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STABILITY ANALYSES OF FLYING PLATFORM IN HOVERING AND FORWARD FLIGHT

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STABILITY ANALYSES OF FLYING
PLATFORM IN HOVERING AND
FORWARD FLIGHT

October 12, 1956

Report No. 112

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No. of pages 41

ADVANCED RESEARCH DIVISION
OF
HILLER HELICOPTERS

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LIST OF SYMBOLS

A	$= \int (y^2 + z^2)\, dm$
a	= distance from center of platform to pilot's platform support spring ft. Also $= K \left[S^2 + \frac{2\Omega}{K} S + 2\Omega^2 \right]$
a_1	= fore-aft tilt of gyrobar tip path plane; + aft
B	$= \int (z^2 + x^2)\, dm$
b	= distance from pilots platform to pilot c.g. Also = $2\Omega S$
b_1	= lateral tilt of gyrobar tip path plane; + to right
C	$= \int (x^2 + y^2)\, dm$
c	= distance from total c.g. to pilot's platform
D	$= \int yz\, dm$ $\left(\text{also } \frac{d}{dt}\right)$
d	= distance from total c.g. to c.g. of platform less pilot
E	$= \int zx\, dm$
F	$= \int xy\, dm$
g	= acceleration of gravity - Ft/sec^2
H	= distance from bottom of duct to c.g., ft.
h	= distance from point of application of aerodynamic drag force to c.g., ft.
h_f	= distance from bottom of duct to pilot platform, ft.
h_1, h_2, h_3	= angular momentum about X, Y, Z, axes respectively
I_1	= moment of inertia of platform less pilot about own axis slug-ft^2
I_2	= moment of inertia of pilot about own axis slug-ft^2
K	= damping ratio of gyrobar
k	= spring constant of pilot's platform support spring lb/ft
K_A^2	$= A/m$

K_B^2	=	B/m
K_C^2	=	C/m
K_D^2	=	D/m
K_E^2	=	E/m
K_F^2	=	F/m
K_2^2	=	radia of gyration squared, see page 16
K_2^2	=	radia of gyration squared, see page 16
L	=	distance from vanes to total c.g.
L,M,N	=	moments of external forces about X,Y,Z,axes divided by the mass
m	=	total mass of platform plus pilot-slugs
m_1	=	mass of platform less pilot-slugs
m_2	=	mass of pilot - slugs
m_r	=	m_2/m_1
M_{a_1}	=	pitching moment set up by change in gyrobar tilt, ft-lb/rad
$M_q m(=L_p m)$	=	pitching (rolling) moment developed for change in pitching (rolling) angular velocity ft-lb/rad/sec
$M_u m(=-L_v m)$	=	pitching (rolling) moment developed for change in forward (lateral) velocity ft-lb/ft/sec
$M_w m$	=	pitching moment developed for change in vertical velocity ft-lbs/ft/sec
n	=	a/a_1
p	=	$d\phi/dt$
q	=	$d\theta/dt$
r	=	distance from total c.g. to c.g. of pilot
r	=	$d\psi/dt$

S	= Laplace - transformation complex variable
T_o	= steady thrust component
u	= dx/dt
v	= dy/dt
w	= dz/dt
X,Y,Z	= components of external forces along X,Y,Z, axes divided by the mass
x,y,z	= displacements above the X,Y,Z, axes
$+x$	= horizontally forward
X_o	= steady aerodynamic force - also system input
X_{a_1}	= fore-aft force due to change in gyrobar tilt, lb/rad
$X_w m$	= fore-aft force developed for change in vertical velocity lb/ft/sec
$X_u m(=Y_v m)$	= fore-aft (lateral) force developed for change in pitching (rolling) angular velocity lb/rad/sec
$+y$	= horizontally to right
$+z$	= vertically down
Z_o	= steady aerodynamic force
$Z_q m$	= vertical force developed for a change in pitching angular velocity ft/rad/sec
$Z_u m$	= vertical force developed for a change in forward velocity lb/ft/sec
$Z_w m$	= vertical force developed for a change in vertical velocity lb/ft/sec
α	= angle of attack of gyrobar stabilizing vanes
δ_1	= flapping deflection of gyrobar number 1
δ_2	= flapping deflection of gyrobar number 2
θ	= angular displacement about Y axis
θ_o	= steady angle that X axis makes with horizontal
ϕ	= angular displacement about X axis
Ψ	= angular displacement about Z axis

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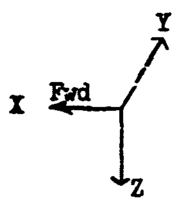
TABLE OF CONTENTS

I. SUMMARY

This report presents a summary of various analyses of the dynamic stability characteristics of the Model 1031B Rotorcycle (Flying Platform), in both hovering and forward flight conditions. To establish the notation, the derivation of equations of motion for a hovering rigid body is first outlined. To introduce the factors affecting the platform's stability, a hovering analysis consisting of both two and four degrees of freedom is presented. A spring-mounted pilot is considered, and finally an investigation is made of the problems associated with installing two gyro bars to stabilize both the hovering and forward flight conditions.

II. THE EQUATIONS OF MOTION



A right handed system of Cartesian coordinates are used, where:

x, y, z are the displacements along the X, Y, Z axes.

$u = dx/dt$, $v = dy/dt$, $w = dz/dt$ are the velocities of translation along the axes.

du/dt, dv/dt, dw/dt are the accelerations along the axes.

ϕ, Θ, Ψ are the angular displacements about the axes (roll, pitch, yaw).

$p = d\phi/dt$, $q = d\Theta/dt$, $r = \frac{d\Psi}{dt}$ are the angular velocities about the axes.

X, Y, Z are the components of the external force divided by the mass (accelerations), F_x/m, F_y/m, F_z/m.

h_1, h_2, h_3 are the angular momentum about the respective axes.

L, M, N are the moments of the external force about the respective axes, divided by the mass (rolling moment/mass, pitching moment/mass, yawing moment/mass).

The six equations of motion of the platform, considered as a rigid body, and relative to axes fixed in space, are

$$X = \frac{du}{dt} \qquad Y = \frac{dv}{dt} \qquad Z = \frac{dw}{dt}$$

$$\frac{1}{m}\frac{dh_1}{dt} = L \qquad \frac{1}{m}\frac{dh_2}{dt} = M \qquad \frac{1}{m}\frac{dh_3}{dt} = N \tag{1}$$

Moments of momentum can be written as

$$
\begin{aligned}
h_1 &= Ap - Fq - Er \\
h_2 &= Bq - Dr - Fp \\
h_3 &= Cr - Ep - Dq
\end{aligned}
\tag{2}
$$

where

$$
\begin{aligned}
A &= \int (y^2 + z^2)\, dm \\
B &= \int (z^2 + x^2)\, dm \\
C &= \int (x^2 + y^2)\, dm \\
D &= \int yz\, dm \\
E &= \int zx\, dm \\
F &= \int xy\, dm
\end{aligned}
\tag{3}
$$

If we describe the motion relative to fixed axes, then as the platform moves through space the moments and products of inertia relative to these axes change with time. To avoid this difficulty Eulerian axes (or moving axes) are used which at any instant are fixed in space but which change their position from instant to instant, coinciding at any instant with a definite set of axes fixed in the platform. As a result of this choice of axes, the expressions for the true acceleration and angular momentum relative to fixed axes become

$$
\begin{aligned}
a_x &= \frac{du}{dt} - vr + wq \\
a_y &= {}^{dv}/_{dt} - wp + ur \\
a_z &= {}^{dw}/_{dt} - uq + vp \\
dH_x/dt &= dh_1/dt - h_2 r + h_3 q \\
dH_y/dt &= dh_2/dt - h_3 p + h_1 r \\
dH_z/dt &= dh_3/dt - h_1 q + h_2 p
\end{aligned}
\tag{4}
$$

where a_x, a_y, a_z, dH_x/dt, dH_y/dt, dH_z/dt are all measured relative to fixed axes, and u, v, w, h_1, h_2, h_3 are all measured relative to Eulerian axes.

If we combine equations (1), (2) and (4) and introduce the radii of gyration by $K_A^2 = A/_m$, $K_B^2 = B/_m$, etc., there results the following equations of motion relative to Eulerian axes:

$$du/dt - vr + wq = X$$

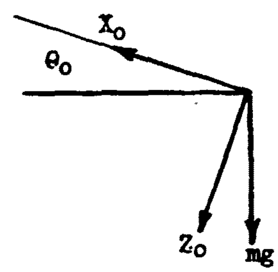
$$dv/dt - wp + ur = Y$$

$$dw/dt - uq + vp = Z$$

$$
\begin{aligned}
&K_A^2\, dp/dt - K_F^2\, dq/dt - K_E^2\, dr/dt + qr\left[K_C^2 - K_B^2\right] + K_D^2\, r^2 \\
&\qquad + K_F^2\, pr - K_E^2\, pq - K_D^2 q^2 = L \\
&K_B^2\, dq/dt - K_D^2\, dr/dt - K_F^2\, dp/dt + pr\left[K_A^2 - K_C^2\right] + K_E^2 \\
&\qquad \left[p^2 - r^2\right] + K_D^2\, pq - K_F^2\, rq = M
\end{aligned}
\tag{5}
$$

$$K_C^2\, dr/dt - K_E^2\, dp/dt - K_D^2\, dq/dt + pq\left[K_B^2 - K_A^2\right] + K_F^2\, q^2 + K_E^2\, qr + K_D^2\, pr - K_F^2\, p^2 = N \tag{5}$$

The external forces and moments must now be considered. Since the X axis will be taken as being in the direction of motion, K_A^2 and K_C^2 will be slightly different for every flight condition. If θ_o is the angle that the X axis makes with the horizontal, then the equilibrium equations for steady motion are



$$X_o + T_o - g \sin \theta_o = 0$$
$$Z_o + g \cos \theta_o = 0 \tag{6}$$

If small deviations from steady flight are considered, there is the possibility of the 36 stability derivatives:

$$(X, Y, Z, L, M, N)_{u,\, v,\, w,\, p,\, q,\, r}$$

Because of symmetry, and the fact that the z motion will not be considered, only eight deviatives are of interest in the hovering analyses:

$$X_u = Y_v$$
$$M_u = -L_v$$
$$X_q = -Y_p$$
$$M_q = L_p$$

In the disturbed state, the axes are displaced from the steady state by the small angular rotations ϕ, θ, Ψ. The components of gravity relative to the new axes are

$$
\begin{aligned}
O_x &: -g \sin \theta_o - g \cos \theta_o \, \theta \\
O_y &: \;\; g \, \psi \sin \theta_o + g \, \phi \cos \theta_o \qquad (7) \\
O_z &: -g \sin \theta_o \, \theta + g \cos \theta_o
\end{aligned}
$$

The net component of all external forces are then (including Z)

$$
\begin{aligned}
X &= -g \cos \theta_o \, \theta + X_u \, u + X_q \, q \\
Y &= g \sin \theta_o \, \psi + g \, \phi \cos \theta_o + Y_v \, v + Y_p \, p \qquad (8) \\
Z &= -g \sin \theta_o \, \theta + Z_u \, u + Z_v \\
L &= L_v \, v + L_p \, p \\
M &= M_u \, u + M_q \, q \\
N &= 0
\end{aligned}
$$

If equations (5) and (8) are combined, powers and products of small quantities are neglected, and $K_D^2 - K_E^2$ is assumed zero for the platform, the resulting equations of motion are (neglecting yaw, and vertical motion)

$$
\begin{aligned}
&(D - X_u)u - (X_q \, D - g \cos \theta_o) \, \theta = 0 && \text{(a)} \\
&(D - Y_v)v - (Y_p \, D + g \cos \theta_o) \, \phi = 0 && \text{(b)} \\
&-L_v \, v + (K_A^2 \, D^2 - L_p \, D) \, \phi - K_F^2 \, D^2 \, \theta = 0 && \text{(c)} \qquad (9) \\
&-M_u \, u + (K_B^2 \, D^2 - M_q \, D) \, \theta - K_F^2 \, D^2 \, \phi = 0 && \text{(d)}
\end{aligned}
$$

These are the equations that will be used in the hovering analyses. In every case, $\cos \theta_o$ will be assumed one. The first two equations are equations of forces (actually linear accelerations as written) and the last two are moment equations (angular accelerations). If the stability derivatives are found in terms of forces in pounds, and moments in ft-pounds, they must be divided by the mass in slugs before being used in the above equations.

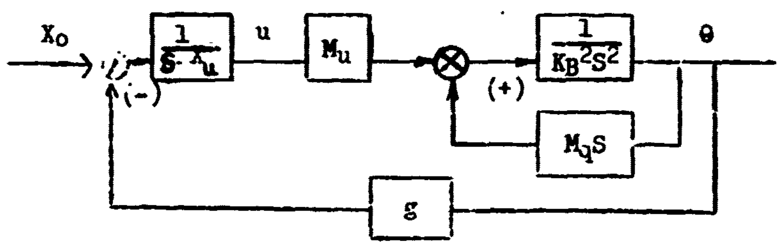
III. TWO DEGREE OF FREEDOM ANALYSIS WITH $X_q = 0$

The change in drag due to a pitching velocity (i.e. X_q) is obviously small and also occurs in the equations in such a way as to be unimportant. In the following analysis only the pitching and forward displacements will be considered, with $X_q = 0$. Section IV will show the effects of X_q. Under these assumptions, the equations are

$$\begin{array}{ccc} u & \theta & \\ S - X_u & g & = X_o \\ -M_u & S(K_B^2 S - M_q) & = 0 \end{array} \qquad (10)$$

where X_o is the Laplace - Transformation of an arbitrary forward acceleration input and S is the Laplace - Transformation complex variable.

The block diagram for this system is



and the open loop transfer function can be written

$$\text{O.L.T.F.} = \frac{\frac{M_u\, g}{K_B^2}}{S(S - X_u)(S - \frac{M_q}{K_B^2})} = \frac{\frac{M_u\, g}{K_B^2}}{S(S + .222)(S + .0726)} \; * \qquad (11)$$

* See following table for numerical derivatives.

Numerical calculations based on Hiller report No. 680.2 will show the following variations of the stability derivatives and constants with center of gravity location (See also Report ARD No. 111):

h_f	I	K_B^2	$\frac{M_q}{K_B^2}$	$\frac{M_{u}g}{K_B^2}$	X_u
31	109	7.56	-.0748	+.0343	-.222
32.2	112.5	7.8	-.0748	0	-.222
33.0	115	7.98	-.0748	-.224	-.222
34	118	8.18	-.0748	-.0491	-.222
					m = 14.43 slugs

Fortunately, M_q/K_B^2 does not change with c.g. location, which greatly simplifies the problem of determining the effect of c.g. (M_u) variation. In the range of c.g. locations considered, M_u changes sign, making the system regenerative feedback.

Figure 1 shows the root locus of the system. Positive M_u variations are shown in red and negative M_u variations in blue. At a gain of .00529 neutral stability exists at a frequency of 0.13^+ rad/sec. Increasing the c.g. location height (i.e., raising the pilot), makes M_u less positive and the platform stable. Theoretically, at a gain of .00081 the oscillitory roots would be .5 critically damped, which would give a reasonable response. (The real pole at 0.24 would affect the response only slightly).

The mM_u at neutral stability is

$$mM_u \approx \frac{(.00529)(14.43)}{4.26} = .0179 \ ^{Ft\text{-}lb}/Ft/sec.$$

At .5 damping

$$mM_u \approx \frac{(.00081)(14.43)}{4.26} = .00274 \ ^{Ft\text{-}lb}/Ft/sec.$$

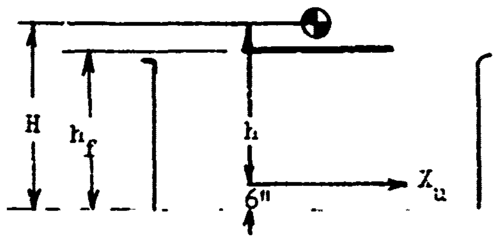
Although this represents a 6.5 to 1 change in mM_u, from Figure 2 it is seen to occur over a very small range of c.g. variations near zero M_u. The analysis thus shows that the platform is theoretically very sensitive to vertical c.g. location, stable only for a very small range of positive M_u's near zero, and unstable for all negative M_u's.

The platform as designed has a c.g. location of 19.5 inches above the bottom of the duct, and $M_u = 1.3/14.42 = +.0902$ (Reference Figure 3). The resulting stability equation is

$$S^3 + .295\,S^2 + .0161S + .516 = \left[S + .906\right] \left[(S - .305)^2 + .69^2\right]$$

which would give an unstable response.

IV. TWO DEGREE OF FREEDOM ANALYSIS WITH $X_q \neq 0$



The transverse component of the angular velocity at X_u due to a rotation q about the c.g. is hq.

The fore-aft force due to this rotation is $X_u hq$, and hence

$$X_q = X_u h$$

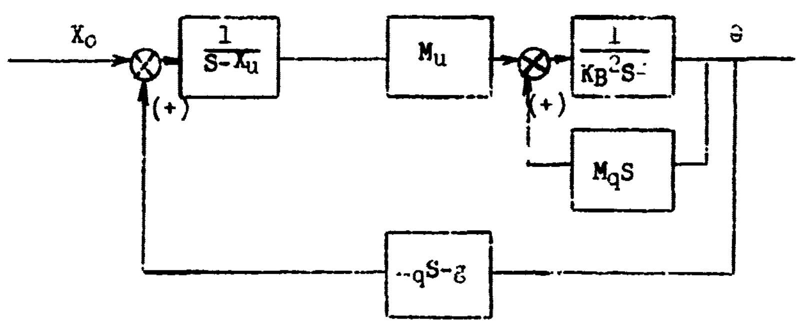
With reference to Figure 3, the following table can be constructed:

h_f	H	X_u	h(ft)	X_q
31	34.83	-.222	2.4	-.532
32.2	35.3	-.222	2.44	-.541
33	35.6	-.222	2.465	-.548
34	35.96	-.222	2.495	-.554
				Ave. -.544

If X_q is not neglected, the equations are then

$$\begin{array}{ccc} u & \theta & \\ S-X_u & -(X_q S-g) & =X_0 \\ -M_u & K_B^2 S^2-M_q S & =0 \end{array} \qquad (12)$$

The block diagram for this system is



where the outer loop has been made regenerative (X_q however, is negative). For this system

$$\text{O.L.T.F.} = \frac{\frac{M_u}{K_B^2} X_q \left(S - \frac{g}{X_q}\right)}{S(S - X_u)\left(S - \frac{M_q}{K_B^2}\right)} = \frac{\frac{M_u X_q}{K_B^2} (S + 59.2)}{S(S + .222)(S + .0726)} \tag{13}$$

Consideration of X_q thus adds the zero at -59.2, which has almost a negligable effect on the low frequency behavior of the system. (Calculations will show, for example, that neutral stability will occur for $M_u = .0167$ at $\omega = .129$ rad/sec rather than at a frequency of $\omega = .12^+$ for $M_u = .0179$). The high frequency behavior is considerably different, however, since the system now approaches infinity as $1/S^2$ rather than as $1/S^3$. Since the asymptote is now vertical, the theoretical possibility exists of changing the system to make the asymptote intersect the <u>negative</u> real axis. The platform would then be stable for <u>all</u> c.g. locations that give any positive M_u.

Since the asymptote intersects the axis at the point $1/2 \left[\Sigma \text{poles} - \Sigma \text{zeros}\right]$ the intersection will be positive if

$$\left| X_u + \frac{M_q}{K_B^2} \right| > \left| \frac{g}{X_q} \right| \tag{14}$$

assuming that X_u, M_q, and X_q maintain their negative sign. For the platform as now designed this inequality results in

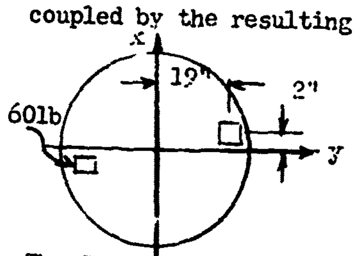
$$\left| .222 + .0726 \right| \not> 59.2$$

which is far from being satisfied.

One obvious, though perhaps impractical, method of achieving stability would be to hang a flat plate below the platform. Then both M and X would increase with pitching velocity, achieving the desired stability.

V. FOUR DEGREE OF FREEDOM ANALYSIS WITH $X_q = 0$

The platform has its two engines mounted to either side of the pitch and roll axes as shown in the sketch below. The motions are then coupled by the resulting product of inertia about the vertical axis.



$$F = \Sigma Mxy = \frac{(2)(60)(2)(19)}{(32.2)(144)} = 0.984 \text{ slug/ft}^2$$

The four hovering equations (neglecting $X_q = Y_p$) are

u	v	ϕ	θ		
$S-X_u$	0	0	$+g$	$=X_o$	
0	$S-Y_v$	$-g$	0	$=0$	(15)
0	$-L_v$	$K_A^2S^2-L_pS$	$-K_F^2S^2$	$=0$	
$-M_u$	0	$-K_F^2S^2$	$K_B^2S^2-M_qS$	$=0$	

If these are solved for the pitch and roll responses the results can be put in the form

$$\frac{\theta}{M_uX_o} = \frac{\frac{1}{K_B^2}\bar{M}}{M\bar{M} - \frac{(K_F^4S^4)}{K_A^2K_B^2}(S-X_u)(S-Y_v)} \qquad (16)$$

$$\frac{\phi}{M_uX_o} = \frac{\frac{K_F^2}{K_A^2K_B^2}S^2(S-Y_v)}{M\bar{M} - \frac{K_F^4S^4}{K_A^2K_B^2}(S-X_u)(S-Y_v)} \qquad (1\ldots)$$

where $$M = S(S - \frac{M_q}{K_B^2})(S - \text{[illegible]}_u) + \frac{M_u g}{K_B^2}$$

$$\bar{M} = S(S - \frac{L_p}{K_A^2})(S - \text{[illegible]}_v) - \frac{L_v g}{K_A^2}$$

If symmetry is assumed, $M = \bar{M}$. Furthermore, since the second term in the denomination subtracts a negligible amount from M^2, the response contains double roots and is thus unstable.

For example, if the c.g. were raised to the stable height such that $M_u = .00274$ ft-lb/ft/sec (Reference Fig. 2) then M^2 would be $S^6 + .5936\ S^5 + .1214\ S^4 + \ldots$ and the second terms in the denomination mentioned above would subtract $0.0000816\ S^6 + .000036\ S^5 + .0000004\ S^4$ from this. As K_F^2 becomes larger, the roots would spread and eventually give a stable response, but most likely one containing large amplitude transients. Flight tests have indicated a marked improvement in response when F was made equal to zero.

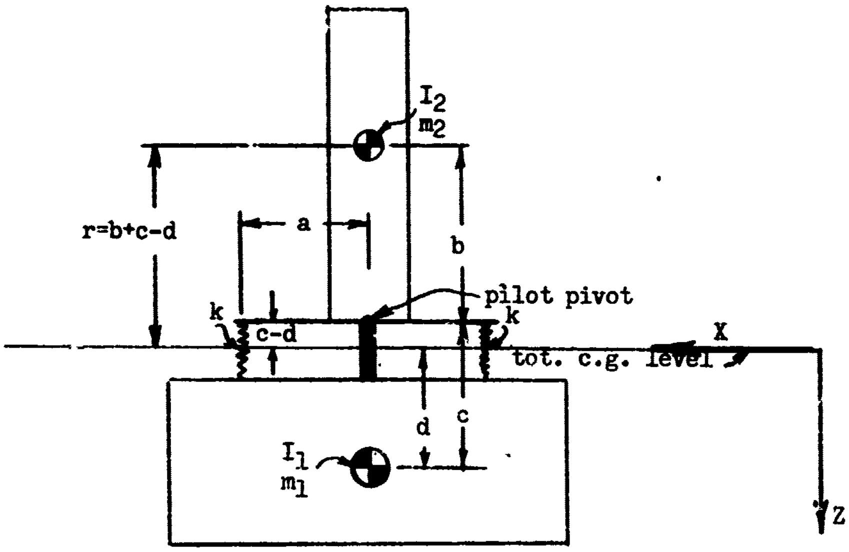
VI. ANALYSIS OF A SPRING MOUNTED PILOT PLATFORM

The pilot's platform is considered to be mounted on two springs, each of spring constant k $^{lb}/ft$ located at a distance "a" ft from the center of the platform. Only pitching and horizontal displacements will be considered; the three degrees of freedom being:

1) x - displacement of total c.g. from fixes axes,

2) θ - rotation of m_1 relative to vertical (+ nose up),

3) α_r - rotating of m_2 (pilot) relative to platform (+ pilot tilts back).



The kinetic energy associated with the motion of the mass m_1 is

$$T_1 = \frac{1}{2} I_1 \dot{\theta}^2 + \frac{1}{2} m_1 \left[\dot{x} + d\dot{\theta}\right]^2 \tag{18}$$

The kinetic energy associated with the motion of the mass m_2 is

$$T_2 = \frac{1}{2} I_2 (\dot{\theta} + \dot{\alpha}_r)^2 + \frac{1}{2} m_2 \left[\dot{x} - (b + c - d)\,\dot{\theta} - b\,\dot{\alpha}_r\right]^2 \tag{19}$$

The total potential energy of the spring deflection is

$$V_s = k\, a^2\, \alpha_r^{\,2} \tag{20}$$

Under the assumption of small angles the potential energy associated with a tilt back of mass m_2 is

$$V_2 = -m_2 g \left[\frac{1}{2}\, r\, \theta^2 + \frac{1}{2}\, b\, \alpha_r^{\,2} + b\, \alpha_r\, \theta\right] \tag{21}$$

And for a rotation of m_1 the energy function is

$$V_1 = \frac{1}{2}\, m_1\, g\, \theta^2\, d \tag{22}$$

If Lagrangian equations are applied to the above energies and the result combined with the equations previously derived for the platform (also applicable to fixed axes) the resulting equations are

$$\begin{array}{llll} u & \theta & \alpha_r & \\ s - X_u & g & -m_r\, b\, s^2 & = X_o \\ -M_u & K_B^{\,2} s^2 - M_q s & \bar{K}_2^{\,2} s^2 - m_r g b & = 0 \\ -m_r b s & \bar{K}_2^{\,2} s^2 - m_r g b & K_2^{\,2} s^2 + \left(\dfrac{2k\, a^2}{m} - m_r g b\right) & = 0 \end{array} \quad (2$$

where

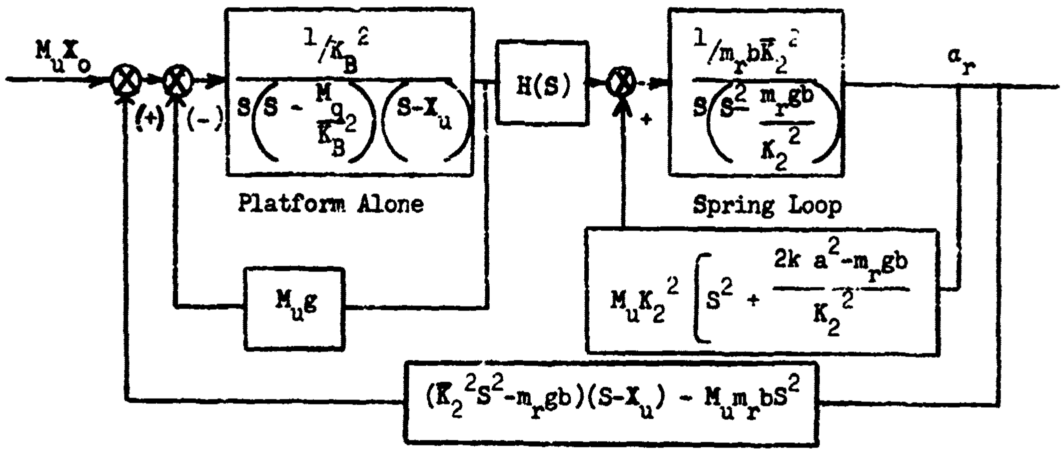
$$\frac{m_2}{m} = m_r$$

$$\bar{K}_2^2 = \frac{I_2 + m_2 r b}{m}$$

$$K_2^2 = \frac{I_2 + m_2 b^2}{m}$$

$$K_B^{\,2} = \frac{I_1 + I_2 + m_1 d^2 + m_2 r^2}{m}$$

The block diagram of this system is



$$H(S) = m_r b K_B^2 \left\{ S^3 - \left[\frac{M_q}{K_B^2} + \frac{M_u}{K_B^2}\,\frac{\bar{K}_2^2}{m_r b}\right] S^2 + \frac{M_u g}{K_B^2} \right\}$$

Although the system is complicated, qualitative results can readily be found if it is handeled numerically and only the responses for variation in the spring constant are investigated.

For the present platform the constants are

$X_u = -0.222$

$M_q = -0.407$

$M_u = +0.0902$

$m_2 = \frac{175}{32.2} = 5.43$

$m_1 = \frac{290}{32.2} = 9.00$

$m_r = \frac{5.43}{14.43} = .376$

$K_B^2 = 5.62$

$\frac{M_q}{K_B^2} = -0.0725$

$I_2 = 1.645$

$b = 3.33$

$c = .542$

$d = 1.46$

$r = 2.42$

$K_2^2 = 4.28$

$\bar{K}_2^2 = 3.11$

The C.L.T.F. of the Platform alone is $= \dfrac{1/K_B^{\,2}}{(S+.906)\left[(S-.305)^2+.69^2\right]}$ (24)

$$H(S) = m_r B K_B^{\,2}\left[(S + .8134) \quad (S - .390)^2 + .695^2\right]$$

The feedback T.F. $= \bar{K}_2^{\,2}\ (S + .222)(S + 3.582)(S - 3.618)$

The open loop T.F. of the spring loop is

$$\text{O.L.T.F.} = \frac{M_u K_2^{\,2}}{m_r b \bar{K}_2^{\,2}} \; \frac{\left[S^2 + \frac{2ka^2}{61.7} - 9.42\right]}{S\left[S^2 - 12.97\right]} \tag{25}$$

Since $2ka^2$ must be greater than 580 $^{\text{Ft-lb}}/\text{rad}$, any variation in k only moves the complex zeros up and down the jw axis. Furthermore, since the above gain is small (.0991) for the present M_u, the open loop poles move very little. For example, if $2ka^2 = 700$ $^{\text{Ft-lb}}/\text{rad}$, the closed loop T.F. is

$$\text{C.L.T.F.} = \frac{\dfrac{1}{m_r b \bar{K}_2^{\,2}}}{(S+.01475)(S-3.66)(S+3.54)} \tag{26}$$

For higher values of the spring constant, the small real root would become more negative. The complete O.L.T.F. for $2ka^2 = 700$ is

$$\frac{(S+.8134)(S+.222)(S+3.582)(S-3.618)\left[(S-.390)^2+.695^2\right]}{(S+.906)(S+.01475)(S+3.54)(S-3.66)\left[(S-.305)^2+.69^2\right]} \tag{27}$$

Since the gain is one, the pole at +3.66 goes to infinity and does not enter into the response. The unstable roots of the helicopter will still be present with the additional possibility of an aperiodic root from the small spring loop pole. If the spring constant is greater than 2370 $^{\text{Ft-lb}}/\text{rad}$, the spring pole will be to the left of .222 and the possibility of divergent aperiodic motion is eliminated. The system, however, is still unstable and the conclusion is reached that mounting the pitot on springs does not appear to be a promising method of improving stability.

VII. IN PLANE ANALYSIS OF GYROBAR STABILIZING DEVICE

In this section a free pivoted, air damped gyrobar is analyzed. The bar senses rate of pitching motion ($\dot{\theta}$), and by linkages, controls vanes located below the platform that set up correcting moments. An identical system controls the roll rate ($\dot{\phi}$). Pitch alone will be analyzed here and in Section VIII. coupled roll and pitch will be considered.

If δ_1 is the amplitude of the flapping deflection of the pitch control bar, and δ_2 the flapping amplitude of the roll control bar then*

$$\begin{aligned} \ddot{\delta}_1 + 2K\Omega\dot{\delta}_1 + \Omega^2\delta_1 = & -2\Omega\dot{\theta}\sin\Psi_1 + 2K\Omega\dot{\theta}\cos\Psi_1 \\ & +2\Omega\dot{\phi}\cos\Psi_1 + 2K\Omega\dot{\phi}\sin\Psi_1 \end{aligned} \tag{28}$$

$$\begin{aligned} \ddot{\delta}_2 + 2K\Omega\dot{\delta}_2 + \Omega^2\delta_2 = & -2\Omega\dot{\theta}\sin\Psi_2 + 2K\Omega\dot{\theta}\cos\Psi_2 \\ & +2\Omega\dot{\phi}\cos\Psi_2 + 2K\Omega\dot{\phi}\sin\Psi_2 \end{aligned} \tag{29}$$

where $\Psi_2 = 90^\circ + \Psi_1$

Under the assumptions

$$\delta_1 = -a_1\cos\Psi_1 - b_1\sin\Psi_1$$

$$\delta_2 = -a_1\cos\Psi_2 - b_1\sin\Psi_2$$

where $+a_1$ is + tilt back

$+b_1$ is + tilt to right

* "The Frequency Response of the Ordinary Rotor Blade, the Hiller Servo Blade, and the Young-Bell Stabiliser" by G. J. Sissingh, Royal Aircraft Establishment Report No. Aero 2367, May 1950.

the above two equations reduce to

$$
\begin{array}{ccccc}
\theta & \phi & a_1 & b_1 & \\
2\Omega S & -2K\Omega S & 2\Omega S+2K\Omega^2 & -S\left[S+2K\Omega\right] & = 0 \\
+2K\Omega S & +2\Omega S & S\left[S+2K\Omega\right] & 2\Omega S+2K\Omega^2 & = 0
\end{array}
\qquad (30)
$$

If only a pitch ($\dot{\theta}$ sensing) bar is considered, the equation representing the bar is

$$K\Omega a_1 + \dot{a}_1 + \dot{\theta} = 0 \qquad (31)$$

This, together with the platform equation (page 7), result in the following group representing the system:

$$
\begin{array}{cccc}
u & \theta & a_1 & \\
S-X_u & g & -X_{a_1} & = X_o \\
-M_u & K_B S^2-M_qS & -M_{a_1} & = 0 \\
0 & S & S+K\Omega & =0
\end{array}
\qquad (32)
$$

where X_{a_1} and M_{a_1} are the force and moment derivatives set up by bar motion.

If the angle of attack of the vane is denoted by α, the linkage ratio n is defined by

$$n = \frac{\alpha}{a_1}$$

then

$$X = X_\alpha \alpha = X_\alpha n a_1 = X_{a_1} a_1$$

$$M = M_\alpha \alpha = M_\alpha n a_1 = M_{a_1} a_1$$

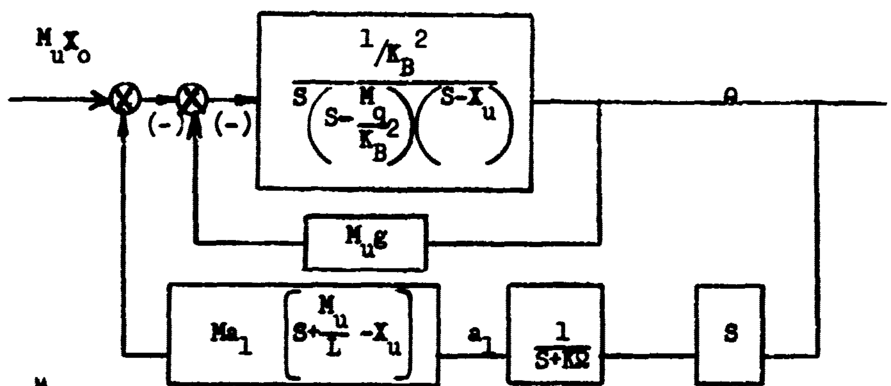
If it is assumed that

$$M_{a_1} = X_{a_1} L$$

where L is the distance from the vane to the c.g. (3.04 ft. with h_f = 19.5"), then

$$\frac{X_{a_1}}{M_{a_1}} = \frac{1}{L} = \text{constant}$$

The block diagram for the system is



$$\text{O.L.T.F.} = \frac{\dfrac{M_{a_1}}{K_B^2}\, S \left[S + \dfrac{M_u}{L} - X_u\right]}{\left[S+K\Omega\right]\left[S\left(S - \dfrac{M_q}{K_B^2}\right)\left(S-X_u\right) + \dfrac{M_u g}{K_B^2}\right]}$$

$$= \frac{\dfrac{M_{a_1}}{K_B^2}\, S \left[S + .252\right]}{\left[S+K\Omega\right]\left[S+.906\right]\left[(S-.305)^2 + .69^2\right]} \qquad (33)$$

The system is very sensitive to changes in $K\Omega$. A value of $K\Omega$ = .4 and M_{a_1}/K_B^2 of about 3.0 results in a reasonable response (See Fig. 4). With Ω = 2550 rpm (267 rad/sec) K = .0015, a very small value.

If $M_{a_1}/K_B^2 = 3.0$ then

$$M_{a_1} = (3)(5.62)\frac{(14.43)}{57.3} = 4.25\ ^{\text{Ft-lb}}/\text{degree}$$

The M_α realizable from the present vane configuration is about $6.4^{\text{Ft-lb}}/$ degree. Therefore, the linkage ratio is

$$n = \frac{4.25}{6.40} = .65$$

If the bar were allowed 15° maximum deflection, the vanes would then be at approximately 10°, which is about stall.

VIII. COUPLED PITCH AND ROLL ANALYSIS OF GYROBAR STABILIZING DEVICE

The gyrobars couple the pitch and roll responses of the platform. The six equations are then

$$
\begin{array}{ccccccc}
u & v & \phi & \theta & a_1 & b_1 & \\
S-X_u & 0 & 0 & -(X_q S-g) & -X_{a_1} & 0 & =X_o \\
0 & S-Y_v & -(Y_p S+g) & 0 & 0 & -Y_{b_1} & =0 \\
0 & -L_v & K_A{}^2 S^2-L_p & 0 & 0 & -L_{b_1} & =0 \\
-M_u & 0 & 0 & K_B{}^2 S^2-M_q S & -M_{a_1} & 0 & =0 \\
0 & 0 & -2K\Omega S & 2\Omega S & 2\Omega S+2K\Omega^2 & -\left[S^2+2K\Omega S\right] & =0 \\
0 & 0 & 2\Omega S & 2K\Omega S & S^2+2K\Omega S & \left[2\Omega S+2K\Omega^2\right] & =0
\end{array}
\tag{31}
$$

If symmetry is assumed and $X_q = -Y_p = 0$ then

$$K_A{}^2 = K_B{}^2$$

$$X_u = Y_v$$

$$M_q = L_p$$

$$X_{a_1} = -Y_{b_1}$$

$$M_{a_1} = L_{b_1}$$

$$M_u = -L_v$$

The six equations can be reduced to the following four:

$$
\begin{array}{ccccc}
\phi & \theta & a_1 & b_1 & \\
0 & M & -N & 0 & = M_u X_o \\
M & 0 & 0 & -N & =0 \\
-2K\Omega S & 2\Omega S & 2\Omega\left[S+K\Omega\right] & -S\left[S+2K\Omega\right] & =0 \\
2\Omega S & 2K\Omega S & S\left[S+2K\Omega\right] & 2\Omega\left[S+K\Omega\right] & =0
\end{array}
\qquad (35)
$$

where
$$M = K_B{}^2\left[\left(S^2 - \frac{M_q}{K_B{}^2}\right)\left(S - X_u\right) + \frac{M_u g}{K_B{}^2}\right] \qquad (36)$$

$$N = M_{a_1}\left[S + \frac{M_u X_{a_1} - M_{a_1} X_u}{M_{a_1}}\right] \qquad (37)$$

If the additional notation is used that

$$a = K\left[S^2 + \frac{2\Omega}{K} S + 2\Omega^2\right]$$

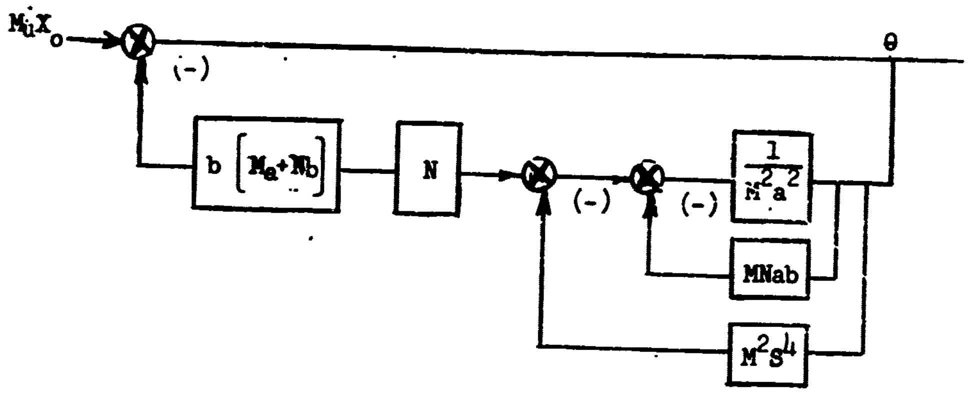
$$b = 2\Omega S$$

the responses for the pitch axis are

$$\frac{\theta}{M_u X_o} = \frac{Ma(Ma+Nb) + M^2 S^4}{(Ma+Nb)^2 + M^2 S^4} \qquad (38)$$

$$\frac{a_1}{M_u X_o} = \frac{-b\,(Ma+Nb)}{(Ma+Nb)^2 + M^2 S^4} \qquad (39)$$

The block diagram for the system is



For the platform as designed

$$M = K_B^{\ 2} \left[(S+.906) \left[(S-.305)^2 + .69^2 \right] \right] = K_B^{\ 2} \left[S^3 + .295S^2 + .0161S + .516 \right]$$

$$N = M_{a_1} \left[S + .252 \right]$$

$$a = K \left[S^2 + 356,475S + 142,578 \right] = K \left[S + 356,475 \right] \left[S + .4 \right]$$

$$b = 534S$$

Since $M^2 S^4 \langle\langle\langle\langle\langle (M_a + N_b)^2$, the system contains double roots and is unstable. Numerical calculation will show that

$$M_a + N_b = S^5 + 3.55 \times 10^5 \, S^4 + 2.475 \times 10^5 \, S^3 + 11.14 \times 10^5 \, S^2 + 4.55 \times 10^5 \, S + .7358 \times 10^5 = \left[(S+.214)^2 + .1565^2 \right] \left[(S+.135)^2 + 1.71 \right] \left[S + 3.55 \times 10^5 \right]$$

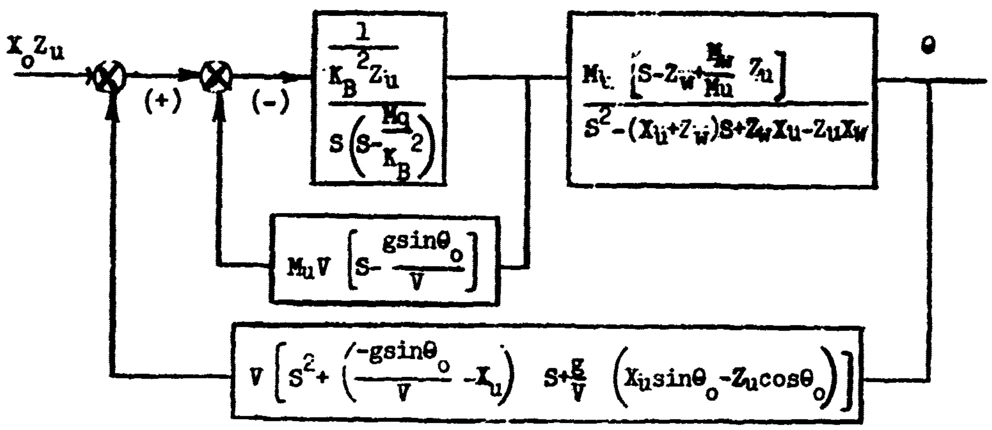
In order to remove the theoretical instability due to the assumption of symmetry, it will be necessary to make $M_{a_1} \neq I_{b_1}$. Since the system is sensitive to changes in that derivative, flight tests will be made to determine those factors (values of the linkage ratio, for example) that will spread the roots sufficiently to achieve good response.

IX. FORWARD FLIGHT ANALYSIS; $X_q = Z_q = 0$

The equations of motion for the platform in forward flight are the conventional vertical plane motion equations of the airplane. These are as follows:

$$
\begin{array}{cccc}
u & \theta & w & \\
S-X_u & -(-g\cos\theta_o+X_qS) & -X_w & =X_o \\
-Z_u & -(-g\sin\theta_o+Z_qS+VS) & S-Z_w & =0 \\
-M_u & S(K_B^2S-M_q) & -M_w & =0
\end{array}
\qquad (40)
$$

The block diagram of the system with $X_q = Z_q = 0$ is



If for ease of writting, the following notation is adopted:

$$\bar{A} = S^2 - (X_u+Z_w)\,S + Z_wX_u - Z_uX_w \qquad (41)$$

$$\bar{B} = S\left(S - \frac{M_q}{K_B^2}\right) + \frac{M_uV}{K_B^2 Z_u}\left(S - \frac{g\sin\theta_o}{V}\right) \qquad (42)$$

$$\bar{C} = S - Z_w + \frac{M_w}{M_u} Z_u \tag{43}$$

$$\bar{D} = S^2 + \left(\frac{-g\sin\theta}{V} - X_u\right) S + \frac{g}{V}\left(- Z_u\cos\theta_o + X_u\sin\theta_o\right) \tag{44}$$

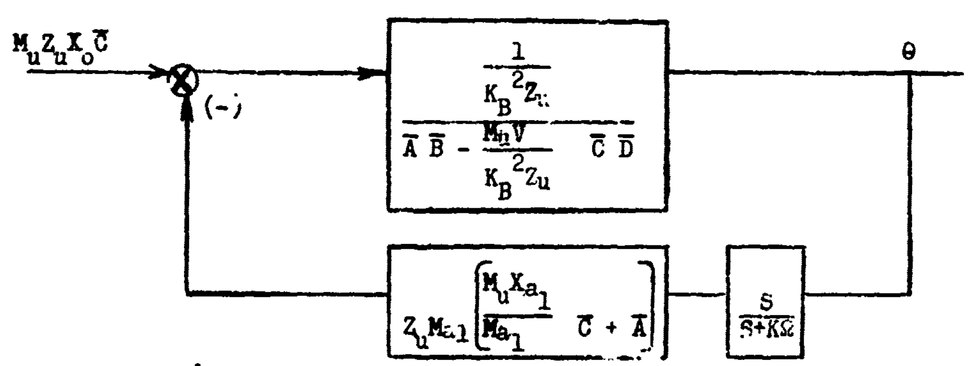
Then the final open loop transfer function is

$$\text{O.L.T.F.} = \frac{M_u V}{K_B{}^2 Z_u} \; \frac{\bar{C}\,\bar{D}}{\bar{A}\,\bar{B}} \tag{45}$$

If the gyrobars are added, the equations of motion are (with $X_q = Z_q = 0$)

$$\begin{array}{ccccc}
u & \theta & w & a_1 & \\
S-X_u & +g\cos\theta_o & -X_w & -X_{a_1} & =X_o \\
-Z_u & -(-g\sin\theta_o+VS) & S-Z_w & 0 & =0 \\
-M_u & S(K_B{}^2 S-M_q) & -M_w & -M_{a_1} & =0 \\
0 & S & 0 & S+K\Omega & =0
\end{array} \tag{46}$$

The block diagram with the bars present can be put in the form



$$\text{O.L.T.F.} = \frac{\frac{M_{a_1}}{K_B{}^2}\, S \left[M_u \frac{X_{a_1}}{M_{a_1}}\,\bar{C} + \bar{A}\right]}{(S+K\Omega)\left[\bar{A}\,\bar{B} - \frac{M_u V}{K_B{}^2 Z_u}\,\bar{C}\,\bar{D}\right]} \tag{47}$$

The above diagram brings into evidence the feedback function of the gyrobar which changes the characteristics of the basic platform located in the feed forward loop.

Hiller Engineering Report 680.2 will show the following stability derivatives for the five chosen flight conditions:

Cond. No.	V Ft/sec	θ_o Deg.	X_u	M_u	Z_u	X_w	M_w	Z_w
1	27.4	-11°	-.288	+.581	-.0789	.0421	.0557	-.114
2	44.8	-21	-.316	+.690	-.178	.0561	.282	-.059
3	59.5	-31	-.325	+.210	-.137	.071	.578	-.251
4	66.8	-36	-.322	+.0964	-.148	.0459	.518	-.414
5	74.4	-42	-.315	+.0469	-.1503	.022	.430	-.434

If center of gravity locations other than those in the truck tests are conceived, the new moment derivatives will be given approximately by

$$M_q = {}_oM_q + (H - 15.3)^2 \left[X_u\cos\theta_o - Z_u\sin\theta_o\right]$$

$$M_u = {}_oM_u + (H - 15.3) \left[X_u\cos\theta_o - Z_u\sin\theta_o\right] \qquad (46)$$

$$M_w = {}_oM_w + (H - 15.3) \left[X_w\cos\theta_o - Z_w\sin\theta_o\right]$$

The following table gives a summary of moment derivative variation with pilot platform height and center of gravity location.

h_f in.	H in.	Condition 1 $\alpha = -11^\circ$ V = 27.4Ft/sec			Condition 2 $\alpha = -21^\circ$ V = 44.8Ft/sec			Condition 3 $\alpha = -31^\circ$ V = 59.5Ft/sec			Condition 4 $\alpha = -36^\circ$ V = 66.8Ft/sec			Condition 5 $\alpha = -42^\circ$ V = 74.4Ft/sec		
		Mu	Mw	Mq	Mu	Mw	Mq	Mu	Mw	Mq	Mu	Mw	Mq	Mu	Mw	Mq
19.5	30.5	.203	.0805	-.530	.235	.322	-.627	-.232	.492	-.611	-.344	.257	-.608	-.377	.083	-.588
25.0	32.5	.154	.0838	-.663	.176	.327	-.788	-.290	.481	-.768	-.401	.223	-.763	-.433	.037	-.739
30.0	34.4	.107	.0869	-.806	.118	.332	-.962	-.346	.470	-.936	-.456	.190	-.930	-.486	-.006	-.900
35.0	36.3	.059	.0900	-.965	.062	.337	-1.150	-.401	.459	-1.120	-.511	.157	-1.113	-.539	-.050	-1.077
40.0	38.2	.012	.0931	-1.138	.004	.342	-1.361	-.457	.448	-1.325	-.567	.125	-1.317	-.593	-.093	-1.273
45.0	40.1	-.036	.0963	-1.328	-.053	.347	-1.589	-.512	.437	-1.546	-.622	.092	-1.537	-.646	-.137	-1.486

STABILITY DERIVATIVES FOR VARIOUS FLIGHT CONDITIONS

The function of the gyrobar is to stabilize the platform, with the degree of stability achieved a function of M_{a_1} for a fixed damping ϑ. Since the high frequency asymptote of the closed loop system with the bars present is vertical and in the left half plane, any conjugate complex unstable roots of the platform without the bars can be made stable at <u>some</u> value of M_{a_1}. This then causes no difficulty, at least theoretically.

However, if the platform alone without the bars has an unstable real root, it will travel toward the zero at the origin that has been added by the bars, and regardless of the value of M_{a_1} the platform will always be unstable with aperiodic divergence. This situation can be avoided under the following conditions:

Consider first M_u positive.

Inspection of the functions $\bar{D}$ and $\bar{A}$ will show that

1. They contain only force derivatives and thus are independent of c.g. location.
2. The coefficients of $\bar{D}$ and $\bar{A}$ will always be positive for the platform under all flight conditions since the force derivatives do not change sign with forward speed. The roots of $\bar{D}$ and $\bar{A}$ will therefore always be in the left hand plane.

Since for a $+M_u$, $\bar{B}$ will always have one unstable real root it is necessary to investigate $\bar{C}$.

If $-Z_w + \frac{M_w}{M_u} Z_u$ is positive, the zero of $\bar{C}$ will be in the left hand plane and there is the possibility (if the second condition below is fulfilled) that the unstable pole of $\bar{B}$ will become stable. Therefore, the first necessary condition, that the platform not contain a positive real root, is that

$$-Z_w + \frac{M_w}{M_u} Z_u > 0$$

or since Z_u is negative

$$\frac{M_w}{M_u} < \frac{Z_w}{Z_u} \tag{49}$$

If the zero frequency amplitude of the O.L.T.F. without the bars at unity $\frac{M_u V}{K_B^2 Z_u}$ is greater than $1/\frac{M_u V}{K_B^2 Z_u}$ then the unstable root of $\bar{B}$ will be located in the left half plane when the loop is closed. This will occur when

$$\left| \frac{\left[-Z_w + \frac{M_w}{M_u} Z_u \right] \left[\frac{g}{V} (-Z_u \cos\theta_o + X_u \sin\theta_o) \right]}{\left[Z_w X_u - Z_u X_w \right] \left[- \frac{M_u V}{K_B^2 Z_u} \frac{g}{V} \sin\theta_o \right]} \right| > \left| \frac{1}{\frac{M_u V}{K_B^2 Z_u}} \right|$$

or

$$\left| \frac{\left[-\frac{Z_w}{Z_u} + \frac{M_w}{M_u} \right] \left[\frac{Z_u}{X_u} \frac{1}{\tan\theta} + 1 \right]}{\left[\frac{Z_w}{Z_u} - \frac{X_w}{X_u} \right]} \right| > 1 \tag{50}$$

For a negative Mu, it can be shown that $\bar{B}$ will always have negative roots, and the condition that $-Z_w + \frac{M_w}{M_u} Z_u > 0$ does not need to be satisfied to insure a stable real root, but that Equation (50) is necessary and sufficient with the sense of the inequality reversed.

...re 5 is a plot of the left side of Equation (50) set equal to R for the five flight conditions. Only for $\frac{M_w}{M_u}$ ratios where $R > 1$ (+ M_u) can the gyrobars stabilize the platform. Inspection of the table on page 29 indicates that this state exists only under Condition 1 with the pilot platform below a point around 30" from the bottom of the duct.

If M_u is negative, then M_w must be negative to insure the possibility of $R < 1$, since a negative ratio $\frac{M_w}{M_u}$ would never allow $R < 1$ because of the slope of the curves. Inspection of the table on page 29 will show that a negative M_u, M_w combination appears to be impractical.

There thus appears to be some indication that the derivatives under various flight conditions should be studied in some detail before design to insure that the gyrobars can stabilize the platform.

X. CONCLUSIONS

The results of the present investigations seem to indicate the following conclusions:

1. A 2 degree of freedom hovering analysis shows that the platform is stable for a small range of positive M_u (c.g. locations).

2. Under the design condition

$$\left| X_u + \frac{M_q}{K_B^{\,2}} \right| > \left| \frac{g}{X_q} \right|$$

   the platform would be stable for all c.g. locations that give a positive M_u.

3. If symmetry is assumed but the product of inertia (F) has a small value, the four degree of freedom analysis indicates an unstable response with double roots.

4. It seems impractical to mount the pilot's platform on springs to achieve stability.

5. A damped gyrobar can stabilize hovering flight, although the coupled pitch-roll response will be theoretically unstable if symmetry is assumed. If the pitch-and-roll gyrobar linkage ratios are different, stabilized hovering flight can be achieved.

6. A preliminary forward flight analysis with the gyrobars installed indicates that the [illegible] can stabilize flight if the platform

without the bars does not contain an aperiodic divergent root. If it does, the bars can not stabilize forward flight. The basic platform must therefore be carefully designed to eliminate aperiodic divergent roots. If M_u is positive, this can be avoided if

$$\frac{M_w}{M_u} < \frac{Z_w}{Z_u}$$

and

$$\frac{\left[-\frac{Z_w}{Z_u} + \frac{M_w}{M_u}\right]\left[-\frac{Z_u}{X_u}\,\frac{1}{\tan\theta} + 1\right]}{\left[\frac{Z_w}{Z_u} - \frac{X_w}{X_u}\right]} > 1$$

XI. LIST OF FIGURES

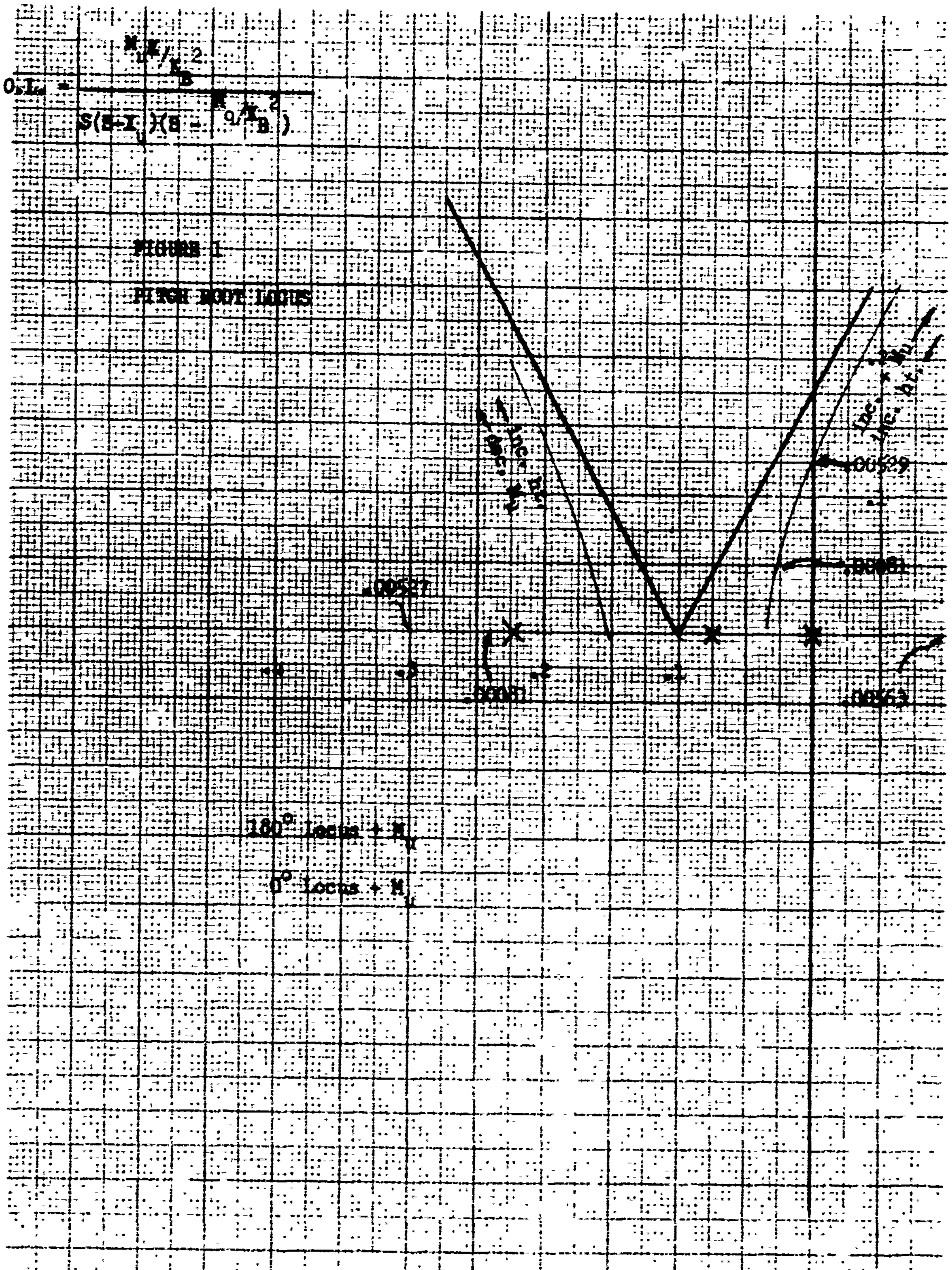


FIGURE 1
PITCH ROOT LOCUS
.00527
.00081
.00529
.00081
.00563
180° Locus + M_u
0° Locus + M_u

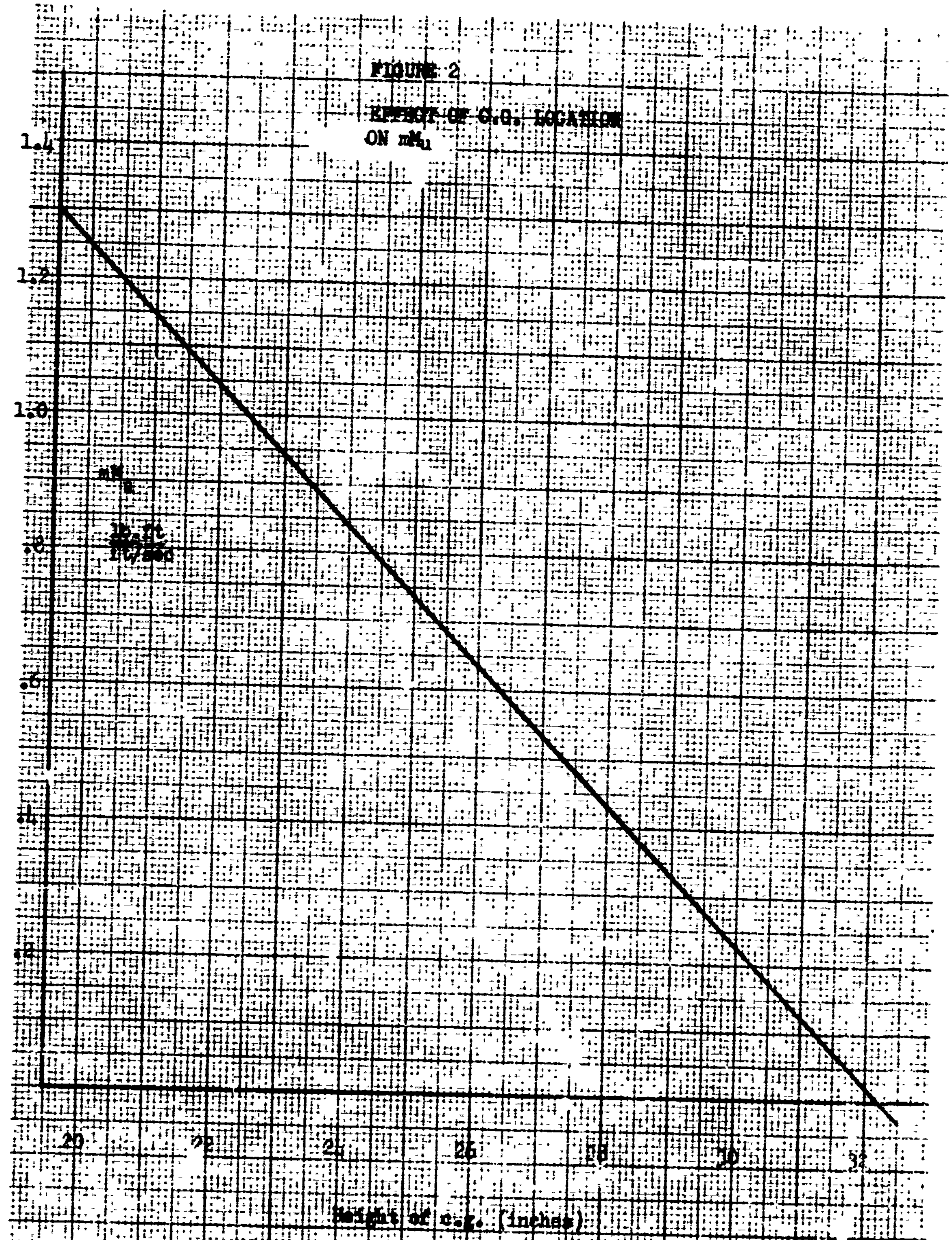
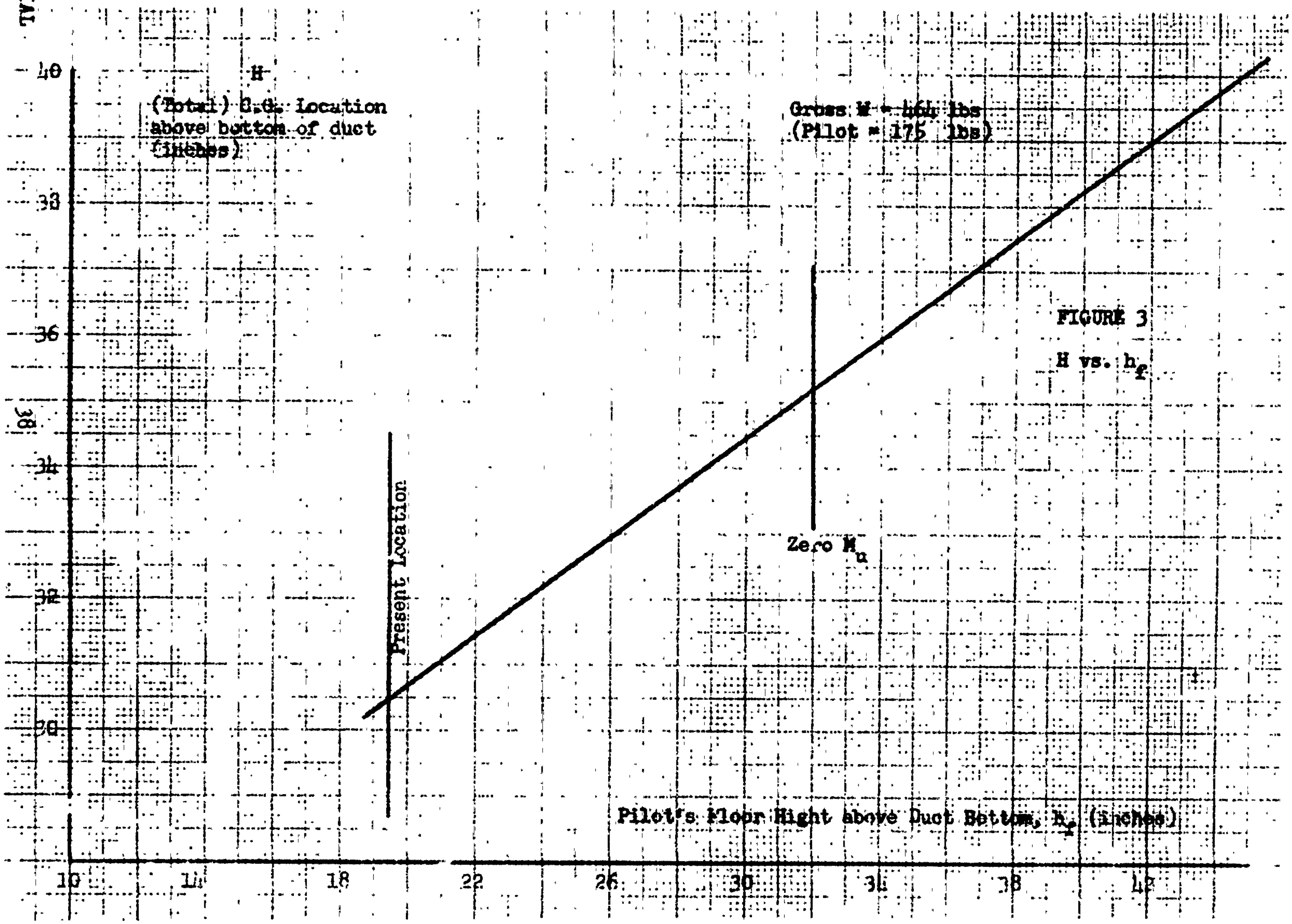


FIGURE 2

EFFECT OF C.G. LOCATION ON mM_1


H
(Total) C.G. Location above bottom of duct (inches)
Gross W = 464 lbs
(Pilot = 175 lbs)
FIGURE 3
H vs. h_f
Present Location
Zero M_α
Pilot's Floor Hight above Duct Bottom, h_f (inches)
40
38
36
34
32
30
10
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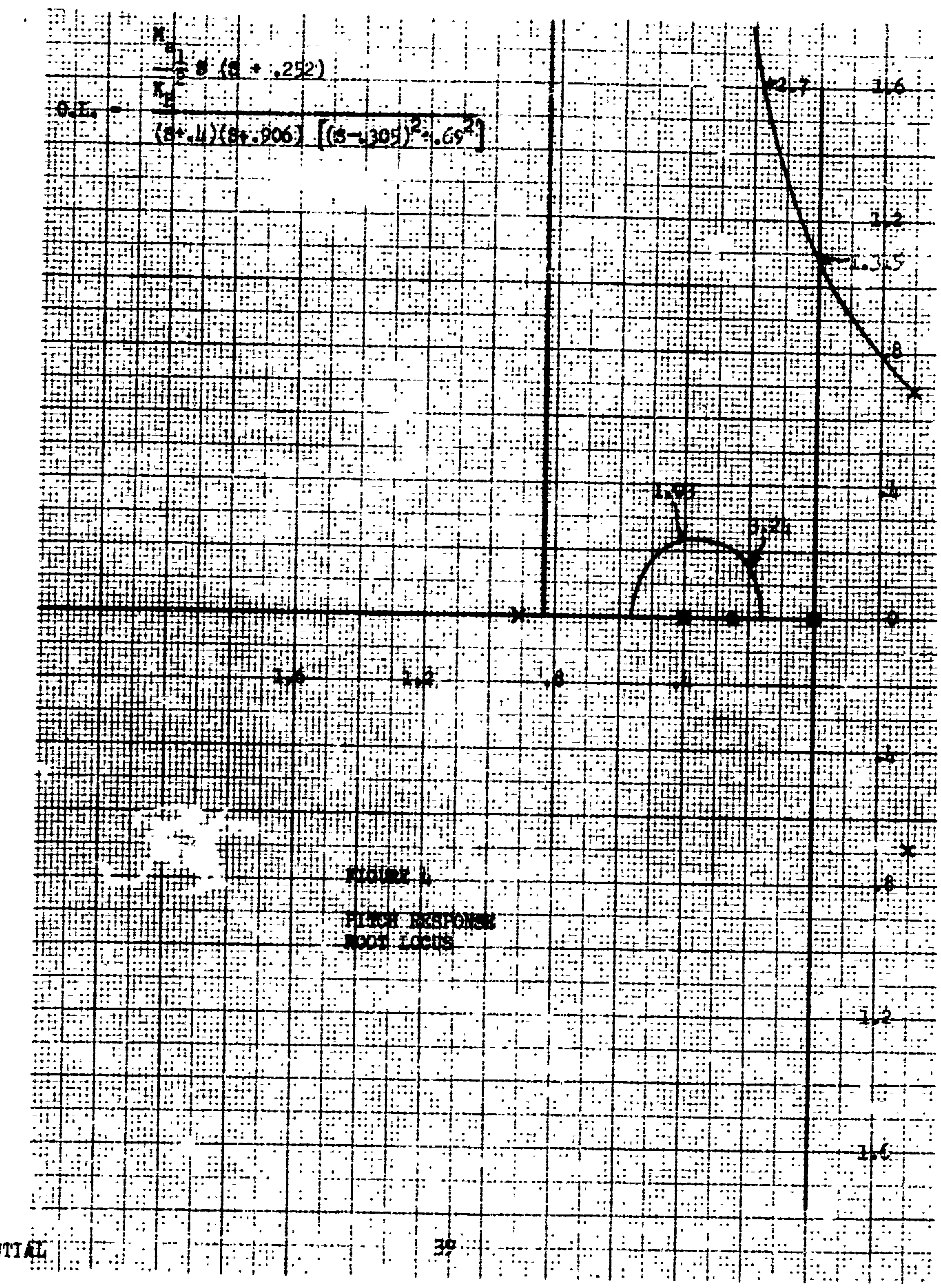

FIGURE 4

PITCH RESPONSE
ROOT LOCUS

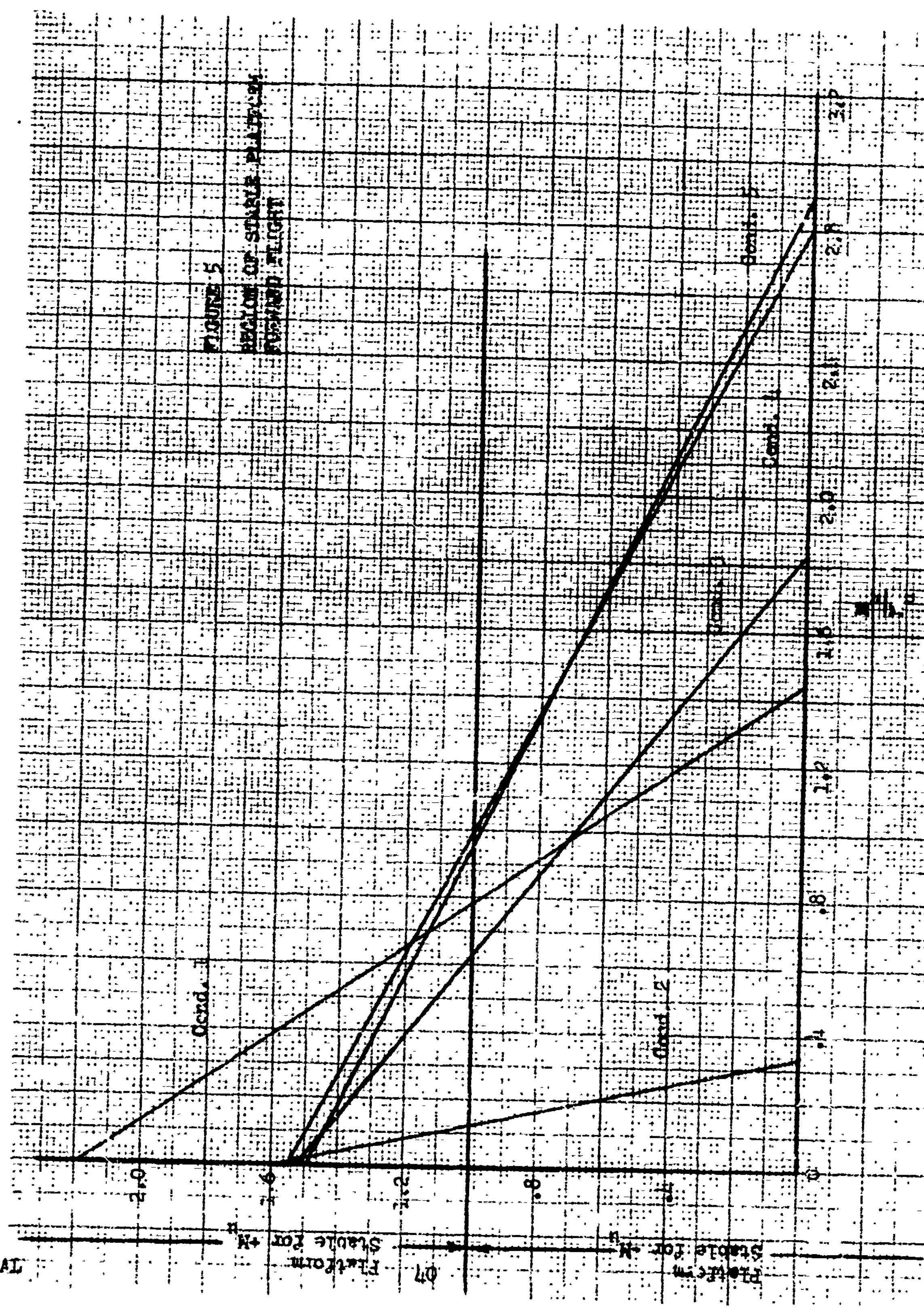

FIGURE 5
REGION OF STABLE PLATFORM
PUSHED FLIGHT

XII. REFERENCES

Frequency Response of the Ordinary Rotor Blade, the Hiller o Blade, and the Young-Bell Stabiliser" by G. J. Sissingh, 1 Aircraft Establishment Report No. Aero 2367, May 1950.