite eratic both in time and space. Thus, the detection probability under such conditions may be expected to vary considerably and will in general not be as great nor as predictable as it is under more normal conditions.

The refraction which occurs under near-trapping conditions results in an outward displacement of the lobe pattern, such that the lowest lobe will occupy a position farther out than usual. This will increase the range of initial detection at low angles. Since the lowest lobe is displaced more than the next higher one, there will be a larger gap between lobes, but one can conclude that the overall initial detection probability at extreme range and at low elevation angles will be somewhat improved, although the precise determination of elevation angle will deteriorate because of the large δ's and its standard deviations which occur under these conditions.

Terrain and Sea Clutter

Examination of the surface ducts occurring in January in Fairbanks (strongest ducting conditions found in the analysis) indicate that the ducts present will cause a moderate increase in clutter range during the percentage of time listed in Table VI. Although the ducts found are not capable of trapping radar energy at frequencies below 800 mc, the attenuation rates in near-trapping ducts are lower for frequencies lower than this limit, hence one can expect somewhat greater extensions in clutter range at 400 mc than at 1000 mc. If one assumes a radar capable of detecting a target of 1 sq ft radar cross section at 1600 n.mi., one can determine the detection range of a typical terrain feature as a function of radar wavelength and duct characteristics. Measurements indicate that a steep sided mountain face 2000 ft high and 30 miles long has a radar cross section of about 77 db above 1 sq ft.

Applying measured attenuation rates in surface ducts to the ducts found at Fairbanks, indicates that maximum detection ranges of the above type of mountain face will be about 300 n.mi. at 1000 mc and 350 n.mi. at 400 mc. Fewer cases of extreme ranges will be found however at 400 mc than at 1000 mc because only the thickest ducts will give maximum ranges at 400 mc, whereas at 1000 mc, the range is relatively insensitive to duct thickness from 150 - 400 ft, which is the range of heights found. Similar analyses were carried out for Thule and Leuchars, and the results are also indicated in Table VI. It can be seen that the clutter effects are less severe at these stations than at Fairbanks.

Sea clutter will be no problem at any of these locations. At 400 mc, maximum range will be about 50 miles or less. At 1000 mc one can expect maximum sea clutter ranges of about 100 mi.

IONOSPHERIC EFFECTS

SRA Report #68 describes in detail a simple and accurate method for computing refractive effects of the troposphere and the ionosphere. A brief review of this method and its application to ionospheric refraction is given below.

As pointed out in the previous section, a ray passing through a layer bounded by heights h_j and h_k and whose index of refraction varies linearly from n_j and n_k, is refracted by a small angle $\Delta\gamma_{jk}$ which is given by

$$\Delta\gamma_{jk} = \frac{2\ (n_j - n_k)}{(\tan\beta_j + \tan\beta_k)} \tag{8}$$

where β = ray inclination angle as determined from Snell's law.

The contributions to γ due to the passage of the ray through other layers are directly additive. Thus the total bending γ in milliradians is given by

$$\gamma\ (mr) = \sum_{0}^{k} \frac{N_k - N_{k+1}}{500\ (\tan\beta_{k+1} + \tan\beta_k)} \tag{9}$$

where $N = (n - 1) \times 10^6$

The application of Equation 9 to the tropospheric bending has already been discussed. Its application to the ionospheric bending is essentially the same but some modifications are desirable. In the case of the ionosphere, the refractivity N is a negative quantity and is inversely proportional to the square of the signal frequency. It is given by

$$N^i = -4.03\ (N_e/f^2) \times 10^{-6} \tag{10}$$

where N_e = electron density per cubic meter

f = signal frequency in megacycles

The superscripts "i" and "t" will be used in this section to differentiate between the ionospheric and the tropospheric quantities.

N^i is also a function of the Earth's magnetic field. However at frequencies above 100 mc, the effect of the terrestrial field on refractive bending is negligible, but it is important since it is responsible for the Faraday rotation of the plane of polarization. More will be said about this later.

In the case of the ionospheric bending, the minimum value of β even for a tangentially departing ray ($\alpha_o = 0$) is about 200 milliradians. Under these conditions the difference between the ray inclination angles of the refracted and the unrefracted rays is very small such that very little error is introduced if Equation 9 is written as,

$$\gamma^i \text{ (mr)} = \sum_k \frac{|N^i_{k+1}| - |N^i_k|}{500 \ (\tan \alpha_{k+1} + \tan \alpha_k)} \qquad (11)$$

The limits of the summation are the bottom edge of the F layer and the target height. The number of terms in the summation that are required to approximate an analytic model of the ionosphere is surprisingly small. For instance, for the case of a parabolic layer, where γ^i can be evaluated by a direct (but tedious) integration, five equal steps between the base and the vertex yield γ^i which agrees within a small fraction of one percent with the exact value. Evaluation of Equation 11 can be greatly facilitated by the use of Figure 4 which is a plot of 500 tan α, corrected for tropospheric refraction, versus height for various angles of elevation in milliradians.

The refractive bending can also be expressed in terms of the angle subtended at the Earth's center between the refracted and the unrefracted rays. This angle, ε, is given by

$$\begin{aligned} \varepsilon &= \delta - (\alpha - \beta) \\ &= \delta - (N_o - N) \cot \alpha \end{aligned} \qquad (12)$$

where $\varepsilon = \varepsilon^t + \varepsilon^i$

$\delta = \delta^t + \delta^i$

N_o = surface value of the refractivity

N = value of the refractivity at the target height

At infinite distances cot α approaches zero and ε and δ become equal to each other. For computational reasons it is usually convenient to split Equation 12 into the tropospheric and the ionospheric components.

$$\varepsilon^t = \delta^t - N_o \cot \alpha \quad (13a)$$

$$\varepsilon^i = \delta^i + N^i \cot \alpha \quad (13b)$$

The quantity of the greatest practical interest is the elevation angle error δ, which can conveniently be expressed in terms of ε

$$\delta = \frac{\varepsilon \tan \alpha + \varepsilon^2/2}{\varepsilon + \tan \alpha - \tan \alpha_o} \quad (14)$$

Equation (14) is an alternate form of Equation 4. The details of the derivation of both forms are given in SRA 66. In general Equation 4 is more convenient for low angle tropospheric work. At greater heights or higher angles the numerator of Equation 4 involves subtraction of two nearly equal terms and therefore the form of Equation 14 is preferable.

It should be noted that the tropospheric and the ionospheric contributions to δ are not strictly additive. However, in nearly all practical cases ε/2 is much less than tan α and ε is much less than tan α - tan α_o, so that only a negligible error is introduced if δ is considered to be directly proportional to ε. This approximation is extremely convenient since it permits a separate treatment of the tropospheric and the ionsopheric δ's. The ratio of δ/ε versus height for various values of α_o is plotted in Figure 5.

The range errors can be evaluated by a method analogous to that used for computing γ

$$\Delta r^{i} = \sum_{k=o}^{m} \frac{(|N_k| + |N_{k+1}|)\ (h_{k+1} - h_k)}{1000\ (\sin \alpha_{k+1} + \sin \alpha_k)} \qquad (15)$$

where h = height in kilometers

and Δr^{i} = range error in meters

The accuracy of Equation 15 is adequate for all practical purposes. For layer laminations of less than 100 km, and a tangentially departing ray, the values of Δr^{i} are within 5% of the exact values. For thinner laminations, (which is normally the case) or higher angles of elevation, the agreement is even better. Equation 15 can be readily evaluated with the aid of Figure 6, which is a plot of 1000 sin α corrected for tropospheric refraction versus height for various angles of elevation in milliradians.

Equation 15 also offers a convenient method for computing the Faraday rotation. It turns out that the number of rotations of the plane of polarization is very nearly proportional to Δr^{i}. For the case of a thin layer the relationship between the Faraday rotation and Δr^{i} is

$$\Delta\Omega = \frac{2 \cos \theta}{\lambda_g}\ \Delta r^{i} \qquad (16)$$

where Ω = the number of rotations of the plane of polarization for a double passage through the layer

θ = the angle between the wave normal and the magnetic field

λ_g = gyro wavelength

Both θ and λ_g are functions of position. However the rate of change of these quantities is sufficiently slow so that F region values of these parameters may be treated as constants applicable to the entire path. This point will be discussed in more detail in the section dealing with Faraday rotation.

Ionospheric Model

For the purpose of computation of refractive effects from ionosonde data, it was necessary to postulate a model of the ionosphere which would approximate the observed data. The normally available data contain information from which one can obtain the height of the base of the layer, h_o, the height of the maximum electron density, h_m, and the critical frequency of the layer, f_c. It is therefore desirable to choose a model which has three degrees of freedom and is also in accord with available experimental data regarding the electron density profiles.

The shape of the ionospheric electron density profile below the maximum can be fairly well approximated by a parabolic distribution. The shape of the profile above the region of the maximum density is virtually unknown. It is believed that the electron density above the peak of the F region does not fall off as rapidly as it might have been expected from the Chapman distribution. Also, the Faraday rotation experiments indicate that the total electron content above the maximum density is about three times as large as below it. Using these facts the following model was selected.

$$
\begin{aligned}
N_e/N_o &= 1 - (1 - \sigma)^2 \qquad & 0 \le \sigma \le 1 \\
&= \operatorname{sech} \tfrac{1}{4}\pi\,(\sigma - 1) \qquad & \sigma \ge 1
\end{aligned}
\tag{17}
$$

where N_e = electron density per cubic meter

N_o = maximum density

$\sigma = (h - h_o)/\,y_m$

y_m = half thickness of the parabolic layer

$= h_m - h_o$

h = height above the ground

h_o = height of the base of the layer

h_m = height of the maximum electron density

This model has the following desirable characteristics.

a. The model has three degrees of freedom, (h_o, y_m and N_m,) which can be obtained from ionogramic data. These parameters uniquely specify the entire distribution.

b. The distribution is parabolic below the maximum density, nearly twice as thick parabolic immediately above the maximum and exponential at great heights.

c. The electron content of the distribution above the maximum is three times that below it.

d. The entire electron density profile and its derivative are continuous everywhere.

Figure 7 is a plot of the ionospheric model. The heights of the base, the maximum density and the point of interest define σ, and the ionospheric N^i unit is obtained from

$$N^i = \frac{1}{2} (N_e / N_m) \quad (f_c / f)^2 \times 10^6 \tag{18}$$

where f_c = critical frequency of the layer

$= 8.97 \, N_m^{\frac{1}{2}} \times 10^{-6}$ megacycles

f = signal frequency in megacycles

The h_o, h_m and f_c parameters refer to the F Layer.

In this report the refractive effects of the D and E layer are not singled out. The reason for this is that they are quite small in comparison with those due to the F layer and are approximately accounted for by allowing the electron density at the bottom edge of the F layer to be zero. Furthermore, the shape of the electron density profile above the maximum is not too well known and since this region, as far as the refractive effects of the ionosphere are concerned, is probably much more important than the D and E layers, it was felt that the introduction of a more complicated ionospheric model was not justifiable.

Operational Procedure

Using the computational method described and assuming the ionospheric model of the previous section, the elevation angle error, δ^i, and range retardation Δr^i were computed for a variety of layer heights and thicknesses. Since the magnitude of these effects is proportional to the square of the ratio of critical to signal frequencies, it was convenient to normalize the magnitude by assuming the critical frequency of 1 mc and signal frequency of 100 mc.

Figures 8 - 11 are plots of normalized δ^i for various layer configurations. Computations were carried out for angles of elevation, α_o of 0, 100, 300 and 1000 milliradians and target heights h, of 300 and 1000 n.miles. Since above 1000 n.miles the ionosphere effects are presumably negligible, δ^i for this region may be readily obtained from Figure 5 by noting that ϵ^i beyond the layer is constant.

Figures 12 - 15 are plots of normalized Δr^i for α_o's of 0, 100, 300 and 1000 milliradians. Above 1000 n.miles Δr^i is assumed to remain constant. For elevation angles greater than 1000 mr, Δr^i is very nearly proportional to .86 csc α, where α is determined in the F region. At vertical incidence csc α is equal to unity.

From Figures 8 through 15, δ^i and Δr^i can be readily obtained if the ionospheric parameters are given. This procedure becomes exceedingly tedious if it is employed on a large amount data. The following alternate procedure has been adopted. Since the accuracy of the data, as well as that of the ionospheric model is limited, very little is lost if δ^i and Δr^i are quantized in 10% steps. Thus, for a given combination of h_o, y_m, α_o and the target height h, an integer "p" is defined so that normalized δ^i (or Δr^i) is approximately equal to 1.1^p.

For a fixed value of α_o and a target height h, a matrix of "p" can be constructed where rows and columns represent equal values of h_o and y_m with 10 km increments. Similarly, an integer "q" is defined so that $f_c^2 = 1.1^q$. The total δ^i (or Δr^i) is then approximately equal to 1.1^{p+q}. This method is particularly convenient since the values of p range from about -5 to +25 and those of q from about 5 to 60 such that for any one ionogram where h_o, y_m and f_c are known, p + q can be obtained immediately. Since it is preferable to plot δ^i (or Δr^i) on a logarithmic scale, p + q can be plotted directly on an appropriate linear scale and thus minimize data handling problems.

It was also found that over a considerable range of h_o and y_m, ratios of δ^i and Δr^i as computed for $\alpha_o = 0$ to those at other elevation angles remain reasonably constant. Since at any one location, time of day and year the values of h_o and y_m from day to day do not differ appreciably, very little error is introduced by computing δ^i and Δr^i for the case of $\alpha_o = 0$ and scaling them to other angles of elevation by multiplying by an appropriate factor based on the average values of h_o and y_m. The statistical parameters obtained by this method were spot checked against those obtained using the more accurate but much more time consuming procedure and the agreement was found to be within a few percent.

Reduction of Data

As stated in the introductory remarks, one of the aims of this investigation was to obtain statistical information on elevation angle and range errors at three different locations; Inverness or Leuchars, Scotland; Thule, Greenland; and Fairbanks, Alaska. Since ionospheric parameters are subject to wide fluctuations, it was necessary to select representative situations and group them in some logical fashion. The three most important parameters are the sun-spot cycle, the seasonal, and the diurnal variations. In order to keep the amount of detailed analysis down to a reasonable amount and still obtain meaningful results, it was decided that the analysis would be carried out for March, June, September and

December of 1954 and 1957. These two years represent, respectively, low and high sunspot activity. Since there is no 1954 data available for Thule, the data for Resolute Bay, Canada were used instead. Thule and Resolute Bay lie in reasonably comparable geographic and geomagnetic latitudes and thus ionospheric data from the two locations are similar.

Three types of ionospheric data were used to obtain the statistical information on elevation angle and range errors. These were,

a. 35 mm ionogramic film

b. Detailed ionospheric tabulation from the U S and the Canadian operated ionospheric stations

c. Detailed ionospheric tabulations from the British operated ionospheric stations.

The problem was to obtain from this data the parameters of the approximating parabolic distributions.

The British data which were used to obtain information on Inverness during 1954 and the first half of 1957, were already tabulated in terms of the parabolic parameters obtained by the Appleton-Beynon method. Consequently, aside from a magnetic correction which is described later the reduction of the British data was quite straightforward.

The American and the Canadian tabulations posed more of a problem since the approximating parabolic parameters had to be computed indirectly from the listings of the h'f and the M (3000) F_2 parameters. The details of this procedure are described in SRA Report #80, but a very brief resumé is given below. It has been shown by Shimazaki(2) that there is a very good one-to-one correspondence between the M(3000)F_2 factor and the h_m. One might also expect that since h'F and h_o represent the height of the bottom edge of the F layer, barring a serious retardation in the E region, the two quantities should be identical, if the layer were truly parabolic. Unfortunately this is not the case. In general, there is a difference between the two and this difference is due primarily to the fact that the approximating parabolic distribution is derived on the basis of the best fit, in the region of the greater electron density. It is, however, reasonable to expect that fluctuations in one quantity should be approximately equal to the fluctuations in the other. It was noticed from the examination of the British tabulations that at higher latitudes the average value of y_m was reasonably insensitive to change in location. Thus a correction to h'F for obtaining h_o was specified by requiring that the average value of y_m at a particular time should correspond to the value observed at the British stations. The results using this procedure appeared to compare quite well with the results obtained from the examination of the 35 mm film. This particular procedure was used for the following data: 1954 Fairbanks and Resolute Bay; 1957 Thule; March and June 1957 Fairbanks; and September and December 1957 Inverness. The reason for using this method on the Inverness data was that starting with the IGY the British began to tabulate their data in terms of the h'F and the M(3000) F_2. The remaining (and incidentally the most important) parameter, the critical frequency of the F layer, is tabulated by both the British and the American sources.

The 35 mm ionogramic film data were used to reduce September and December 1957 Fairbanks data. The procedure used was a hybrid between the Appleton-Beynon and the modified Kelso methods. Essentially what was done was to reduce the ionogram into the best fitting parabolic distribution by the use of the Appleton-Beynon method and then to apply a magnetic correction which was obtained in the following manner.

Using the no-field vertical heights of idealized parabolic distributions, which fits surprisingly well more than 80% of ionograms, true height profiles were obtained by the use of the modified Schmerling-Kelso coefficients as published in the NBS Report #6031.[4] The resulting true height distributions were then fitted into parabolic curves and the differences between the original and the modified parabolic parameters were tabulated as a function of layer height, thickness, and critical frequency. It was found that the main effect of the magnetic correction was to reduce the no-field value of y_m by 30, 25 or 20% depending whether the critical frequency was less than 4.5 mc, between 4.5 and 8.0 mc or greater than 8.0 mc. This very simple rule of thumb, which was also used to correct the tabulated data, results in the maximum error in y_m of ±5 km.

Ionospheric Elevation Angle Error at Finite Heights

A rather interesting phenomenon can be seen if δ^i is plotted versus α_o for a variety of target heights; h, as shown in Figure 16. For all finite target heights the slope, $d\delta^i/d\alpha_o$ is positive for small α_o. For target heights of 200 or 300 n.miles the value of maximum δ^i occurs between 100 and 200 milliradians and is about 10% higher than the value at zero elevation angle. Due to the obvious implications that this type of behavior would have on tracking problems, this phenomenon was investigated in some detail.

If it is assumed that the ionosphere consists of a layer defined by heights h_1 and h_2 and whose refractivity decreases linearly from zero to $-|N^i|$, it follows from equations 11, 12 and 14 that

$$\delta^i = \frac{|N^i| (\tan \alpha_2 - \tan \alpha_1)}{(\tan \alpha_2 + \tan \alpha_1) (\tan \alpha_2 - \tan \alpha_o)} \qquad (19)$$

where subscripts 1 and 2 refer to the value of the parameters at the base and the top of the layer.

It was shown by L. Colin of R.A.D.C. that when equation 19 is differentiated with respect to α_o and evaluated for the special case of α_o equal to zero the result is

$$\left.\frac{d\delta^i}{d\alpha_o}\right|_{\alpha_o = 0} = \frac{\varepsilon^i \tan \alpha_1}{(\varepsilon^i + \tan \alpha_1)^2} \qquad (20)$$

Since ε^i is always a positive quantity it follows that the slope of δ^i at finite heights evaluated at $\alpha_o = 0$ must always be positive. On the other hand, the tropospheric counterpart of equation 20, equation 21 is always negative.

$$\left.\frac{d\delta^t}{d\alpha_o}\right|_{\alpha_o = 0} = \frac{-2N_o \times 10^{-6}}{(\varepsilon^t + \tan \alpha_T)^2} \qquad (21)$$

where N_o = surface refractivity

and α_T = inclination angle at the top of the troposphere.

Colin estimated that under typical conditions the signal frequency would have to be below 20 or 30 mc before the initial slope of the total δ becomes positive. This does not, however, preclude the possibility of a hump or a shoulder in δ^i vs α_o at higher frequencies at angles of elevation greater than zero.

To determine the value of α_o at which δ^i is maximum, equation 19 is differentiated with respect to α_o and the derivative is set equal to zero. The resulting expression is quite complicated but it can be greatly simplified if it is assumed quite correctly, that the required value of α_o is small. Under these conditions the following approximations can be made.

$$\sin 2\alpha_2 \pm \sin 2\alpha_1 \cong 2 \sin (\alpha_2 \pm \alpha_1)$$

and

$$\sin 2\alpha_2 - \sin 2\alpha_o \cong 2 \sin (\alpha_2 - \alpha_o)$$

The resulting expression for α_o at which δ^i is maximum is

$$\alpha_o = \left[(\rho^2 - a^2) \,/\, (4\rho^2 - a^2)\right]^{\frac{1}{2}} \qquad (22)$$

where α_o = elevation angle in radians

a = Earth's radius

ρ = distance from the Earth's center to the base of the layer

For typical F region parameters one might expect maximum elevation angle error due to the ionospheric refraction at ionospheric target heights to occur between 100 and 200 milliradians.

Figure 16, which is a plot of δ^i vs α_o for the much more realistic models of the ionosphere shown in Figure 7, is in good agreement with the rough estimate of Equation 22.

Figure 17 is a plot of Δr^i vs α_o for several different ionospheric models and target heights. Unlike δ^i, the range error continuously decreases with an increase in the elevation angle.

Faraday Rotation

It was shown in SRA Report #68 that the number of rotations due to the Faraday rotation is very nearly proportional to the ionospheric range error, Δr^i. It is given by

$$\Omega = 2\,\lambda_g^{-1} \cos\theta\, \Delta r^i \qquad (23)$$

where Ω = number of rotations due to a double passage through the ionosphere

θ = angle between the magnetic field and the direction of propagation

λ_g = gyro wavelength in meters

= 107/B

B = Intensity of the Earth's magnetic field in gauss

Δr^i = range error in meters

The value of λ_g and θ vary sufficiently slowly along a path so that little error is introduced if they are treated as constants based on the F region values. For target heights within the ionosphere, these constants were evaluated at 300 km height and for target heights above the ionosphere the values are based on the 500 km height.

Table VII lists the values of $2\,\lambda_g^{-1} \cos\theta$ for a variety of azimuths and elevation angles, at the three sites. The magnetic field orientation was assumed to be the same as on the surface and the intensity was assumed to be inversely proportional to the cube of the distance from the Earth's center. This table has to be used in conjunction with corresponding Δr^i tables (XII through XV). For example, at Fairbanks in midday December 1957 at elevation angle σ_o = 100 mr, target height 300 n.mi., azimuth + 30° (measured East from true North) and signal frequency 100 mc, the expected median number of rotations of the polarization plane for a double passage through the ionosphere is

$$(3.44 \times 10^3)\ (2.30 \times 10^{-3}) = 7.9 \text{ rotations}$$

Interpretation of Δr^i and δ^i Distribution Plots

The distributions of Δr^i and δ^i for zero elevation angle and a target height of 1000 n.mi. are shown in Figures 18 through 21. These figures are plots of the number of points with a given error at 100 mc (horizontal scale) as a function of time of day (vertical scale) for each station-month analyzed.

These distribution plots show the following interesting characteristics:

a. When there is a diurnal variation it is strongest in December and weakest in June.

b. The shape of the diurnal variation is the same throughout the sunspot cycle, but the values of the errors are approximately five times as great during the sunspot maximum as during the sunspot minimum.

c. The diurnal variation is greatest at the station farthest from the geomagnetic pole and almost non-existent near the pole.

d. The relative spread of values at any one hour is greater during the night time hours than during the daylight hours and is larger near the pole than away from the pole.

e. The diurnal variation may exceed an order of magnitude.

Of the three factors which affect the magnitude of δ^i and Δr^i, the critical frequency of the layer appears to be the most dominant. It is for this reason that the diurnal shapes of δ^i and Δr^i are quite similar. The second factor which influences the magnitude of the error is h_o. The lower the layer the more oblique is the angle of entrance into the layer and consequently the greater the error. The third factor, y_m, has a greater effect on Δr^i than δ^i.

In general the critical frequency has a more pronounced diurnal variation in winter than in summer, is greater at Inverness than at Thule, and is generally higher during the peak of the sunspot cycle than at its minimum. However, the effects of h_o on δ^i and Δr^i cannot be neglected. In December, h_o decreases during the day and increases at night approximately in phase with f_c. The combination of effects greatly accentuates the diurnal variation in δ^i and Δr^i. In June, h_o is approximately constant, but the variation which does occur is in such a direction as to partially cancel out the smaller effects of f_c, resulting in almost no diurnal variation of δ^i. The variations in y_m have a similar effect on Δr^i.

This behavior is most clearly illustrated by means contours of constant δ^i and Δr^i plotted versus time of day and year. These contours, shown in Figures 22, 23 and 24 have been prepared by taking the monthly mean values of h_o, y_m and the critical frequency of the F layer and computing δ^i and Δr^i for the case of zero elevation angle and 1000 n. mile target heights.

The diurnal variations are most accentuated at Inverness and during the winter months. A three dimensional pictorial representation gives a "hill" in midday winter and a valley during the winter night. During the summer, diurnal variation is almost nonexistent. Fairbanks displays a similar situation but it is less pronounced. The situation at Thule is very poorly defined. Figure 22 is a contour plot of δ^i and Δr^i for Inverness during 1954. The overall shape is quite similar to that of 1957 although the magnitudes are very much smaller. An interesting point about these contours is a tendency for a second minimum which occurs at 0300 in September. Both the 1954 and the 1957 data exhibit this behavior. The reason for this not clear.

Explanation of Tables for δ^i and Δr^i

The distribution plots of δ^i and Δr^i, Figures 18 thru 21, show a pronounced diurnal trend on most of them. Because of this diurnal behavior, it was thought advisable to split the day into parts in order to arrive at more meaningful means and standard deviations. The procedure was to choose a few hours around the maximum which appeared fairly homogeneous for one part, a few hours around the minimum for a second part, and the remaining hours for the third part. In the case of Thule and Resolute Bay, which showed no pronounced diurnal behavior, the entire day was lumped together. Since it was found that most of the distributions obtained in this manner are approximately normal distributions, they were plotted on probability paper. From such plots the medians and standard deviations were obtained.

The results are given in Tables VIII thru XV, where the medians and standard deviations σ, of these distributions are given as a function of elevation angle, α_0, target height, h, station, year, month and time of day. These results are for a signal frequency of 100 mcs. To find the value of the error at any other frequency these values must be divided by the square of the frequency in hundreds of megacycles.

COMBINATION OF IONOSPHERIC AND TROPOSPHERIC RESULTS

As previously stated, both the ionospheric and tropospheric distributions of δ and Δr are approximately normal. Furthermore, under the assumptions made, the tropospheric δ^t's and Δr^t's and the ionospheric δ^i's and Δr^i's are additive.

From the elementary theory of probability[3] it can be shown that the sum of two independent normal distributions is itself a normal distribution with

$$\mu_{A+B} = \mu_A + \mu_B$$

$$\sigma_{A+B} = (\sigma_A^2 + \sigma_B^2)^{\frac{1}{2}}$$

where μ = median

σ = standard deviation

Subscripts A, B and A + B refer to the separate and the combined distributions.

Consequently, to obtain the statistics of the combined tropospheric and ionospheric effects it is only necessary to add the individual medians to find the median value of the combined effects, and take the square root of the sum of the squares of the individual standard deviations to obtain the standard deviation of the combined effects. This has been done for a few of the more significant cases covered in this study. The extreme results are shown in graphical form in Figures 25 thru 28, which are plots of the mean and standard deviation of δ at Inverness for the months of December and June as a function of α_o with frequency as a parameter for a given target height. The tropospheric distributions of δ^t for target heights of 100, 300, 1000 and 2500 n.mi. are tabulated in Tables XVI, XVII and XVIII. These distributions were made by month and station only, since no significant diurnal variations occurred.

Tables XIX thru XXIX give the combined results at 100 mc and 400 mc for the extremes (December, day and night) and for a more normal situation (June) for each site in 1957. In addition, the same results are given for Inverness for the year of 1954. The combined results at all sites are close to the tropospheric values even at a signal frequency of 100 mc in 1954.

As an example of the procedure for combining components, let us find the combined elevation angle error, δ, at Fairbanks, for December 1957 noon for a target height of 300 n.mi., an elevation angle of 100 mr and a signal frequency of 100 mc.

Looking at Tables XVIII and X we find

$$\text{median } \delta^t = 2.84 \text{ mr}$$

$$\text{median } \delta^i = 5.38 \text{ mr}$$

$$\sigma^t = .18 \text{ mr}$$

$$\sigma^i = 2.25 \text{ mr}$$

The median value of the combined δ's is 2.84 + 5.38 = 8.22 mr, and the combined standard deviation is $\left[(.18)^2 + (2.25)^2\right]^{\frac{1}{2}} = 2.26$ mr.

APPLICATION OF OPERATIONAL PROCEDURES

The following is a detailed illustration of the operational procedures which may be used to compute refractive elevation angle error, range error and the Faraday rotation from available radiosonde and ionogramic data.

A typical radiosonde ascent was selected as an example. This data, as well as point by point computation of tropospheric bending γ^t, and the elevation angle error δ^t, is shown in Table XXX.

The first and second columns list the value of N as a function of h as found from the radiosonde data. Columns three and four give the values of $N_o - N_k$ and $N_j - N_k$ at various heights. Columns five and six give the values of 500 tan β for α_o's of 0 mr and 300 mr at these heights. The values of 500 tan β have been obtained from Figure 2. The detailed procedure for the use of Figure 2 is given in the section on tropospheric refraction. These values are combined to give the values of $\Delta\gamma$ given in the seventh and eighth columns. The $\Delta\gamma$'s are then summed to give the values of γ given in the ninth and tenth columns. Table XXX also lists the steps used to find δ from γ at target heights of 10 K ft, 50 K ft and 300 K ft respectively at the two elevation angles.

As explained in the tropospheric section, these δ's must be extended to greater heights in order to combine with the ionospheric δ's. To do this we must use the expression for δ^t_H given in the tropospheric section,

$$\delta^t_H = \delta_{300} + (\gamma_{300} - \delta_{300}) \frac{\tan \alpha_H - \tan \alpha_{300}}{\tan \alpha_H - \tan \alpha_o}$$

The total tropospheric range error Δr^t in this case is

$$\Delta r^t = \frac{.000136\ (313.1)^2}{\sin \alpha_o + (\sin^2 \alpha_o + .001879)^{\frac{1}{2}}}$$

$$= \frac{13.3}{\sin \alpha_o + (\sin^2 \alpha_o + .001879)^{\frac{1}{2}}}$$

For $\alpha_o = 0$; $\Delta r^t = 308$ ft, and for $\alpha_o = 300$ mr; $\Delta r^t = 22$ ft.

The reduction of the ionospheric data requires some knowledge of ionogram interpretation. This subject is treated in detail in a number of different places. An excellent introductory treatment may be found in Vol. 3 pt. I of the Annals of the International Geophysical Year (Pergamon Press 1957). For the purpose of this illustration a few very brief remarks will be made.

The ionogram shown in Figure 29 is essentially a plot of virtual height of reflection versus frequency. The virtual height is defined as the height that the radio pulse would have reached had it travelled with a free space velocity. Due to the Earth's magnetic field, the signal upon entering the ionospheric layer splits up into two elliptically polarized components with opposite sense of rotation and slightly different signal velocities. These components labelled O and X are called the ordinary and the extraordinary components. The critical frequency is the minimum frequency at which the signal penetrates the ionospheric layer and the virtual height becomes infinite. In the illustration, the critical frequency of the ordinary component is 10.4 mc. The trace in the 500 to 700 km region is a double reflection. The trace between 600 and 900 km and in the vicinity of 5 mc is an interfering signal.

To obtain the approximating parabolic parameters (by the Appleton-Beynon method) we plot virtual height versus the parameter $\varphi(f/fc)$ which is defined by

$$\varphi(f/fc) = \frac{1}{2}\,(f/fc)\,\ln \frac{1 + f/fc}{1 - f/fc} \qquad (24)$$

where fc is the critical frequency of the ordinary component.

If the electron profile were truly parabolic the plot of virtual height h' versus $\varphi(f/fc)$ would result in a straight line given by

$$h' = h_m + y_m \varphi(f/fc)$$

The details of the ionogram reduction are given by the following table.

f/fc	.648	.725	.757	.834	.867	.901	.925
φ(f/fc)	-1/2	-1/3	-1/4	0	1/4	1/3	1/2
h'	270	280	290	305	330	340	350

From the plot of $\varphi(f/fc)$ vs h' shown in Figure 29 and Equation 24 we obtain y_m = 85 km h_o = 225 km.

Since the critical frequency is greater than 8 mc, the magnetic correction for y_m is 20% and the corrected values are

y_m = 68 km

h_o = 225 km

Using figures 8, 10, 12 and 14, we find the values of normalized δ^i and Δr^i

α_o \ h	normalized δ^i (mr) 300 n.mi.	1000 n.mi.	normalized Δr^i (meters) 300 n.mi.	1000 n.mi.
0 mr	.041	.042	245	269
300 mr	.037	.0248	190	205

Since the critical frequency is 10.4 mc and assumed signal frequencies are 100 and 400 mc, the factors by which the normalized parameters have to be multiplied are $(10.4/1)^2\ (100/100)^2$ and $(10.4/1)^2\ (100/400)^2$ respectively. The results are:

α_o \ h	δ^i (milliradians) 100 mc 300 n.mi.	100 mc 1000 n.mi.	400 mc 300 n.mi.	400 mc 1000 n.mi.
0 mr	4.44	4.54	.278	.284
300 mr	4.00	2.68	.250	.167

α_o \ h	Δr^i (meters) 100 mc 300 n.mi.	1000 n.mi.	400 mc 300 n.mi.	1000 n.mi
0 mr	2650	2910	166	181
300 mr	2060	2220	129	153

The total elevation angle and range error are simply the sum of the tropospheric and the ionospheric components.

The Faraday rotation is obtained by multiplying Δr^i by the appropriate factor from Table VII. Thus, if the true azimuth is 300° and the elevation angle 300 mr, the multiplying factors for Δr^i to obtain the number of rotations for a double passage through the ionosphere are .00367 for the 300 n.mi. target height and .00380 for the 1000 n.mi target height. Using these values one obtains the following:

α_o \ h	Ω (revolutions) 100 mc 300 n.mi.	1000 n.mi.	400 mc 300 n.mi.	1000 n.mi.
0 mr	9.71	11.05	0.61	0.69
300 mr	7.55	8.43	0.47	0.51

SUMMARY AND RECOMMENDATIONS

As a result of the work done under Engineering Changes A and B of Contract AF30(602)-1624, and reported herein, simplified methods have been developed for computing radar elevation angle and range errors, as well as total ray bending and Faraday rotation. These methods are in a form to utilize radiosonde and ionogram data for defining refractivity profiles. Details of these methods have been presented in this report, as well as sample calculations on a nearly simultaneous set of radiosonde and ionogram data.

The methods were used on radiosonde and ionogram data for Fairbanks, Thule and Northern Scotland. Statistical distributions were derived for these locations, which allow one to assess the radar refraction errors as a function of radar frequency, target height, elevation angle and time. The occurrence of surface ducts, was also determined at each location and the effects on terrain and sea clutter were determined at various radar frequencies.

It is recommended that the methods presented in this report be used in conjunction with radiosonde and ionogram data for a particular time and station to obtain the refraction effects to be expected for a particular radar location.

The statistical data presented, together with the procedures given for its use, will give an adequate picture of the average values (and the standard deviations) of the effects to be expected at the three specified sites.

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(2) Shinazaki, T.; Jour. Radio Res. Lab. 2, 85 (1955)

(3) Munroe, M. E.; Theory of Probability McGraw--Hill 1951 pp 92, 93

(4) Wright, J. W. and Norton, R. B.; NBS Report 6031 (Feb. 1959)

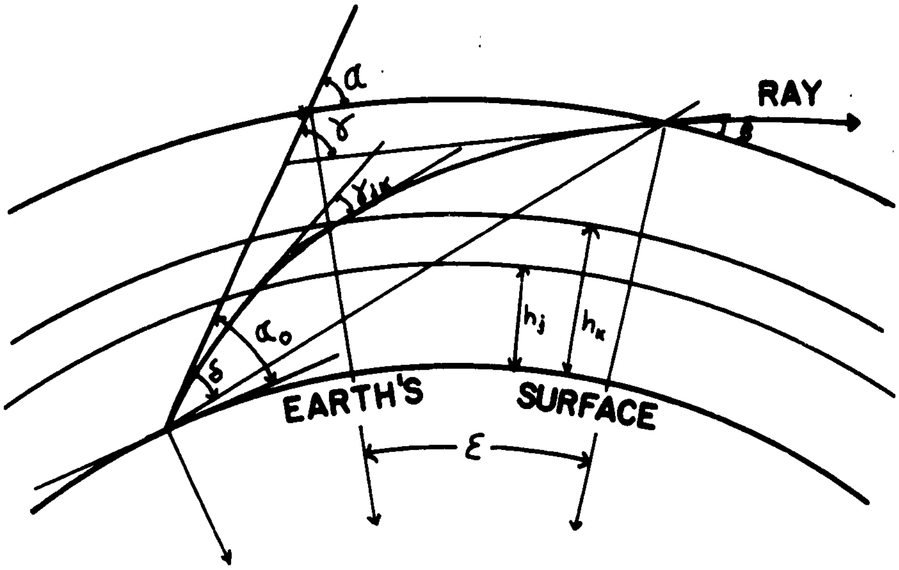


FIG. I GEOMETRY OF REFRACTION

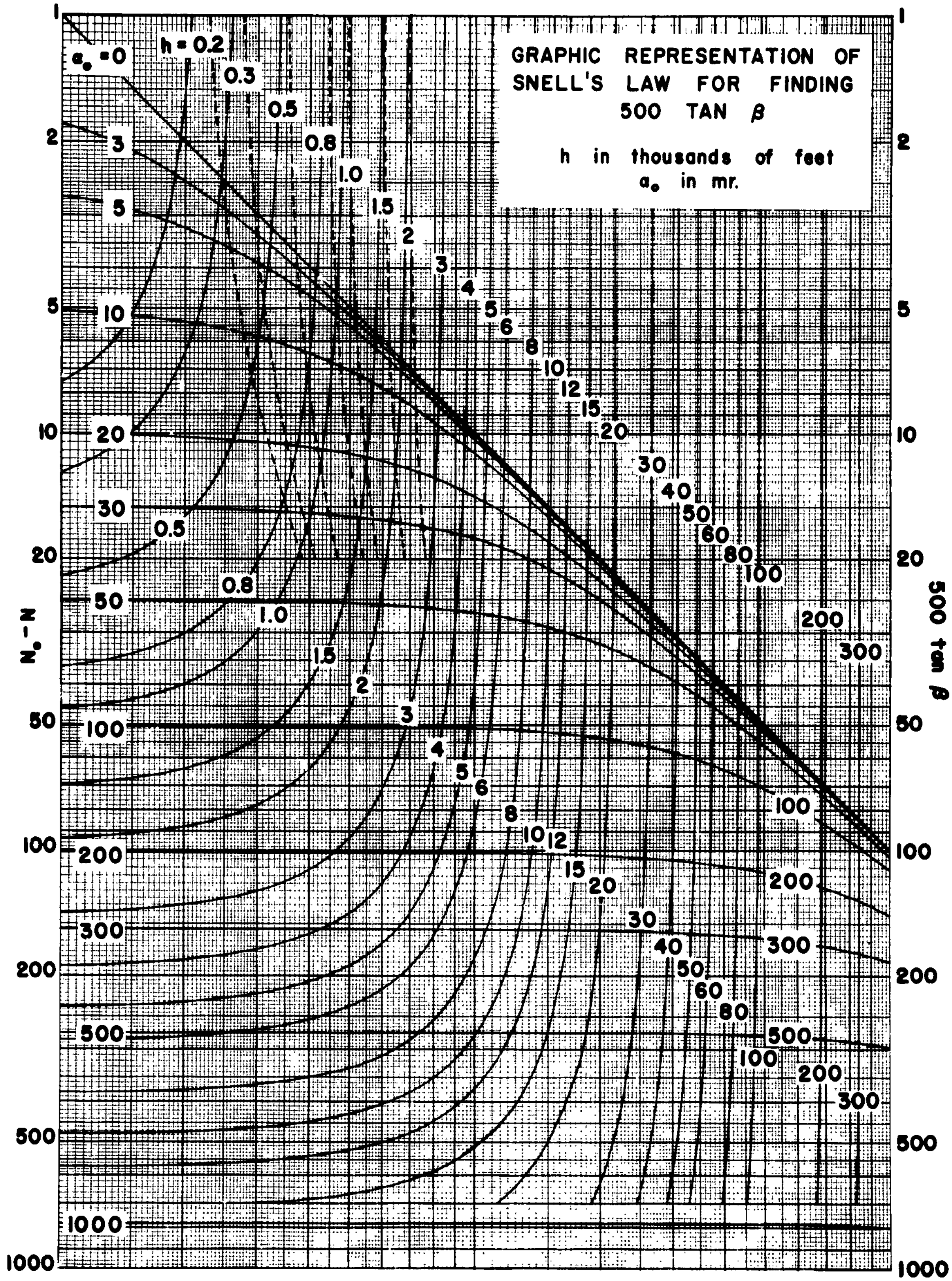

GRAPHIC REPRESENTATION OF SNELL'S LAW FOR FINDING 500 TAN β
h in thousands of feet
a_o in mr.
$N_o - N$
500 tan β

FIGURE 2

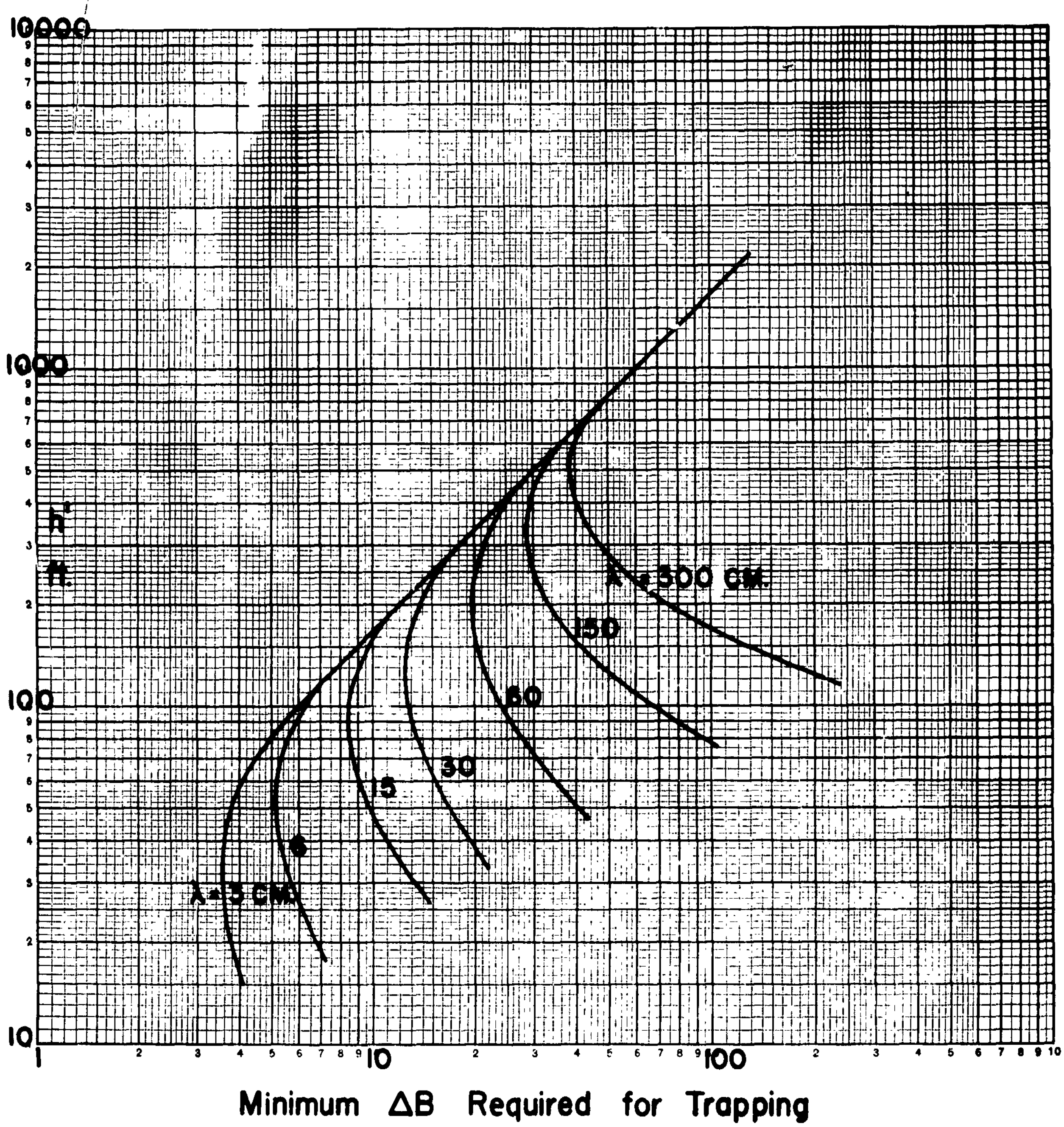
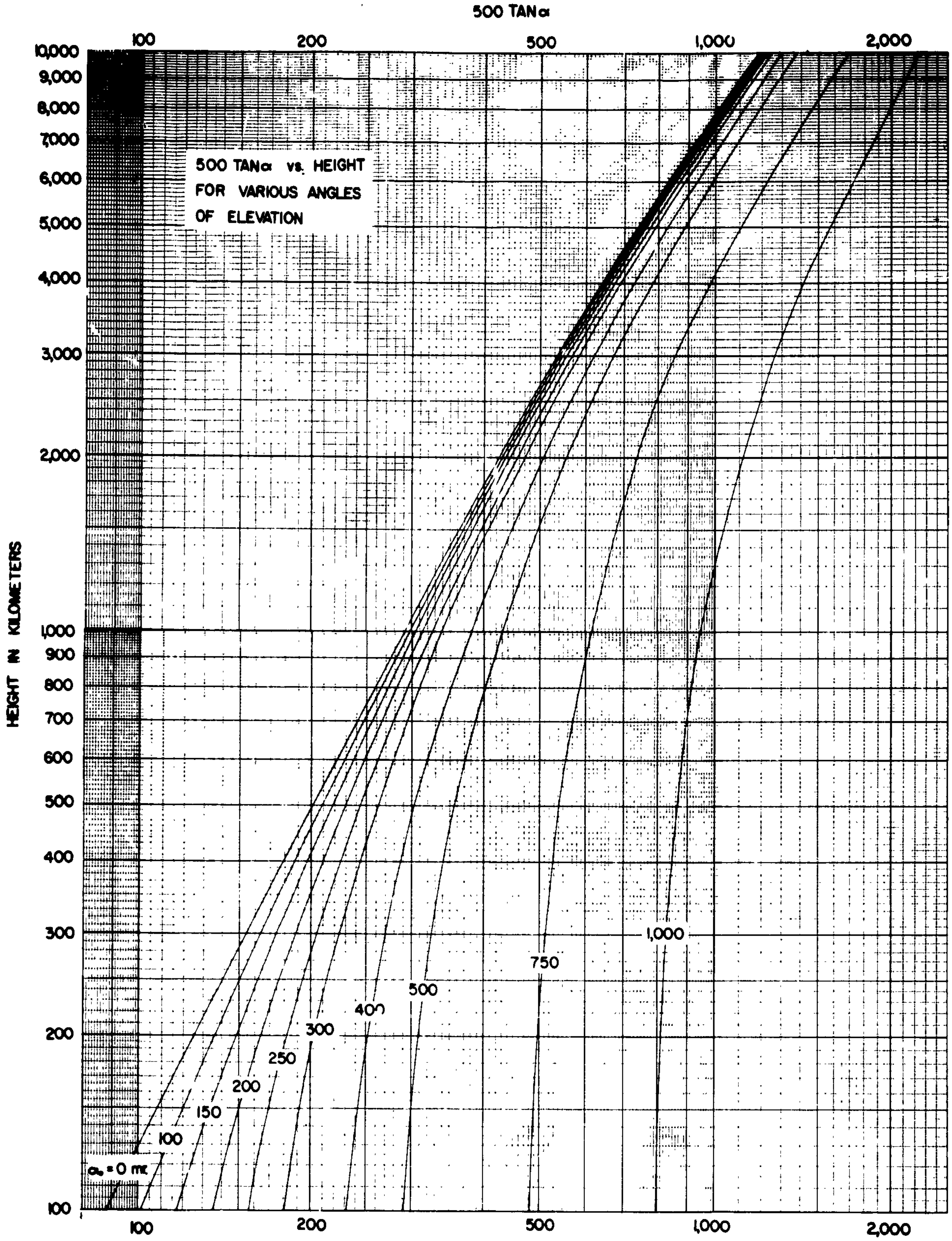

h'
ft.
λ = 300 CM.
150
60
30
15
6
λ = 3 CM.
Minimum ΔB Required for Trapping

FIG. 3



500 TAN α
500 TAN α vs. HEIGHT
FOR VARIOUS ANGLES
OF ELEVATION
HEIGHT IN KILOMETERS
α₀ = 0 mr
100
150
200
250
300
400
500
750
1,000

FIG. 4

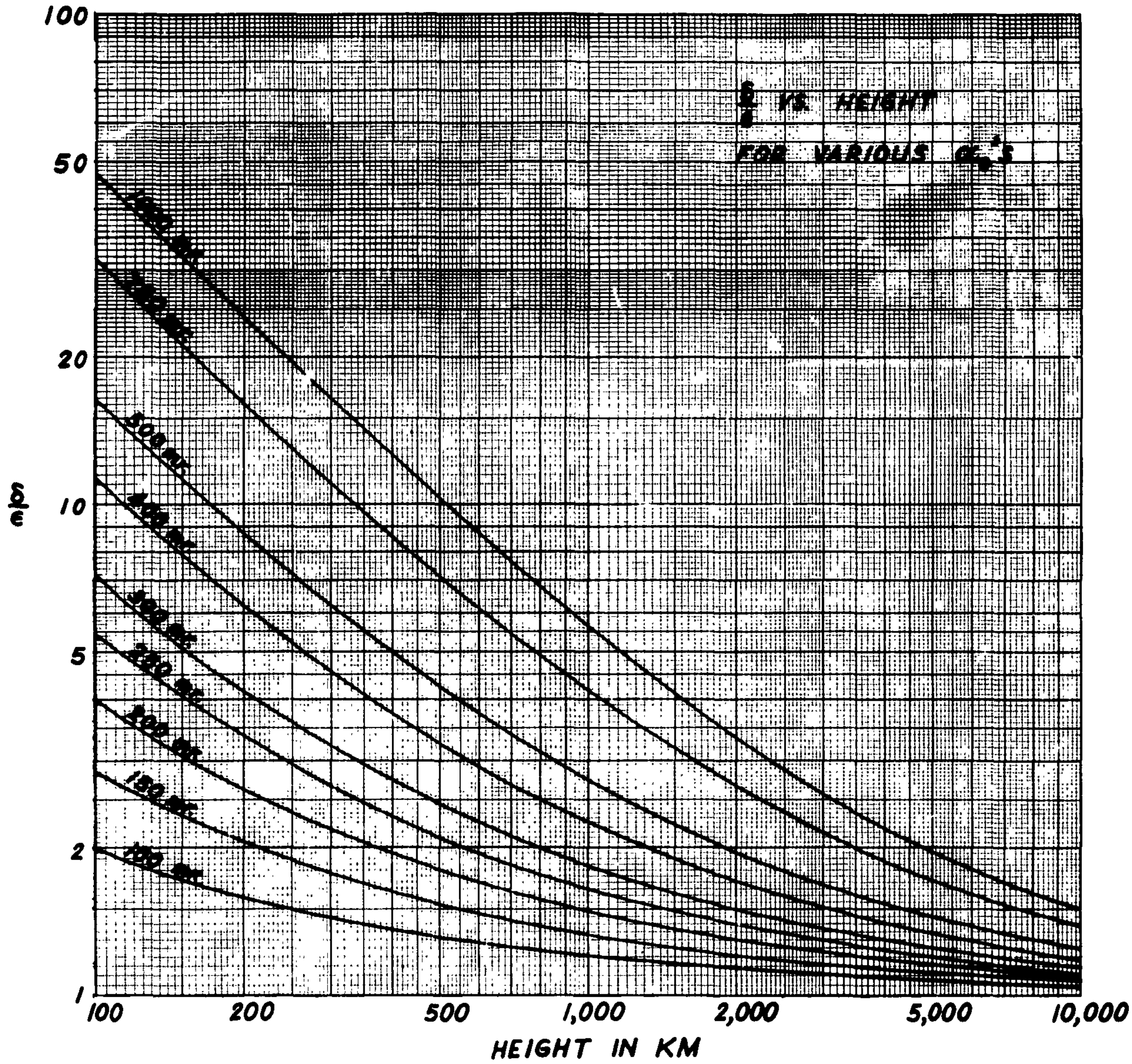
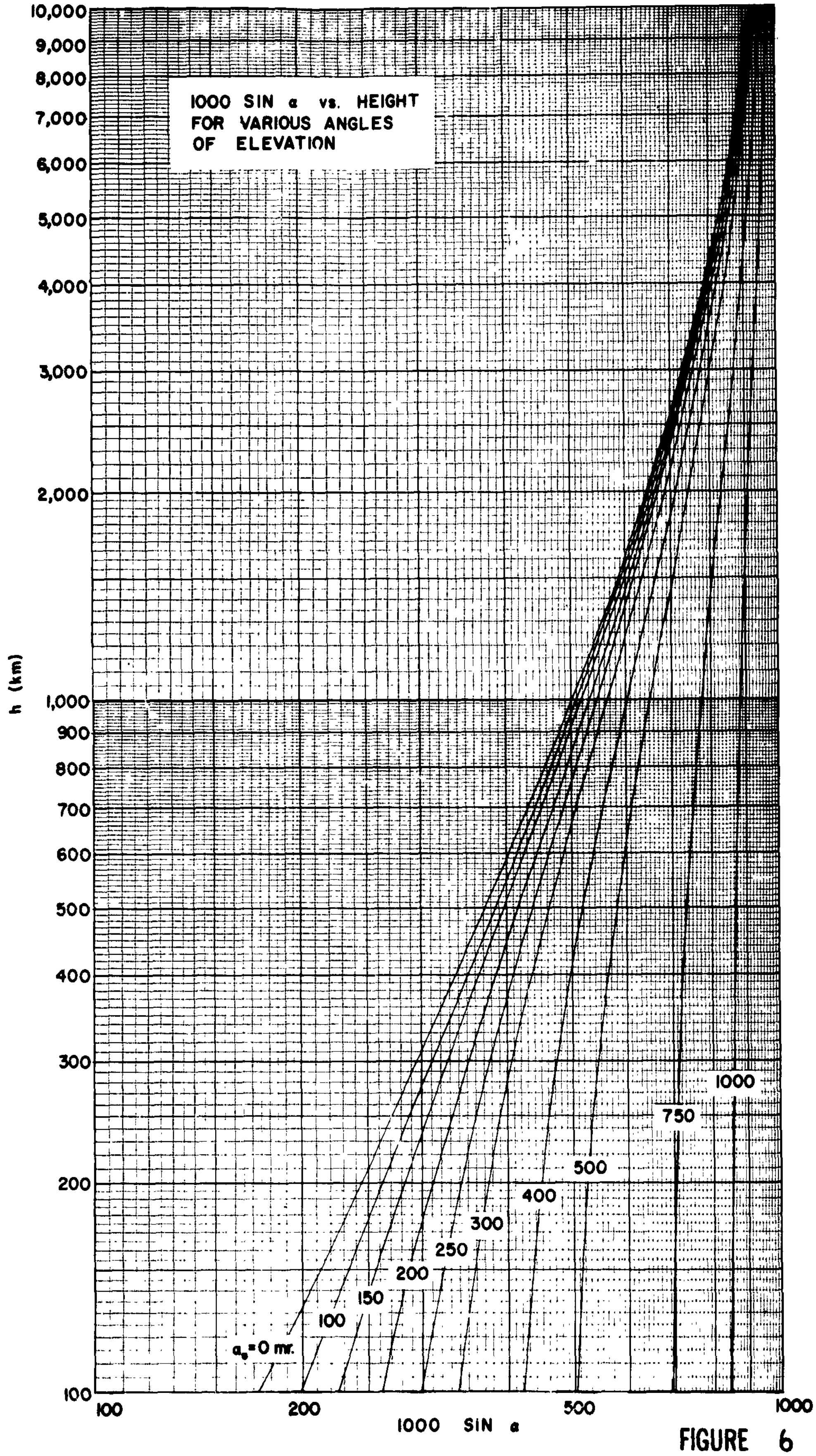

VS. HEIGHT
FOR VARIOUS
δ/ε
100
50
20
10
5
2
1
100
200
500
1,000
2,000
5,000
10,000
HEIGHT IN KM
500 MC

FIG. 5



1000 SIN α vs. HEIGHT
FOR VARIOUS ANGLES
OF ELEVATION
h (km)
10,000
9,000
8,000
7,000
6,000
5,000
4,000
3,000
2,000
1,000
900
800
700
600
500
400
300
200
100
1000 SIN α
100
200
500
1000
α_0 = 0 mr.
100
150
200
250
300
400
500
750
1000

FIGURE 6

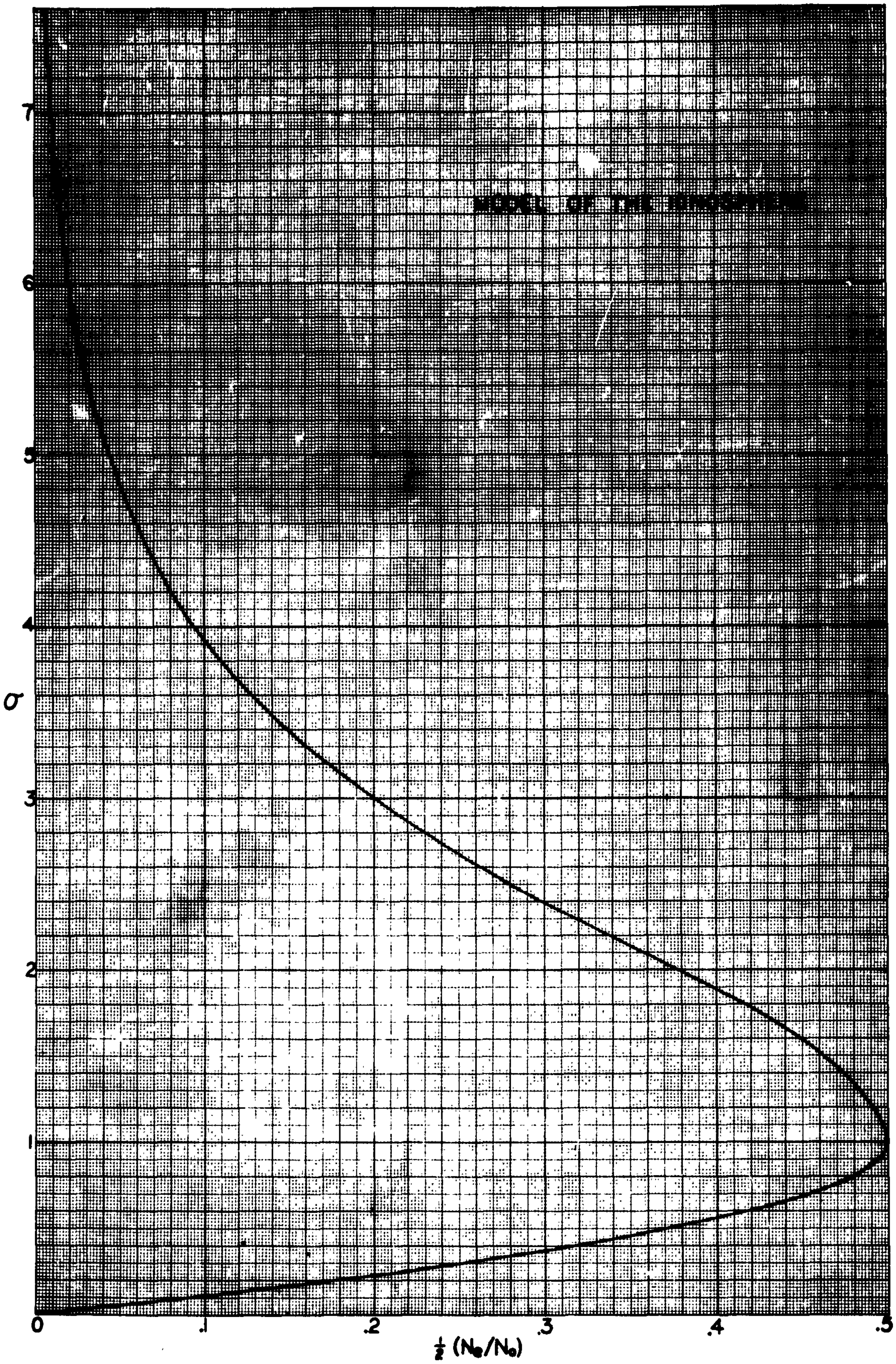
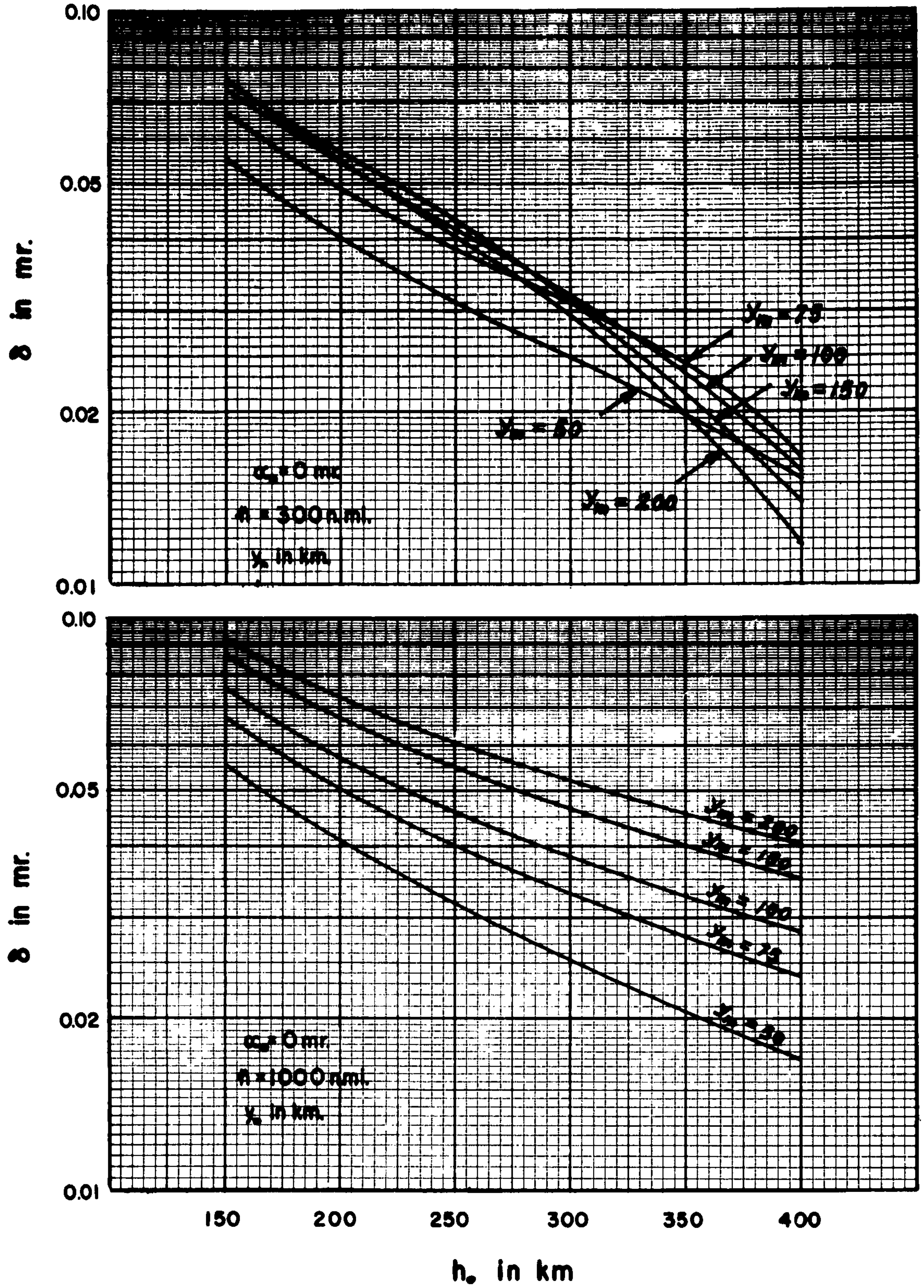


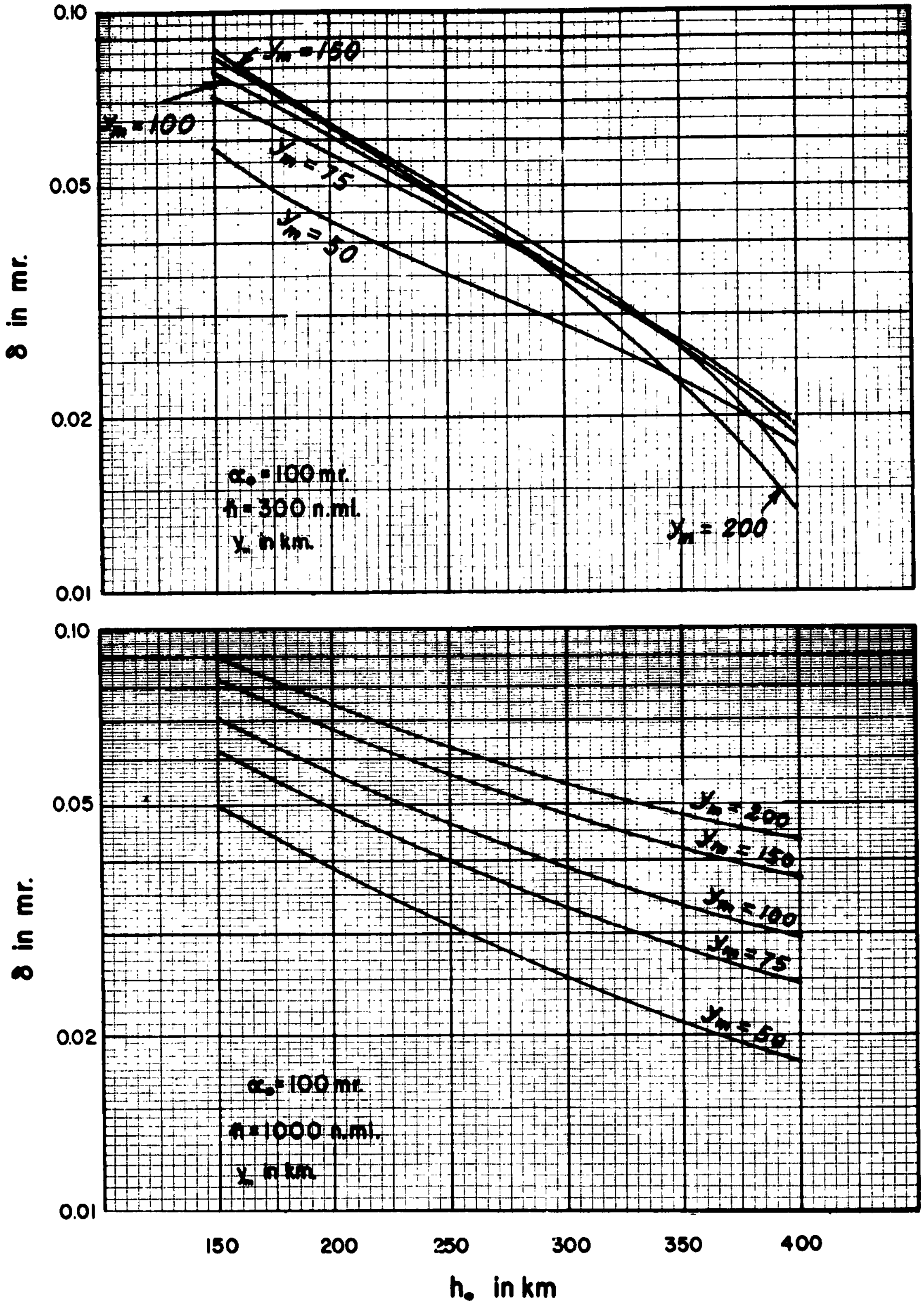
FIG. 7



ELEVATION ANGLE ERROR

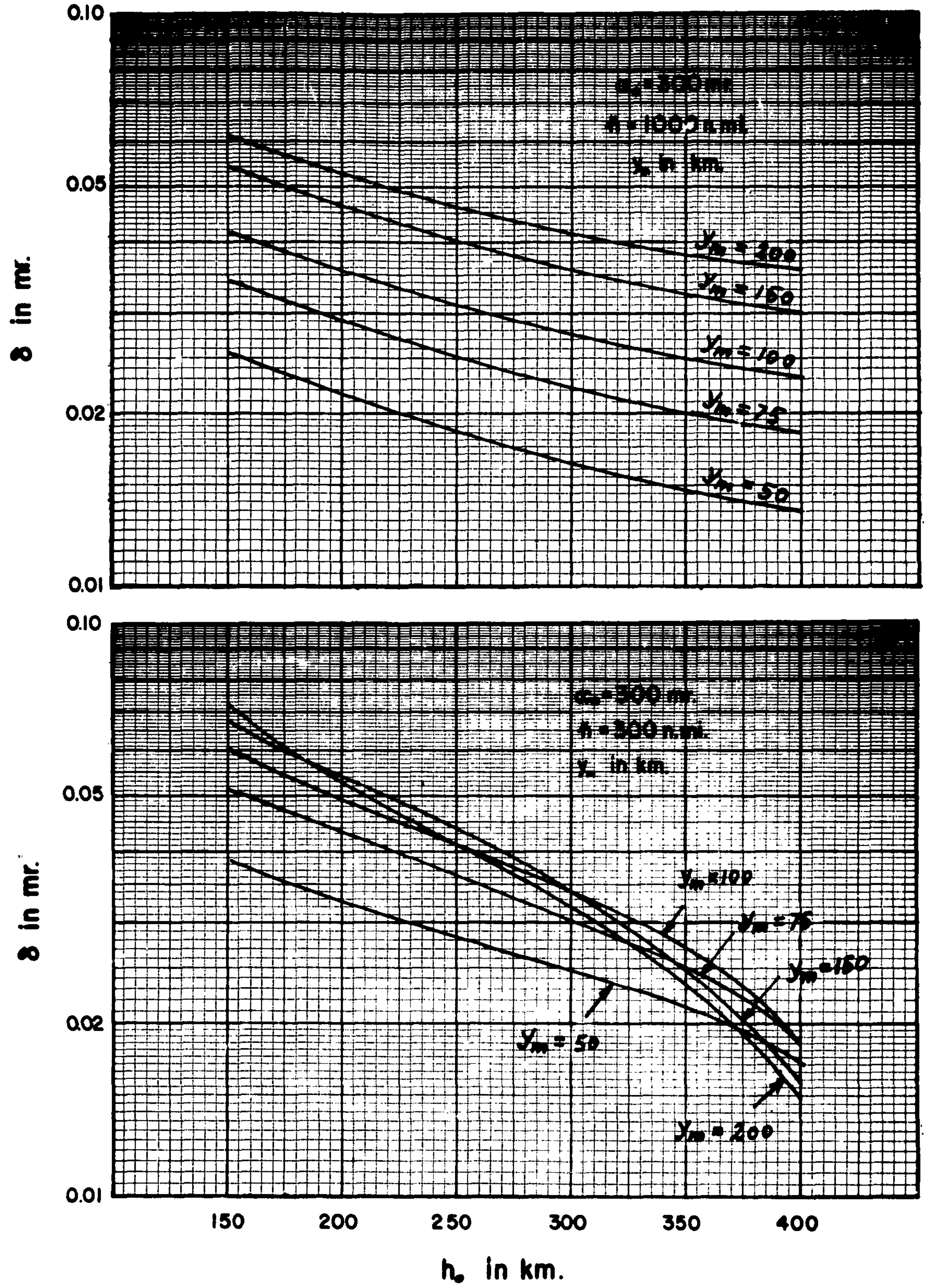
CRITICAL FREQUENCY —— 1.0 MC.
SIGNAL FREQUENCY —— 100 MC.

FIG 8



ELEVATION ANGLE ERROR
CRITICAL FREQUENCY —— 1.0 MC.
SIGNAL FREQUENCY —— 100 MC.

FIG 9

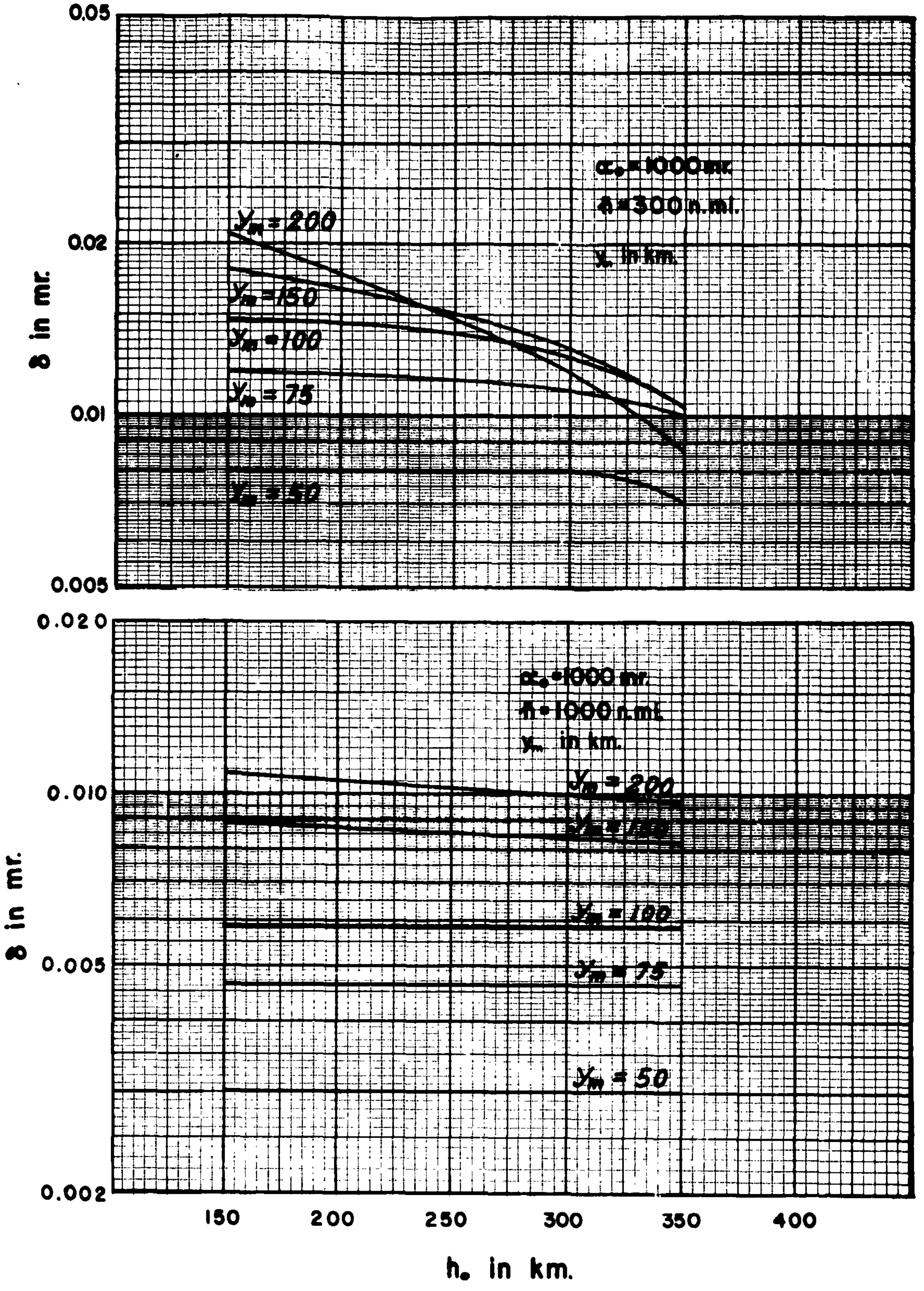


ELEVATION ANGLE ERROR

CRITICAL FREQUENCY —— 1.0 MC.

SIGNAL FREQUENCY —— 100 MC.

FIG 10

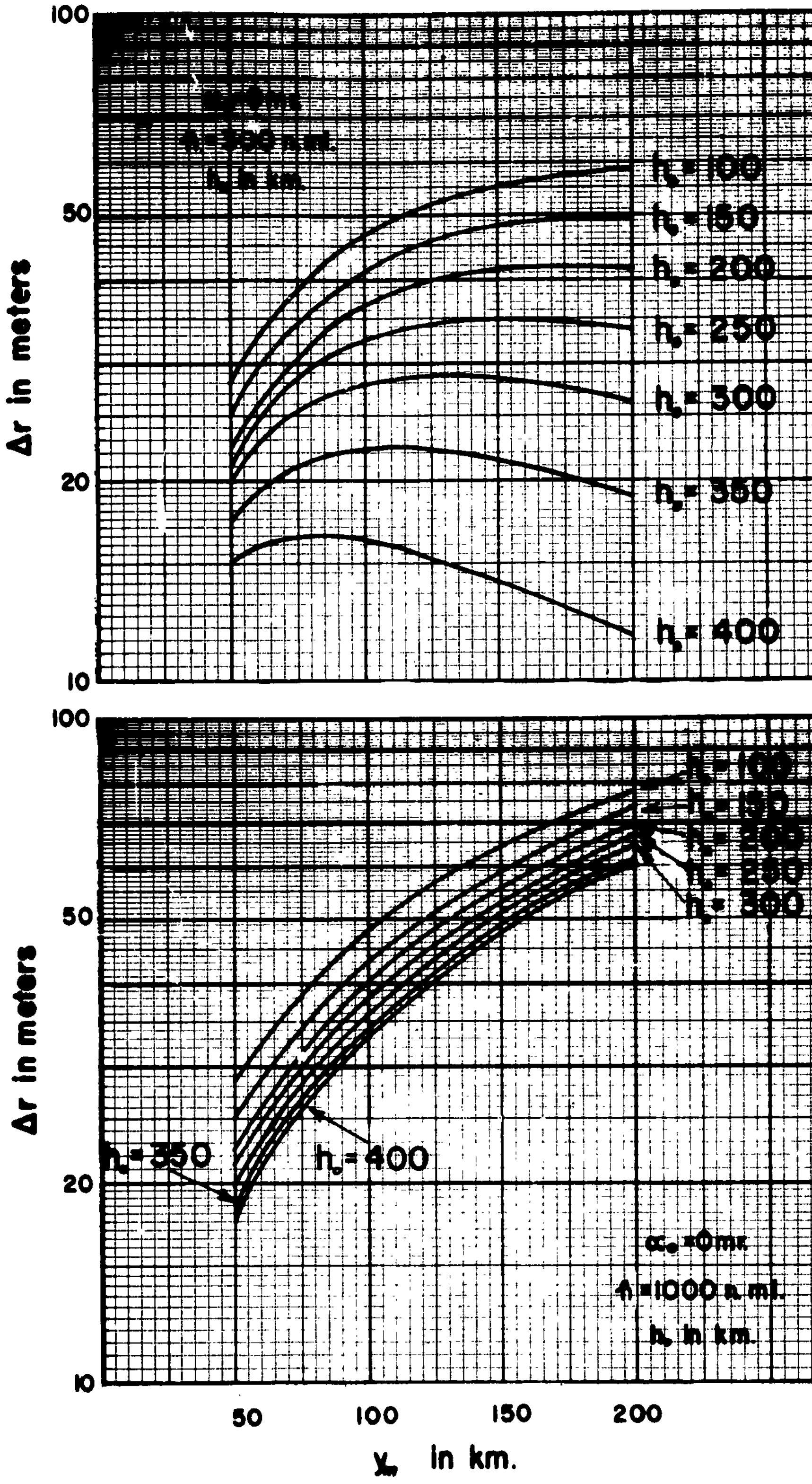


ELEVATION ANGLE ERROR

CRITICAL FREQUENCY——1.0 MC.

SIGNAL FREQUENCY——100 MC.

FIG 11

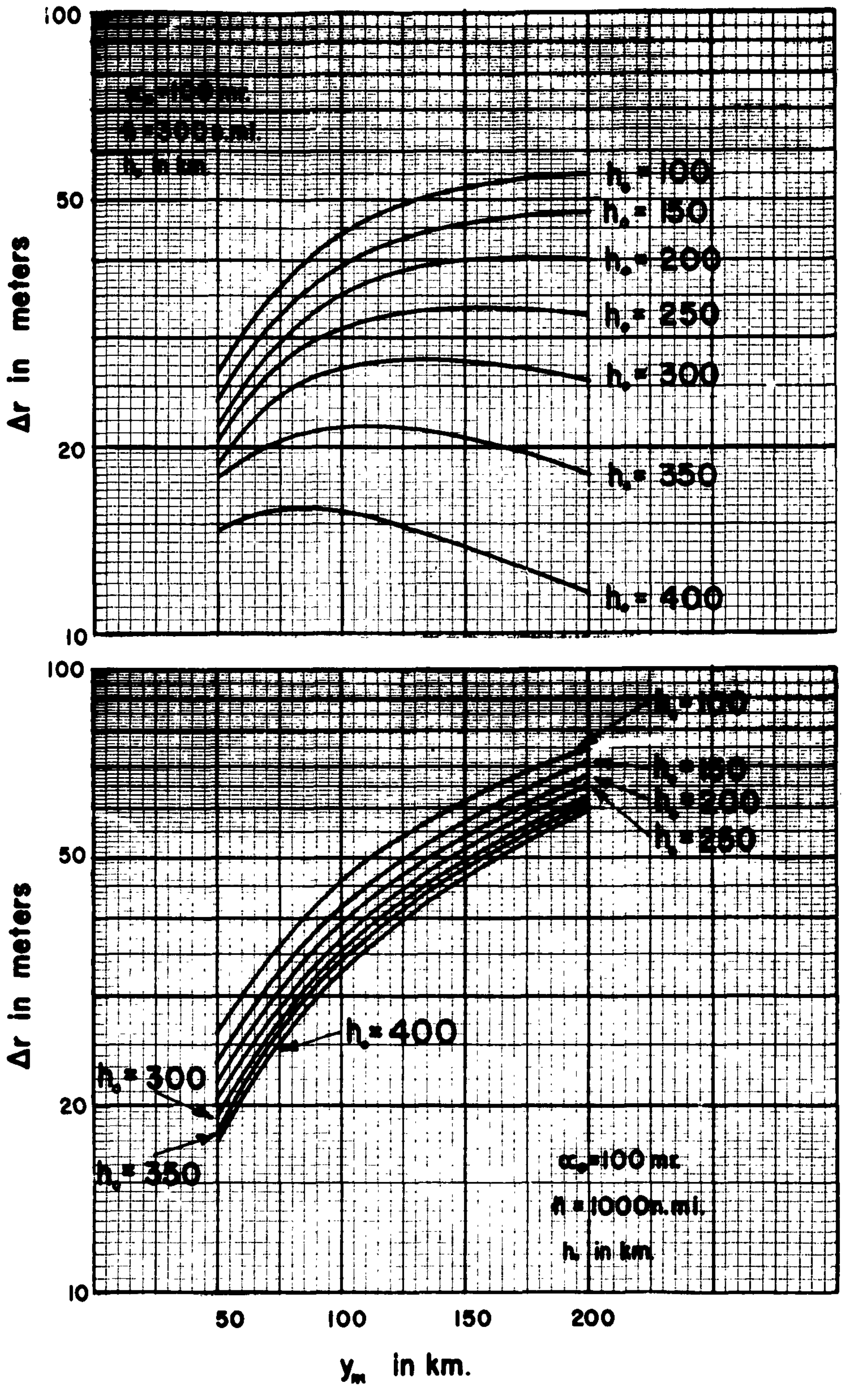


RANGE ERROR

CRITICAL FREQUENCY ——— 1.0MC.

SIGNAL FREQUENCY ——— 100MC.

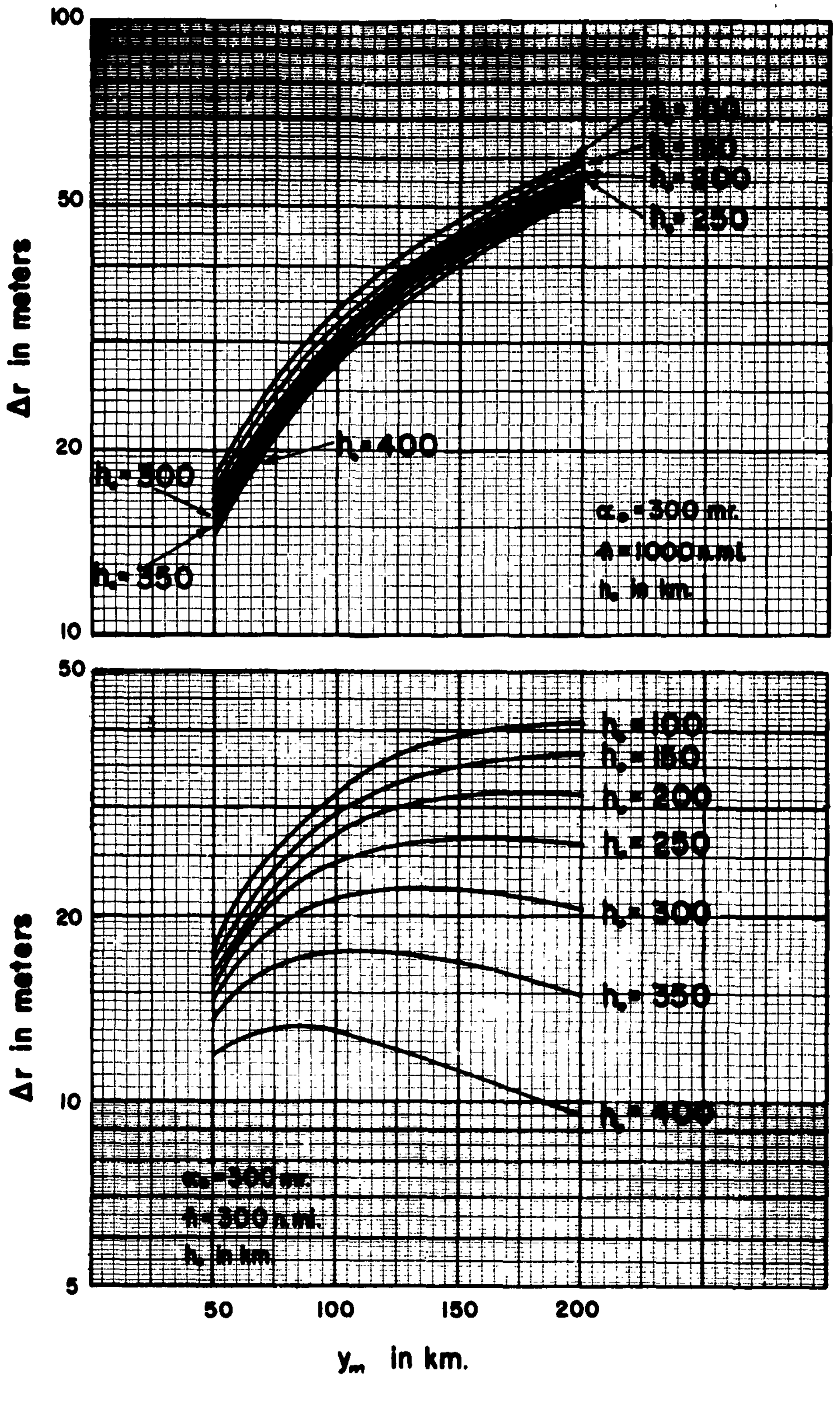
FIG 12



RANGE ERROR

CRITICAL FREQUENCY ——— 1.0 MC.
SIGNAL FREQUENCY ——— 100 MC.

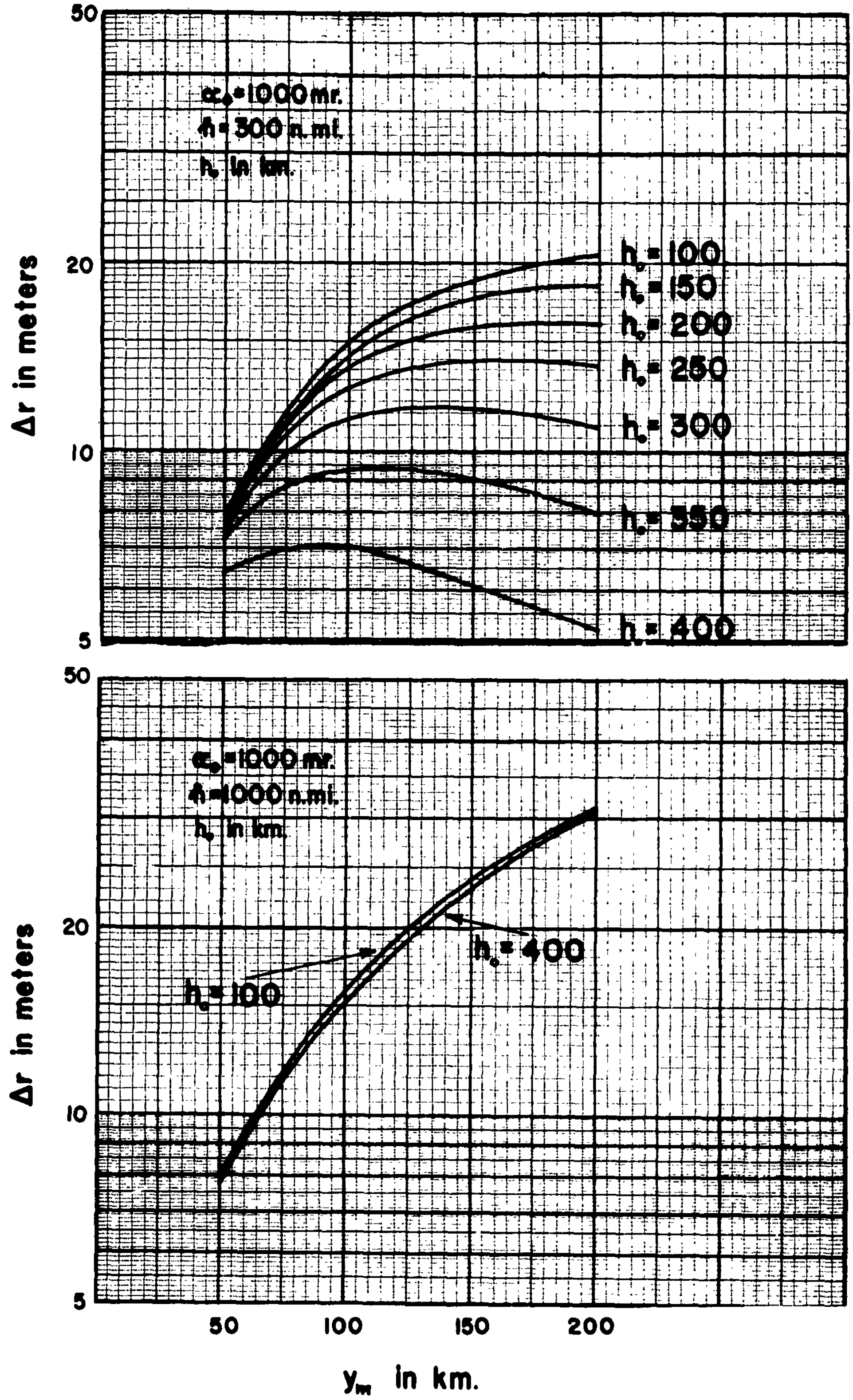
FIG 13



RANGE ERROR

CRITICAL FREQUENCY ——— 1.0 MC.
SIGNAL FREQUENCY ——— 100 MC.

FIG 14



RANGE ERROR

CRITICAL FREQUENCY ——— 1.0 MC.
SIGNAL FREQUENCY ——— 100 MC.

FIG 15

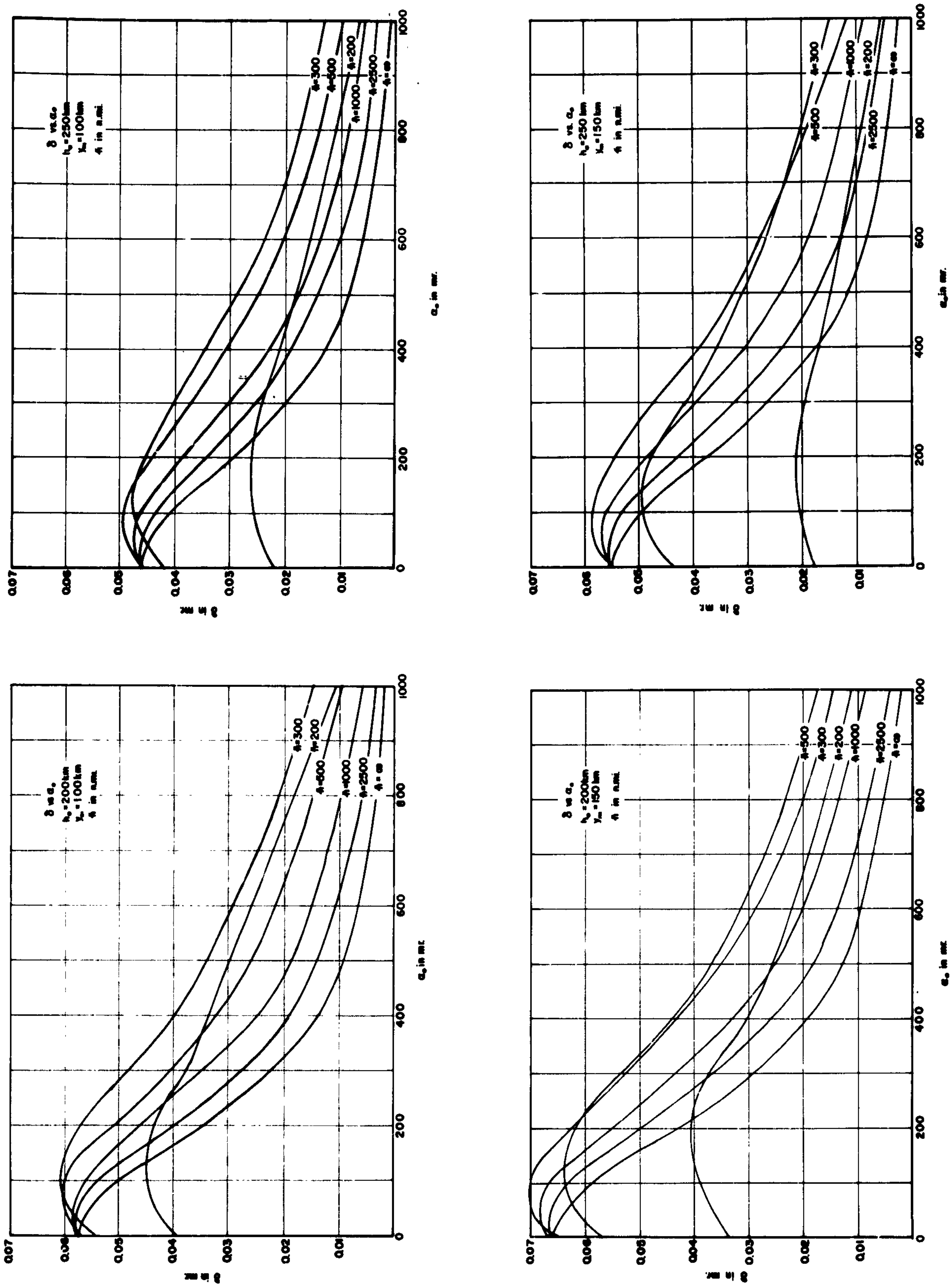

δ vs a₀
δ in mr.
a₀ in mr.
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
200
400
600
800
1000

FIG. 16

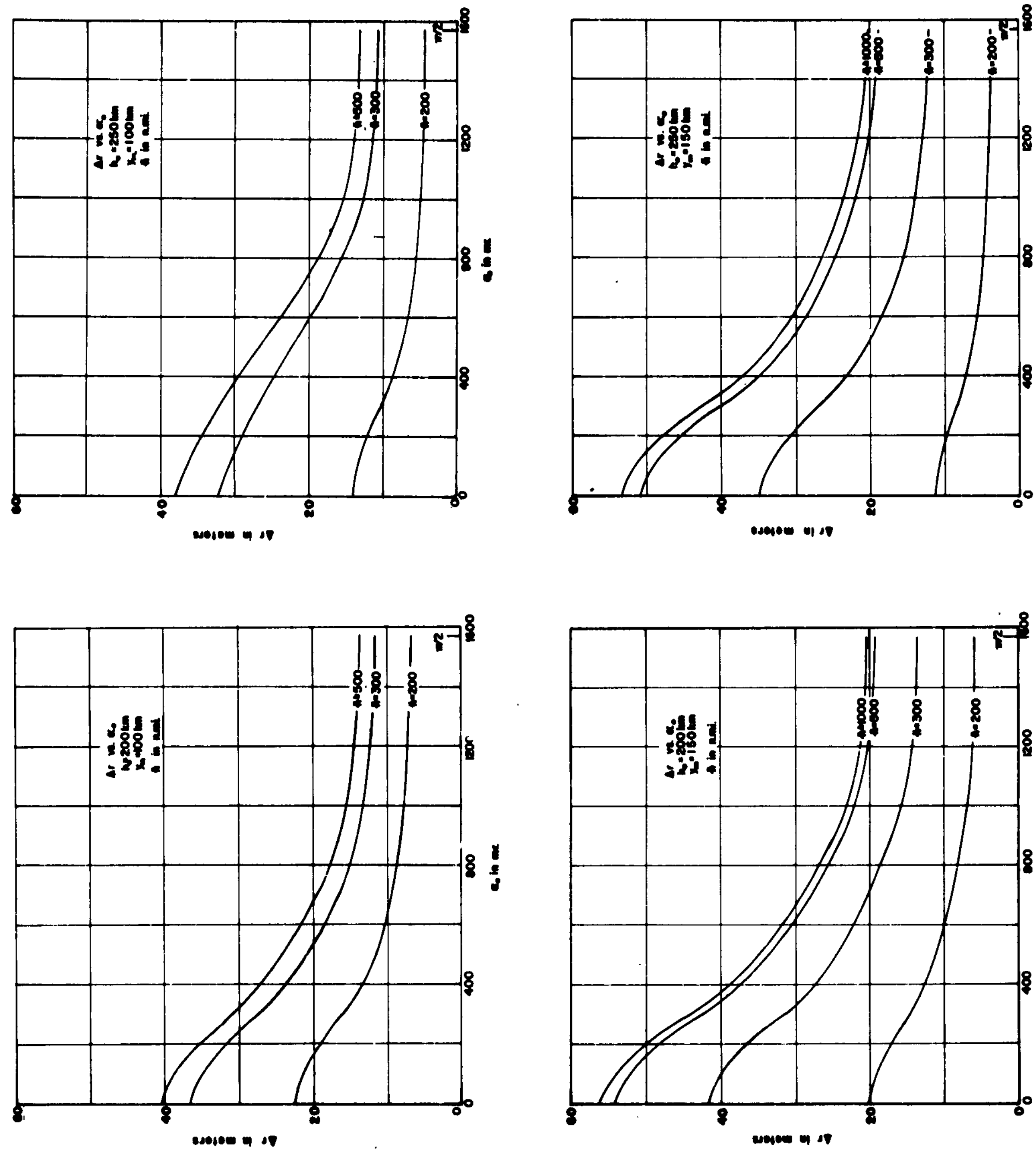
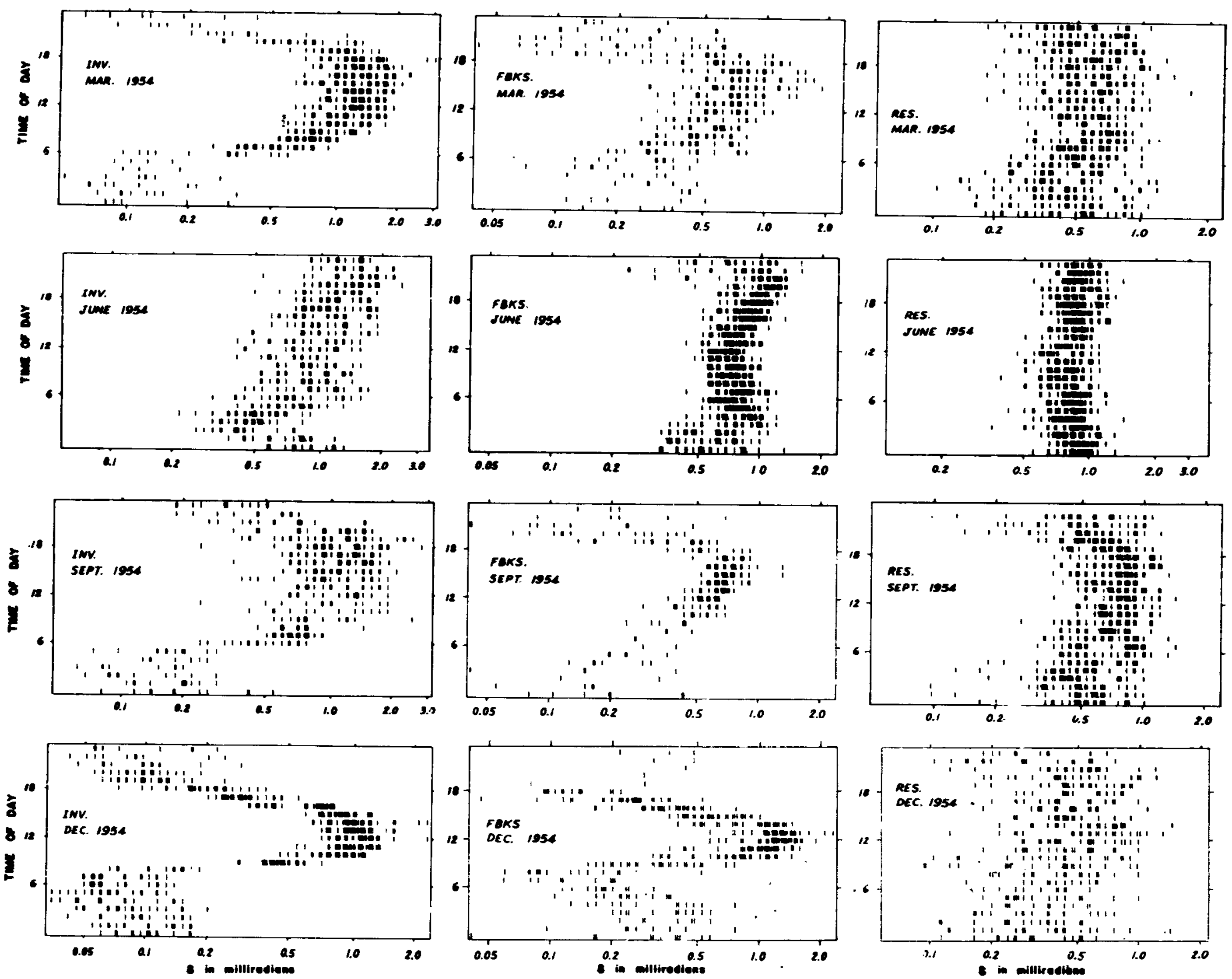


FIG. 17



DISTRIBUTION OF δ

$\alpha_0 = 0$ mc H=1000 n.mi.

FIG. 18

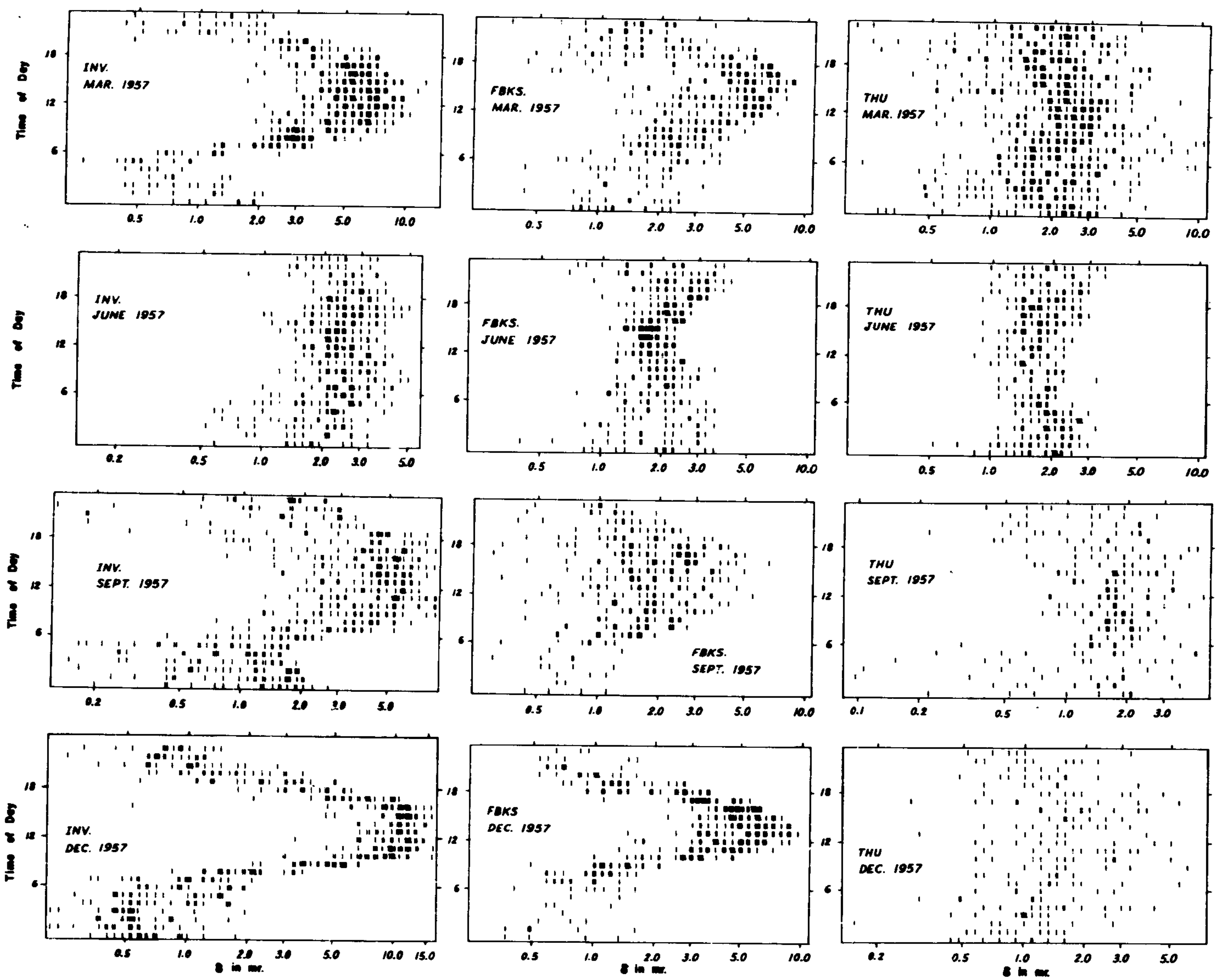
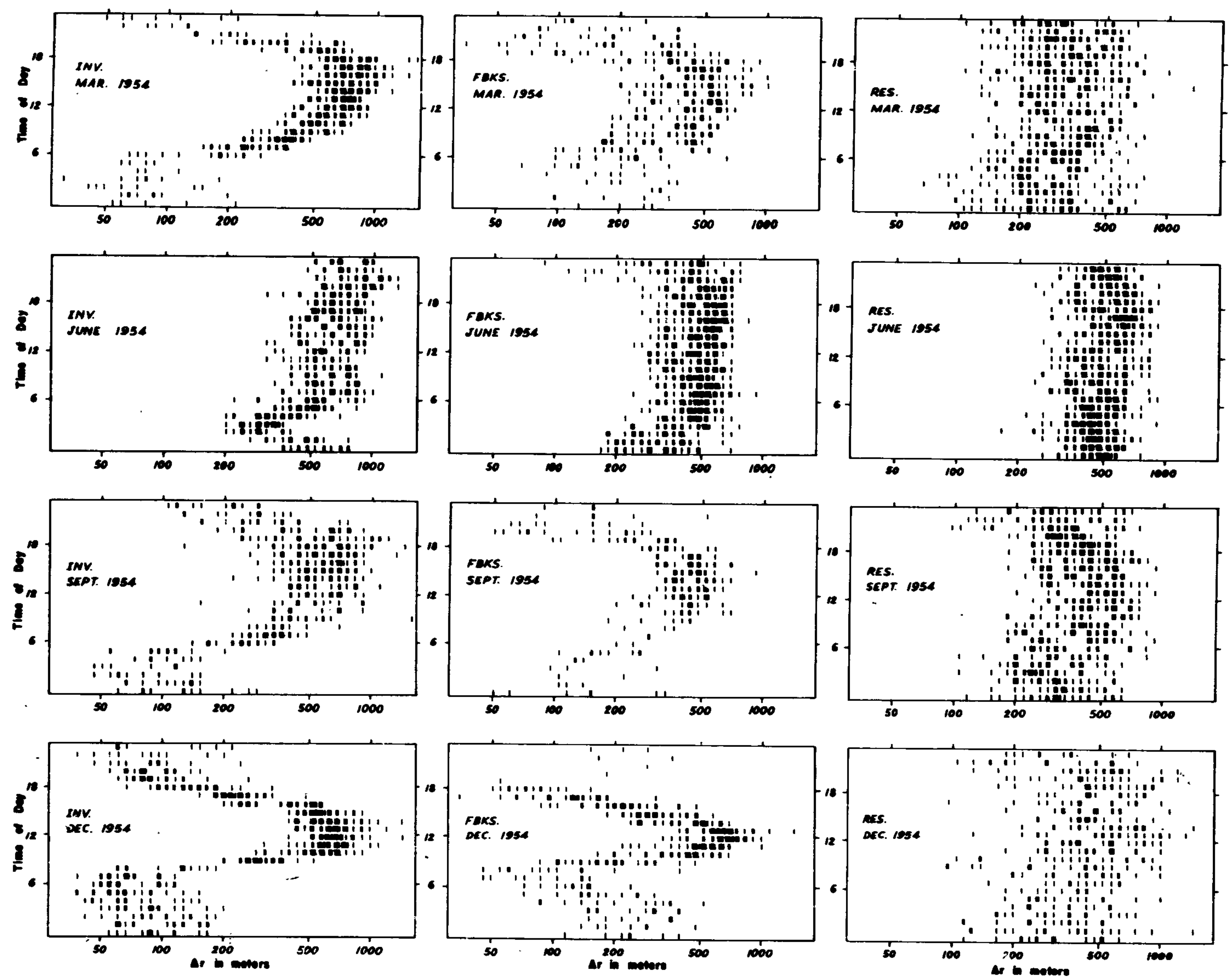


FIG. 19

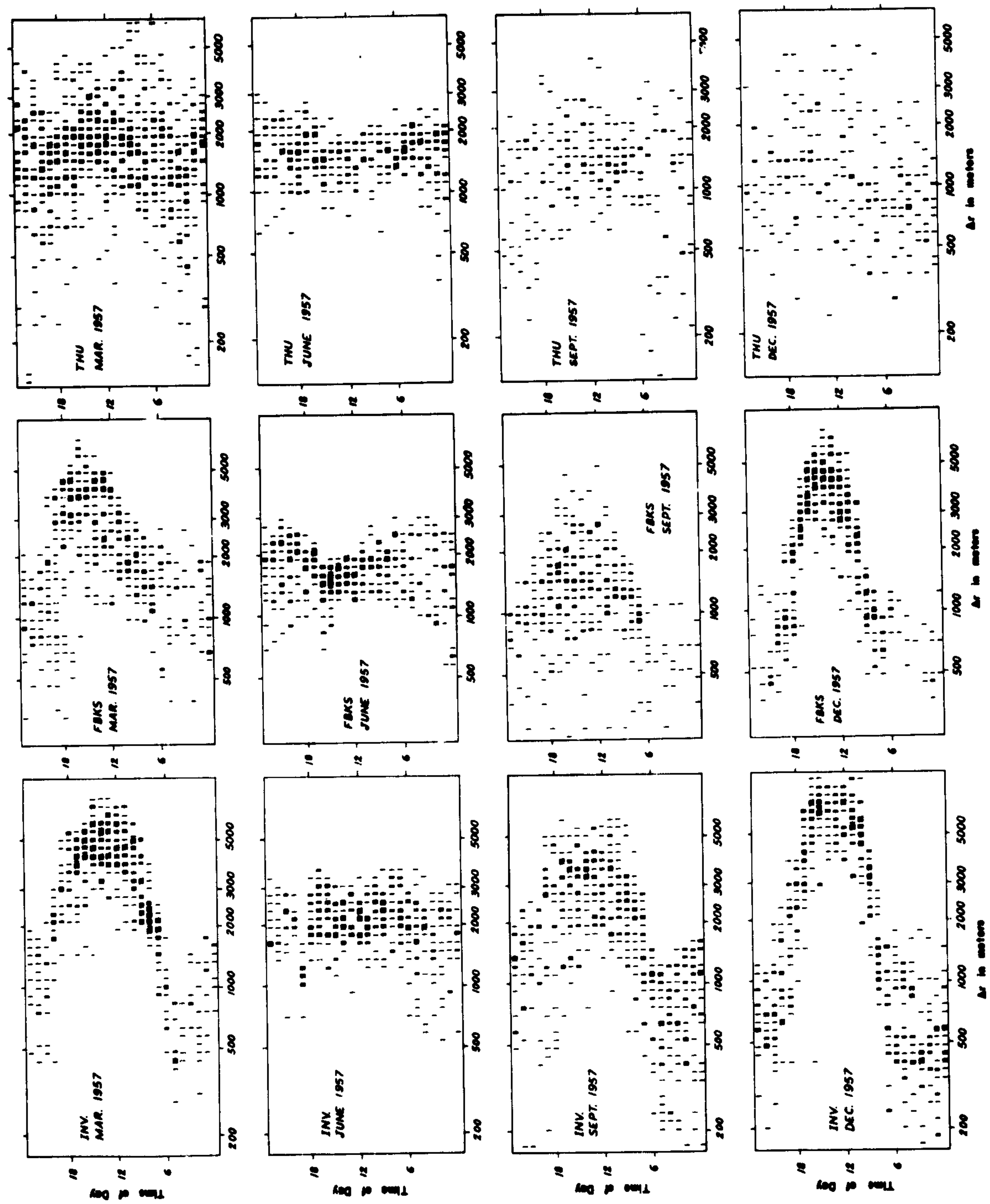
DISTRIBUTION OF δ

α_o = 0 mc h=1000 n.mi.



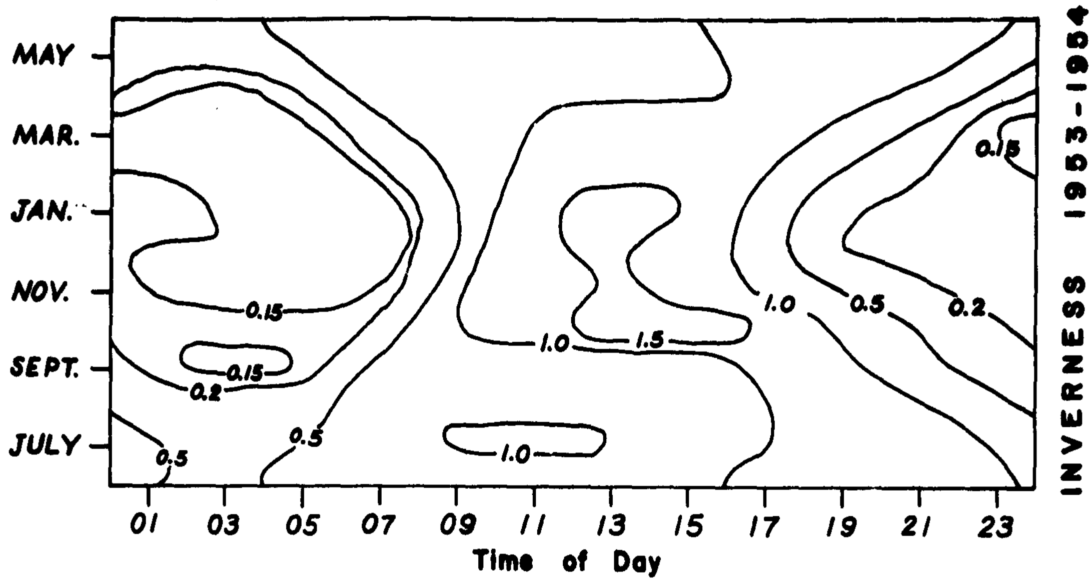
DISTRIBUTION OF Δr
α_o = 0mr. h = 1000 n.mi.

FIG. 20



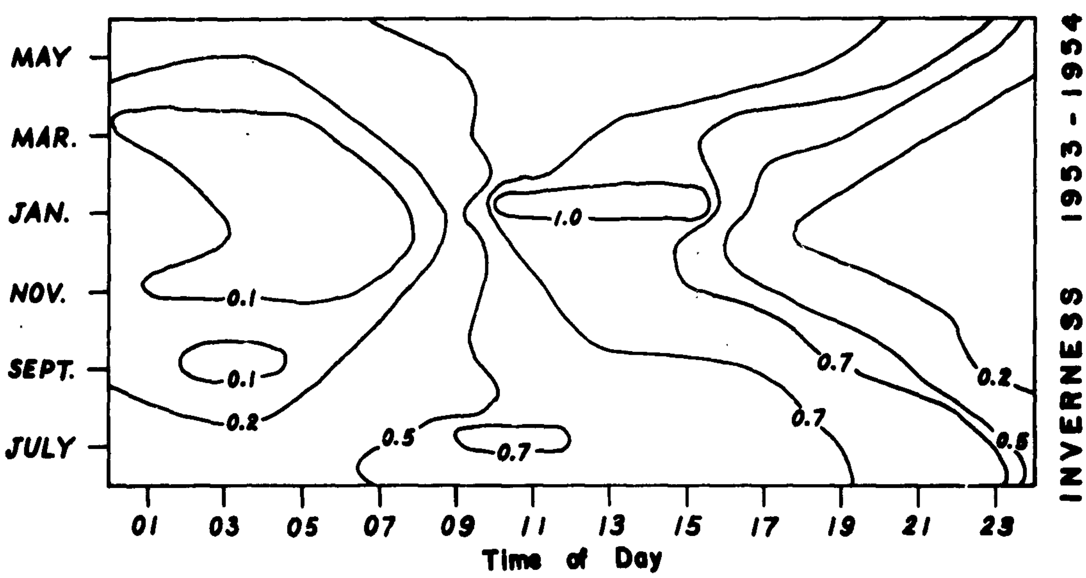
DISTRIBUTION OF Δr
$\alpha_o = 0$ mc $h = 1200$ n.mi.

FIG. 21



CONTOURS OF CONSTANT δ (in mr.)

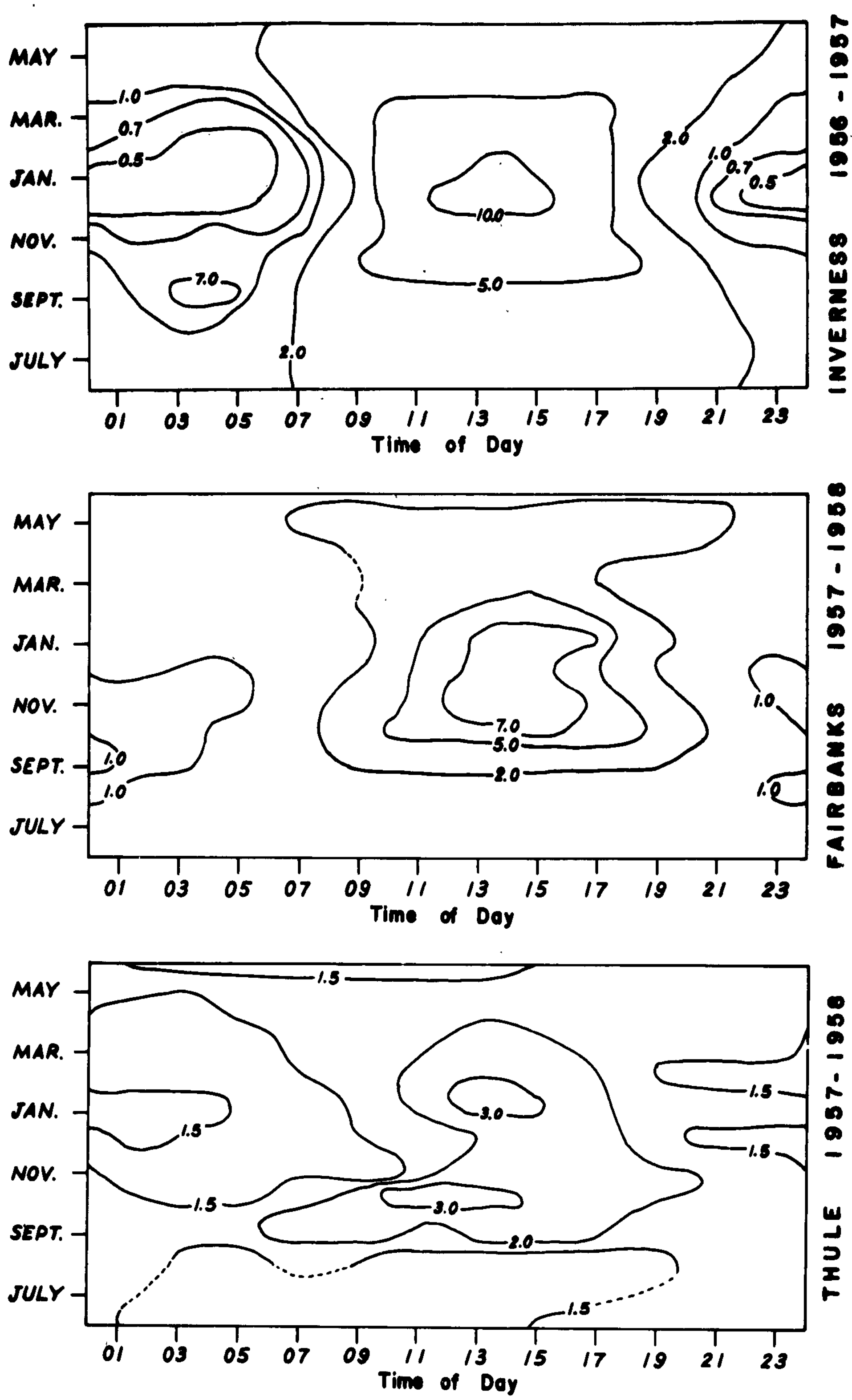
α_0 = 0 mr. h = 1000 n.mi.



CONTOURS OF CONSTANT Δr (in km.)

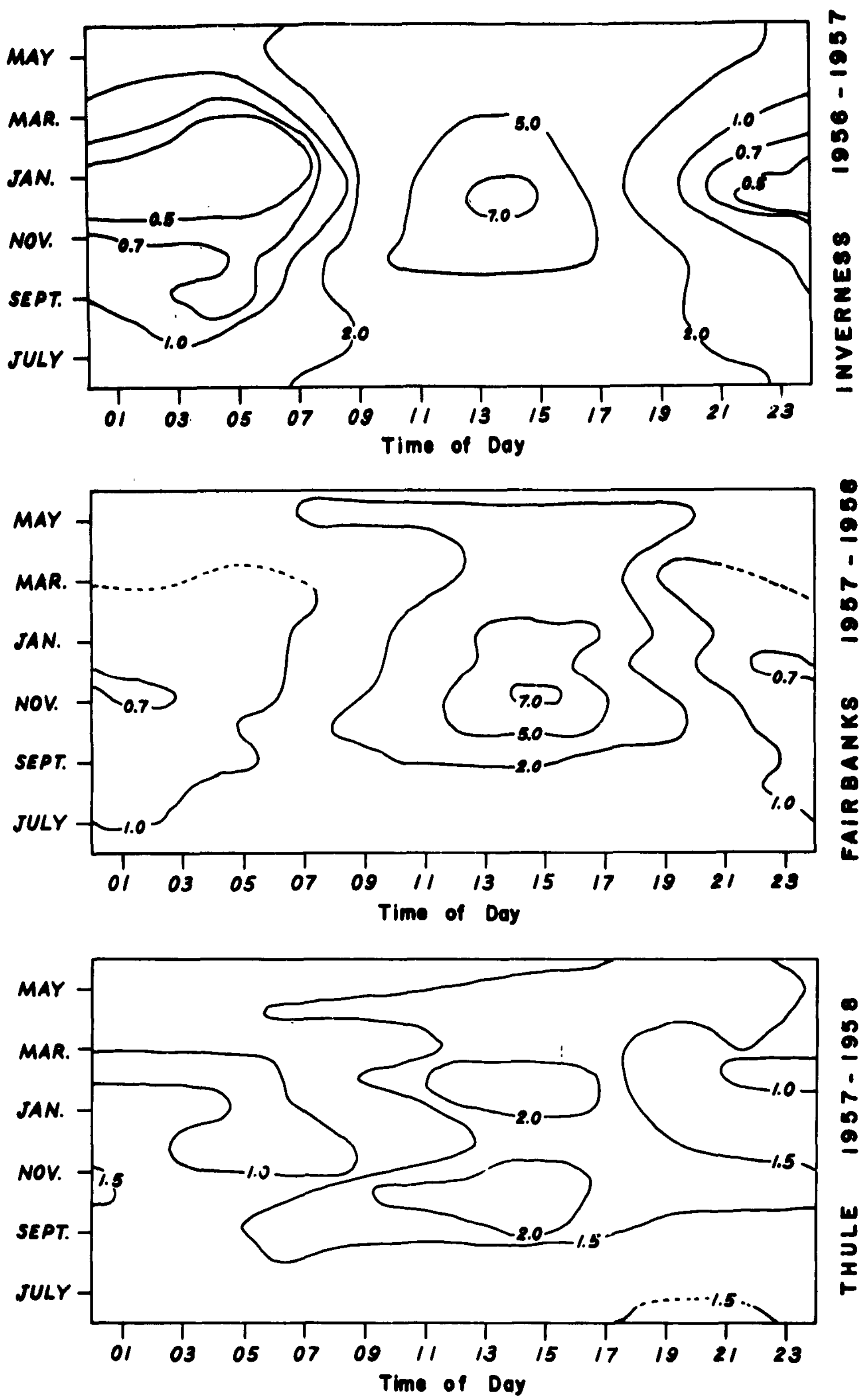
α_0 = 0 mr. h = 1000 n. mi.

FIG. 22



CONTOURS OF CONSTANT S (in mr.)
α_o = 0 mr. H = 1000 n.mi.

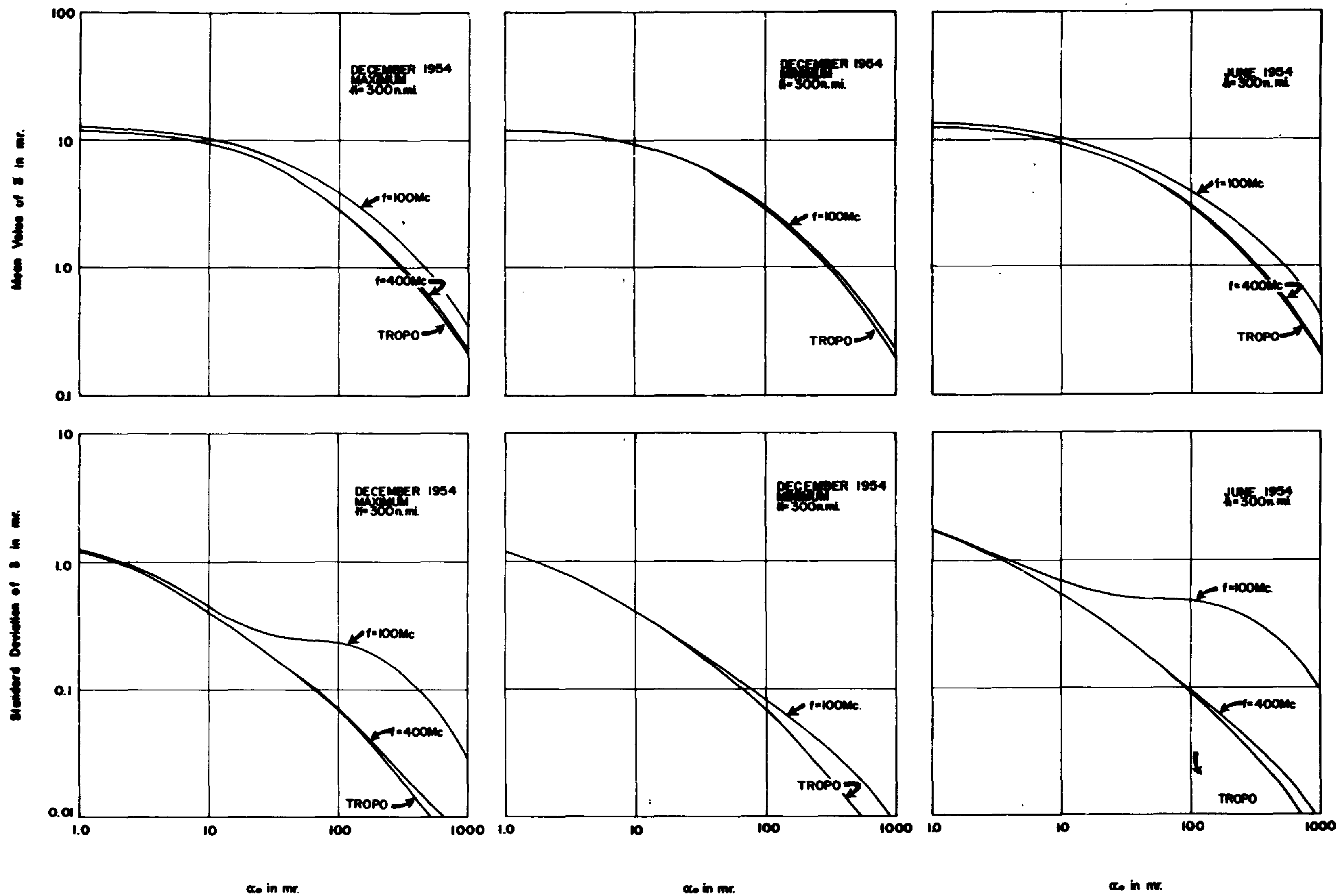
FIG. 23



CONTOURS OF CONSTANT Δr (in km.)

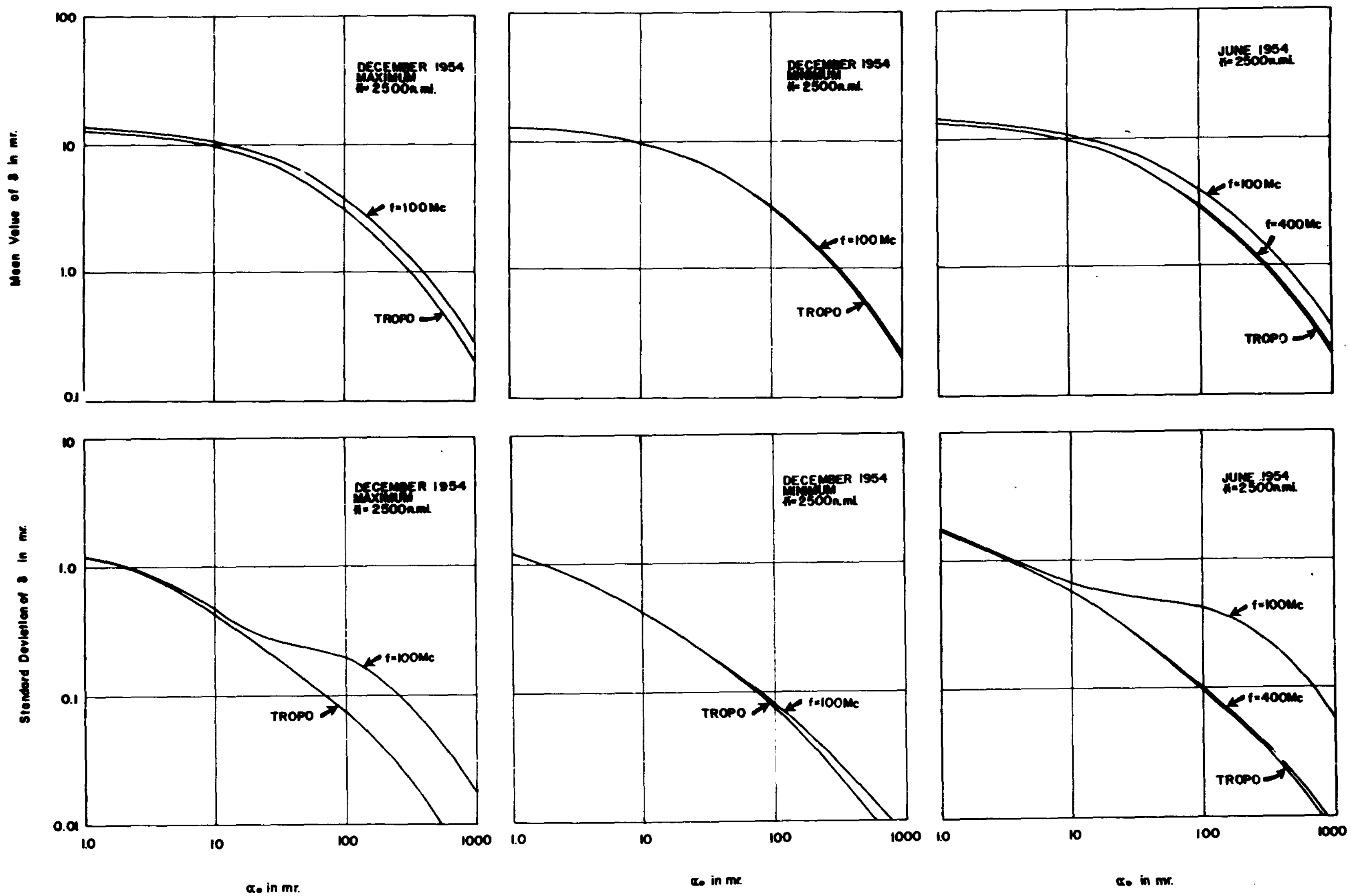
α_o = 0 mr. h = 1000 n. mi.

FIG. 24



COMBINATION OF TROPOSPHERIC AND IONOSPHERIC ERROR AT INVERNESS

FIG. 25



COMBINATION OF TROPOSPHERIC AND IONOSPHERIC ERRORS AT INVERNESS

FIG. 26

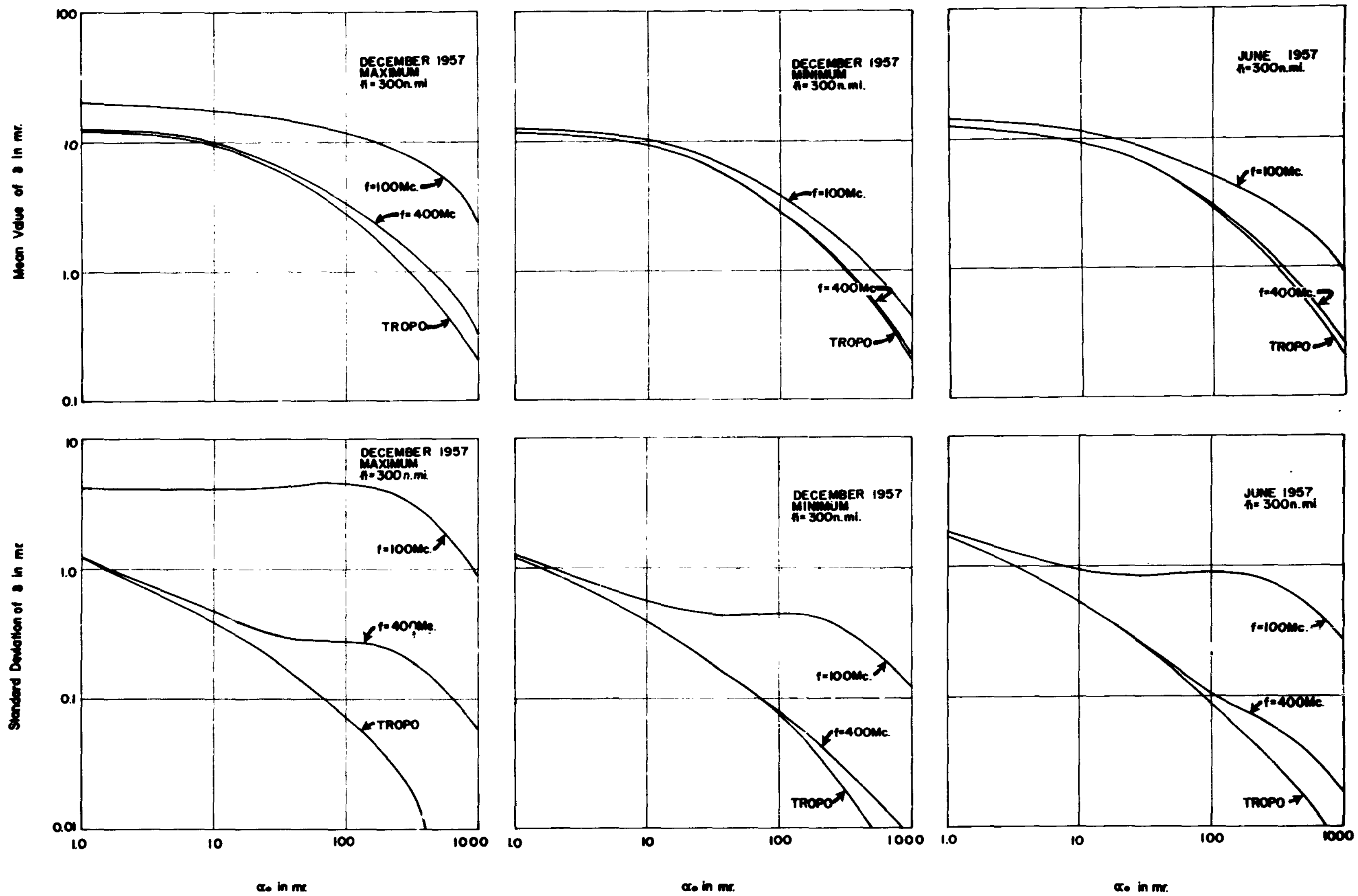
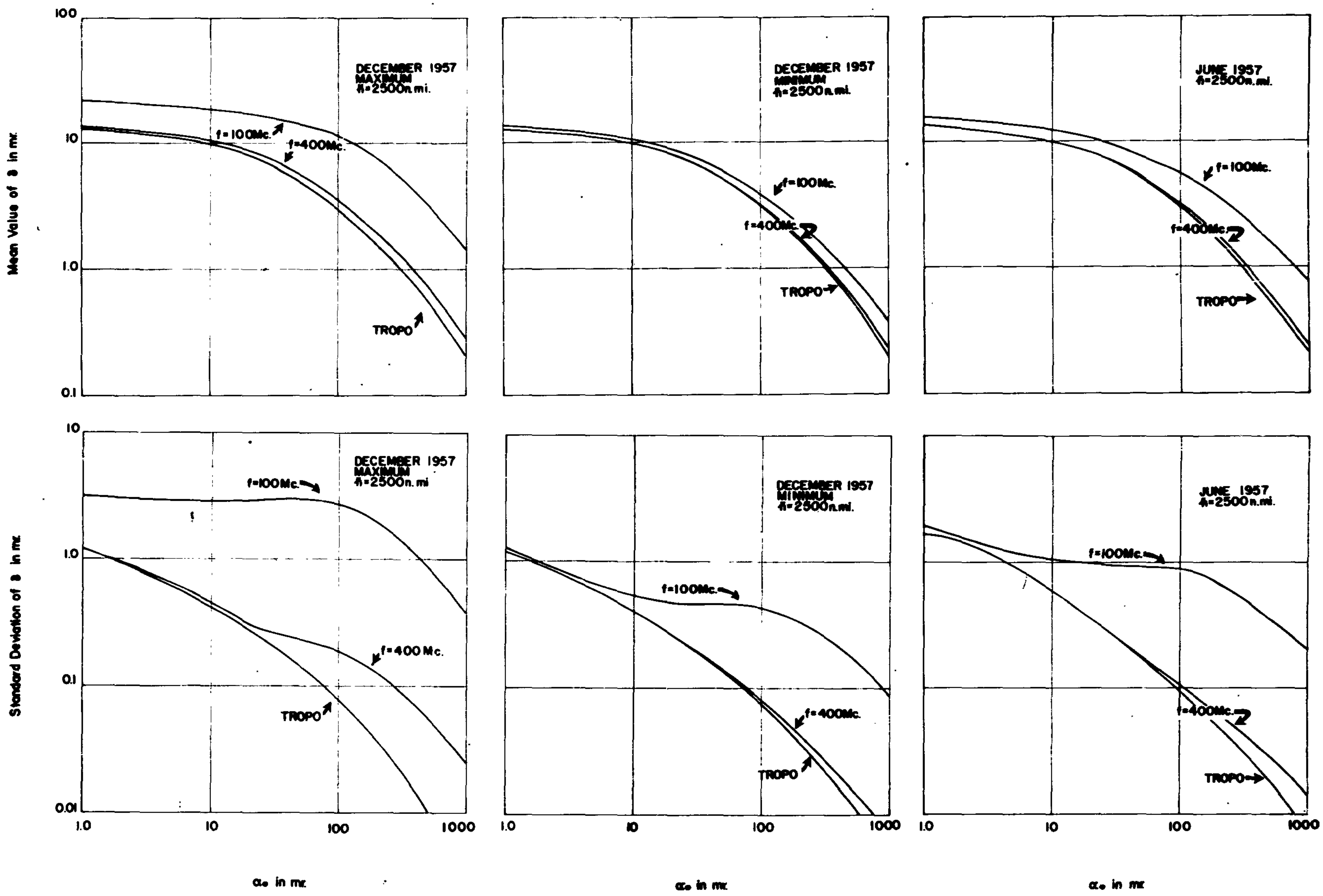


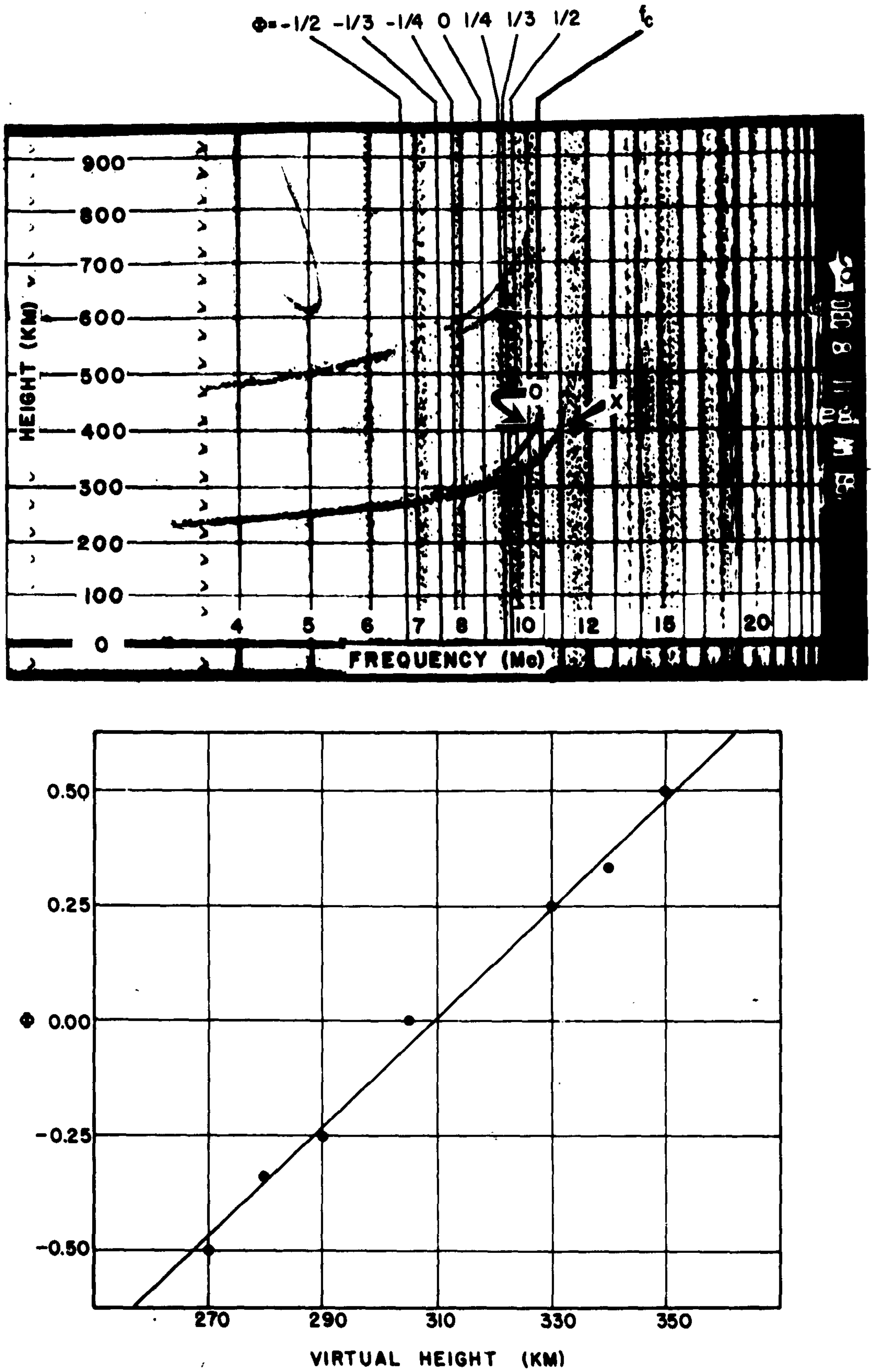
FIG. 27

COMBINATION OF TROPOSPHERIC AND IONOSPHERIC ERRORS AT INVERNESS



COMBINATION OF TROPOSPHERIC AND IONOSPHERIC ERRORS AT INVERNESS

FIG. 28



Φ=-1/2 -1/3 -1/4 0 1/4 1/3 1/2
f_c
HEIGHT (KM)
900
800
700
600
500
400
300
200
100
0
O
X
4
5
6
7
8
10
12
15
20
FREQUENCY (Mc)
0.50
0.25
Φ 0.00
-0.25
-0.50
270
290
310
330
350
VIRTUAL HEIGHT (KM)

FIG. 29

TABLE 1

TROPOSPHERIC δ's (mr)

Leuchars, Scotland

h(kft)	α_o=0mr med	σ	α_o=10mr med	σ	α_o=30mr med	σ	α_o=100mr med	σ	α_o=300mr med	σ	α_o=1000mr med	σ
January												
10	4.5	0.8	2.97	.25	1.53	.06	0.52	.020	.174	.007	.029	.0009
50	7.8	1.0	6.08	.30	3.96	.14	1.64	.050	.549	.011	.105	.0034
100	9.3	1.0	7.21	.35	4.90	.16	2.15	.059	.742	.013	.146	.0037
300	10.7	1.0	8.26	.45	5.77	.18	2.58	.065	.915	.021	.182	.0043
February												
10	4.7	1.0	3.00	.40	1.54	.06	0.53	.020	.175	.007	.029	.0009
50	8.3	1.0	6.09	.42	3.98	.15	1.66	.050	.544	.010	.105	.0030
100	9.7	0.9	7.22	.43	4.92	.17	2.18	.059	.745	.013	.146	.0033
300	10.8	1.0	8.37	.43	5.81	.19	2.61	.065	[illegible]	.020	.182	.0040
March												
10	3.6	1.0	3.06	.40	1.55	.07	0.54	[illegible]	.176	[illegible]	.029	.0007
50	7.9	1.1	6.10	.40	4.00	.16	1.70	.050	.552	.010	.107	.0026
100	9.2	1.1	7.22	.43	4.95	.18	2.21	.059	.746	.013	.148	.0029
300	10.6	1.2	8.48	.42	5.82	.20	2.65	.065	.922	.020	.184	.0034
April												
10	3.8	1.2	3.48	.50	1.55	.07	0.53	[illegible]	.175	.008	.029	.0009
50	7.8	1.2	6.05	.50	4.00	.18	1.67	.053	[illegible]	.015	.107	.0034
100	9.2	1.2	7.23	.60	4.95	.20	2.18	.063	.745	.016	.148	.0037
300	10.7	1.3	8.42	.57	5.82	.22	2.62	.070	.920	.025	.184	.0043
MAY												
10	4.4	1.3	3.02	.45	1.56	.08	0.54	.022	.177	.009	.029	.0010
50	8.0	1.4	6.00	.50	4.04	.20	1.69	.057	.554	.014	.106	.0041
100	9.2	1.4	7.30	.51	5.00	.23	2.20	.067	.750	.017	.147	.0044
300	11.0	1.5	8.45	.55	5.87	.25	2.64	.075	.926	.026	.183	.0053
June												
10	4.9	1.4	3.10	.65	1.60	.08	0.54	.026	.182	.011	.030	.0012
50	8.6	1.5	6.40	.58	4.15	.19	1.71	.066	.560	.017	.108	.0047
100	10.0	1.6	7.65	.63	5.12	.22	2.22	.078	.759	.021	.150	.0051
300	11.5	1.8	8.55	.50	6.02	.24	2.67	[illegible]	[illegible]	.032	.187	.0061

TABLE 1
(cont.)

h(kft)	α_c=0mr med	α_c=0mr σ	α_c=10mr med	α_c=10mr σ	α_c=30mr med	α_c=30mr σ	α_c=100mr med	α_c=100mr σ	α_c=300mr med	α_c=300mr σ	α_c=1000mr med	α_c=1000mr σ
						July						
10	5.9	1.7	3.53	.41	1.72	.08	0.58	.024	.200	.011	.031	.0010
50	8.4	1.6	6.95	.45	4.41	.20	1.78	.061	.585	.017	.114	.0041
100	9.9	1.5	8.17	.45	5.42	.23	2.32	.072	.790	.021	.157	.0044
300	12.8	1.4	9.43	.54	6.35	.25	2.77	.080	.986	.032	.194	.0053
						August						
10	6.0	2.0	3.50	.48	1.70	.09	0.56	.025	.193	.012	.031	.0012
50	9.8	2.0	6.82	.60	4.36	.21	1.73	.062	.577	.019	.113	.0046
100	11.3	2.0	8.40	.44	5.35	.24	2.25	.074	.779	.023	.156	.0051
300	12.3	1.5	9.30	.60	6.26	.26	2.70	.082	.970	.035	.193	.0060
						September						
10	4.8	1.4	3.26	.44	1.63	.10	0.55	.025	.185	.014	.030	.0013
50	8.5	1.6	6.50	.57	4.20	.24	1.70	.065	.565	.021	.109	.0050
100	9.8	1.6	7.72	.64	5.18	.27	2.22	.077	.764	.025	.152	.0056
300	11.2	1.6	8.92	.70	6.08	.30	2.67	.085	.949	.039	.189	.0066
						October						
10	4.8	1.3	3.13	.32	1.63	.11	0.55	.023	.185	.009	.030	.0009
50	8.6	1.3	6.52	.50	4.19	.27	1.71	.059	.564	.013	.109	.0035
100	10.0	1.4	7.58	.58	5.17	.30	2.23	.070	.763	.016	.152	.0040
300	11.6	1.3	8.85	.63	6.07	.32	2.68	.078	.948	.025	.189	.0048
						November						
10	4.6	0.9	3.17	.36	1.58	.06	0.53	.017	.179	.007	.030	.0008
50	6.[illegible]	1.[illegible]	6.30	.38	4.09	.14	1.67	.044	.555	.010	.107	.0029
100	7.5	1.[illegible]	7.46	.40	5.05	.16	2.18	.052	.750	.012	.149	.0032
300	10.8	1.1	8.69	.42	5.94	.18	2.62	.058	.930	.019	.186	.0038
						December						
10	4.7	1.0	3.12	.40	1.57	.06	0.53	.020	.173	.006	.030	.0009
50	8.2	1.1	6.24	.38	4.06	.13	1.65	.052	.552	.010	.10[illegible]	.0035
100	9.5	[illegible]	7.42	.40	5.02	.15	2.17	.061	.746	.012	.148	.0039
300	11.0	1.2	8.55	.40	5.9[illegible]	.17	2.60	.068	.925	.018	.185	.0040

TABLE II

TROPOSPHERIC δ's (mr)

Thule, Greenland

h(kft)	α_o=0mr med	σ	α_o=10mr med	σ	α_o=30mr med	σ	α_o=100mr med	σ	α_o=300mr med	σ	α_o=1000mr med	σ
						January						
10	6.3	2.5	3.41	.55	1.57	.06	0.53	.016	.176	.006	.029	.0009
50	9.7	2.6	6.46	.44	4.07	.13	1.65	.040	.551	.009	.107	.0033
100	11.1	2.6	7.58	.47	5.03	.15	2.17	.048	.748	.011	.148	.0038
300	12.6	2.5	8.77	.47	5.92	.17	2.60	.053	.924	.017	.184	.0045
						February						
10	5.9	2.3	3.50	.60	1.57	.06	0.53	.016	.173	.006	.029	.0008
50	9.3	2.6	6.51	.65	4.05	.13	1.65	.041	.547	.008	.107	.0030
100	10.8	2.7	7.50	.60	5.01	.15	2.17	.049	.741	.010	.148	.0033
300	12.3	2.6	8.85	.56	5.90	.17	2.60	.054	.915	.016	.184	.0040
						March						
10	5.4	1.9	3.40	.50	1.55	.06	0.53	.016	.173	.006	.029	.0007
50	8.9	2.1	6.45	.51	4.01	.13	1.65	.042	.548	.008	.106	.0026
100	10.5	1.8	7.50	.52	4.98	.15	2.17	.050	.742	.010	.146	.0030
300	12.0	2.0	6.80	.50	5.85	.17	2.60	.055	.916	.016	.182	.0035
						April						
10	4.5	2.7	3.20	.46	1.54	.06	0.53	.016	.173	.006	.029	.0007
50	7.8	2.6	6.16	.45	4.00	.13	1.64	.042	.546	.008	.106	.0028
100	9.3	2.9	7.30	.47	4.96	.15	2.16	.050	.740	.010	.146	.0027
300	10.8	3.0	8.50	.48	5.82	.17	2.59	.055	.914	.016	.182	.0033
						May						
10	3.9	0.6	2.73	.32	1.46	.04	0.50	.011	.164	.005	.028	.0005
50	6.8	0.8	5.62	.24	3.84	.10	1.58	.028	.535	.007	.101	.0020
100	7.9	0.9	6.72	.25	4.71	.11	2.08	.033	.725	.008	.142	.0022
300	9.6	0.9	7.90	.25	5.56	.12	2.50	.037	.890	.013	.177	.0026
						June						
10	4.0	0.6	2.76	.31	1.47	.04	0.50	.013	.164	.005	.028	.0005
50	7.1	0.6	5.63	.29	3.86	.10	1.58	.032	.535	.007	.101	.0020
100	8.4	0.7	6.74	.31	4.73	.11	2.08	.038	.725	.009	.142	.0022
300	9.9	0.6	8.00	.29	5.58	.12	2.50	.042	.890	.014	.177	.0026

TABLE II
(cont.)

h(kft)	α_0=0mr med	σ	α_0=10mr med	σ	α_0=30mr med	σ	α_0=100mr med	σ	α_0=300mr med	σ	α_0=1000mr med	σ
July												
10	4.2	0.7	2.90	.35	1.48	.06	0.50	.020	.164	.007	.028	.0009
50	7.4	0.8	5.72	.35	3.89	.14	1.59	.050	.535	.011	.102	.0032
100	8.7	0.8	6.91	.36	4.77	.16	2.09	.059	.725	.014	.144	.0037
300	10.2	0.8	8.07	.37	5.62	.18	2.51	.065	.890	.021	.179	.0043
August												
10	4.9	0.6	2.78	.34	1.48	.05	0.50	.014	.165	.005	.028	.0005
50	7.3	0.7	5.67	.33	3.89	.11	1.60	.036	.537	.007	.102	.0018
100	8.6	0.8	6.85	.33	4.77	.13	2.10	.042	.729	.009	.144	.0020
300	9.9	0.7	8.02	.34	5.62	.14	2.52	.047	.895	.013	.179	.0024
September												
10	3.8	0.5	2.60	.29	1.42	.04	0.48	.013	.160	.005	.028	.0006
50	6.8	0.5	5.43	.25	3.73	.10	1.56	.033	.528	.007	.099	.0022
100	8.1	0.5	6.58	.25	4.62	.11	2.05	.040	.716	.009	.141	.0025
300	9.5	0.5	7.74	.25	5.46	.12	2.47	.044	.877	.014	.175	.0029
October												
10	3.4	1.0	2.63	.31	1.42	.05	0.48	.014	.160	.005	.028	.0006
50	6.4	1.0	5.46	.27	3.73	.12	1.55	.034	.528	.007	.099	.0023
100	7.8	1.[illegible]	6.58	.27	4.64	.14	2.04	.040	.717	.009	.141	.0026
300	9.5	0.8	7.80	.26	5.48	.15	2.46	.045	.878	.014	.175	.0030
November												
10	3.9	1.7	2.84	.40	1.47	.04	0.50	.014	.164	.005	.028	.0004
50	7.1	1.8	5.68	.36	3.86	.10	1.59	.035	.535	.008	.099	.0015
100	8.3	1.8	6.77	.39	4.74	.11	2.09	.041	.726	.010	.14[illegible]	.0017
300	10.0	1.8	7.98	.37	5.59	.12	2.51	.046	.892	.015	.175	.0020
December												
10	6.0	2.4	3.23	.46	1.50	.04	0.51	.018	.167	.006	.029	.0008
50	7.5	2.8	6.12	.40	3.89	.10	1.62	.046	.540	.009	.101	.0029
100	10.9	2.7	7.20	.43	4.84	.12	2.12	.054	.734	.012	.145	.0033
300	12.0	2.3	8.43	.45	5.70	.13	2.54	.060	.901	.018	.180	.0039

TABLE III

TROPOSPHERIC δ's (mr)

Fairbanks, Alaska

h(kft)	α_o=0mr med	α_o=0mr σ	α_o=10mr med	α_o=10mr σ	α_o=30mr med	α_o=30mr σ	α_o=100mr med	α_o=100mr σ	α_o=300mr med	α_o=300mr σ	α_o=1000mr med	α_o=1000mr σ
					January							
10	5.7	4.2	3.42	1.60	1.58	.15	0.52	.04	.172	.015	.029	.0018
50	9.3	4.2	6.50	1.50	4.08	.36	1.64	.11	.546	.022	.102	.0067
100	10.7	4.0	7.60	1.55	5.04	.40	2.15	.12	.740	.028	.147	.0077
300	12.1	4.0	8.85	1.65	5.92	.45	2.58	.14	.912	.043	.182	.0090
					February							
10	5.6	3.4	3.35	.65	1.54	.10	0.51	.02	.170	.009	.029	.0010
50	9.2	3.3	6.10	.90	4.00	.23	1.62	.06	.543	.013	.103	.0040
100	10.5	3.7	7.00	1.04	4.92	.26	2.15	.07	.736	.016	.145	.0045
300	11.9	3.5	8.55	.55	5.81	.29	2.56	.08	.907	.025	.180	.0053
					March							
10	5.0	6.2	3.45	.90	1.48	.09	0.50	.029	.166	.015	.029	.0016
50	8.5	6.3	6.32	1.15	3.89	.22	1.54	.072	.537	.019	.102	.0060
100	9.7	6.5	7.40	1.20	4.77	.25	2.09	.090	.729	.023	.143	.0068
300	11.5	5.0	8.30	.60	5.62	.28	2.51	.095	.895	.036	.178	.0080
					April							
10	5.4	1.5	2.97	.42	1.45	.06	0.49	.017	.160	.007	.028	.0008
50	8.4	2.1	5.83	.45	3.77	.14	1.56	.043	.527	.010	.099	.0028
100	9.8	2.3	6.96	.47	4.69	.15	2.05	.051	.715	.013	.141	.0032
300	11.1	1.9	7.97	.46	5.53	.17	2.47	.057	.876	.020	.175	.0038
					May							
10	5.2	2.1	2.84	.46	1.44	.06	0.49	.018	.159	.007	.029	.0008
50	8.2	2.0	5.68	.52	3.74	.14	1.55	.046	.525	.010	.102	.0031
100	9.6	2.1	6.72	.56	4.66	.16	2.04	.054	.713	.013	.142	.0035
300	10.8	2.3	7.90	.52	5.50	.18	2.46	.060	.873	.020	.177	.0041
					June							
10	4.0	5.0	3.36	.57	1.52	.09	0.51	.03	.168	.010	.029	.0011
50	8.0	5.2	6.30	.37	3.93	.22	1.61	.08	.540	.016	.103	.0043
100	9.0	5.4	7.49	.68	4.87	.25	2.11	.09	.731	.020	.145	.0048
300	11.5	4.5	8.60	.67	5.74	.28	2.53	.10	.901	.030	.180	.0057

TABLE III
(cont.)

h(kft)	α_o=0mr med	α_o=0mr σ	α_o=10mr med	α_o=10mr σ	α_o=30mr med	α_o=30mr σ	α_o=100mr med	α_o=100mr σ	α_o=300mr med	α_o=300mr σ	α_o=1000mr med	α_o=1000mr σ
						July						
10	6.0	5.0	3.60	.70	1.60	.11	0.54	.03	.180	.011	.030	.0012
50	9.2	6.2	6.55	.70	4.12	.26	1.68	.08	.558	.016	.108	.0047
100	11.4	7.0	7.75	.72	5.09	.29	2.20	.09	.755	.020	.150	.0053
300	12.6	4.3	9.02	.90	5.98	.32	2.63	.10	.934	.031	.186	.0062
						August						
10	5.6	1.4	3.42	.47	1.63	.06	0.54	.020	.180	.008	.030	.0009
50	9.2	1.5	6.50	.60	4.19	.14	1.70	.050	.560	.012	.108	.0034
100	10.6	1.5	7.72	.55	5.17	.15	2.22	.059	.756	.014	.151	.0038
300	12.1	2.0	8.99	.50	6.07	.17	2.66	.065	.937	.022	.187	.0045
						September						
10	5.5	2.4	3.29	.61	1.54	.10	0.52	.028	.171	.011	.029	.0012
50	8.9	2.7	6.20	.70	4.00	.24	1.64	.070	.545	.016	.103	.0048
100	10.4	2.7	7.39	.71	4.95	.27	2.14	.083	.738	.020	.146	.0054
300	12.0	2.9	8.61	.77	5.82	.30	2.57	.092	.909	.031	.181	.0064
						October						
10	5.2	2.0	3.12	.45	1.48	.05	0.50	.015	.165	.006	.029	.0007
50	8.6	2.0	6.04	.37	3.85	.11	1.58	.039	.535	.009	.102	.0025
100	9.9	1.9	7.15	.40	4.78	.13	2.08	.046	.725	.011	.142	.0029
300	11.1	2.0	8.29	.38	5.63	.14	2.50	.054	.889	.017	.177	.0034
						November						
10	5.5	2.5	3.45	.65	1.54	.11	0.51	.032	.169	.013	.029	.0013
50	9.0	2.8	6.17	.89	3.98	.27	1.61	.081	.542	.017	.103	.0049
100	10.4	2.8	7.25	.92	4.93	.30	2.12	.096	.734	.020	.145	.0055
300	12.0	4.0	8.43	.95	5.80	.34	2.54	.106	.904	.031	.180	.0065
						December						
10	7.2	8.6	3.85	.95	1.64	.20	0.53	.05	.185	.021	.030	.0024
50	8.2	10.0	7.24	1.75	4.22	.47	1.68	.14	.565	.032	.108	.0090
100	11.2	9.4	8.22	1.78	5.20	.52	2.20	.16	.764	.040	.150	.0100
300	12.2	6.0	9.44	1.45	6.10	.58	2.64	.18	.948	.061	.186	.0120

TABLE IV

TROPOSPHERIC RANGE ERROR IN FEET

Month	α_0=0mr med	α_0=0mr σ	α_0=10mr med	α_0=10mr σ	α_0=30mr med	α_0=30mr σ	α_0=100mr med	α_0=100mr σ	α_0=300mr med	α_0=300mr σ	α_0=1000mr med	α_0=1000mr σ
Leuchars												
Jan.	310	12	247	9	163	7	64.8	2.8	22.75	1.	8.0	.38
Feb.	312	12	248	8	164	6	65.3	2.6	23.0	.9	8.0	.35
Mar.	314	13	250.5	8	166	6	65.8	2.5	23.1	.88	8.16	.30
Apr.	314	13	251	10	166	7	66.0	3.1	23.2	1.10	8.49	.38
May	312	12	249	10	166	9.5	66.4	4.0	23.3	1.30	8.27	.53
June	323	15	258	12.5	171	9	67.7	4.6	23.8	1.65	8.40	.60
July	340	18	273.4	11.0	182.6	7	73.2	3.4	25.8	1.25	9.09	.43
Aug.	335	18	268	11	178	8	71	4	25.2	1.3	9.0	.45
Sept.	329	18	264.5	10.8	174.2	10.9	69.9	4.4	24.55	1.50	8.67	.52
Oct.	326	13	261.2	10.0	173.8	7.2	69.2	3.2	24.35	1.10	8.59	.40
Nov.	318	11	254.2	7.8	167.4	6.0	67.0	2.5	23.45	.98	8.29	.33
Dec.	317	13	253.5	10.2	167.2	7.2	66.4	3.1	23.40	1.11	8.26	.40
Thule												
Jan.	310	9	251.4	7.3	165.8	5.5	65.8	2.0	23.2	.83	8.19	.29
Feb.	312	10	255	7.5	167	6.0	66.5	2.2	23.6	.9	8.4	.3
Mar.	310	9	250	7.1	164	5.3	65.2	[illegible]	23.1	.8	8.2	.3
Apr.	311	8.0	247.3	7.0	163.2	5.1	64.8	2.3	22.70	.80	8.03	.30
May	299	7.0	237.0	6.2	156.0	4.6	61.5	1.8	21.55	.70	7.62	.23
June	300	8	235.8	6.3	155.8	4.2	61.9	2.2	21.72	.77	7.68	.26
July	302	10.0	240.0	8.6	158.5	6.8	62.5	2.8	22.00	.98	7.73	.32
Aug.	301.5	6.5	239.5	5.5	157.6	4.0	62.7	1.8	21.98	.61	7.73	.21
Sept.	293.1	6.7	232.8	6.0	152.4	4.1	60.4	1.9	21.33	.68	7.47	.24
Oct.	292.0	7.0	232.0	6.0	151.0	3.0	59.8	1.9	21.0	.7	7.4	.25
Nov.	302	6.5	238	5.0	157	4.5	61.5	2.0	21.5	.7	7.65	.25
Dec.	304	8.5	240	8.0	159	5.0	63.0	2.5	22.1	.9	7.8	.29
Fairbanks												
Jan.	309	16	247	17	162	14	65.5	6.2	22.7	2.2	8.0	.8
Feb.	307	15	245	12	161	9	63.8	3.8	22.4	1.2	7.9	.45
Mar.	299	12	239	20	158	14	62.3	6.4	21.9	2.4	7.8	.75
Apr.	287	9	228	6.5	152	6	59.8	2.5	21.0	0.9	7.4	.32
May	290	9	230	7.5	152	3.0	59.7	2.7	21.0	0.9	7.5	.30
June	304	10	242	13	159	9	63.5	3.9	22.0	1.3	7.9	.47
July	317	15	254	12	168	9	66.5	3.5	23.5	1.3	8.3	.40
Aug.	318	16	256	13	171	10	68.0	4.2	23.9	1.5	8.5	.55
Sept.	308	12	247	13	162	10	64	4.4	21.3	2.1	8.0	.51
Oct.	297	9	237	8	155	5.6	61	2.3	21.6	.85	7.6	.30
Nov.	307	15	244	13	159	10	63.5	4.0	22.2	1.5	7.8	.51
Dec.	316	17	257	19	167	10	68	5.5	23.8	2.2	8.4	.8

TABLE VI

DUCT CHARACTERISTICS

	Fairbanks	Thule	Leuchars
Maximum % occurrence of surface $\Delta N/h > 45/1000'$	19 (Jan)	3 (Jan)	4 (July)
Average thickness	250'	165'	270'
Minimum frequency trapped	800 mc	3000 mc	3000 mc
% Occurrence of trapping at 1000 mc	3	0	0
at 3000 mc	7	1	0.3
Maximum terrain clutter range n. mi. (mountain face 2000' high, 30 miles long)			
400 mc	350	200	350
1000 mc	300	200	250
% Occurrence of maximum clutter ranges			
400 mc	2	0.3	1
1000 mc	5	0.3	3

TABLE VII

$2 \lambda_g^{-1} \cos \theta \times 10^3$ (meters)$^{-1}$

Inside Ionosphere

STATION	α_o mr \ Az	+90°	+60°	+30°	0°	330	300	270
THULE	0	4.01	3.65	3.21	2.92	3.37	3.69	3.91
	100	3.96	3.62	3.21	2.96	3.15	3.36	3.46
	300	4.61	4.11	3.84	3.78	3.73	3.72	3.70
	1000	8.10	7.96	7.81	7.61	7.47	7.47	7.50
	Vert				9.06			
FAIRBANKS	0	3.07	2.70	2.76	2.62	2.44	2.94	4.10
	100	2.74	2.24	2.30	2.36	2.30	2.72	4.09
	300	3.29	2.76	2.65	2.79	3.02	3.67	4.71
	1000	7.36	6.60	6.65	6.77	7.01	7.55	8.06
	Vert				8.96			
INVERNESS	0	3.26	1.81	1.31	1.12	1.07	1.28	2.24
	100	3.10	1.90	1.24	.945	.644	1.31	2.14
	300	3.69	2.64	1.81	1.40	1.42	1.85	2.72
	1000	6.64	6.00	5.47	5.19	5.20	5.55	6.01
	Vert				7.46			

Above Ionosphere

STATION	α_o mr \ Az	+90°	+60°	+30°	0°	330	300	270
THULE	0	4.36	4.04	3.55	3.62	3.90	4.34	4.62
	100	4.26	3.94	3.54	3.41	3.82	4.05	4.31
	300	4.65	4.39	3.79	3.86	4.07	4.24	4.25
	1000	7.46	7.35	7.24	7.14	7.05	7.11	7.11
	Vert				8.29			
FAIRBANKS	0	3.76	3.51	3.56	3.02	3.05	3.54	4.43
	100	3.45	3.21	3.21	3.14	2.98	3.41	4.36
	300	3.67	3.27	3.24	3.44	3.38	3.80	4.76
	1000	6.80	6.43	6.45	6.49	6.61	7.05	7.47
	Vert				8.20			
INVERNESS	0	3.56	2.76	1.97	1.94	1.95	2.08	2.79
	100	3.54	2.28	1.83	1.72	1.67	1.87	2.68
	300	3.72	2.78	2.26	1.96	1.94	2.24	3.06
	1000	6.20	5.66	5.20	5.06	4.97	5.26	5.60
	Vert				6.82			

TABLE VIII

IONOSPHERIC δ's (mr)

Thule, Greenland

h(n.mi.)	α_0 = 0 mr med	α_0 = 0 mr σ	α_0 = 100 mr med	α_0 = 100 mr σ	α_0 = 300 mr med	α_0 = 300 mr σ	α_0 = 1000 mr med	α_0 = 1000 mr σ
				March 1957 0000-2300				
300	1.95	.90	2.34	1.08	1.95	.90	.624	.29
1000	2.10	.90	2.31	.99	1.47	.63	.683	.29
2500	2.10	.90	2.16	.93	1.21	.52	.392	.17
inf.	2.10	.90	1.94	.83	.860	.38	.136	.059
				June 1957 0000-2300				
300	1.60	.52	1.92	.62	1.60	.52	.520	.17
1000	1.85	.51	2.04	.56	1.30	.36	.647	.18
2500	1.85	.51	1.91	.53	1.06	.29	.372	.10
inf.	1.85	.51	1.71	.48	.754	.21	.129	.055
				September 1957 0000-2300				
300	1.52	.78	1.82	.94	1.37	.70	.486	.25
1000	1.60	.74	1.76	.81	1.12	.52	.520	.24
2500	1.60	.74	1.65	.76	.920	.43	.298	.14
inf.	1.60	.74	1.48	.68	.654	.31	.103	.048
				December 1957 0000-2300				
300	1.25	.55	1.50	.66	1.13	.50	.400	.18
1000	1.35	.66	1.49	.73	.877	.43	.439	.21
2500	1.35	.66	1.39	.68	.720	.35	.252	.12
inf.	1.35	.66	1.25	.61	.511	.25	.087	.042

ALL TIMES IN MEAN LOCAL TIME

TABLE IX

IONOSPHERIC δ's (mr)

Resolute Bay, Canada

h(n.mi.)	α_o = 0 mr med	α_o = 0 mr σ	α_o = 100 mr med	α_o = 100 mr σ	α_o = 300 mr med	α_o = 300 mr σ	α_o = 1000 mr med	α_o = 1000 mr σ
March 1954 0000-2300								
300	.495	.190	.545	.209	.408	.157	.108	.040
1000	.525	.200	.525	.200	.315	.120	.112	.042
2500	.525	.200	.493	.188	.258	.099	.0638	.024
inf.	.525	.200	.442	.169	.185	.070	.0221	.0083
June 1954 0000-2300								
300	.778	.145	.656	.160	.641	.120	.165	.0305
1000	.820	.155	.820	.155	.492	.093	.172	.0326
2500	.820	.155	.770	.145	.404	.076	.0990	.0190
inf.	.820	.155	.691	.130	.286	.054	.0352	.0066
September 1954 0000-2300								
300	.622	.198	.685	.218	.514	.163	.131	.0416
1000	.660	.220	.660	.220	.396	.132	.139	.0462
2500	.660	.220	.620	.206	.325	.108	.0800	.0270
inf.	.660	.220	.556	.185	.231	.077	.0277	.0093
December 1954 0000-2300								
300	.440	.220	.485	.242	.362	.181	.0925	.0436
1000	.450	.215	.450	.215	.270	.129	.0945	.0452
2500	.450	.215	.422	.202	.222	.106	.0545	.0260
inf.	.450	.215	.379	.181	.156	.075	.0188	.0090

ALL TIMES IN MEAN LOCAL TIME

TABLE A

IONOSPHERIC δ's (mr)

Fairbanks, Alaska

h(n.mi.)	α_o = 0 mr med	σ	α_o = 100 mr med	σ	α_o = 300 mr med	σ	α_o = 1000mr med	σ
				March 1954 1200-1800				
300	.685	.150	.754	.143	.565	.107	.164	.0312
1000	.742	.243	.742	.243	.445	.146	.163	.0535
2500	.742	.243	.696	.228	.366	.120	.094	.0310
inf.	.742	.243	.625	.204	.260	.085	.032	.011
				March 1954 1900-0600				
300	.277	.223	.332	.268	.250	.201	.069	.0557
1000	.281	.238	.281	.238	.183	.155	.073	.0619
2500	.281	.238	.266	.224	.150	.127	.042	.0360
inf.	.281	.238	.238	.201	.106	.090	.014	.0125
				March 1954 0700-1100				
300	.477	.158	.573	.190	.429	.142	.117	.0387
1000	.512	.215	.512	.215	.333	.140	.128	.0537
2500	.512	.215	.490	.202	.274	.115	.074	.0310
inf.	.512	.215	.439	.181	.194	.082	.026	.0107
				June 1954 0000-2300				
300	.595	.150	.655	.165	.535	.135	.149	.0375
1000	.665	.175	.665	.175	.432	.114	.173	.0455
2500	.665	.175	.625	.164	.355	.094	.100	.0260
inf.	.665	.175	.560	.147	.252	.067	.035	.0090
				September 1954 1000-1900				
300	.585	.180	.645	.198	.526	.162	.143	.0442
1000	.635	.172	.635	.172	.413	.112	.159	.0440
2500	.635	.172	.596	.161	.340	.092	.091	.0250
inf.	.635	.172	.535	.144	.242	065	.031	.0086
				September 1954 2000-0900				
300	.200	.165	.240	.198	.180	.149	.056	.0462
1000	.225	.170	.248	.187	.146	.110	.064	.0485
2500	.225	.170	.233	.176	.120	.090	.036	.0280
inf.	.225	.170	.209	.158	.085	.064	.012	.0097

TABLE X
(cont.)

h(n.mi.)	α_0 = 0 mr Med	σ	α_0 = 100 mr med	σ	α_0 = 300 mr med	σ	α_0 = 1000 mr med	σ
December 1954 1000-1500								
300	.960	.410	1.060	.451	.720	.307	.163	.0695
1000	.960	.370	.960	.370	.528	.204	.157	.0610
2500	.960	.370	.902	.348	.454	.168	.090	.0550
inf.	.960	.370	.808	.303	.308	.119	.031	.0121
December 1954 0500-0800 and 1800-1900								
300	.155	.067	.171	.074	.128	.055	.039	.0168
1000	.155	.085	.155	.084	.093	.051	.041	.0225
2500	.155	.085	.146	.080	.076	.042	.024	.0129
inf.	.155	.085	.131	.072	.055	.030	.008	.0048
December 1954 1600-1700, 0900 and 2000-0400								
300	.340	.140	.374	.154	.280	.116	.075	.0308
1000	.340	.135	.340	.135	.221	.088	.075	.0297
2500	.340	.135	.320	.127	.182	.072	.043	.0171
inf.	.340	.135	.287	.113	.129	.051	.015	.0059
March 1957 1000-1900								
300	3.90	2.00	4.68	2.40	3.51	1.80	1.170	.600
1000	4.00	2.40	4.40	2.64	2.60	1.56	1.200	.720
2500	4.00	2.40	4.13	2.48	2.14	1.28	0.690	.414
inf.	4.00	2.40	3.70	2.22	1.52	.91	0.239	.143
March 1957 2000-0900								
300	1.70	.90	2.04	1.08	1.53	.81	.420	.270
1000	1.79	.67	1.94	.96	1.18	.57	.537	.261
2500	1.79	.87	1.82	.90	.954	.47	.308	.150
inf.	1.79	.87	1.63	.81	.68	.33	.108	.052
June 1957 0000-2300								
300	1.69	.89	2.03	1.07	1.69	.89	.558	.294
1000	1.98	.59	2.18	.65	1.39	.41	.704	.209
2500	1.98	.59	2.04	.61	1.14	.34	.405	.120
inf.	1.98	.59	1.83	.54	1.00	.24	.140	.042

TABLE X
(cont.)

h(n.mi.)	α_0 = 0 mr		α_0 = 100 mr		α_0 = 300 mr		α_0 = 1000 mr	
	med	σ	med	σ	med	σ	med	σ
				September 1957 1200-1900				
300	1.81	.96	2.18	1.18	1.63	.88	.561	.304
1000	2.00	.98	2.20	1.08	1.40	.69	.620	.304
2500	2.00	.98	2.06	1.01	1.15	.57	.356	.175
inf.	2.00	.98	1.86	.90	.62	.41	.123	.061
				September 1957 2200-0600				
300	.82	.32	.98	.38	.82	.29	.268	.106
1000	.87	.31	.96	.34	.61	.22	.296	.106
2500	.87	.31	.90	.32	.50	.18	.170	.061
inf.	.87	.31	.81	.29	.35	.13	.059	.021
				September 1957 2000-2100 and 0700-1100				
300	1.45	.59	1.74	.71	1.51	.53	.457	.186
1000	1.61	.65	1.77	.72	1.05	.42	.515	.208
2500	1.61	.65	1.66	.68	.86	.34	.296	.119
inf.	1.61	.65	1.49	.61	.61	.24	.102	.041
				December 1957 1100-1700				
300	4.48	1.87	5.38	2.25	4.03	1.67	1.295	.543
1000	4.93	1.62	5.43	1.78	3.20	1.05	1.493	.470
2500	4.93	1.62	5.10	1.67	2.63	.86	.860	.270
inf.	4.93	1.62	4.57	1.44	1.80	.61	.298	.093
				December 1957 1800-1000				
300	1.05	.45	1.26	.54	.95	.41	.541	.144
1000	1.21	.59	1.33	.65	.85	.41	.400	.195
2500	1.21	.59	1.25	.61	.70	.34	.230	.112
inf.	1.21	.59	1.12	.55	.50	.24	.079	.039

ALL TIMES IN MEAN LOCAL TIME

TABLE XI

IONOSPHERIC δ's (mr)

Inverness, Scotland

h (n.mi.)	$\alpha_o = 0$ mr med	$\alpha_o = 0$ mr σ	$\alpha_o = 100$ mr med	$\alpha_o = 100$ mr σ	$\alpha_o = 300$ mr med	$\alpha_o = 300$ mr σ	$\alpha_o = 1000$ mr med	$\alpha_o = 1000$ mr σ
March 1954 0900-1900								
300	1.13	.325	1.25	.392	.944	.221	.239	.0682
1000	1.18	.385	1.13	.323	.655	.160	.248	.0809
2500	1.18	.385	1.06	.304	.539	.131	.142	.0465
inf.	1.18	.385	.952	.273	.382	.0910	.0491	.0161
March 1954 2100-0600								
300	.133	.147	.155	.155	.128	.100	.0366	.0405
1000	.133	.147	.129	.143	.0876	.0955	.0372	.0412
2500	.133	.147	.121	.134	.0720	.0785	.0214	.0236
inf.	.133	.147	.109	.120	.0511	.0556	.0074	.0082
March 1954 2000 and 0700-0800								
300	.565	.195	.585	.168	.454	.147	.119	.410
1000	.550	.185	.550	.185	.330	.111	.113	.0378
2500	.550	.185	.516	.174	.271	.091	.0650	.0217
inf.	.550	.185	.464	.156	.192	.065	.0225	.0075
June 1954 0000-2300								
300	.950	.415	1.02	.466	.766	.342	.214	.0955
1000	.990	.440	.990	.440	.643	.286	.218	.0970
2500	.990	.440	.930	.414	.529	.235	.125	.0560
inf.	.990	.440	.835	.372	.375	.167	.0433	.0194
September 1954 1000-1900								
300	.860	.347	.945	.382	.710	.286	.193	.077
1000	1.15	.400	1.15	.400	.690	.240	.258	.090
2500	1.15	.400	1.08	.376	.567	.197	.148	.052
inf.	1.15	.400	.970	.338	.403	.140	.0513	.018
September 1954 0100-0500								
300	.130	.051	.156	.061	.117	.046	.0338	.0152
1000	.156	.075	.156	.075	.101	.049	.0405	.0195
2500	.156	.075	.146	.070	.083	.040	.0232	.0110
inf.	.156	.075	.131	.063	.059	.028	.0080	.0038

TABLE XI
(cont.)

h(n.mi.)	α_o = 0 mr med	σ	α_o = 100 mr med	σ	α_o = 300 mr med	σ	α_o = 1000 mr med	σ
				September 1954 0600-0900 and 2000-0000				
300	.455	.205	.500	.226	.375	.169	.102	.0461
1000	.560	.250	.560	.250	.336	.150	.126	.0562
2500	.560	.250	.525	.235	.276	.123	.0724	.0320
inf.	.560	.250	.471	.211	.196	.087	.0251	.0111
				December 1954 1000-1500				
300	.900	.200	.990	.220	.630	.140	.155	.030
1000	.955	.200	.955	.200	.478	.100	.145	.030
2500	.955	.200	.898	.188	.395	.082	.0821	.017
inf.	.955	.200	.806	.169	.279	.058	.0284	.0059
				December 1954 1900-0800				
300	.080	.031	.088	.034	.072	.028	.0196	.0076
1000	.090	.039	.090	.039	.049	.021	.0225	.0098
2500	.090	.039	.085	.037	.040	.017	.0129	.0056
inf.	.090	.039	.076	.033	.028	.012	.0045	.0019
				December 1954 0900 and 1600-1800				
300	.330	.185	.363	.204	.247	.139	.0595	.0333
1000	.369	.165	.369	.165	.240	.107	.0665	.0297
2500	.369	.165	.346	.155	.197	.088	.0382	.0170
inf.	.369	.165	.310	.139	.140	.063	.0136	.0059
				March 1957 1000-1800				
300	5.33	1.84	6.40	2.21	4.89	1.66	1.63	.561
1000	5.80	1.85	6.39	2.04	4.05	1.29	1.83	.585
2500	5.80	1.85	6.00	1.92	3.33	1.06	1.05	.336
inf.	5.80	1.85	5.49	1.73	2.36	.753	.364	.116
				March 1957 2100-0600				
300	.800	.50	.960	.60	.880	.55	.332	.21
1000	1.05	.55	1.16	.61	.797	.41	.451	.24
2500	1.05	.55	1.09	.57	.655	.34	.260	.14
inf.	1.05	.55	.980	.51	.465	.24	.090	.048

TABLE XI
(cont.)

h(n.mi.)	α_o = 0 mr med	α_o = 0 mr σ	α_o = 100 mr med	α_o = 100 mr σ	α_o = 300 mr med	α_o = 300 mr σ	α_o = 1000 mr med	α_o = 1000 mr σ
March 1957 1900-2000 and 0700-0900								
300	3.42	1.52	4.10	1.83	3.08	1.37	.956	.425
1000	3.10	1.15	3.10	1.15	2.02	.750	1.18	.437
2500	3.10	1.15	2.91	1.08	1.66	.617	.678	.252
inf.	3.10	1.15	2.61	.970	1.18	.439	.234	.087
June 1957 0000-2300								
300	1.85	.75	2.22	.90	1.85	.75	.684	.28
1000	2.47	.85	2.72	.94	1.85	.64	.986	.34
2500	2.47	.85	2.56	.88	1.52	.53	.561	.20
inf.	2.47	.85	2.30	.79	1.06	.38	.194	.069
September 1957 0900-1900								
300	4.25	2.11	4.67	2.32	3.51	1.74	1.04	.495
1000	4.45	2.02	4.45	2.02	2.89	1.31	1.11	.580
2500	4.45	2.02	4.19	1.90	2.38	1.08	.639	.330
inf.	4.45	2.02	3.76	1.70	1.69	.766	.221	.114
September 1957 0000-0500								
300	1.05	.600	1.23	.720	.922	.540	.323	.266
1000	1.05	.650	1.16	.715	.682	.422	.315	.260
2500	1.05	.650	1.09	.672	.560	.350	.181	.150
inf.	1.05	.650	.979	.604	.396	.248	.063	.052
September 1957 0600-0800 and 2000-2300								
300	2.05	1.17	2.26	1.29	1.69	.965	.482	.275
1000	2.20	1.27	2.20	1.27	1.43	.830	.528	.309
2500	2.20	1.27	2.07	1.20	1.17	.680	.303	.180
inf.	2.20	1.27	1.86	1.08	.830	.483	.087	.062
December 1957 1000-1700								
300	8.10	3.90	8.90	4.30	6.68	3.22	1.86	.896
1000	9.20	2.85	9.20	2.85	5.51	1.71	2.12	.656
2500	9.20	2.85	8.65	2.68	4.53	1.41	1.22	.380
inf.	9.20	2.85	7.76	2.41	3.22	1.00	.423	.131

TABLE XI
(cont.)

h(n.mi.)	α_o = 0 mr med	α_o = 0 mr σ	α_o = 100 mr med	α_o = 100 mr σ	α_o = 300 mr med	α_o = 300 mr σ	α_o = 1000 mr med	α_o = 1000 mr σ
				December 1957 2100-0700				
300	.675	.375	.810	.450	.606	.338	.219	.122
1000	.900	.450	.990	.495	.630	.315	.297	.148
2500	.900	.450	.930	.415	.519	.260	.171	.085
inf.	.900	.450	.835	.372	.368	.185	.059	.029
				December 1957 1800-2000 and 0800-0900				
300	2.60	1.65	2.86	1.82	2.34	1.49	.650	.412
1000	2.80	1.70	2.80	1.70	1.82	1.11	.742	.462
2500	2.80	1.70	2.63	1.60	1.50	.910	.426	.280
inf.	2.80	1.70	2.36	1.44	1.07	.645	.147	.097

ALL TIMES IN MEAN LOCAL TIME

TABLE XII

IONOSPHERIC RANGE ERROR IN METERS

Thule, Greenland

h(n.mi.)	α_o = 0 mr med	α_o = 0 mr σ	α_o = 100 mr med	α_o = 100 mr σ	α_o = 300 mr med	α_o = 300 mr σ	α_o = 1000 mr med	α_o = 1000 mr σ
March 1957 0000-2300								
300	1420	690	1350	656	1066	518	530	260
>1000	1650	740	1568	703	1239	555	660	300
June 1957 0000-2300								
300	1250	370	1188	351	936	277	478	140
>1000	1550	420	1472	399	1161	315	635	170
September 1957 0000-2300								
300	1220	650	1150	617	914	488	466	250
>1000	1310	650	1245	620	982	490	524	250
December 1957 0000-2300								
300	1000	530	950	503	750	398	383	200
>1000	1100	650	1044	617	825	488	440	260

ALL TIMES IN MEAN LOCAL TIME

TABLE XIII

IONOSPHERIC RANGE ERROR IN METERS

Resolute Bay, Canada

h(n.mi.)	α_o = 0 mr med	α_o = 0 mr σ	α_o = 100 mr med	α_o = 100 mr σ	α_o = 300 mr med	α_o = 300 mr σ	α_o = 1000 mr med	α_o = 1000 mr σ
			March 1954 0000-2300					
300	290	130	286	124	203	91	101	45
>1000	312	172	290	135	218	121	109	60
			June 1954 0000-2300					
300	455	85	432	81	318	60	159	30
>1000	505	120	455	108	354	84	176	42
			September 1954 0000-2300					
300	360	118	342	112	252	83	126	41
>1000	400	160	360	144	280	112	140	56
			December 1954 0000-2300					
300	260	140	247	133	182	98	91	49
>1000	270	165	250	149	189	116	95	58

ALL TIMES IN MEAN LOCAL TIME

TABLE XIV

IONOSPHERIC RANGE ERRORS IN METERS

Fairbanks, Alaska

	α_o = 0 mr		α_o = 100 mr		α_o = 300 mr		α_o = 1000 mr	
h(n.mi.)	med	σ	med	σ	med	σ	med	σ
March 1954 0900-1900								
300	400	160	380	152	280	112	140	56
>1000	425	155	404	147	318	116	154	56
March 1954 2000-0800								
300	205	115	195	109	154	86	74	41
>1000	225	140	214	133	169	105	84	52
June 1954 0000-2300								
300	400	95	380	90	300	71	144	34
>1000	455	125	432	119	341	94	170	47
September 1954 1000-1800								
300	405	87	385	83	304	65	146	31
>1000	445	105	422	100	334	79	166	39
September 1954 1900-0900								
300	165	85	157	81	124	64	62	32
>1000	200	110	190	105	150	83	75	41
December 1954 1100-1400								
300	562	103	534	98	394	72	197	36
>1000	585	195	540	176	410	137	204	68
December 1954 0500-0800 and 1700-2000								
300	120	60	114	57	84	42	42	21
>1000	120	36	114	34	90	27	43	13
December 1954 2100-0400, 0900-1000 and 1500-1600								
300	250	145	238	138	175	102	88	51
>1000	283	135	254	122	198	95	99	47
March 1957 1100-1900								
300	2650	1270	2500	1210	1970	950	985	476
>1000	3150	1540	2990	1460	2360	1150	1230	600

TABLE XXV
(cont.)

h(n.mi.)	$\alpha_o = 0$ mr med	$\alpha_o = 0$ mr σ	$\alpha_o = 100$ mr med	$\alpha_o = 100$ mr σ	$\alpha_o = 300$ mr med	$\alpha_o = 300$ mr σ	$\alpha_o = 1000$ mr med	$\alpha_o = 1000$ mr σ
March 1957 2000-1000								
300	1220	470	1160	445	915	352	457	176
>1000	1430	670	1360	635	1070	500	557	261
June 1957 0000-2300								
300	1390	440	1320	417	1040	330	542	171
>1000	1650	490	1570	465	1240	370	676	200
September 1957 1000-1900								
300	1440	740	1370	701	1080	555	540	280
>1000	1470	800	1400	760	1100	600	560	320
September 1957 2000-0900								
300	880	380	835	360	660	285	334	140
>1000	920	400	875	380	690	300	368	160
December 1957 1200-1600								
300	3620	1160	3440	1100	2720	870	1370	440
>1000	3950	1210	3750	1150	2960	910	1410	450
December 1957 2000-0900								
300	710	300	674	284	532	225	276	120
>1000	790	230	790	230	591	175	284	130
December 1957 1700-1900 and 1000-1100								
300	1920	840	1820	796	1440	630	730	320
>1000	2020	1120	1920	1060	1520	840	749	330

ALL TIMES IN MEAN LOCAL TIME

TABLE XV

IONOSPHERIC RANGE ERROR IN METERS

Inverness, Scotland

h(n.mi.)	$\alpha_o = 0$ mr med	$\alpha_o = 0$ mr σ	$\alpha_o = 100$ mr med	$\alpha_o = 100$ mr σ	$\alpha_o = 300$ mr med	$\alpha_o = 300$ mr σ	$\alpha_o = 1000$ mr med	$\alpha_o = 1000$ mr σ
March 1954 1300-1800								
300	710	160	674	152	497	112	248	56
>1000	730	175	694	157	511	123	252	60
March 1954 2200-0600								
300	72	24	68	23	55	18	27	9
>1000	81	84	77	80	61	63	32	33
March 1954 1900-2100 and 0700-1200								
300	469	166	445	158	328	116	164	58
>1000	469	180	445	162	328	126	164	63
June 1954 0700-2300								
300	665	168	632	159	465	118	236	60
>1000	670	205	636	195	503	153	240	74
June 1954 0000-0600								
300	405	110	384	104	284	77	142	38
>1000	415	150	394	142	311	113	149	54
September 1954 1000-2000								
300	520	170	494	161	364	119	182	60
>1000	565	175	536	166	425	131	197	61
September 1954 0100-0500								
300	96	29	91	28	72	22	35	11
>1000	96	35	91	33	72	26	36	13
September 1954 0600-0900 and 2100-0000								
300	270	107	256	102	189	75	95	37
>1000	290	125	276	119	218	94	103	44
December 1954 1000-1600								
300	550	160	522	152	385	112	181	53
>1000	613	222	552	200	428	155	202	73

TABLE XV
(cont.)

h(n.mi.)	α_o = 0 mr med	α_o = 0 mr σ	α_o = 100 mr med	α_o = 100 mr σ	α_o = 300 mr med	α_o = 300 mr σ	α_o = 1000 mr med	α_o = 1000 mr σ
December 1954 1900-0800								
300	66	22	63	21	46	15	24	8
>1000	84	49	80	46	63	37	31	18
December 1954 0900 and 1700-1800								
300	197	70	187	67	138	49	65	23
>1000	225	88	203	79	157	62	75	29
March 1957 1000-1800								
300	3900	1300	3700	1240	2920	960	1480	495
>1000	5750	2100	5750	2100	4310	1580	2300	840
March 1957 2100-0600								
300	700	300	665	285	525	225	294	126
>1000	1000	550	1000	550	850	470	430	236
March 1957 0700-0900 and 1900-2000								
300	2050	650	1950	620	1540	490	784	242
>1000	3250	1000	3090	950	2444	750	1240	385
June 1957 0000-2300								
300	1590	580	1510	550	1190	440	644	234
>1000	1970	640	1970	610	1670	520	837	260
September 1957 1000-1700								
300	2900	1100	2760	1050	2180	830	1040	400
>1000	2950	1110	2800	1050	2220	830	1100	410
September 1957 2100-0600								
300	840	460	798	436	630	330	314	170
>1000	860	460	816	436	645	335	329	180
September 1957 0700-0900 and 1800-2000								
300	1780	1000	1690	950	1250	700	623	350
>1000	1950	1070	1740	961	1450	800	695	390

TABLE XV
(cont.)

h(n.mi.)	α_o = 0 mr med	α_o = 0 mr σ	α_o = 100 mr med	α_o = 100 mr σ	α_o = 300 mr med	α_o = 300 mr σ	α_o = 1000 mr med	α_o = 1000 mr σ
				December 1957 1000-1600				
300	6200	1770	5890	1680	4340	1200	2160	620
>1000	6300	1770	5980	1680	4410	1200	2280	600
				December 1957 0000-0700				
300	470	300	446	290	352	225	179	116
>1000	650	300	617	290	488	225	260	120
				December 1957 1700-2300 and 0800-0900				
300	1430	1380	1360	1310	1070	1040	544	519
>1000	1600	1500	1520	1430	1200	1130	610	570

ALL TIMES IN MEAN LOCAL TIME

TABLE XVI

TROPOSPHERIC δ's AT GREAT HEIGHTS (δ_H^t) (mr)

Leuchars, Scotland

H (n.mi.)	α_o=0mr med	α_o=0mr σ	α_o=10mr med	α_o=10mr σ	α_o=30mr med	α_o=30mr σ	α_o=100mr med	α_o=100mr σ	α_o=300mr med	α_o=300mr σ	α_o=1000mr med	α_o=1000mr σ
January												
100	11.10	1.04	8.50	.45	5.91	.19	2.68	.065	.958	.021	.192	.0043
300	12.00	1.07	9.07	.45	6.23	.20	2.78	.066	.986	.021	.199	.0044
1000	12.35	1.09	9.44	.45	6.44	.20	2.84	.066	.999	.021	.201	.0044
2500	12.62	1.10	9.62	.45	6.55	.21	2.87	.067	1.003	.022	.202	.0045
February												
100	11.2	.97	8.45	.45	5.93	.19	2.65	.065	.960	.020	.195	.0040
300	12.0	.99	9.10	.45	6.30	.20	2.76	.065	.990	.021	.200	.0041
1000	12.4	1.02	9.50	.45	6.50	.20	2.83	.065	1.002	.021	.202	.0042
2500	12.6	1.05	9.60	.45	6.57	.21	2.86	.066	1.005	.021	.203	.0043
March												
100	10.90	1.10	8.66	.40	6.01	.20	2.72	.064	.966	.020	.194	.0034
300	11.7	1.10	9.24	.40	6.39	.21	2.81	.064	.994	.020	.201	.0034
1000	12.3	1.10	9.62	.40	6.62	.22	2.86	.064	1.007	.020	.203	.0035
2500	12.6	1.10	9.79	.40	6.72	.23	2.88	.064	1.011	.021	.204	.0035
April												
100	11.0	1.33	8.61	.55	5.98	.22	2.71	.071	.965	.025	.194	.0043
300	11.7	1.36	9.21	.55	6.31	.23	2.81	.071	.993	.025	.201	.0043
1000	12.2	1.38	9.61	.55	6.52	.24	2.86	.072	1.006	.026	.203	.0044
2500	12.5	1.39	9.79	.55	6.61	.25	2.89	.073	1.010	.026	.204	.0045
May												
100	11.3	1.56	8.64	.56	6.07	.25	2.73	.076	.971	.026	.193	.0053
300	12.0	1.57	9.23	.57	6.43	.25	2.81	.077	.999	.026	.200	.0053
1000	12.6	1.58	9.61	.59	6.58	.26	2.86	.078	1.012	.027	.202	.0054
2500	12.9	1.59	9.79	.61	6.65	.26	2.89	.079	1.016	.027	.203	.0055
June												
100	11.8	1.70	8.71	.52	6.16	.25	2.77	.088	.985	.032	.197	.0061
300	12.5	1.70	9.30	.54	6.49	.25	2.87	.088	1.013	.032	.204	.0062
1000	13.1	1.70	9.67	.57	6.74	.26	2.93	.089	1.026	.033	.206	.0062
2500	13.4	1.70	9.85	.60	6.75	.27	2.96	.090	1.030	.033	.207	.0063

TABLE XVI
(cont.)

H(n.mi.)	α_o=0mr med	σ	α_o=10mr med	σ	α_o=30mr med	σ	α_o=100mr med	σ	α_o=300mr med	σ	α_o=1000mr med	σ
July												
100	13.0	1.47	9.66	.56	6.53	.25	2.87	.080	1.031	.032	.204	.0053
300	13.7	1.44	10.20	.58	6.82	.25	2.97	.080	1.059	.032	.211	.0054
1000	14.3	1.46	10.63	.60	7.03	.25	3.03	.080	1.072	.033	.213	.0054
2500	14.6	1.48	10.82	.62	7.13	.25	3.06	.080	1.076	.033	.214	.0055
August												
100	12.7	1.55	9.52	.65	6.38	.25	2.87	.080	1.031	.032	.204	.0053
300	13.7	1.60	10.08	.67	6.70	.25	2.97	.080	1.059	.032	.211	.0054
1000	14.5	1.70	10.48	.70	6.90	.26	3.03	.080	1.072	.033	.213	.0054
2500	14.8	1.80	10.65	.73	6.99	.27	3.06	.080	1.076	.033	.214	.0055
September												
100	11.6	1.65	9.17	.70	6.22	.27	2.77	.085	.994	.039	.199	.0066
300	12.4	1.68	9.73	.71	6.57	.27	2.87	.085	1.022	.039	.206	.0067
1000	13.1	1.72	10.10	.72	6.77	.27	2.93	.085	1.035	.039	.208	.0067
2500	13.4	1.77	10.28	.73	6.86	.27	2.96	.085	1.039	.040	.209	.0068
October												
100	11.8	1.2	9.10	.63	6.22	.29	2.78	.078	.993	.025	.199	.0048
300	12.3	1.2	9.66	.64	6.57	.29	2.88	.078	1.021	.025	.206	.0048
1000	12.9	1.2	10.07	.64	6.77	.29	2.94	.078	1.034	.026	.208	.0049
2500	13.2	1.2	10.28	.65	6.87	.29	[illegible]	.078	1.038	.026	.209	.0049
November												
100	11.2	1.15	8.93	.42	6.09	.18	2.72	.058	.975	.019	.196	.0038
300	12.0	[illegible]	9.44	.43	6.42	.18	2.82	.058	1.003	.019	.203	.0038
1000	12.6	1.24	9.84	.44	6.63	.18	2.88	.059	1.016	.020	.205	.0039
2500	12.9	1.28	10.02	.45	6.72	.18	2.91	.060	1.020	.020	.206	.0039
December												
100	11.3	1.2	8.78	.40	6.08	.17	2.70	.070	.970	.020	.195	.0046
300	11.9	1.2	9.32	.40	6.39	.17	2.80	.073	.998	.021	.202	.0046
1000	12.5	1.2	9.70	.41	6.60	.18	2.86	.075	1.011	.022	.204	.0047
2500	12.8	1.2	9.89	.42	6.69	.18	2.89	.077	1.015	.023	.205	.0048

TABLE XVII

TROPOSPHERIC δ's AT GREAT HEIGHTS (δ_H^t) (mr)

Thule, Greenland

H(n.mi.)	α_o=0mr med	α_o=0mr σ	α_o=10mr med	α_o=10mr σ	α_o=30mr med	α_o=30mr σ	α_o=100mr med	α_o=100mr σ	α_o=300mr med	α_o=300mr σ	α_o=1000mr med	α_o=1000mr σ
January												
100	12.9	2.5	8.99	.47	6.07	.17	2.70	.060	.969	.017	.194	.0045
300	13.6	2.5	9.53	.47	6.40	.17	2.80	.063	.997	.017	.201	.0045
1000	14.2	2.6	9.87	.48	6.61	.17	2.86	.065	1.010	.018	.202	.0046
2500	14.5	2.7	10.05	.49	6.69	.17	2.89	.068	1.014	.018	.203	.0047
February												
100	11.9	2.3	9.07	.45	6.05	.17	2.70	.070	.960	.018	.194	.0045
300	12.8	2.4	9.57	.45	6.38	.17	2.80	.070	.988	.018	.201	.0045
1000	13.4	2.5	10.00	.45	6.59	.17	2.86	.070	1.001	.018	.202	.0046
2500	13.7	2.6	10.15	.45	6.66	.17	2.89	.070	1.005	.018	.203	.0047
March												
100	12.3	2.0	9.01	.46	6.00	.17	2.70	.070	.960	.018	.194	.0045
300	13.0	2.0	9.50	.46	6.33	.17	2.80	.070	.988	.018	.201	.0045
1000	13.6	2.1	9.88	.46	6.54	.17	2.86	.070	1.001	.018	.202	.0046
2500	13.9	2.1	10.06	.46	6.62	.17	2.89	.070	1.005	.018	.203	.0047
April												
100	11.1	3.0	8.72	.48	5.97	.17	2.69	.050	.959	.016	.192	.0033
300	11.9	3.0	9.25	.48	6.31	.17	2.79	.050	.967	.016	.199	.0033
1000	12.3	3.0	9.61	.48	6.52	.17	2.85	.050	1.000	.016	.201	.0034
2500	12.6	3.1	9.78	.48	6.60	.17	2.88	.050	1.004	.017	.202	.0035
May												
100	9.8	.9	8.10	.25	5.72	.12	2.60	.038	.935	.013	.187	.0026
300	10.6	.9	8.66	.25	6.05	.12	2.70	.039	.963	.013	.194	.0026
1000	11.1	.9	9.02	.25	6.25	.13	2.76	.040	.976	.014	.196	.0027
2500	11.4	.9	9.18	.25	6.33	.13	2.79	.041	.980	.014	.197	.0028
June												
100	10.2	.63	8.19	.29	5.73	.13	2.60	.042	.935	.014	.187	.0026
300	10.9	.64	8.72	.29	6.05	.14	2.70	.042	.963	.014	.194	.0026
1000	11.4	.65	9.10	.29	6.24	.15	2.76	.043	.976	.014	.196	.0027
2500	11.7	.66	9.26	.30	6.32	.15	2.79	.044	.980	.015	.197	.0028

TABLE XVII
(cont.)

H(n.mi.)	α_o=0mr med	α_o=0mr σ	α_o=10mr med	α_o=10mr σ	α_o=30mr med	α_o=30mr σ	α_o=100mr med	α_o=100mr σ	α_o=300mr med	α_o=300mr σ	α_o=1000mr med	α_o=1000mr σ
					July							
100	10.5	.81	8.24	.38	5.76	.18	2.61	.046	.935	.021	.189	.0043
300	11.2	.82	8.82	.39	6.06	.18	2.71	.046	.963	.021	.196	.0043
1000	11.7	.83	9.20	.39	6.28	.18	2.77	.046	.976	.022	.198	.0044
2500	11.9	.84	9.36	.40	6.36	.18	2.80	.046	.980	.022	.199	.0045
					August							
100	10.2	.65	8.22	.34	5.77	.14	2.62	.046	.940	.013	.189	.0024
300	10.9	.65	8.78	.34	6.10	.14	2.72	.046	.968	.013	.196	.0024
1000	11.4	.65	9.16	.35	6.30	.14	2.78	.046	.981	.014	.198	.0025
2500	11.7	.65	9.33	.36	6.38	.14	2.81	.046	.985	.014	.199	.0026
					September							
100	9.8	.51	7.93	.25	5.62	.12	2.57	.045	.922	.014	.185	.0029
300	10.5	.52	8.47	.25	5.95	.12	2.67	.046	.950	.014	.192	.0029
1000	11.0	.53	8.82	.26	6.15	.12	2.73	.047	.963	.014	.194	.0030
2500	11.3	.54	8.99	.27	6.23	.12	2.76	.048	.967	.015	.195	.0031
					October							
100	9.8	.8	8.01	.26	5.64	.15	2.56	.045	.923	.014	.135	.0050
300	10.4	.8	8.52	.26	5.96	.15	2.66	.045	.951	.014	.192	.0050
1000	10.9	.8	8.87	.26	6.16	.16	2.72	.046	.964	.014	.194	.0051
2500	11.2	.8	9.03	.27	6.24	.17	2.75	.047	.968	.015	.195	.0052
					November							
100	10.3	1.8	8.17	.37	5.75	.12	2.61	.042	.937	.015	.185	.0020
300	11.0	1.8	8.73	.37	6.08	.12	2.71	.042	.965	.015	.192	.0020
1000	11.5	1.8	9.10	.37	6.28	.12	2.77	.042	.978	.015	.194	.0021
2500	11.8	1.8	9.27	.38	6.36	.13	2.80	.042	.982	.016	.195	.0022
					December							
100	12.3	2.3	8.65	.45	5.86	.14	2.64	.055	.946	.018	.190	.0039
300	1[illegible].9	2.3	9.16	.45	6.20	.15	2.74	.055	.974	.018	.197	.0039
1000	13.5	2.3	9.54	.45	6.40	.16	2.80	.055	.987	.018	.199	.0040
2500	13.8	2.4	9.70	.46	6.49	.17	2.83	.055	.991	.019	.200	.0041

TABLE XVIII

TROPOSPHERIC δ's AT GREAT HEIGHTS (δ_H^t) (mr)

Fairbanks, Alaska

H(n.mi.)	α_o=0mr med	α_o=0mr σ	α_o=10mr med	α_o=10mr σ	α_o=30mr med	α_o=30mr σ	α_o=100mr med	α_o=100mr σ	α_o=300mr med	α_o=300mr σ	α_o=1000mr med	α_o=1000mr σ
January												
100	12.3	4.3	9.05	1.66	6.06	.45	2.68	.15	.957	.043	.192	.009
300	12.7	4.5	9.52	1.67	6.39	.45	2.78	.16	.985	.043	.199	.009
1000	13.1	4.7	9.89	1.68	6.59	.46	2.84	.16	.998	.044	.201	.009
2500	13.2	4.9	10.06	1.69	6.67	.47	2.87	.17	1.002	.045	.202	.009
February												
100	12.2	3.5	8.76	.56	5.95	.29	2.66	.08	.952	.025	.190	.0053
300	12.8	3.6	9.26	.57	6.27	.29	2.76	.08	.980	.025	.197	.0053
1000	13.3	3.7	9.63	.58	6.46	.29	2.82	.08	.993	.026	.199	.0054
2500	13.6	3.9	9.81	.59	6.54	.30	2.85	.08	.997	.027	.200	.0055
March												
100	11.7	5.2	8.49	.62	5.77	.28	2.61	.095	.940	.036	.185	.0038
300	12.3	5.5	8.91	.63	6.10	.28	2.71	.095	.968	.036	.192	.0038
1000	12.9	5.8	9.44	.64	6.29	.29	2.77	.096	.981	.037	.194	.0039
2500	13.1	6.2	9.61	.66	6.38	.30	2.80	.097	.985	.038	.195	.0040
April												
100	11.3	2.0	8.17	.46	5.67	.17	2.57	.059	.921	.020	.184	.0038
300	11.9	2.1	8.70	.46	6.10	.17	2.67	.060	.949	.020	.191	.0038
1000	12.4	2.2	9.07	.46	6.19	.17	2.73	.061	.962	.020	.193	.0039
2500	12.6	2.4	9.24	.46	6.27	.18	2.76	.063	.966	.021	.194	.0040
May												
100	11.[illegible]	2.3	8.09	.52	5.66	.17	2.56	.061	.918	.020	.187	.0041
300	11.8	2.3	8.61	.52	5.98	.17	2.66	.062	.946	.020	.194	.0041
1000	12.4	2.4	8.96	.52	6.18	.17	2.72	.063	.959	.020	.196	.0042
2500	12.6	2.5	9.12	.52	6.26	.17	2.75	.064	.963	.021	.197	.0043
June												
100	11.7	4.7	8.81	.68	5.89	.28	2.63	.10	.946	.030	.190	.0057
300	12.3	4.9	9.30	.69	6.21	.28	2.73	.10	.974	.030	.197	.0057
1000	12.9	5.2	9.66	.70	6.41	.29	2.79	.10	.987	.030	.199	.0058
2500	13.2	5.5	9.84	.71	6.49	.30	2.82	.10	.991	.031	.200	.0059

TABLE XVIII
(cont.)

H(n.mi.)	α_o=0mr med	σ	α_o=10mr med	σ	α_o=30mr med	σ	α_o=100mr med	σ	α_o=300mr med	σ	α_o=1000mr med	σ
						July						
100	12.8	4.4	9.24	.90	6.13	.32	2.73	.10	.979	.031	.196	.0062
300	13.3	4.5	9.77	.90	6.48	.32	2.83	.10	1.005	.031	.203	.0062
1000	14.0	4.6	10.16	.91	6.68	.33	2.89	.10	1.020	.032	.205	.0064
2500	14.2	4.8	10.36	.92	6.76	.34	2.92	.10	1.024	.033	.206	.0065
						August						
100	12.4	2.0	9.20	.53	6.21	.17	2.76	.062	.982	.022	.197	.0045
300	13.1	2.0	9.71	.56	6.56	.17	2.86	.062	1.010	.022	.204	.0045
1000	13.7	2.0	10.10	.58	6.75	.17	2.92	.062	1.023	.023	.206	.0046
2500	13.9	2.0	10.29	.59	6.84	.18	2.95	.062	1.027	.024	.207	.0047
						September						
100	12.3	3.0	8.78	.79	5.98	.30	2.67	.094	.954	.031	.191	.0064
300	12.9	3.1	9.38	.80	6.33	.30	2.77	.096	.982	.032	.198	.0065
1000	13.5	3.2	9.77	.81	6.53	.31	2.83	.098	.995	.032	.200	.0066
2500	13.8	3.3	9.93	.82	6.61	.32	2.86	.099	.999	.033	.201	.0067
						October						
100	11.4	2.2	8.50	.38	5.78	.14	2.60	.052	.954	.017	.187	.0054
300	12.2	2.3	9.01	.39	6.11	.15	2.70	.053	.962	.017	.194	.0055
1000	12.9	2.4	9.37	.39	6.29	.15	2.76	.054	.975	.018	.196	.0055
2500	13.2	2.5	9.54	.40	6.36	.16	2.79	.055	.979	.018	.197	.0056
						November						
100	12.2	4.3	8.66	.96	5.94	.34	2.64	.106	.949	.031	.190	.0065
300	12.8	4.5	9.20	.97	6.26	.34	2.74	.106	.977	.032	.197	.0066
1000	13.4	4.7	9.58	.98	6.45	.35	2.80	.107	.990	.032	.199	.0067
2500	13.6	4.9	9.77	.99	6.53	.35	2.83	.108	.994	.033	.200	.0068
						December						
100	12.4	6.5	9.64	1.47	6.24	.58	2.74	.18	.993	.062	.196	.0012
300	13.0	7.0	10.16	1.48	6.58	.59	2.84	.18	1.021	.063	.203	.0012
1000	13.6	7.5	10.44	1.49	6.78	.59	2.90	.18	1.034	.064	.205	.0013
2500	13.8	8.0	10.76	1.50	6.87	.60	2.93	.18	1.038	.065	.206	.0013

TABLE XIX

COMBINED δ's (mr)

THULE DECEMBER 1957

α_o mr	Trop. med	Trop. σ	Ion. 100 mc med	Ion. 100 mc σ	Ion. 400 mc med	Ion. 400 mc σ	Total 100 med	Total 100 σ	Total 400 med	Total 400 σ
300 N. Miles										
1	12.9	2.3	1.25	0.55	0.0781	0.034	14.2	2.4	13.0	2.3
10	9.16	0.45	1.35	0.59	0.0843	0.037	10.5	0.74	9.24	0.45
30	6.20	0.15	1.39	0.61	0.0869	0.038	7.59	0.63	6.29	0.15
100	2.74	0.055	1.50	0.66	0.0959	0.041	4.24	0.66	2.83	0.069
300	0.974	0.018	1.125	0.495	0.0704	0.031	2.10	0.50	1.044	0.036
1000	0.197	0.0039	0.400	0.18	0.0250	0.01125	0.597	0.18	0.222	0.012
1000 N. Miles										
1	13.5	2.3	1.35	0.66	0.0843	0.041	14.9	2.4	13.6	2.3
10	9.54	0.45	1.48	0.723	0.0925	0.045	11.0	0.85	9.63	0.45
30	6.40	0.16	1.50	0.735	0.0959	0.046	7.90	0.76	6.49	0.17
100	2.80	0.055	1.485	0.726	0.0930	0.045	4.29	0.73	2.89	0.071
300	0.987	0.018	0.877	0.429	0.0548	0.027	1.86	0.43	1.042	0.032
1000	0.199	0.0040	0.439	0.21	0.0274	0.013	0.638	0.21	0.226	0.014
2500 N. Miles										
1	13.8	2.4	1.35	0.66	0.0843	0.041	15.2	2.5	13.9	2.4
10	9.70	0.46	1.47	0.72	0.0919	0.045	11.2	0.85	9.79	0.46
30	6.49	0.17	1.49	0.73	0.0930	0.046	7.98	0.75	6.58	0.18
100	2.83	0.055	1.39	0.682	0.0869	0.043	4.22	0.68	2.92	0.070
300	0.991	0.019	0.720	0.35	0.0450	0.022	1.71	0.35	1.036	0.029
1000	0.200	0.0041	0.252	0.12	0.0157	0.0075	0.452	0.12	0.216	0.0085

TABLE XX

COMBINED δ's (mr)

THULE JUNE 1957

α_o mr	Trop. med	Trop. σ	Ion. 100 mc med	Ion. 100 mc σ	Ion. 400 mc med	Ion. 400 mc σ	Total 100 med	Total 100 σ	Total 400 med	Total 400 σ
					300 N. Miles					
1	10.9	0.64	1.60	0.52	0.100	0.033	12.5	0.82	11.0	0.64
10	8.72	0.29	1.72	0.56	0.108	0.035	10.44	0.63	8.83	0.29
30	6.05	0.14	1.78	0.58	0.111	0.036	7.83	0.60	6.16	0.14
100	2.70	0.042	1.92	0.624	0.120	0.039	4.62	0.62	2.82	0.057
300	0.963	0.014	1.60	0.52	0.100	0.033	2.56	0.52	1.063	0.036
1000	0.194	0.0026	0.520	0.17	0.0325	0.011	0.714	0.17	0.227	0.011
					1000 N. Miles					
1	11.4	0.65	1.85	0.51	0.116	0.032	13.3	0.85	11.5	0.65
10	[illegible]	0.29	2.04	0.56	0.128	0.035	[illegible]	0.63	9.23	0.29
30	6.24	0.15	2.07	0.57	0.130	0.036	8.31	0.59	6.37	0.15
100	2.76	0.043	2.04	0.56	0.128	0.035	4.80	0.56	2.89	0.046
300	0.976	0.014	1.30	0.36	0.0813	0.023	2.28	0.36	1.057	0.027
1000	0.196	0.0027	0.647	0.18	0.0404	0.011	0.843	0.18	0.236	0.011
					2500 N. Miles					
1	11.7	0.66	1.85	0.51	0.116	0.032	13.6	0.83	11.8	0.66
10	9.26	0.30	2.01	0.56	0.126	0.035	11.27	0.63	9.39	0.30
30	6.32	0.15	2.02	0.56	0.126	0.035	8.34	0.58	6.45	0.15
100	2.79	0.044	1.91	0.53	0.120	0.033	4.70	0.53	2.91	0.055
300	0.980	0.015	1.06	0.29	0.0664	0.018	2.04	0.29	1.046	0.023
1000	0.197	0.0025	0.372	0.10	0.0232	0.0063	0.569	0.10	0.220	0.0068

TABLE XXI

COMBINED δ's (mr)

FAIRBANKS DECEMBER 1957 MID-DAY

α_o mr	Trop. med	Trop. σ	Ion. 100 mc med	Ion. 100 mc σ	Ion. 400 mc med	Ion. 400 mc σ	Total 100 med	Total 100 σ	Total 400 med	Total 400 σ
300 N. Miles										
1	13.0	7.0	4.48	1.87	0.270	0.117	17.48	7.4	13.27	7.0
10	10.16	1.48	4.84	2.02	0.302	0.126	15.0	2.50	10.46	1.485
30	6.58	0.59	4.98	2.09	0.311	0.131	11.56	2.17	6.89	0.74
100	2.84	0.18	5.38	2.25	0.336	0.141	8.22	2.25	3.176	0.224
300	1.021	0.063	4.03	1.68	0.252	0.105	5.05	1.68	1.273	0.123
1000	0.203	0.012	1.295	0.543	0.081	0.034	1.498	0.543	0.284	0.0366
1000 N. Miles										
1	13.6	7.5	4.95	1.62	0.308	0.101	18.55	7.65	13.908	7.5
10	10.44	1.49	5.43	1.78	0.339	0.110	15.87	2.32	10.779	1.49
30	6.78	0.59	5.50	1.80	0.344	0.112	12.28	1.89	7.124	0.60
100	2.90	0.18	5.43	1.78	0.339	0.110	8.33	1.79	3.239	0.211
300	1.034	0.064	3.20	1.05	0.200	0.066	4.234	1.05	1.234	0.092
1000	0.205	0.013	1.493	0.470	0.093	0.029	1.698	0.47	0.298	0.0318
2500 N. Miles										
1	13.8	8.0	4.93	1.62	0.308	0.101	18.73	8.2	14.108	8.0
10	10.76	1.50	5.39	1.76	0.337	0.110	16.15	2.31	11.097	1.50
30	6.87	0.60	5.40	1.77	0.338	0.111	12.27	1.86	7.208	0.615
100	2.93	0.18	5.10	1.67	0.319	0.104	8.03	1.68	3.249	0.216
300	1.038	0.065	2.63	0.864	0.164	0.054	3.668	0.864	1.202	0.084
1000	0.206	0.013	0.860	0.270	0.0538	0.0169	1.066	0.270	0.2598	0.0214

TABLE XXII

COMBINED δ's (mr)

FAIRBANKS DECEMBER 1957 NIGHT

α_o mr	Trop. med	Trop. σ	Ion. 100 mc med	Ion. 100 mc σ	Ion. 400 mc med	Ion. 400 mc σ	Total 100 med	Total 100 σ	Total 400 med	Total 400 σ
300 N. Miles										
1	13.0	7.0	1.05	0.45	0.066	0.028	14.05	7.0	13.07	7.0
10	10.16	1.48	1.135	0.485	0.071	0.030	11.30	1.56	10.23	1.48
30	6.58	0.59	1.168	0.500	0.073	0.031	7.75	0.77	6.65	0.59
100	2.84	0.18	1.26	0.54	0.079	0.034	4.10	0.57	2.92	0.183
300	1.021	0.063	0.945	0.405	0.059	0.025	1.966	0.44	1.080	0.068
1000	0.203	0.012	0.341	0.144	0.021	0.0090	0.544	0.144	0.224	0.015
1000 N. Miles										
1	13.6	7.5	1.21	0.59	0.076	0.037	14.81	7.5	13.68	7.5
10	10.44	1.49	1.33	0.65	0.083	0.0406	11.77	1.62	10.52	1.49
30	6.78	0.59	1.345	0.66	0.084	0.041	8.13	0.89	6.86	0.59
100	2.90	0.18	1.33	0.65	0.083	0.0406	4.23	0.67	2.98	0.183
300	1.034	0.064	0.846	0.413	0.053	0.026	1.88	0.418	1.087	0.069
1000	0.205	0.013	0.400	0.195	0.025	0.012	0.605	0.195	0.230	0.018
2500 N. Miles										
1	13.8	8.00	1.21	0.59	0.076	0.037	15.01	8.0	13.88	8.0
10	10.76	1.50	1.32	0.644	0.082	0.040	12.08	1.63	10.84	1.50
30	6.87	0.60	1.32	0.644	0.082	0.040	8.19	0.88	6.95	0.60
100	2.93	0.18	1.25	0.61	0.078	0.038	4.18	0.64	3.01	0.184
300	1.038	0.065	0.696	0.34	0.044	0.021	1.734	0.346	1.082	0.068
1000	0.208	0.013	0.230	0.112	0.014	0.0070	0.436	0.113	0.220	0.015

TABLE XXIII

COMBINED δ's (mr)

FAIRBANKS JUNE 1957

α_o mr	Trop. med	Trop. σ	Ion. 100 mc med	Ion. 100 mc σ	Ion. 400 mc med	Ion. 400 mc σ	Total 100 med	Total 100 σ	Total 400 med	Total 400 σ
300 N. Miles										
1	12.3	4.9	1.69	0.89	0.106	0.056	13.99	4.98	12.41	4.90
10	9.30	0.69	1.84	0.96	0.115	0.060	11.14	1.18	9.42	0.69
30	6.21	0.28	1.88	0.99	0.117	0.062	8.09	1.05	6.33	0.287
100	2.75	0.10	2.05	1.07	0.127	0.067	4.76	1.07	2.86	0.108
300	0.974	0.030	1.69	0.89	0.106	0.056	2.66	0.89	1.080	0.064
1000	0.197	0.0057	0.558	0.294	0.035	0.0184	0.755	0.294	0.232	0.0192
1000 N. Miles										
1	12.9	5.2	1.98	0.59	0.124	0.037	14.88	5.25	13.02	5.20
10	9.66	0.70	2.18	0.65	0.136	0.0406	11.82	0.95	9.80	0.70
30	6.41	0.29	2.20	0.66	0.137	0.041	8.61	0.72	6.55	0.293
100	2.79	0.10	2.18	0.65	0.136	0.0406	4.97	0.66	2.93	0.108
300	0.987	0.30	1.39	0.41	0.087	0.026	2.38	0.51	1.074	0.30
1000	0.199	0.0058	0.704	0.209	0.044	0.0131	0.903	0.209	0.243	0.0143
2500 N. Miles										
1	13.2	5.5	1.98	0.59	0.124	0.037	15.18	5.53	13.32	5.50
10	9.84	0.71	2.16	0.645	0.135	0.040	12.00	0.96	9.98	0.71
30	6.49	0.30	2.16	0.645	0.135	0.040	8.65	0.71	6.63	0.30
100	2.82	0.10	2.04	0.61	0.127	0.038	4.86	0.62	2.95	0.107
300	0.991	0.031	1.14	0.34	0.071	0.021	2.13	0.34	1.062	0.037
1000	0.200	0.0059	0.405	0.120	0.025	0.0075	0.605	0.120	0.225	0.0079

TABLE XXVI

INVERNESS JUNE 1957

COMBINED δ's (mr)

α_o mr	Trop. med	Trop. σ	Ion. 100 mc med	Ion. 100 mc σ	Ion. 400 mc med	Ion. 400 mc σ	Total 100 med	Total 100 σ	Total 400 med	Total 400 σ
300 N. Miles										
1	12.5	1.70	1.85	.75	.116	.047	14.35	1.86	12.62	1.70
10	9.50	.54	1.88	.763	.118	.048	11.18	.955	9.42	.54
30	6.49	.25	1.95	.80	.122	.050	8.44	.857	6.61	.255
100	2.87	.086	2.22	.90	.139	.056	5.09	.90	3.01	.104
300	1.013	.032	1.85	.75	.116	.047	2.863	.75	1.129	.057
1000	.204	.0062	.684	.278	.043	.0174	888	.278	.247	.0184
1000 N. Miles										
1	13.1	1.70	2.47	.83	.155	.053	15.57	1.90	13.26	1.70
10	9.67	.57	2.49	.86	.156	.054	12.16	1.03	9.83	.57
30	6.74	.26	2.52	.87	.158	.054	9.26	.907	6.90	.266
100	2.93	.069	2.72	.94	.170	.059	5.65	.94	3.10	.107
300	1.026	.033	1.85	.64	.116	.040	2.876	.64	1.142	.052
1000	.206	.0062	.986	.34	.0617	.021	1.192	.34	.268	.022
2500 N. Miles										
1	13.4	1.70	2.47	.85	.155	.053	15.87	1.90	13.56	1.70
10	9.85	.60	2.48	.854	.155	.053	12.33	1.04	10.01	.60
30	6.75	.27	2.49	.856	.156	.0535	9.24	.905	6.91	.27
100	2.96	.090	2.56	.885	.160	.055	5.52	.888	3.12	.105
300	1.030	.033	1.52	.530	.0951	.033	2.55	.530	1.125	.047
1000	.207	.0063	.561	.20	.0352	.0125	.767	.20	.242	.014

TABLE XXVII

INVERNESS DECEMBER 1954 MID-DAY

COMBINED δ's (mr)

α_o mr	Trop. med	Trop. σ	Ion. 100 mc med	Ion. 100 mc σ	Ion. 400 mc med	Ion. 400 mc σ	Total 100 med	Total 100 σ	Total 400 med	Total 400 σ
500 N. Miles										
1	11.9	1.2	.900	.20	.056	.0125	12.8	1.22	11.96	1.2
10	9.32	.40	.900	.20	.056	.0125	10.22	.45	9.38	.40
30	6.39	.17	.940	.21	.059	.013	7.33	.27	6.45	.17
100	2.80	.073	.990	.22	.062	.014	3.79	.23	2.86	.074
300	.998	.021	.630	.14	.039	.0087	1.628	.14	1.037	.023
1000	.202	.0046	.135	.030	.0084	.0048	.337	.030	.210	.0066
1000 N. Miles										
1	12.5	1.2	.955	.20	.060	.0125	13.5	1.22	12.56	1.2
10	9.70	.41	.989	.21	.062	.013	10.69	.46	9.76	.41
30	6.60	.18	.995	.21	.062	.013	7.60	.28	6.66	.18
100	2.86	.075	.955	.20	.060	.0125	3.82	.21	2.92	.076
300	1.011	.022	.478	.10	.030	.0063	1.489	.103	1.041	.023
1000	.204	.0047	.143	.030	.0089	.0048	.347	.030	.213	.0067
2500 N. Miles										
1	12.8	1.2	.955	.20	.060	.0125	13.8	1.22	12.86	1.2
10	9.89	.42	.964	.20	.060	.0125	10.85	.46	9.95	.42
30	6.69	.18	.964	.20	.060	.0125	7.65	.27	6.75	.18
100	2.89	.077	.898	.19	.056	.012	3.79	.20	2.95	.078
300	1.015	.023	.393	.082	.025	.0051	1.408	.085	1.040	.024
1000	.205	.0048	.0821	.017	.0051	.0011	.287	.018	.210	.0049

TABLE XXVIIA

INVERNESS DECEMBER 1954 NIGHT

COMBINED δ's (mr)

α_o mr	Trop. med	Trop. σ	Ion. 100 mc med	Ion. 100 mc σ	Ion. 400 mc med	Ion. 400 mc σ	Total 100 med	Total 100 σ	Total 400 med	Total 400 σ
500 N. Miles										
1	11.9	1.2	0.080	0.051	0.0050	0.0019	12.0	1.2	11.9	1.2
10	9.32	0.40	0.080	0.051	0.0050	0.0019	9.40	0.41	9.32	0.40
30	6.39	0.17	0.0835	0.032	0.0052	0.0020	6.47	0.17	6.39	0.17
100	2.80	0.073	0.088	0.034	0.0055	0.0021	2.89	0.80	2.81	0.073
300	0.998	0.021	0.072	0.028	0.0045	0.0017	1.070	0.035	1.002	0.021
1000	0.202	0.0046	0.0196	0.0076	0.0012	0.00048	0.222	0.0089	0.203	0.0046
1000 N. Miles										
1	12.5	1.2	0.090	0.039	0.0056	0.0024	12.6	1.2	12.51	1.2
10	9.70	0.41	0.0932	0.040	0.0058	0.0025	9.79	0.41	9.71	0.41
30	6.60	0.18	0.0937	0.041	0.0059	0.0026	6.69	0.18	6.61	0.18
100	2.86	0.075	0.090	0.039	0.0056	0.0024	2.95	0.084	2.87	0.075
300	1.011	0.022	0.049	0.021	0.0051	0.0013	1.06	0.030	1.014	0.022
1000	0.204	0.0047	0.0225	0.0098	0.0014	0.00061	0.227	0.0109	0.205	0.0047
2500 N. Miles										
1	12.8	1.2	0.090	0.039	0.0056	0.0024	12.9	1.2	12.81	1.2
10	9.89	0.42	0.091	0.040	0.0057	0.0025	9.98	0.42	9.90	0.42
30	6.60	0.18	0.091	0.040	0.0057	0.0025	6.78	0.18	6.70	0.18
100	2.89	0.077	0.085	0.037	0.0053	0.0023	2.98	0.082	2.90	0.077
300	1.015	0.023	0.040	0.017	0.0025	0.0011	1.055	0.029	1.018	0.023
1000	0.205	0.0048	0.0129	0.0056	0.00081	0.00035	0.218	0.0074	0.206	0.0048

TABLE XXIX

COMBINED δ's (mr)

INVERNESS JUNE 1954

α_o mr	Trop. med	Trop. σ	Ion. 100 mc med	Ion. 100 mc σ	Ion. 400 mc med	Ion. 400 mc σ	Total 100 med	Total 100 σ	Total 400 med	Total 400 σ
500 N. Miles										
1	12.5	1.70	0.950	0.42	0.058	0.026	13.45	1.75	12.56	1.7
10	9.50	0.54	0.950	0.43	0.058	0.027	10.23	0.69	9.36	0.54
50	6.49	0.25	0.968	0.45	0.060	0.028	7.46	0.51	6.55	0.25
100	2.87	0.088	1.02	0.47	0.064	0.029	3.89	0.48	2.93	0.093
300	1.013	0.032	0.766	0.34	0.048	0.021	1.779	0.34	1.061	0.038
1000	0.204	0.0062	0.214	0.098	0.013	0.0060	0.418	0.098	0.217	0.0086
1000 N. Miles										
1	13.1	1.70	0.990	0.44	0.062	0.028	14.09	1.76	13.16	1.7
10	9.67	0.57	1.025	0.46	0.064	0.029	10.70	0.61	9.73	0.57
50	6.74	0.26	1.03	0.46	0.064	0.029	7.77	0.53	6.80	0.26
100	2.93	0.089	0.990	0.44	0.062	0.028	3.92	0.45	2.99	0.093
300	1.026	0.033	0.643	0.29	0.040	0.018	1.669	0.29	1.066	0.038
1000	0.206	0.0062	0.218	0.097	0.014	0.0061	0.424	0.097	0.220	0.0087
2500 N. Miles										
1	13.4	1.70	0.990	0.44	0.062	0.028	14.39	1.76	13.46	1.7
10	9.85	0.60	0.996	0.44	0.062	0.028	10.85	0.66	9.91	0.60
50	6.75	0.27	0.998	0.44	0.062	0.028	7.75	0.52	6.81	0.27
100	2.96	0.090	0.930	0.41	0.058	0.026	3.89	0.42	3.02	0.094
300	1.030	0.033	0.529	0.24	0.033	0.015	1.559	0.24	1.063	0.036
1000	0.207	0.0063	0.125	0.056	0.0073	0.0035	0.332	0.056	0.214	0.0072

TABLE XXX

SAMPLE COMPUTATION OF γ and δ [illegible]

Fairbanks December 8, [illegible]

h-feet	N	N_o-N	N_j-N_k	500 tan β, $\alpha_o=0$	500 tan β, $\alpha_o=300$
0	313.1	0	13.4	0	155
669	299.7	13.4	16.0	3.13	155
2316	283.7	29.4	6.9	6.30	155
2779	276.8	36.3	17.2	6.80	155
4507	259.6	53.5	7.0	9.10	155
5246	252.6	60.5	10.0	9.7	155
6131	242.6	70.5	12.7	10.6	155
7535	229.9	83.2	10.6	11.6	155
8674	219.3	93.8	1.5	12.4	155
8844	217.8	95.3	22.1	12.8	155
11780	195.7	117.4	39.7	14.6	155
18128	156.0	157.1	10.7	19.0	155.5
19723	145.3	167.8	9.6	19.5	155.5
21464	135.7	177.4	30.8	20.5	155.5
28124	104.9	208.2	20.1	23.2	156
32303	84.8	228.3	16.2	25.7	156
36886	68.6	244.5	6.7	27.2	156.5
38828	61.9	251.2	21.4	28.0	157
47984	40.5	272.6	14.4	31.2	158
60000	26.1	287.0	22.2	35.6	159.5
100000	3.9	309.2	3.9	41.5	161
300000	0	313.1		84.0	185

H = 10 K

	$\alpha_o=0$	$\alpha_o=300$
	9.87	.3358
γ_H (mr)	104.1	104.1
N_o-N_H	27.0	310.
1000 tan β_H	266.5	104.1
γ_H tan β_H	48.8	.056
$\gamma^2/2$	211.2	.06
$\gamma \tan\beta + \gamma^2/2 - (N_o-N_H)$	36.9	.34
γ_H · tan β_H – tan α_o	5.72	.18
δ (mr)		

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