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THE ZENITH ANGLE VARIATION OF COSMIC RAY MU-MESON INTENSITY

COSMIC RAY PROJECT
Project No. 101
Report No. 101.17
March 1, 1954
Prepared for
NUCLEAR PHYSICS BRANCH
OFFICE OF NAVAL RESEARCH
Washington, D.C.
Contract No. N6 ONR 279 T.O.2

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ABSTRACT

We have examined the zenith angle variation of cosmic ray mu-mesons at two different altitudes and two different geomagnetic latitudes. No perceptible altitude variation was found, but a difference with latitude has been detected. An equation for the variation has been derived which takes into account the decay of the mesons. The experimental results have been checked against both this equation and the empirical $\cos^{2.1}\psi$ variation, where ψ is the angle from the vertical. This equation has not been found to be more accurate than the empirical expression, but it predicts a latitude shift which is in qualitative agreement with the observed change.

Introduction

Shortly after the coincidence counter technique was devised by Rossi[1], several investigators employed this circuit to measure the variation of cosmic-ray intensity with zenith and azimuth angles. Their counter telescopes detected only a negligible azimuthal variation at geomagnetic latitudes away from the geomagnetic equator, as predicted by Lemaitre and Vallarta. However, even these earliest investigators[2] found a marked variation of directional intensity with zenith angle. The low altitude measurements appeared to fit a $\cos^2\psi$ law, that is, that the intensity falls off as the square of the cosine of the angle between the axis of the counter telescope and the zenith. Subsequent determinations by Greisen[3] and Zar[4] gave the same result, but with a slightly greater exponent for the cosine: 2.1.

An analysis of the zenith angle distribution to give a $\cos^2\psi$ law was made by Beiser[5], and, in order to examine its validity, the present series of experiments were carried out. This work is described below.

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1. B. Rossi, N. Cimento 8, 49 and 85 (1931), Zeits. f. Phys. 68, 64 (1931).
2. L. Tuwim, Akad. Wiss. Preuss., Sitz.-Ber. 19, 91, 360 (1931); G. Bernardini, Nature 129, 518 (1932); G. Medicus, Zeits. f. Phys. 74, 350 (1932); D. Skobelzyn, Compt. rend. 194, 118 (1932); T.H. Johnson, Revs. Mod. Phys. 10, 205 (1938).
3. K. Greisen, Phys. Rev. 61, 212 (1942).
4. J.L. Zar, Phys. Rev. 83, 761 (1951).
5. A. Beiser, J. Geophys. Research, in press.

I. Theory of the Zenith Angle Effect.

From sea-level to mountain elevations the hard (i.e. penetrating) cosmic radiation consists predominately of μ-mesons resulting from decay of π-mesons, themselves originating in high energy nuclear interactions at the top of the atmosphere. A contribution by locally produced mesons is also possible, although probably quite small. A small number of protons at the mountain altitude is also present.

It is convenient to make use of the usual assumptions that the bulk of pi-meson production occurs in a sufficiently restricted region near the top of the atmosphere. At low elevations production may be considered as occurring in a single layer at a constant altitude. The π-mesons are assumed to be produced isotropically, and their lifetime may be neglected in comparison with the μ-lifetime. According to Sands[6] the first approximation is satisfactory at low elevations for mesons of energy greater than $3mc^2$, a condition satisfied in zenith angle measurements employing the usual 10 cm. or more Pb absorber.

Now, μ-decay in terms of the distance z from the point of production is given by

$$-dN(z)/dz = N(z)/R, \qquad (1)$$

where $R = \tau p/m$ is the mean range of a meson corresponding to its lifetime τ, with p and m the meson momentum at elevation z and meson rest-mass respectively. Integrating,

$$N(z)_p = N(0)_p \exp(-mz/\tau p),$$

for a given p. A meson incident at an angle γ with the vertical

COMPARISON OF IONIZATION LOSS ALONE AS AGAINST IONIZATION PLUS DECAY LOSS

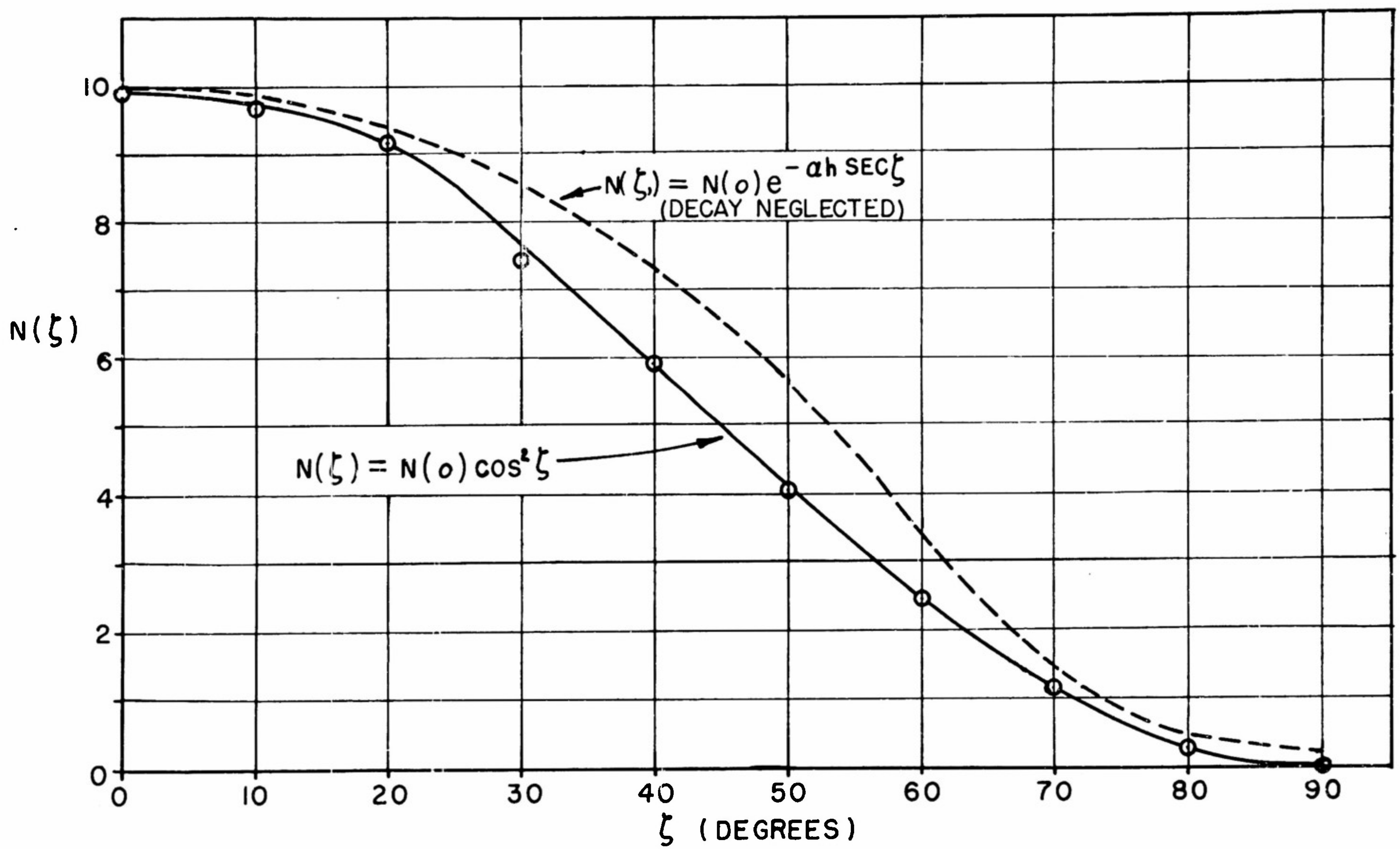

FIGURE 1

has $z = h \sec \vartheta$, h being the height of the production layer, and so

$$N(\vartheta)_p = N(0)_p \exp(-mh\sec\vartheta/\tau p). \qquad (2)$$

To find the total variation of N with ϑ it is necessary to integrate over the meson momentum spectrum at production, $F(p)dp$. Taking $F(p)dp = Ap^{-n}dp$, where $n \sim 3$, and absorbing the constant A into $N(0)$,

$$N(\vartheta) = N(0)\int_{p_{min}}^{\infty} p^{-n} \exp(-mh\sec\vartheta/\tau p)dp, \qquad (3)$$

with p_{min} the minimum momentum required to penetrate the atmosphere and the experimental apparatus and, of course, varying somewhat with ϑ. For an approximate result p_{min} can be taken as 0, and, letting $p = 1/x$,

$$N(\vartheta)_{approx} = N(0)\Gamma(n-1)/(mh\sec\vartheta/\tau)^{n-1},$$

or,

$$N(\vartheta)_{approx} = N(0)' \cos^{n-1}\vartheta. \qquad (4)$$

Hence this approximation gives for the variation a form identical with experimental findings. For a $\cos^2\vartheta$ distribution, a p^{-3} production spectrum is implied, which is in quite good agreement with the spectra deduced by Sands[6] and others from various considerations.

Eq. (3) can be evaluated exactly for integral and half-integral values of n, giving for $n = 3$

$$N(\vartheta) = \frac{N(0)\left\{1-\left[(mh\sec\vartheta/\tau p_{min}) + 1\right]\exp(-mh\sec\vartheta/\tau p_{min})\right\}}{(mh\sec\vartheta/\tau)^2} \qquad (5)$$

In the region of interest here, that is for $p_{min} \sim 2$ Bev at $\vartheta = 0°$, p_{min} is almost directly proportional to the atmospheric absorption

6. M.L. Sands, Phys. Rev. 77, 180 (1950)

depth, itself proportional to sec ψ. Hence sec ψ / p_{min} remains very nearly constant, centainly for $0^{\circ} \leqslant \psi \leqslant 60^{\circ}$, $1 \leqslant \sec \psi \leqslant 2$ which is the interval in which experimental measurements are usually made. This means that the numerator in eq. (5) is essentially constant. Since eq. (4) can also be shown in this way to hold for $n = 2$ and $n = 4$, it may be concluded that eq. (4) is quite generally valid in the vicinity of $n = 3$. However, if the thickness of absorber used in the counter telescope is significant compared with the atmospheric absorption depth p_{min} is no longer proportional to sec ψ and eq. (4) is not valid. For the thicknesses of absorber ordinarily employed, this is not a problem.

If one neglects decay loss, and tries to explain the zenith angle variation on the basis of absorption alone, the theoretical variation is plotted in figure (1). The experimental curve, $N(\psi) = N(0) \cos^2\psi$, is also plotted therein for purposes of comparison.

III. Apparatus.

In order to test the validity of the foregoing equations, a counter telescope was constructed. The purpose of such an instrument is to determine the direction of arrival of particles actuating the counters which are connected in coincidence. In the design of a counter telescope, it is necessary to find the best compromise between the two desirable but mutually exclusive characteristics of high counting rate and good angular resolution, while retaining the portability of the apparatus. Figure 2 shows the essential features of the telescope, which was supported in a light wooden frame, and could be tipped from the vertical to one of a series of zenith angles. The telescope had an angular resolution of 23° in the zenith direction. Small brackets were provided, so that 10 centimeters of lead could be incorporated within the counter-train itself, in order to prevent the recording of the soft component. It was also possible to add a further thickness of lead above the top-most counter, in order to facilitate the evaluation of p_{min}.

The Geiger-Muller counters that were used were $1\frac{1}{2}$" in diameter, and had an active length of 12". The cathodes were made of copper, and the anodes were three mil tungsten wires. This value for the active length made the angle subtended in a direction perpendicular to the zenith 52°. The self-quenching counters were filled with neon and ethyl acetate, and had an estimated useful life of 10^8 counts.

TABLE 1.

COUNTER	PLATEAU LENGTH (volts)	SLOPE
1	1100 to 1250	5% / 100v
2	1120 1325	5%
3	1100 1210	5%
4	1100 1300	5%
5	1100 1220	2%
6	1100 1250	5%
7	1100 1300	5%
8	1100 1300	2%
9	1100 1210	4%
10	1100 1250	3%
11	1150 1300	5%
12	1180 1300	3%

Table 1 gives the plateau lengths and slopes of the plateaus of the twelve counters that were used during the course of the experiment. The plateaus overlap at about 1200 volts, which was chosen as the operating potential of the counter telescope.

A commercial 4-channel coincidence analyzer (Atomic Instrument Co. Model 502A) was employed. Each channel has an individual pulse-height discriminator, which could be set so as to accept only pulses resulting from the discharge of the counter or counters to which it was connected. It is possible to switch one of the four channels into anti-coincidence, while the other three are in coincidence. The four counters labelled "A" in figure 2 were connected in parallel and fed into the anti-coincidence channel. In this way, the extraneous counts due to fast electrons and sideways showers were minimized. An energetic electron, upon passing through the lead, gives rise to a number of secondaries which discharge one or more of the anti-coincidence counters. Uncollimated showers, because of their lateral extent, almost always discharge the anti-coincidence counters if they can traverse the entire coincidence train. The resolving time of the circuit was set at two microseconds, which provides the best compromise between a reasonable counting rate and a minimum of accidental coincidences(See section V.).

The output of the coincidence circuit was fed into a standard scaling circuit, which also contained an input discriminator. This further guarded against spurious pulses due to noise. The output of the scaler was then fed into a mechanical register.

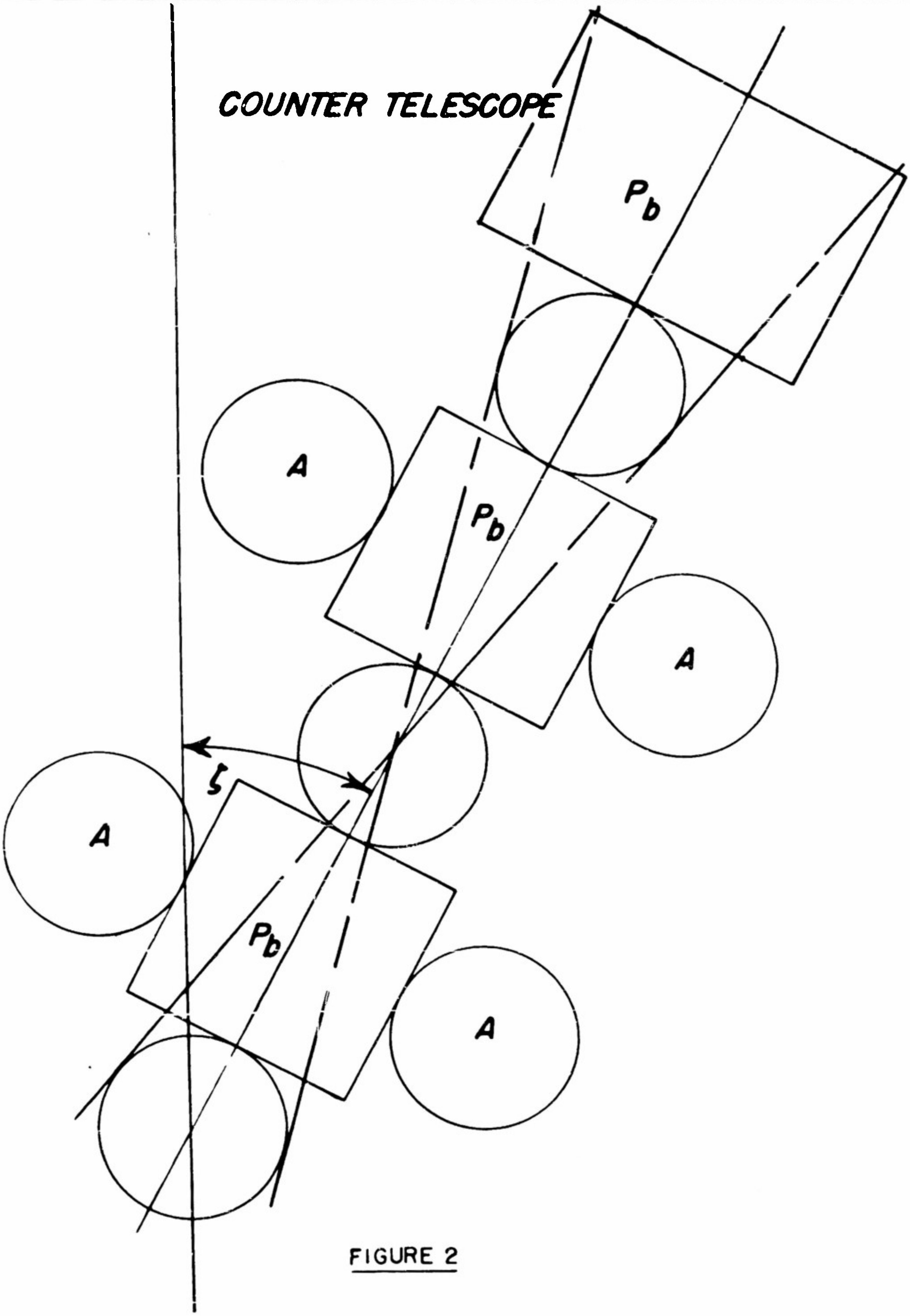

FIGURE 2

IV. Experimental Technique.

It was decided to perform the experiment at different elevations and latitudes, in order to examine the actual dependence of N(γ) on n and p_{min} and to compare it with that predicted by Equation (5). The Inter-University High Altitude Laboratory, at the summit of Mount Evans, Colorado, which is 4,350 meters above sea-level, was used as a high-altitude station. The experiment was also run at New York University, which is at the same geo-magnetic latitude as Mount Evans, but is at sea-level. The following summer, the experiment was repeated, this time at the Geophysical Institute of the University of Alaska, which is also effectively at sea-level. The Mount Evans phase of the work was done during the summer of 1952, and the New York work was performed in the autumn of that year.

At Mount Evans, the thicknesses of absorber employed were 0, 5, 10, and 20 centimeters of lead, in addition to the wall-thickness of the counters, while at New York the amounts used were 0, 5, 10, and 15 centimeters. The Alaska work used 10, 20, and 30 centimeters. The 0 and 5 cm. thicknesses were used in order to determine the effect of both soft and hard components of the cosmic radiation.

The telescope was pointed south at all three places in order to avoid the complicating effect of the east-west assymetry. All of the discriminators were checked twice daily, in order to prevent any drifting which might have introduced spurious counts from noise generated in the circuits.

If one continuously monitors the vertical intensity, and uses this data as a normalization factor, one avoids the necessity of calculating approximate corrections for changes in the temperature and pressure of the atmosphere. Professor Mario Iona, Jr., of Denver University, was kind enough to supply us with data on the hourly fluctuations in the intensity of the hard component on Mount Evans (Figure 3). In New York, monitoring runs (usually 6 hours in length) were taken of the vertical intensity before and after every angle setting and the average of the two was employed as the normalizing factor (Figure 4). In Alaska, the data was monitored simultaneously, and a typical day is depicted in Figure 4a.

V. Results.

The experimental results are plotted in figures five through twelve. When a straight line is fitted to the points by the Least Squares method, one finds the slope, which is indicated on the graphs. The experimental value for the slope (n - 1) is then employed in equations (4) and (5). The results of the calculations, and their comparison with the observed counting rates, are summarized in tables 2, 3, and 3a.

On Mount Evans, the apparatus was housed in a hut made of pine logs. This was done in order to avoid the effects of weather on the apparatus, and to minimize the temperature effects in the counters[7]. The logs were equivalent to about 6 gm cm^{-2}, which provides a negligible correction to the absorber thickness. In New York, a canvas tarpaulin was draped over the apparatus, while in Alaska the telescope was placed in a thermally insulated wooden box.

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7. S.A. Korff, Electron and Nuclear Counters (New York, D. Van Nostrand and Company, Inc., 1946), p. 114.

VERTICAL INTENSITY FLUCTUATIONS (USING 8 CM. Pb.), JULY 19-20, 1952 AT MOUNT EVANS, COLORADO

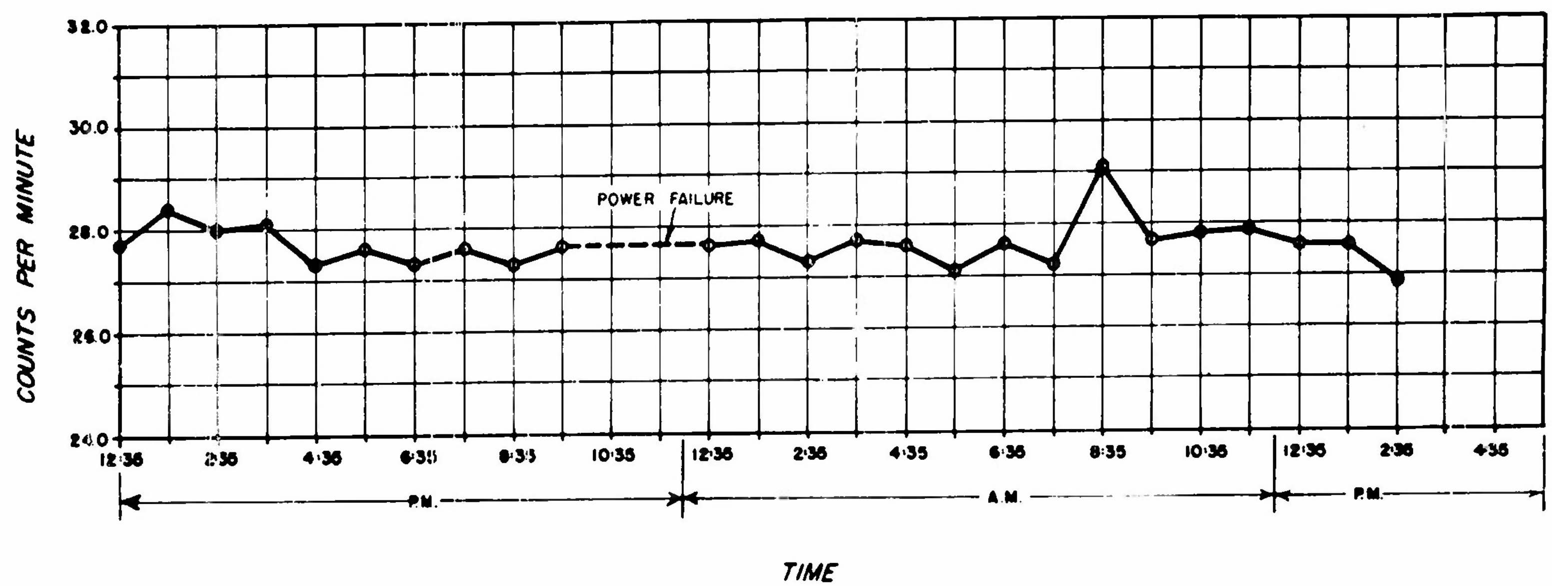


FIGURE 3

VERTICAL INTENSITY FLUCTUATIONS (USING 15 CM. Pb.), NOVEMBER 12-13, 1952, AT NEW YORK UNIVERSITY

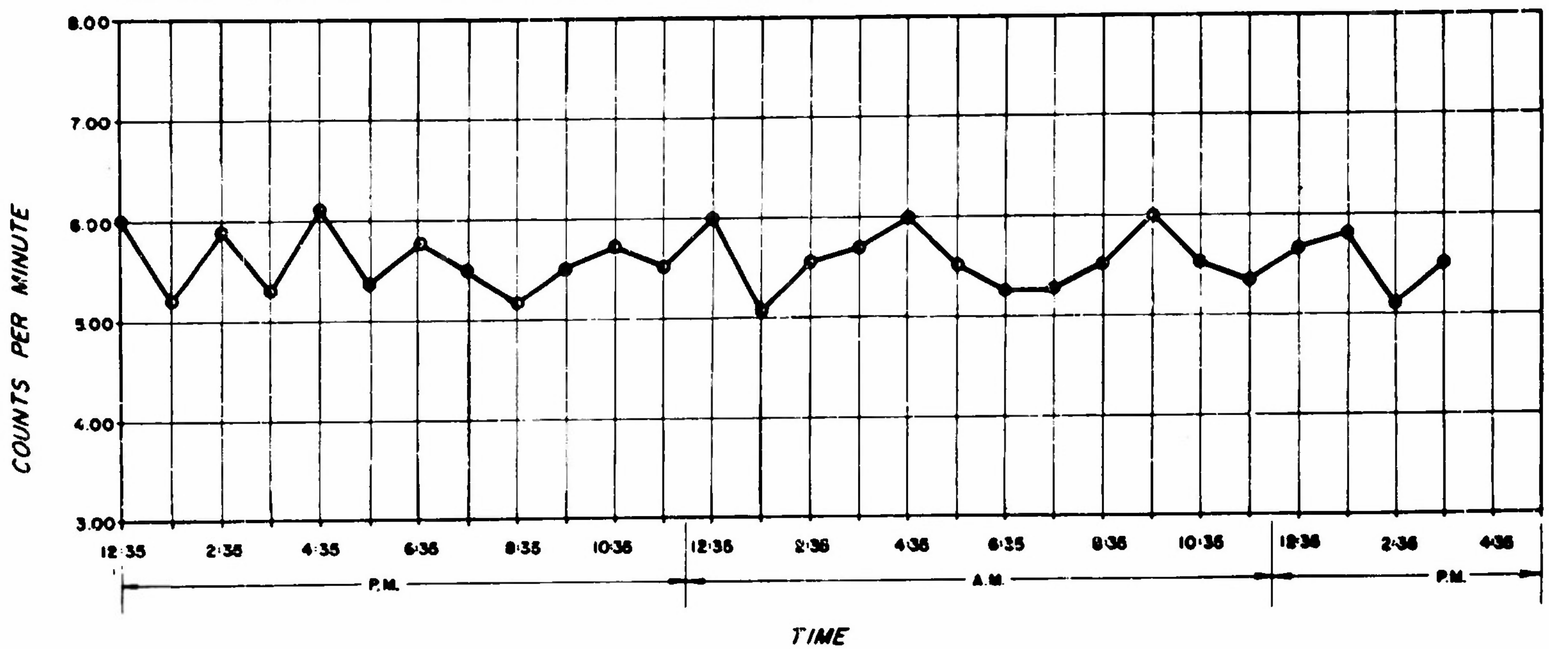


FIGURE 4

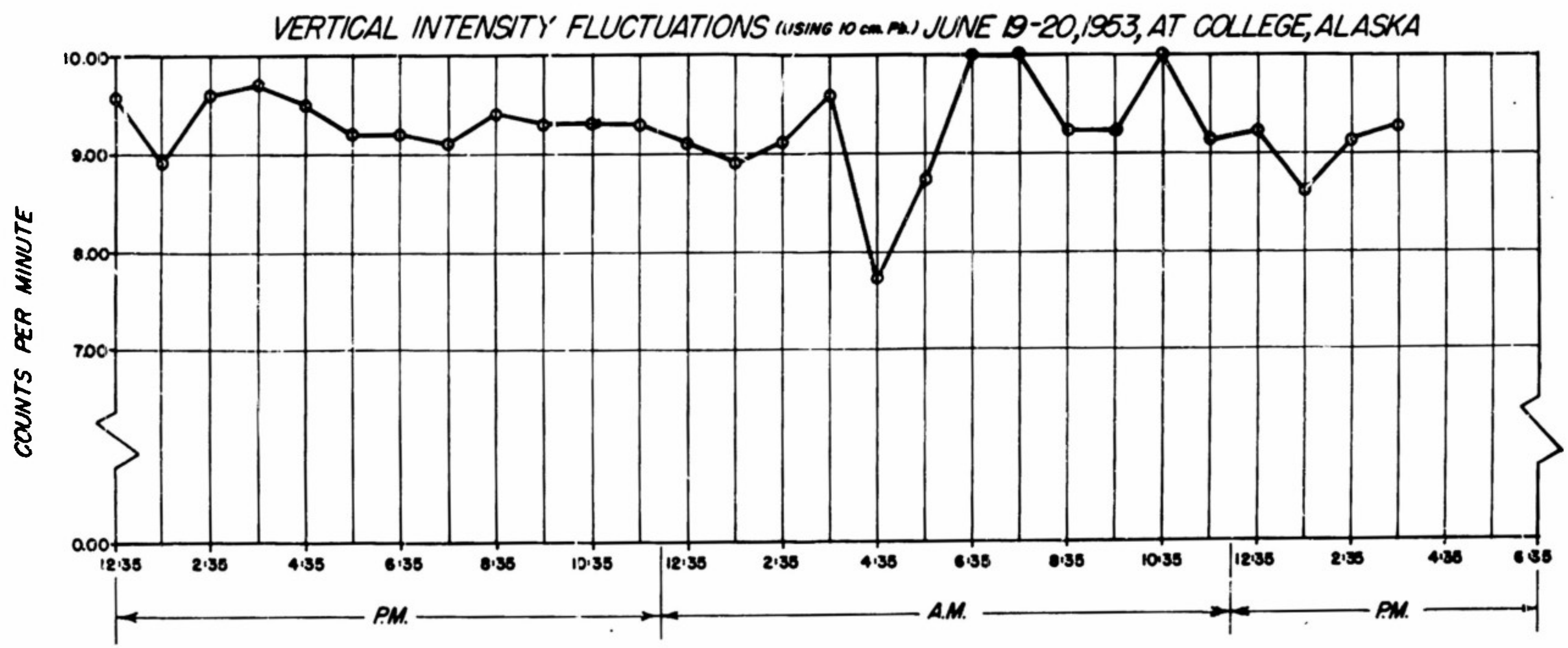

VERTICAL INTENSITY FLUCTUATIONS (USING 10 cm. Pb.) JUNE 19-20, 1953, AT COLLEGE, ALASKA
COUNTS PER MINUTE
10.00
9.00
8.00
7.00
0.00
12:35
2:35
4:35
6:35
8:35
10:35
12:35
2:35
4:35
6:35
8:35
10:35
12:35
2:35
4:35
6:35
P.M.
A.M.
P.M.

FIGURE 4a

Mount Evans, Colorado (4,300 meters)

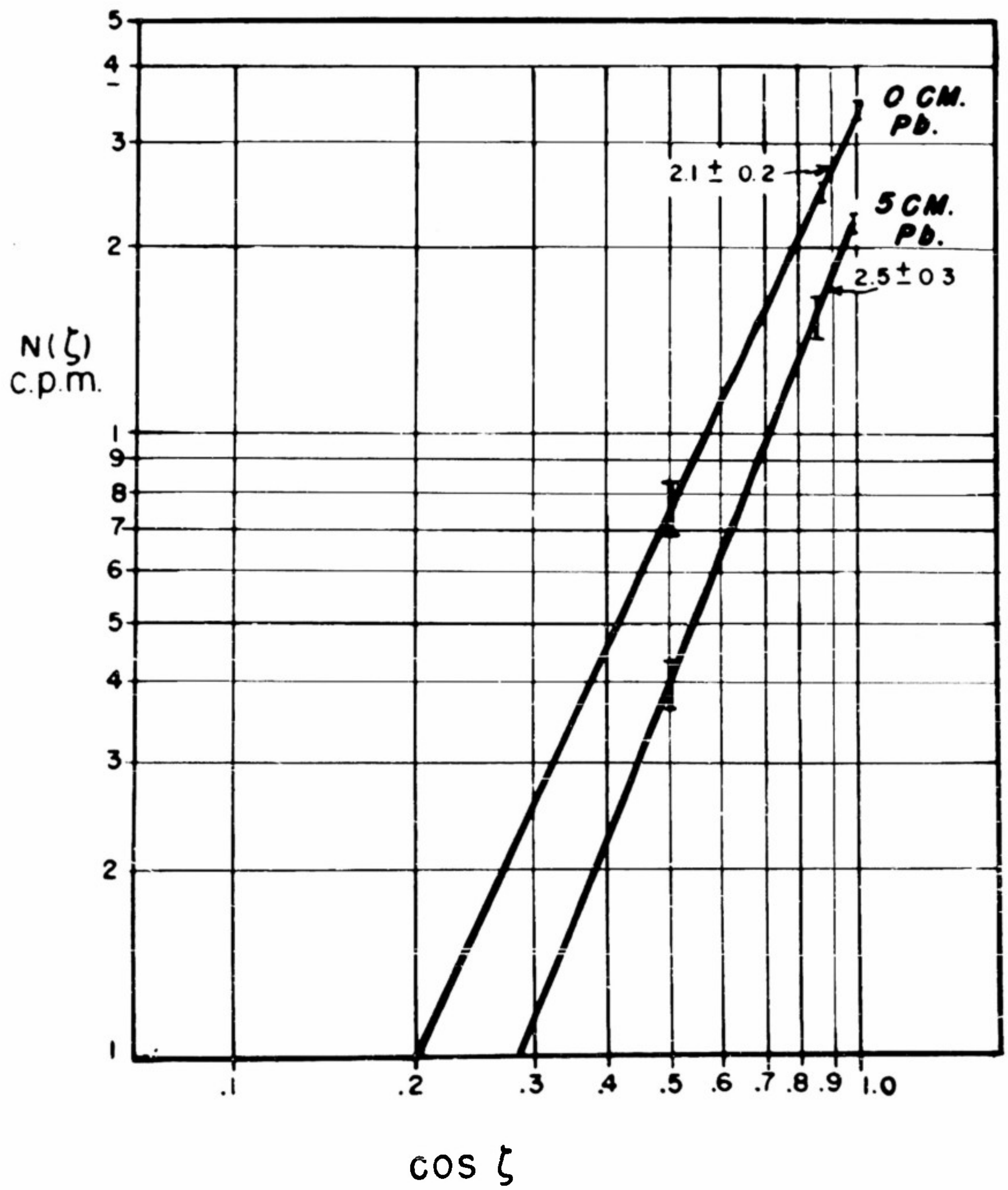


FIGURE 5

Mount Evans, Colorado (4,300 meters)

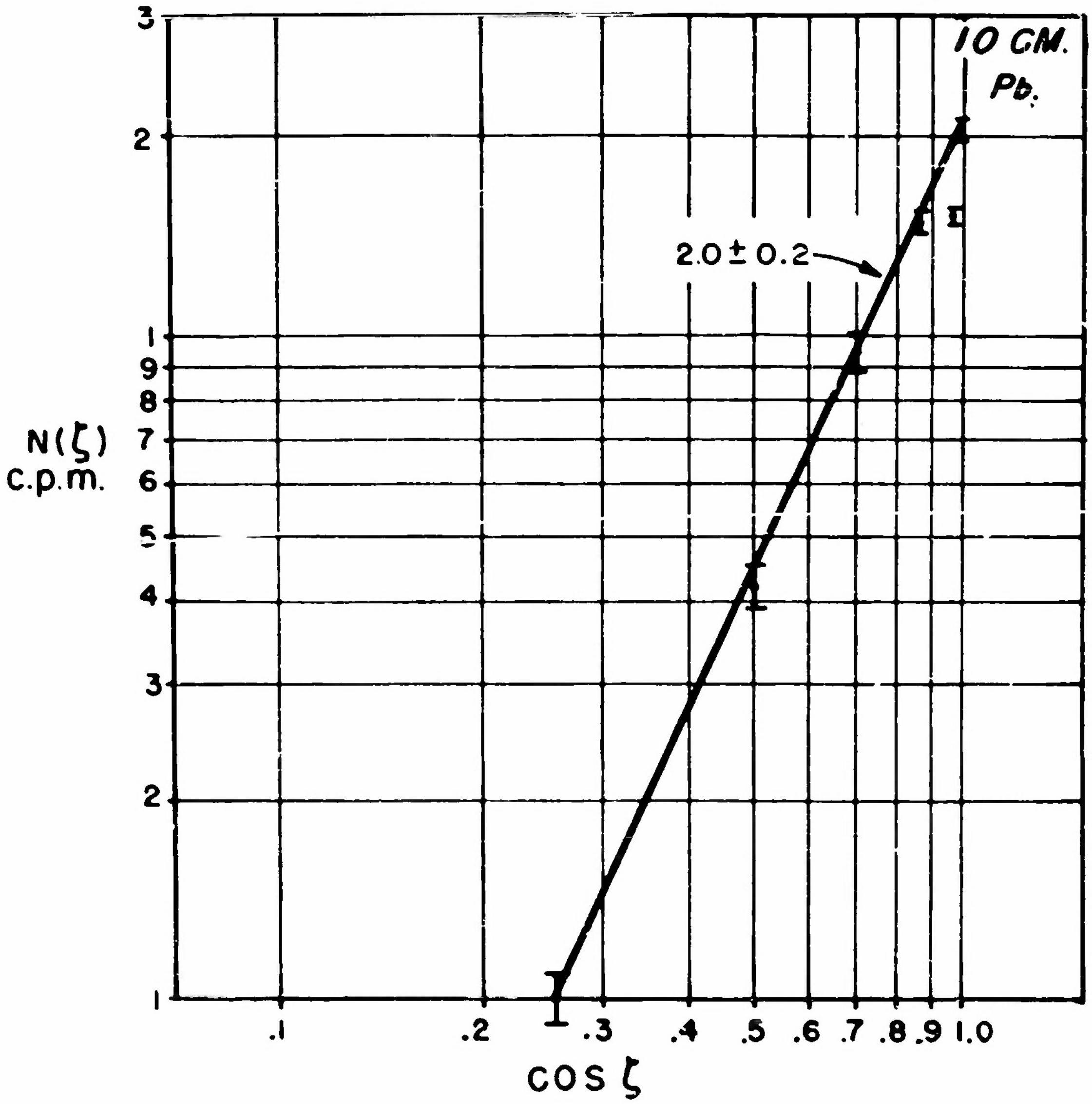


FIGURE 6

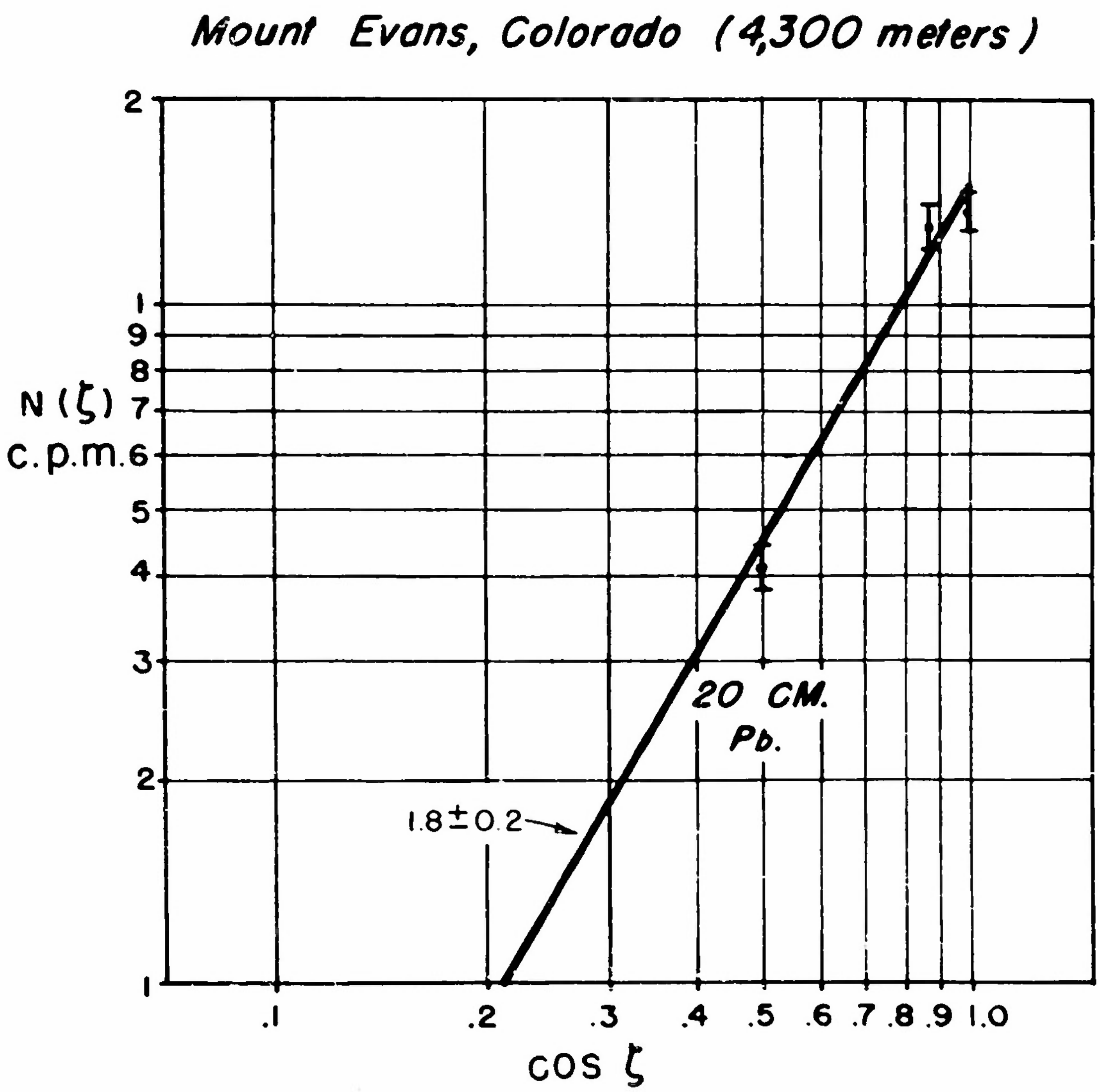
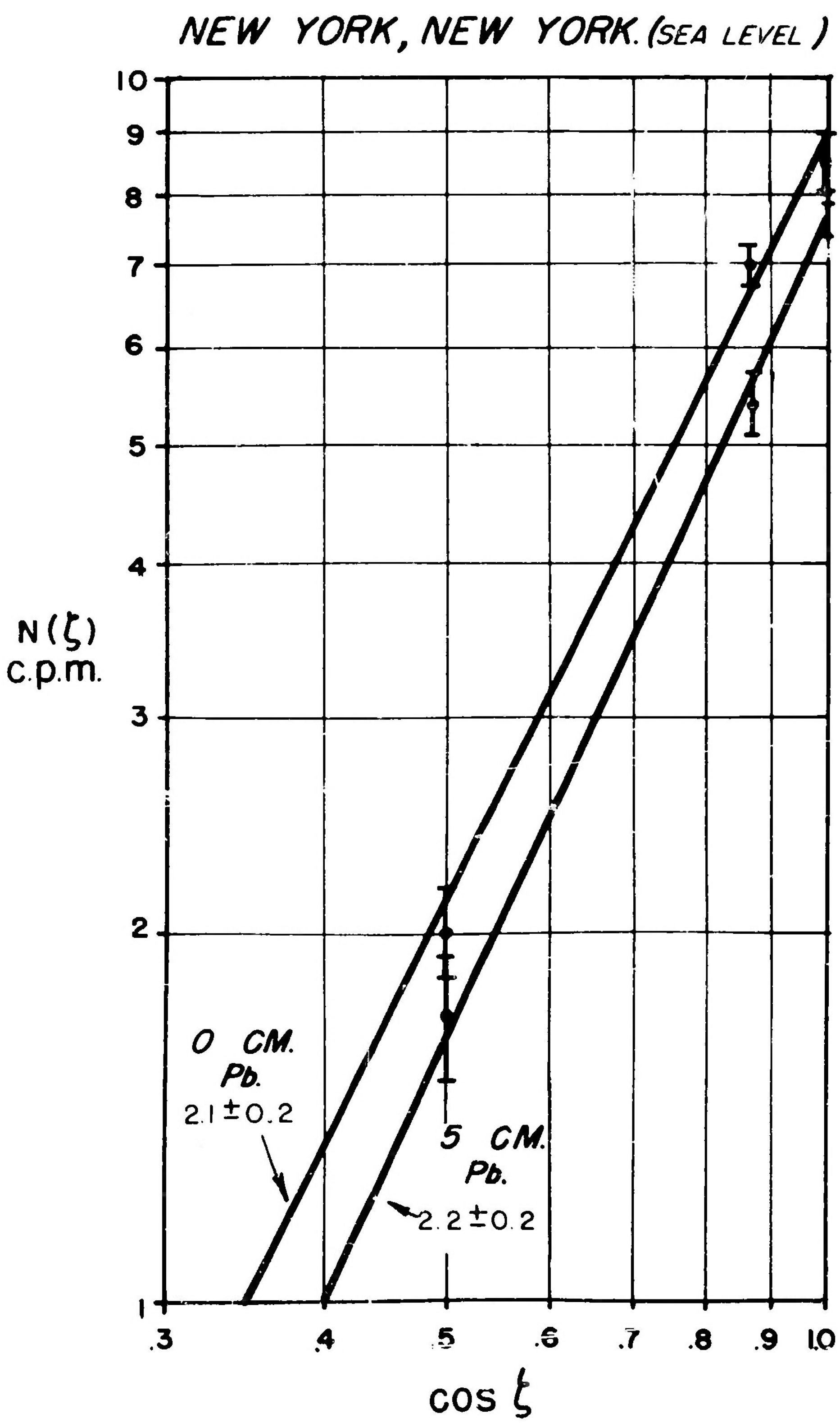

Mount Evans, Colorado (4,300 meters)
N (ζ)
c.p.m.
2
1
9
8
7
6
5
4
3
2
1
20 CM.
Pb.
1.8±0.2
.1
.2
.3
.4
.5
.6
.7
.8
.9
1.0
cos ζ

FIGURE 7



NEW YORK, NEW YORK. (SEA LEVEL)
10
9
8
7
6
5
4
3
2
1
N(ζ)
c.p.m.
0 CM.
Pb.
2.1 ± 0.2
5 CM.
Pb.
2.2 ± 0.2
.3
.4
.5
.6
.7
.8
.9
1.0
cos ζ

FIGURE 8

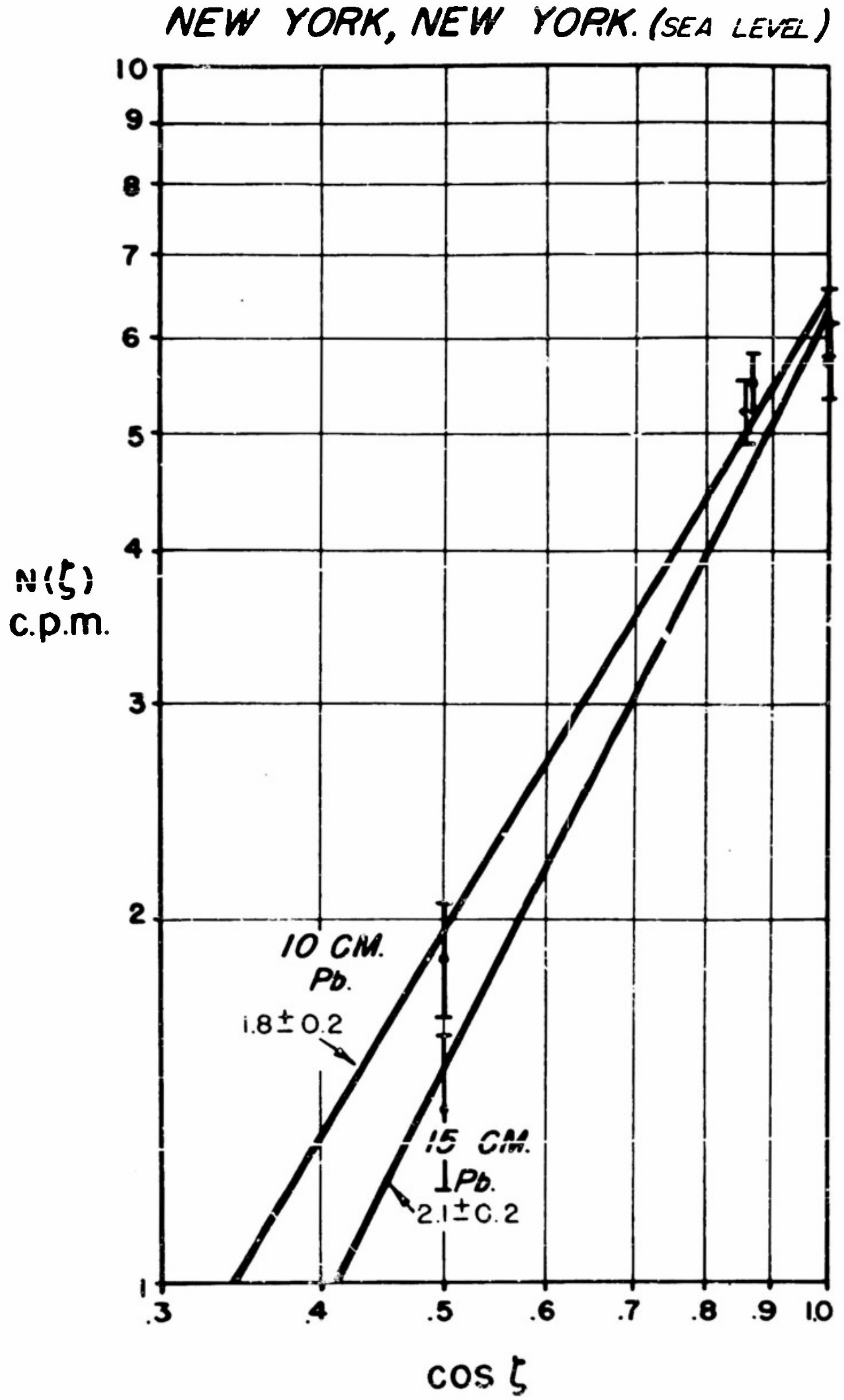
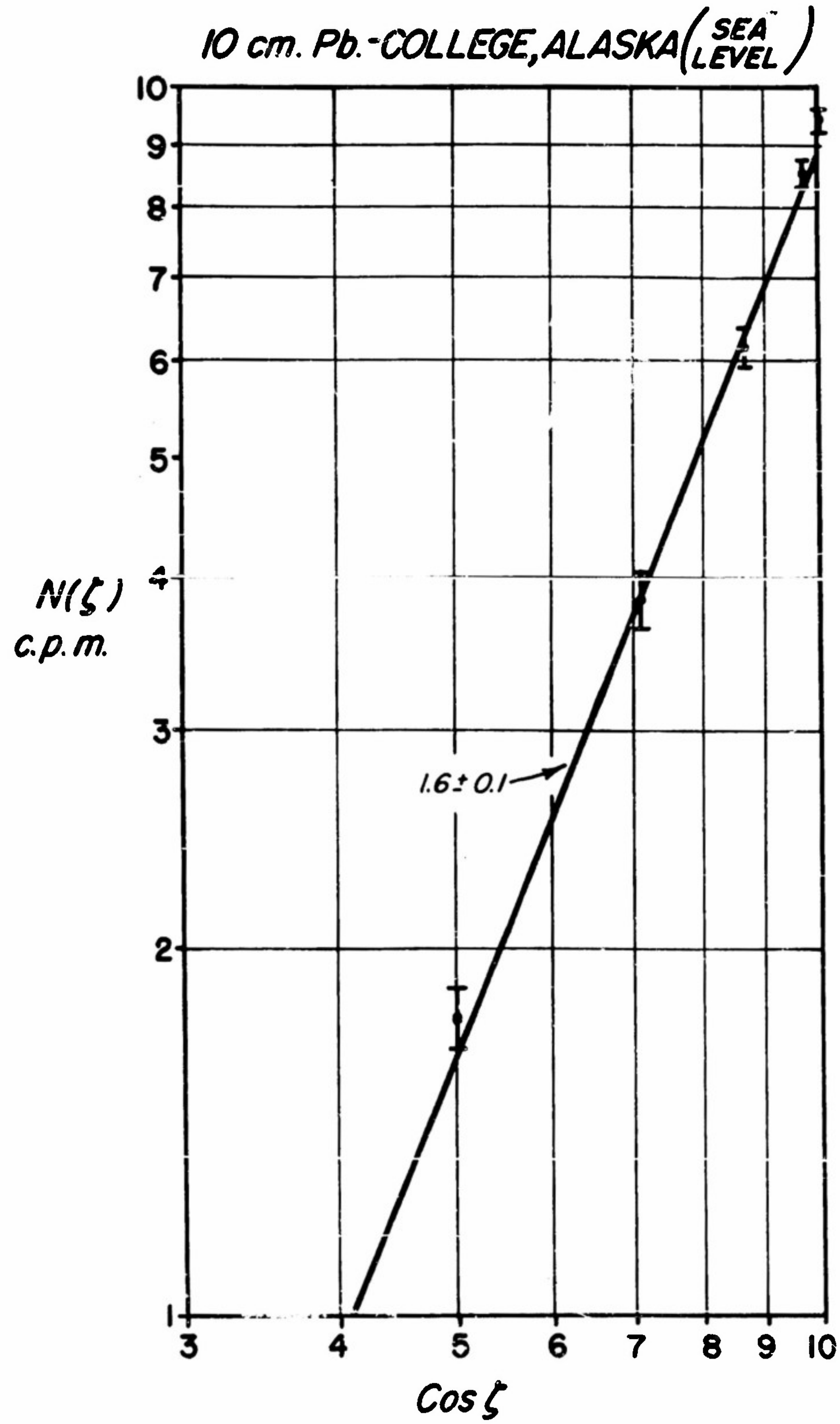

NEW YORK, NEW YORK. (SEA LEVEL)
10
9
8
7
6
5
4
3
2
1
N(ζ)
c.p.m.
10 CM.
Pb.
1.8 ± 0.2
15 CM.
Pb.
2.1 ± 0.2
.3
.4
.5
.6
.7
.8
.9
1.0
cos ζ

FIGURE 9



10 cm. Pb.-COLLEGE, ALASKA (SEA LEVEL)
N(ζ) c.p.m.
1.6 ± 0.1
Cos ζ
10
9
8
7
6
5
4
3
2
1
3
4
5
6
7
8
9
10

FIGURE 10

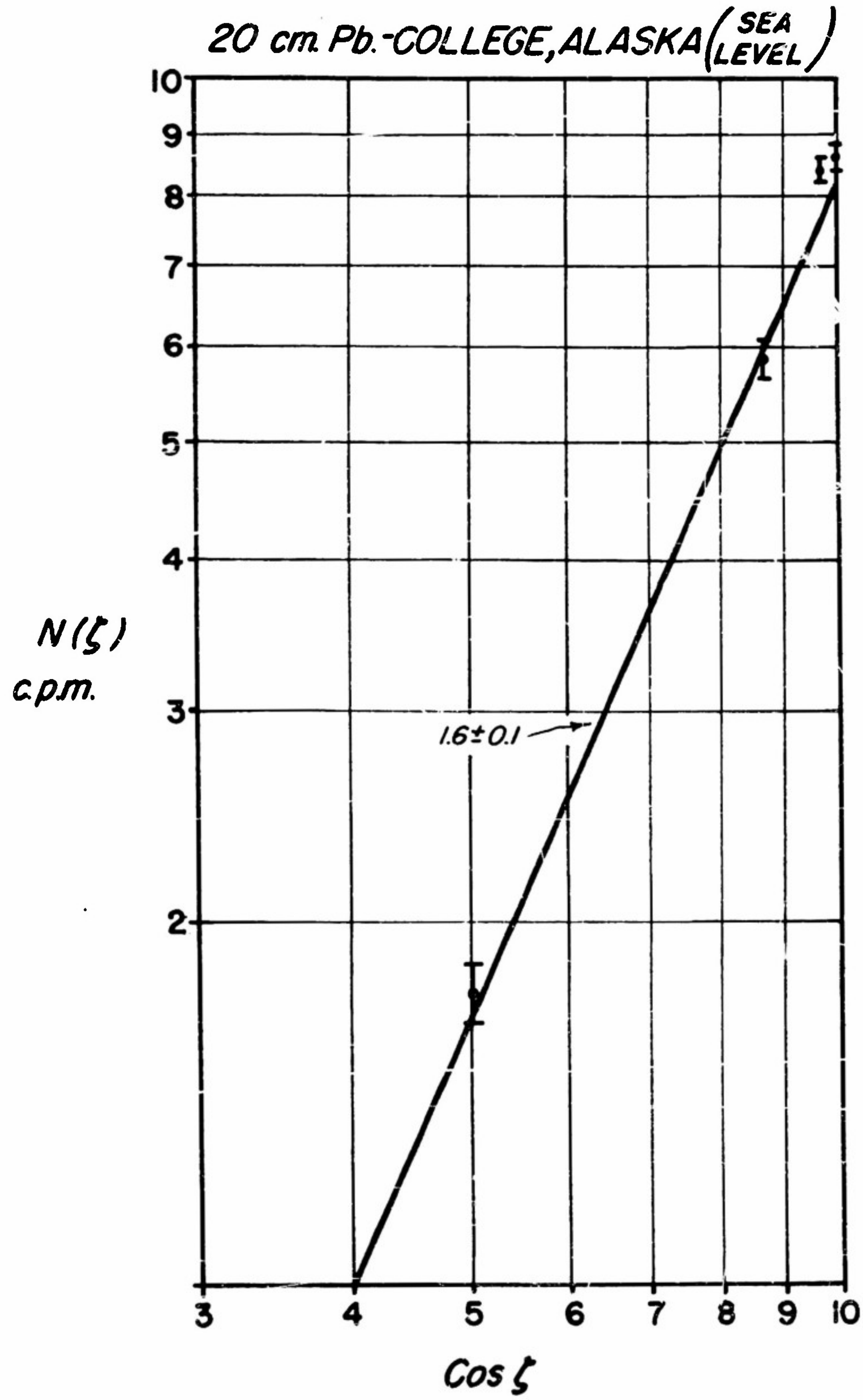
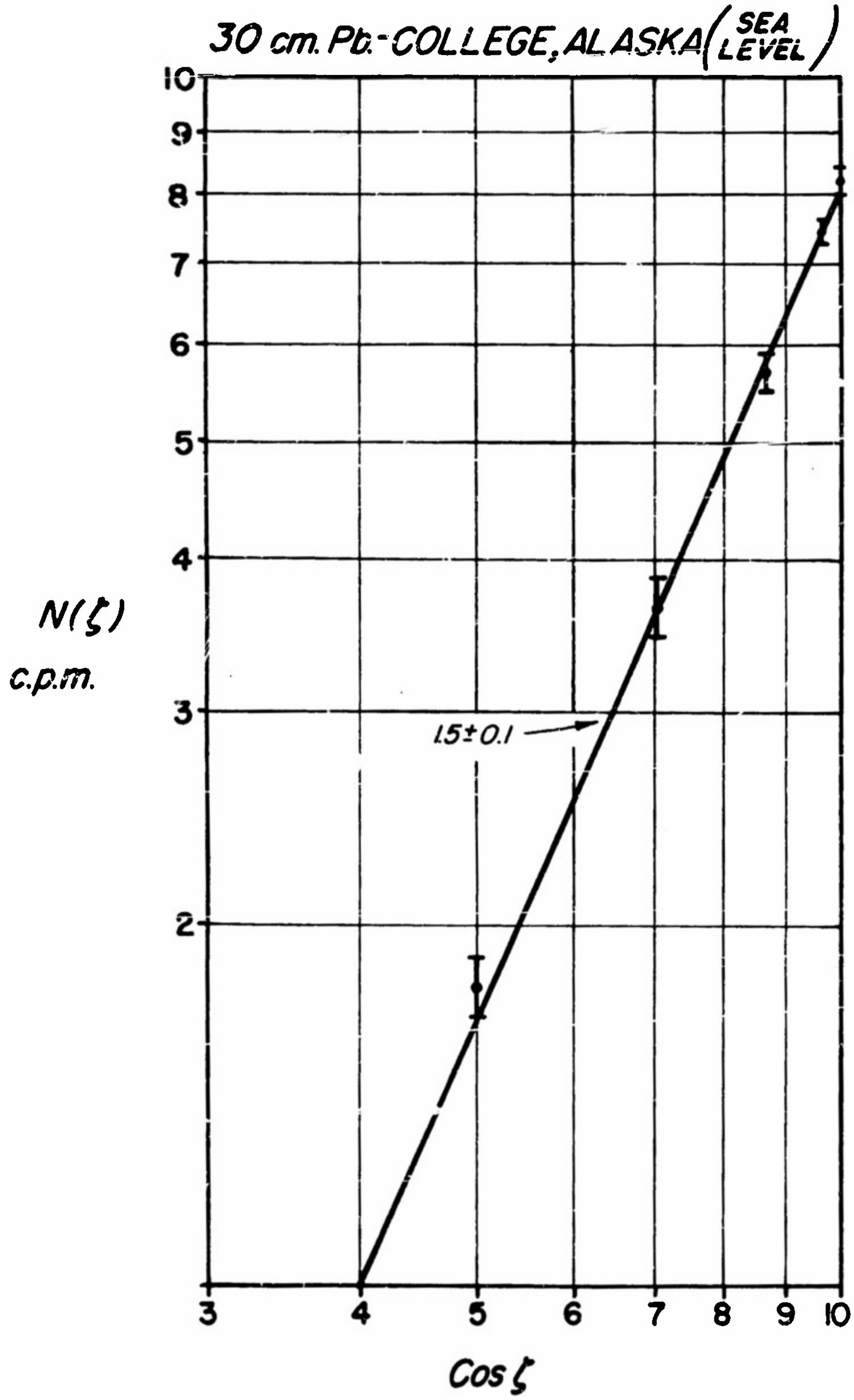

20 cm. Pb.-COLLEGE, ALASKA (SEA LEVEL)
10
9
8
7
6
5
4
3
2
N(ζ)
c.p.m.
1.6±0.1
3
4
5
6
7
8
9
10
Cos ζ

FIGURE 11



30 cm. Pb-COLLEGE, ALASKA (SEA LEVEL)
10
9
8
7
6
5
4
3
2
N(ζ)
c.p.m.
1.5±0.1
3
4
5
6
7
8
9
10
Cos ζ

FIGURE 12

PLOT OF N VERSUS P_{MIN}

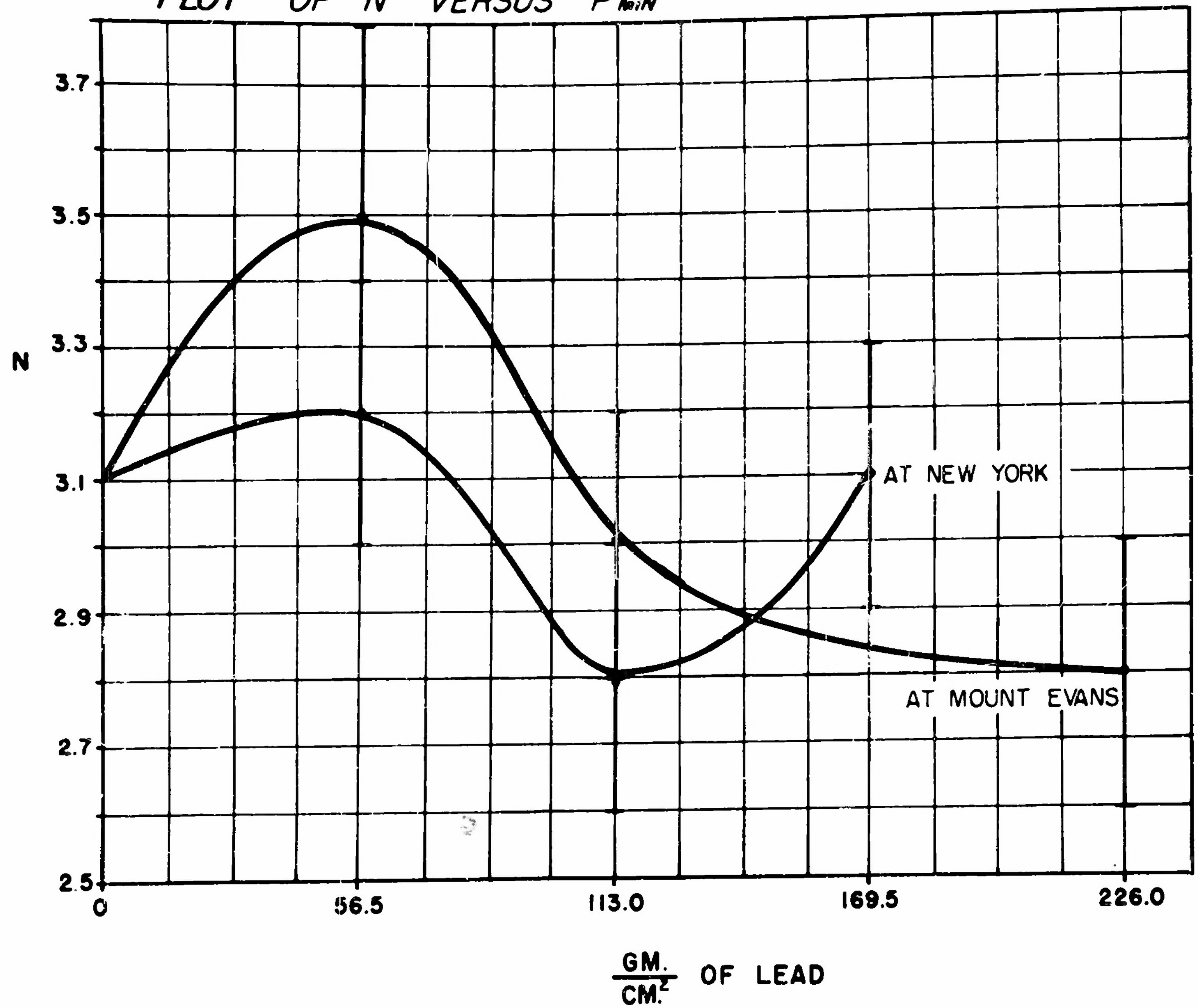


FIGURE 13

TABLE 2.

(USING EQUATION 5)

STATION	N(γ) OBSERVED COUNTS/MINUTE	N(γ) PREDICTED COUNTS/MINUTE	PER-CENT DIFFERENCE (O – P)/P X 100
Mt. Evans	N(0°): 34.13	34.13	0
0 cm. Pb.	N(30°):24.40	22.46	+8.64
	N(60°): 7.75	7.16	+8.24
Mt. Evans	N(0°) :23.06	23.06	0
5 cm. Pb.	N(30°):15.68	14.17	+10.65
	N(60°): 4.09	3.67	+11.44
Mt. Evans	N(0°) :20.43	20.43	0
10 cm.Pb.	N(15°):15.13	15.92	-4.93
	N(30°):14.89	13.29	+12.03
	N(45°): 9.54	9.00	+ 6.00
	N(60°): 4.21	4.47	-5.81
	N(75°): 0.94	1.20	-21.66
Mt. Evans	N(0°) :13.80	13.80	0
20.2 cm. Pb.	N(30°):13.03	9.09	+43.3
	N(60°): 4.14	3.34	+23.9
New York	N(0°) : 8.54	8.54	0
0 cm. Pb.	N(30°): 7.06	5.45	+29.6
	N(60°): 2.03	1.71	+19.88
New York	N(0°) : 7.82	7.82	0
5 cm. Pb.	N(30°) :5.39	4.99	+8.01
	N(60°) :1.72	1.56	+11.68

TABLE 2 (CONTINUED).

STATION	N(γ) OBSERVED	N(γ) PREDICTED	PER-CENT DIFFERENCE
New York	N(0°) : 6.23	6.23	0
10 cm. Pb.	N(30°): 5.51	4.25	+29.6
	N(60°): 1.86	1.59	+17.0
New York	N(0°) : 5.71	5.71	0
15 cm. Pb.	N(30°): 5.19	3.76	+38.0
	N(60°): 1.39	1.17	+18.8
College	N(0°) : 9.40	9.40	0
10 cm. Pb.	N(15°): 8.49	33.00 (?)	
	N(30°): 6.13	6.61	-7.26
	N(45°): 3.81	4.84	-21.2
	N(60°): 1.75	2.77	-36.8
	N(75°): 0.51	0.97	-47.4
College	N(0°) : 8.66	8.66	0
20 cm. Pb.	N(15°): 8.40	31.04 (?)	
	N(30°): 5.88	6.17	-4.70
	N(60°): 1.75	2.61	-33.0
	N(75°): 0.48	0.92	-47.8
College	N(0°) : 8.23	8.23	0
30 cm. Pb.	N(15°): 7.44	31.31 (?)	
	N(30°): 5.69	7.36	-22.6
	N(45°): 3.65	4.48	-18.5
	N(60°): 1.76	2.65	-33.6
	N(75°): 0.46	0.70	-34.2

TABLE 3.

(USING EQUATION 4)

STATION	N(𝛾) OBSERVED COUNTS/MINUTE	N(𝛾) PREDICTED COUNTS/MINUTE	PER-CENT DIFFERENCE (O-)/P x 100
Mt. Evans	N(0°): 34.13	34.13	0
0cm. Pb.	N(30°):24.40	25.36	-3.79
	N(60°): 7.75	8.53	-9.14
Mt. Evans	N(0°) :23.06	23.06	0
5 cm. Pb.	N(30°):15.68	17.13	-8.46
	N(60°): 4.09	5.77	-29.11
Mt. Evans	N(0°) :20.43	20.43	0
10 cm. Pb.	N(15°):15.13	18.18	-16.78
	N(30°):14.89	15.18	-1.91
	N(45°): 9.54	10.28	-7.20
	N(60°): 4.21	5.11	-17.61
	N(75°): 0.94	1.37	-31.39
Mt. Evans	N(0°) :13.80	13.80	0
20.2 cm. Pb.	N(30°):13.03	10.25	+27.12
	N(60°): 4.14	3.45	+20.00
New York	N(0°) : 8.54	8.54	0
0 cm. Pb.	N(30°): 7.06	6.35	+11.19
	N(60°): 2.03	2.14	-5.14
New York	N(0°) : 7.82	7.82	0
5 cm. Pb.	N(30°): 5.39	5.81	-7.23
	N(60°): 1.72	1.96	-12.24

TABLE 3 (CONTINUED).

STATION	$N(\psi)$ OBSERVED	$N(\psi)$ PREDICTED	PER-CENT DIFFERENCE
New York	$N(0°)$: 6.23	6.23	0
10 cm. Pb.	$N(30°)$: 5.51	4.63	+19.01
	$N(60°)$: 1.86	1.56	+19.23
New York	$N(0°)$: 5.71	5.71	0
15 cm. Pb.	$N(30°)$: 5.19	4.24	+22.41
	$N(60°)$: 1.39	1.43	- 2.78
College	$N(0°)$: 9.40	9.40	0
10 cm. Pb.	$N(15°)$: 8.49	8.89	- 4.49
	$N(30°)$: 6.13	7.47	-17.9
	$N(45°)$: 3.81	5.40	-29.4
	$N(60°)$: 1.75	3.10	-43.5
	$N(75°)$: 0.51	0.22	+131.8
College	$N(0°)$: 8.66	8.66	0
20 cm. Pb.	$N(15°)$: 8.40	8.20	+ 2.43
	$N(30°)$: 5.88	6.90	-14.7
	$N(60°)$: 1.75	2.90	-39.6
	$N(75°)$: 0.48	0.21	+128.5
College	$N(0°)$: 8.23	8.23	0
30 cm. Pb.	$N(15°)$: 7.44	7.81	- 4.73
	$N(30°)$: 5.69	6.63	-14.2
	$N(45°)$: 3.65	4.90	-25.5
	$N(60°)$: 1.76	2.91	-39.5
	$N(75°)$: 0.46	0.19	+142.1

VI. Sources of Error.

The first, and most important source of error to be considered is statistical in nature. This is caused by the short term random fluctuations in the intensity of the cosmic radiation. It was found that 24 hours provided sufficient time for the majority of such fluctuations to average out, in any given run.

If several counters are used in a coincidence arrangement, there will be an error caused by accidental coincidences in the resolving time of the circuit, T. For our case, where three counters were used, it can be shown[8] that the number of accidentals per second is given by:

$$A_{123} = 3T^2N_1N_2N_3,$$

where N_1, N_2, and N_3 are the individual counting rates. This error was investigated for our two microsecond resolving time, and found to be negligible, since it gave rise to a counting rate of 7×10^{-7} counts per minute.

In New York, it was found necessary to employ a constant voltage transformer in the AC power line, after an hourly check on the vertical intensity was taken over a 24 hour period. The power stability at the Geophysical Institute in Alaska was much worse, necessitating the use of an electronic regulator in the line. Figures 4 and 4a illustrate the counting rate after these remedial measures had been taken. If a constant voltage transformer had not been used in New York, it would have been essential to time the vertical intensity monitoring runs so that they did not occur at a line voltage peak or minimum.

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8. Korff, reference 7, p. 145.

Because the telescope has a finite opening, there is an error in its zenith resolution. Greisen[3] shows that the error is very small for openings such as the one employed here, and may safely be neglected.

Another possible error lies in the accuracy of the zenith angle measurement. However, sufficient precautions were taken so that this error is felt to be insignificant.

Table 4.

Absorber	gm. cm.$^{-2}$	P_{min}
5 cm. Pb.	56.5	92 Mev/c
10 cm. Pb.	113.0	225 Mev/c
15 cm. Pb.	169.5	300 Mev/c
20 cm. Pb.	228.3	370 Mev/c
30 cm. Pb.	339.0	510 Mev/c
Above Mt. Evans; average vertical	620	1.13 Bev/c
Above New York and College; average vertical	1040	2.05 Bev/c

9. D.J.X. Montgomery, Cosmic Ray Physics (Princeton, Princeton University Press, 1949), Appendix E.

VII. Discussion.

In the calculation of the counting rates predicted by Equation (5), the mu-meson rest mass was taken[10] as 215 ± 5 m_e, where m_e is the mass of the electron. The mean lifetime (at rest) was taken[11] as 2.15 ± 0.07 microseconds.

The minimum momenta required to penetrate the atmosphere and the absorber are listed in Table 4. In the calculations, the unit of momentum used was $mh/\tau_0 = 17.8 \times 10^{-14}$ gm. cm./sec.

The per-cent differences listed in tables 2 and 3 appear to be quite large. However, over half of the difference is due to the procedure employed in the calculations. Since the vertical counting rate is most accurate (because the largest number of counts/unit are observed at 0°), it was assumed that each of the curves passes through it. This gives rise to larger per-cent differences at the other angles. An alternate procedure is to assume that the curve passes some arbitrary distance from $N(0^\circ)$, which would minimize the other differences. This, however, is statistically questionable.

Figures five and eight indicate that, within the limits of the experimental error, there is no appreciable difference in the zenith angle variation of the soft component as against the hard (meson)

10. R.B. Brode, Revs. Mod. Phys. 21, 37 (1949)

11. B. Rossi and N. Nereson, Phys. Rev. 62, 417 (1942); 64, 199 (1943)

component. Greisen[3] finds an exponent of 3.6 for a soft component with energies in the interval 4×10^6 ev to 2×10^7 ev, and says that the "very soft" component has an exponent which is less than 3 because of scattering. He finds an exponent of 2.1 for the hard component at mountain elevations, while Zar working at sea-level, reports that the exponent decreases from 2.04 to 1.85 as P_{min} increases from 283 Mev/c to 960 Mev/c. Our results are in agreement with both of these papers (within the limits of the experimental error) (See figure 13). However, Zar[4] remarks that he used several different methods in his work, and that the results varied. Indeed, for one of the experiments, the "decrease" of n with increasing P_{min} disappeared, leaving the result in some doubt.

Now since a straight line may be fitted reasonably well to the data, we may say that Equation (4) is valid for this momentum range within the limits of the experimental errors. In fact, on the basis of the Colorado and New York data alone, there would not appear to be any momentum dependence, although the experimental errors mask the effect predicted by Equation (5).

However, the results from the campus of the University of Alaska (see figures 10, 11, and 12) indicate a distinctly lower value of n at 64° geomagnetic latitude. That is, for comparable thickness of absorber, the northern value of the exponent is lower than that found at New York and Mount Evans.

Del Rosario and Davila-Aponte[12] have reported a distinctly different shape for the spectrum at a latitude of 29° from what is found at 50°. It is possible that the spectrum at 64° is much flatter than it is at 50° since the cut-off there (0.8 Bev) allows many more low energy mesons to be created. We are interested only in mesons with a production momentum of at least 2 Bev/c, so that they may reach sea-level. Thus, the effective primary cut-off is 2 Bev, which is not much less than the 2.6 Bev value at 50°. Thus, the spectrum change appears to be larger than the Geomagnetic Analysis would predict.

A conclusion regarding one of the assumptions involved in the derivation of Equation (2) may be reached. Since the data do not fall very far from a straight line, even at large angles from the vertical, we may conclude that local meson production (even at mountain altitudes) is not an appreciable factor, because local production would tend to "smear out" the effect.

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12. L. Del Rosario and J. Davila-Aponte, Phys. Rev. 88, 998 (1952)

VIII. Conclusions

We have found that Equation (5) does not lead to a more accurate prediction of the zenith angle variation (at a given latitude), but is at least as valid as the $\cos^{2.1}\gamma$ variation. On the basis of this, we have evaluated the meson momentum spectrum at production, and have found that the exponent shows a slight tendency to decrease with increasing minimum momentum, although the experimental errors are large.

Further, we have been able to conclude that local meson production is not an appreciable factor, even at mountain altitudes. Thus, one may regard the mesons as coming from a layer at a very high altitude (about 20,000 meters).

A variation of the meson spectrum with geomagnetic latitude has been found, on the basis of a change in the zenith angle effect. This has led to the conclusion that the primary spectrum has changed with latitude. The change is in qualitative agreement with the Geomagnetic Analysis, but appears to be larger than predicted.

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