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# Monte Carlo Computer War Gaming (U) 

OPERATIONS RESEARCH OFFILE

The Johns Hopkins University

Operaíing Under Contract with the DEPARTMENT OF THE ARMY

A Feasibility Study


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## WORKING PAPER

This is a working paper of members of the technical staff of the Tactics Division concerned with ORO Study 15.1, but the calculations were completed in Project ARMOR under the previous ORO organization.

It is the objective of Study 15.1 to develop and apply analytical techniques for the comparison of tactics, organization, and weapons systems within the context of a realistic twosided battle situation. This paper, "Monte Carlo Computer War Gaming: A Feasibility Study," represents a preliminary investigation of a technique for simulating the effects of fire and maneuver in a small-unit action. The findings and analysis are subject to revision as may be required by new facts or by modification of basic assumptions. Comments and criticism of the contents are invited. Remarks should be addressed to:

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TACTICS DIVISION
ARMOR \& TACSPIEL GROUPS
Technical Memorandum ORO-T. 325
Publishod March 1956

# Monte Carlo Computer War Gaming (U) A Feasibility Study 

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OPERATIONS RESEARCH OFFICE The Johns Hopkins University Chevy Chase, Maryland

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FOREWORD

This is the first of one series of memoranda to be prepared by the Tacspiel Group, Tactics Division, dealing with the TO\&E of small units. It describes the basic features of a war-gaming technique which, when fully developed, is expected to contribute to the design of improved TO\&Es.

The memorandum is published to acquaint the Army agencies responsible for research and combat developments with the nature of a specialized form of a war game. It is hoped that this will facilitate a critical review of the technique itself as well as contribute to an understanding of its strengths and weaknesses and the nature of the data required for its use.

The immediate area of application of this war-gaming approach will be to assess in relative terms the performance of untried proposals for new smallunit TO\&Es in a realistic two-sided combat action. It should aid the timely identification of proposals deserving no further attention and promising ideas that merit more thorough (and expensive) study.

As will be evident from the study, use of a war-gaming technique, like any other technique, imposes the most severe requirements on the analyst in providing for the necessary comprehensiveness and realism of each phase in the analysis. The ultimate source of that comprehensiveness and realism must be experienced members of the combat arms whose judgment is essential to identify relevant battle factors as well as in the design and conduct of field experiments, maneuvers, and (in a limited sense) CPXs testing promising organizations.

Since the memorandum discusses only a technique of analysis, no formal recommendations for DA action are offered.

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## ACKNOWLEDGMENTS

The writer gratefully acknowledges the inspiration of G. Gamow (ORO consultant), who first proposed the essential features of this methodology, ${ }^{1}$ and W. W. Nicholas, who aided greatly in the formulation of the principles applied in these calculations and also strongly supported the detailed study portions of the research that he supervised as chairman of Project ARMOR; the work of G. Cramer and E. Joseph of Engineering Research Associates, who, while under subcontract to ORO, not only contributed their special technical and mathematical skills in applying the ERA 1101 computer to the calculation of these battles, but also aided in the development of the special computing techniques used; and the help of the many ORO staff members and consultants who contributed data, advice, and encouragement.

Special mention should be made of N. M. Smith (ORO), who led the early discussions that produced the guidelines applying to this study; S. Ulam of Los Alamos Scientific Laboratory (ORO consultant), who contributed basic and original thinking on Monte Carlo techniques; Col Billingslea, who supplied authoritative advice and proposals on the tactical aspects of the trail combat action; V.V. McRae and M. C. Grabau, who kindly permitted the use of a very large quantity of original and unpublished tentative performance data for the armored vehicles; G.J. Blakemore, Jr , who supplied certain of the statistical computations; and J. Federico, who prepared certain tabulated data.

The writer is especially grateful to V. V. McRae for his valuable criticism of several drafts of the memorandum. However, the writer alone is responsible for all errors of expression and content.

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## SUMMARY

## PROBLEM

To develop and test the feasibility of a high-speed computational system permitting the simulation of small-unit combat actions in detail in order to improve the numerical evaluation of proposed new weapons, weapons systems, and tactical doctrines.

FACTS

The rate at which unproved weapons of radical or unconventional nature are becoming available to our military forces is increasing tremendously, compounding the difficulty of evaluating new weapons and weapons systems in the absence of actual combat. Some of these weapons may strongly influence the organization and tactical doctrine of the military forces. Thus the effectiveness of all weapons, even those already tested in combat, may be altered. To be adequate, analysis of weapons must be made in the context of the weapons system containing them. Proving-ground data, although necessary, are not enough; yet full field tests of all proposed weapons systems are prohibitively expensive.

There is, therefore, a requirement for a computational technique capable of simulating the operation of a weapons system, and economically screening large numbers of such proposed TO\&Es, eliminating quickly impractical proposals and clarifying elements of strength and weaknesses in promising ones.

Conventional mathematical analysis has not yielded a satisfactory or convincing simulation of an entire combat action. There is a widespread belief that this is due in part to the oversimplification required in practice before the operation of a complex system can be reduced to a set of equations.

Recently a technique yielding approximate solutions to problems involving multiple probabilities has come into use. This technique, called "Monte Carlo," has been applied successfully by mathematicians to important problems that had been "unsolvable" owing to the length of time required when using conventional techniques. The system permits a large electronic computer to be employed to carry out the calculations.

Large electronic computers are now available that, in addition to their well-known ability to solve arithmetic problems at great speed, have also a capability for solving "logical" problems rapidly. That is, they can be caused to determine the logical consequences of a given set of facts and/or assumptions.

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## DISCUSSION

Tests of new weapons or weapons systems require that proper account be taken of the important battle factors, including terrain, weather, doctrine, enemy strength, troop formations, mobility, human factors, and supporting weapons. Traditionally the world's military establishments have used the war gameor map exercise-as one means of incorporating all these battle factors, and more, into an analysis of the strengths and weaknesses of an existing or proposed TO\&E. At the same time attempts to simulate a complete military operation using conventional mathematical techniques have been only partly successful, owing to the multitudinous and interdependent battle factors. Basically this appears to result from the enormous lengths of time required for the complete solution of sufficiently general equations involving the necessary number of variables, even using the most modern computing machinery. It is also true that the merely conceptual task of translating complete military operations into the special forms required for conventional analysis offers formidable difficulties.

The computational system described is in the form of a very detailedtwosided war game but avoids the troublesome mathematical systems previously applied.

## Battle Factors

Ten basic factors that may be used in simulation of battle are proposed. For each weapon or unit there are six physical performance characteristics: (1) kill probability; (2) rate of fire (includes logistics); (3) probability of seeing enemy; (4) communication probabilities; (5) mobility (includes mechanical reliability, weather effects); and (6) human factors influencing or limiting the physical capabilities of a weapon. Following these are (7) physical terrain features of the battlefield, in terms of their influence on the first six factors, and (8) the missions of the opposing forces, particularly their terrain objectives. The actual and estimated enemy situation influences the latter factor. The particular principles of tactical doctrine selected to govern the way in which the first eight are to be combined are the bases for (9) doctrine for selecting targets (includes support-fire plan), and (10) doctrine for properly relating the scheme of maneuver to the weapons, the battlefield and mission. This list of variables can be tefoted as including, directly or indirectly, all the information contained in a complete operations order.

The Monte Carlo computing techniques applied to the simulation of battle provide a capability for simulating in detail the battle factors deemed essential, in a conceptually simple manner and within reasonable time limits, but at the expense of complete mathematical accuracy. Thus the Monte Carlo calculations may be considered to provide feasible approximations to (time-wise) infeasible mathematical calculations.

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## Trial Combat Action

One hypothetical combat operation, consistent with the statement of the problem, serves to aid in the refinement of the model of battle and to test the computational technique. The combat action selected was formulated with the aid of knowledgeable Army officers and civilian technicians. It is patterned aft $t=r$ the "Reinforced Tank Company in the Attack" problem performed frequently at the Armored Center, Ft Knox, to illustrate armored small-unit tactics. The attacking forces include a medium tank company, three squads of infantry mounted in personnel carriers, with a battery of $4.2-\mathrm{in}$. mortars in support. The defenders are assumed to have a company of 10 medium tanks, a company of 5 SP guns and 9 squads of dismounted infantry.

The action is put in the context of an over-all tactical situation and takes place on a piece of terrain patterned closely after an area a little over a mile square in Bavaria, 30 miles north of Wurzburg. The major terrain features of this area are similar to those in the area at Ft Knox where the attack problem is demonstrated.

As the first step in refining the model of battle, the combat action is broken down into its essential elements of fire and maneuver. A precise statement of the calculations the computer must perform in order to simulate the actions of fire and maneuver is formulated.

Stated briefly, these fundamental actions of the separate combat elements are reduced to (a) a decision to move from one small $100-\mathrm{m}$ by $100-\mathrm{m}$ square, which is its present position, to a selected neighboring square, taking proper account of the factors of terrain and combat that must influence the selection; and (b) a decision to deliver a single unit of fire against a selected enemy target in accordance with the terrain factors and combat situation, which must influence the selection of a target and the effectiveness of the unit of fire. The majority of the calculations involve probabilities in a natural and necessary way. Hence any single battle calculation can have any one of a large number of possible results. Thus more than one battle calculation must be carried out to determine the average, or typical result.

These two types of fundamental activities by the combat elements in the battle are properly ordered in time by the computer; i.e., are caused to be performed in a sequence that is militarily sensible and at a rate consistent with the capabilities of the weapon and weapon crew.

## Statistical Analysis

The results of 114 trial battles are analyzed to determine the nature and statistical accuracy of conclusions that can be derived from a short series of battle calculations.

The fact that chance plays an important part in the calculations raises certain statistical questions that must be investigated, and the trial battle results provide useful answers to these questions. Essentially, one question is

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*How many times must a particular battle be repeated to give an acceptable measure of the 'average' outcome of the battle?" Related to the answer to this question is the spread in the results of a given battle; i.e., the likelihood with which "nonaverage" or exceptional outcomes of the battle occur. The study shows that 50 repetitions, or less, of the computer battle in its present form are sufficient to determine the "average" outcome with acceptable accuracy (about $\pm 10$ percent in the average number of casualties) and shows that the spread of the battle results is fairly measured by the same number of repetitions.*

This first group of test battles is also applied to a test of the ultimate mission of battle simulation, the comparison of effectiveness $\dagger$ of two weapon designs. For this purpose 50 additional battle calculations were obtained for the case where the medium tanks of the assaulting force were replaced by a set of hypothetical tanks with (a) lower kill probability of its gun against the enemy armor, (b) an increased rate of (effective) fire, (c) higher vulnerability to the enemy weapons, and (d) an increased mobility (speed of movement). The change in the numbers that specify the performance of (a) and (c) above follows roughly the difference between M48 medium tanks and M41 light tanks. The changes made under (b) and (d) were hypothetical. Therefore the results of the trial battles computed in this feasibility study cannot be taken in any sense as a comparison of the effectiveness of the M48 medium tanks with the M41 tank. However, the comparison made is a concrete illustration of the area of application of battle simulation. Such a comparison can be made as soon as refined battle codes are devised and acceptable performance data are available.

An additional 14 battles were computed for the case where hypothetical "heavy" tanks replaced the mediums. The performance data for these tanks followed roughly, but only in part, tentative performance data for the T43 tank supplied by the staff of the ARMOR Group.

In both cases, the modification in the performance characteristics of the tanks caused a major variation in the outcome of the "average" battle, which was measurable with useful accuracy and gave rise to a spread of results not so wide as to make predictions impossible.

## Application of Model of Battle

With the test-battle code as a base line, the flexibility of the model of battle is discussed and a refined model of battle is developed, which is feasible on computers now available and, it is believed, possesses sufficient detail and realism to permit its immediate application to the solution of real and pressing problems relating to the TO\&E of small combat units. Formulation of such a detailed code need take no more than 6 months, once agreement has been reached on the type of combat to be investigated.

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It builds directly on the lessons learned from this feasibility study and takes into account such refinements as appear necessary for the production of an immediately usable computer battle code. In particular the necessary flexibility in the tactical doctrine governing the actions of the combat units is provided for. The command-control structure of subordinate units is an integral part of the proposed code and permits inclusion of the important command-control problems in a realistic fashion, including modification of the mission and of the tactical means used in the execution of the mission during the battle calculations. Since such command decisions are made on the basis of the commander's knowledge at the time, the operation and effectiveness of the commander's data-gathering system, including his radio net, are a part of the proposed computations.

Although this model of battle was developed expressly to simulate smallunit combat actions, the model is not necessarily restricted to that use. The components of the model-such as combat elements, grid squares, terrain objectives, and kill probabilities per unit of time-can also be applied to largescale combat operations, provided reasonable estimates of the corresponding performance characteristics of entire platoons, companies, or battalions are available.

## CONCLUSIONS

1. The Monte Carlo technique enables a very large number of battle factors to be introduced into a feasible analysis of the performance of alternative weapons and weapons systems. The number of battle factors is sufficient to warrant designation of the computing system as a "battle simulator."
2. The technique permits direct participation of nonmathematical per-sonnel-most importantly, officers with extensive combat experience-at every significant step of the design and criticism of controlled, scientific war gaming.
3. The battle factors used are sufficiently comprehensive and basic to permit great flexibility in the manner of their combination into various combat situations involving a variety of weapons and at any echelon for which the performance characteristics of the weapons systems may be specified.

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## MONTE CARLO COMPUTER WAR GAMING A FEASIBILITY STUDY

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## INTRODUCTION

## PROBLEM

The most pressing military problems currently requiring analysis result from the apparent necessity of countering the Soviet military threat with unproved weapons. Tactical and organizational innovations that may appear desirable to exploit such new weapons fully may cause, throughout the organization, unexpected chain reactions that could nullify the expected improvements. As the expected tempo of battle is stepped up, the command-control-communication system becomes more critical. As the weapons themselves become more complex the nature and degree of logistical support and training required acquire an increasingly critical bearing on the selection of the best weapon system.

The potential of the existing and predicted technology not only produces very complex weapons systems but produces them in substantial variety. There is almost an "embarrassment of riches" among the competing proposals for new weapons systems. It is clearly out the question to subject every one of the proposed TO\&Es to the heavy expense of full-scale field tests. Yet many of the proposals involve such radical and untried weapon systems and tactical innovations that neither experience nor conventional theoretical analysis appears capable of adequately screening them for merit.

The methods of screening numerous proposed weapon systems is the problem area to which this memorandum is addressed.

## WAR GAMING

The use of war games by the world's military establishments to aid in planning and training has a long history. ${ }^{2}$ Map exercises and war games used as a part of the war planning process are not usually expected to predict accurately the outcome of some future battle or war. Rather the process is useful in pointing up elements of strength and weakness in existing troop formations, weapon stocks, and/or logistical operations. Such apparent strengths and weaknesses can then be made the subject of more detailed study by the responsible officers and their staffs to determine if the apparent strengths and weaknesses are real and, if so, to determine what action is indicated to exploit more fully the strong points and to remedy the weak points.

The results of such war games are not ordinarily accepted in an absolute sense. Rather they are used as a basis of comparison with the results of war gaming an alternative action. This characteristic of war gaming is common to

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all theoretical computations; i.e., the relative (in the nonmathematical sense) standing of two proposed weapon systems is apt to be a great deal more meaningful than the absolute effectiveness values computed.

The special contribution of the war game to planning is, of course, the natural way in which the influence of one phase of the operation on another phase may be shown. Traditionally war gaming in the form of map studies (and to a much lesser extent, command post exercises) has required the continuous participation of experienced senior officers to provide authoritative umpire decisions at crucial times during the game. Attempts to condense such authoritative judgments to a limited set of rules have in the past met with a general lack of success. Consequently many personnel engaged in the scientific analysis of military problems have experienced great difficulty allowing for the influence of necessary battle factors.

Clearly the utilization of war games to provide for a more realistic assessment of the capabilities of men-weapon teams in scientific analysis requires that the judgment of experienced officers be a part of the analysis. But also such judgments should be in a form amenable to the special techniques of scientific analysis.

The technical analysis of weapon effectiveness has always depended on the availability of performance data such as kill probability and range. Such per formance data can frequently be determined on the proving ground. However, proving-ground data do not always take adequate account of combat conditions, particularly those involving human factors. For example, the kill probabilities of small-arms fire is known to be significantly less in actual combat than that deduced directly from their performance on the firing range. So long as historical records of the effectiveness of small-arms fire are available, appropriate corrections to the measured performance of a new small arm can be made on the assumption that the new small arm would be used in combat very much like the old one. However, when a new weapon has radically different characteristics from the old, or when the battlefield conditions are radically altered, it becomes unlikely that such an assumption is justified. It then is extremely difficult to estimate in a simple way the necessary corrections for human factors and the influence of other weapons in the combat team.

The war game described in this technical memorandum is especially designed to provide a means so that (a) man-weapon performance data as determined from field experiments and (b) the judgment of experienced officers regarding important human factors may be incorporated in a natural way into theoretical calculations of weapons systems effectiveness.

Naturally no war game or calculation can be used uncritically to predict the future. The results of any analysis in advance of actual combat-whether technical or military-is at best only one of the many factors the commander on the spot must take into account when battle is joined. However, results of technical analysis that have had the benefit of treatment in the more realistic context of a war game should be of material assistance to the responsible of ficers in the design and testing phase of new weapons systems.

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## FACTS AND ASSUMPTIONS

This section lists basic facts and assumptions on which this study rests. In general the facts relate to the capabilities of computers and past experience with special mathematical computing techniques such as the Monte Carlo approximation. More numerous are the assumptions adopted (a) to characterize the simulation of battle, (b) the list of battle factors which comprise the simulation, and (c) time considerations limiting the scope of the calculations.

## SIMULATION OF BATTLE

The essential features of combat that must be simulated appear to be:
(a) Opposing combat units and their capabilities (battle factors 1 to 6 , next section).
(b) A battlefield (battle factor 7).
(c) An over-all mission for both sides (battle factor 8).
(d) The technique or doctrine of fighting to be applied (battle factors 9 and 10 ).

To have great flexibility and wide areas of application the rules for conducting the war game must permit a variety of choices in each of these four areas. The mission of this study is to demonstrate by example that it is feasible to compose such a set of rules with the necessary flexibility and that modern high-speed computing machinery can be used to conduct any desired portion of the resulting war game.

This memorandum first lists in general terms the physical variables believed to be required to implement the construction of a war game. These physical variables correspond to the first three essential features of combat listed above. They must be defined in a fashion that leaves completely open the fourth factor, i.e., the choice of a technique for applying the available military force toward the accomplishment of the (combat) mission, but at the same time permits straightforward implementation of any desired tactical doctrine.

## BATTLE FACTORS

The battle factors (or variables) that were selected as essential for the simulation of fire and maneuver are:
(1) Kill probability per round, burst or salvo, of a particular weapon or crew, vs opposing weapons or crews.

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(2) Rates of fire of weapon crews (includes necessary logistic factors).
(3) Probability of seeing enemy (depends strongly on terrain features but includes other combat factors).
(4) Communication system (determines how quickly and accurately combat information known to one battle participant is shared with another).
(5) Mobility (particularly of combat vehicles but including entire combat units where appropriate; includes effects of weather and mechanical reliability of vehicles).
(6) The human factors (which influence performance levels of weapons and weapon systems so far as the identified battle factors are concerned).
(7) Physical terrain features of battlefield (includes influence of weather).
(8) Mission of units (including terrain objectives if necessary).
(9) Priority system (for selecting a target among available enemy units).
(10) Tactical doctrine for maneuver (as it relates movement to terrain features, the mission, and progress of battle).

The first six of these factors are pure performance characteristics of a weapon, or more generally, a weapons system, and constitute the raw material of military force. Each of these six, singly or in combination, call for objective data that may be determined from historical battle accounts, proving-ground experiments, field exercises, and maneuvers or theoretical studies.

The seventh battle factor, physical terrain features of battlefield, represents the point of application of military force. It too is represented by objective performance data, in the sense that a given terrain feature derives its battle significance only from its influence on the performance characteristics of combat units.

The eighth factor is a quantitative statement of the mission of each of the opposing units. For the attacking unit this is conventionally a terrain objective, and for the defending forces the mission is conventionally to frustrate the efforts of the attacker. Clearly other missions may be assigned and, further, the mission may be qualified by self-imposed limitations on acceptable casualty levels, specific time limitations, or similar qualifications. In any event the missions are capable of being precisely and quantitatively described.

The remaining two "factors," however, are something else entirely. They are not intrinsically measurable quantities. Rather they represent the intent of a set of rules, or expression of doctrine, that permits a selective application of the available military force on the battlefield. Without these last two factors there could, of course, be no sensible combination of the first six battle factors (force) with the seventh (terrain). Indeed the ultimate purpose of this war-gaming technique requires the quantification of tactical doctrine in terms of the performance characteristics of the opposing weapon systems and within the framework of specified missions on specified terrain.

Therefore a method of computation that allows the influence of approximate forms of the battle factors to be made a natural part of battle simulation must be described. The study does not concern itself with determining accurately the numerical values of the factors. However, all the factors used are so defined that they may be determined by field experiments to a useful accuracy. In case the weapons under consideration are not yet in existence the simulator would be used to determine the importance of a proposed performance level of a selected battle factor (e.g., mobility), so as to determine how much relative emphasis should be placed on improving competing performance levels, e.g., armor protection.

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Concerning these battle factors, three assumptions basic to the development of the model of battle here presented are:
(a) Simulation of battle requires that each of the 10 battle factors appear explicitly in the model of battle; this list of battle factors is sufficiently comprehensive to warrant classifying the resulting model of battle a "battle simulator."
(b) The numerical values of these battle factors are too imperfectly known at this time to justify applying the model of battle to any but the lowest echelons -the individual tanks in a tank company and the individual squads in an infantry company.
(c) The battle factors are so numerous, and their interdependence so complex, that completely accurate solutions cannot be found by conventional mathematical means.

It is obvious that only time will tell whether these three assumptions are completely justified. However, one of the purposes of the material presented in this memorandum is to demonstrate, so far as is practicable, the plausibility of these assumptions.

To ensure clarity, the 10 factors are further refined in the context of a concrete example-a sample combat action. It is desirable to do this-not only because it is difficult to discuss combat in a tactical vacuum but also because the special strength of the Monte Carlo method is best shown by an actual application of the technique.

However, after the factors have been refined in terms of the example, their flexibility and generality is discussed.

Before developing the trial combat action the remaining facts and assumptions must be reviewed.

## THE MONTE CARLO APPROXIMATION

A basic assumption is that, to simulate battle successfully, the battle factors used must refer directly to the individual participants in a combat actionat least so far as the major combat elements are concerned. For example, tanks were selected as the combat element to be emphasized in this feasibility study. Thus it is assumed to be necessary to treat the tanks individually. That is, their movement, firing, and other important actions must be treated as distinct actions -not averaged out over a platoon or other tactical unit.

This assumption appears attractive for at least three reasons: First, the physical characteristics of weapons are (usually) best determined on an individual basis and are (usually) the most accurate information available. The results of calculations starting from such data are apt to be more believable than calculations starting from less well-known data.

Second, the proposed methodology will be the more flexible the more readily weapons and equipment are added, altered, or removed from the weapon systems. It is more convenient to do this when the battle model includes the weapons and their characteristics explicitly than when weapons and equipment must be combined in some average way before insertion into the model of battle.

Finally, one of the primary purposes of constructing this new model is to render the interactions between distinct weapons susceptible to calculation.

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Thus to the extent that these interactions are averaged out prior to insertion into the model they are not subject to analysis, and part of the purpose of the methodology would be frustrated.

To carry out such extensive calculations requires that the most powerful and rapid computing facilities be employed. The use of modern high-speed computers is therefore a necessity.

Some compromise is required in this regard. A computer does not have an infinite capacity to treat all weapons and other equipment separately. The necessary compromises grow out of specific limitations of the computing machinery. They are discussed in later sections.

When describing separate actions of an individual combat unit, e.g., a tank, it appears inescapable that probabilistic notions are required. Thus with a given round a tank will either hit an enemy tank or it will fail to do so. The difference between various tanks in this regard can only be in the probability of a hit. Similar though more complex notions apply to the probability of a kill.

Once probabilities are injected into a calculation the outcome of that calculation cannot be a certainty. Thus if a model of battle that assigns probabilities to describe the performance of a weapon is constructed, then a single simulation of any given battle could have any one of a large number of outcomes, according to the play of chance. The difference in effectiveness of competing weapon designs can therefore be measured only by means of the difference in the average outcome of the battle or by similar factors. This is a basic limitation on the use to which this battle simulator may be put. It is a natural one. It is tempting to interpret it as a general inability of humans to $h$ ow the present in such detail as to be able to predict the future with certainty.

There is every reason to believe that a model of battle including the proposed number of battle factors, and in the great detail required to treat each combat element distinctly, would require a prohibitive cost in time and/or money for its complete accurate solution using conventional mathematical techniques. ${ }^{3}$

However, a technique for providing approximate solutions to such complicated problems in much less time is known and has been in use by applied mathematicians for some time, particularly since 1943. ${ }^{3}$ This approximate method of problem solution is called the "Monte Carlo technique" and is based on the everyday concept of probabilities and the science of statistics. It is therefore particularly useful when the problem to be solved itself involves many complicated probability actions such as kill probabilities and the probability that one combat unit will detect an enemy unit.

A Monte Carlo calculation can be considered as a straightforward substitute for the use of a probability equation. For example, suppose it is desired to determine the probability $P$ that a tank will be knocked out by either the first or second shot from an antitank gun, assumed to possess a constant kill probability per round of $p=1 / 2$. The correct answer is given by the simple equation

$$
P=p+(1-p) p=2 p-p^{2}=1-0.25=0.75
$$

If the Monte Carlo method is used in place of the equation, the value of $P$ could be found by simulating each round of the antitank gun by the flip of a coin, calling a hit when a head comes up (probability of a head is $1 / 2$ ), a miss for the tail.

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Any "honest" gambling device displaying the proper "odds" could be used. If a record were kept of the results when the coin was flipped, say 1000 times, then the value of $P$ so determined would almost certainly be very nearly the correct answer, 0.75.* Clearly the Monte Carlo method would be a poor substitute for the equation in the simple problem above. However, some systems are so extremely complicated that it is all but impossible to write down and solve the required equations. In many such cases the Monte Carlo technique has proved an effective means for approximating their solution, ${ }^{3}$ particularly since high-speed electronic computers can be used to simulate the flip of a coin (or other mechanical actions) involving the play of chance.

There is a second important reason for investigating a methodology using Monte Carlo calculations. This memorandum demonstrates that the Monte Carlo system of calculation permits a very close and detailed correlation to be maintained between each separate operation in the real battle and that part of the battle simulation corresponding to it. Perhaps this is due to the nature of the human thought and decision processes necessarily included in a battle simulator. Human reasoning appears to depend more on a system of "logical computation" than on an arithmetic or mathematical system, and the model of battle described in this memorandum makes important use of such "logical" computations.

Still, a compromise between the use of Monte Carlo operations and conventional mathematics is frequently desirable. Such compromises usually result from the limited capacity of the computing machinery. They are discussed as the need arises.

## TIME LIMITATIONS

Use of the Monte Carlo technique results in the necessity of repeating the battle for every distinct set of initial conditions (such as choice of weapons, terrain, and mission) a number of times sufficient to determine the average battle outcome and other related factors. The number of repetitions required depends on the accuracy with which it is desired that the average shall be determined, and on the spread of the results. The science of statistics applies to this determination. A necessary part of this feasibility study is therefore the series of trial calculations described in later sections, which indicates the spread of battle outcomes to be expected when this model of battle is used in analysis and permits determination of the approximate number of battle repetitions required. $\dagger$

The accuracy with which the average battle outcome must be determined is dependent in part on how similar are the battle results when two alternative courses of action are compared. For example, it may be desired to determine which of two proposed tank designs is more effective. In this case the battle must be repeated a number of times, increasing as the more nearly identical are the effectiveness values of the competing tank designs. That is, a few repetitions may be sufficient to demonstrate an overwhelming superiority of one
*Statistically there would be only about 1 chance in 1000 that the value of $P$ calculated by the above procedure would fall outside the interval 0.70 to 0.80 , i.e., be in error by more than $\pm 7$ percent.
$\dagger$ No attempt is made here to investigate possible application of certain refined statistical theories that may further reduce the required number of battle repetitions.

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tank over another, but a much larger number of repetitions is necessary if it is required to demonstrate a marginal superiority of one tank over another.

It is therefore important to compute a second series of trial battles so as to indicate the sensitivity of the battle results to a significant variation in the performance characteristics of the tanks involved.

From the preceding discussion it is clear that a major restriction on the scope of the combat action to be simulated is the length of time that can be allowed for the computer to simulate a single battle. This does not depend on details of the design of the battle simulation. It depends only on the way in which the battle simulator may be applied to the solution of military problems and on the decision to use Monte Carlo calculations.

To establish an approximate limit on the length of time to be allowed the computer, suppose that it is desired to investigate the relative desirability of different calibers of tank guns when mounted on the same basic tank chassis. Thus factors of mobility and armor protection may have been fixed at some value, and within wide limits the caliber of the main gun may be varied. As the caliber of the main gun is varied, certain related factors must also change. Thus the base load of ammunition generally decreases as the caliber of the weapon increases if, for all calibers, the best high-pressure designs are used. Also the effective rate of fire may decrease as the rounds become heavier and more cumbersome to handle. The diameter of the turret ring may also change and, with it, certain proportions of the tank design.

A straightforward application of a battle simulator to this design problem would involve simulating the battle between a fixed enemy force in a fixed tactical situation while varying the caliber of the main tank gun. If a measure of the effectiveness of a tank were agreed on,* the caliber of tank gun yielding the best performance in this regard could be selected. Simulating the battles resulting from 10 different choices of main gun caliber might be sufficient to identify (by interpolation) the gun caliber yielding the best performance in the particular battle situation selected.

In general the choice of any weapon requires that it perform well in a variety of tactical situations. Thus the performance of each caliber of tank gun needs to be tested in perhaps 10 different tactical contexts. For example, these might include (a) attack of heavily fortified position, (b) exploitation, (c) airborne assault, (d) delaying action, (e) mobile defense, and (f) counterattack to restore a position. This second factor of 10 raises the number of battles requiring simulation to 100 . Finally it is necessary to test how critically each of these 100 battle results depends on certain major assumptions, such as the level of training of the crews and the quality of enemy equipment to be expected. Perhaps 10 such assumptions would require some variation, which when multiplied by the factor of 100 already derived indicates that as many as 1000 battles may need to be simulated in the process of a thorough investigation of the main armament of a tank.

This is of course an extremely detailed application of battle simulation. Much of value could be learned with a less extensive analysis. However, if it is

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to be possible to carry out such a detailed program in 6 months to a year, so as to permit timely solutions, then each battle simulation can consume no more than the order of 1 hr .*

The limitation on a single battle calculation is therefore in the order of a few minutes since, as was discussed above, each battle simulation requires a number of repetitions of a single battle so that the average battle results may be determined. Thus if on the average 30 repetitions were sufficient to determine the average battle result of a single combat situation, then each separate battle can consume but 2 min of computer time so as to provide one average battle result each hour.

It must be emphasized that the example discussed above is not the only way in which the methodology may be applied to analysis. It represents what is thought to be the most extreme case among possible applications and therefore results in the most stringent limitations on the time that should be available to the computer.

Since the computing machinery that may be used in the application of the proposed methodology is at least 10 times as fast as the ERA 1101 computer used in this feasibility study, a time limit 10 times larger than that calculated above can be used here. Thus the average computer battle on the 1101 computer should be completed within 20 min . Since the basic assumption was to treat the battle participants and equipment in as much detail as is feasible, no lesser time is considered for purposes of this feasibility study.

## CAPABILITIES OF ELECTRONIC COMPUTERS

Certain facts and assumptions previously described strongly influence the methodology. The nature of the proposed methodology also depends critically on the capabilities of the computer. A discussion of only the most essential features of a computer follows.

## Nature of Computers in General

The essential difference between a desk calculation machine and the electronic computers used for computer battles is the "automatic" nature of the latter. That is, an electronic computer can not only add, subtract, multiply, and divide but can also be instructed to perform a long series of such operations in any desired sequence with no further attention required from the human operators. It can be instructed to carry out such an extensive number of mathematical operations that a special means-a "flow diagram"-is used throughout this study wherever it is necessary to show the order in which these mathematical operations are performed by the computer. The general character of a flow diagram is illustrated by Fig. 1. Each block indicates some simple calculations that the computer must carry out. The arrow or arrows leading from a box then indicate the next operation. By following along the arrows in a flow diagram every step of the computations can be traced out. In principle such a flow diagram relates only to the logical structure of the computation itself and not at all to the particular computer for which it is designed. In practice, however, the particular way in which the over-all problem is broken down into simpler parts will depend on the special characteristics of the particular computer.
${ }^{*} 1000 \mathrm{hr}=2540$-hr weeks or about 6 months of computer time.

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Fig. 1-Example of a Flow Diagram


Fig. 2-Schematic Diagram of Typical Electronic Computer

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The Computer. The computer itself can be described generally without detailed reference to its actual physical structure. Thus all general-purpose electronic computers can be considered as composed of four functional subgroups. These are:
(a) The arithmetic units (those in which adding, subtracting, etc., is actually carried out).
(b) The memory units (those in which numbers are retained before, during, and after use).
(c) The control units (the source of the instructions telling the arithmetic units what to do next, where to get the numbers to be used, and where to store the answers).
(d) The input-output units (the machinery used by the human operator to tell the computer what to do and which numbers to use; and the machinery used by the computer to "tell" the human operator what it has done, and what the answers are).

These functional units are usually, but not necessarily, associated with separate electrical or mechanical units. In the case of the ERA 1101 computer used for this feasibility study, the physical equipment performing the four functions (Fig. 2) are (a) arithmetic unit: about 600 ordinary (radio-type) vacuum tubes; (b) memory unit: a rotating cylinder, covered with a magnetic substance similar to that used on magnetic tape (phonograph) recorders, which records voltage pulses interpreted as numbers and has a capacity of 16,384 seven-digit numbers; (c) control unit: about 400 ordinary (radio-type) vacuum tubes; and (d) input-output unit: input is by paper tape having holes punched in it by a special typewriter; output is by the same type of paper tape and/or a direct connection from the computer to a fast electric typewriter.

The computer can be caused to perform any stated sequence of arithmetic operations (add, subtract, multiply, and divide) and certain special forms of these arithmetic operations, usualiy called "logical" operations. These are more completely described in App A.

The procedure to be described for making use of the computer in this study has six stages: (a) a sample military engagement is broken down into simple understandable steps, each involving a single elementary action by a small combat unit; (b) each step is translated into an equivalent mathematical or logical operation that the computer can perform; (c) a number code that will cause the computer to carry out all the calculations in the desired order and that includes all the numbers necessary for the calculations is prepared; (d) a punched-hole paper tape of the necessary length is then prepared by typing the number code on a special typewriter; (e) the prepared paper tape is fed into the computer, which then starts its calculations; and (f) selected results of the calculations are caused to be typed out directly onto a special typewriter as they are obtained. At the same time a more detailed record of the calculations is also punched out by the computer on paper tape that can be inspected at a later time.

However, this more detailed account of the calculations cannot be conveniently read directly. The paper tape must be rerun through the computer while the computer reinterprets what had been punched out originally on the tape. In so doing, the computer can directly cause the special typewriter to type out the detailed results stored in the tape.

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For this memorandum, Steps $a$ and $b$ above are discussed in the next section, "Trial Combat Action."

This completes the consideration of the general features and restrictions of the proposed methodology. Appendix A gives a more detailed discussion of the capabilities of the computer. The remainder of the study will develop the methodology within the limits imposed by the facts and assumptions now identified.

## TRIAL COMBAT ACTION

The battle factors selected to simulate combat have been listed in general terms. To establish the adequacy of this list and aid in the formulation of the manner of their simulation a sample military situation is desirable. In this section the details of the "computer battle" are related to such a situation. It must be emphasized that the resulting system for calculation is not restricted to any specific combat situation. The rules for computation have in all possible cases been so stated that a change in the battle situation requires only that certain characteristic (coded) numbers be changed. The resulting codification of battle will be reexamined from the point of view of generality and flexibility in the last section.

## SCOPE OF TRIAL CALCULATIONS

It is desirable to use the simplest possible trial combat action in establishing the feasibility of the proposed methodology. Contrariwise, the combat action to be analyzed must be large enough to be self-contained; i.e., it should include as many as possible of the significant factors that influence the battle once the forces are joined. Thus, if the action is to include a grouping of tanks, the battlefield must be large enough to include all, or most, of the elements that interact with those tanks. That armored vehicles are of special interest to this study is suggested by these considerations:
(a) Tanks represent the largest capital outlay and give rise to one of the major logistical problems of the Army, particularly under the conditions of atomic warfare.
(b) Current doctrine implies strongly that the decisive combat actions will involve strong tank forces.
(c) Tanks are combat elements severely restricted by their mechanical characteristics and thus are more clearly susceptible to mathematical analysis than less mechanical systems.

If tanks are to be included, then the smallest self-contained battlefield will be of a size comparable to the maximum effective range of their guns, i.e., about 1 to 2 miles on a side.

An intense combat action on such a battlefield could involve about a company of tanks; a smaller unit would lack tactical flexibility. Since the smallest meaningful action is desired it follows that the trial combat action should be of company size.

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Even at this small scale, operation of tanks without infantry is unlikely; therefore, along with appropriate infantry units the major anti-infantry weapon (indirect fire) must also be included.

A complete combat action on such a battlefield could conceivably be completed in as little as 30 min , if the action were of sufficient intensity. In this case the action would not involve logistical problems during the action, and they could be properly left out of these trial calculations by taking certain levels of supply as basic assumptions.

Similarly, TAC is outside the necessary scope of a feasibility study. The much larger range of TAC aircraft requires that their influence be assessed in a war game on a scale where alternative allocations of strikes during an assault are feasible.*

These considerations are suggestive of the type of combat action that may serve as a test vehicle. The trial combat action should (a) emphasize tanks; (b) feature intense action-lasting about $1 / 2 \mathrm{hr}$; (c) include company-sized units with reasonable attachments of infantry and indirect-fire weapons; and (d) take place on battlefield of one or a few square miles.


Fig. 3-General Military Situation Leading to Trial Combat Action

## THE MILITARY SITUATION

A military adviser to ORO during the early part of the study formulated a military situation that might generate a combat action having the characteristics just described. It is a hypothetical situation constructed for these special purposes and is not presented as being either a typical situation in some future war or as representing a typical mission for the troops involved.

A heavily reinforced Blue infantry battalion is given the mission of delaying a Red mechanized division, in column, for a period of 12 hr at Münnerstadt, which lies about 30 miles south of the zonal (East German) boundary at Meiningen on a railroad line to Würzburg (Fig. 3). Delay is to be effected by forcing the Red
*Since both logistical and TAC air support may be important parts of the smallest unit actions, proper provision for these factors must be made when the methodology is applied. It will be shown that the methodology is flexible enough to permit inclusion of these factors, when necessary, by using more capable modern computers.

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forces to deploy under heavy fire at the river line, which is the northern boundary of Münnerstadt.

Attached to the reinforced infantry battalion is a reinforced medium tank company of M48's, and a heavy mortar company. Figure 4 shows the troop disposition.

Before the Blue battalion had fully occupied its position in and about Münnerstadt, the point of the main Red column approached and was brought to a halt under fire. The Red point began to deploy, sending a strong force to cross the river on the right flank of the Blue position. Red combat engineers succeeded in quickly erecting a temporary bridge, and a company of $10 \mathrm{~T}-34$ 's, a company of $5 \mathrm{SU}-100$ 's and a company of infantry crossed the river and assembled on a hill nearby. They could then bring direct fire on sections of the road south of Münnerstadt along which the Blue forces must soon withdraw. Further, they would quickly attempt to cut that road in an enveloping maneuver.

In the face of this threat, the Blue forces committed their reserve tank company reinforced by three squads of infantry mounted in three armored personnel carriers (a "scratch" force, since TO\&E does not include carriers). The mission of this force was to push the Red force back across the river in preparation for the withdrawal of the Blue forces in Münnerstadt. The resulting armored assault is the action programmed for the computer.

## THE BATTLE

The tactics of the counterattacking Blue force are to provide (a) an assaulting group composed of two platoons of M48's ( 10 tanks) and the platoon of infantry ( 3 armored vehicles, one squad each); and (b) a covering force (overwatching) composed of the CO, FO, and one tank platoon (total 7 tanks). In addition, the company of heavy mortars ( $4.2-\mathrm{in}$.) is available in direct support. The remainder of the Blue force is heavily engaged elsewhere with the Red point and cannot be assumed to assist in this operation.

The assault group moves toward the Red bridgehead, keeping in a draw as far as possible and then making a frontal assault (Fig. 4). The overwatching tanks provide support fire from cover and concealment at a range of about 1500 yd.

The Blue infantry dismount from their carriers when the Red position is closely approached. The Blue mortar fire is lifted at the same time. The mission of the Blue forces is to move on through the Red position, firing as they go. Since the battle will feature intense action with one or the other force decimated in a half hour or less to meet the requirements generated in the section, "Time Limitations," no further mission for the Blue forces is required.

The analysis of the tank attack consists of three major steps: (a) identify the interdependence of the selected battle factors for each of the combat elements on the battlefield; (b) develop a system whereby the computer can compute the basic activities of each of the combat elements on the battlefield, using experimental data giving the capabilities of the individual combat elements; and (c) provide the means for the computer to arrange the possible basic actions of the individual combat elements into a sequence of fire and maneuver activities reflecting the sense of a stated tactical doctrine.

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Fig. 4-Initial Disposition of Forces in Trial Combat Action
Reproduction of Overlay to AMS M841 Sheet 5727, Münnerstadt 1:25,000 (Fig. 6)

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This study carries out these three steps for the trial battle described only to the extent required for demonstrating the feasibility of the methodology within the physical limitations of the ERA 1101 computer.

## TERRAIN FACTORS

Since all the battle factors depend basically on the terrain factor, they cannot be discussed in detail until a means for inserting terrain factors into the machine is selected.

The major alternative means of including factors of terrain considered are discussed in App B, and the reasoning behind the method selected for this feasibility study is presented.

Essentially, the choice made is to dissect the battlefield into the largest number of small grid squares consistent with the capacity of the computer to be used. With the battlefield under consideration this results in each square being 100 m on a side for a total of 576 squares over the entire battlefield of about 2 square miles.

The average terrain factors for each square are listed and stored in the memory of the computer. These factors include the average elevation of the grid square and the average concealment afforded by the vegetation on the square, in five steps from completely open fields to dense forest. The presence of selected special characteristics such as swamp, military crest, steep slope, and a road or trail is noted.

The data giving the terrain features that are stored in the machine's memory are used by the computer in accounting for the battle factors associated with each separate action of fire or maneuver.

For example, one of the 10 battle factors is the probability that one combat element will "see" another. One essential component of this factor (but not the only one) is the identification of those enemy units in plain view of the tank attempting to pick up a target. If the elevation of all squares is known, then the computer can determine whether any square between shooter and any enemy unit is so high as to cut off the view of the shooter. If there is one such square, then that particular enemy unit could not possibly be a potential target. Similarly, if the enemy unit is in the midst of dense forest, then it cannot be seen by the shooter, even though no intervening ground interrupts the line of sight.

Dissecting the battlefield into squares also serves to make specific the fundamental actions of movement that (it is proposed) when assembled comprise maneuver. Thus the maneuver of the fundamental combat units may be considered as being made up of a series of elemental decisions, each one of which can be reduced to the following form: A combat element is on square $A$ on the battlefield. It has the capability of moving to any one of the eight adjacent squares (Fig. 5) in some brief interval of time. (It may also remain in its present position, making a total of nine possible courses of action.) Formulate the rules that will permit the combat element to make a realistic choice among these nine possible courses of action.

Thus it is seen that, if the terrain of the battlefield map is put into the machine's memory in the form of the average terrain features of distinct (small) squares, it is possible to provide approximate but specific answers to the type

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of terrain problems one expects in the course of computing each separate elementary combat action of fire or movement.

## Coding the Terrain for the Computer

Figure 6 is a one-color copy of the battlefield section on a standard military contour map. The map has superimposed $100-\mathrm{m}$ grid lines that divide the map into 576 separate terrain squares. Figure 7 shows more clearly the contours. Using this figure, the elevation of each square was determined by interpolation to the nearest meter.


Fig. 5—Elemental Move Decision
A tank on square $A$ has the option of moving to any of the eight neighboring squares or remaining on its present position

Figure 8 shows the average terrain features of each $100-\mathrm{m}$ grid square, both natural and artificial, except for vegetation. Figure 9 shows the average concealment offered by each $100-\mathrm{m}$ grid square as inferred from the vegetation indicated on the original contour map. The significance of the various degrees of concealment is in their influence on the probability that a combat unit within that square will be picked up as a target; or, once picked up, on the kill probability of another weapon against that combat unit due to the influence of partial concealment on hit probability; and also on the speed with which a combat unit can (or will) move across that square. The quantitative effect of all these terrain features on the assumed performance characteristics of the combat units is described in App C for each of the combat elements involved. The "slow" squares were determined by inspection of the contour map. Detailed performance data for the armored vehicles together with doctrinal discussion would be required to support any final evaluation of which grid squares may be properly termed "slow."

## Special Terrain Calculations

Numerous references must be made to the line of sight between two squares during the course of the battle. During the firing calculations, one of the criteria for selecting a target must be that there is no intervening terrain feature that would cut off vision between the square containing the shooter and the square containing the potential target. In the present formulation of the battle, the treat ment of this factor is by far the most critical portion of the calculations. No

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Fig. 6--One-Color Copy of Battlefield Section from Military Contour Map AMS M841 Sheet 5727, Münnerstadt 1:25,000, with 100-m Grid Lines Superimposed

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other factor so strains the capacity of the computer. Nowhere else is the importance of the advantages that accrue from the use of logical calculations so clearly demonstrated.

The calculation required to show whether or not there is a terrain feature cutting off vision is straightforward but tedious. It involves computing for each intervening square the angle between the horizontal and a line drawn from the surface of the shooter's square to the surface* of the intervening square. As is shown in Fig. 10, if this angle is everywhere smaller than the vertical angle formed by the shooter and the target with the horizontal, then there does exist a line of sight and it is at least physically possible for the shooter to pick up this potential target.

This calculation could have been carried out during the battle only as the need arose. In the present series of calculations, using the ERA 1101 computer, it proved much more economical (timewise) to determine the existence of a line of sight between all possible squares before the series of battles begins. Once done the calculation need not be repeated for additional battles so long as the same terrain section is used and the limits on the positions of the Red forces are not changed. However, storage and use of such large quantities of data do pose problems. Appendix B discusses these problems in detail.

It is conceivable that for other types of battle and/or using different computers the advantage of carrying out this calculation in advance of the battle will disappear.

## SIMULATION OF MANEUVER

The battle factors that should influence the movement of a tank or infantry squad from grid square to grid square on the battlefield have been briefly mentioned. The division of the battlefield into small squares, 100 m on a side, has been made. It remains to describe the specific manner in which the terrain factors, enemy actions, and tactical decisions shall influence the movement of the combat elements.

A partial list of the factors that must influence movement are (a) desirability of remaining in present position and firing; (b) direction to terrain objective; (c) whether or not currently under enemy fire; (d) character of terrain differences among possible new position, e.g., swamp, thick concealment, crest of hill, steep slope; and (e) presence of enemy fire on neighboring positions.

To do this, each of the eight neighboring squares plus the square presently occupied is scored separately on its desirable characteristics. For example, one neighboring square might be allowed 25 points if movement to that square is directly toward the terrain objective. Another square might be given a score of only 5 points if movement to that square is directed to one side of the terrain objective, and a square on the opposite side of the present position away from the terrain objective could be scored as 0 , or even negatively, so far as contributing toward reaching the ter rain objective is concerned.

Thus a series of scores-or ratings-is adopted, which is to be associated with each square on various counts of possessing desirable or undesirable terrain features, or exposure to enemy fire, or other factors that are proposed to contribute to the desirability of movement to that square. The total of all the
*In the event that the intervening square is covered with dense forest, the surface is taken to be the top of the tree stand. For these test battles, all tree stands are assumed to be 20 ft in height.

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individual scores for each square gives a number. The higher this number the more desirable movement to that square at that particular time appears to be, insofar as the tank commander can determine. At this point it would be possible to have the computer select, as the next position for the combat unit, that square which has the highest rating. It is essential that the reasoning behind rejecting this possibility receive careful attention.

There are several different reasons for the rejection and several different ways of looking at those reasons. One way of putting it is to assert that all performance data to be inserted into the battle must be capable of being determined by field experiments or by a study of history. But it is clear that, were a number of different tank commanders put in the same position on the battlefield under identical circumstances so far as could be determined, then all the men would not choose the same square (or even a square in the same general direction) as their next position. Thus the ratings could not be completely determined by experiment, not even in principle, since in the experiments there would surely be some variation in choice among different men.

Another way of looking at the same problem is to consider the case where two squares in quite different directions have nearly the same total rating, e.g., differing by only 1 percent. If the computer always chooses the square with the highest rating, then this is tantamount to asserting that the rating numbers are so accurately known that it is reasonably certain which is the more desirable. It would seem to be overly optimistic to assert that experiment in such a complex matter (or a study of history) could ever produce answers with such certainty.

A third point of view is to consider whether it may be important to determine the influence on the outcome of a battle of various assumed degrees of variability in the response of men to the same situation. Thus it might be argued that weapon system $A$ is better than weapon system $B$ because $A$ functions better with men who have received only 6 months of training than does B ; although if all men could receive 6 yr of training $B$ would be the better choice. In other words, the extent of the variation in the response of different tank crews to the same situation might be considered as related in part to the thoroughness of training. An investigation of the influence of nonuniform responses to similar situations may therefore be given a practical interpretation.

Each of the three points of view presented above points toward the inadequacy of a system where the computer always chooses that square which acquires the highest rating. The simplest alternative to such a rule is to cause the computer to interpret the rating numbers as the relative probabilities with which the combat element will choose its next position among the adjacent squares. This is what is done in the proposed model of battle.

Probabilities in this model of battle are always treated by the Monte Carlo technique. Thus the computer actually chooses only one of the nine possible squares as its next position, but the probability that any particular one is chosen is caused to be the same as the probabilities computed for that grid square on the basis of the appropriate battle factors.

On the other hand there will undoubtedly be some situations where it is desirable to remove even a slight chance of moving into some particular square. This is accomplished in the present calculations by allowing negative ratings to be assigned for certain special situations. If these negative values are made sufficiently large they will cancel out any possible positive score the square

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might acquire from other considerations. The computer then is instructed to consider only positive ratings as a valid relative probability, and hence there is no chance of selecting that (negative value) square.

There is also the possibility of suspending the entire rating process in emergency cases and making selection of a particular square a certainty. This has been done in the present calculations for the special case where a tank has just moved from a concealed position and has been fired on. If the target discovers that it is under fire then the target always returns to the concealed position.

Thus the methodology is flexible enough to permit considerable modification of the maneuver calculations should that prove desirable for special cases.

Appendix C gives an example of a move decision based on the rating system described here as well as the numerical values used in rating all grid squares during the trial calculations.

## SIMULATION OF FIRING

Consider first the problem of simulating the fire of the main gun on a tank. Given the correct "kill" probability for the circumstances applying to any particular round, a Monte Carlo (coin-flipping) decision can easily be made by the computer to determine whether the given round did "kill" its target. Thus suppose that the correct kill probability for the round is known to be 0.4. Then if the computer chooses a number at random between 0 and 1 , there is a 40 percent chance that the number so chosen will be less than 0.4 and a 60 percent chance that it will be greater than 0.4 . Thus the computer will be using the proper probabilities if it makes its decision as to whether the target was killed by the given round by calling a "kill" if the randomly selected number is less than 0.4 , a miss if greater than 0.4 .

Appendix A describes a standard procedure by which the computer can "choose a number at random."

The above discussion clearly leaves out most important factors in the "firing" action. In particular, it is also necessary to (a) select a target and (b) decide to fire at the target.

The decision to fire or not to fire at the selected target depends on (a) whether the tank is physically capable of firing, i.e., has a loaded gun that has been laid on the target, and (b) a tactical decision on the desirability of firing at that particular time.

The selection of a target means that the potential targets already picked up by the tank commander are made the subject of a priority system that eliminates all but one of the potential targets.

This suggests a systematic statement of the time sequence of events in an elemental action of firing; at least for a tank firing its main armament. It is summarized by the flow diagram shown in Fig. 11.

It will be recognized that the diagram does not allow for all eventualities. For example, there is the real possibility that something may occur to change the tank commander's mind during Step 4 while the turret is being rotated, or that Step 1 should follow Step 2, so that the decision to fire depends in part on

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what types of target are available. Tentatively, however, the flow diagram in Fig. 11 will be taken as approximately describing the essential character of the elemental combat action of firing the main gun on a tank.

Step 1 requires a (tactical) decision as to whether the particular tank was on a fire mission. This is accomplished in this study by a decision made in advance of the actual computations.


Fig. 11-Steps in Firing Calculations
A systemotic description of various steps leading up to firing a single round

All tanks will fire, given a target, as soon as physically possible to do so except* that (a) no firing by assaulting combat units is permitted until one of their number reaches the edge of the Red position, and all other combat units open fire at this same time; or (b) firing begins 15 (battlefield) min after start of battle, whichever is the earlier.

Step 2 involves those computations listing all potential enemy targets known at the time to the tank commander who is searching for a target. Thus it is necessary to determine which enemy units it is possible for the tank commander to see by reason of cover (elevation) and concealment (vegetation). Other factors involved are:
(a) Identification of those enemy units which disclosed their position by fire or maneuver to any member of the opposing side, together with the probabilities that all units of either side will share such knowledge through the radio net.
(b) Recollection of those enemy units which have previously been actually noted by the tank commander.
(c) Identification of those enemy units which are placing fire on the tank in question.

Step 3 involves selecting among the potential targets that one which has the highest priority. The priority system used in the present battle is, from the highest to the lowest class of targets:
(a) The tank firing at the shooter (random choice if more than one).
(b) Tank that was last target.
(c) Any tank (make random choice if more than one).
(d) The infantry unit firing at the shooter (random choice if more than one).
(e) The infantry unit last fired at.
(f) Any infantry unit (make random choice if more than one).

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Step 4 involves establishing that the gun has been reloaded and is laid on the target. Time has already passed sufficient for the gun to have been reloaded and for minor adjustments of the gun's sighting before the tank was selected by the computer for processing. However, if the target selected in Step 3 is a new target, then an additional time delay is required while the turret is traversed and the gun accurately laid on target. In the trial calculations a constant delay of 8 sec is allowed for this when necessary.* When the tank is selected again for firing, it has thereby been allowed the necessary time for laying its gun on target and will be able to fire immediately, unless in the meantime the target has disappeared from sight or has been killed, or another target of higher priority has become known to the shooter.

Step 5 involves the actual firing. The principal calculation at this point is to determine the correct kill probability for the particular set of circumstances. The kill probabilities are stored in the computer's memory and depend on the following seven factors:
(a) Type of shooter (weapon).
(b) Shooter moving or not.
(c) Type of target (armor thickness-size).
(d) Target moving or not.
(e) Range to target.
(f) Cover and concealment available to target (e.g., hull defilade or in edge of forest).
(g) First or subsequent round by shooter.

The last section of Step 5 carries out three calculations:
(a) Records casualties, if any.
(b) Determines the time interval required to reload and relay the gun.
(c) Determines whether shooter has disclosed his position to enemy.

This completes the general description of the basic firing action by tanks. In the case where an infantry unit is firing, the computations are exactly the same, although the interpretation is somewhat altered.

Small-arms fire is considered as being lumped into discrete units of fire (or bursts) delivered at the same rate as the main armament for the tanks and at comparable rates for the infantry units. Since the infantry units in the present model of battle are taken to be squads and involve more than one discrete fighting unit (more than a single man), on the average, one burst of small-arms fire would not totally destroy the entire infantry unit. Thus infantry targets are treated differently than tank targets. Most generally the fighting potential of such a unit would be reduced by a fraction. For example, under the proper circumstances, one $30-\mathrm{sec}$ burst of machine-gun fire from a tank might reduce the effectiveness of an infantry squad by one-fourth. Determination of the proper fraction involves not only the number of casualties but also the influence of such a loss on the effectiveness of the remainder of the squad.

With the above difference noted, the general treatment of firing suggested by the five steps in Fig. 11 will be considered to apply to all combinations of tanks and infantry, with suitable adjustment of the performance characteristics.
*Existing higher-capacity computers will permit the time delay to depend on the angle through which the turret is to be moved.

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The mortar fire is treated as a special case in the present model of battle. Only an "average" treatment is given of the five steps just outlined for firing the tank gun. The steps are:
(a) Grid square (to be impact area of a salvo of 12 mortar rounds) is selected at random from the general area occupied by the Red forces.
(b) If there are infantry units within the selected $100-\mathrm{m}$ grid square, then a degradation factor (which is a function of the cover and concealment afforded by this area) is applied to the infantry strength of the unit.
(c) The time interval before the next salvo will be fired is computed.

## BATTLEFIELD TIME

Step 4 in the systematic treatment proposed for the elementary combat action of firing (Fig. 11) requires that the computer "allow time for loaded gun to be laid on target." Computations of the movement of tanks and other combat units require that the proper time be allowed for the combat unit also to reach its new position before the computer considers still another change of position. Thus both elementary combat actions require reference to the passage of time in the simulated battle.


Fig. 12-Flow Diagram Showing Way in Which Computer Maintains Order in the Sequence of Moving and Firing Calculations

The computer keeps the calculations of the various elements on the battlefield in a proper time sequence by the use of what will be called "alarm clock words" or "clocks" for short. Ignoring for the moment certain complications arising from compromises made in this first coding of the battle, the treatment of time, using the "alarm clocks" is indicated by Fig. 12.

In the operations box, called "select clock," the computer checks the times when each tank or other combat unit is able to perform its next activity, either movement to an adjacent square or a search for a target, firing when possible. The tank or unit that is able to act at the earliest time is chosen by the computer for the next calculation. The clock box determines what this action will be, either a move or an attempt to fire.

The firing or moving box then carries out the necessary calculations to select a target and shoot, or to decide where the tank will move next. The results of these actions, including any effect they will have on the future activities of other tanks, are also recorded.

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The reset boxes end the firing or moving calculations by determining when the particular unit will have completed the action just started and hence may be chosen again by the clock routine for additional treatment.

For example, when a moving calculation results in a move to a neighboring square, there must be a certain delay before the tank reaches this new position. The delay will depend on the distance to the next position and on the speed with which the tank moves, which in turn depends on the character of the ground between these positions and also on whether the tank stops to fire along the way. The reset calculation takes these factors into account and determines what the necessary delay must be. Then the move clock of this tank will be set up the necessary amount in time preventing subsequent moves from occurring prematurely.

After leaving the reset operation boxes the course of the two types of combat action calculations rejoin, entering the battle-over-test operation box. At this point it is determined whether the battle computations should be terminated. Tests are made to establish whether the appropriate criteria have been met. In the present battle the calculations are stopped if either (a) all the tanks on either side have become casualties or (b) the battle time has reached 30 min .

If the battle has not ended in the battle-over-test box, then the computer returns to the clock operation box and selects the next combat unit to be carried through the calculations. Thus the closed loop indicated in Fig. 12 by the thickened arrows is traversed repeatedly throughout the battle, along one or the other of the two major branches, firing and moving. In the present battle this computation loop is repeated, on the average, about 1600 times for each battle and consumes about 0.75 sec of the computer's time per loop.

## SUMMARY FLOW DIAGRAM

Figure 13 summarizes the principal steps carried out in the course of simulating the combat action, which have been discussed. The flow diagram is largely self-explanatory.

The reset operation indicated explicitly in Fig. 12 has been absorbed into the over-all firing and moving operation boxes. The reset calculation is still performed but at various stages of the calculation, depending on the circumstances. For example, there are four different outcomes (or "exits") of the firing calculations indicated in the flow diagram. One of these, outcome 3, is used only when the battle is over, hence no reset calculation is necessary. Each of the other three outcomes-1, 2, and 4-requires a separate and distinct reset calculation since the battle continues. For outcome 1 the reset calculation must allow an appropriate time delay for the tank crew to "survey the terrain" and receive a few radio reports giving the position of enemy units before being given another opportunity to fire. In the present battle a time delay varying between 30 and 62 sec is imposed for this purpose.

Outcome 2 in the firing routine imposes an $8-\sec$ delay whenever the target selected requires that the turret be rotated and the gun relaid.

Outcome 4 imposes a variable delay that allows time for the gun to be reloaded and minor aiming adjustments made for a second shot. The delay depends on the type of tank doing the shooting and also on a probability distribution to take

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account of minor variations in the crews and other circumstances (see Table C5, App C). Data from Project STALK (tests conducted jointly by BRL and OCAFF) were not available at the time these calculations were made but may prove useful for future application of the methodology.

A special operations box, barrage, is used to control the firing of the mortar battery. The clocks box includes a test on whether the mortar battery has been selected to fire (outcome 3). Although this fire could have been controlled by the firing box, the details of the calculation are quite different. Thus time would be wasted were the computer to combine into one series of calculations


Fig. 13-General Flow Diagram for Battle
Indicated are the major components of the calculations controlling the progress of the combat action. Several of the blocks have more than one "exit" as shown by the numbers in parentheses. The choice of the proper "exit" varies as the battle prograsses and is determined by detailed calculations not shown.
both types of firing. A reset calculation is included in this operation and imposes a time delay before the next salvo is fired, varying between 0 and 64 sec according to a probability distribution. The average delay is 32 sec . The target area is selected at random from the general area known to include the Redforces.

The battle-over-test box indicated in Fig. 12 has disappeared. Part of its function has been absorbed into the firing routine at outcome 3 where the test is made determining whether either side has been annihilated. A check on whether the battle has exceeded the time limit is made in the clocks operation box. For most of the test battles included in this report, the time limit was 30 min , battlefield time. Some of the battles using heavy tanks were allowed to continue for an additional $41 / 3 \mathrm{~min}$ to compensate partly for the delay resulting from their lower cross-country speed.

The detailed flow diagram of the computer battle is given in App C along with a running commentary on the various operations. The diagram has the same general format as that shown in Fig. 13.

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## PERFORMANCE CHARACTERISTICS OF WEAPONS

The performance data assumed for all combat units in the trial calculations are tabulated in App C.

The values used were the best readily obtainable but all values used include a modifying factor to account for combat and terrain factors not ordinarily a part of proving-ground data or theoretical calculations. This modifying factor was in all cases an estimate obtained from a limited number of experienced officers and civilian analysts.

In summarizing, a military situation that generates a small combat action meeting the requirements developed in the introduction has been described. The combat action itself has been "dissected" into simple combat actions occurring on small sections of terrain. A series of precise calculations and decisions have been proposed, which, taken together, afford a systematic means for calculating the outcome of each separate elementary combat action of fire and maneuver. Finally a system for recording the passage of battlefield time that will permit the computer to maintain a sensible sequence in the order with which the separate elementary combat actions are computed has been described. The rules and numerical values used are described in great detail in App C.

## CONCLUSION

The preceding description (including Apps A, B, and C) of how the trial combat action is designed constitutes the evidence supporting the second conclusion of the study: The technique permits direct participation of nonmathematical personnel-most importantly, officers with extensive combat experience -at every significant step of the design and criticism of controlled, scientific war gaming.

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## RESULTS OF TEST BATTLES

Results of the trial calculations are required for two purposes:
(a) To establish the spread* of battle outcomes deriving from the nature of the model of battle, and from the spread of results to assess the statistical reliability of average battle results.
(b) To establish the sensitivity of the average battle outcome to a signifi cant alteration in the performance characteristics of the Blue tanks only.

Once these two parameters are determined it is possible to specify the number of repetitions of the battle that are required to indicate, for instance, the better of the two tank designs.

The principal results of the trial caiculations are applied in this section to this determination. A detailed tabulation of all available battle results is given in App D.


Fig. 14-Distribution of Blue Medium Tank Losses in 50 Battles with Red Tanks $\sigma=2.33$ tanks per battle

## SPREAD OF BATtLE RESULTS

The most basic characteristic of the model of battle described in this memorandum is the influence of the play of chance that is included. Figure 14 shows the variation in the number of tank casualties suffered by the Blue side,

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equipped with medium tanks, in 50 battle calculations that differed only by virtue of the play of chance. Figure 15 shows the variation in Red tank losses (T-34's and $\operatorname{SU}-100$ 's) during the same 50 battles. Although on the average Red suffered 7.1 tank casualties per battle compared to Blue's average losses of 10.4, it is


Fig. 15-Distribution of Red Vehicle Losses in 50 Battles with Blue Medium Tanks $\sigma=2.74$ tanks per battle
evident there were many departures from this average. Figure 16 shows that in 6 of the 50 battles the Red losses were actually larger than the Blue losses. This fact is indicated in Fig. 16 by the 6 points above the dashed line, along which the losses on both sides are identical.


Fig. 16-Scatter Diagram for Comparing Red and Blue Tank Losses in 50 Medium Tank Battles $\oplus$ indicates point for average losses, viz., 7.1 tanks and SP guns, and 10.4 Blue tanks

If the number of battles were increased beyond 50 , the spiead in tank losses indicated by Figs. 14 and 15 would in all likelihood not be changed significantly. There is only 1 chance in 1000 that it should vary by more than plus or minus 30 percent. Hence the degree of spread in the results is mainly characteristic of the battle model and the performance characteristics of the man-weapon teams alone.

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## TESTING COMPETING TANK DESIGNS

The important corollary to the spread in results effected by any given weapon design is the concomitant number of times the battle computations must be repeated to reveal differences among competing tank designs.


Fig. 17—Distribution of Red Vehicle Losses in 50 Blue Light Tank Battles $\sigma=1.66$ tanks per battle


Fig. 18-Distribution of Blue Tank Losses in 50 Blue Light Tank Battles
$\sigma=2.38$ tanks per botile
To investigate this feature of the methodology, 50 additional battles were computed for the case where the Blue medium tanks were replaced by the same number of hypothetical light tanks. All other features of the battle situation remained as before. Figures 17 and 18 show the distribution of the number of

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tank casualties experienced by both sides in this second series of battles. On the average, Red lost 8.4 tanks in each battle, whereas Blue lost an average of 6.5 light tanks per battle. Thus, based on the average number of tank casualties alone, the Blue hypothetical light tank was more effective than the Blue medium tank. In particular the average effectiveness ratio* for the Blue medium tank battles was 0.6 (to the disadvantage of Blue) whereas for the hypothetical Blue light tank the effectiveness ratio was 1.14 (to the advantage of Blue).


Fig. 19-Variation in Computed Effectiveness Ratio (ER) of Blue Tanks over Red Tanks as the Number of Battles Used for Computation Is Increased

$$
E R=\frac{\text { average number Red tanks killed by each Blue tank }}{\text { average number Blue tanks killed by each Red tank }}
$$

It is at this point that the degree of spread in the number of tank casualties in the various battles must be considered. The two effectiveness ratios 0.61 and 1.14 calculated above are statistical approximations to the "correct" values that would have been produced had the battle computations been repeated an "infinite" $\dagger$ number of times. Thus there is always the chance, however remote, that both these numbers are so much in error that, in fact, the Blue light tank is actually less effective than the Blue medium tank. It is possible to reduce the risk that such an erroneous conclusion would be drawn to any size however small, at the expense of increasing the number of test battles.

Application of standard statistical tests on the reliability of these test results shows that the odds are overwhelming against (better than 360,000:1) the possibility that either one of the two series of 50 battles incorrectly identified the winning side. Appendix $\mathbf{D}$ describes these statistical calculations.

The conclusion is that a sample size of 50 battles was sufficient to demonstrate the superior killing powers of the Red tanks in this series of battles. Indeed, a substantially reduced number of repetitions would probably have been acceptable. Figure 19 shows what the computed effectiveness ratio for the Blue medium tanks would have been had the battle calculations been stopped after
*A simple definition of tank effectiveness has been used by V. McRae and A. Coox in ORO-T-278, "Tank-vs-Tank Combat in Korea." There, tank effectiveness was defined as the ratio of the average number of enemy tanks killed by each friendly tank to the average number of friendly tanks killed by each enemy tank. Other definitions of effectiveness have been proposed, including cost effectiveness, which includes the elements of production and logistical costs.
$\dagger$ For practical purposes, "infinite" can be taken to mean a very large number, e.g., $1,000,000$.

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each of the 50 battles in turn. It will be noted that the computed effectiveness ratio varies by only about $\pm 3$ percent as the number of battle computations is increased beyond 30. It is evident that sequential sampling techniques may be applied to minimize the quantity of calculations.

The previous discussion does not require that the distributions of tank losses shown in Figs. 14, 15, 17, and 18 be normal. However, in view of the unusual character of the distribution of Red casualties shown in Fig. 15, a test on the statistical hypothesis that each of the four distributions was normal gives the results shown in Table 1. The results show that all four distributions are

Table 1
STATISTICAL TEST ON THE SIGNiFICANCE OF OBSERVED DEVIATIONS FROM NORMAL ERROR CURVE FOR FOUR DISTRIBUTIONS OF TANK CASUALTIES

|  | Mean <br> losses | Standard <br> deviation | Probability of observed <br> departure from normal <br> curve by chance alone |
| :---: | :---: | :---: | :---: |
| Blue medium | $10.42^{\mathrm{a}}$ | 2.33 | 0.90 |
| tank battles | $7.08^{\mathrm{b}}$ | 2.74 | 0.15 |
| Blue light | $6.46^{\mathrm{c}}$ | 2.38 | 0.21 |
| tank battles | $8.36^{\mathrm{d}}$ | 1.66 | 0.29 |

ablue (Fig. 14). cBlue (Fig. 18).
bred (Fig. 15). dhed (Fig. 17).
well within the 0.05 level of significance.* If there were serious concern regarding whether these distributions may be approximated by normal error curves, then an appeal to statistical rigor could only be supported by the results of additional computer calculations.

Fourteen additional battles were computed for the case where the Blue forces were equipped with a hypothetical heavy tank. The Blue forces were the winners in terms of casualties in this series of battles, losing an average of 5.4 tanks per battle compared to the average Red losses of 8.8 tanks per battle. The sample size of 14 is so small as to cast doubt on the reliability of the results however. The detailed results are given in App D.

The conclusion is that a series of 50 battle calculations for each tank design may be expected to be sufficient to identify the superior tank design features in the present instance when significant variations in major tank design features are assumed.

## DISCUSSION OF RESULTS

It must be emphasized that the superiority is stated only in terms of some battle result that it has been agreed will indicate superiority. Clearly there are different aspects of superior performance. For example, in the preceding calculations, relative tank killing power has been used as indicating superiority.

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Other factors could have been used in its place. Thus, superior Blue performance could have been measured solely in terms of the destruction of the Red forces regardless of the Blue losses sustained in the attack. Or superior Blue tank performance could have been taken as being indicated solely in terms of the number of Blue tanks that were able to reach the terrain objective. Or any combination of these features could have been used to measure superior performance. The purpose of this feasibility study is not to formulate the criteria of superior performance but to provide the means for simulating battle so as to permit identification of superior performance once it has been defined.

Appendix D gives in considerable detail the history of the battle calculations.

## CONCL USION

The trial battle calculations just presented (and in App D) constitute the evidence supporting the first conclusion of this memorandum, viz., the Monte Carlo technique enables a very large number of battle factors to be introduced into a feasible analysis of the performance of alternative weapons and weapons systems. The number of battle factors warrants designation of the computing system as a battle simulator.

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## APPLICATION OF METHODOLOGY TO TO\&E AND TACTICAL STUDIES

In the preceding section a detailed set of rules (culminating in the flow diagram, App C) was constructed for the express purpose of enabling a computer to war-game a particular counterattack of a reinforced Blue tank company. Limitations on the applicability of these war-gaming rules to other situations must be identified. In the light of the four essential components of combat listed in the section "Facts and Assumptions," the flexibility of the war-gaming rules will be examined with respect to each component in turn.

## FLEXIBILITY OF WAR GAME

The first of these components refers to the opposing combat forces. The manner in which the Blue reinforced tank company was injected into the war game involved breaking the combat unit down into subordinate units (in this case individual tanks and infantry squads) and storing in the computer the values of each parameter to be associated with the several battle variables listed in the section "Battle Factors." Owing to the extensive use of logical computer operations, the number of distinct subunits that could be conveniently processed was limited to 24 , which is the number of "bits"* in a computer number. Thus, so far as the war-gaming rules are concerned, the subunits in the war game can correspond to any military fighting units whatsoever, so long as their killing power, rates of fire, vulnerability, "seeing" prot bility, communicating ability, and mobility may be specified. For example, the 20 combat units that constituted the order of battle of the Blue forces could be caused to correspond instead to 20 separate infantrymen, with appropriate values for the performance characteristics.

On the computer it is entirely feasible to scale all the physical measurements up so that the calculations may be interpreted as involving the killing power, vulnerability, and mobility of platoons of tanks instead of individual tanks, and platoons of infantry in place of squads of infantry. With 24 such units the overall combat action then involves a heavily reinforced battalion maneuvering on a battlefield of perhaps 6 to 10 miles on a side with grid squares of perhaps 300 m on a side.

However, for such calculations to be significant, it is necessary that fairly accurate performance data for the fundamental combat units be available. One possible source of such data is the careful study of the smaller company-sized actions proposed for CARMON.

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Thus there is no technical limitation to any desired expansion of the scale of the calculations, retaining approximately the same time limitations. The problem is solely in terms of the availability of performance data for the subunits.

If time restrictions are relaxed, then a number of complete company-sized actions may be joined to form a battalion-sized action, while retaining the present detailed dependence on individual tanks and squads of infantry.

However, as the scale of combat is increased (i.e., involves either longer combat actions, or higher combat echelons), the lack of nonfighting units in this type of battle code becomes serious. This is particularly true of resupply operations. The present battle code does not allow for resupply. However, it is evident that inclusion of such noncombat resupply units does not raise any new problems. Thus the same attention to the mobility of resupply subunits is required as is already provided for combat subunits. The "terrain objectives" of such resupply subunits would, of course, be the combat units themselves or a supply dump. For present purposes, however, the scale of combat it appears feasible to treat does not involve any large resupply effort.

Earlier it was mentioned that TAC was ignored in the feasibility study. It should be evident at this point that selected subunits in the computer order of battle could have such numerical values assigned for their battle factors as to cause them to correspond to TAC aircraft or TAC units. However, as was also mentioned earlier, the simulation of a combat action is useful only insofar as it permits investigation of the variation of combat results for significant alternatives. For such small-unit actions as are under consideration here, alternative TAC strikes cannot be made. The only choice would be a decision to lay on a TAC strike or to refrain from doing so. In the latter course the impact of the alternative TAC missions on the war is not being assessed by the simulator itself. It follows that weapons whose scale of potential application far exceeds the scope of the battlefield being simulated should be inserted into the battle simulation only indirectly as boundary conditions or in terms of influence on doctrine.

All 20 subordinate units of the Blue forces could be interpreted as individual infantry battalions if the numerical values used for the six classes of performance characteristics were appropriately selected. Doubtless this is currently difficult, if not impossible, owing to the lack of comprehensive data. But the war-gaming rules themselves do not prohibit such use, were the data available.

It would therefore appear that, so far as the first essential component of combat is concerned, the war-gaming rules developed for the tank counterattack are not generally restricted to any particular type of unit.

Although it is clearly possible to design much more complex models of battle than that used here, including both additional battle factors and much more detailed relations between them, such a venture would ordinarily involve construction of a second code for the computer and is beyond the scope of this study. The mission of this study is to test only the feasibility of constructing a single code for the computer which may be applied to many situations, within wide limits, with no modification of the code, but only modification of the numerical value of selected parameters.

Consider the second "essential component of combat" -the battlefield. The use of a grid-square system of storing terrain information is clearly not restricted to the particular battlefield chosen for these sample combat calculations. Neither must their size be restricted to 100 m on a side. The limitation on the

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use of grid squares is in the number of squares that can be processed quickly and the quantity of detailed terrain information stored for each square. The size of a grid square can be varied to suit the circumstances, consistent with the accuracy required in specifying the position of any particular combat unit.

The third "essential component of combat" -the mission-clearly may be varied in any specified manner. Any number of terrain objectives may be identified, or other missions stated.

The fourth component-tactical doctrine-enters into the war-gaming rules in a more subtle fashion.

## MODIFICATIONS OF TACTICAL DOCTRINE

The war-gaming rules require three major types of decisions of each simulated battle participant repeatedly throughout the calculations. These are:
(a) Choice of which (adjacent) grid square is to be next occupied.
(b) Selection of a target from among available enemy combat units.
(c) Decision to move or fire, or to refrain from doing so.

The rationale motivating this series of decisions constitutes the tactical doctrine being applied. However, the numerical values inserted into the computer to govern and control this series of decisions include the influence of whatever human factors are assumed to limit or otherwise modify the "pure" expression of doctrine.

For example, the doctrine governing the first class of decisions above for the Blue assaulting tanks was very elementary, being simply to attack, rather directly, the enemy position after debouching from the draw. However, the numerical values of the parameters used to cause such a series of move decisions (the "terrain weighting" numbers) permitted considerable transient variation in each individual decision. This variation (i.e., the use of probabilities) is to be interpreted as representing the combined effect of the individual's military training (i.e., his knowledge and application of doctrine), modified by his own personal inclinations (i.e., his departure from the use of doctrine). The particular set of numbers to be used for the simulation of any particular series of combat actions represent the assumed level of training, morale, and the selection of a scheme of maneuver. The choice is entirely at the discretion of the operator of the battle simulator.

If desired, the numbers used may correspond to "robotlike" tank commanders whose responses show no variation from tank to tank and show no departure whatever from a rigid interpretation of a proposed doctrine.

At the other extreme the numerical values may be selected to correspond to a "disorganized mob," with extreme variation among individual tank behavior and extreme departures from the tactical doctrine under study.

The same considerations may apply to the second and third classes of decisions. However, as was discussed in App B, for the special purposes of this feasibility study certain practical limitations were placed on the convenience with which such alterations could be made.

Thus the fourth and last component of battle simulation-tactical doctrinemay be varied at will in the same manner, and within the same limits as the first three components, i.e., the order of battle, the terrain, and missions of the opposing forces.

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## APPLICATION TO ERA 1103 COMPUTER

The scale on which the Monte Carlo methodology may be applied to analysis of TO\&E studies must be assessed in terms of a particular design of computer. The one selected for present purposes is the ERA 1103 now installed at ORO. Of course, any high-capacity computer* could be applied but the details would likely be quite different.

This computer is similar to the 1101 computer used for the trial calculations insofar as a part of its memory is the same 16,384 -word magnetic drum. However, in addition there is a very fast memory of 1024 words (magnetic core) and a slow auxiliary tape memory of 262,000 words. The order structure is similar for the two machines. The 1103 computer words are 36 bits $\dagger$ long.

Owing to the extensive use of logical operations, the number of distinct combat elements on each side should not exceed the number of bits in each number used by the computer. For the 1101 computer this was 24 ; hence the maximum number of combat units was 24 . In the 1103 , these numbers are 36 bits long, and hence each side may have up to 36 distinct combat units and still retain the speed associated with logical operations.

The 36 -bit words therefore allow inclusion of heavily reinforced companysized combined-arms teams to be used on both sides. For example, Blue may consist of a tank company ( 17 tanks) with a platoon of heavy tanks attached ( 5 tanks) and two platoons of armored infantry ( 6 squads plus 2 platoon HQ plus 2 LMG sections plus 2 mortar sections) for a total of 34 distinct combat elements.

For similar reasons, the main battlefield may consist of up to $36 \times 36=1296$ grid squares, provided storage of necessary data is not prohibitive. If $100-\mathrm{m}$ grid squares continue to appear to be a useful approximation, this could result in a battlefield of up to 3600 m on a side.

Although the auxiliary magnetic tape memory of the 1103 has a large capacity, access to this storage is time consuming. Thus its contents can only be consulted infrequently throughout the battle. Therefore at any given time during the battle calculations the action under consideration by the computer should be entirely within some 36 - by 36 -grid-square area. On the other hand there is adequate storage to permit the battle to progress gradually from one 36-by 36-grid-square area to another, if the time restrictions are relaxed somewhat.

## CARMON: A REFINED BATTLE CODE

The battle code applying the techniques developed in this feasibility study has been designated by the name CARMON.

## Refined Moving Calculations

With the increased speed of the ERA 1103 computer, it is practicable to make a number of refinements in the calculation of move probabilities. Thus the move probabilities of any given combat unit can be made to depend on the
*For example, the IBM 704, the Remington Rand UNIVAC, the Bureau of Standards SEAC, the Aberdeen ORDVAC.
$\dagger$ This is equivalent roughly to a 12 -digit number. The term "bits" is required since the computer uses binary numbers. Appendix A discusses this feature of electronic computers.

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fields of fire of known or suspected enemy positions. The raw data for effecting this were already present in the calculations for the feasibility study. However, time limitations prevented the use of the precalculated line-of-sight data for this purpose.

It is also practicable to improve the calculation of the delay that firing activities impose on moving. Thus, whereas moving in the feasibility study was adjusted for the average case of 2 rounds fired for every $100-\mathrm{m}$ advance, the delay in moving can now be tied directly to the time lost while firing.

In addition the cross products of two terrain features may be included. Thus the presence of two distinct terrain features on the same square, e.g., edge of heavy forest plus swampy area, may be accorded a rating independent of the rating allowed for these two terrain features separately.

Also, the increased capability of the 1103 computer permits the influence of the terrain features of each grid square to be related to each adjacent square in turn. Thus, if one grid square contains a road, the influence of that road on movement from an adjacent square depends on whether the pair of squares in question is linked by the road. In other words the existence of a road or river is less a significant characteristic of a single square, and more a characteristic of a pair of squares. The number of such pairs of squares is four times greater than the number of squares alone, and hence the need for a higher-capacity computer than the 1101 to include this effect.

Finally, provision can be made to impose a formation on a group of combat units. This can be done by means of the dependence of the move probabilities on a terrain objective. Thus if the platoon leader (tanks or infantry) takes account of the terrain objective in his calculations, while each member of the platoon moves with reference to the changing position of the platoon leader, then the members of the platoon can be caused to maintain a specified posture relative to the platoon leader, while the entire group advances toward the terrain objective.

## Firing Calculations

With the increased capacity of the ERA 1103 computer numerous refinements may also be made in the firing calculations.

One such refinement is the angular orientation of a target at the time it is struck by a round. By allowing each tank always to have established an orientation relative to the battlefield that depends on its own direction of motion, or its recent firing activities, the angular aspect of a target relative to the shooter can also be determined. Once determined it can be allowed to influence the kill probabilities and perhaps also the tactics.

A second refinement is possible relative to the priority lists used in selecting a target. It is practicable to classify units in terms of the range and allow this factor to influence the selection of a target.

## Infantry Calculations

The added capacity of the ERA 1103 computer will permit the infantry squads to be accorded three different sets of kill probabilities corresponding to (a) the rifle components, (b) the automatic weapon component, and (c) the

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antitank component. The significance of distinguishing between these factors is that the different components have distinct vulnerabilities and performance characteristics.

The military posture of the infantry squads may also be related in more detail to the battle circumstances. For example, the squads may be (a) advancing rapidly, firing, and upright; or (b) advancing slowly, firing, and crawling; or (c) stationary and firing from hoxholes; or (d) stationary but not firing owing to the volume of fire they are receiving.

## Communication System

Information about the enemy or friendly forces acquired during the battle calculations is shared with other friendly elements depending on the facility of the entire communication system. The techniques applied in the feasibility study permit such sharing of information to be limited by parameters that reflect the operational performance characteristics of the communication system.

Specifically, each act of sharing information may be delayed by a time $t$, after which the bulk of the information is available to some other combat element with probability $R$; there is a probability $D$ that the critical part of this information will be incorrectly interpreted. The three parameters $t, D$, and $R$ thus characterize the capabilities of the communication system.

Imposing a time delay on the sharing of information requires that the clock technique be used. Hence a third clock, the communication clock, is to be added to the two already present: the moving clock and the firing clock.

The application of the probabilities $D$ and $R$ follows the same general pattern as that already established in these feasibility calculations (App C).


Fig. 20-Simplified Flow Diagram for CARMON

## INTRODUCING COMMAND DECISIONS

The feasibility calculations implement a simple tactic at the lowest level by means of the tables of values to be used in the calculation of move probabilities, the priority list used in selecting a target, and the choice between moving and firing. For example, the rating numbers used in this feasibility study correspond to a frontal assault by the Blue assault group. Other tactics (defend,

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envelop, or infiltrate) would require a different set of values to be used in the rating calculations that produce the move probabilities. Thus the feasibility study already contains the system required to implement a variety of tactical alternatives. It is therefore feasible to allow appropriate variations in the tactics of different combat groups as the battle progresses. All that is required is that, on the basis of information compiled by a unit commander at any echelon (using the communication system already discussed), the commander select among the tables of values associated with these several tactics that one which he desires his subordinates to follow. So that his decisions are properly ordered in time, a decision clock that functions in the same way as the other three clock types must be provided.

Figure 20 shows the flow diagram for the principal calculations (similar to Fig. 12). As a concrete example of the workings of this generalized system, Table 2 shows a sample application of the system. This hypothetical calculation illustrates the manner in which the interactions due to the communication system are taken account of, while retaining the basic features of the 1101 battle.

Table 2
HYPOTHE TICAL CARMON CALCULATION


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Table 2 (continued)

| Battlefield time, sec | Calculations | Results of reset calculations |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Communication clocks | Decision clocks | Firing clocks | Moving clocks |
| T-54; checks criteria and decides to ask B3 for repeat; query is computed to get through with $9-\mathrm{sec}$ delay |  |  |  |  |  |
| 209 | B 2 stops moving and decides to return to original position, reaching there in $9 \mathrm{sec}($ at 218) | B2 218 |  |  |  |
| 214 | B3 hears Bl request for repeat of message; computes this will be done in 15 sec | B1 229 |  |  |  |
| 218 | B2 arrives back at square 9; rerates squares, searching for position covered from Rl , chooses square 2 , commences move, computing will arrive at border in 45 sec (at 263) |  |  |  | B2 263 |
| 220 | B3 gets off first shot at R1; misses; computes that R1 will become aware of this with $10-\mathrm{sec}$ delay at 230 and that B3 itself can get off second shot in 8 sec at 228 | R1 230 |  | B3 228 |  |
| 228 | B3 misses second shot at R1; computes third shot in 8 sec (at 236) |  |  | B3 236 |  |
| 229 | Bl receives correct message from B3; checks battle decision criteria, noting that this is 10 th T-54 tank in area in front of B2 and B3 noted and still alive; decides that assault group which includes B 2 and B 3 will take up covered firing positions immediately while second assault group maneuvers around to the flank of T-54's to -et on objective; computes that B2 ond B3 group will react to this order in 50 sec (at 279) |  | $\begin{array}{ll} \text { B2 } & 279 \\ \text { B3 } & 279 \end{array}$ |  |  |
| 230 | 7. lespns of fire by B3, backs up to p.evious covered position calculating that it will leave line of sight to B3 in 25 sec (at 255) |  |  |  | R1 255 |
| 236 | B3 checks on firing third shot at R1, discovers R1 is now moving target and delays firing for 7 additional sec (at 243) |  |  | B3 243 |  |

## SPECIAL "TELEVISION" DISPLAY SYSTEMS

The utility of a battle simulator fashioned with the techniques described in this memorandum is twofold. First, the outcome of a series of battles may indicate the superiority of a proposed weapon or system design. Second, once identified, the superior performance indicated by the battle outcome may be

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justified and explained; i.e., the reason that superior performance results from the performance characteristics may be identified clearly so that the results (a) may be examined for plausibility, and (b) then be exhaustively checked by detailed analysis and field or proving-ground experiments.

To facilitate the second application it is necessary that the progress of a series of battle computations be easily inspected as well as the outcome of the battle. The presently available equipment records the progress of the battle calculations in very lengthy tabulations. It is expensive in time and money to reconstruct in a meaningful way the battle situations from such tabulated data.

This inspection process may be enormously simplified if certain television-type cathode-ray-tube systems are connected to the computer in such a way that the progress of the battle is continuously displayed on the face of the tube in terms of conventional military map symbols. Such equipment is well developed and can be adapted to most large general-purpose computers, including the ERA 1103 computer.

TRAINING
Particularly with the addition of appropriate display devices, the system could have application as a training device. In this case an external operator would replace the "automatic commander" in the battle. The external operator would have available only the summary information that would otherwise be available to the "automatic commander" and would make the same type of decisions; i.e., fix terrain objectives, specify the scheme of maneuver, and make such alterations in these orders as is appropriate as the battle progresses. The system could be used with a human operator on only one side or with both sides so controlled. The battle calculations could be made at a rate consistent with real battle or at an accelerated rate. The electronic modifications of the computer required to facilitate such control over the progress of the battle are easily carried out.

## CONCLUSION

The preceding description of how the scope and detail of the model of battle may be changed constitutes the evidence in support of the third conclusion of the study in the section immediately following.

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## CONCLUSIONS

The conclusions drawn from this study relate only to the limited objective of the memorandum, i.e., to test the feasibility of the Monte Carlo small-unit war game.

The design and results of the trial combat action justify the first conclusion below. The actual, detailed process of designing the trial combat action justifies the second. The entire discussion of alternative procedures and general areas of application justifies the third.

1. The Monte Carlo technique enables a very large number of battle factors to be introduced into a feasible analysis of the performance of alternative weapons and weapons systems. The number of battle factors is sufficient to warrant designation of the computing system as a "battle simulator."
2. The technique permits direct participation of nonmathematical personnel-most importantly, officers with extensive combat experienceat every significant step of the design and criticism of controlled, scientific war gaming.
3. The battle factors used are sufficiently comprehensive and basic to permit great flexibility in the manner of their combination into various combat situations involving a variety of weapons and at any echelon for which the performance characteristics of the weapons systems may be specified.

Owing to the limited objective of this study, there are no formal recommendations.

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## Appendix A <br> CAPABILITIES OF THE ELECTRONIC COMPUTER

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Selected performance data of the computer is given in Table A1.
Table Al
PERFORMANCE DATA FOR ERA 1101 COMPUTER

| Item | Amount |
| :--- | :---: |
| Memory capacity, 7-digit numbers | 16,384 |
| Max additions (or subtractions) per sec | 15,000 |
| Max multiplications per sec | 3,000 |
| Total number of possible distinct operations | 43 |
| Time to fill memory from tape, min | $81 / 2$ |
| Digits (or alphabetic characters) typed per sec | 10 |

aThese maximum performance levels cannot always be achieved for reasons discussed in the next section, "The Magnetic Drum."

## Computer Calculation

The significant types of calculations, or operations, which the 1101 computer can perform may be listed under three general categories:
(a) Arithmetic Operations: Ordinary addition, subtraction, multiplication, and division.
(b) Logical Operations: A special form of arithmetic designed for carrying out a type of calculation akin to "logical reasoning."
(c) "Jump" Operations: A special class of operations that makes it possible for the computer to alter the scheme of calculation depending on the result of some previous numerical or logical calculation.

Every automatic calculating machine has at least a few operations of each type listed above. In addition there are other less significant operations, which stop and start calculation of the computer, and which cause the computer to punch or type out selected results, to "read" numbers punched into the input tape, and perform other necessary but subordinate functions.

Since the ERA 1101 computer has a definite list of possible operations, every step in the computer battle is ultimately stated in terms of one or a few of these operations. Annex A1 lists these orders in some detail.

The computer battle described in this memorandum makes important use of all three classes of operations. In general any calculation expressed in terms

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of a logical operation could also be reduced to an arithmetic one. However, tremendous savings in time and memory capacity, as well as an increased simplicity of conception, is possible when logical operations are used.

For the present a simple example of each of these three basic types of operations is given. Elsewhere the combat action is described, dissected into its components, and finally translated into a series of instructions for the computer in terms of these basic operations.

A Simple Arithmetic Operation. The computer could be caused to carry out this operation: "add the number of Blue tanks killed (number is stored at place $X$ in memory) to the number of Red tanks killed (number is stored at place $Y$ in memory) and store the sum at place $Z$ in memory." It takes three separate steps for the computer to perform this simple addition:

Step 1. Take number stored in place $X$ (memory) and put into "adder" (in arithmetic unit).

Step 2. Take number stored in $Y$ and add it into the adder.
Step 3. Take sum now in adder and put into place $Z$.
A special number code is used to cause the computer to perform each step.

A Simple Logical Operation. The computer could also be caused to carry out this operation: "there is a number composed of 5 digits that may be either 1's or 0's, e.g., 10110." The first digit* is a 1 if the first tank has been killed, a 0 otherwise. The second digit is a 1 if the second tank has been killed, it is a 0 otherwise; and similarly for the interpretation of each position in the number; with the third digit relating to the third tank, the fourth digit to the fourth tank, and the fifth digit to the fifth tank. This 5-digit number is stored at the place $X$ in the computer's memory. Question: is the third tank a casualty?

Step 1. Take number in $X$ and put in adder.
Step 2. Take number in $Y$ (number is 00100 ) and put into adder; form the "sum" of the number in $X$ with the number in $Y$ using the convention that the number expressing the sum will contain a 1 in a given position if both the number in $X$ and the number in $Y$ have a 1 in that same position. Otherwise the digit in that position in the sum is to be a 0 . Carrying this out shows:

| Number in $X$ | 10110 |
| :--- | :--- |
| Number in $Y$ | 00100 |
| Sum | 00100 |

Step 3. Is the sum in adder different from 0 ? If it is then the third tank is a casualty.

The preceding example identifies the number resulting from combining the number in $X$ with the number in $Y$ as a sum. Logicians call it a "logical product." This type of operation plays an extremely important role in the computer battle. It is discussed in much greater detail in the remainder of the memorandum. A special number code causes the computer to perform these three steps in about ${ }^{1 / 6000}$ of a sec.

A Simple "Jump" Operation. The computer may be caused to calculate this: "stop the computations and type out the letter $R$ if the total number of
-The digits in a number composed only of 0's and l's are usually called "bits" when used with computers.

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tanks killed (e.g., calculated in the preceding example of an arithmetic operation) is as much as 17." The steps for carrying out this calculation are listed below.

Step 1. Put the number from place $U$ (this is the number 17) into the adder.

Step 2. Subtract the number from place $Z$ (this is the total number of tanks killed computed in the previous example of an arithmetic operation) from the number in the adder. The adder now contains the difference between the number in $U$ and the number in $Z$.

Step 3. Test the size of the number in the adder. If it is exactly 0 , go to Step 4. If it is not 0 , go to Step 5.

Step 4. Cause the typewriter to type out the letter signified by the number in place $W$. Then stop the computer. (Note: The place $W$ must have inserted into it before the start of the computations that number which will cause the typewriter to print R.)

Step 5. Go to the next proper step for continuing the battle.
There is a precise number code that will cause the computer to carry out these steps in as little as $1 / 5000 \mathrm{sec}$.

## The Magnetic Drum

The body of this memorandum includes a general description of the ERA 1101 computer and its capabilities. Both the strength and weakness of the computer derive from the fact that its entire memory is in the form of a rotating drum. The strength results from the large capacity of the drum. The weakness results from the time delays required while the computer waits for the rotating drum to bring a desired number into a position where the control (Fig. 2 in text) can "send" it to the arithmetic unit.

Other sections of this memorandum indicate the necessity of the large memory capacity and the reason for choosing the ERA 1101 computer for these calculations.

Since the drum rotates with an angular speed of 3500 rpm , each revolution of the drum consumes 16 msec . Thus any given position on the drum will, on the average, require a delay of half a drum revolution or 8 msec before it swings under the heads which "read" the number and "send" it to the arithmetic unit. Since the arithmetic operations themselves are carried out very rapidly, the delay imposed on the computer while waiting for the drum to rotate is the principal contribution to the calculation time. In order to reduce such delays as far as possible, very sophisticated schemes of coding are used under the general category of "minimum access programming." It would be outside the scope of this memorandum to discuss this further. However, the operations of the control and arithmetic units are discussed. Since application of this methodology is expected to make use of the ERA 1103 computer which largely eliminates the minimum access problem, the omission is appropriate.

## LOGICAL OPERATIONS

An important feature of the ERA 1101 computer (and to some extent most modern high-capacity computers) is the system of logical operations it can perform. An essential factor in this capability is the fact that most modern

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computers use the so-called "binary number system," with which all numbers are indicated by a series of 1 's and 0's; e.g., $1100101011=811$. Annex A2 describes the way in which this system of numbers (and arithmetic with the numbers) operates.

For present purposes the significant feature of the binary system is the ease with which it can be altered so as to carry out logical operations with enormous speed. This capability will be demonstrated by an example that has direct application to the methodology described in this feasibility study.

## TARGET IDENTIFICATION

Let there be associated with a given combat element on the Blue side (e.g., a tank) a binary number consisting of 24 positions (or bits). The convention is established that the leftmost bit of this number will be a 1 if the combat unit has previously seen the Red (enemy) combat unit 1. Otherwise this bit will be a 0 . Similarly, the second bit from the left in this binary number will be a 1 if the Blue combat unit has previously seen Red combat unit 2. Similarly for each bit in turn, each one being a 1 if the Blue combat unit has previously seen the corresponding Red combat unit. For example, given a total of five enemy combat units, the binary number associated with a particular Blue combat unit might be (for convenience let this be termed the "A number"):

$$
A=10110
$$

indicating that the Blue unit has previously seen enemy combat units numbers 1,3 , and 4 but not numbers 2 and 5 .

If at some time during a battle calculation it is necessary to select a target for firing purposes and therefore refer to the number and position of enemy combat elements visible at that moment to the same Blue combat unit, then the information stored in the binary number just described would need to be brought up to date. Thus those enemy combat elements which had since become casualties could be removed from the list. Also those combat elements which are currently in a concealed position are not at the moment available as targets. Finally, additional combat elements may have recently betrayed their position but are not yet included in the binary number.

By making use of logical operations the binary word describing enemy combat elements previously seen may be brought up to date so as to indicate, by the same convention, those enemy combat units now seen.

The first step is to add to the binary word those combat elements which have recently done something which can be assumed to betray their position to all within sight. Suppose the list of such combat elements has been kept current in a single binary number word (for convenience, called the "B number") with the convention that the leftmost bit shall be a 1 if enemy combat unit 1 has recently betrayed its position, and a 0 otherwise. Similarly the bit second from the left shall be a 1 if enemy combat unit 2 has recently betrayed its position, and the same for all five enemy combat units. For example the $B$ number may be

$$
B=01100
$$

indicating that enemy combat units 2 and 3 have recently betrayed their position.

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If the $B$ number is added to the $A$ number in a special way, the sum will have a 1 in all positions corresponding to enemy combat units whose position is currently known to the Blue combat unit.

$$
\begin{aligned}
A & =10110 \\
B & =01100 \\
A \oplus B & =11110
\end{aligned}
$$

Inspection of the sum will show that the addition indicated by the symbol " $\oplus$ " is carried out by writing a 1 in the sum provided that either or both $A$ and $B$ have a 1 in the same position. This special version of addition is one of the two basic logical operations.

The significant feature of the above operation is that only one operation was required to add to the list of enemy combat units previously seen those which recently betrayed their position. In the trial calculations carried out in the course of this feasibility study there are 24 enemy combat units. There are also 24 bits in the binary numbers handled by the computer. Hence the above system can be applied to bring the list of potential targets up to date for any combat unit in only one operation.

The next step in the calculations to bring up to date the list of targets available to a selected Blue combat unit is to subtract from the list those Red combat units which are now in a fully covered or concealed position.

For this purpose let there be a third binary number, to be called the " C number," which has 1's in those positions where the corresponding enemy combat unit is not now concealed and a 0 for those which are concealed. For example, suppose the $C$ number is the following:

$$
C=10111
$$

Then this indicates that enemy combat units $1,3,4$, and 5 are not concealed, but that 2 is concealed. If the number $\mathrm{A} \oplus \mathrm{B}$ previously calculated is multiplied by the number $C$ in a special way then the product will have 1's in every position corresponding to an enemy combat unit that (a) can currently be seen by the Blue combat and (b) whose position is known to the Blue combat unit. Thus,

$$
\begin{array}{r}
A \oplus \begin{array}{l}
B \\
=11110 \\
C
\end{array}=10110
\end{array}
$$

$(A \oplus B) \otimes C=10110$
where the symbol $\otimes$ has been used to indicate the special system of multiplication where a 1 is written in any position in the product provided that both the multiplier $(\mathrm{C})$ and the multiplicand $(\mathrm{A} \oplus \mathrm{B})$ have a 1 in that position. This special form of multiplication is the second basic logical operation, here called "logical" multiplication.

As a last step in deriving an "up-to-date" list of the enemy combat units available to a selected Blue combat unit as targets, the list must be corrected by removing from consideration those enemy combat units which have become casualties. This requires that the product derived in the last step be logically multiplied with a D number that has a 1 in those positions corresponding to

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tanks not yet casualties, a 0 otherwise. To complete the example, suppose enemy combat unit 3 is the only casualty. Then

$$
\begin{aligned}
(A \oplus B) \otimes C & =10110 \\
D & =11011
\end{aligned}
$$

## $[(A \oplus B) \otimes C] \otimes D=10010=$ current targets

Thus three simple logical operations have sufficed to correct an out-ofdate list of potential targets. This system is applied, using 24 Red combat elements and 20 Blue combat elements, throughout the model of battle calculations and much of the speed of the computer calculation is dependent on its use.

The preceding logical operations may also be used to inject an element of chance into such calculations. To effect this, the use of random numbers is required. The properties and use of random numbers are discussed in Annex A3.

Note that the simplicity of this system is not without cost. For it is necessary that the computer continuously correct the $B, C$, and $D$ numbers as the battle progresses so they may be current.

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## Annex A1

ORDER STRUCTURE OF ERA 1101 COMPUTER

The computer used in this technical memorandum has a total of 42 nontrivial orders. These may be classified as shown in Table A2.

Table A2
TYPES OF ORDERS USED WITH ERA 1101 COMPUTER

| Operation | No. of orders <br> in class |
| :--- | :---: |
| Add and subtract orders with variations | 15 |
| Multiply and divide with variations | 3 |
| Transmit | 5 |
| Address substitution | 1 |
| Stop with variations | 3 |
| Shift | 2 |
| Print out | 3 |
| Jump instructions | 6 |
| Logical operations | 4 |
| $\quad$ Total | 42 |

Table A3
SPECIAL LOGICAL OPERATIONS WITH ERA 1101 COMPUTER

| Designation | Explanation |
| :--- | :--- |
| $Q$ jump | If leftmost bit of $Q$ word is 1 , jump to $y$ for next instruction, <br> otherwise continue in normal order sequence; in either <br> case shift bits in $Q$ word one place to left, with leftmost <br> bit going into rightmost position <br> Replace each bit of $y$ with corresponding bit of $A$, provided <br> the corresponding bit of $Q$ is a $l$; the remaining bits of $y$ <br> will not be disturbed |
| Clear logical multiply | Clear A and then add the 48 -bit number whose left-hand 24 <br> bits are all 0 and whose right-hand 24 bits consist of the <br> bits of y corresponding to the l's of $Q$ with 0 's in all <br> other positions |
| Hold logical multiply | Same as above except that $A$ is not cleared initially <br> Complement those bits of $A$ which correspond to l's in $y$ |

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These orders are nearly all of conventional type and need not be further discussed. An exception is the group of logical operations together with one of the jump instructions that play an important part in this feasibility study. Table A3 describes these five instructions. In the description of these orders, $Q$ and $A$ are the two registers in the arithmetic unit, $y$ is any memory position.

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## Annex A2

BINARY NUMBERS AND ARITHMETIC

Owing to the degree to which the methodology proposed in this memorandum depends on logical operations as distinct from common arithmetic, it is necessary to describe the binary number system employed by many electronic computers including the ERA 1101 computer. In fact nearly all fast computers use the same number system.

In the computer all numbers may be considered as being denoted by a series of 1's and 0's alone called "bits" in contradistinction to "digits" in the decimal system. The actual number indicated may be found by adding a series of small numbers, each a different power of 2, depending on whether a given part of the number is a 1 or a 0 . As an example, the binary number 1101 is interpreted to mean 13 by the following process.


The reason for using what may appear to be a clumsy notation is that much simpler computing machines of much greater reliability may be built on this basis. Notice however the basic similarity between the interpretation of binary numbers based on powers of 2 and the familiar decimal notation which is based on powers of 10 . Thus, using the system;

$$
143=\left(1 \times 10^{2}\right)+\left(4 \times 10^{1}\right)+\left(3 \times 10^{0}\right)
$$

Addition and subtraction of binary numbers follows a series of straightforward rules similar to those for "carry" in ordinary arithmetic. Two examples of each will suffice to illustrate these rules.

| Addition |  | Subtraction |  |
| :---: | :---: | :---: | :---: |
| Example 1 | Example 2 | Example 1 | Example 2 |
| $1 \equiv 01$ | $7 \equiv 0111$ | $2 \equiv 10$ | $7 \equiv 0111$ |
| $+1 \equiv 01$ | $+6 \equiv 0110$ | $-1 \equiv 01$ | $-6 \equiv 0110$ |
| $=2 \equiv 10$ | $-13 \equiv 1101$ | $-1 \equiv 01$ | $=1 \equiv 0001$ |

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Multiplication is effected by repeated addition, and division is effected by repeated subtraction.

The ERA 1101 computer uses binary numbers with 24 bits or positions. Hence, if the decimal point is at the right, the most significant bit (the leftmost) contributes a value of $2^{23}=8,388,608$ to the over-all number. Thus about seven significant figures may be indicated by a 24 -bit binary number if such accuracy is required.

Fractions in the binary system are interpreted to involve negative exponents. For example:

$$
\begin{aligned}
\text { means } & \left(0 \times 2^{-1}\right)+\left(1 \times 2^{-2}\right)+\left(0 \times 2^{-3}\right)+\left(1 \times 2^{-4}\right) \\
\text { or } & (0 \times 1 / 2)+(1 \times 1 / 4)+(0 \times 1 / 8)+(1 \times 1 / 18) \\
\text { or } & 0+1 / 4+1 / 18+5 / 18=0.3125 \text { (decimal) }
\end{aligned}
$$

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Annex A3

## RANDOM NUMBERS FOR MONTE CARLO CALCULATIONS

The term "random number" as used in this memorandum may be taken to mean simply that numbers are selected by some process such that each digit of the number has an equal chance that it will be a $0,1,2,3,4,5,6,7,8$, or 9 ;* and that, if a series of random numbers is to be written down, the digits that appear are in no way dependent on the digits preceding it. The crux of the definition is that any group of numbers will be taken as satisfying this definition if accepted statistical measures of the frequency with which any given digit appears in an extended list of numbers, singly and in combination with other numbers, show the hypothesis to be a tenable one.

It is essential to recognize that, whereas drawing numbers out of a hat can be made to satisfy the definition, a scheme for the generation of numbers is possible that violates part of the definition, yet can pass the test stated in the preceding sentence.

It is not within the scope of this memorandum to discuss this apparent contradiction. It suffices to say that, for all practical purposes, numbers that are "so nearly random" to permit their use as such may be generated by the computer. They are properly called "pseudorandom" numbers.

One such scheme was used in this feasibility study. A new random number was produced when required from the previous one using a revised Lehmer ${ }^{4}$ method:

$$
\mathrm{R}_{k+1}=\mathrm{R}_{k} \cdot \mathrm{C}(\bmod \mathrm{M})
$$

where $\mathrm{R}_{k+1}=$ new (pseudo) random number
$\mathrm{R}_{k}=$ previous (pseudo) random number
$C=28$
$M=6,236,449$
The pseudorandom numbers generated by this method are all less than $M$, and the sequence of numbers will not repeat itself before 1 million such numbers have been produced but will, of course, repeat sometime before $6,236,450$ numbers have been generated.

RANDOMNESS AND LOGICAL OPERATIONS
For many logical operations associated with probabilities, it is desirable to have binary numbers having a stated probability of a 0 at every position. Each position in the binary form of every pseudorandom number generated by a process such as was described above has a 50 percent chance of being a 1 and a 50

* The fact that the computer uses binary numbers does not basically influence this definition.


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percent chance of being a 0 . If 0 's are desired with some other probability, this can be brought about by various combinations of logical operations on these pseudorandom numbers. For example, if the logical product (Annex A1) of two pseudorandom numbers is formed, then there is a 75 percent probability that each and every position, separately, in the product will contain a 0 . This follows from the fact that there is only a 1 in any given position in the product if there is a 1 in the same position in both the multiplier and the multiplicand. But there is a 0.5 chance that either of these should happen or a ( $0.5 \times 0.5=0.25$ ) 25 percent chance that both would happen; hence a $(100-25=75) 75$ percent chance that this should fail to come about, yielding a 0 in the product.

By repeated logical operations, binary numbers may be produced which have, to within a stated accuracy, any desired probability of possessing at each and every position, separately, a 0 . Thus, if logical multiplication is denoted by $\otimes$, logical addition by $\oplus$, and every random number by $r$, the following list shows some of the possibilities (exponent -1 indicates that all 0 's are changed into 1's and all 1's into 0, i.e., the "complement bits" order in Annex A1).

| Operation | Chance of $0, \%$ |
| :---: | :---: |
| $r$ | 50 |
| $r \otimes r$ | 75 |
| $(r \otimes r)^{-1}$ | 25 |
| $r \otimes r \otimes r$ | 87.5 |
| $(r \times r \otimes r)^{-1}$ | 12.5 |
| $r \otimes r \oplus r$ | 37.5 |
| $(r \otimes r \oplus r)^{-1}$ | 62.5 |

Annex A1 shows the two logical operations. Logical addition is performed using the substitute order in Table A3.

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## Appendix B

## ALTERNATIVE BATTLE MODELS

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## INTRODUCTION

In the body of the memorandum a series of decisions are made to use various particular techniques to restate the combat action in a form amenable to computer calculations. In most cases there are several alternative means of effecting this. This study has made a particular choice among these alternatives consistent with the limitations on the feasibility study. But from time to time it will be reasonable to review the alternatives to determine whether additional experience and modified circumstances indicate a different choice to be advantageous. To aid in such periodic reviews of the methodology, the most important alternatives will be described in this appendix along with the reasoning that prompted each choice made.

The alternatives discussed include the various ways in which the terrain features may be introduced into the computer, the use of tabulated weapon performance data contrasted with formulas, and the use of special logical operations. These alternatives are keyed to the capabilities of different types of computers and to the nature of the problem areas to be analyzed.

## INTRODUCING TERRAIN FEATURES INTO THE COMPUTER

The most basic feature of the proposed methodology involves a scheme for effecting direct reference to the terrain features during the battle calculations. The major alternatives to the proposed battle model therefore naturally involve the way in which the terrain features are made a part of the calculations.

There appear to be three essentially different means of including the effects of terrain: (a) the use of mathematical formulas, (b) the use of a gridsquare system that can also provide a basis for measuring distance or range, and (c) the use of a system where the various areas coextensive with the major terrain features are the basic units. Each of the systems has been used in previous work. Mathematical formulas and the grid-square system have been applied extensively by many nations to war gaming and map exercises for many years. A recent ORO technical memorandum ${ }^{5}$ discusses an application of system $c$.

Each of the three systems for computing terrain effects has certain advantages and disadvantages in the present case depending on (a) the scope of the military problems to be investigated, (b) the types of electronic computers available, (c) the type of weapon performance data available, and (d) the facility with which the same methodology handles other battle factors such as communications, training, and tactical doctrine.

A choice among these systems is made by identifying the one that best meets the requirements of the methodology when stated in the above terms. To make this identification requires that specific instances of the application of

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these systems be considered. Other sections of the memorandum show that frequent reference to the terrain features by one of these methods is required for two general purposes: First, influence on movement, which is subdivided into (a) determining the trafficability of terrain over which the combat element is moving or is considering movement, (b) determining the degree of cover and concealment (referred to enemy positions) offered by the terrain over which the combat unit is moving or is considering movement, and (c) control of the general maneuver so that progress toward the terrain objectives results.

Second, influence on firing, which is subdivided into (a) identifying those enemy units which are not visible, or only partly visible, owing either to their defiladed position (intervening terrain features cut off the line of sight) or to the concealment afforded by the vegetation surrounding the enemy combat unit, and (b) determining the range between combat units.

These purposes may be reduced to two general requirements: (a) the significant terrain features that characterize the actual position (or a proposed future position) of the combat unit must be known, and (b) the visibility of a remote enemy unit from the position of any given combat unit must be known. The best system will be the one that yields acceptable approximations in the least computing time.

## Mathematical Formulas

As an example of the application of conventional formulas to the coding of terrain features for the computer, consider the following simplified case. A tank is moving across an area open except for a large tree stand. The speed


Fig. B1-Approximating the Perimeter of a Tree Stand on an Open Plain by on Elliptical Figure
For the approximate elliptical figure the equation is

$$
x^{2}+c y^{2}+d x y+e x+f y+g=0
$$

The coofficients $c, d, e, f$, and $g$ are adjusted for the best approximation
with which the tank moves depends on whether it is within or without the tree stand. The computer must therefore be provided with directions permitting the determination of this fact. The current grid coordinates of the tank are known to an accuracy of $\pm 1 \mathrm{~m}$.

To apply the system of mathematical formulas requires that the perimeter of the tree stand be approximated by an equation. A simple approximation is to fit an ellipse to the perimeter (Fig. B1). Annex B1 shows how the computer may determine for any position of the tank whether it is within or without the tree

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stand. To do this requires that five constants be stored in the computer to define the equation for the elliptical figure. The calculations require seven multiplications. If more than one ellipse were required to yield an acceptable approximation to the perimeter of the tree stand, then additional constants, five for each ellipse, would have to be stored in the computer and, on the average, three to four additional multiplications would be required for each added ellipse.

Before considering the other factors that were stated above to characterize the three systems, each of the other two systems will be applied to the same example.

## Grid-Square System

Application of the grid-square system by the computer in order to determine whether the tank is within or without the tree stand is straightforward. Before the computations are started, the entire battle area is divided up into a system of grid squares (Fig. B2).


Fig. B2-Approximating the Perimeter of a Tree Stand (Shaded Area) on an Open Plain by the Grid-Square System (Heary Lines)

In order to be able to determine, for an arbitrary position of the combat unit, whether it is within or without the tree stand it is necessary for the computer first to identify the grid square including the combat unit's position and then to refer to data previously stored in the computer, which gives the desired information. Figure B3 gives a flow diagram for accomplishing this. By measuring the position coordinates in units of the length of one side of a grid square, the coordinates of the grid square including the position of the combat unit are found by ignoring their fractional parts.

The flow diagram in Fig. B3 accomplishes the same determination as does the flow diagram in Fig. B7 of Annex B1, yet involves no multiplications. All the operations together involve only the equivalent of approximately one-fourth of a

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multiplication time. Therefore the grid-square determination is, for the case of one tree stand in an open plain, computed at least 16 times as quickly compared with the use of an elliptical approximation, provided that no significant time is lost while the computer searches through the memory to locate the proper constants. The most important gain, however, results from the fact that, although additional multiplications are required as the number of tree stands increases, when using approximating formulas, no increase in computation time results as the number of tree stands is increased when using the grid-square system.


Fig. B3-Flow Diagram Showing Calculations to Determine Whether Combat Unit Is Inside or Outside Tree Stand (Fig. B2) Using Grid-Square System

Thus, if there were five separate tree stands, the grid-square system would permit determination whether a combat unit was within or without a tree stand at least 40 times as quickly as the formula system would.

The disadvantage of the grid-square system is the much larger number of constants that must be stored in the computer before the calculation is started. The flow diagrams in Figs. B3 and B7 show that, although the formula system requires only five constants to be stored, the grid-square system requires the storage of 256 constants to supply the same information. Thus, in general, the large savings in calculation time the grid system provides is only acquired at the cost of an increased demand on the memory capacity of the computer.

In general the same compromise obtains everywhere in the computer battle. Virtually every separate action during the battle can be speeded up significantly when a grid-square system is used but only at the expense of the capacity of the memory and only if data in the memory can be located quickly.

## System of Terrain-Feature Areas

In contrast with the formula system and the grid system, this method does not include at any time an accurate specification of the position ( $x$ and $y$ coordinates) of the combat unit. The position of the combat unit is known only in terms of the terrain-feature area it is within. The problem of determining which terrain-feature area includes the combat unit never arises. The problem the computer is concerned with is to cause the combat unit to move from one terrain feature to another in a sensible manner. Since the exact position of any combat unit within the area coincident with the terrain feature is not known, the range from the combat unit to some other combat unit cannot be accurately determined. To the extent that particular weapons effects (which depend on range) must be

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included, this is a serious disadvantage. However, the system does permit concentration on an element that is very prominent in any statement of tactical doctrine, i.e., the relation between the military effectiveness of a combat unit and the terrain feature it occupies. For a more detailed discussion of this system, refer to the memorandum previously mentioned. ${ }^{5}$

## Comparison of Systems

A preliminary discussion of the relative desirability of the three systems for inserting terrain features into a computer is now possible. This discussion is in terms of three of the four factors previously listed: (a) type of military problem, (b) type of computers available, and (c) type of weapon performance data available. (The fourth factor, treatment of other battle factors, is taken up in the section "Tactical Doctrine," of this appendix.)

The nature of presently available weapon performance data (e.g., from the proving grounds) rules out the third system for the immediate purposes of this methodology, since the method does not include specific reference to the range separating combat units. Thus, insofar as the killing power of weapons as a function of range is a primary measure of effectiveness of weapon systems, such data cannot be included directly in a calculation using the third system.

Table Bl
DESIRABLE COMPUTER FEATURES FOR
THE TWO PRINCIPAL SYSTEMS OF
CODING TERRAIN FEATURES

| Computer feature | Emphasis required for system |  |
| :--- | :--- | :--- |
|  | Formulas | Grid squares |
| Size of memory | Not emphasized | Very large |
|  | Not emphasized | Very fast |
| Speed of multiplication | Very fast | Not required |
| Speed of addition | Not emphasized | Emphasized |
| Special operations | Not required | Emphasized |

Both of the remaining systems do include some reference to the position of a combat unit. Hence the range to some other combat unit can be computed, permitting use of weapons effectiveness data, which has a strong dependence on range.

A choice between the remaining two systems can be made tentatively on the basis of which will better exploit the capabilities of the best modern computers for rapid calculation. It will be recalled that the grid-square system of calculation was the more rapid (compared to the use of formulas) when the memory of the computer was large enough to store the much larger number of constants, and provided that the computer was able to locate desired constants rapidly in this memory. Also the use of formulas requires the computer to perform numerous multiplications, whereas the grid-square system uses only the simpler operations such as addition.

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It is therefore reasonable that, since computers usually compromise one or another of their capabilities in favor of emphasizing some other capability, there would be, among all available computers, one that would perform best with the formula systems and a second that would perform best using the gridsquare system. Which of these two "system-computer" combinations is the better for simulating battle is not obvious a priori.

However, at present general-purpose high-speed computers tend to differ mainly in the size of their memory and to perform multiplications much more slowly than additions. General-purpose computers seem to be best suited for battle simulation work-owing to the great flexibility in their use. Tentatively then, the grid-square system appears most attractive, in which case the most desirable feature of a computer to be used for battle simulation is a large memory. Table B1 summarizes the desirable features of a computer in terms of the preceding discussion.

## Application of Grid System to Maneuver

Limitation of Grid System. Unlike the formula system, the grid-square system cannot be used to indicate accurately the position of small terrain features. For example, a small clump of vegetation may offer important concealment to a combat unit. But if the clump is smaller than a single grid square, its exact position within the square cannot be designated. As a consequence, only movement from one square to another can be related directly to the terrain features. So far as the calculations are concerned, every position within a square is influenced in the same way by the average terrain features of the whole square.

It follows that in general the only meaningful option available to a combat unit is the selection of an adjacent grid square to be presently occupied.

Also, if the grid-square system is used to measure the range between units, then this range cannot be specified more accurately than the size of the grid square permits.

Mixed Systems. It is not essential that only one of the several systems be selected for codifying the terrain features. Nevertheless a mixed system would be expected to take a longer time for calculation. None was considered for this feasibility study. However, it may well be that some critical action within the battle requires a special treatment, and some loss in speed can be tolerated.

A possibility in this regard is the action of small infantry groups. Thus, whereas a knowledge of the range between tanks to the nearest 100 yd may be sufficient for most cases, the range to nearby infantry groups may sometimes require refinement to within only a few yd.

There are three important qualifications to be added: First, the utility of the grid-square system regarding other battle factors must be taken into account. Second, as the memory capacity of the computer is applied to various grid-square calculations, eventually the residual memory capacity will be insufficient to support additional grid-square calculations, forcing dependence on the use of formulas. Third, new developments in computers may so facilitate multiplication operations as to tip the balance in favor of the extensive use of formulas.

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## KILL PROBABILITIES

Much the same series of alternatives, as was available in the coding of terrain features, applies to the storing of kill-probability data for the various combat elements. The principal alternatives are (a) to store formulas from which the kill probability can be computed, or (b) to store extensive tables providing separate entries for all variations in range, target type, type of shooter, etc., which are adjudged significant. Again the formula system is characterized by (a) the requirement to store only a few constants that define the equation, and (b) calculation of any given kill probability using an equation which is apt to involve numerous time-consuming multiplications.

On the other hand, although the use of tables of kill probabilities avoids lengthy calculations, the extensive tables require a large memory capacity. A special advantage of tabulated kill probabilities is the ease with which irregular or nonuniform kill probabilities may be included. Thus, once set up for the use of tables, no new formulas need to be designed and inserted into the coding to take account of newly proposed weapon characteristics.

Again, as the memory capacity of a computer approaches exhaustion, there may come a time when there is no room for the tabulated kill probabilities required for some special circumstance. Such a situation might force the use of formulas for special cases.

The battle code used in this memorandum affords an example of such a case. The kill probabilities of an infantry squad of fractional strength $f$ are found by multiplying the kill probability of the full-strength squad, stored in table form, by the factor $f$.

## TACTICAL DOCTRINE

Actual battle involves (a) physical weapons and their performance characteristics, (b) the maneuver of weapon systems wherein the human weapon operators are very prominent, and (c) a codified set of principles-tactical doc-trine-which serve as guidelines to the unit commanders as they develop their plan of action (or concept). To be of use, a tactical doctrine must be stated in such general terms as to permit application to a variety of specific combat situations; i. e., it cannot be too specific.

Thus a battle model must be capable of implementing the general type of rules-or axioms-that are the substance of an element of tactical doctrine.

The three systems of introducing terrain features have been briefly discussed in terms of the facility with which they treat of weapons (e.g., range dependence) and weapon systems (e.g., terrain objectives). A final choice must, however, depend also on their facility in implementing tactical doctrine.

A statement of an element of doctrine is a (tentative) identification of a course of action depending on the relation between such elements as a covered approach, fields of fire, built-up enemy position, military crest, and ratio of forces. All these elements involve, directly or indirectly, areas on the battlefield of various shapes and sizes to which such statements can be applied; e.g.,

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this area is within the enemy fields of fire; that area provides a covered approach to the enemy position; the ratio of forces in this area is 2 to 1 . It follows that implementation of a tactical doctrine with a battle simulator requires that such areas be easily identified during the course of a battle (or just preceding the battle). Thus the discussion of terrain-feature areas in the previous sections applies here. The formula system involves lengthy calculations whenever areas of arbitrary shape and extent are considered. The grid-square system avoids such lengthy calculations if the computer has a large memory capacity. The terrain-feature area has as its basic elements the very areas required to implement (and discover!) elements of a tactical doctrine.

## EVALUATION OF GRID-SQUARE SYSTEM

Clearly the terrain-feature-area system is favored by the ease with which it deals with the type of statements-the building blocks-used in formulating a tactical doctrine. Unfortunately this system does not also provide in a convenient way the means for including weapon characteristics. Thus the grid-square system appears to be a usable compromise between the requirement that the system can handle weapon performance characteristics and the elements of a tactical doctrine.

Table B2
RELATIVE ADVANTAGES AND DISADVANTAGES OF THREE SYSTEMS OF CODING TERRAIN FOR A COMPUTER

| System | Best suited for what <br> type of analysis | Suited for what <br> type of computer | Fits presently <br> available weapon <br> performance data | Fits other <br> factors in <br> battle |
| :--- | :---: | :---: | :---: | :---: |
| Formulas | Limited-memory <br> general-purpose <br> computer | Very well | No |  |
| Grid squares | Mane uver of weapon <br> systems <br> Large-memory <br> general-purpose <br> computer | Well | Fair |  |
| Terrain-feature <br> areas | Tactical doctrine | Special logical <br> computer | No | Very well |

Table B2 summarizes the discussion of the merits of the three systems. If one of the three systems is to be chosen, it appears that the grid-square system is the best compromise.

On the other hand it might be expected that improved battle codes are possible if the best elements of all three systems are combined in a mixed system.

## LINE-OF-SIGHT CALCULATIONS

An essential step in the firing calculations requires that it be known which of the opposing combat units are visible to the combat unit preparing to fire. The grid-square method of calculating this is indicated in Fig. 10. An important alternative in the design of the battle refers to the possible desirability of carrying

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out this extensive calculation for all possible positions of the combat units in advance of the battle calculations themselves. If the memory capacity of the computer is sufficient to contain all the results of such a calculation, and if the results once stored are readily available for use, then significant savings in the time to compute a large series of battles on the same terrain are possible.

In this feasibility study it proved possible to store the results of this precalculation in the memory since the Red combat units were confined to a region embracing only 96 of the $100-\mathrm{m}$ grid squares. Thus, by using each bit in the 24-bit numbers to indicate whether or not a "line of sight" did exist between a pair of grid squares, the results of the precalculation required only 2304 numbers* for storage, which was well within the 16,000 -number capacity of the 1101 drum. However, if the Red combat units had not been confined to only a portion of the battlefield, then it would have required a minimum of three times this number of words of storage, or 6900 words. Further, if the data were required to be easily accessible, then six times this amount ( 13,800 numbers) is required. Owing to the quantity of other data also required, this would have been impossible.

If any significant variation in the system is made, precalculation of the line of sights between grid squares may become unfeasible. In any such case it will be necessary to consider the desirability of an increased number of grid squares as opposed to the cost in time of carrying out line-of-sight calculations. This last depends on the particular capabilities of the computer to be used but also on the nature of the terrain and the tactics to be used by the opposing forces since the calculations are drastically reduced if the combat units spend any appreciable amount of time in heavy cover or concealment.

It may be expected that optimum solutions of this time-saving problem will in general involve a combination of precalculations plus calculations during the battle itself for special or unusual circumstances. This in turn suggests that grid squares of nonuniform sizes may find future application.

## FLEXIBILITY VS SPEED

If a logical scheme of calculation is to be used, an important pair of alternatives in the manner used to carry out that calculation occurs.

As an example of such a pair of alternatives, consider a logical operation, necessary whenever one combat unit (e.g., a tank) must choose a single target from among a group of possible targets (e.g., several enemy tanks). To make such a selection the computer must be supplied with a priority list stating all the rules necessary to eliminate as a target all the enemy tanks but one. The pair of alternatives arises from the existence of two extreme methods of implementing such a priority list. After the nature of the two methods, respectively, these alternatives are identified as implicit logical calculation and explicit logical calculation.

## Implicit Logical Calculation

In an implicit logical calculation the priority system to be used is rigidly fixed by the particular sequence in which the computer carries out the calculation. Suppose for example that there are three different types of enemy

* Number words $=(576$ squares $\times 96$ squares $) /(24$ pairs / number) $=2304$ numbers. If economy in storage space had been sufficiently important, this could have been reduced to 2082 numbers. However, access to the data would then have been much more time consuming.


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tanks: type A, type B, and type C. Suppose further that the desired priority system is as follows:

Priority 1: Always select type A as a target when the group of available targets includes this type. If there is more than one such tank, select one at random.

Priority 2: If there are no type A tanks, then select a type B tank if present; if more than one such tank, select one at random.

Priority 3: If there are no type A or type B tanks in the group of available targets, select any type $C$ tank at random.


Fig. B4-Flow Diagram That Implements Implicit Priority System Governing Selection of Target

A sequence of operations permitting the computer to select a target under this priority system is illustrated by the flow diagram in Fig. B4.

The important characteristic of this flow diagram is that nowhere does it indicate that type A tanks have first priority. Type A acquires first priority only because of the order in which the various operations are performed, i.e., the priority system is implicit not in what the orders say but rather in the order in which they are performed.

This is the system used in the present battle to implement the priority systems used in selecting a target. The computer used can carry out such a sequence of operations at great speed.

The disadvantage of this system is that the only feasible way of changing the priority system is to make the necessary changes in the entire series of basic orders used to code for the computer. This can be a very time-consuming operation.

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## Explicit Logical Calculation

This system of logical calculation has characteristics opposite to those of the implicit type. Whereas the previous method is carried out very quickly, this one is done rather slowly; however, whereas the previous system allows new priority systems to be added only with great difficulty, the explicit system permits rapid modification.


Fig. B5—Flow Diagram That Implements Explicit Priority System Governing Selection of Target

A flow diagram for an implicit logical calculation can easily be altered to serve for explicit calculation. (The system of orders required for the computer to implement the flow diagram will, in general, be much more complicated.)

To modify the flow diagram in Fig. B4 for explicit calculation is straightforward. The modified flow diagram is shown in Fig. B5.

The scheme of logical calculation illustrated permits any one of the six possible priority systems involving three types of tanks to be used. Changing

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from one system to another is effected by selecting the appropriate priority system number. Once set up, no modification in the numerous basic orders used to control the computer is required.

## Choice between Methods

Essentially all the logical computations used in the computer battle could be set up for either method. A choice in each case is a compromise between the desirability of speed and economical use of the memory capacity (favors the implicit system) and the desirability of flexibility (favors explicit system).

Owing to the limited capacity of the 1101 computer most of the logical calculations for the battles computed in this study were set up using the implicit system. The much larger 1103 computer, which can be used for application of the computer battle methodology, will permit extensive use of the more flexible explicit system.

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## Annex B1

## DETERMINATION OF POSITION RELATIVE TO TERRAIN FEATURES COMPUTED BY FORMULAS*

It is to be determined, using conventional mathematical formulas whether a given point $x_{o}, y_{o}$, which represents the coordinates of a combat unit, is within or without a terrain feature whose perimeter is approximated by an ellipse with major axis $2 B$, and minor axis $2 A$; whose center is at $x_{1}, y_{1}$; and with the major axis inclined by an angle $\theta$ to the $x$ axis (Fig. B6).


Fig. B6-Parameters Defining an Arbitrary Ellipse Used to
Approximote the Perimeter of a Terrain Feature

$$
h(x, y)=x^{2}+a y^{2}+b x y+c x+d y+e=0
$$

$$
\text { where } a=1 / f\left(B \sin ^{2} \theta+A \cos ^{2} \theta\right)
$$

$$
b=1 / f\left[\left(B^{2}-A^{2}\right) \sin 2 \theta\right]
$$

$$
c=1 / f(-2 x, f-y, b)
$$

$$
d=1 / f\left(-x_{1}, b-2 y_{1} a\right)
$$

$$
e=1 / f\left(x_{1}^{2} f+x_{1} y_{1} b+y_{1}^{2} a-A^{2} B^{2}\right)
$$

$$
f=B^{2} \cos ^{2} \theta+A^{2} \sin ^{2} \theta
$$

The point $x_{0}, y_{0}$ falls within the area if and only if

$$
h\left(x_{0}, y_{0}\right) \leq 0 .
$$

The flow diagram describing the determination of whether the point $x_{o}, y_{o}$ is within the ellipse is shown in Fig. B7.
*As suggested by R. Durfee (ORO).

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Inspection of Fig. B7 will show that a total of seven multiplications are required to carry out the complete calculation. If more than one such area is involved, then the same routine must be repeated for each such area until the point is shown to be in one of them, or in none of them. On the average, therefore, this operation must be repeated a number of times at least as great as one-half the number of ellipses since, if the areas are mutually exclusive and taking the excluded area as an additional area, then the point is just as likely to be in one ellipse as in any other.


Fig. B7-Flow Diagram to Determine if Combat Unit at $x_{0}, y_{0}$ Is Within Arbitrary Ellipse:

$$
x^{2}+a y^{2}+b x y+c x+d y+d y+c=0
$$

On general-purpose digital computers, the time to carry out a multiplication is about five times as long as is required for additions, subtractions, and "test for inequality." Hence only the multiplications in the subroutine need be considered when estimating the time required for calculation.

## CONCLUSION

Use of generalized ellipses to locate a combat unit with respect to $n$ terrain features will generally require between $7 / 2 n$ and $7 n$ multiplications each time the point is located with reference to the ellipses. Five constants must be stored for each ellipse included.

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## Appendix C <br> DETAILS OF COMPUTER BATTLE

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## Appendix C

## DETAILS OF COMPUTER BATTLE

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## INTRODUCTION

In this appendix all the militarily significant operations that are the subject of computation during the battle are described. The detailed flow diagram, which lists the operations and the order in which they are performed, is followed by a running commentary on the diagram. Then four sections present all the performance characteristics used for the battle. These include the various kill probabilities, see probabilities, speed of movement, and move probabilities.

Finally the two different formats used by the computer in presenting the results of a battle are described.

## DESCRIPTION OF DETAILED FLOW DIAGRAM

Figure C1 is the flow diagram showing each militarily significant step in the calculations. About 100 different steps are indicated in the various major operation boxes. To carry these out requires about 7000 separate orders to the computer. A list of these 7000 orders is not given in this memorandum. Appendix A lists the basic orders used by the computer in carrying out these calculations as well as an example of their application to the target selection operation.

## Start Routine

S1 This group of orders sets up the initial values required to start a battle. For example, if the computer has just finished one battle and is ready to start another, then all the dead tanks must be revived; the locations in the computer that record the number of rounds expended by moving tanks must be set back to zero; etc. This group of orders assembles the line-of-sight data, which apply to the combat units in their starting positions. To do this it uses the same group of orders, M21, which are used by the moving routine throughout the entire battle to assemble the same data.

## Clocks Routine

C1 This group of orders searches through the clocks of all units and identifies that clock which has the lowest time "on" it. This has the next turn and will be carried through the computer routine.

C3, 4,5 This group of orders makes a test each time a unit is chosen to move, to determine whether the first 15 min of the battle are up (C3). C4 sets up all the firing clocks, except the moving Blue units, to within

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a random amount of 3 sec of the battlefield time shown on the unit being processed. C5 then sets C2 permanently to the $B$ position, bypassing this test for the rest of the battle.
C6 This operation is a check to determine whether the $1 / 2-\mathrm{hr}$ limit on battles is up. If so the computer goes into the repeat routine. If not, the computer continues the calculations.

C7 This operation determines whether the clocks routine selected the mortar battery to fire. If it did, the computer turns next to the barrage routine. Otherwise it goes to the writing routine.

Barrage Routine
B1 These orders select a square at random from the $800-$ by $1200-\mathrm{m}$ area containing the Red units.

B2, 3 These orders check to see if there is a Red infantry unit on the square. If so their effectiveness is reduced by the appropriate factor as determined from the table of kill probabilities (Annex C1, Table C16).

B4 B4 resets the firing clock of the mortars and records the results of these computations. Control is returned to the clocks routine.

## Writing Routine

W1, 2 This group of orders establishes which clock was chosen by the clocks routine and therefore whether the unit is to be processed by the moving routine or the firing routine. It then sets up the order sequence for the computer in the manner required for processing the particular unit chosen. A complete list of all the preparations made in the writing routine will not be given at this time. A study of the firing and moving routines themselves is necessary to understand such a list. For example, if the unit has been selected as ready to fire its main armament again, then the unit's selection of a target depends partly on whether the unit knows that it has been fired at. Every time a unit is fired at during the firing routine, operation F29 decides whether the target shall be aware of that fact. If the decision is to so inform the target, then the memory is so adjusted. The way in which this is actually done is to record the fact that the writing routine will choose the "yes" exit path from operation F8 when that target itself is selected to fire.
In describing other operations in the various routines, reference is not usually made as to whether the operation makes use of the writing routine. It will generally be clear from the context whether the operation is to be done only for some specified unit when that unit is treated next, or whether the operation will henceforth affect every unit, thus being carried out immediately and permanently without involving the writing routine.

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## Firing Routine

F1, 2 In this group of orders each unit selected to fire first identifies those enemy units which have been picked up before and are still visible. This requires the use of a "line-of-sight" word ( $S_{1}$ ) with a "have-seen" word $\left(S_{2}\right)$. Combined, these give the units previously seen which are still visible. The dead tank units are then removed from this list, using the "dead" word $\mathrm{S}_{3}$. F2 then checks to see whether this calculation has produced any potential targets. The section "Target Identification" in App A describes this operation in detail.

F3-F7 If the above routine does not produce any targets then this group of orders allows the unit an opportunity to "survey" the battlefield, with a view toward picking up a target. Since this process must take time, the only thing accomplished by this group of orders is, at most, to select one or more targets which the unit will be able to take under fire after the required time delay. To do this, the memory is first checked to determine whether any unit has disclosed its position. If there are such, then F4 gives those units a 0.5 chance of being added to the "have seen or heard of " memory of the target seeker. After this, F7 computes when the unit will be given a chance to fire again, using a constant time delay ( 30 sec ) plus a small random number of seconds, totaling between 30 and 62 sec . If no units have disclosed their positions, then F3 goes to F5, where it is determined whether there is a line of sight from the unit to any enemy units. If there is not, then F7 is used as before to reset the firing clock. If there is a line of sight to one or more enemy units, then $F 6$ gives the target seeker a 0.5 chance of seeing each of them, and those seen are added to the unit's "have seen or heard of" memory. Finally the code proceeds to F7, where the fire clock is reset as described above.
F8-F17 If the group of orders F1, F2 does produce some possible targets from previous knowledge, then this group of orders is used to add to the list certain special targets of high priority and to make a choice among all targets so as to take one of them under fire. The routine actually adds to the list and implements a priority system at the same time. Therefore the most compact way in which to describe the effect of this group of orders is to give the priority system applied. It is as follows:

Priority 1. Return fire of visible tank known to be firing at target seeker. The knowledge of being fired at comes from F21 (computed in F10, F11, and F12).
2. Continue firing at last tank target (F14 and F15).
3. Make a random choice on all visible tanks (F17).
4. Return fire of infantry unit (F11, F12, and F13).
5. Continue firing at last infantry unit (F14, F15, and F16).
6. Random choice of visible infantry units (F17).

F18, 19 At this point it is determined whether the target selected is a new target. If so then the F18 switches to F19. This later group of orders is used to delay firing on a new target until sufficient time has elapsed

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for the turret to be swung around and the gun laid on the new target. In the present routine the delay is taken to be a constant 8 sec . (Since the unit must be processed again by the target selection process before it is actually allowed to fire at the selected target, there is a chance that a target of higher priority will interrupt before fire is actually delivered.) F19 terminates the firing calculations and control is returned to clocks.

In F20 the firing routine is set up for the actual firing, in the event the selected target does not require any delay for switching the turret and laying on the target. The correct kill probability is selected from tables held in the memory. To do so the computer refers to all the information describing the shooter, target, range, whether moving or stationary, influence of partial cover, etc. All the other information required has been stored during the computations and must be inspected. For example, M25 may have decided that the shooter is stationary and the target is moving, which will affect the kill probability.
F21 F21 calculates when the unit will be able to fire the next time, although the firing clock will not actually be reset till the end of firing routine, in F33. The reset time is, of course, a function of what type of unit (tank or infantry) is firing, and includes a small random number, distributed uniformly between certain limits. For a medium tank the total will be between 11 and 19 sec with an average of 15 sec . This group of orders also keeps track of the ammunition expenditure of the moving tanks, and, if the shot being processed is the last round of the tank, it is prevented from firing ever again, by setting its firing clock up to a very large value that can never be reached during the battle. In any event the firing routine continues to process the present shot.

F22-F25 This group of orders determines whether there has been a kill when the target is a tank (F25) and reduces the effectiveness of the target when the target is an infantry unit (F23). F24 takes into account the reduced kill probability required when an understrength infantry unit is the shooter. To compute this, the kill probability from the tables is multiplied by the fractional effectiveness of the unit.

F26 F26 records the results when a tank is killed and checks to see whether the battle is over by reason of all the tanks on one side being dead. In that event the computer proceeds to the repeat routine.

F27, 28 This group of orders carries out two operations: First, F27 takes note of the first tank casualty (either side) and, by changing M8, thereafter the moving routine is caused to check whether there are any dead tanks in any of the squares to which a unit is considering moving. Second, F28 sets up the Red moving clocks by a small random amount each time a Red unit is killed. Since the Red move clocks are originally set to a very large time, they will never be selected to move until this setting-up operation has been repeated enough times to lower


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the clock value on one of the Red units to a point where the clocks routine selects it for moving. As soon as one Red unit has been selected to move, M3 changes F27 so that the Red move clocks are no longer set up for each Red casualty.

F29-33 Every unit allowed to fire goes through this last group of orders. F29 decides whether the target will become aware that it is being shot at. This is done even though the target has been killed, since, if the choice is to let the target know it is being fired at, F30 is used to include the shooter in the "disclosed-position" memory. The effect of this is to give the shooter the same chance of disclosing its position for the battlefield as a whole as it has of disclosing its position to the target it is shooting at. The probabilities used in this routine are given in Table C1. (See Annex A3 for the way in which these probabilities are combined with the other logical operations.) In the course of making known the fact that a target is under fire, F30 sets M5 to "yes," F10 to "yes," and adds the shooter to the list of units the target has seen or heard of. These three orders have a practical effect only if the target was not killed. From this point the computer proceeds to F33 where the firing clock for the shooter is reset using the time calculated by the order group F20 already described. In the event that F30 decides not to let anyone know that

Table Cl
PROBABILITY OF DISCLOSING POSITION
TO TARGET AND ALSO BEING PUT IN DISCLOSED-POSITION MEMORY

| Probability | Range intervals, m |  |
| :---: | :---: | :---: |
|  | Inf target | Tank target |
| 1.00 | $0-100$ | $0-500$ |
|  | $100-200$ | $500-1000$ |
| 0.25 | $200-1500$ | $1000-1500$ |
| 0.125 | 1500 and over | 1500 and over |

the shooter was firing at the target, then the computer proceeds to F31, where, if this is the second or later shot fired from the same position, the shooter is definitely added to the list of units that have disclosed their position. Finally the computer goes to the last operation as before (F33) and resets the firing clock.

## Moving Routine

There are only two possible outcomes of this computation; either the unit changes its position (exit from order group 28) or it does not (exit from order group 19).

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M1, 2, 3 This group of orders stops F28 from effecting further reductions in the Red moving clocks as soon as any Red tank is selected to move. Since all the Red move clocks are set down, on the average, about the same amount, as soon as any one Red move clock is low enough to be selected by clocks, the others will not be far behind. M3 causes the use of this group of orders to be canceled for the remainder of the battle.

M4, 5, 6 This group of orders checks whether the "emergency move" is called for. This move is used by a combat unit in the special case when (a) it has just moved from cover and (b) it knows that it has been fired at; then it automatically chooses to return to the covered position. M6 causes this return move to consume the same additional time as had elapsed since the move from cover. If the emergency move is called for, M6 bypasses M7 through M20.
M7, 9, This group carries out the move calculations proper; i.e., the move 15,16 probabilities are determined. M7 assembles the precalculated ratings of each adjacent square as they depend on the terrain features of that square alone. The ratings used in the present battle are given in

Table C2
VALUES USED TO WEIGHT SQUARE ON
BASIS OF ITS TERRAIN FEATURES

| Weighting values | Concealment | Special terrain features | Weighting values | Concealment | Special terrain features |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | Normal |  | Crest of Mill (continued) |  |  |
| +15 | Three-quarter |  | - 5 | Full | Forest |
| +13 | Half |  | +15 | Half | Edge of forest |
| +11 | Quarter |  | + +5 | Zero | Trail |
| $+9$ | Zero |  |  |  |  |
| - 5 | Full | Forest | Steep Hill |  |  |
| +13 | Half | Edge of forest |  |  |  |
| $+9$ | Zero | Edge of swamp | $+10$ | Three-quarter |  |
| -15 | Zero | Swamp | $+8$ | Half |  |
| + 15 | Zero | Trail | + 6 | Quarter |  |
|  |  |  | $+4$ | Zero |  |
| Crest of Hill |  |  | -10 | Full | Forest |
| +15 | Three-quarter |  | +8 | Half | Edge of forest |
| +15 | Half |  | + 4 | Zero | Edge of swamp |
| +15 | Quarter |  | +10 | Zero | Trail |
| -10 | Zero |  | -13 | 1 or more tank | alties on square |

Table C2. M9 modifies the local terrain ratings, computed in M7, according to whether there are any tank casualties in the vicinity. In the present battle a square containing one or more tank casualties is penalized 13 points. No test is made for the color of the tank casualty in this battle since, during most of the fighting, tanks of opposite colors do not occupy the same general area.

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The last probability factor used in computing the move depends on the direction of the terrain objective. Each adjacent square is rated on this basis, using the scheme describedin Fig. C3 and under "Terrain Objective." Application of this system results in a high rating for the square in the direction of the terrain objective, a large negative rating for the square directly opposite to the terrain objective, and intermediate ratings for the remainder of the adjacent squares. M15 then combines the three separate ratings for each square, resulting in a single over-all rating for each adjacent square. Interpreted as probabilities, M16 then uses these ratings to make a "Monte Carlo" choice among the nine possibilities.

M11 M11 keeps the Blue assaulting tanks and infantry from firing until at least one of their number has penetrated east into the north-south band composed of all squares whose $x$ coordinate is 17 . In the event this occurs before 15 (battlefield) min have elapsed then none of the combat units are yet firing, in which case M11 will start all units firing.

M12 M12 causes the computer to skip the calculation in M11 once a combat unit has crossed the no-fire-line.

M17 M17 checks on whether a given combat unit has approached its current terrain objective closely enough to require substituting the next terrain objective for future calculations. In the battle computed here the actual location of the first terrain objective for each assaulting combat unit is several thousand yd east of the Red position. However, the terrain objective is changed several thousand yd north of the Red position as soon as each Blue unit reaches the neighborhood of the actual Red position. This was done for mathematical reasons. The effect of these calculations is the same as if the terrain objectives were actually in the Red positions. The system is described in detail in the section "Terrain Objective." The mortar fire is stopped as soon as any one combat unit approaches the first terrain objective so closely as to require a change in the axis of advance. Also the infantry squads are caused to dismount from their armored carriers at this point. The practical effect of this change is to substitute an altered set of "kill probabilities." There is no change in the moving calculations.

M18, 19 Here the computer checks on whether the preceding calculations resulted in a change of position. If there was no change in position, then, since no adjustments of the memory are necessary, the moving calculations terminate with M19 and control is returned to clocks.

M20 Using the terrain of the selected square, the time delay required for the necessary movement is computed using the data in Table C3, and includes an additional time, which varies between 0 and 32 sec . Included is a correction for the extra distance involved when the combat unit has selected one of the "corner" squares (Fig. C2).

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M21 As a result of each move, all the line-of-sight information previously assembled into the line-of-sight memory word relating to the moving unit and each enemy unit must be corrected for the new position.
Since line-of-sight calculations were made for every necessary pair of squares before the battle commenced, this operation involves only

Table C3
ASSUMED SPEEDS OF MOVEMENT OF ALL UNITS

| Tank speed, mph |  |  | Terrain | Concealment |
| :---: | :---: | :---: | :---: | :---: |
| Medium (and all other units) | Light | Heavy |  |  |
| 2 | 4 | 1.0 | No hill | Three-quarter |
| 3 | 6 | 1.5 |  | Half |
| 4 | 8 | 2.0 |  | Quarter |
| 5 | 10 | 2.5 |  | Zero |
| 1 | 2 | 0.5 | Forest | Full |
| 3 | 6 | 1.5 | Edge of forest | Half |
| 5 | 10 | 2.5 | Edge of swamp | Zero |
| 0 | 0 | 0 | Swamp | Zero |
| 15 | 20 | 5.0 | Road | Zero |
| 1.0 | 2 | 0.5 | Steep hill or | Three-quarter |
| 1.5 | 3 | 0.8 | crest of hill | Half |
| 2.0 | 4 | 1.0 |  | Quarter |
| 2.5 | 5 | 1.3 |  | Zero |
| 0.5 | I | 0.3 | Forest | Full |
| 1.5 | 3 | 0.8 | Edge of forest | Half |
| 6.0 | 12 | 3.0 | Road | Zero |

sorting out from the memory the results of the line-of-sight precalculation for the particular squares occupied by the combat units involved. Once sorted out, these data are combined into a single large "number," which is used in the firing calculations until another move occurs.

"Corner" Squore

Fig. C2-Movement to "Corner" Square is Corrected for the Longer Distance Involved as Compared to Movement Along B

M22-28 This group adjusts the "disclosed-position" information about the moving combat unit according to the nature of the change in concealment resulting from the move. Also decided is whether the unit shall be considered stationary or moving if it becomes involved in a shooting action. Finally the move clock is reset with the value previously computed in M20 and control is returned from M28 to clocks.

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## Repeat Battle

R1 This group of orders causes the final data to be printed out. This is actually the total number of Red tanks killed and the total number of Blue tanks killed.

R2 These orders check on whether the computer is to compute another battle. If so then the computer stores the last random number (which is used by the computer to start off the next battle) and control is returned to start where another battle commences.

R3 If the computer is not to compute another battle, R3 causes the computer to type out the last random number and stop operation.

## CONTENTS OF MEMORY

The content and nature of the computer battle is only partly indicated by a flow diagram showing the order in which the important operations are performed. Appendix B gives a discussion of the importance of the information stored in a computer's memory when logical calculations are involved. Therefore as a necessary companion to the detailed flow diagram discussed in the previous section, the general nature of the computer's memory contents is described.

Of the 16,384 numbers (or "words") in the computer's memory, roughly 7000 are required to direct the operations of the computer. The remaining 9000 numbers are used mainly for the storage of information. Appendix A discussed the special (binary) number system used by the computer for these purposes.

Table C4 gives the principal data stored in the memory of the computer.

## FIRING PERFORMANCE CHARACTERISTICS

## Kill Probabilities

All the kill probabilities used in the trial calculations are given in the eight tables in Annex C1. They are not directly supported by experimental data, since, at the time of calculations there were no data taken under the necessary field conditions. Since that time Project STALK (BRL and OCAFF) results have become available and seem to meet some of the requirements for such data. In the absence of valid experimental data, the various kill probabilities were estimated, based on unpublished theoretical calculations made available by V.V. McRae and M. Grabau of ARMOR Group (ORO). For purposes of these feasibility calculations it is not necessary that the assumed kill probabilities have high accuracy. However, before such computer battles can be put to their most efficient use, improved data related to the proper field conditions will be required.

The entries in the tables are the kill probabilities expressed as a fraction, with the number indicating the number of sixteenths that most closely approximate the true kill probability. This is the form in which the computer most easily handles the data. A higher accuracy was not deemed necessary in these calculations, but can be imposed when required.

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Table C4
PRINCIPAL DATA STORED IN COMPUTER MEMORY

| Memory locations required | Data | Remarks |
| :---: | :---: | :---: |
| 467 | Kill probabilities | Contents Table C9 to Cl6; six kill probabilities are stored in each word |
| 2304 | Precalculated line of sight | Results in terrain precalculation and each of the 24 bits in a word is associated with one pair of squares; it is a 1 if there is a line of sight between that pair, a 0 otherwise |
| 576 | Local terrain weights | One word is used for the rating each square receives by virtue of the local terrain features; values used are given in Table C2 |
| 48 | Terrain objectives | The assaulting combat units can be provided with nine successive and distinct terrain objectives; data stored are the coordinates of the grid square for each terrain objective; also included are the alternative firing positions provided for the defending Red forces |
| 2 | Dead word | One memory location is used by each of the two forces to keep a record of which tanks have become casualties; the bit in each of the 24 positions is a 1 or a 0 according to whether the associated combat unit is yet a casualty; since the Blue forces have only 20 combat units, 4 of the positions in the Blue dead word are not used |
| 44 | Have-seen-or-heard-of word | Each combat unit uses one such word to keep a record of which enemy units it has seen or heard of; "heard" implies it was advised by the communication system of the whereabouts of the enemy unit; the bit in each of the 24 positions in the number is a 1 if the assoc iated enemy combat unit has been detected, a 0 otherwise |
| 44 | Line-of-sight word | Each combat unit uses one such word to keep current the record of all enemy units on a terrain square that can be seen by the unit; the bit in each of the 24 positions is a 1 if the associated enemy combat unit can be seen, a 0 otherwise |
| 2 | Disclosedposition word | Each of the two forces uses one of these words; the bit in each of the 24 positions is a 1 if the associated combat unit has been deemed to have disclosed its position, either by moving across an open space or firing repeatedly; it is a 0 otherwise |
| 18 | Mobility | These memory units contain the constant time delay to be used in setting up the move clock for motion across a particular terrain type of square; it does not include the correction necessary when motion is along a diagonal |
| 89 | Clocks | Each combat unit uses one of these words to retain the future time when it will be able to fire again and a second word serving the same purpose for future moves; there is also a firing clock for the mortars |
| 44 | Position | Gives the $x$ and $y$ coordinate of the current position of the combat unit |
| 132 | Firing counter | Each combat unit uses one of these to keep a running total of the total number of shots fired; another for the total shots fired from the same position; and a third for the total number of rounds fired at the same target |
| 12 | Infantry strength | One of these words is used for each of the 12 infantry squads in the battle to keep a record of its current strength |
|  | Miscellaneous constants | In general, one word is used to retain the value used for each of the many constants: e.g., the $15-\mathrm{min}$ delay before firing; the nofire line of $x=17$; the time limit of the battle ( 1800 sec ); the maximum value of the $x$ and $y$ coordinates (24); the time delay required when a tank must swing its turret to a new target |

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The lowered kill probabilities applied to the first round, when the target is partly concealed, are meant to account for a lack of exact knowledge of the position of the target tank, which is sometimes the case when a target tank has fired from a concealed position.

The kill probabilities calculated for the light tank are patterned after the M41 light tank; those for the heavy tank after the T43 heavy tank. The infantry capability against tanks assumes the use of rocket launchers fired at approximately $30-\mathrm{sec}$ intervals at ranges of not over 100 m .

The kill probabilities of infantry against infantry and tanks against infantry are assumed to take account of small-arms fire, mainly machine-gun, lumping together these effects at roughly $30-\mathrm{sec}$ intervals. These kill probabilities are also cut off at 100 yd in these calculations. The kill probabilities against infantry units are not actually used in a probability sense. Instead the effectiveness of the infantry unit is reduced by a factor equal to the kill probability. If by repeated hits an infantry unit is reduced in effectiveness below one sixteenth, it is taken as totally destroyed.

It was further assumed for purposes of this trial calculation that the infantry mounted in carriers would be assumed to move with great caution, and that hence they would not present a target to enemy antiarmor weapons over 100 yd distant; i.e., the enemy AT weapons were given a 0 kill probability whenever they were separated from the Armd Pers Carr by one or more intervening squares. A different set of kill probabilities is used according to whether the Blue infantry are inside or outside their carrier. In both cases, however, the infantry effectiveness is degraded by the kill probability factor, rather than totally destroyed.

Since a moving tank was not assumed to have a significant capability against other tanks, no entries are made in the tables for this circumstance. However, the machine guns on a moving tank are assumed to have an anti-infantry capability.

To a first approximation it was assumed that the larger size of a T43 canceled out the advantage of the somewhat heavier armor compared to the M48. Hence the kill probabilities of the enemy armor against the T43 and M48 were taken to be the same.

Each of the tanks were assumed to be firing the best ammunition known to be available to it, either HVAP or APC.

## Rates of Fire

The time delays to be associated with firing are of three kinds: (a) delays while turret is traversed and gun laid on new target-8 sec, (b) delays before a combat unit is given a new opportunity to acquire a target whenever the firing calculations are terminated owing to the lack of a target-average 46 sec with a probability uniformly distributed between 30 and 62 sec , and (c) delays for all combat units before firing a second or later shot against the same target (given in Table C5).

## Ammunition Supply

The infantry units, mortar battery, and stationary tanks in the battle are permitted an inexhaustible ammunition supply. The last are the SU- 100 Red tanks and the seven overwatching Blue tanks in each action. The moving tanks

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are limited to their base load of ammunition. Once this ammunition is expended, the moving tanks can no longer fire during the battle. Table C6 gives the assumed quantities for these base loads.

Table C5
RATES OF FIRE, ALL COMBAT UNITS

| Combat unit | Time delay, sec |  |  | Average rounds <br> per minute |
| :--- | ---: | :---: | :---: | :---: |
|  | Minimum | Average | Maximum |  |
| Blue mortar battery | 0 | 32 | 64 | 2 (ank |
| Blue medium tand | 11 | 15 | 19 | 4 |
| Blue light tank | 6 | 10 | 14 | 6 |
| Blue heavy tank | 26 | 30 | 34 | 2 |
| T-34 | 11 | 15 | 19 | 4 |
| SU-100 | 11 | 15 | 19 | 4 |

Table C6
BASE LOAD OF AMMUNITION FOR MOVING TANKS

| Tank | Rounds |
| :--- | :---: |
| Blue medium tank | 60 |
| Blue light tank | 60 |
| Blue heavy tank | 30 |
| T-34 | 60 |

## No-Fire-Line and Time

The limitations on firing effected by the no-fire-line of $x=17$ and the time limit of 15 min before any of the units (except the mortars) are allowed to fire are not strictly meaningful in the military sense. These restrictions were imposed mainly to reduce the time of calculation so as to permit the calculation of a larger number of battles within the budgetary limits. They should not be interpreted as performance characteristics.

## MOVING PERFORMANCE CHARACTERISTICS

In the following subsections are listed all the numerical values which were assumed for the performance characteristics of the combat units relating to movement. The breakdown of the contour map into squares and the list of terrain features to be associated with these squares are given in Figs. 8 and 9. The mathematical treatment of these parameters is illustrated by a sample move calculation in Annex C2.

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## Mobility Factors

Table C3 gives the speeds of movement that the various combat units were assumed to have, according to the character of the terrain over which they are moving at the time. These numbers permit the computer to calculate the number of seconds the move clocks must be advanced to allow time for the combat unit to move to its new position. The computer selects the speed corresponding to the terrain features of its new position. The longer distance to be associated with a move diagonally across the grid lines is taken into account by multiplying the time delay computed for an east-west or north-south move by the factor sec $45 \mathrm{deg}=\sqrt{2}=1.41$.

To the time delay computed from the mobility alone is added a variable time, varying uniformly between 0 and 32 sec with an average value of 16 sec . This variation may be adjusted to account for variations among tank crews.

## Firing by Assaulting Tanks

After the assaulting Blue tanks have passed the no-fire-line they begin firing. Thereafter a correction factor must be applied to the time of arrival at the next position computed from the mobility factors just discussed. This correction factor allows for the time lost by the assaulting tanks when they stop to fire. To take account of this, 64 sec is added to the computed time delay. This is approximately sufficient for the tank to stop and fire 2 rounds in the course of its movement to the next position.

## Weight Factors for Computing Move Probabilities

In the present battle all moving tanks are assumed to use the same system of weights for computing the move probabilities to be associated with each of the neighboring squares and the current position of the tank. Table C2 gives the weights used for taking account of the terrain features and also the presence of a tank casualty.

## Terrain Objectives

The weight assigned to a square to account for the direction toward the terrain objective is a variable number and is computed by the method shown in Fig. C3. For purposes of comparison with the values given in Table C2, Fig. C4 shows the values of the ter rain-objective weight factor on each of the eight neighboring squares for two extreme positions of a Blue assaulting tank.

All assaulting Blue tanks are given the same theoretical terrain objectives. Initially it is at 23,09 . As it approaches to within 400 m in the eastwest direction and, simultaneously, to within 400 m in the north-south direction, each Blue tank has its theoretical terrain objective changed. This criterion is equivalent, except in special situations, to designating the military objective of the assaulting Blue tanks as a north-south line of grid squares 800 m long, starting at 18,05 and extending south to 18,13 .

The second theoretical terrain objective given to the assaulting Blue tanks is at 20,00 . This grid square does not actually exist on the map. However, the same $400-\mathrm{m}$ criterion already mentioned is used again. Thus the north-bound assaulting Blue tanks need only reach the east-west line of grid

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Fig. C3-Fquations for Calculating the Weighting
Values for Each Adjacent Square

$$
\text { Where } \begin{aligned}
a & =\left|Y_{0}-Y\right| \\
b & =\left|X_{0}-X\right|
\end{aligned}
$$



Fig. C4-Representative Weights Computed to Account for Direction of Terrain Objective Using Method in Annex C2
a, Approximate maximum values for weight factor: Blue tank in initial position, coordinates $(05,18)$; $b$, approximate minimum values for weight factor: Blue tank, at coordinates $(18,12)$ is one square away from military objective

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squares running from $(16,04)$ to $(24,04)$ to have their terrain objectives changed once more.

In the present battle, the battle is usually halted before any combat units have approached the second objective. However, the code provides for a sequence of nine distinct terrain objectives. For purposes of this code the above two objectives are simply repeated, alternately.

Table C7
POSITIONS FOR RED MOVING TANKS and INF ANTRY
A. First Alternate Position for Particular Red Moving Tanks and Infantry

| Combat unit | Alternate position <br> coordinates |  |
| :---: | :---: | :---: |
| T-34 no. | 33 | 20,08 |
|  | 34 | 19,04 |
|  | 35 | 22,04 |
|  | 36 | 20,12 |
|  | 37 | 20,09 |
|  | 38 | 19,05 |
|  | 39 | 20,10 |
|  | 40 | 18,11 |
|  | 41 | 19,08 |
|  | 42 | 20,12 |
| Infantry | 48 | 23,03 |
| squad no. | 49 | 19,08 |
|  | 50 | 21,04 |
|  | 51 | 22,06 |
|  | 52 | 21,10 |
|  | 53 | 20,12 |
|  | 54 | 19,05 |
|  | 55 | 22,04 |
|  | 56 | 22,04 |

The primary alternate positions to which the moving Red combat units are caused to move when they begin taking heavy casualties are treated as terrain objectives. Table C7 gives the coordinates of these primary alternate positions.

As each Red moving combat unit reaches its primary alternate position it is assigned additional alternate positions as listed in Table C7. In the present battle the calculations rarely reached a stage where these additional alternate positions came into use.

## OTHER RESTRICTIONS AND PERFORMANCE CHARACTERISTICS

## Length of Battle

The restriction on the length of the battle ( 30 min ) was, in part, arbitrary. It was selected partly on the basis of budgetary considerations. The battle

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itself was designed to cause heavy (approximately 50 percent) casualties to be suffered within the time limit so as to show clearly variations in the effectiveness of the combat units.

For 7 of the 21 heavy tank battles the battle was permitted to continue for an additional $4^{1 / 3}$ min to compensate for the slowness with which the heavy tanks approached the enemy position.

For the last 14 of the heavy tank battles, the length of the battle was again restricted to 1800 sec . To speed up the computations the heavy tanks were permitted to move more rapidly during the approach before the shooting commenced. During such a period, when there is no firing, the scale of time is arbitrary and speeding up the movement does not affect the results.

## Communications

The influence of the communications system on the progress of the battle is included in an approximate fashion. This results from two calculations, both of which involve the sharing of information concerning the existence and position of enemy units among several friendly combat units. These two calculations are (a) the operations F4, F6, and F9 (Fig. C1), which give a combat unit a 0.5 chance to become aware of the existence of an enemy tank that has disclosed its position by an overt action of movement (across an open field) or firing; and (b) the operation M5 (Fig. C1), where the choice of movement depends on whether a combat unit is aware it is being fired at. M5, in turn, depends in part on F32. Each of these operations assumes that the combat unit can become aware of the existence of an enemy combat unit either by observing that unit or by receiving a communication from another (friendly) combat unit. Thus the correct probability required for these calculations must include the effectiveness of the communication system.

## Enemy_Area

Throughout the battle the Red elements are restricted to an area 1200 by 800 m in extent. The squares in this area are on the interior of the rectangular area at $17,02,24,02,17,13$, and 24,13 . This restriction has the effect of reducing drastically the number of pairs of squares for which the line-of-sight calculation had to be made and the quantity of results to be stored in the computer. It can be relaxed when the more capacious computers now available are used.

## Initial Clock Settings

At the start of the battle all move clocks of the assaulting Blue tanks are set to 0 , and a time to the nearest $1 / 64 \mathrm{sec}$ selected at random from the interval of 0 to 3 sec is added into each move clock. This serves to give every combat unit an equal chance at the start of each battle of being selected first, second, third, etc., for its first move. The time is computed to such an accuracy, not because it is known to have a real significance, but to establish an order of moving with a negligible chance for ties to develop.

Initially all firing clocks (except the mortar) are set at a very large value to prevent firing. As soon as the operation C3 or M11 (Fig. C1) determines that (some) firing should commence, the firing clocks of all appropriate com-

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bat units are first set at the then current battlefield time (i.e., the same time as the move clock of the combat unit then being processed), and then to each is added a (different) time selected at random from the interval 0 to 3 sec . This serves to give every eligible firing unit an equal chance of being selected first, second, third, etc., to begin firing activities.

At the start of the battle all the move clocks of the Red moving tanks (T-34) are set to a very high value. Thus, unless these clocks are reduced, the T-34's will not be selected to commence movement toward their alternate positions. However, each time a Red tank becomes a casualty, F27 reduces every Red move clock by variable times selected at random from the interval 0 to 4 min . Thus as the battle proceeds and additional Red tanks become

Table Cy
INITIAL POSITION OF ALL COMBAT UNITS

| Forces | Moving vehicles |  |  | Stationary vehicles |  |  | Infantry squads |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Unit | Code | Coord | Unit | Code | Coord | Code | Coord |
| Blue | Tank | 01 | 02,18 | Tank | 11 | 02,05 | 26 | 06,23 |
|  |  | 02 | 03,18 |  | 12 | 02,04 | 27 | 04,18 |
|  |  | 03 | 05,18 |  | 13 | 03,03 | 28 | 05,18 |
|  |  | 04 | 04,18 |  | 14 | 05,07 |  |  |
|  |  | 05 | 04,20 |  | 15 | 06,23 |  |  |
|  |  | 06 | 05,18 |  | 16 | 01,11 |  |  |
|  |  | 07 | 04,18 |  | 17 | 02,04 |  |  |
|  |  | 08 | 06,23 |  |  |  |  |  |
|  |  | 09 | 07,20 |  |  |  |  |  |
|  |  | 10 | 07,23 |  |  |  |  |  |
|  |  | 33 | 17,05 | SU-100 | 43 | 20,07 | 48 | 20,08 |
|  |  | 34 | 17,04 |  | 44 | 21,04 | 49 | 23,03 |
|  |  | 35 | 20,07 |  | 45 | 20,08 | 50 | 23,07 |
|  |  | 36 | 20,07 |  | 46 | 18,10 | 51 | 20,07 |
|  |  | 37 | 23,04 |  | 47 | 20,09 | 52 | 21,04 |
|  |  | 38 | 23,07 |  |  |  | 53 | 20,05 |
|  |  | 39 | 23,06 |  |  |  | 54 | 23,06 |
|  |  | 40 | $2 l, 04$ |  |  |  | 55 | 20,09 |
|  |  | 41 | 20,05 |  |  |  | 56 | 18,10 |
|  | 42 | 23,03 |  |  |  |  |  |  |

casualties the Red move clocks are reduced to progressively lower (but different) times. Eventually one of these move clocks reaches a value so low that it is selected to move. At that point this reduction calculation is suspended. Thereafter the Red tanks are selected by the clocks operation for movement on the same basis as are the assaulting Blue tanks. Since the Red move clocks were reduced by variable amounts, they will not all commence moving at the same time.

## Initial Position of Combat Elements

Table C8 gives the coordinates of the initial positions of all the combat units in the battle. (See Fig. D1 in App D). The mortar battery has no specified position.

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## FORMAT FOR RESULTS

Only a limited quantity of results of the battle can be typed out by the ERA 1101 computer during battle computations without seriously delaying the computations. However, there will be a continuing requirement that the detailed progress of selected battles be available for study. To meet this requirement for each battle the computer is caused to record the progress and results of each battle in two distinct forms: a short form and a long form.

## Short Form (Casualty Data)

The computer types out directly on a single line the major items of interest relating to each casualty at the time of its occurrence. The following data are indicated:
(a) The letter $\underline{r}$ or $\underline{b}$ depending on which side suffered the casualty.
(b) The code number of the combat unit that became a casualty ( 2 digits).
(c) The time, to the nearest sec, when the casualty occurred ( 3 to 4 digits).
(d) The position coordinates of the combat unit that became a casualty ( 3 to 4 digits).
(e) The letter $\underline{r}$ or $\underline{b}$ and the code number of the combat unit causing the casualty ( 1 letter and 2 digits).
(f) The position coordinates of the shooter ( 3 to 4 digits).
(g) If the casualty were a tank, the number of consecutive rounds fired to produce the casualty ( 1 or 2 digits) or if the casualty were an infantry unit, the numerator of a fraction with a denominator of 16 that gives the remaining effectiveness of the combat unit ( 1 or 2 digits).
(h) The letter and code number of any tank that runs out of ammunition before the next casualty is recorded ( 1 letter and 2 digits). For example, when the computer types out the following two lines:

| r37 | 247 | 20,7 | b11 | 11,5 | 3 | r39 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| b27 | 253 | 22,4 | r41 | 22,3 | 8 |  |

then the occurrence of two casualties is indicated with one additional tank running out of ammunition. Specifically the first line indicates that red tank, code number 37, was killed at $\underline{247} \mathrm{sec}$, when at position $x=\underline{20}, y=\underline{7}$, by blue tank, code number 11 , at position $x=11, y=\underline{5}$, by the third consecutive round. The last element of this first line, $\underline{\underline{r}} \mathbf{3 9}$, indicates that red tank, code number $\mathbf{3 9}$ ran out of ammunition at some time during the $6-\mathrm{sec}$ interval before the occurrence of the casualty indicated by the second line. The second line indicates the blue infantry squad, code number 27 , was hit 253 sec after the battle started, when at the position $x=\underline{22}, y=\underline{4}$, by red tank, code number 41 , while at position $x=22, y=\underline{3}$; and that the infantry unit was reduced in effectiveness to eightsixteenths of its full strength by this hit The absence of any entry in the last column of this line indicates that no tank ran out of ammunition before the time of the next casualty.

## Long Form (Moving or Firing Actions)

During the calculation of the battle the computer causes a detailed record of each combat action to be recorded in the form of a specially coded punched paper tape. This tape is many feet long for a complete battle. In order to

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make the data stored on this tape intelligible, it must be run back through the computer. The computer then deciphers the pattern of holes punched in the tape and causes the associated typewriter to type out one line of data for each separate moving or firing calculation effected during the battle. With present equipment this operation is very time consuming, taking about three times as long as the original battle calculation; i.e., about 1 hr .*

The typewritten record of the combat actions computations is arranged to provide one line for each action. The five different possible actions are:
(a) unit moved to new position,
(b) unit considered movement but rejected it and remained in the same position,
(c) unit fired at enemy unit and missed,
(d) unit fired at enemy unit and hit, and
(e) unit had an opportunity to fire but failed to do so, either because it had no target or had to delay firing while traversing its turret.

As an example of the form used for each of these five cases, samples of each are arranged in the accompanying tabulation in the same order as in the preceding paragraph.

Column

| 1,2 | 3 | 4 | 5,6 | 7 |
| :---: | :---: | :---: | :---: | :---: |
| b09 | 0124.36 | 15,07 |  |  |
| b01 | 0126.07 | nc |  |  |
| r36 | 0139.25 |  | b09 |  |
| r36 | 0172.10 |  | b09 | Yes |
| r36 | 0205.64 |  | nsf |  |

The first line indicates that Blue tank 09 at 124.36 sec moved to square 15,07 . (The square from which it moved would be found by checking the preceding move.) The second line indicates that Blue tank 01 at 126.07 sec considered moving but decided against it (nc = no change in position). The third line indicates that Red tank 36 at 139.25 sec fired at Blue tank 09 and missed. The fourth line indicates that Red tank 36 at 172.10 sec fired at Blue tank 09 and (yes) killed it. (The short form tells which consecutive round this was.) The fifth line indicates that Red tank 36 at 205.64 sec did not accept an opportunity to fire (nsf = no shot fired.)

In the present battle each battle involves the calculation of about 1600 separate combat actions. About 1200 of these are for the trivial case when dismounted infantry are fired on at long range with machine guns having 0 kill probability. The remaining 400 significant combat operations involve movement and firing calculations where the kill probabilities are not 0 .

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## Annex C1 <br> TABULATED KILL PROBABILITIES

Table C9
RED INFANTRY KILL PROBABILITIES FOR ANY SHOT
(Probabilities to nearest sixteenth; i.e., $15=15 / 16$ )

| Target | Target movement | Range, m | Concealment of target |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | Zero | '2uarter | Half | Three-quarter | Full |
| Any Inf | Moving | 0 | 15 | 11 | 8 | 6 | 3 |
|  |  | 100 | 6 | 3 | 1 | 0 | 0 |
|  | Stationary | 0 | 5 | 2 | 1 | 0 | 0 |
|  |  | 100 | 0 | 0 | 0 | 0 | 0 |
| Med tank | Moving | 0 | 4 | 4 | 4 | 4 | 4 |
|  |  | 100 | 2 | 2 | 2 | 2 | 2 |
|  | Stationary | 0 | 8 | 8 | 4 | 4 | 4 |
|  |  | 100 | 4 | 4 | 4 | 4 | 4 |
| Light tank | Moving | 0 | 5 | 5 | 5 | 5 | 5 |
|  |  | 100 | 3 | 3 | 3 | 3 | 3 |
|  | Stationary | 0 | 9 | 9 | 5 | 5 | 5 |
|  |  | 100 | 5 | 5 | 5 | 5 | 5 |
| Heavy tank |  | Same as medium tank above |  |  |  |  |  |

Table Cl0
BLUE INFANTRY KILL PROBABILITIES FOR ANY SHOT ${ }^{\text {a }}$
(Probabilities to nearest sixteenth; i.e., $8 \equiv 8 / 16$ )

| Target | Target movement | Range, m | Conc ealment of target |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | Zero | Quarter | Half | Three-quarter | Full |
| Red Inf | Moving | 0 | 8 | 5 | 4 | 3 | 2 |
|  |  | 100 | 3 | 2 | 1 | 0 | 0 |
|  | Stationary | 0 | 5 | 2 | 1 | 0 | 0 |
|  |  | 100 | 0 | 0 | 0 | 0 | 0 |
| T-34 and SU-100 | Moving | 0 | 4 | 4 | 4 | 4 | 4 |
|  |  | 100 | 2 | 2 | 2 | 2 | 2 |
|  | Stationary | 0 | 8 | 8 | 4 | 4 | 4 |
|  |  | 100 | 4 | 4 | 4 | 4 | 4 |

${ }^{\text {aRegardless of state of motion of Blue Infantry. }}$

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Table Cll
BLUE MEDIUM TANK (T48) KILL PROBABILITIES
(Probabilities to nearest sixteenth; i.e., $2 \equiv 2 / 16$ )

| Target | Shot | State of motion |  | Range, m | Concealment of target |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Shooter | Target |  | Zero | Quarter | Half | Three-quarter | Full |
| Red Inf | 1 orn | Moving | Stationary | 0 | 2 | 1 | 1 | 1 | 1 |
|  |  |  |  | 100 | 2 | 1 | 1 | 1 | 1 |
|  | 1 or n | Stationary | Stationary | 0 | 2 | 1 | 0 | 0 | 0 |
|  |  |  |  | 100 | 2 | 1 | 0 | 0 | 0 |
|  | 1 or $n$ | Stationary | Moving | 0 | 4 | 2 | 1 | 1 | 1 |
|  | $1 \text { or } n$ | Moving | Moving | 0 | 2 | 1 | 1 | 1 | 1 |
| T-34 | 1 | Stationary | Stationary | 0-500 | 14 | 11 | 7 | 4 | 0 |
|  |  |  |  | 500-1000 | 10 | 8 | 5 | 3 | 0 |
|  |  |  |  | 1000-1500 | 6 | 5 | 3 | 2 | 0 |
|  |  |  |  | 1500-2500 | 3 | 2 | 2 | 1 | 0 |
|  | n | Stationary | Stationary | 0-500 | 14 | 14 | 14 | 14 | 0 |
|  |  |  |  | 500-1000 | 13 | 13 | 13 | 13 | 0 |
|  |  |  |  | 1000-1500 | 10 | 10 | 10 | 10 | 0 |
|  |  |  |  | 1500-2500 | 6 | 6 | 6 | 6 | 0 |
|  | 1 | Stationary | Moving | $0-500$ |  | 8 | 2 | 1 | 0 |
|  |  |  |  | $500-1000$ | $10$ | $6$ | 2 | $1$ | 0 |
|  |  |  |  | 1000-1500 | 7 | 4 | 1 | 1 | 0 |
|  |  |  |  | 1500-2500 | 3 | 2 | 1 | 0 | 0 |
|  | $n$ | Stationary | Moving | 0-500 | 14 | 14 | 14 | 14 | 0 |
|  |  |  |  | $500-1000$ | 10 | 10 | 10 | 10 | 0 |
|  |  |  |  | $1000-1500$ | 5 | 5 | 5 | 5 | 0 |
|  |  |  |  | 1500-2500 | 2 | 2 | 2 | 2 | 0 |
| SU-100 | 1 | Stationary | Stationary | 0-500 | 14 | 11 | 7 | 4 | 0 |
|  |  |  |  | 500-1000 | 8 | 6 | 4 | 2 | 0 |
|  |  |  |  | 1000-1500 | 5 | 4 | 3 | 1 | 0 |
|  |  |  |  | 1500-2500 | 2 | 2 | 1 | 1 | 0 |
|  | n | Stationary | Stationary | $0-500$ | 14 | 14 | 14 | $14$ | 0 |
|  |  |  |  | $500-1000$ | 11 | 11 | 11 | $11$ | 0 |
|  |  |  |  | 1000-1500 | 8 | 8 | 8 | 8 | 0 |
|  |  |  |  | 1500-2500 | 5 | 5 | 5 | 5 | 0 |

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Table Cl2
BLUE LIGHT TANK KILL PROBABILITIES
(Probabilities to nearest sixteenth; i.e., $13 \equiv 13 / 16$ )

| Target | Shot | Motion of target | Range, m | Concealment of target |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  | Zero | Quarter | Half | Three-quarter |
| Red Inf | Same as Blue medium tank against Red Inf, Table C11 |  |  |  |  |  |  |
| T-34 | 1 | Stationary | 0-500 | 13 | 10 | 6 | 3 |
|  |  |  | 500-1000 | 10 | 8 | 5 | 3 |
|  |  |  | 1000-1500 | 5 | 4 | 2 | 1 |
|  |  |  | 1500-2500 | 2 | 2 | 1 | 1 |
|  | n | Stationary | 0-500 | 14 | 14 | 14 | 14 |
|  |  |  | 500-1000 | 12 | 12 | 12 | 12 |
|  |  |  | 1000-1500 | 9 | 9 | 9 | 9 |
|  |  |  | 1500-2500 | 5 | 5 | 5 | 5 |
|  | 1 | Moving | $0-500$ | 12 | 9 | 5 | 2 |
|  |  |  | $500-1000$ | 8 | 6 | 4 | 2 |
|  |  |  | 1000-1500 | 2 | 1 | 1 | 0 |
|  |  |  | 1500-2500 | 1 | 1 | 0 | 0 |
|  | n | Moving | 0-500 | 14 | 14 | 14 | 14 |
|  |  |  | 500-1000 | 9 | 9 | 9 | 9 |
|  |  |  | 1000-1500 | 4 | 4 | 4 | 4 |
|  |  |  | 1500-2500 | 1 | 1 | 1 | 1 |
| SU-100 | 1 | Stationary | 0-500 | 13 | 10 | 6 | 3 |
|  |  |  | $500-1000$ | 8 | 6 | 4 | 2 |
|  |  |  | $1000-1500$ | 4 | 3 | 1 | 1 |
|  |  |  | 1500-2500 | 2 | 1 | 1 | 0 |
|  | n | Stationary | 0-500 | 14 | 14 | 14 | 14 |
|  |  |  | 500-1000 | 12 | 12 | 12 | 12 |
|  |  |  | 1000-1500 | 8 | 8 | 8 | 8 |
|  |  |  | 1500-2500 | 4 | 4 | 4 | 4 |

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Table Cl3
BLUE HEAVY TANK KILL PROBABILITIES
(Probabilities to nearest sixteenth; i.e., $14 \equiv 14 / 16$ )

| Target | Shot | Motion of target | Range, m | Concealment of target |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  | 7.ero | Quarter | Half | Three-quarter |
| Red Inf | Same as Blue medium tank against Red Inf, Table C11 |  |  |  |  |  |  |
| T-34 | 1 | Stationary | 0-500 | 14 | 11 | 8 | 5 |
|  |  |  | 500-1000 | 10 | 8 | 5 | 3 |
|  |  |  | 1000-1500 | 7 | 6 | 4 | 3 |
|  |  |  | 1500-2500 | 5 | 4 | 3 | 2 |
|  | n | Stationary | 0-500 | 14 | 14 | 14 | 14 |
|  |  |  | 500-1000 | 13 | 13 | 13 | 13 |
|  |  |  | 1000-1500 | 12 | 12 | 12 | 12 |
|  |  |  | 1500-2500 | 8 | 8 | 8 | 8 |
|  | 1 | Moving | 0-500 | 13 | 10 | 7 | 3 |
|  |  |  | 500-1000 | 8 | 6 | 4 | 2 |
|  |  |  | 1000-1500 | 4 | 3 | 2 | 1 |
|  |  |  | 1500-2500 | 2 | 2 | 1 | 0 |
|  | n | Moving | 0-500 | 14 | 14 | 14 | 14 |
|  |  |  | 500-1000 | 10 | 10 | 10 | 10 |
|  |  |  | 1000-1500 | 7 | 7 | 7 | 7 |
|  |  |  | 1500-2500 | 4 | 4 | 4 | 4 |
| SU-100 | 1 | Stationary | 0-500 | 14 | 11 | 8 | 5 |
|  |  |  | 500-1000 | 10 | 8 | 5 | 3 |
|  |  |  | 1000-1500 | 7 | 6 | 4 | 2 |
|  |  |  | 1500-2500 | 4 | 3 | 2 | 1 |
|  | $n$ | Stationary | 0-500 | 14 | 14 | 14. | 14 |
|  |  |  | 500-1000 | 12 | 12 | 12 | 12 |
|  |  |  | 1000-1500 | 10 | 10 | 10 | 10 |
|  |  |  | 1500-2500 | 7 | 7 | 7 | 7 |

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Table Cl4
T-34 KILL PROBABILITIES
(Probabilities to nearest sixteenth; i.e., $7 \equiv 7 / 16$ )

| Target | Shot | Motion of target | Range, m | Concealment of target |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  | Zero | Quarter | Half | Three-quarter | Full |
| Dismounted Inf | Same as Blue medium tank against Red Inf, Table Cll |  |  |  |  |  |  |  |
| Mounted Inf | 1 | Stationary | 0-100 | 7 | 5 | 4 | 2 | 0 |
|  | n | Stationary | 0-100 | 7 | 7 | 7 | 7 | 7 |
|  | 1 | Moving | 0-100 | 7 | 5 | 4 | 2 | 0 |
|  | n | Moving | 0-100 | 7 | 7 | 7 | 7 | 7 |
| Blue medium tank (also Blue heavy tank) | 1 | Stationary | 0-500 | 14 | 11 | 7 | 4 | 0 |
|  |  |  | 500-1000 | 8 | 6 | 4 | 2 | 0 |
|  |  |  | 1000-1500 | 3 | 2 | 2 | 1 | 0 |
|  |  |  | 1500-2500 | 1 | 1 | 1 | 0 | 0 |
|  | n | Stationary | 0-500 | 14 | 14 | 14 | 14 | 0 |
|  |  |  | 500-1000 | 11 | 11 | 11 | 11 | 0 |
|  |  |  | 1000-1500 | 8 | 8 | 8 | 8 | 0 |
|  |  |  | 1500-2500 | 4 | 4 | 4 | 4 | 0 |
|  | 1 | Moving | 0-500 | 13 | 10 | 7 | 3 | 0 |
|  |  |  | 500-1000 | ó | 5 | 3 | 2 | 0 |
|  |  |  | 1000-1500 | 1 | 1 | 1 | 0 | 0 |
|  |  |  | 1500-2500 | 0 | 0 | 0 | 0 | 0 |
|  | n | Moving | 0-500 | 14 | 14 | 14 | 14 | 0 |
|  |  |  | 500-1000 | 8 | 8 | 8 | 8 | 0 |
|  |  |  | 1000-1500 | 2 | 2 | 2 | 2 | 0 |
|  |  |  | 1500-2500 | 1 | 1 | 1 | 1 | 0 |
| Blue light tank | 1 | Stationary |  | 14 | 11 | 7 | 4 | 0 |
|  |  |  | $500-1000$ | 9 | 7 | 5 | 3 | 0 |
|  |  |  | 1000-1500 | 4 | 3 | 3 | 2 | 0 |
|  |  |  | 1500-2500 | 2 | 2 | 1 | 0 | 0 |
|  | n | Stationary | 0-500 | 14 | 14 | 14 | 14 | 0 |
|  |  |  | 500-1000 | 12 | 12 | 12 | 12 | 0 |
|  |  |  | 1000-1500 | 9 | 9 | 9 | 9 | 0 |
|  |  |  | 1500-2500 | 5 | 5 | 5 | 5 | 0 |
|  | 1 | Moving |  | 13 | 10 | 7 | 3 | 0 |
|  |  |  | $500-1000$ | 8 | 6 | 4 | 3 | 0 |
|  |  |  | 1000-1500 | 1 | 0 | 0 | 0 | 0 |
|  |  |  | 1500-2500 | 0 | 0 | 0 | 0 | 0 |
|  | n | Mloving | 0-500 | 14 | 14 | 14 | 14 | 0 |
|  |  |  | 500-1000 | 9 | 9 | 9 | 9 | 0 |
|  |  |  | 1000-1500 | 3 | 3 | 3 | 3 | 0 |
|  |  |  | 1500-2500 | 2 | 2 | 2 | 2 | 0 |

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Table Cl5
SU-100 KILL PROBABILITIES
(Probabilities to nearest sixteenth; i.e., $14 \equiv 14 / 16$ )

| Target | Shot | Motion of target | Range, m | Concealment of target |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  | Zero | Quarter | Half | Three-quarter |
| Dismounted Inf | Same as Blue medium tank against Red Inf, Table Cll |  |  |  |  |  |  |
| Mounted Inf | Same as T-34 against mounted Inf, Table C14 |  |  |  |  |  |  |
| Blue medium and heavy tanks | 1 | Stationary | 0-500 | 14 | 11 | 7 | 4 |
|  |  |  | 500-1000 | 8 | 6 | 4 | 2 |
|  |  |  | 1000-1500 | 4 | 3 | 2 | 1 |
|  |  |  | 1500-2500 | 2 | 2 | 1 | 1 |
|  | n | Stationary | 0-500 | 14 | 14 | 14 | 14 |
|  |  |  | 500-1000 | 11 | 11 | 11 | 11 |
|  |  |  | 1000-1500 | 8 | 8 | 8 | 8 |
|  |  |  | 1500-2500 | 5 | 5 | 5 | 5 |
|  | 1 | Moving | 0-500 | 12 | 9 | 6 | 4 |
|  |  |  | 500-1000 | 6 | 5 | 3 | 2 |
|  |  |  | 1000-1500 | 2 | 2 | 1 | 1 |
|  |  |  | 1500-2500 | 1 | 1 | 1 | 0 |
|  | n | Moving | 0-500 | 14 | 14 | 14 | 14 |
|  |  |  | 500-1000 | 8 | 8 | 8 | 8 |
|  |  |  | 1000-1500 | 2 | 2 | 2 | 2 |
|  |  |  | 1500-2500 | 2 | 2 | 2 | 2 |
| Blue light tanks | 1 | Stationary | 0-500 | 14 | 11 | 7 | 4 |
|  |  |  | 500-1000 | 9 | 7 | 5 | 3 |
|  |  |  | 1000-1500 | 5 | 4 | 3 | 2 |
|  |  |  | 1500-2500 | 3 | 2 | 2 | 1 |
|  | n | Stationary | $0-500$ | $14$ | 14 | 14 | 14 |
|  |  |  | $500-1000$ | 12 | 12 | 12 | $12$ |
|  |  |  | 1000-1500 | 9 | 9 | 9 | 9 |
|  |  |  | 1500-2500 | 6 | 6 | 6 | 6 |
|  | 1 | Moving | 0-500 | 12 | 9 | 6 | 3 |
|  |  |  | 500-1000 | 8 | 6 | 3 | 2 |
|  |  |  | 1000-1500 | 2 | 2 | 1 | 1 |
|  |  |  | 1500-2500 | 1 | 1 | 1 | 0 |
|  | n | Moving | 0-500 | 14 | 14 | 14 | 14 |
|  |  |  | 500-1000 | 9 | 9 | 9 | 9 |
|  |  |  | 1000-1500 | 3 | 3 | 3 | 3 |
|  |  |  | 1500-2500 | 3 | 3 | 3 | 3 |

Table Cl 6
BLUE MORTAR KILL PROBABILITIES
AGAINST RED INF ANTRY
(Probabilities to nearest sixteenth; i.e., $8 \equiv 8 / 16$ )

| Concealme at | Kill probability |
| :--- | :---: |
| Zero | 8 |
| Quarter | 6 |
| Half | 5 |
| Three-quarter | 3 |
| Full | 2 |

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Annex C2
SAMPLE MOVE CALCULATION

Consider that a Blue tank is on grid square $(07,19)$ and the decision as to which neighboring square is to be occupied next must be made. Figure C5 lists the pertinent terrain-feature data as indicated by Figs. 8 and 9.


Fig. C5-Terrain Features for Sample Move Calculation


Fig. C6—Weighting Values Derived from Terrain Features Alone for Sample Move Calculation

Reference to Table C2 shows the grid squares are rated on the basis of the terrain features alone as shown in Fig. C6.

Additional weightings are calculated to account for the direction to the terrain objective located at $(23,09)$ using the method shown on Fig. C3. The weights calculated are shown in Fig. C7.

Finally, the presence of a (friendly) tank casualty on grid square $(08,20)$ shown on Fig. C5, requires that the grid square also be weighted by -13 (from Table C2). Combining all these weightings gives the final ratings shown in Fig. C8.

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Fig. C7-Weighting Values Derived from Direction to Terrain Objective Alone for Sample Move Calculation


Fig. C8-Comhined Weights for Sample Move Calculation


Fig. C9-Move Probabilities Computed for Sample Move Calculation

The move probabilities are calculated from the combined weights and are expressed as the percentage chance that the move will be made to the indicated square. Negative combined weights are given a 0 chance probability. The percentages are indicated in Fig. C9.

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Appendix D

TABULATED BATTLE RESULTS

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## INTRODUCTION

This appendix gives selected results of 141 repetitions of the computer battle. Table D1 describes the general purposes served by the calculation of these battles.

Table DI
REPETITIONS OF COMPUTER BATTLE

| Group | Repetitions | Type of Blue tank | Battle code | Purpose |
| :---: | :---: | :---: | :---: | :---: |
| 1 | 4 | Medium tank | A | Establish correctness of code |
| 2 | 4 | Medium tank | A | Test for battle intensity |
| 3 | 5 | Medium tank | C | Test for length of battle |
| 4 | 50 | Medium tank | $\mathrm{B}_{1}$ | Test for accuracy of average number of casualties from a set of battles |
| 5 | 50 | Light tank | $B_{2}$ | Test for influence of significantly altered performance characteristics on outcome |
| 6 | 7 | Heavy tank | $\mathrm{B}_{2}$ | Same |
| 7 | 7 | Heavy tank | $\mathrm{B}_{3}$ | Same |
| 8 | 14 | Heavy tank | $B_{4}$ | Same |
|  | 141 |  |  |  |

Table D2
DIFFERENCES AMONG BATTLE CODES

| Battle code ${ }^{\text {a }}$ | Comparison |
| :---: | :---: |
| $\mathrm{B}_{1}$ | Type A; plus (a) delay for turret switching, (b) lifting of mortar barrage, and (c) change in terrain objectives to mathematical type |
| $\mathrm{B}_{2}$ | Type $B_{1}$, plus orders for computer to type out number of tank when it runs out of ammunition |
| C | Type $B_{1}$, without delay in firing till second half of battle |
| $\mathrm{B}_{3}$ | Type $B_{2}$ $\min$ , with time limit on battle extended from 30 to $34 \frac{1 / 3}{}$ |
| $\mathrm{B}_{4}$ | Type $B_{2}$, with heavy tanks moving at same speed as mediums until the shooting starts, after which they revert to performance characteristics of heavy tanks |

${ }^{\text {a }}$ The type B battle code is the one described in detail in App C.
Table D2 lists the differences among the various battle codes used.
The results of the eight battles computed with the type A battle code will not be discussed. This code was superseded by the later types for most of the computations. These first eight battles were required mainly to (a) check out

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the bulk of the code, (b) establish that the computations gave the same results when repeated starting with the same random number, and (c) that the computations were within the time limits established earlier.

The third group of battles (five repetitions) demonstrated that, without the limitation on firing during (approximately) the first half of the battle, the calculations took about 1 hr for each battle. This resulted from the low kill probabilities causing a large number of rounds to be fired at each target, multiplying greatly the quantity of calculations required for each kill.

## DETAILS OF A SINGLE MEDIUM TANK BATTLE

Battle 10 of the fourth group of calculations (Table D1) was typed out by the computer in the maximum available detail using the type $B$ battle code (see section "Long Form" in App C). Annex D1 gives the significant part of the record for this battle as it was printed out by the computer.

Figures D1 to D7 show the progress of this battle at 300-sec intervals.

## Movement

Tank b09 moves quite erratically; whereas b02 usually moves quite directly toward the enemy (Fig. D8). Clearly the degree to which the path of one of these units deviates from a straight line leading to the objective is governed by how strongly local terrain features are allowed to influence the tank movements relative to the "strength" of the influence of the terrain objective. The particular relative values used for these calculations (see Annex C2) provided, in some cases, a strong dependence on the local terrain features. Other values may be more representative. The point is that the erratic movement may be reduced or removed by merely adjusting the table of ratings.

## Firing Activities

Table D3 summarizes the major firing activities that took place during the same medium tank battle.

## Influence of Communication System

Table D3 indicates one of the outstanding characteristics of these calculations. Thus, in every case but one, two or more opposing tanks were delivering fire against each casualty, immediately preceding the kill. This results from the speed with which one tank's knowledge of the position of an enemy tank was shared with other friendly tanks. In fact the detailed print-out of this battle in Annex D1 shows not a single case where a tank, once brought under fire by the enemy, was ever able to return that fire. In other words, the number of shooters increased so rapidly that even when the kill probabilities were small the volume of fire was always sufficient to cause a kill before the target could reply.

Clearly the character of the results might be radically altered if the communication system were not assumed to function so rapidly.

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Fig. D1-Initial Position (000 Sec) of Combat Units (All Battles) Numbers indicate code number of combat unit

- Blue tank ○ Blue infantry squad O Red tank (T-34) $\oplus$ Red SP gun (SU-100) (D) Red infantry squad


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Fig. D2-Position of Blue Assault Group ( 300 Sec Battlefield Time)
Battle no. 10; group 4; Blue medium tanks

- Blue assaulting tanks oblue assaulting infantry


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Fig. D3—Position of Blue Assault Group ( 600 Sec Battlefield Time)
Battle no. 10; group 4; Blue medium tanks

- Blue assaulting tanks O Blue as saulting infantry


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Fig. D4-Position of All Combat Units Just before Firing Starts ( 900 Sec Battlefield Time) Battle no. 10; group 4; Blue medium tanks

- Blue tanks o Blue infantry squad

ORed tank (T-34) $\oplus$ Red SP gun (SU-100) ©Red infantry squad

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Fig. D5—Position of All Combat Units Including Casualties Occurring since Start of Firing ( 1200 Sec Battlefield Time)

Battle no. 10; group 4; Blue medium tanks

- Blue tank ○ Blue infantry squad XTank casualty $n X \operatorname{Infantry}$ squad reduced effectiveness
© Red tank (T-34) $\oplus$ Red SP gun (SU-100) © Red infantry squad


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Fig. D6-Position of All Combat Units Including Units Which Became Casualties during Previous 300-Sec Interval ( 1500 Sec Battlefield Time)

Battle no. 10; group 4; Blue medium tanks

- Blue tank o Blue infontry squad X Tank casualty $\bigcirc$ Red tank (T-34) $\oplus$ Red SP gun (SU-100) © Red infantry squad


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Fig. D7-Position of All Combat Units at End of Battle Calculations Including Units Which Became Casualties during Previous 300 -Sec Interval ( 1800 Sec Battlefield Time)

> Battle no. 10; group 4; Blue medium tanks

- Blue tank ○ Blue infantry squad X Tank casualty $n \mathrm{X}$ Infantry squad reduced effectiveness $\bigcirc$ Red tank (T-34) $\oplus$ Red SP gun (SU-100) © Red infantry squad


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Fig. D8-Firing and Moving Activities of Two Blue Medium Tanks Battle no. 10; group 4; test battles
A graphic representation of the complete moving and firing history of two selected Blue cambat units in the same battle
Moving sequence - Firing

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Table D3
MAJOR FIRING ACTIVITIES
BATTLE NUMBER 10: BLUE MEDIUM TANKS

| Group | Target tank | Firing tank | Total rounds received | Total rounds fired |
| :---: | :---: | :---: | :---: | :---: |
| Blue assault tanks | 1 | 45,46 ${ }^{\text {a }}$ | 2 | 0 |
|  | 2 |  | 0 | 7 |
|  | 3 | $35^{\text {a }}$ | 2 | 2 |
|  | 4 | 35, 41, $33^{\text {a }}$ | 3 | 2 |
|  | 5 | 37, ${ }^{\text {a }} 34,36,42$ | 9 | 1 |
|  | 6 | $44^{\text {a }}$ | 5 | 4 |
|  | 7 | 41, ${ }^{\text {a }} 43$ | 4 | 4 |
|  | 8 | 42, 40, 44 ${ }^{\text {a }}$ | 3 | 0 |
|  | 9 |  | 0 | 4 |
|  | 10 | 43, ${ }^{\text {a }}$ 44, 34, 41 | 5 | 3 |
| Blue overwatching tanks | 11 |  | 0 | 3 |
|  | 12 |  | 0 | 4 |
|  | 13 |  | 0 | 4 |
|  | 14 | 38, $46^{\text {a }}$ | 3 | 1 |
|  | 15 | 41, 34, 43, 44 | 9 | 4 |
|  | 16 | 39, ${ }^{\mathbf{a}} 43,47,39$ | 4 | 2 |
|  | 17 |  | 0 | 2 |
| Red T-34's | 33 | $6,13,7,12,17^{a}$ | 12 | 1 |
|  | 34 |  | 0 | 6 |
|  | 35 | $15,{ }^{\text {a }} 3$ | 3 | 3 |
|  | 36 | $11^{\text {a }}$ | 2 | 2 |
|  | 37 | 2, ${ }^{1} 9$ | 4 | 3 |
|  | 38 | $2^{\text {a }}$ | 2 | 2 |
|  | 39 | 2, ${ }^{9} 9$ | 3 | 2 |
|  | 40 | 15, 4, 16, ${ }^{9} 10,3,5,9,14$ | 10 | 1 |
|  | 41 |  | 0 | 8 |
|  | 42 | 10, 6, 7, ${ }^{\text {a }} 13,12,11,9$ | 10 | 2 |
| Red SU-100 | 43 |  | 0 | 6 |
|  | 44 |  | 0 | 10 |
|  | 45 |  | 0 | 1 |
|  | 46 |  | 0 | 1 |
|  | 47 |  | 0 | 1 |

${ }^{\text {a }}$ Indicates killer.

## RESULTS OF 50 BLUE MEDIUM TANK BATTLES

Annex D2 gives the short-form results of the group 4 calculations, involving 50 repetitions of the battle computations. Each battle differs from the others only in the way chance influenced the Monte Carlo calculations.

Table D4 lists the total Red and Blue tank casualties in each of the battles. On the average, Blue lost 10.4 tanks and Red lost 7.1 tanks. The infantry casualties were at all times slight. The average effectiveness ratio for the Blue medium tanks corresponding to the average casualties is 60 percent; i.e., each Blue medium tank in these battles caused on the average only 60 percent as many casualties as did each Red armored vehicle.*
*The average effectiveness ratio computed from the average casualties as above is not necessarily identical to the average of the effectiveness ratios for each battle. This is because the latter is independent of the absolute number of casualties. The average of the Blue effectiveness ratios in those 50 battles is, however, almost exactly the same; 61 percent.

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Table D5 gives a detailed breakdown of the casualties occurring in the various battles. It will be noted that the Red overwatching tanks (SU-100) imposed a very unfavorable exchange rate on the Blues, losing only 11 out of a possible 250 , while accounting for 242 Blue casualties.

Table D4
TANK LOSSES IN 50 MEDIUM TANK BATTLES

| Battle <br> no. | Blue <br> losses | Red <br> losses | Battle <br> no. | Blue <br> losses | Red <br> losses | Battle <br> no. | Blue <br> losses | Red <br> losses |
| ---: | ---: | ---: | ---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 11 | 10 | 18 | 9 | 10 | 35 | 14 | 7 |
| 2 | 12 | 9 | 19 | 9 | 7 | 36 | 9 | 5 |
| 3 | 6 | 4 | 20 | 9 | 5 | 37 | 14 | 4 |
| 4 | 8 | 10 | 21 | 12 | 8 | 38 | 11 | 10 |
| 5 | 10 | 11 | 22 | 12 | 7 | 39 | 13 | 6 |
| 6 | 10 | 5 | 23 | 10 | 9 | 40 | 17 | 5 |
| 7 | 12 | 5 | 24 | 11 | 9 | 41 | 8 | 11 |
| 8 | 15 | 12 | 25 | 11 | 9 | 42 | 9 | 9 |
| 9 | 10 | 10 | 26 | 14 | 8 | 43 | 6 | 3 |
| 10 | 11 | 8 | 27 | 14 | 9 | 44 | 11 | 10 |
| 11 | 7 | 8 | 28 | 11 | 8 | 45 | 8 | 10 |
| 12 | 8 | 4 | 29 | 12 | 5 | 46 | 12 | 10 |
| 13 | 11 | 2 | 30 | 9 | 4 | 47 | 8 | 1 |
| 14 | 7 | 4 | 31 | 9 | 5 | 48 | 7 | 1 |
| 15 | 10 | 5 | 32 | 9 | 6 | 49 | 10 | 10 |
| 16 | 10 | 6 | 33 | 11 | 7 | 50 | 9 | 5 |
| 17 | 12 | 7 | 34 | 13 | 11 |  |  |  |



Fig. D9-Distribution of Red and Blue Tank Losses as a Function of Time, Expressed as a Fraction of Total Losses in 50 Medium Tank Battles

Fig. D9 shows the rate at which tank casualties occurred at different times during the battles. Note that roughly half the casualties occurred before the 15min time limit, which indicates that the medium tanks had reached their first terrain objective before the overwatching tanks had opened fire.

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Table D5
DISTRIBUTION OF INDIVIDUAL TANK CASUALTIES IN 50 BLUE MEDIUM TANK BATTLES ${ }^{a}$

${ }^{\text {a Key: }} \quad 40 \begin{gathered}38 \\ 4 \\ 2\end{gathered}$ Blue tank 4 killed Red tank 38, 3 times; Red tank 38 killed Blue tank 4, 2 times.

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## Infantry Activities

The activities of the infantry units during the 50 battles did not significantly affect the principal results, which were the number of tank casualties. Table D6 summarizes the casualties occurring during the battles that did involve the infantry units. There was at least one example of every possible interaction among tank and infantry units, but nowhere was a Red tank killed by a Blue infantry unit.

## Statistical Reliability

A common measure of the uncertainty that must be associated with an average $m$ computed from a limited series of tests is given by

$$
c=3 s / \sqrt{n},
$$

where $s$ is the standard deviation about the mean (average) of the distribution and $n$ is the number of repetitions. For normal distributions the odds are then about 300 to 1 that the true average lies somewhere in the interval $m-c$ to $m+c$ (plus or minus three standard deviations from the mean).

For the case of the Red tank losses while defending against Blue medium tanks, the value of $s$ is 2.74 tanks per battle. Hence $c=1.16$, and the odds are about 300 to 1 that the "correct" average Red tank loss lies in the interval 7.1$1.2=5.9$ to $7.1+1.2=8.3$ tanks per battle.

Similarly the distribution of Blue medium tank losses yields a value of $s=$ 2.33 tanks per battle. Hence $c=1.0$ tank per battle, and the odds are about 300 to 1 that the "correct" average Blue tank losses lies in the interval 10.4-1.0= 9.4 to $10.4+1.0=11.4$ tanks per battle. Since Fig. 16 indicates no strong dependence of the number of Blue tank casualties on Red tank casualties these probabilities may be assumed to be substantially independent.* It follows that the
*The assumption that the two distributions are statistically independent is only an approximation. Actually the coefficient of correlation(see, for example, Johnson ${ }^{6}$ ) for the medium tank battles is

$$
\begin{aligned}
r_{x y} & =[(\overline{x y})-(\bar{x})(\bar{y})] / s_{x} s_{y}, \\
& =0.278,
\end{aligned}
$$

where $(\overline{x y})=$ average of the products of the Red and Blue losses in each battle
$\bar{x}(\bar{y})=$ average Red (Blue) loss
$S_{x}\left(S_{y}\right)=$ standard deviation in the Red (Blue) losses.
This result is significant at the 0.05 level ( $r_{0.05}=0.269$ ). However, a positive correlation coefficient increases the significance of the observed difference in the mean losses of the Red and Blue forces comparea to the case where the Red and Blue losses are independent, because

$$
S_{\bar{x}-\bar{y}}=\sqrt{\left(S_{x}^{2}+S_{y}^{2}-2 r_{x y} S_{x} S_{y}\right) / N},
$$

where $S_{\bar{z}-\bar{y}}=$ standard deviation of the difference in the mean between two samples drawn from the same population
$S_{x}=$ standard deviation of the Red tank losses
$S_{y}^{x}=$ standard deviation of the Blue tank losses
$N=$ number of test battles.
Clearly as $r$ becomes larger, the standard deviation in the difference of the means becomes smaller, so that the observed difference between the means includes an increasing number of standard deviations. In this case $S_{\bar{x}-\bar{y}}=0.44$ so that the Red and Blue mean losses are observed to be separated by $3.34 / 0.44=7.6$ standard deviations. That this should happen by chance alone is many times more unlikely even than the approximate calculation given in the text above of 1 chance in 360,000 .

Thus if the null hypothesis is taken to be that the true difference between the mean Red and Blue losses is 0 , then the hypothesis may be rejected. If the selected null hypothesis had asserted any superiority of Blue over Red, it would have been rejected with even more confidence.

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Table D6
ACTIVITIES ${ }^{\text {a }}$ OF INFANTRY UNITS IN 50 BLUE MEDIUM TANK BATTLES

| Battle no. | Red Infantry casualties from: |  |  | Blue Infantry casualties from: |  | Tank casualties from: |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Mortars | Tanks | Infantry | Tanks | Infantry | Red Infantry | Blue Infantry |
| 1 | 0 |  |  |  |  |  |  |
| 2 | 4 (6) |  |  |  |  |  |  |
| 3 | 4 (5) |  |  |  |  |  |  |
| 4 | 3 |  |  |  |  |  |  |
| 5 | 2 |  |  | 1 (2) |  |  |  |
| 6 | 3 (4) |  |  |  |  |  |  |
| 7 | 3 |  |  | 1 (2) | 1 (2) |  |  |
| 8 | 3 |  |  |  |  |  |  |
| 9 | 3 (5) |  |  |  |  |  |  |
| 10 | 1 | 1 (6) |  |  |  |  |  |
| 11 | 2 | 1 (4) |  |  |  |  |  |
| 12 | 5 |  |  |  |  |  |  |
| 13 | 1 |  |  |  |  |  |  |
| 14 | 1 |  |  |  |  |  |  |
| 15 | 2 |  |  |  |  |  |  |
| 16 | 1 |  |  |  |  |  |  |
| 17 | 1 |  |  |  |  |  |  |
| 18 | 2 |  |  | $1(4)$ |  |  |  |
| 19 | 1 |  |  |  |  |  |  |
| 20 | 2 |  |  |  |  |  |  |
| 21 | 0 |  |  |  |  |  |  |
| 22 | 2 |  |  |  |  |  |  |
| 23 | 2 |  |  |  |  |  |  |
| 24 | 2 |  | 1 (1) |  |  |  |  |
| 25 | 4 (6) |  |  |  |  |  |  |
| 26 | 2 |  |  | 1 (2) | 1 (2) |  |  |
| 27 | 1 |  |  |  |  |  |  |
| 28 | 2 (3) |  |  |  |  |  |  |
| 29 | 3 |  |  | 1 (4) |  |  |  |
| 30 | 4 |  |  |  |  |  |  |
| 31 | 1 |  |  |  |  |  |  |
| 32 | 5 |  |  |  |  |  |  |
| 33 | 2 |  |  |  |  |  |  |
| 34 | 3 |  |  |  |  |  |  |
| 35 | 0 |  |  |  |  | 1 (1) |  |
| 36 | 1 |  |  | $1(4)$ |  |  |  |
| 37 | 3 |  |  |  |  |  |  |
| 38 | 0 |  |  |  |  |  |  |
| 39 | 0 |  |  |  |  |  |  |
| 40 | 0 |  |  |  |  |  |  |
| 41 | 2 (3) |  |  |  |  |  |  |
| 42 | 2 |  |  |  |  |  |  |
| 43 | 1 |  |  |  |  |  |  |
| 44 | 0 |  |  |  |  |  |  |
| 45 | 2 |  |  |  | 1 (6) |  |  |
| 46 | 3 |  |  |  |  |  |  |
| 47 | 1 |  |  |  |  |  |  |
| 48 | 2 |  |  |  |  |  |  |
| 49 | 3 |  |  |  | 1 (1) |  |  |
| 50 | 2 |  |  | 1 (4) |  |  |  |

${ }^{\text {a }}$ Table entries in parentheses are number of hits on indicated number of units.

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probability that the correct mean Blue losses should be as low, or lower, than 9.4 tanks per battle is 1 in 600 ; and the probability that the correct mean Red losses should be as high, or higher, than 8.3 tanks per battle is 1 in 600 ; and the probability that the two circumstances be simultaneously true is the product of these two probabilities or 1 in 360,000 .

These calculations show that the odds overwhelmingly indicate that the Red forces would, on the average, suffer fewer casualties than the Blue forces, no matter how many additional battles were computed. Therefore the sample size of 50 battles may be presumed to be sufficiently large to identify the winning side correctly.

## RESULTS OF 50 BLUE LIGHT TANK BATTLES

All performance characteristics referring to the type of Blue tank were altered so that Blue might be equipped with a hypothetical light tank. The killing power of its gun and the vulnerability of its armor were derived from tentative

Table D7
TANK LOSSES IN 50 LIGHT TANK BATTLES ${ }^{\text {a }}$

| Battle <br> no. | Blue <br> losses | Red <br> losses | Battle <br> no. | Blue <br> losses | Red <br> losses | Battle <br> no. | Blue <br> losses | Red <br> losses |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 6 | 9 | 18 | 5 | 9 | 35 | 8 | 10 |
| 2 | 9 | 9 | 19 | 9 | 9 | 36 | 8 | 8 |
| 3 | 6 | 9 | 20 | 2 | 8 | 37 | 5 | 9 |
| 4 | 4 | 10 | 21 | 9 | 10 | 38 | 6 | 7 |
| 5 | 7 | 10 | 22 | 4 | 8 | 39 | 3 | 8 |
| 6 | 7 | 7 | 23 | 5 | 7 | 40 | 4 | 7 |
| 7 | 6 | 4 | 24 | 9 | 7 | 41 | 6 | 7 |
| 8 | 9 | 10 | 25 | 6 | 9 | 42 | 5 | 9 |
| 9 | 5 | 9 | 26 | 9 | 10 | 43 | 7 | 7 |
| 10 | 4 | 3 | 27 | 11 | 9 | 44 | 6 | 7 |
| 11 | 8 | 7 | 28 | 6 | 10 | 45 | 3 | 6 |
| 12 | 6 | 10 | 29 | 3 | 4 | 46 | 6 | 8 |
| 13 | 11 | 9 | 30 | 10 | 8 | 47 | 9 | 9 |
| 14 | 6 | 8 | 31 | 6 | 10 | 48 | 6 | 10 |
| 15 | 5 | 9 | 32 | 4 | 10 | 49 | 7 | 8 |
| 16 | 5 | 8 | 33 | 8 | 8 | 50 | 6 | 10 |
| 17 | 7 | 11 | 34 | 11 | 10 |  |  |  |

${ }^{\text {a }}$ All Blue medium tanks replaced with light tanks.
calculations for the T41 tank and are discussed in App C. Entirely hypothetical were the assumed doubled speed and rate of fire (both referred to the Blue medium tank). Except for performance data no other changes were made in the code.

## Distribution of Tank Casualties

Table D7 gives the tank losses of both sides for these battles. On the aver age Red lost more tanks ( 8.4 tanks per battle) than Blue ( 6.5 tanks per battle). On this basis, Blue may be said to have "won" the battle when equipped with the

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hypothetical light tanks in contrast to "losing" the battle when equipped with mediums. The average exchange rate for these battles was 1.1 in favor of Blue, almost twice as great as the value of 0.6 previously computed for the Blue exchange rate in the medium tank battles.

## Statistical Reliability

The chance that the observed "superiority" of Blue over Red (average excess Red loss of $8.4-6.5=1.9$ tanks per battle more than the Blue losses) does not represent the true state of affairs and was due to chance alone must be determined. The calculation is the same as for the medium tank battles.

Figure D10 gives the results of the series of battles in the form of a scatter diagram. No strong correlation is indicated.


Fig. D10-Scatter Diagram Indicating Degree of Independence of Blue and Red Tank Losses
Each point corresponds to outcome of one battle of the 50 battles computed. Data from Table D7.

$$
\oplus=\text { average los ses (8.4 Red; 6.5 Blue) }
$$

Using the same equations for the same purposes as in the case of the medium tanks, three standard deviations in the mean Red casualties are subtracted from the observed mean number of Red tank casualties. This gives

$$
8.4-3(1.66 / \sqrt{50})=7.7 \text { tank casualties per battle }
$$

The quantity in parenthesis is the standard deviation in the mean, computed by dividing the standard deviation of distribution of Red casualties by the square root of the number of battle repetitions. The chance that the true mean would be less than the observed mean ( 8.4 tanks per battle) by as much or more than three standard deviations in the mean is about 1 in 600 . Similarly the chance that the true mean number of Blue light tank casualties should be as much as, or more than, three standard deviations in the mean greater than the observed number of Blue light tank casualties is 1 in 600 ; this puts an upper confidence limit on the mean number of Blue casualties of

$$
6.5+3(2.38 / \sqrt{50})=7.6 \text { casualties per battle }
$$

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Table D8
DISTRIBUTION OF RED AND BLUE TANK LOSSES IN 50 BLUE LIGHT TANK BATTLES ${ }^{\text {a }}$



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If the distributions are taken as independent,* the combined chance that both the average Red losses should be less than 7.7 tanks per battle and, simultaneously, the average Blue tank losses more than 7.6 tanks per battle, is the product of these two probabilities or $(1 / 600) \times(1 / 600)$ equals $1 / 360,000$. Thus the odds overwhelmingly favor the hypothesis that the Blue forces would suffer less casualties than the Red forces no matter how many additional calculations of this battle were made.

## Discussion of Results

Table D8 gives the performance of each individual tank in these 50 battles, both in terms of the enemy tanks they "killed" as well as the enemy tanks by which they were killed. This shows that the most striking reduction in Blue casualties has been among the overwatching tanks, only one of which (tank 15) was ever a casualty. There may have been a modest reduction in the number of casualties among the light tank assault group. Thus the average number of battles in which an assaulting Blue medium tank was killed is calculated from the data in Table D5 and found to be 30.4. The corresponding average for the assaulting light tanks is 27.5 . However, the reduction in number of battles is only barely significant, being a drop of 2.5 standard deviations in the mean. The chance against this happening by chance alone is about 1 in 100. This factor is usually taken to be large enough to reject the hypothesis that the difference between 30.4 and 27.5 may be considered as due to chance alone. However, the difference is not large in any event, and, withall, a probability of 0.01 is not so small as would be desirable.


Fig. Dll-Distribution of Red and Blue Tank Losses as a Function of Time, Expressed as a Fraction of Total Losses in 50 Light Tank Battles

This is a striking example of the requirement that a detailed investigation of the reason for superior performance be carried out. Figure D11 illustrates the most obvious difference between the series of medium and light tank battles.

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For the light tank battles the highest rate of casualties occurred between 600 and 700 sec . For the medium tank battles, the highest rate of casualties occurred between 900 and 1000 sec . This suggests a possible mechanism to account for the lack' of casualties a mong the overwatching tanks during the 50 light tank battles. Evidently the light tanks appeared at the edge of the Red position so rapidly that they distracted the attention of the Red tanks from the more distant and stationary overwatching tanks to the extent that Red tanks concentrated their fire on the assaulting elements only. Due to the limited number of calculations permitted by the feasibility study, no detailed investigation of this unusual circumstance was possible.

Annex D3 gives the detailed (long form) moving and firing record of one of the light tank battles.

Annex D4 gives the detailed (short form) results of the 50 Blue light tank battles.

## RESULTS OF 14 BLUE HEAVY TANK BATTLES

Table D9 gives the results of 14 battle calculations, where the Blue forces were assumed to be equipped with heavy tanks. Although the sample is too small to lend weight to the results, the Blue forces did impose an unfavorable exchange ratio on the Red forces, losing an average of 5.4 tanks per battle to the Red forces' loss of an average of 8.8 tanks per battle.

Table D9
TANK LOSSES IN 14 BATTLE CALCULATIONS
(Blue equipped with heavy tanks)

| Battle <br> no. | Blue <br> losses | Red <br> losses | Battle <br> no. | Blue <br> losses | Red <br> losses |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 5 | 8 | 8 | 3 | 9 |
| 2 | 4 | 9 | 9 | 8 | 9 |
| 3 | 7 | 9 | 10 | 6 | 6 |
| 4 | 5 | 10 | 11 | 8 | 8 |
| 5 | 6 | 11 | 12 | 6 | 9 |
| 6 | 4 | 10 | 13 | 4 | 9 |
| 7 | 4 | 9 | 14 | 6 | 8 |

Figure D12 shows the distribution of tank casualties for both sides. Due to the limited number of battles and the observed spread of results, the statistical reliability of these results is not so high as was the case for the medium tank and light tank battles. Even so, the standard deviation in the Red losses is 1.24 tanks per battle and in the mean is 0.332 tanks per battle. Hence three standard deviations in this mean is 1.00 tank per battle and hence the odds against the true Red losses being as low as 7.8 tanks per battle are about 300 to 1 . Similarly the odds against the true Blue losses being as high or higher than

$$
5.4+3(1.40) /(\sqrt{14})=6.5 \text { tanks lost per battle }
$$

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are also 300 to 1 . Thus, if the losses may be considered as being statistically independent (as was the case for the previous 100 battles) then the odds against both of these being simultaneously true are, as before, 360,000 to 1. Hence the


Fig. D12-Distribution of Tank Losses in 14 Blue Heavy Tank Battles
odds are overwhelming that the Blue tanks would continue to lose the lesser number of tanks no matter how many additional battles were calculated.

Note that the difference in the mean losses for these heavy tanks was 3.4 tanks per battle, which is the largest difference noted in these three sets of battles.

Annex D5 gives the (short form) results of these 14 Blue heavy tank battles.

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## Annex D1

## LONG-FORM PRINT-OUT OF BLUE MEDIUM TANK BATTLE 10

In the short-form print-out the $\mathbf{R}$ No. is the random number selected as the point from which the battle begins. The short-form print-out preceding the long forms has been included to facilitate identification of critical events. For key to print-outs see "Format for Results" in App C.

SHORT-FORM SUMMARY OF RESULTS OF BATTLE 10, GROUP 4

R No. 11321055

| b08 | 918 | 12,15 | rud |  |
| :---: | :---: | :---: | :---: | :---: |
| b01 | 923 | 13,08 | r46 | 18,10 1 |
| r53 | 925 | 20,05 | b31 | 0, |
| b04 | 926 | 16,11 | r33 | 17 |
| 6 | 930 | 21,04 | bl6 | 1, |
| r36 | 953 | 20,05 | b11 | 2, |
| b16 | 953 | 1,11 | r39 | 23, |
| 5 | 955 | 11, 14 | r37 | 23, |
| 33 | 955 | 1?,05 | b17 | 2, 42 |
| b14 | 959 | 5,07 | r 46 | 18,10 1 |
| b03 | 966 | 14,15 | r 35 | 20, 72 |
| r37 | 967 | 23,04 | b02 | 12, 83 |
| r35 | 972 | 20,07 | b15 | 6,23 2 |
| r42 | 1000 | 23,03 | b07 | 13,142 |
| b07 | 1000 | 13,14 | r 41 | 20, 53 |
| 06 | 1013 | 3,15 | rul | 21, 4 |
| bl5 | 1062 | 6,23 | r 4 | 20, 53 |
| blo | 1098 | 16,15 | r 43 | 20, |
| r38 | 1298 | 22,06 | b02 | 15, 8 |
|  | 1503 | 17,06 | b02 | 17, ? 12 |
|  | 1517 | 17,06 | b02 | 17, |
|  | 1531 | 17,06 | b02 | 17, 76 |
|  | 1558 | 17,06 | b02 | 17, |
| r 51 | 1601 | 17,06 | r02 | 17, 70 |
| r 51 | 1617 | 17,07 | b02 | $1 \%, 70$ |
| r39 | 1680 | 22,06 | b02 | 18,8 2 |

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LONG-FORM PRINT-OUT OF BATTLE 10


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|  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |

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Annex D2

## SHORT-FORM PRINT-OUT OF 50 BLUE MEDIUM TANK BATTLES

In all the print-outs the $R$ No. is the random number selected as the point from which the battle begins. For key to print-outs see "Format for Results" in App C.


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Annex D3

## LONG-FORM PRINT-OUT OF BATTLE 21 (Blue equipped with hypothetical light tanks)

In the short-form print-out the R No. is the random number selected as the point from which the battle begins. For key to print-outs see "Format for Results" in App C.

SHORT-FORM PRINT-OUT OF BATTLE 21

| R No. 13276404 |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| b04 | 495 | 13,12 | r 47 | 20, 11 | b07 | 629 | 17,12 | r39 | 23, 61 |
| r36 | 500 | 20,05 | b07 | 16,131 | rli | 630 | 20,05 | b06 | 13,16 2 |
| r35 | 508 | 20,07 | b16 | 1,112. | r39 | 630 | 23,06 | bll | 2, 52 |
| b03 | 518 | 12,12 | r34 | 17, 41 | r33 | 644 | 17,05 | b08 | 12,16 4 |
| r38 | 541 | 23,07 | b17 | 2, 42 | b15 | 656 | 6,23 | r 43 | 20, 72 |
| b09 | 573 | 18,16 | r 45 | 20, 81 | b08 | 665 | 13,15 | r 47 | 20, 91 |
| b01 | 593 | 11,13 | r 47 | 20, 91 | r 40 | 667 | 21,04 | b14 | 5,72 |
| r 42 | 594 | 23,03 | b02 | 11,15 2 | r34 | 720 | 17,04 | b06 | 13,15 2 |
| blo | 606 | 17,15 | r 43 | 20, 71 | r37 | 768 | 23,04 | bl3 | 3, 32 |
| r 53 | 612 | 20,05 | b31 | 0, 08 | bs r | 0 hi |  |  |  |
| b02 | 620 | 10,15 | r 4 | 20, 52 |  |  |  |  |  |

LONG-FORM PRINT-OUT OF BATTLE 21


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22,09
16,09
22,03
23,05

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|  |  | n2,06 |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |

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## Annex D4

## SHORT-FORM PRINT-OUT OF 50 BLUE LIGHT TANK BATTLES

In all print-outs the $R$ No. is the random number selected as the point from which the battle begins. For key to print-outs see "Format for Results" in App C.

| R No. 115607072 |  | $\begin{array}{llll}33 & 1514 \\ 603 & 17616 \\ 21,11\end{array}$ |  |  | 213 74500,7 | ${ }^{1803} \quad 34127$, |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Battle 1 |  |  |  |  |  |  |  |
| r56 23218,10 | D31 0, 010 |  | (0) 23,142 |  | ¢ 461818120 | ${ }_{606} 6261413$ |  |
| ${ }^{544}$ | b31 0,08 |  |  | ${ }_{540} 795120$ | 51612112 | +62 634 20, 5 | b13 3, 32 |
| $\begin{array}{llll}\text { r38 } & 406 \\ 6006 & 13,13\end{array}$ |  |  |  | B07 00512,4 | 14120, 52 |  | 5044,72 |
| 507 494 10,16 | r 43 20, 71 | P:10.00145362 |  | 539 229 23, 6 | 5094.402 | 539 648 23, | boll 13,162 |
| 50150010,14 | 534 27, 42 |  |  | ${ }^{236} 867{ }^{20,5}$ | 612 2, 42 | ${ }^{236} 749300$ | 11 |
| ${ }^{8} 500616.16$ | $5^{546}$ 28,10 2 |  |  |  |  | 542 23321313 | ${ }_{513} 17.51$ |
| ${ }_{536} 5072005$ |  | 620055314.16 | rut 21, 41 |  |  | ${ }_{233} 133317$, 5 | 300 12,10 1 |
|  | ${ }_{\text {bog }}^{609}$ 11, 172 | 54255923.3 | b16 2,112 | P No. 01435630 |  | 540 1219 20, 3 | 500817,215 |
| r33 53517 , 5 | b16 1,114 | 502509 | 5430097 |  |  |  |  |
| 002 5369 9,12 | r37 23, 42 | ${ }_{609} 509150$ | 54580.81 | ${ }_{549} 52002313$ | ${ }_{632} 0,08$ |  |  |
|  | $\begin{array}{ll}614 & 51 \\ 016 \\ 1,42\end{array}$ | F39 599 23,6 | b15 6, 233 | ז49 306 23, 3 | b31 0, 04 | п Ho .15525 |  |
| 542 637 <br> 556 675 <br> 18,10  | ${ }^{616} 11,312$ | 507 61315.14 | F45 20, 81 | 6154736,23 | 54720,91 | Batt. |  |
| 605 1204 11,16 | 3372033 |  | ${ }^{831} 01618$ | ${ }^{512} 4078{ }^{23}$ | $\operatorname{LbO}_{5} 12,162$ | 551 038 <br> 551  <br> 530  | ${ }^{631} 0931085$ |
| r35 124. 19, 6 | 503 28,122 | ${ }_{5}^{54} 465923,6$ | b31 0, 04 |  | $\mathrm{F}_{541} \mathbf{4} 20,52$ | ${ }_{552} 5392104$ | ${ }_{632} 0,08$ |
|  | 50825,128 8, | +40 2020 23,4 | b04 15,152 | 102 5069 | r44 18,101 | 542 $4 \% 23$ 2, | 40212,152 |
|  |  | 235 1135 20,7 | 80312.163 | 1035159412 | 536 20, 51 | 50149812,15 | [43 20, 71 |
|  |  | 233 21899 17, 5 | ${ }^{603} 125,152$ | [37 533 23, 4 | 6212,33 | 402 305 9,19 | 54720,92 |
| R Mo. 22107017 |  | r36 220720,5 | 10116142 | [35 540 20, ? | bou 23,172 | 61052216015 | 53620,52 |
| Battie 2 |  | ${ }^{537} 125722,3$ | bos 16,123 | 5368302005 | b16 5, 74 | 53353217,5 | $6^{613} 313$ |
|  | 131 0, 08 | ${ }^{234}$ 1656 21, 7 | 50618,121 | 0637213,17 | 531023, 72 | 1095154 15,13 | T41 20032, |
|  | 831 0, 0154 | 4 rio hit |  | $\pm 6 \mathrm{IL} \mathrm{hlt}$ |  |  | ©0¢ 15,10 2,08,00,05 |
| 104644, 615 | [47 20, 91 |  |  |  |  |  |  |
| 53667520,5 | b15 6,23 2 | R No. 26025002 |  | No.0522 |  |  |  |
| 6206943.15 |  |  |  |  |  | P No. 234 |  |
| b05 69 16, ${ }^{\text {ch }}$ | r34 17, ${ }^{4} 1$ |  | 631 620 2, 2, 4 | 55935621 | b31 0, 0 |  |  |
|  |  |  | r40 21, 62 | $\begin{array}{llll}534 & 609 & 17 \\ \text { b15 } & 615 & 6,23\end{array}$ |  | $\begin{array}{llll}541 & 293 & 20,7 \\ 533 & 602 & 17,5\end{array}$ | ${ }^{1031} 80,0$ |
| F35 735 20, 7 | 4172,43 | r364019,4 | ${ }^{60} 1317162$ | 538 616 23,7 | b10 14, 42 | 602 61512,4 | =4021, 42 |
| 5417888 | B03 26, ${ }^{\text {a }}$ 2 | ${ }^{003} 44913,1$ | r33 17, 51 | $609620.15,16$ | r 46 18,10 1 | 231 621 23,7 | ${ }^{216}$ 1,112 |
| 100 79014.15 | $54520,8 \frac{1}{2}$ | 507461010 |  | 420863314,14 | 735 20, ? | 60767818,15 | 74820,81 |
|  |  |  | 304 12,162 | $\begin{array}{llll}302 & 636 \\ 805 & 639 & 13,14\end{array}$ |  | $\begin{array}{lllll}335 & 637 & 20,7 \\ 825 & 639 & 6,23\end{array}$ |  |
| r38 820023 , 7 | b27 2, 42 | 1001468 9,16 | 247 20, 92 | r39 646 23, 6 | b17 2, 43 | 100 64513,13 | x422, 42 |
| 507822314,22 | [40 21, 42 | $535471{ }^{20}$, 7 | 6172,43 | 14264620,5 | $6^{616} 1,172$ | г39 647 23, 6 | 802 22,10 2 |
| 133 830 17, 5 | b14 5,73 | 100 47614.18 | 54320,72 | 74265623,3 | ${ }^{600} 17,162$ | 10664717,18 | r46 18,20 1 |
|  |  | [39 509313,15 | $\pm 1320,71$ | $\begin{array}{llll}406 & 667 & 14,15 \\ & 35 & 668 & 20,7\end{array}$ | 53317,53 00316,14 |  | 545 617 20, 2, |
| 534 179,5 | b16 1,113 | 10651313,16 | 745 20, 81 | ${ }_{607} 67317,18$ | 54780,91 | 105 6989 | r34, 27,42 |
| 59 xo hit |  | 536853005 | W313, 32 | 82066121,4 | 501 12,102 | 14 705 20, 5 | 611 2, 52 |
|  |  | 512625 23, 3 | 1361,112 | 10869117,16 | r43 20, 72 | ${ }_{137} 70523,4$ | \$16 1,113 |
|  |  | 53760023 , |  | 50370216,14 | 53620,51 | 103 730 21,15 | 24618,10 2 |
| R No. $24012 \%$ |  | $\mathrm{xLl}_{4} 71300,5$ | ${ }^{613} 3,35$ | ${ }^{237} 72383$. | $\square_{0} 17,152$ | 58 rl nit |  |
|  | b31 0, 08 | O7 510 mit |  | 536116420, | b0\% 25,152 |  |  |
|  | 13120,010 |  |  |  | 600 15,162 |  |  |
| 100 590 10,16 | 54320, | a Ho. 26730757 |  |  |  | a No. 4773101 |  |
| 548500808 | ${ }^{\text {b31 }} 000818$ |  |  |  |  |  |  |
|  | b07 12, 122 |  | ${ }_{631} 0,010$ |  |  | 29940 | 100210101 |
| 18180000005 |  | ז49 $29223 ;$ | b31 0, 01 | Batille 9 |  | 801 \% 12, 4 | 54233,32 |
| 10064041516 | 53620,52 | 55155320,7 | b31 0, 0 | 55925320,5 | 1320008 | ${ }^{33} 660175$ |  |
| 106 628 12,16 | I $47{ }^{2} 20,92$ | W15 $729.60{ }^{6}$ | 5M1 22, 42 | ${ }_{551} 51{ }^{304}$ |  | ${ }^{204} 8898$ |  |
|  | 505 23,133 |  | ${ }^{603} 129.172$ |  |  | H15 6971080 | 1\% 20, |
|  |  |  | b11 2, 53 | r5s 48180 | b33 0,010 |  | 54120,52 |
| r36 672 20, 5 | 50712,214 | ${ }^{0} 085314.16$ | 547 20.91 | 549 sto 23, ${ }^{3}$ | 633 0, 04 | $\begin{array}{llll}388 & 936 & 23,7\end{array}$ | 40918,151 |
| 542 239722,2 | 420 29, 12 | b02 75614.17 | 537 23, 42 | 605 500 | 54, 2, 41 | ${ }^{336} 973$ 20, 7 |  |
| r35 145020,7 | 30713,122 | 534 75817, 4 | b4 3,72 | ¢37 590 23, 6 | b14 5, 72 | 3359020,7 | 806 37, 212 |

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| ${ }^{108}$ | r | ${ }^{56} 68680$ |  | ${ }^{60}$ |  | sos |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | 515090 |  | ${ }_{102} 0$ |  |  |  |
|  | bos 212,104 | 806 65015121 | 546 28,102 | ${ }_{53} 5$ | ${ }_{013}{ }^{2} 3,312$ |  |  |
| ${ }_{142} 920{ }^{2} 31$ | 805 12,112 | 604 67413,13 | ris 20,181 | r52 460 | ${ }^{312} 0008$ | ${ }_{515} 5$ |  |
|  |  |  | 515 ${ }^{15}$ | 60 | ris |  |  |
|  |  |  |  | ${ }^{1008}$ | ${ }^{12}$ |  |  |
|  |  | ${ }_{737} 100923.3$ a |  | ${ }^{2} 3385$ |  | 53960 |  |
|  |  | 54233120 | 610 27,132 | ${ }^{2} 2$ |  | 530682 |  |
|  |  | $\mathrm{br}_{6} \mathrm{rlo} \mathrm{hit}$ |  | ${ }_{5}^{536} 593129$ |  | 33 |  |
| [53 09820,5 | ${ }^{631} 00,08$ |  |  | bet 179316 , 12 |  | ${ }_{r}$ |  |
| ${ }^{\text {bos }}$ sod 31315 |  | R No. 1261742 |  |  |  | 504 | 1 |
| 39120, ${ }^{12}$ | ${ }^{009} 12,1612$ |  |  |  |  | 540923 |  |
|  |  |  |  |  |  | ${ }^{217} 9$ |  |
|  |  |  |  | Battie 50 |  |  |  |
| ${ }_{615} 628$ 6,23 | [33 27,52 | bis ${ }^{\text {un }}$ 6,23 | r63 20, ${ }^{\text {2 }}$ | 83457817 | 209 15,12 |  |  |

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Annex D5
SHORT-FORM PRINT-OUT OF 14 BLUE HEAVY TANK BATTLES
In all print-outs the $R$ No. is the random number selected as the point from which the battle begins. For key to print-outs see "Format for Results" in App C.


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[^0]:    *The evidence tends to support the possibility of a much reduced number of repetitions being sufficient in many cases.
    $\dagger$ "Effectiveness" is used throughout to refer to the ability of a weapon to accomplish its mission. This is a necessary step before determining the more meaningful and basic "effectiveness per unit cost" of the weapon.

[^1]:    *A simple definition of tank effectiveness has been used by V. McRae and A. Coox in ORO-T-278, "Tank-vs-Tank Combat in Korea." There, tank effectiveness was defined as the ratio of the average number of enemy tanks killed by each friendly tank to the average number of friendly tanks killed by each enemy tank. Other deficitions of effectiveness have been proposed, including cost effectiveness, which includes the elements of production and logistical costs.

[^2]:    *This limitation was principally a practical one, so as to stay within the time limits on use of the computing machine for the feasibility calculations. When firing was permitted to start with the onset of the assault, the computer calculations consumed an hour per battle, three times too long.

[^3]:    *"Spread" as used here is equivalent to the standard deviation of the distribution. For normal distributions this is the interval about the mean which includes 63 percent of the cases: Table 1 gives the spread of all the casualty distributions presented.

[^4]:    *So long as the probabilities are greater than 0.05 that the observed deviation from a normal curve could be due to chance alone, the assumption that the distributions are normal is tenable.

[^5]:    *"Bits" are to the special computer numbers as "digits" are to ordinary numbers (see App A for details).

[^6]:    *Improved equipment is now available to speed up this process enormously.

[^7]:    *As in the case of the medium tanks this is only an approximation. The coefficient of correlation $r$ is computed to be 0.327 , which is significant at the 0.05 level, though not at the $0.01\left(r_{0.01}=0.354\right)$. Again the effect of this positive correlation coefficient is to increase the odds computed above ( 360,000 to 1 ). In fact the mean Hed and Blue losses are computed to be separated by 5.9 standard deviations, which is many times larger than is required to reject the null hypothesis that the Red and Blue mean losses were "actually" the same.

