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Studies of the Electric Potential between Key West, Florida, and Havana, Cuba

by

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Director

ABSTRACT

The mass transport of the Florida Current across the Key West - Havana section has been studied by means of the associated electrical potential due to the geomagnetic field. An analysis of the seasonal, daily, and short term variations of the transport is presented, as well as a critical discussion of the limitations of the method of measurement employed.

The average mass transport for the period of August 1952 through July 1953 was found to be $27 \times 10^{6}\ m^{3}$/sec. Rapid seasonal variations, possibly of meteorological origin, were observed. The daily variations in the transport due to influence of the tides in the Atlantic and Gulf of Mexico reach 50% of the average transport. Short term variations in the measured potential are related to the rate of change of the geomagnetic field.

I Introduction

Motion of an electrolyte in the geomagnetic field leads to the development of an induced potential difference at right angles to the direction of motion. It has recently been pointed out (Malkus and Stern, 1952) that the potential, φ, across a stream is related to the velocity, v, of the stream:

$$\int_0^d [\varphi(\infty,d) - \varphi(-\infty,d)]\,dz = \int_0^d \int_{-\infty}^{+\infty} H_z v \, dz \, dx \qquad (1)$$

If the bottom is flat and nonconducting, and the horizontal variation of the earth's magnetic field is neglected this reduces to

$$T = \frac{Vd}{H_z} \qquad (2)$$

where T - transport, cubic meters per second

V - electrical potential difference, volts

d - the depth of the water, meters

H_z - vertical component of the geomagnetic field, webers per square meter

An analysis for an elliptical sea bed of finite conductivity (Longuet-Higgins, 1949) shows that the effect of sea bed conductivity is to reduce the potential between the edges of the stream by the factor

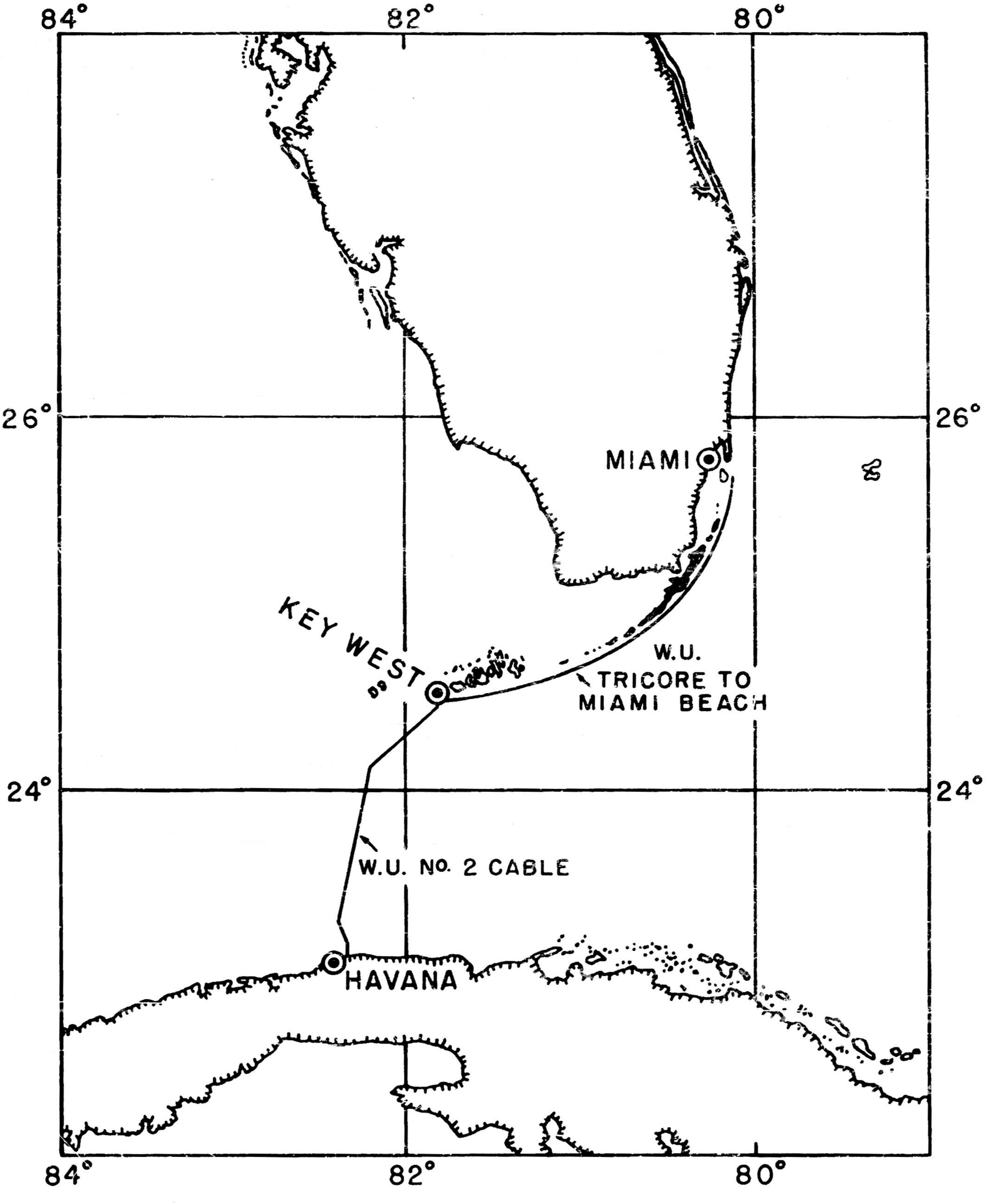
$$1 \Big/ 1 + \frac{w}{2d}\frac{\sigma_B}{\sigma_S} \qquad (3)$$

where w - width of stream bed

d - maximum depth

σ_B - electrical conductivity of sea bed

σ_S - electrical conductivity of the water



CABLE LOCATION

FIG. I

Application of equations (2) and (3) makes possible the continuous measurement of the net transport across any section of an ocean current, provided that the potential difference V across the section can be determined by establishing a suitable electrical connection across the stream. The kind cooperation of the Western Union Telegraph Company which operates communications cables between Key West and Havana (Figure 1) has made possible such measurements of the Florida Current during the past year.

The equipment used consists of two silver - silver chloride electrode clusters (von Arx, unpublished, 1952) placed respectively on the Northern and Southern edge of the Florida Current, connected by Western Union Cable to a recording potentiometer located in Key West, Florida (Figure 2). Over sixty hours of continous data have been obtained on every week-end, with minor interruptions, since August 1952.

The data are obtained as a continous curve of potential versus time. This curve exhibits fluctuations which may be divided into a number of categories according to their origin.

A 24-hour record will have the following characteristics: (a) It has an average value which is always positive and represents the average transport during the given interval. A study of changes in the 24-hour averages gives a picture of the seasonal variation in the transport. (b) The data shows variations of tidal period which may be ascribed to the effect of the Atlantic and Gulf tides on the transport through the Florida straits. (c) If the tidal variations are subtracted from the record, there remain irregular, short term fluctuations which on some days are wholly absent and on others are

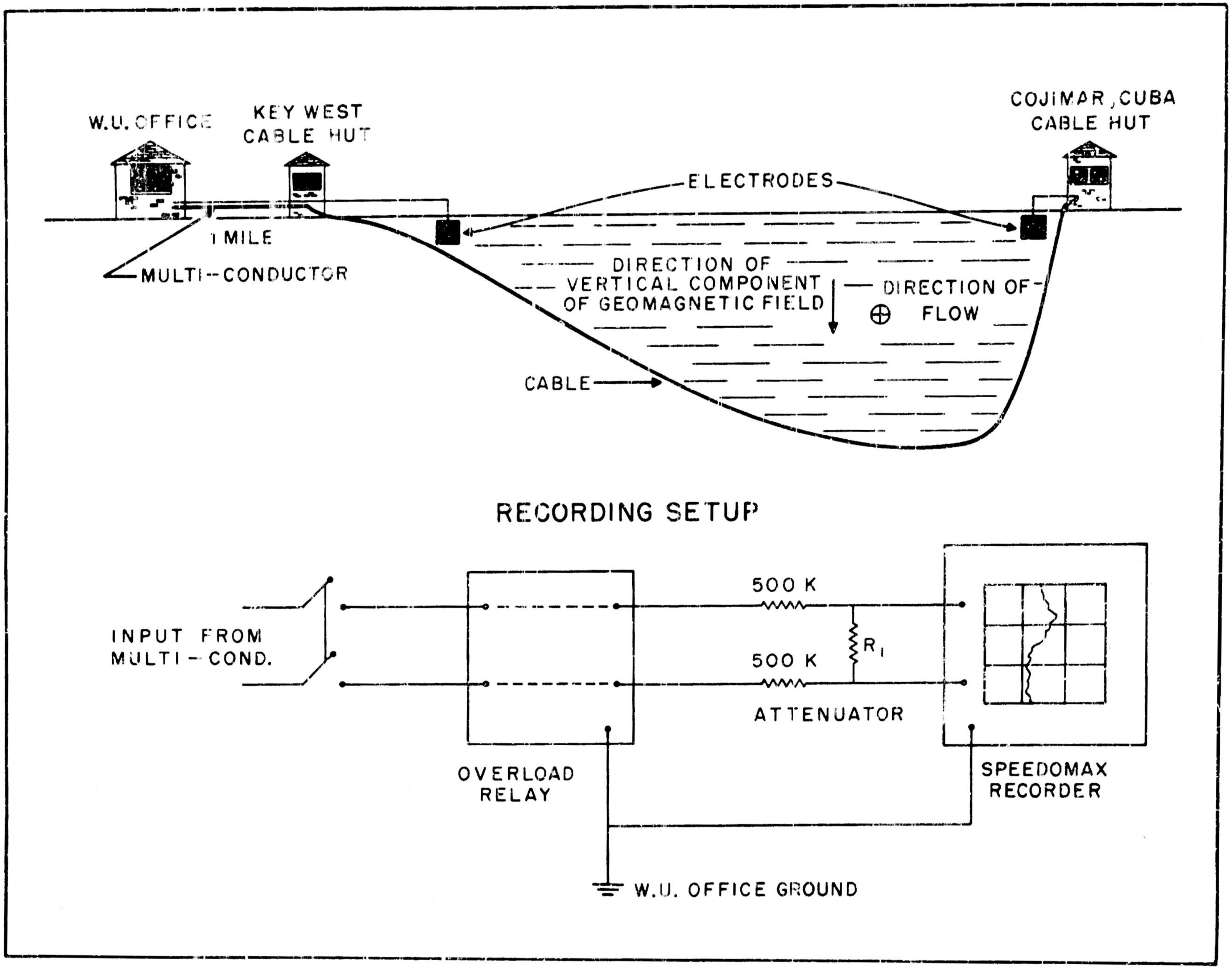

W.U. OFFICE
KEY WEST CABLE HUT
COJIMAR, CUBA CABLE HUT
ELECTRODES
1 MILE
MULTI--CONDUCTOR
DIRECTION OF VERTICAL COMPONENT OF GEOMAGNETIC FIELD
DIRECTION OF FLOW
CABLE
RECORDING SETUP
INPUT FROM MULTI-COND.
OVERLOAD RELAY
500 K
500 K
R1
ATTENUATOR
SPEEDOMAX RECORDER
W.U. OFFICE GROUND

FIG. 2

greater in amplitude than either the tidal component or the average value. These fluctuations are found to be due to the variations in the horizontal component of the geomagnetic field. (d) Finally, there is noise of small amplitude and short duration which is ascribed to man-made interference.

These various aspects of the record will be discussed in the following sections.

II Computation of Transport

In order to compute the transports from the measured electrical potentials, it is necessary to estimate the importance of bottom conductivity in the Straits of Florida. Let the emf induced in the absence of bottom conductivity by a uniform stream filling the whole channel be **E** (this is equal to the potential measured across the stream for any distribution of velocity having the same transport). The effect of finite bottom conductivity may then be estimated from the equivalent circuit shown in Figure 3.

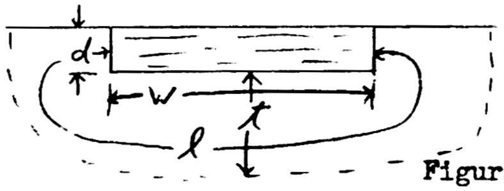
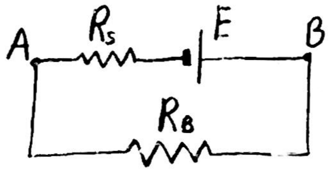


Figure 3



Equivalent Circuit

where the subscripts s and b refer to sea and bed respectively. We now approximate the effective resistance of the sea and bed by

$$R_s = \frac{W}{\sigma_s d} \quad ; \quad R_B = \frac{l}{\sigma_B t} \qquad (4)$$

Then the potential measured between points A and B is

$$V = \frac{E}{1 + \frac{w}{2d}\frac{\sigma_B}{\sigma_s}\frac{2t}{l}} \qquad (5)$$

It is instructive to compare this result with the one obtained by Longuet-Higgins for the elliptical sea bed. If we let the effective length of the sea bed path be twice its thickness, (5) reduces to Longuet-Higgin's solution. Since l must be of the order of w, we note that the sea bed current penetrates significantly to a depth equal to half the width of the stream bed. In other words, in estimating the effective conductivity of the bed we must actually use the conductivity of the underlying deep rock strata, and therefore only a rough estimate is possible.

If we now insert the following values for the Florida Straits:

w = 160 km

d = 1.68 km, maximum depth

$\sigma_B = 5 \times 10^{-5}$ mho/cm

$\sigma_s = 5 \times 10^{-2}$ mho/cm

we obtain

$$V \approx 0.95E$$

Thus the estimated corrections for the effects of bottom conductivity is of the order of 5%. The indicated correction is so small and its uncertainty so great that it was decided not to apply any systematic correction for the effect of bottom resistivity and compute the transport directly from the measured potential.

A further difficulty in relating transport to potential lies in the fact that equation (2) assumes a flat bottom. As long as the major portion of the Florida Current remains over the central portion of the Straits the use of an effective average depth should be an acceptable approximation. If, however, due to a meander, the axis should shift over the sloping bottom on the Key West side (Figure 4) erroneously high values of transport may result.

A numerical estimate of the magnitude of this effect was made using the velocity profile obtained by direct current measurements (Pillsbury, 1887) and the smoothed bottom profile shown in Figure 4. It was assumed that a shift of the axis of the current may be adequately represented by a rigid translation of the velocity profile. The results show that a meander of 10 km amplitude changes the potential by 8%, while a shift of 20 km changes the potential by 20%, a shift to the north serving to increase the potential. Little is known about the magnitude of the meanders that actually occur in this region so that the significance of this result cannot be evaluated at this time.

It should also be noted that it is possible to have a potential at right angles to the water motion even though there is no net transport across the section if the bottom is not level. Thus an elongated eddy running east over deep water and west over the shallower water close to Key West will indicate transport to the west even though there is no net transfer of water..

III The Average Transport for the Year 1952-1953

Computing in the manner described in Sections I and II the average transport of the Florida Current for the year 1952 - 1953 was found to be 27×10^6 cubic meters per second. The figure lies well within previous estimates. Direct measurements with a current

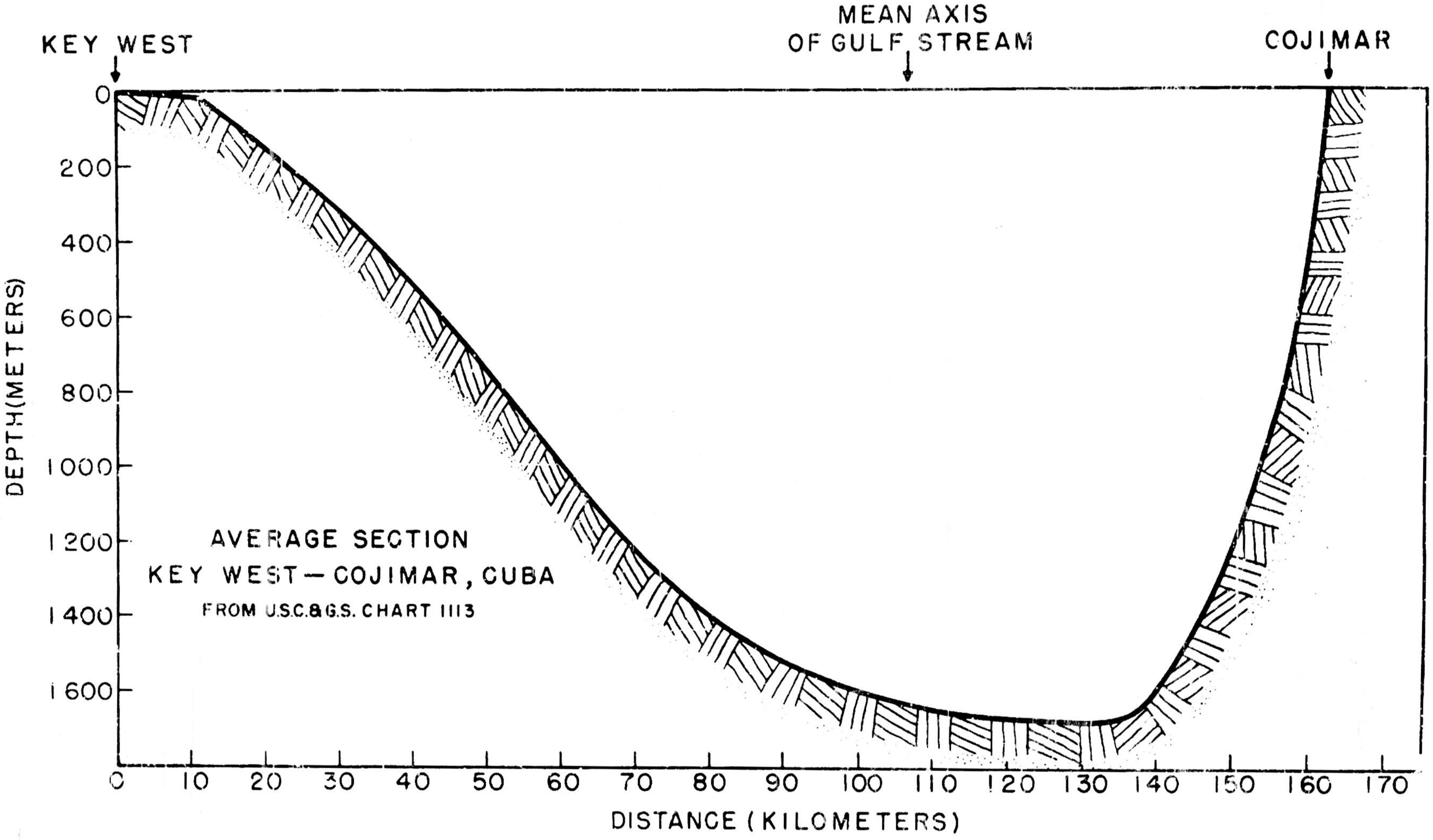

KEY WEST
MEAN AXIS OF GULF STREAM
COJIMAR
AVERAGE SECTION
KEY WEST — COJIMAR, CUBA
FROM U.S.C.&G.S. CHART 1113
DEPTH (METERS)
0
200
400
600
800
1000
1200
1400
1600
DISTANCE (KILOMETERS)
0
10
20
30
40
50
60
70
80
90
100
110
120
130
140
150
160
170

FIG. 4

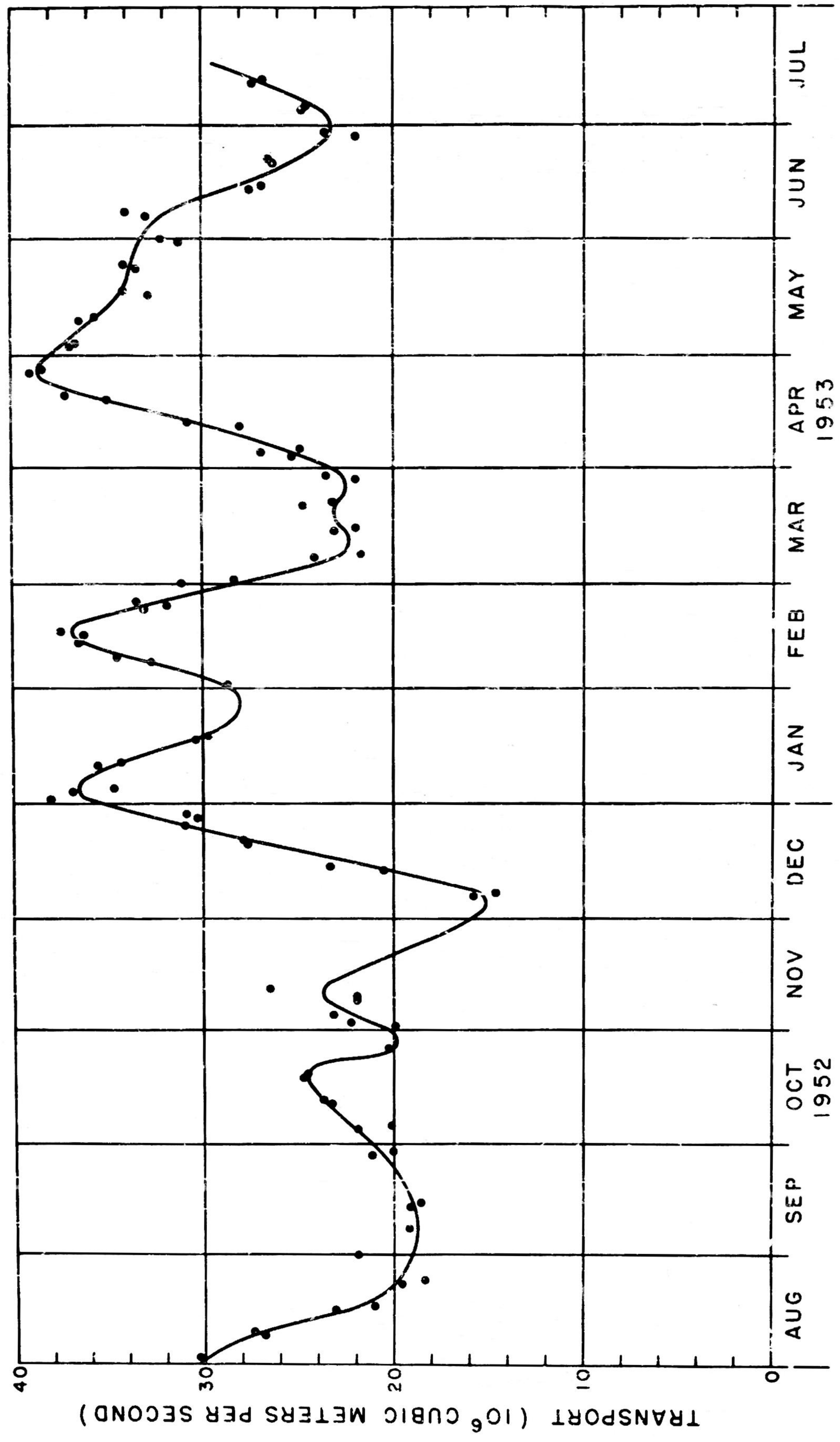
meter from an anchored ship (Pillsbury, 1887) gave a transport of $28\times10^6\ m^3/sec$. An estimate obtained from a series of dynamic sections (Montgomery, 1941) yielded a transport of 26 to $30\times10^6\ m^3/sec$. The agreement between the electrical measurements and previous estimates of transport confirms that the correction for bottom conductivity is small.

However, it will be shown in the following paragraphs that estimates of transport based on single measurements may give a very misleading impression of the actual annual average transport because large changes in the transport are observed to take place over intervals of a few weeks.

IV The Long Period Variation of the Transport

A curve of the variation of the transport of the Florida Current was obtained by plotting all 24-hour averages available during the year August 1952 through July 1953. The choice of 24-hour averages was dictated by the desire to eliminate the tidal effect which has harmonic components of approximately 12 & 24^h periods. Twenty-four hour averages were preferred to 48-hour averages in order to be able to include data from incomplete week-ends and holidays which do not have an unbroken 48-hour extent.

The resulting curve is shown in Figure 5. The most striking feature of this curve is the wide range and rapid fluctuation of the transport. For example, early in December 1952 the transport more than doubled in a period of only three weeks. The range of transport values during the year is almost as large as the average transport for the year, the lowest value being $14\times10^6\ m^3/sec$ and



MASS TRANSPORT OF THE FLORIDA CURRENT

FIG. 5

the highest being $39\times10^6\ m^3/sec$. These extreme values persisted only for a few days and are considerably outside of the range of values previously reported for transport. It is interesting to note that the Marine Laboratory of the University of Maimi (Wagner and Chew, 1953) has recently reported high values of transport computed from GEK data. These, however, appear to be partially due to tidal transports.

A close examination of the scatter of the experimental points about the transport curve as drawn for May and June 1953 will reveal that the experimental points for succeeding weekends lie alternately below and above the smooth curve. This scatter may be accounted for by the lunar fortnightly component of the tides, which has a period of 13.66 days.

Taking a 24 hour sample out of a 13.66 day sine wave every seventh day will produce the following characteristic picture:

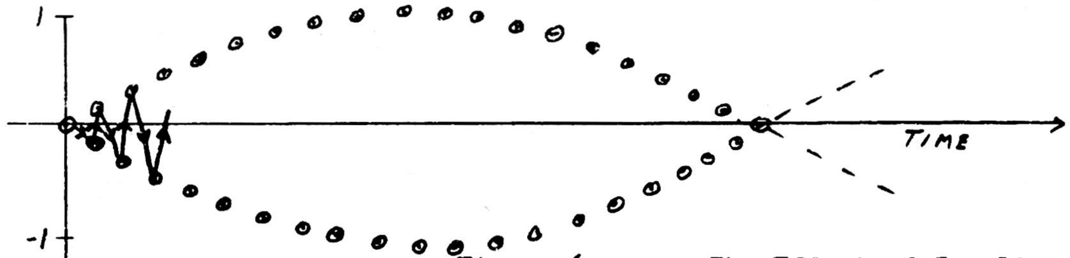


Figure 6 The Effect of Sampling

The lunar fortnightly tide producing force is of the right order of magnitude to account for the observed variation.

i Comparison with Tide Gauge Data.

It is interesting at this point to compare the variation in the transport with the variation in the slope of the sea surface normal to the direction of flow. Neglecting the effects of winds and atmospheric pressure, the slope of the sea surface is related to the surface velocity at any point by

$$\frac{\partial h}{\partial Y} = - \frac{\rho f v_x}{g} \qquad (6)$$

or in terms of the average velocity, $\overline{v_x}$

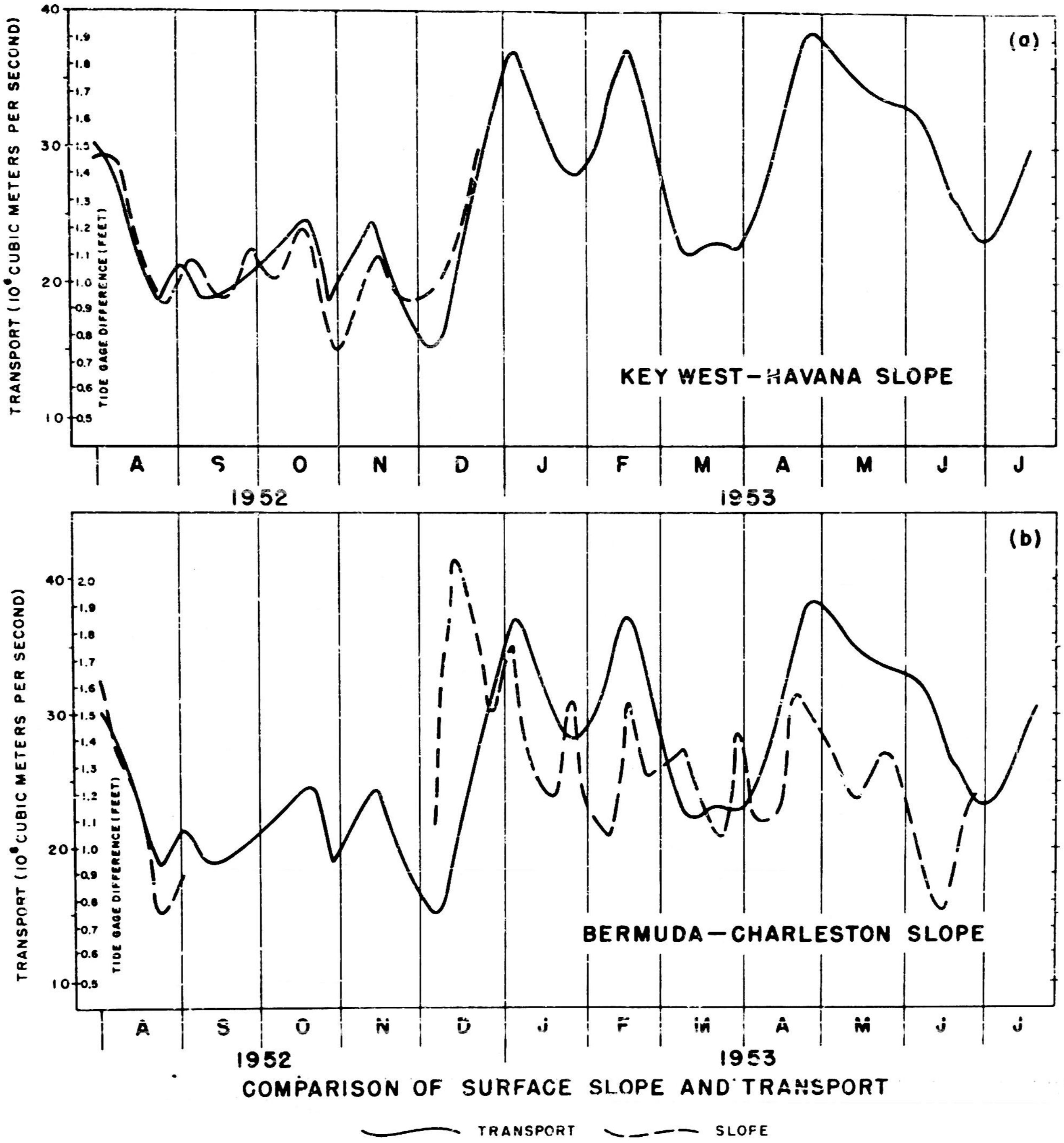
$$h_1 - h_2 = \frac{\rho f \bar{v}_x Y}{g} \qquad (7)$$

where ρ- density of water
f- coriolis parameter
x- distance in direction of stream
y- distance across stream
h- height of sea surface above datum

Unfortunately it is not possible to use this equation to obtain the absolute value of the average surface current because the difference between tide gauge zeroes located on opposite sides of the Florida Current cannot be obtained. However, the change in velocity relative to an arbitrary datum may be obtained from the hourly heights of the tide gauges maintained by the U. S. Coast and Geodetic Survey and Cuba.

Figure 7a shows the transport for the year 1952-1953, as well as the difference between the Havana and Key West tide gauges averaged over one week intervals. Key West tide gauge data terminates in December of 1952, when this gauge ceased operating. The qualitative agreement between the two curves is good, indicating that the surface current in the Straits of Florida is probably a good index of transport. Note that the rapid change in transport observed in December 1952 is also observed on the tidal gauge records and confirms the reality of the electrical observations. Corrections for atmospheric pressure and local wind effects have not been made.

Figure 7b shows a similar curve, comparing the electrically measured transport with changes in the slope of the sea surface



COMPARISON OF SURFACE SLOPE AND TRANSPORT

FIG. 7

between Bermuda and Charleston. The agreement as to detail is poor, although there seems to be general agreement on a large overall scale, leaving the way open for a number of possible explanations. It is possible that the surface current in the open ocean is determined more by local wind effects than by the overall motion of the water. In this case the average surface current (and hence the surface slope) is no longer a measure of transport. On the other hand, it is possible that the mass transport through the Bermuda-Charleston section is not proportional to the transport through the Key West - Havana section, the difference in transport being made up by the Antilles Current. It is also possible that the Charleston tide gauge is affected by local piling up of water by the wind.

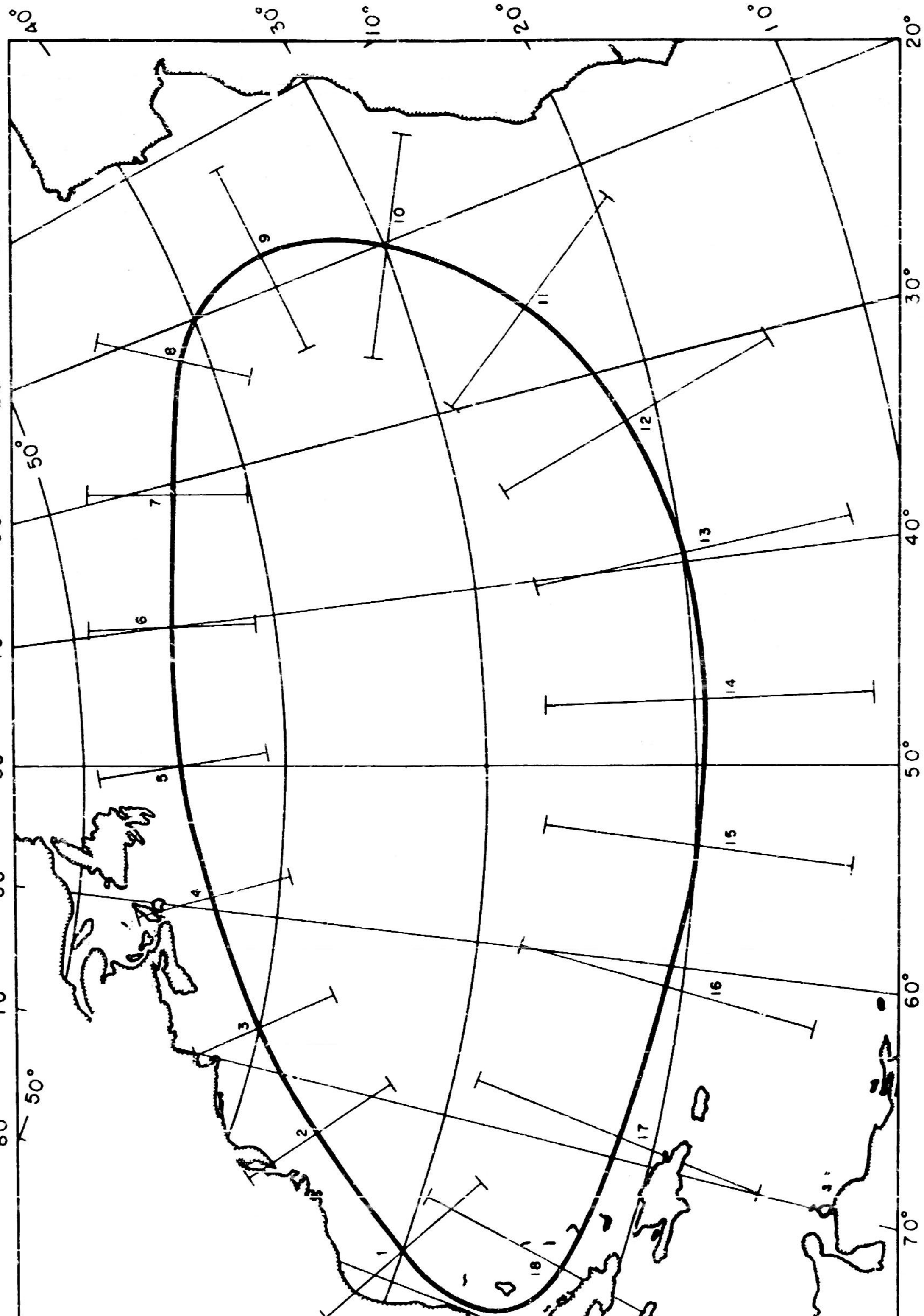
ii Correlation with the Atlantic Wind System

According to a recent paper (Munk, 1950) the transport of the Gulf Stream System is proportional to the curl of the windstress over the Atlantic. Munk was able to obtain good agreement between the theoretical transport computed from the normal windstress over the Atlantic and the known average value of transport. We now examine the correlation between monthly averages of wind and monthly averages of transport and show there is no significant direct correlation over the interval for which data are available.

To obtain a measure of the windstress curl over the Atlantic requires knowledge of the relation between windstress and wind speed as well as information on the actual wind distribution. Neither of these can be accurately obtained. However, even if the exact form of the windstress equation were known it would not be possible to evaluate the time average windstress from the time average atmospheric pressure maps unless stress were a linear function of speed.

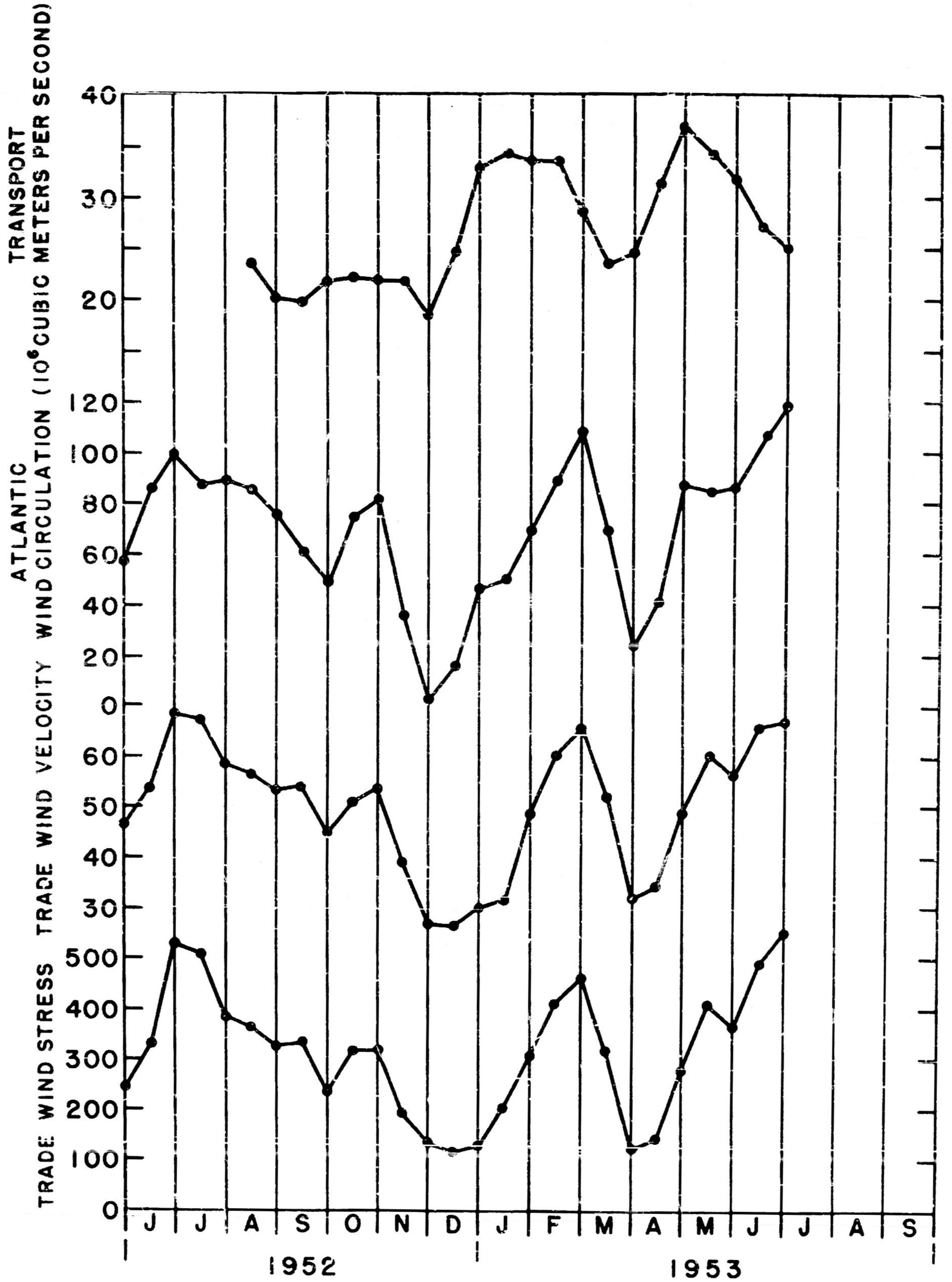
Consequently it was decided to use not the windstress but the wind itself as an index of the driving force of the Atlantic circulation. The reduction of the U. S. Weather Bureau mean monthly sea level pressure maps was made with a geostrophic template (Chase, 1952) constructed to give the components of the geostrophic wind parallel to a given contour. The contour is shown in Figure 8. This method does not take into account the effect of surface friction on the direction of the wind nor the effect of the curvature of the isobars. The results of this analysis as well as the contribution due to the trades alone (segments 11 through 18) are shown in Figure 9. The units of the ordinates in these curves are arbitrary. Figure 9 also shows the trade wind stress obtained by squaring and adding the contributions from the various segments in the trades region. This corresponds to the assumption that the drag coefficient is a constant and that the winds are relatively steady throughout the period over which the average was taken. The close resemblance which this curve bears to the other curves of trade wind velocity and Atlantic wind circulation is taken to indicate that the choice of wind rather than windstress for correlation with the transport is not a serious source of error.

Comparison of these curves with that of transport shown on the same figure indicates that there is no immediate good correlation between the variables in question. In order to obtain a quantitative measure of this preliminary conclusion, a correlogram (Figure 10) was made for the wind circulation and transport. The abscissa represents



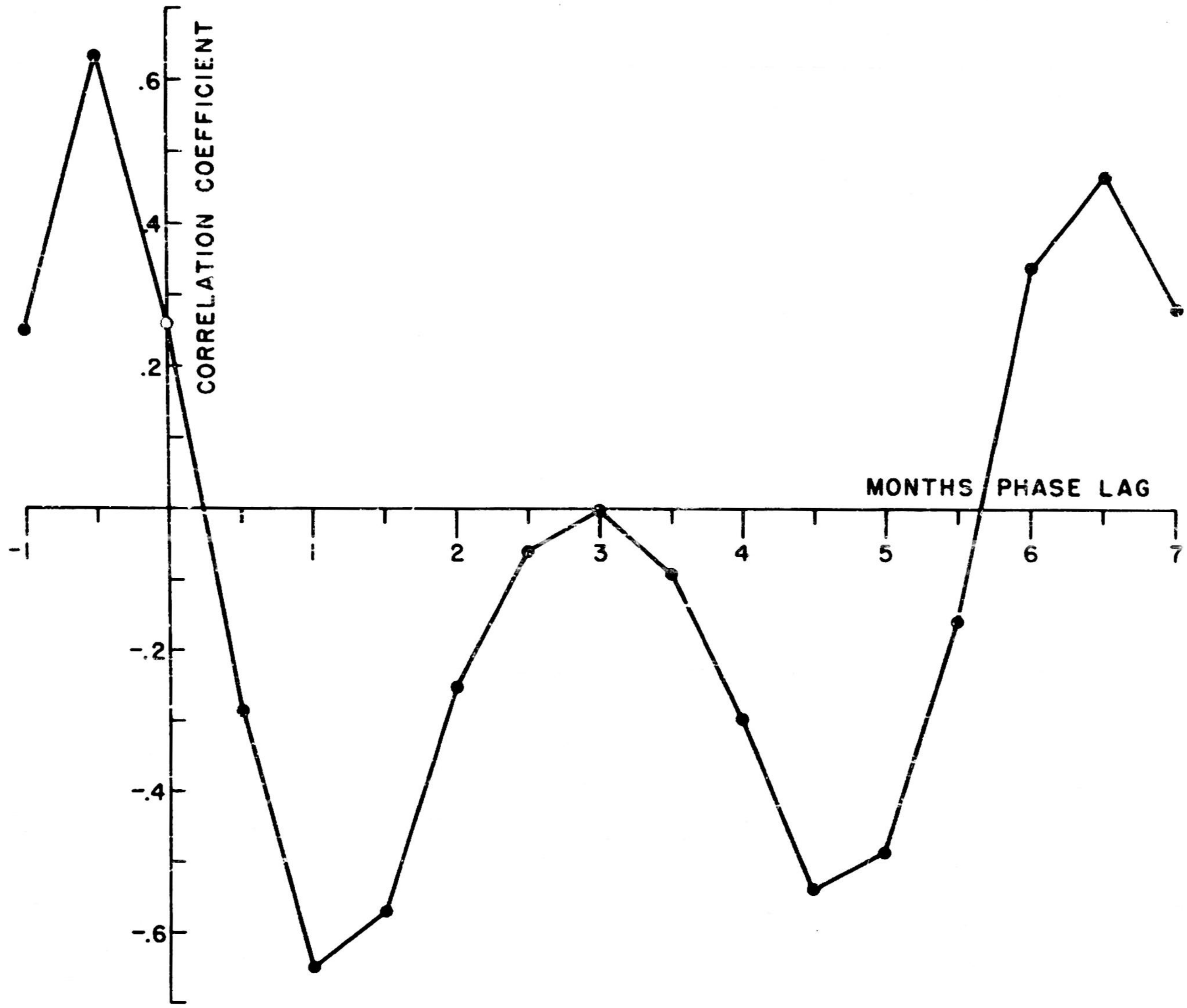
GEOSTROPHIC CONTOUR

FIG.8



COMPARISON OF TRANSPORT AND THE ATLANTIC WIND SYSTEM

FIG. 9



CORRELOGRAM OF WIND STRESS AND TRANSPORT

FIG. 10

the months of phase shift between the two variables. The first positive correlation occurs with a $-\frac{1}{2}$ month phase shift, corresponding to a transport which leads the windstress. It is assumed that this is a fortuitous correlation. Further positive correlations of the order of 0.5 are found at $6\frac{1}{2}$ and 18 months phase lag (the latter is not shown). It is apparent at this point that no positive statement can be made about the relation between monthly average wind and transport on the basis of correlogram analysis. It is quite possible that more data will tend to lower the correlations obtained.

There are, however, a number of serious objections to the analysis as here undertaken. Mathematically we cannot assume that the steady state solution due to Munk will in any sense be applicable to the more rapidly changing regime here encountered. The geostrophic template employed as a means of reducing a mean monthly weather map to a single significant number is not necessarily a reliable and sensitive means of measuring the significant changes in the wind circulation under all possible conditions. Further, it is intuitively clear that simultaneous changes in the wind strength along various portions of the contour will not produce a corresponding change in transport at the same time in the Straits of Florida. Phases should be introduced, but this can be accomplished only on the basis of some mathematical model which will suggest the required magnitudes. Moreover the nearer segments may be more important. Finally it must be pointed out that even on the basis of the most elementary mathematical model one cannot expect simple correlation between driving force and velocity (Stress and Transport).

Consider for example a mass subject to viscous and external driving forces only. The velocity of the mass bears no simple relation to the driving force. If the force is periodic the Fourier components of the force will reappear in the motion of the mass, but the magnitudes of the velocity components relative to the force components depend on the frequency of the particular component, as do their relative phase.

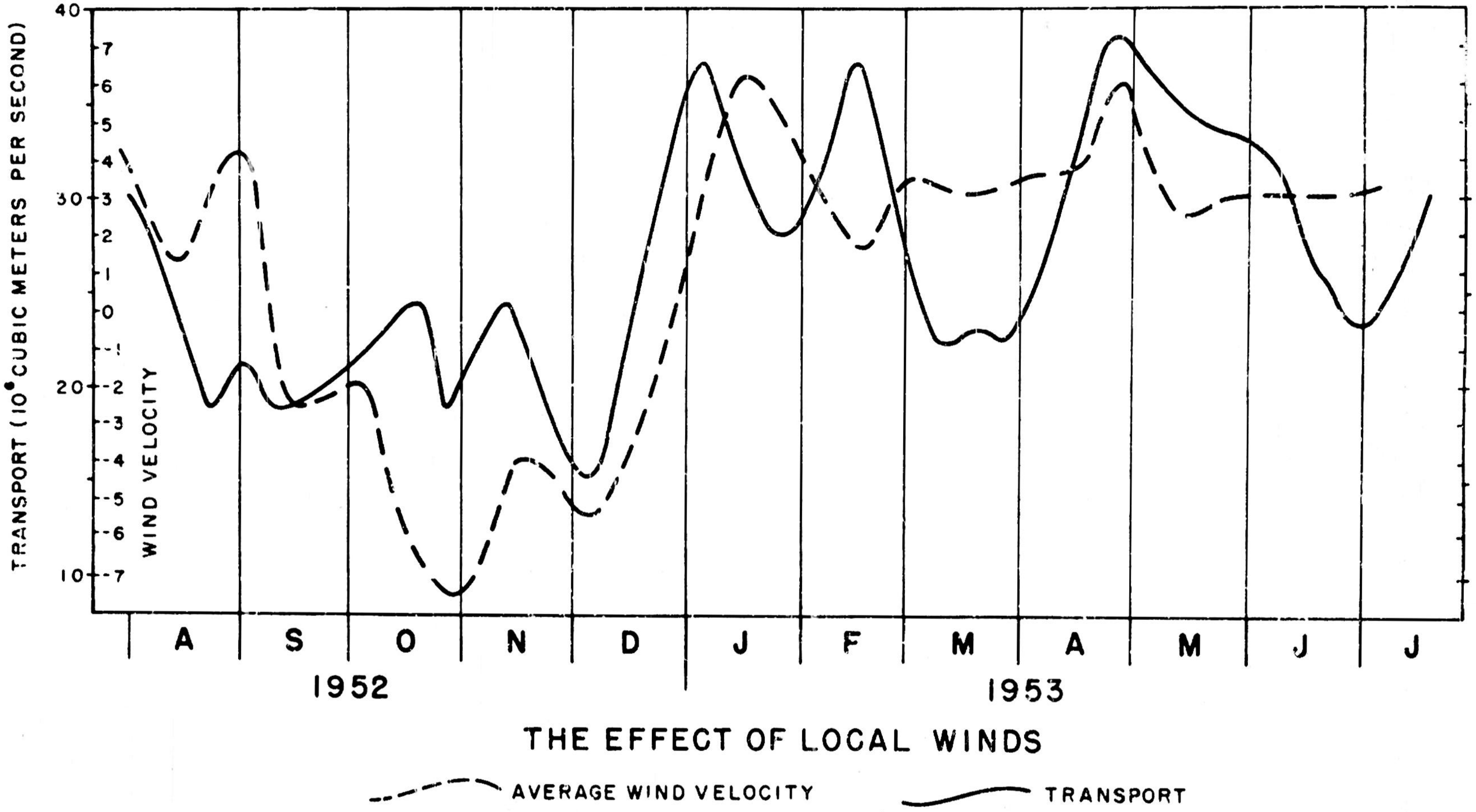
If $$F = \sum_j F_j \cos \omega_j t \qquad (10)$$

then $$v_j(t) = \frac{F_j}{m}\left[\frac{\frac{b}{m}\cos \omega_j t + \omega_j \sin \omega_j t}{\left(\frac{b}{m}\right)^2 + \omega_j^2}\right] \qquad (11)$$

where F - force applied
m - mass
ω - angular frequency
b - viscous damping coefficient

We may say that the system is frequency selective and introduces phase distortion. Under these conditions we cannot expect to find the time variation of the force reproduced in the time variation of the velocity.

As a test of the theory that the segments close to the Straits of Florida exert a dominant influence on the more rapid variations encountered within the seasonal curve of transport we have made a comparison with the mean monthly geostrophic wind given by the first two segments off the coast of Florida and the South Eastern United States. The result is shown in Figure 11. The correlation obtained indicates that the flow through the straits may be predominantly controlled by the winds in this region. However, the data available are not sufficient to make a positive statement as to the significance of this result. In particular it is interesting to note that the rapid increase in transport observed in late December



THE EFFECT OF LOCAL WINDS

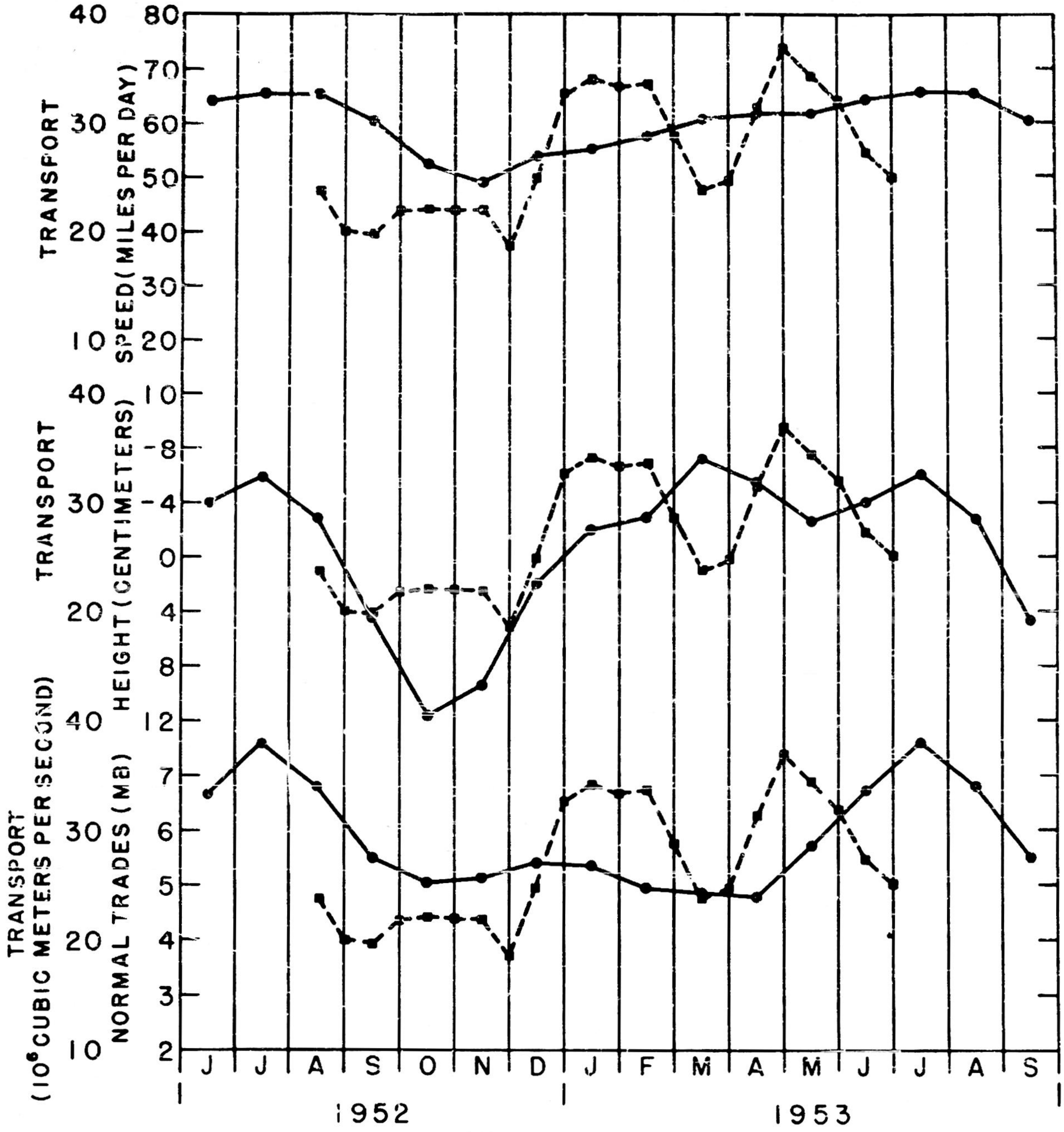
FIG. 11

1952 does correspond to a complete reversal of the wind in the same period. The wind opposing the flow before and aiding the flow after January 1, 1953. Reasonable correlation exists beyond that point.

It is perhaps in order to point out the inherent danger in the blind application or correlation techniques. It should be apparent that correlations can always be obtained between two sets of data having seasonal variations, or variations of similar period, simply by the introduction of suitable phases. The mere existence of a high correlation coefficient in no way demonstrates the existence of a causal relationship.

iii Comparison with Normal Curves.

Previous analyses of the factors contributing to the variations in the transport of the Gulf Stream have been carried out on the basis of "normal" curves, in the sense of month by month averages taken over many years. Figure 12 shows the normal curves of surface current in the Florida Straits (Fuglister, 1951), the Height of the Miami Tide Gauge, and the Atlantic Wind Circulation, (Chase, 1952) taken from various published works. On each there has been superposed the curve of transport as obtained by taking monthly averages. It has been considered significant that the various normal curves show considerable similarity in shape. The 1952-1953 curve of transport which has been superposed shows large, relatively short period, variations about the normal curves. This is particularly noticeable in the case of the surface current which by previous argument appears to be a good index of transport. From the fact that these short term



COMPARISON OF TRANSPORT AND NORMAL CURVES OF SURFACE CURRENT, THE MIAMI TIDE GAUGE AND ATLANTIC WIND CIRCULATION

FIG. 12

variations do not appear in the normal curves it may be deduced that they are not regular annual phenomena but probably depend on essentially random effects, meteorological and otherwise.

It is apparent that additional data will greatly increase our understanding of the significance of these short term variations, and will show to what extent the normal curves represent the mean condition within a given year.

V Variations of Tidal Period

The relation between the lunar transit and the tidal transport was noted by Pillsbury in 1890. This effect is thought to be produced in the following way:

Consider the Straits of Florida as a channel and the Atlantic and Gulf as two large reservoirs. The tide in the Atlantic (Miami) is semi-diurnal, the diurnal components being only 10 percent of the amplitude of the semi-diurnal ones. The Tide in the Gulf is called diurnal, but there are times when the amplitude is greatly reduced and the appearance of the tidal record is essentially semi-diurnal. The effect of the tidal oscillations in the Atlantic will be to send a long wave up the Florida Straits, in a direction opposite to the flow of the Florida Current. This view is justified by the tidal differences tabulated for the various well exposed points along the Florida Keys. High Tide arrives progressively later as one goes down the keys toward Key West, the total difference between Miami and Key West being about 2 hours. This corresponds well to the time of propogation of a long tidal wave, computed from $\sqrt{gh}$. Such a wave which is much longer than the depth of the channel in which it propogates has maximum transport at maximum tidal height, i.e. we expect a minimum in the transport at Key West two hours after high tide at Miami. This expectation is not born out. A continuous picture of the transport as obtained last August shows that the transport varies in a manner which is pictorially speaking between the tidal effect in the Atlantic and that found in the Gulf.

To further substantiate the theory of the progressive tidal wave we will compute the transport of such a wave at a time when the amplitude of the gulf tide is at a minimum. This occurred on August 22, 1952 during the period when continuous data was obtained Examination of the record shows a tidal transport amplitude of approximately $4x10^6$ m^3/sec. On the other hand the amplitude of the transport of a long wave is given by

$$T = \frac{Agh}{v} \qquad (12)$$

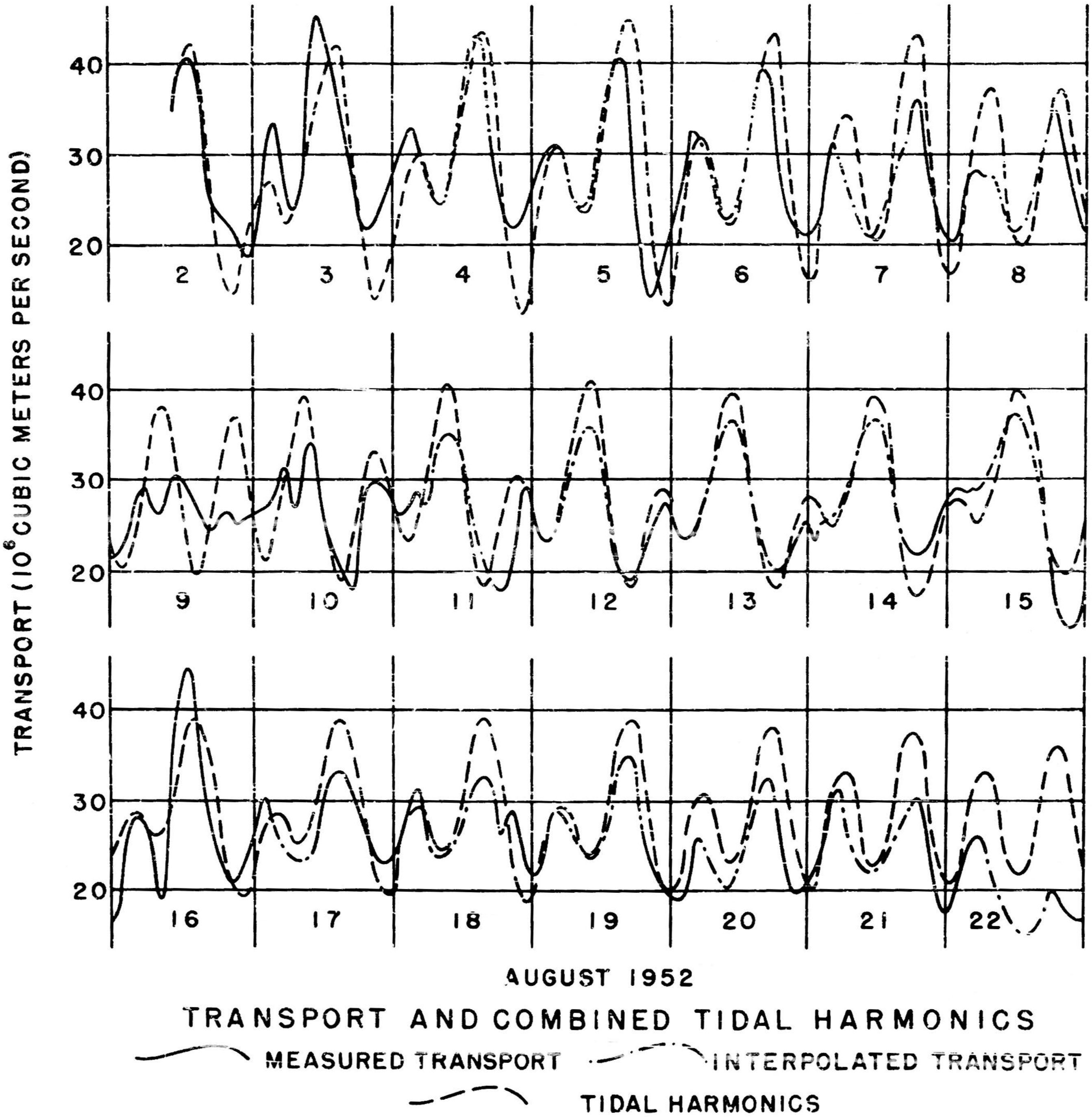
inserting the values for the Florida Straits

A - area of section, $1.6x10^8$ m^2
h - tidal amplitude, 0.22 m
v - velocity of tidal wave, 82 m/sec

we obtain approximately $4x10^6$ m^3/sec, in good agreement with the measured amplitude.

i Comparison with Tide Data

The perturbing influence of the Gulf tide manifests itself in the presence of strong diurnal components in the transport. A solution to the hydrodynamical problem of the Florida Straits -



TRANSPORT AND COMBINED TIDAL HARMONICS

FIG. 13

Atlantic - Gulf of Mexico system (which is useful in interpreting currents in the Florida Straits) has not been obtained. A complete solution must take into account the Coriolis force, stratification of the water and slope of the sea bottom. In order to test the idea that both Atlantic and Gulf tides play a significant role in the diurnal variation of the transport we have combined the tidal harmonic of the Miami and Tampico tide stations with suitable phases so as to give the simultaneous effects at the Key West - Havana section. The result of this rather arbitrary procedure together with a curve of transport for the month of August 1952 is shown in Figure 13. There has been no attempt to weight the two tidal components. The curve is simply a linear combination of existing tidal amplitudes. The Miami tide, representative of the Atlantic, is inverted and displaced by two hours to represent the time of travel of the long wave up the Florida channel. The Tampico tide is displaced by 6 hours for similar reasons.

The resulting agreement is good, except in those regions where the amplitudes of the tidal transport are small. Here other influences, such as local winds, may be significant. It is thought that this agreement may be taken to indicate that the above explanation of the tidal dependence of the transport is essentially correct in so far as it points out the two pertinent parameters. No further significance can so far be attached to the simple linear combination of tidal components employed.

To illustrate the significance of the tidal transport on measurements designed to obtain the average transport consider the values of transport recently published by the University of Miami, Marine Laboratory (Wagner and Chew, 1953)

Date	Miami - Cat Cay Transport	Key West - Havana Transport (Fig. 5)	4-hour KW - H Transport
Dec. 22, 1952	40.5 m^3/sec	28.6 m^3/sec	32.8 m^3/sec
Feb. 17, 1953	43.5	36.2	43.7
Mar. 12, 1953	26.6	22.4	27.5
Mar. 26, 1953	24.5	22.6	25.0
Apr. 28, 1953	23.0	38.0	44.7

Table I. Comparison of Transports Measured at Various Sections of the Florida Current.

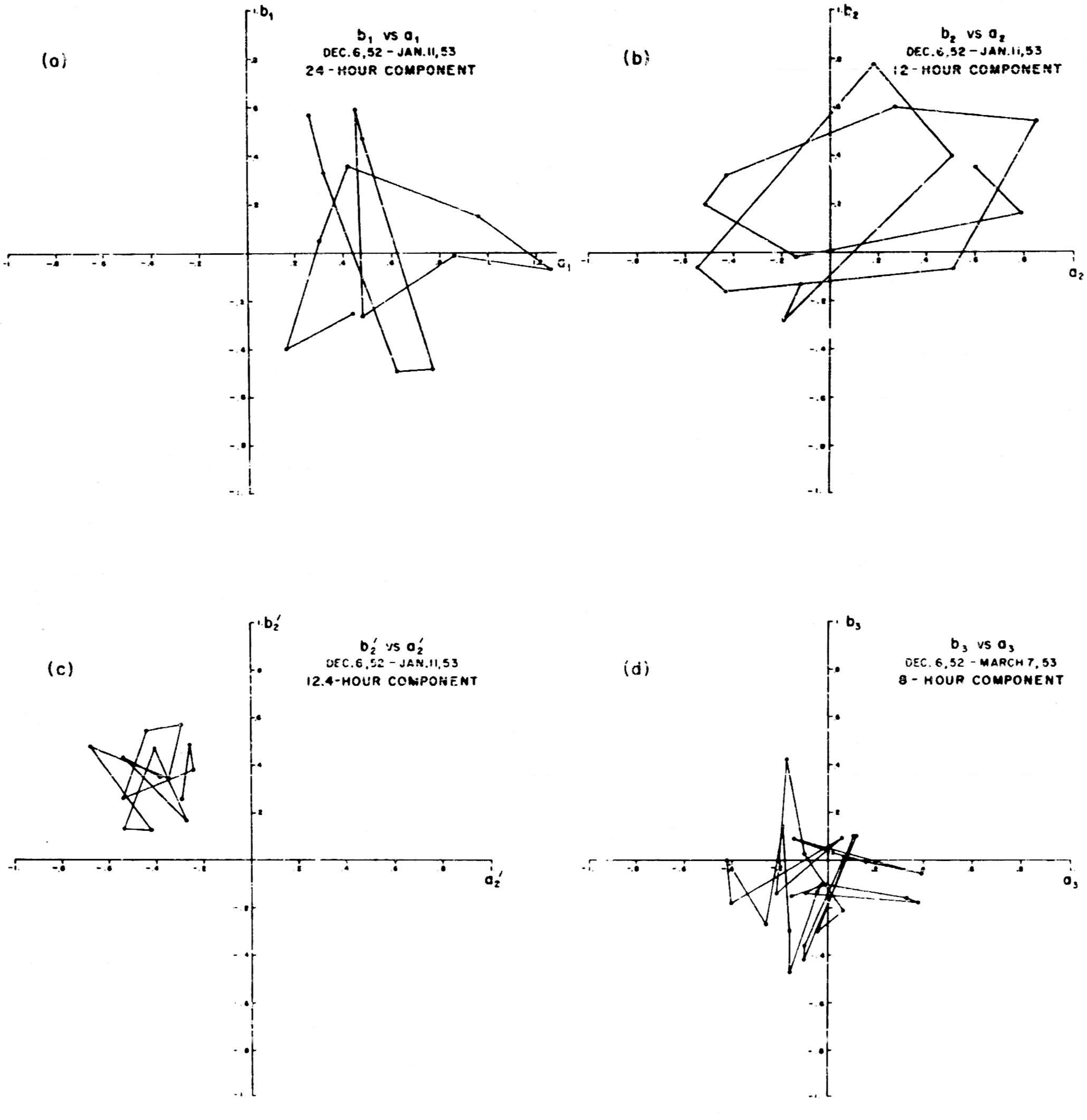
It should be noted that the agreement between the figures of transport quoted for a given day and the corresponding transport read from Figure 5 of this report is poor. The large discrepancies are probably due to the fact that the Miami measurements represent only a small portion of the tidal cycle, the GEK (von Arx, 1950) traverse taking 6 hours, of which only 4 are spent in the main region of the current.

The last column are the Key West - Havana transports corresponding to the four hours during which the GEK traverse was made, allowing for the two hour tidal difference between Key West and Havana as well as the daily shift in the tide since Key West data are available only on weekends. In the first four cases this method leads to significantly better agreement. The large discrepancy in the last case remains unexplained.

ii Harmonic Analysis

To obtain a better picture of the harmonic components of tidal frequency found in the transport curve Fourier analyses were made of various groups of 24-hour data, using midnight to midnight as the standard interval, and employing a 24 ordinate scheme for the numerical analysis. The analysis was carried out to obtain the 24, 12 and 8-hour components using 1-hour averages of transport as ordinates. The attenuating effect on the maximum amplitude due to such an averaging process is only 0.05%. No correction was applied to the amplitudes obtained.

The harmonic components were entered on a harmonic dial as shown in Figure 14. It is noted that the 12 hour component does not remain constant from weekend to weekend in phase or amplitude, but traces out a generally circular path in a counter clockwise direction around a point fixed in the harmonic dial. From the vector nature of such components we conclude that this is due to a 12 hour component, whose phase and amplitude remains constant in time plus a component of longer period. To separate these two components the center of mass of all the points was taken to represent the 12.00 hour component. This fixed component was subtracted from the others. The remaining component rotates with a period of about 14 days. From this it may be shown that the introduction of a new time scale, essentially a lunar one, will reduce this component to constant phase. This has been done and the result replotted in Figure 14c. The remaining scatter is due to smaller components of slightly different frequency known to exist in tidal analyses, and to the effect of geomagnetic disturbances. It might also be noted that the probable error in each point on the harmonic dial is large, due to the many additions and substractions involved in the harmonic analysis.



HARMONIC DIALS

FIG. 14

The 24-hour component was treated similarly to obtain a true 24-hour and 25.83-hour component. The 8-hour component was generally small and irregular such that no further analysis could be carried out. It should be noted that an important approximation is made in this method of analysis. Namely it is assumed that the initial analysis will equally well take out 24.00 and 25.83 hour components. This is not strictly justified, but an estimate based on the usual integral solution for Fourier components shows that the error introduced by this is not larger than 3%. This approximation also accounts for the existence of an 8 hour component which may amount to 5% of the amplitude of the 25.83 h one actually present. In general, a hormonic analysis of a portion of a sine wave whose period is not commensurable with the interval of the analysis will have all components present, though if it be close to one of the periods appearing in the analysis this component will give the major contribution.

The results of the harmonic analyses are summarized by the equation for the transport curve below. This equation should be used only with the greatest caution outside the region on which it is based. The value a_0 that appears in the equation depends on the average Gulf Stream transport and cannot be obtained from the tidal analysis.

$$T = a_0 + 1.6\cos(30.0t + 60°) + 3.8\cos(29.0t + 24.4n + 140°) + \qquad (13)$$
$$+ 3.6\cos(15.0t - 1.05n + 15°) + 3.4\cos(13.9t + 27.8n + 12°)$$

t - time in hours EST

n - number of days from January 1, 1953

A further check was made on the correlation between the magnitude of the 8 hour component and the K index sum for the day in question. No significant correlation was found. This is in keeping with the known fact that the earth current components of a period greater than a few hours are generally small.

The results of the harmonic analysis are shown in Table II, together with the coefficients of the tide producing forces, and the tidal harmonics at Key West, Miami and Galveston. It should be noted that the harmonic components obtained by the analysis of transport are in a ratio which is between those of the Miami and Galveston tide, giving some justification for the arbitrary combination of tidal harmonics of those two stations undertaken above.

Figure 15 shows the data obtained on July 18 and 19 together with a prediction of the tidal variation of the transport based on harmonic analysis of the preceeding 7 weeks data. The Cheltenham K Indices are shown on the record and are generally in the low range indicating that good agreement may be expected.

VI Irregular Fluctuations and Short Term Noise

i Geomagnetic Noise

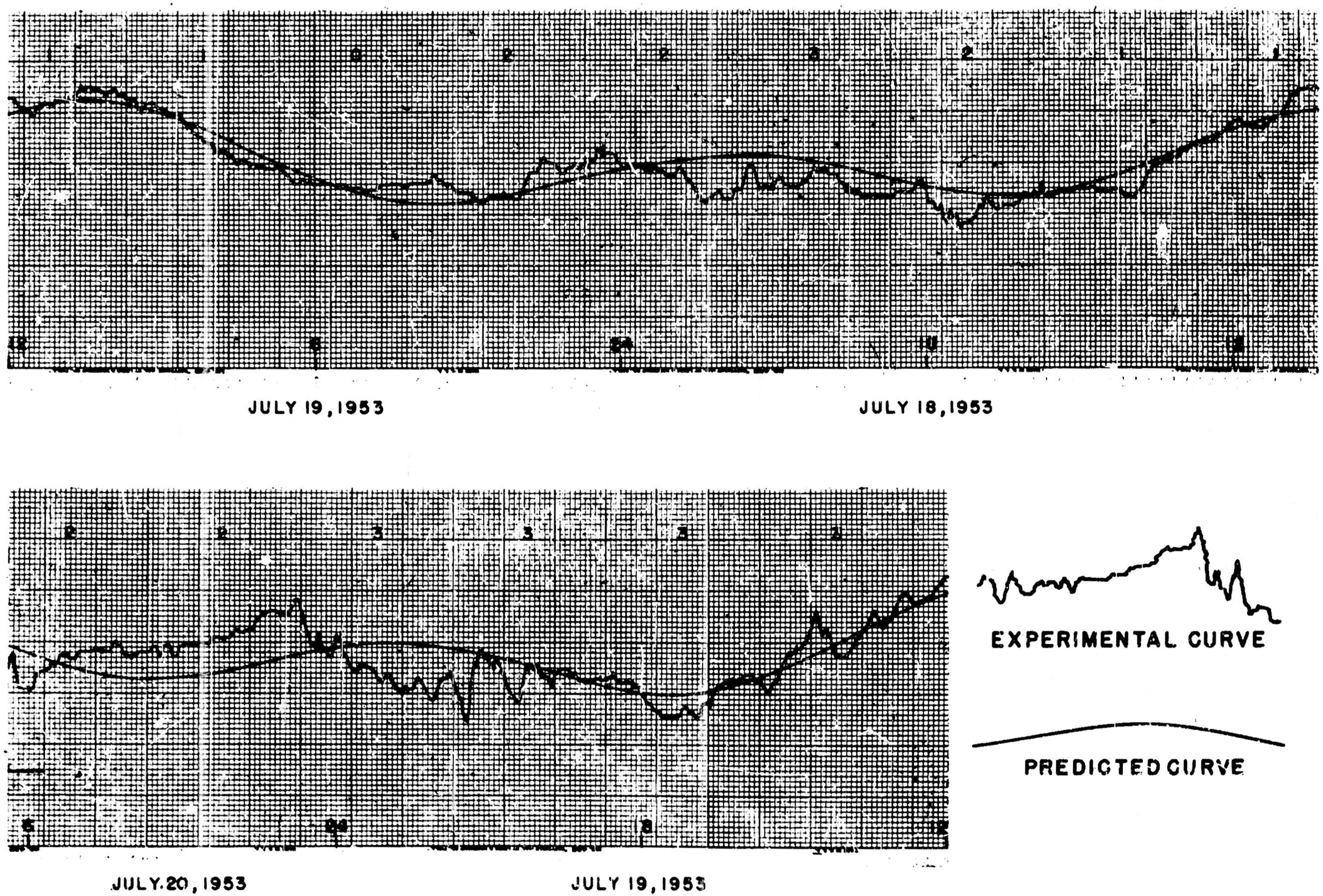
In addition to the components discussed above the records also contain fluctuations of random amplitude and duration. The amplitude of these fluctuations is usually within a given range for as long as six hours but varies unpredictably from day to day. These fluctuations are due to ionspheric or geomagnetic disturbances which produce earth currents whose disturbing influence has long been recognized by the wire communications engineer. The largest amplitude recorded during the year was approximately 1½ volts over a distance of 162 km or 10 mv per km. Potential gradients larger by a factor of 10^3 have been reported on long line communications cables during severe

Table II

Numerical Results of Harmonic Analysis of Transport Data

Name of Tidal Component & Period (hrs)	Principal Lunar 12.42	Principal Solar 12.00	Luni-Solar 23.93	Principal Lunar 25.82
Average Amplitude of Transport Coeff (10^{-6} m^3/sec)	3.8	1.6	3.6	3.4
Amplitude of Tidal Coefficient (feet)				
a) Miami	1.20 ± .37	0.24 ± 0.06	0.13 ± 0.04	0.11
b) Key West	0.56 ± 0.11	0.17 ± 0.04	0.29 ± 0.09	0.29
c) Galveston	0.31 ± 0.08	0.10 ± 0.01	0.38 ± 0.10	0.36
Coefficient of Tide Producing Force	0.45 ± .09	0.21 ± 0.06	0.27 ± 0.09	0.19

The tolerance given for the tidal coefficients represents the effect of smaller coefficients of only slightly different period which could not be separated in the analysis of transport.



RECORD AND PREDICTED CURVE

FIG. 15

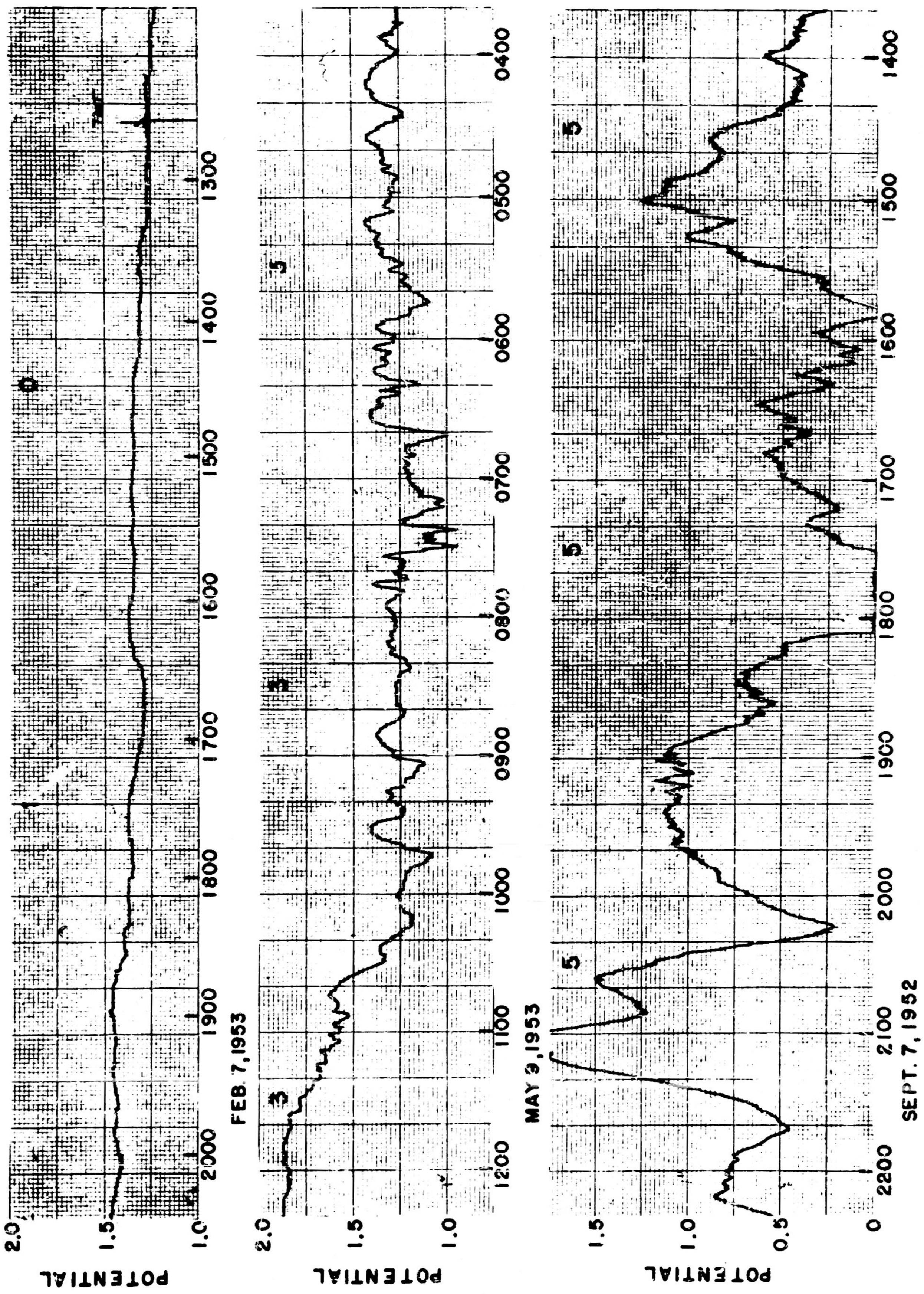
geomagnetic disburbances in years of sun spot maxima.

Figure 16 shows three sample records selected to show the range of geomagnetic effect encountered during the year. The numbers shown along the top edge of each record are the Cheltenham K Indices, which are measures of the intensity of the disturbance of the geomagnetic field during consecutive three-hour periods. The correspondence between high K Index and record noise was consistently observed throughout the year. The large majority of records obtained resembles the middle sample corresponding to a K index of 3. Long periods with index 0 are rare and the one shown is the quietest record obtained.

Elementary electromagnetic theory indicates that the emf measured in the loop consisting of the cable and the water above it may be related to the time rate of change of the horizontal component of the geomagnetic field penetrating the loop. It should be noted that whereas the potential due to water motion in independent of the location of the cable, the geomagnetically induced potential depends on the vertical area of the cross section above the cable. Information on the geomagnetic field is available in the form of magnetograms which present the various components of the field as functions of time. Examination of the magnetogram reproduced in Figure 17 indicates that the time derivative of the horizontal component, H, cannot be accurately obtained by numerical means. As an alternative, however, we may compare the integrated emf with the magnetic field itself.

It follows directly from the law of induction that

$$E = -\mu A \frac{\partial H}{\partial t} \qquad (14)$$



THE EFFECT OF GEOMAGNETIC DISTURBANCE ON THE ELECTRIC POTENTIAL

FIG. 16

where E is the induced emf, A, a fraction of the area of the section and H the horizontal component normal to the section. The potential measured by the apparatus is less than E because of electric current in the water. The value used for H must be an average taken over the cross section in question. If we assume that the geomagnetic disturbances are of ionospheric origin and propagate as electromagnetic waves toward the earth, we may obtain an estimate of the penetration of these disturbances into the water from the usual skin effect calculations. The result indicates that the skin is approximately 6 times as deep as the water in the Straits of Florida for the component of 10 minute period. This means that the disturbance penetrates well to the bottom of the Straits so that little electrical current is set up. Consequently we will let $V \approx E$

$$V = -\mu A \frac{\partial H}{\partial t} \qquad (15)$$

integrating with respect to time

$$\int_0^T V\,dt = \mu A\left[H(0) - H(T)\right] \qquad (16)$$

This equation will now be used to compare the noise with the variation of the horizontal component of the geomagnetic field.

The closest magnetic observatory to the Key West - Havana section is in San Juan, Puerto Rico. We have selected the magnetograms for September 7 and 28, 1952 for the comparison because these days were magnetically active. The required integration was performed numerically by 20 minute intervals in the following way: The 20 minute average ordinates of the transport curve were tabulated, from each there was substracted the average value and the principal tidal component of the transport. The integral given above was approximated by

$$\int_0^T V\,dt = \frac{1}{3}\sum_{j=0}^{3T}\left[V_j - V_{TIDE}\cos\left(\frac{j}{3}\omega+\delta\right) - \overline{V}\right] \tag{17}$$

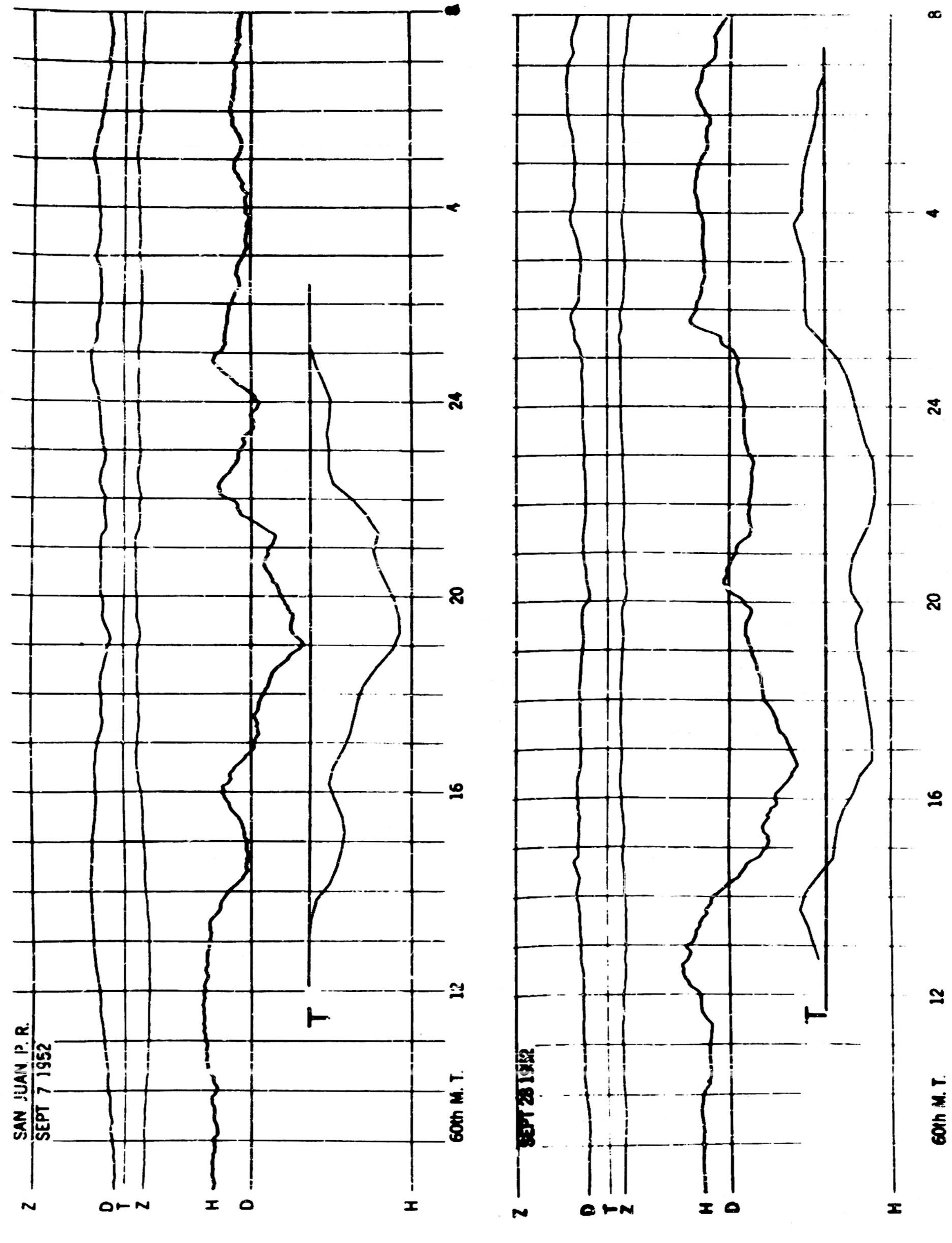
T - time, hours
ω - angular frequency of tidal component, rad/hr
δ - phase angle

The result of this integration appears in Figure 17 as the curve labeled T. The vertical scale for this curve was chosen to approximate the vertical scale of the horizontal component, H, of the geomagnetic field. It should be noted that the agreement with respect to fine detail is limited by the 20 minute increment chosen for the numerical integration. However, the good agreement demonstrated does indicate that the noise observed on days having a high Cheltenham K Index may be ascribed to the variations in the horizontal component of the field. (Note that the transport is measured in terms of the vertical component of the field).

ii Atmospheric Noise

It was also noted that electrical disturbances in the atmosphere, lightening, etc, will affect the record. The usual appearance of the record during times of electrical storms is characterized by short vertical lines apparently due to single electrical discharges. Since the duration of these pulses is short, it is not possible to obtain an estimate of the magnitude of the voltages involved. The inertia of the recording apparatus obscures this information.

At this point it may be in order to caution users of this electrical method of measurement, to determine to what extent geomagnetically induced earth potentials may produce false results.



COMPARISON OF MAGNETOGRAM WITH INTEGRATED TRANSPORT

FIG.17

It is clear that measurements extending over only one hour during periods of extreme earth current activity may be thrown off by as much as a factor of two in the Florida Straits.

iii Man Made Disturbances

Man made noise is frequently noted on the record. A considerable amount of high frequency noise is picked up from Western Union Equipment. This is unavoidable because of the common use of a multiconductor cable from the Cable hut to the Western Union office and unshielded leads carrying square pulses in the office itself. The presence of these influences necessitates the use of low sensitivity on the recording equipment which does obscure some of the fine structure observed in the electrical signal during the hours of little human activity when higher sensitivity could be used.

VII INSTRUMENTATION

The essential details of the instrumental setup are shown in Figure 2. The apparatus consists of two electrodes placed in the sea, one on each edge of the stream, connected through an overload cut-off and an attenuating pad to a recording potentiometer.

The electrodes consist of 18 sets of specially matched GEK electrodes, (von Arx, 1953) packed in glass wool and placed in a perforated fibre container. The electrochemical potential between these electrodes was found to be less than 0.1 mv. This figure includes the contact potentials between the Western Union Office and the Cable hut in Key West, and is close to the limit of resolution of the recording apparatus. It was decided to use a number of sets of matched GEK electrodes in order to obtain partial compensation of the potentials known to exist between individual sets in the presence of salinity, temperature and oxygen concentration differences between their environments. These potentials are less than 0.1 mv under the conditions encountered in waters around the Key West - Havana section.

The electrodes were located as follows: The Key West electrode was put out from the Western Union cable hut at the foot of South Street for a distance of 150 feet into about 4 feet of water. The bottom here is sandy. The Havana electrode was placed in over eight feet of water inside the entrance to Cojimar harbour. All switching in Cuba is done in the Cojimar cable hut. From the Key West cable hut both leads are brought through a multiconductor cable to the Western Union office building for a distance of approximately one mile. This cable also carries communications circuits. The recording equipment was set up in the Western Union cable testing room.

The electrodes were connected to the recording equipment by the Western Union #2 cable whose parameters were as follows at the beginning of the recording program.

Copper Resistance	990 ohms
Dielectric Resistance	2.06 megohms

The recording equipment consists of an overload relay designed to cut off the recorder when the input goes outside specified upper or lower limits, followed by an adjustable attenuating pad, which feeds directly into a Speedomax recorder having sensitivity of 10 mv for full-scale deflection. The resistance R_1 in the attenuating pad was chosen to give a full scale deflection with an input of $2\frac{1}{2}$ volts. The chart speed was 3 inch/hour but has now been reduced to 1 inch/hour.

The effective load impedance at Key West is 1 megohm while the average voltage recorded is approximately 1 volt. Using these values and the cable parameters it may be shown that no correction need be made for cable losses, the error being less than 0.1% which is less than the errors inherent in the recording and reading process.

During July 1952, the Key West - Miami tricore, a cable running along the edge of the Florida Current in Hawk Channel, was used in preliminery studies of noise and geomagnetic disturbance. Continuous data was obtained throughout the month.

During August 1952, Key West - Havana data was obtained daily from 5 p.m. until 6:30 a.m. and on weekends from 5 p.m. Friday until 6:30 a.m. Monday, as well as on Holidays. The daily recording program was abondoned at the end of August and only continuous weekend data retained.

VIII Analysis of data

The data were reduced to numerical form by finding the area under 20 minute segments of the curve. For harmonic analysis one-hour areas were used. The effect of this smoothing process is to eliminate the majority of the geomagnetic disturbances without affecting the peak values of the components with tidal period appreciably. The transport curve in Figure 13 was obtained by plotting one-hour averages and indicates the smoothing afforded by this process.

IX Conclusions and Recommendations

It has been pointed out that there are inherent uncertainities in the computed values of transport because of the effect of bottom conductivity and the possibility of the meandering of the axis of the stream. It is felt that an accurate determination of transport by a hydrographic section taken during the period when tidal variations are small or a series of GEK traverses to locate the axis of the stream may yield significant information on the order of magnitude of the bottom conductivity and the effect of shift in the axis of the current.

The discussion of seasonal variation points out that one years data is not sufficient to either confirm or deny the dependence of the transport on the Atlantic Wind System, or any other parameter so far investigated. Continuation of the measuring program seems definitely called for since it holds promise of yielding, for the first time, continuous data on the transport through the Florida Straits which may serve to clarify the question of the driving force of the Gulf Stream.

The discussion of the tidal effects is incomplete in so far as it has not been attempted to obtain a hydrodynamical analysis of the flow through the Florida Straits. The data presently available will allow evaluation of the validity of any theory produced.

Acknowledgements

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