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SURVEY OF FRICTION COEFFICIENTS, RECOVERY FACTORS, AND HEAT-TRANSFER COEFFICIENTS FOR SUPERSONIC FLOW

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SURVEY OF FRICTION COEFFICIENTS, RECOVERY FACTORS, AND HEAT-TRANSFER COEFFICIENTS FOR SUPERSONIC FLOW*

By

Joseph Kaye[1]

SUMMARY

A brief survey is presented of the progress made in the last four years on theoretical and experimental work on friction coefficients, recovery factors, and heat-transfer coefficients for supersonic flow of air.

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NOMENCLATURE

A	heat-transfer area
c_f	local skin-friction coefficient
c_H	Stanton No. of Van Driest
c_p	specific heat at constant pressure
D	inside diameter of pipe
f	local apparent coefficient of friction, $2\,g\,\tau\,/\,\rho V^2$
G	flow per unit area
g	acceleration given to unit mass by unit force
h	heat-transfer coefficient, $(q/A)/(t_{aw}-t_m)$
M	Mach number, $V/\sqrt{gkRT}$
Nu_L	length Nusselt number, hl/k
Pr	Prandtl number, $c_p \mu g/\lambda$
p	static pressure
q	rate of heat transfer
r	recovery factor, $(t_{aw} - t_m)/(t_o - t_m)$
Re_D	diameter Reynolds number, $DG/\mu g$
Re_L	Length Reynolds number, $LG/\mu g$
St	Stanton number, h/c_pG
T	temperature, ^oF abs
t	temperature, ^oF
V	velocity
ρ	density
λ	thermal conductivity
μ	viscosity
τ	shear stress at wall

Subscripts:

o	refers to stagnation conditions
aw	refers to adiabatic wall conditions
l	refers to mean-stream conditions
∞	refers to free stream conditions
i	refers to flow of incompressible fluid

INTRODUCTION

The importance of designing aircraft for safe operation at higher and higher supersonic speeds is apparent to all. Twenty years ago designers of aircraft were concerned with the effects resulting from an aircraft passing through the speed of sound - "the sonic barrier." But having shown that it is feasible to design an aircraft which can start from rest and exceed sonic speed safely, the designer is at present concerned with safe operation of aircraft at supersonic and hypersonic speeds.

Safe operation of supersonic aircraft implies safety for both the inhabitants of the aircraft and its equipment. It is evident that as the speed increases to five or ten times the sonic speed the function of the human beings on board must be reduced considerably since the human being represents a poorly designed servomechanism at these high rates of action and reaction. Hence, as the speed increases, more and more of the normal human operations will be taken over by electronic gadgets. But here the designer is faced with an obstacle similar to the "sonic barrier."

If the speed of the plane is sufficiently high, its surface temperature may reach a value of the order of 1,000°F. or greater, resulting from "aerodynamic heating." Hence the designer can foresee a "thermal barrier" requiring a tremendous amount of internal cooling in this aircraft in order to keep the inhabitants, the aircraft, and the electronic equipment in safe operating condition. The problem of safe operation of electronic equipment is made even more difficult when it is realized that most of this equipment has been designed only for operation at low ambient temperatures, and not for continuous operation at 1,000°F. or at greater temperatures.

The designers of supersonic aircraft require an extensive knowledge of friction coefficients, recovery factors, and heat-transfer coefficients over a wide range of Mach numbers for both laminar and turbulent boundary layers. It is intended here to present a survey of available data and theory for supersonic velocities and then to discuss two research programs in this field now underway at M.I.T.

Because of the large number of papers published in this field in the last few years, only some of the latest ones are reviewed, and no attempt is made to cover the field completely. In addition, although the problem of transition is closely associated with the above problems for laminar and turbulent boundary layers, it was decided to exclude the large number of papers treating this difficult problem.

OBJECTIVES

The objectives of the present discussion are threefold:

1. Present a brief survey of the state of the art on theoretical and experimental work on friction coefficients, recovery factors, and heat-transfer coefficients for supersonic flow.
2. Present a brief description of the program in supersonic flow of air in a tube at M.I.T. under the sponsorship of the Office of Naval Research.
3. Present a brief description of the program on supersonic flow of air over a flat plate underway at M.I.T. under the sponsorship of the Office of Air Research.

REVIEW

A. Theoretical Work on Laminar Boundary Layer

The analytical investigation of the laminar boundary layer began in 1908 with the thesis of Blasius, who determined the velocity profiles for incompressible flow on a flat plate with zero pressure gradient. In 1921 Pohlhausen used these velocity profiles to calculate temperature profiles in the laminar boundary layer on a flat plate. Since then numerous papers have been published which extended and modified, first the basic assumptions used in the analysis, second the mathematical techniques of reduction of the partial differential equations, and third the methods of obtaining numerical results. The variety of assumptions used in some of these papers is seen from the summary in Table I. Most of the references before 1950 in Table I are given in detail in an excellent review by Kuerti,[1*] which also contains the references to the earlier reviews of Lewis, and of Rubesin and Johnson.

Since the papers listed in Table I represent only a fraction of the available papers, it is obviously possible to give here only a few samples of theoretical work. These will be selected from the most recent papers.

1. Skin-Friction Coefficient for Laminar Boundary Layer

The calculated values of local skin-friction coefficient, c_f, for flow of air over a flat plate are compared in Fig. A. (Moore[2] - Fig. 10.) These curves are for the insulated plate with all air properties evaluated at the free stream temperature.

The effect of evaluating the air properties at the wall temperature is shown in Fig. B. (Van Driest[3] - Fig. 3) by the curve marked $C'_f \sqrt{Re}$. The

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*Superscripts refer to items in Bibliography.

TABLE I

SUMMARY OF ASSUMPTIONS USED IN THEORETICAL WORK ON LAMINAR BOUNDARY LAYER ON A FLAT PLATE

Author	Date	Density	Specific Heat	Thermal Conductivity	Viscosity	Prandtl No.	Pressure Gradient	Radiation	Dissociation	Mach No.	Friction Coefficients	Recovery Factor	Heat-Transfer Coefficient	Remarks
Blasius	1908	c	c	c	c	c	0	0	0	0	$c_f = 0.664 Re_L^{-1/2}$	-	-	
Pohlhausen	1921	c	c	c	c	c	0	0	0	0	$c_f = 0.664 Re_L^{-1/2}$	$r = Pr^{1/2}$	✓	
Busemann	1935	f(T)	c	$\sim T^{1/2}$	$\sim T^{1/2}$	1	0	0	0	8.8	✓	✓	-	
Von Karman and Tsien	1939	f(T)	c	$\sim T^{.77}$	$\sim T^{.77}$	1	0	0	0	0-10	✓	✓	-	
Crocco	1941	f(T)	c	$\sim T^{w}$ (for w = 0.5,.75,1.0,1.25)	$\sim T^{w}$	0.725	0	0	0	0-5	✓	✓	✓	
Brainerd and Emmons	1942	f(T)	c	$\sim T^{.768}$	$\sim T^{.768}$	0.733	0	0	0	0-3.2	✓	✓	-	
Crocco	1946	f(T)	c	$\sim T^{1}$	$\sim T^{1}$	c	0	0	0	>1	✓	✓	✓	
Cope and Hartree	1943	f(T)	c	$\sim T^{.89}$	$\sim T^{.89}$	0.76	0	0	0	>1	✓	✓	✓	
Chapman[18] and Rubesin	1949	f(T)	c	$\sim T^{1}$	$\sim T^{1}$	0.72	0	0	0	0-5	✓	✓	✓	
Lighthill[28]	1950	f(T)	c	f(T)	f(T)	0.7	f(V)	✓	0	>1	✓	✓	✓	Arbitrary surface temperature
Van Driest[3]	1951	f(T)	c	f(T)	Sutherland Rule	0.75	0	0	0	0-20	✓	✓	✓	Arbitrary velocity of main stream and arbitrary surface temperature
Klunker and McLean[29]	1951	f(T)	f(T)	f(T)	f(T)	f(T)	0	0	0	0-10	✓	✓	✓	
Young and Jansen[30]	1952	f(T)	f(T)	f(T)	f(T)	f(T)	0	0	0	0-5	✓	✓	✓	
Moore[2]	1952	f(T)	f(T)	f(T)	f(T)	f(T)	0	0	f(T,P)	0-20	✓	✓	✓	

c ≈ constant

✓ ≈ variable examined

\- ≈ variable not examined

effect of heat transfer on the mean skin-friction coefficient for a flat plate is shown in Fig. B, in terms of the ratio of the wall temperature to free-stream temperature, T_w/T_∞.

2. Recovery Factors for Laminar Boundary Layer

The various theoretical analyses for recovery factors for both subsonic and supersonic flow over a flat insulated plate lead to the simple result,

$$r = \sqrt{Pr}. \tag{1}$$

This approximate rule for laminar boundary layers agrees within one per cent of the more exact and complicated function if the Prandtl number lies between 1.2 and 0.7 and for Mach numbers less than about 8. For large values of the Mach number, however, it is not clear what temperature should be used to evaluate the Prandtl number. Note that this rule predicts the recovery factor is independent of Mach number and Reynolds number.

If dissociation of the fluid at high Mach numbers is taken into account, as Moore[2] has shown for air, the Prandtl number varies strongly with the degree of dissociation, and the above rule does not hold. For this case, the temperature rise of the insulated plate over the free stream temperature, rather than the recovery factor, is shown in Fig. C. (Moore[2] - Fig. 13.) For sea-level conditions, it is seen that dissociation begins to play an important role at a Mach number greater than 8, if radiation effects are ignored. It is also evident that the effects of dissociation at these high Mach numbers will be diminished if the cooling effect of radiation is considered at these high temperatures.

3. Heat-Transfer Coefficient for Laminar Boundary Layer

For incompressible flow, the local coefficient of heat transfer for a flat plate is given by

$$Nu_L = L/k = 0.33\, Re_L^{1/2} Pr^{1/3} \tag{2a}$$

or by

$$St = h/c_pG = 0.33\, Re_L^{-1/2} Pr^{-2/3} \tag{2b}$$

where the coefficient, h, is defined by

$$h = \frac{q/A}{t_w - t_m}. \tag{3}$$

For compressible flow in the boundary layer of a flat plate, the same form of theoretical result as equations (2a) and (2b) is obtained for the case of isothermal plate, provided the "effective" heat-transfer coefficient is defined by

$$h_e = \frac{q/A}{t_w - t_{aw}} \tag{4}$$

and provided the values of the fluid properties are evaluated at the temperature just outside the boundary layer. The "constant" of equations (2a) and (2b) changes with Mach number for compressible flow.

Moore[2] has shown that the calculated value of the heat-transfer coefficient based on dissociation increases with increasing Mach numbers and simultaneously decreasing absolute pressure to such an extent that these values can become double or triple the values of the corresponding heat-transfer coefficient without dissociation.

The variation of the product of Stanton number and square root of length Reynolds number versus Mach number is shown in Fig. D (Van Driest[3] - Fig. 4), for several values of the ratio of wall to free stream temperature. The effect of evaluating the air properties at the wall temperature is also shown by the curve labelled,

$$C_H \sqrt{Re'}.$$

B. Experimental Work on Laminary Boundary Layer

1. Data on Friction Coefficients for Laminar Boundary Layer

The amount of experimental data available on skin-friction coefficient is limited in range of Mach number and of length Reynolds number.

The data of Blue[4] obtained by two methods, one an interferometer study and the other by a total - pressure probe, for a Mach number of 2.0, showed that the measured average friction coefficients were from 7 to 39 per cent larger than the theoretical value of Crocco over the range of Reynolds number from 0.3 to 1.1×10^6.

The data of Higgins and Pappas,[5] obtained by boundary-layer velocity profiles, agreed well with the independent work of Blue, although their Mach number was 2.4. The average friction coefficients of Higgins and Pappas were from 32 to 48 per cent larger than the theoretical values.

Further work by Maydew and Pappas,[6] obtained by impact-pressure surveys of the boundary layer, showed average friction coefficients for the flat plate which were from 37 to 94 per cent larger than the theoretical values at a Mach number of 2.4.

Potter[7] at the Naval Ordnance Laboratory found that average skin-friction coefficients for laminar boundary layers on cones and cone-cylinder combinations agreed with flat-plate values with a maximum deviation of about 20 per cent, when appropriate conversion was made to a flat-plate geometry. These measurements covered the range of Mach number from 1.86 to 4.24. A typical result is shown in Fig. E (Potter[7] - Fig. 7). His friction coefficients are based on force measurements and not on interpretations of data on velocity profiles.

Liepmann and Dhawan,[8] Dhawan,[9] Coles,[10] and Coles and Goddard[11] presented data for local skin-friction coefficients on flat plates based on a sensitive force measuring element floating in the flat plate. The measurements covered the range up to a Mach number of 4.5 and up to a Reynolds number of 10^7. These data are probably the best local friction coefficients available at present and agree within a few per cent with the theoretical values predicted for a laminar boundary layer over a flat plate with zero pressure gradient. A typical result is shown in Fig. F (Coles[10] - Figs. 1, 3).

From this group of experimental data it is evident that the available measurements of skin-friction coefficients extend to a Mach number of 4, and that these data agree with the latest theories over this range of Mach number probably within the experimental error inherent in the particular type of measurement.

2. Data on Recovery Factors for Laminar Boundary Layer

The experimental data on recovery factors for laminar boundary layers, are summarized in Table II.

The local recovery factors of Wimbrow[12] for one cone at a Mach number of 2.0 and for one paraboloid at Mach numbers of 1.5 and 2.0 agreed within one percent of the theoretical value based on the square root of the Prandtl number evaluated at the adiabatic wall temperature. These recovery factors were independent of Mach number, Reynolds number and body shape but appeared to increase slightly with surface roughness.

The recovery factors of Stalder, Rubesin, and Tendeland[13] for a flat plate were independent of Reynolds number and equal to 0.881 ± 0.007. This value is 4 per cent larger than the theoretical value evaluated at adiabatic wall temperature. The experimental data are shown in Fig. G (Stalder, Rubesin, and Tendeland[13] - Fig. 5).

Some typical results of the recent experimental values of Eber[14] for cones and cone-cylinder are shown in Fig. H (Eber[14] - Figs. 3, 4, 5, 7, 8, 9). For the laminar boundary layer, Eber's recovery factors agreed with the square root of the Prandtl number evaluated at the adiabatic wall temperature within one per cent.

The recovery factors of des Clers and Sternberg,[15] of Slack,[16] and of Stine and Scherrer[17] are also shown in Table II.

TABLE II

DATA ON RECOVERY FACTOR FOR LAMINAR BOUNDARY LAYER

Author	Date	Model	Reynolds Number	Mach Number	Recovery Factor
Wimbrow[12]	1949	Cone	2.7×10^6	2.0	$0.855 \pm .008$
		Paraboloid	4.8×10^6	1.5	$0.845 \pm .008$
				2.0	$0.855 \pm .008$
Stalder, Rubesin, Tendeland[13]	1950	Flat plate	$0.2 - 1 \times 10^6$	2.4	$0.881 \pm .007$
Eber[14]	1952	Cones (10^o-80^o) Cone-cylinders	6×10^3 - 5×10^5	0.88-4.65	$0.845 \pm .008$
des Clers and Sternberg[15]	1952	Cone	$0.1 - 1.3 \times 10^6$	2.18	$0.851 \pm .007$
Slack[16]	1952	Flat plate	$0.15 - 3 \times 10^6$	2.4	$0.884 \pm .006$
Stine and Scherrer[17]	1952	Cone	$0.2 - 1.3 \times 10^6$	2.0	0.845

The M.I.T. recovery factors on a flat plate agreed with the cone values of 0.85 and not with the flat-plate values of 0.88 of references 13 and 16.

In summary, up to a Mach number of 3, the experimental recovery factors agree within one per cent of the theoretical value given by the square root of the Prandtl number evaluated at the adiabatic wall temperature. An unexplained discrepancy of about 4 per cent exists in some of the experiments made with flat plates. All evidence indicates the laminar recovery factors are independent of Mach numbers up to 4 and of Reynolds numbers up to the beginning of transition of the laminar boundary layer.

3. Data on Heat-Transfer Coefficients for Laminar Boundary Layer

Experimental data on heat-transfer coefficients for laminar boundary layers in supersonic flow are scarce. The following papers contain some results for supersonic flow.

The local heat-transfer coefficients of Slack,[16] for a flat plate at a Mach number of 2.4, are shown in Fig. I (Slack[16] - Fig. 9). The uncorrected heat-transfer data (not shown in Fig. I) scatter by a factor of 3, whereas the data shown in Fig. I (Slack[16] - Fig. 9) corrected for variable surface temperature, scatter from -15 to +100 per cent relative to the theoretical line of Chapman and Rubesin[18] for constant wall temperature.

The heat-transfer coefficients of Eber,[14] some of which are shown in Fig. H, for cones with Mach numbers ranging from 0.88 to 4.2, scatter from his correlation line by about ± 20 per cent. His correlation for cones agreed well with the theoretical value for flat plates when the appropriate conversion was made from cone-type flow to flat-plate flow.

The local heat-transfer coefficients of Scherrer and Gowen[19] for a cone at a Mach number of 2.0 indicated agreement within 10 per cent of the theoretical value at the base of the cone but showed a large difference of 50 per cent at the nose.

C. Theoretical Work on Turbulent Boundary Layer

The problems of turbulence in general, of the turbulent boundary layer in particular, and of transition from a laminar to a turbulent boundary layer are probably the most important problems in fluid mechanics whose detailed nature and mechanism are not understood. Hence, all theoretical investigations on turbulent boundary layers are based on arbitrary and simplified models or sets of assumptions, which in turn permit the calculation of desired quantities to a first approximation. The extraordinary feature is that the results based on these arbitrary models agree quite well with the available experimental data in many cases.

1. Skin-Friction Coefficient for Turbulent Boundary Layer

Several reviews have been presented of the many and various models used for the turbulent boundary layer in supersonic flow. In order to keep the size of this survey within bounds, the excellent review given by Rubesin, Maydew, and Varga[20] will be the only one discussed here. The basis of these models is to use one set of properties, velocity profiles, etc., for the laminar sublayer and another set of properties, eddy viscosity, velocity profiles, etc., for the turbulent portion of the boundary layer. Most of these models depend on slightly different methods of integration of the von Karman momentum integral for the turbulent boundary layer. The resulting formulas for local and average skin-friction coefficients for a flat plate are not simple; for this reason the theoretical results are shown here only in chart form.

The friction coefficients of Rubesin, Maydew, and Varga,[20] obtained by extending the original analysis of Frankl and Voishel, are shown in Fig. J (Rubesin, Maydew, and Varga[20] - Fig. 9) for a Mach number of 2.5. Wilson[21] extended von Karman's analysis for incompressible flow to include the effects of compressibility. His results are shown in Fig. K (Wilson[21] - Fig. 6). Van Driest[22] derived the continuity, momentum, and energy equations for turbulent flow of a compressible fluid, and used an eddy Prandtl number of unity. He developed a general formula for skin friction including heat transfer. One of his charts for skin friction is shown in Fig. L (Van Driest[22] - Fig. 18).

Donaldson[23] rederived the form of the incompressible turbulent skin-friction law for a plate so as to extend it to compressible flow with an arbitrary set of velocity profiles. His theoretical values compare very well up to a Mach number of 5 with those of Van Driest[22] and Rubesin, Maydew, and Varga,[20] based on more complicated analyses.

2. Recovery Factors for Turbulent Boundary Layer

The theoretical work on recovery factors is limited to the assumption of constant properties in the turbulent portion of the boundary layer, i.e. to a constant value of eddy viscosity and eddy conductivity. The early work of Ackermann, using a kinetic-theory model, showed the recovery factor to be equal to the cube root of the laminar Prandtl number for Prandtl numbers greater than 0.5 and less than 2. Other analyses gave similar results except that a decrease of recovery factor with increasing Mach number was indicated.

The work of Tucker and Maslen[24] extended the incompressible analysis of Squire for a flat insulated plate to compressible flow. Their results can be represented by the following approximate rule within one per cent

$$\ln r = \left[\frac{N + 1 + 0.528 M_1^2}{3N + 1 + M_1^2} \right] \ln Pr,$$

where Pr is the laminar Prandtl number evaluated at the adiabatic wall temperature, M_1 is the free-stream Mach number, and N is the reciprocal of the exponent of the turbulent boundary-layer velocity profile approximated by the power law. This relation holds for Prandtl numbers greater than 0.65 and less than 0.75. This relation reduces to the cube-root rule of Ackermann for zero Mach number and large value of N or large Reynolds numbers.

3. Heat-Transfer Coefficients for Turbulent Boundary Layer

The theoretical work on heat-transfer coefficients for a turbulent boundary layer is limited to the analysis by Van Driest[22] and to the use of the Reynolds analogy for the work on skin-friction coefficients by Rubesin, Maydew, and Varga.[20] The results of Van Driest's analysis are shown in a series of charts in Fig. M (Van Driest[22] - Figs. 1, 2, 3, 5) covering a range of Mach number from 0 - 6 and several values of the ratio of wall temperature to free-stream temperature.

D. Experimental Work on Turbulent Boundary Layer

1. Data on Skin-Friction Coefficient for Turbulent Boundary Layer

Most of the experimental work on skin-friction coefficients for a turbulent boundary layer in supersonic flow is summarized in Table III. Since none of the experimental models had a completely turbulent boundary layer from the leading edge, it was necessary to make a correction for the laminar portion of the total boundary layer. This correction altered the measured value of the local coefficient by 10 to 500 per cent.

TABLE III

DATA ON SKIN FRICTION FOR TURBULENT BOUNDARY LAYER

Author	Date	Model	Reynolds Number	Mach Number	Method
Wilson[21]	1950	Flat Plate	-19×10^6	2.2	Velocity Profiles
Rubesin, Maydew Varga[20]	1951	Flat Plate	-6×10^6	2.5	" "
Potter[7]	1952	Cones Cone-cylinders	-8×10^6	1.86-4.24	Drag Forces
Bloom[25]	1952	Flat Plate	-10×10^6	5.5	Velocity Profiles
Brinich and Diaconis[26]	1952	Hollow cylinder	-14×10^6	3.05	Velocity Profiles Schlieren observations
Coles[10]	1952	Flat Plate	-10×10^6	4.5	Floating Element
Coles and Goddard[11]	1952	Flat Plate	-10×10^6	4.5	Floating Element

With the exception of the work of Potter[7] on cones, the experiments of the other investigators in Table III, showed consistent values of turbulent skin-friction coefficients which agreed within about 10 per cent of the theories of Wilson,[21] of Rubesin, Maydew, and Varga,[20] and of Van Driest.[22] In order to test these theories to better than an error of 10 per cent, it appears necessary to measure turbulent skin-friction coefficients to a much smaller error. If, however, the variable effects of transition are considered, this requirement appears to be a difficult condition to achieve in tests.

2. Data on Recovery Factors for Turbulent Boundary Layer

It is evident from the data in Table IV that, with the exception of Eber's data on cones and cone-cylinders and possibly Slack's data on a flat plate, the recovery factors for a fully turbulent boundary agree within one per cent with the theoretical approximate rule, i.e. equal to the cube root of the Prandtl number. Furthermore the data indicate that these recovery factors are independent of Reynolds number and Mach number over the ranges tested. The theoretical work of Tucker and Maslen[24] does not agree with these experimental results.

3. Data on Heat-Transfer Coefficients for Turbulent Boundary Layer

Experimental data on heat-transfer coefficients for a turbulent boundary layer in supersonic flow are almost nonexistent. Eber[14] has measured a few heat-transfer coefficients on a cone at a Mach number of 2.87 and over a limited range of Reynolds numbers from 0.4 to 0.9 x 10^6; he found his values of the Nusselt number to scatter from +10 to -40 per cent relative to the well known Colburn equation for <u>subsonic</u> flow in a turbulent boundary layer, i.e.

$$Nu = 0.029\, Re^{0.8} Pr^{1/3}. \tag{5}$$

Slack[16] has also measured some heat-transfer coefficients for a flat plate at a Mach number of 2.4 and over a Reynolds number from 1.8 to 3 x 10^6. The twenty odd points are shown in Fig. I (Slack[16] - Fig. 9).

M.I.T. – O.N.R. PROGRAM

A research program, sponsored by the Office of Naval Research has been under way for several years in the Department of Mechanical Engineering with the objective of obtaining reliable data on the rate of heat transfer to air moving at supersponic speeds in a round tube. This program consists of two separate parts. In the first part a well-insulated apparatus was used to measure the values of the local adiabatic wall temperature and local static pressure of a supersonic stream of air in a tube with an entrance Mach number of 2.8. In the second part an apparatus using steam condensing outside a round brass tube was used to measure local coefficients of heat transfer to a similar supersonic stream of air from the same nozzle.

TABLE IV

DATA ON RECOVERY FACTORS FOR TURBULENT BOUNDARY LAYER

Author	Date	Model	Reynolds Number	Mach Number	Recovery Factor
Wimbrow[12]	1949	Cone	-2.7×10^6	2.0	$0.885 \pm .008$
		Paraboloid	-4.8×10^6	1.5	$0.902 \pm .005$
				2.0	$0.894 \pm .008$
Stalder, Rubesin, Tendeland[13]	1950	Flat Plate	-7×10^6	2.4	$0.884 - .897 \pm .007$
Hilton[27]	1951	Flat Plate	-10×10^6	2.0	$0.880 \pm .004$
Eber[14]	1952	Cone and cone-cylinder	-1×10^6 -0.25×10^6	2.87 4.25	0.92 0.97
des Clers Sternberg[15]	1952	Cone and cone-cylinder	-7×10^6	2-3.4	$0.882 + .007$
Slack[16]	1952	Flat Plate	-3×10^6	2.4	0.906
Stine, Scherrer[17]	1952	10° Cone	$0.4\text{-}4 \times 10^6$	1.97-3.77	$0.882 \pm .008$
		40° Cone-cylinder	$0.3\text{-}1 \times 10^6$	3.10-3.77	$0.885 \pm .011$

The results of the first part of this program are available in references 31, 32, and 33; hence these data will not be discussed in detail. The results of some preliminary measurements of heat-transfer coefficients will be given. In addition, preliminary results will be given of some theoretical work for the compressible laminar boundary layer in the entrance region of a round tube.

1. Adiabatic Flow in Tube

Figs. N, O, and P present some typical results for supersonic flow of air in a round tube under adiabatic conditions for the case where a laminar boundary layer appears to exist in the tube. The calculated quantities are determined from two flow models, the first corresponding to the usual one-dimensional flow model based on constant properties at any cross section, and the second based on a two-dimensional flow model where the properties vary over the cross section of the tube. In addition the preliminary calculations obtained from the M.I.T.

Differential Analyzer in the solution of the theoretical problem, from the Doctoral Thesis of Toong,[34] are also shown.

The local coefficients of friction based on the two-dimensional flow model are in fair agreement with the values for a flat plate and the theoretical values for a tube where it appears that a laminar boundary layer exists in the tube. In Fig. P, this agreement is excellent.

The recovery factors based on the two-dimensional flow model vary from 0.87 to 0.89 for that portion of the tube flow where it appears a laminar boundary layer exists. These values are 2-3 per cent larger than the theoretical values for a laminar boundary layer on a flat plate with zero pressure gradient.

The calculated thickness of the laminar boundary layer in the tube, based on the two-dimensional flow model, is also shown in Figs. N, O, and P. The agreement between this quantity and the thickness determined from Howarth's flat-plate formula and from the Differential Analyzer solution is excellent for this type of calculation.

2. Heat-Transfer Measurements in the Tube

Preliminary heat-transfer data are shown in Fig. Q for the case where a laminar boundary appears to exist in the supersonic flow in the tube. The scatter present in these data is typical of most measurements of local heat-transfer coefficients. The data agree well with the theoretical value for a flat-plate with zero pressure gradient and with the Differential Analyzer solution for tube flow.

M.I.T. – O.A.R. PROGRAM

A research program, sponsored by the Office of Air Research, was started about one year ago as a joint effort of the Departments of Aeronautical Engineering and Mechanical Engineering with the objective of obtaining reliable data on heat-transfer coefficients to air moving at supersonic speeds over a flat plate. The experimental program is being carried on at the M.I.T. Naval Supersonic Wind Tunnel, over a range of Mach number from 1.9 to 3.1, and up to a length Reynolds number of 17×10^6.

This program has been divided in two parts. The first part consists of an attempt to secure a flat plate model which corresponds as nearly as possible to an insulated flat plate. On this model the local recovery factors and velocity profiles in the boundary layer could be measured. Thus the type of boundary layer present on the plate could be identified. The second part of the program will be concerned with construction of a separate flat-plate model made of brass, to secure local heat-transfer coefficients by use of condensing steam. Velocity profiles will be measured to determine the type of boundary layer present on the heated plate. The effects of turbulence promoters and of large temperature differences between the free stream and wall will be investigated.

To date the insulated plate tests have indicated that the recovery factor for a laminar boundary layer, properly corrected for the effect of the heated nose resulting from the shock at the nose is 0.850 ± 0.005 for Mach numbers from 1.9 to 3.1. This value agrees within one per cent with the theoretical value based on the square root of the Prandtl number evaluated at the adiabatic wall temperature and also agrees with similar data on cones.

The recovery factor for a turbulent boundary layer is 0.881 ± 0.003 for Mach numbers of 1.9 to 3.1 and up to a Reynolds number of 17×10^6. Within one half of one percent, this recovery factor is independent of Mach number and of Reynolds number over the ranges measured.

CONCLUSIONS

The author examined in 1948 the problem of the transient temperature distribution in a wing moving at supersonic speeds; he found it necessary to review the state of the art relative to heat transfer at supersonic speeds. One conclusion reached at that time was that the state of the art was relatively undeveloped and a considerable amount of theoretical and experimental work was required to improve it significantly.

In the past four years significant contributions have been made to both the theoretical and experimental work on laminar boundary layers up to about a Mach number of 3. Small discrepancies between theory and experiment still exist in this range for friction coefficients and recovery factors. Additional experimental data are greatly needed for heat-transfer coefficients. The greatest need is to extend the experimental data beyond a Mach number of 3 and to investigate problems such as the effect of a pressure gradient, the effect of radiation, the effect of transition, the properties of air at temperatures of the order 3,000°F, etc.

In the past four years several significant contributions have been made to the theory of the turbulent boundary layer for supersonic flow; these have been based on arbitrary models or assumptions because of the lack of knowledge of the details of turbulence. The experimental data on skin-friction coefficients and recovery factors for turbulent boundary layers have been extended to a Mach number of about 3 and a length Reynolds number of about 20×10^6. It is curious that for this case where the basis of the theory of the turbulent boundary layer is fairly uncertain, the agreement between the predicted and measured values of the skin-friction coefficient and recovery factor is the best of all available comparisons at the moment. There is a great need for reliable data on heat-transfer coefficients for turbulent boundary layers since, practically speaking, none are available. The experimental work for the turbulent boundary layer should be extended to higher Mach numbers and to larger Reynolds numbers; the effects of pressure gradient, radiation, etc., should be investigated.

ACKNOWLEDGMENTS

The assistance of Professor T. Y. Toong and George A. Brown in the preparation of Figs. N, O, P, and Q is gratefully acknowledged.

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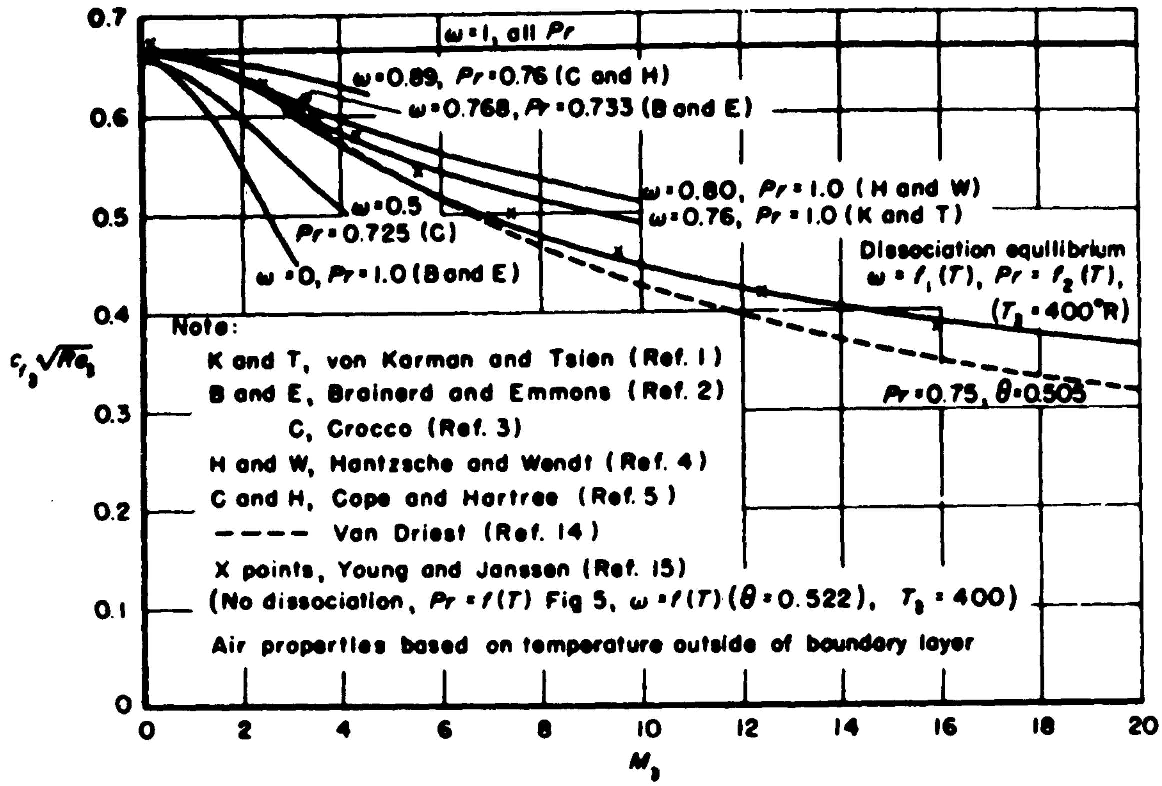


FIG. 10. Skin friction on an insulated flat plate with no radiation.

Fig. A - Moore[2] - 1952

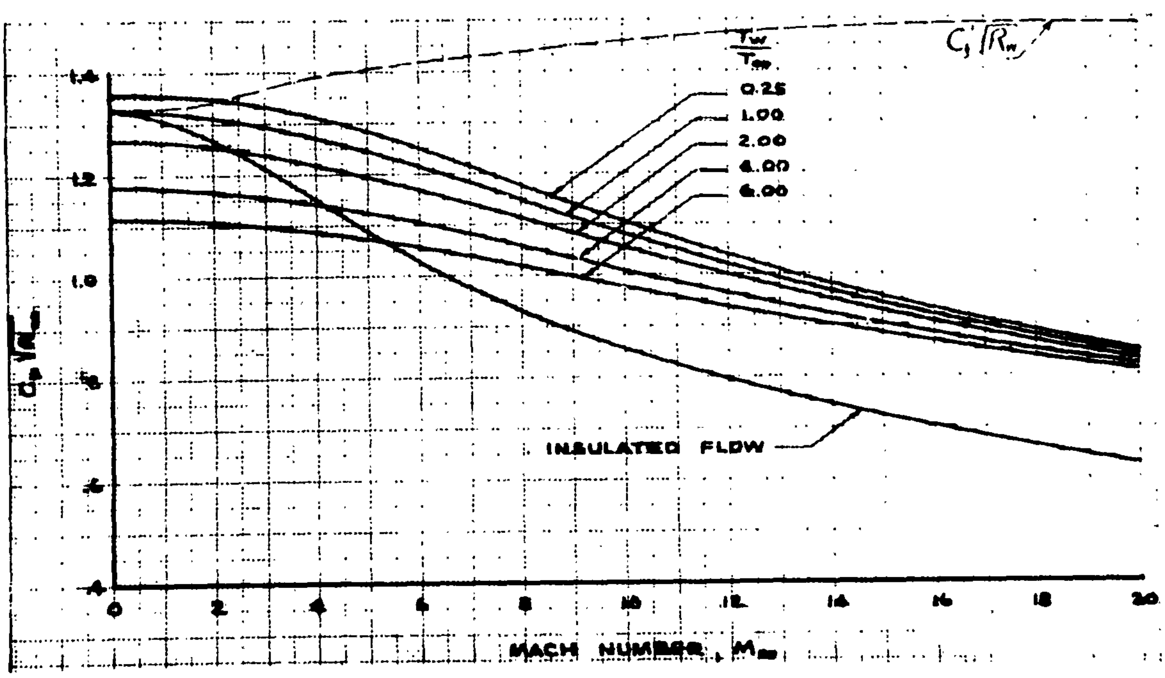


Figure 3.- Mean skin-friction coefficient for laminar boundary layer of a compressible fluid flowing along a flat plate. Prandtl number, 0.75; θ = 0.505.

Fig. B - Van Driest[3] - 1952

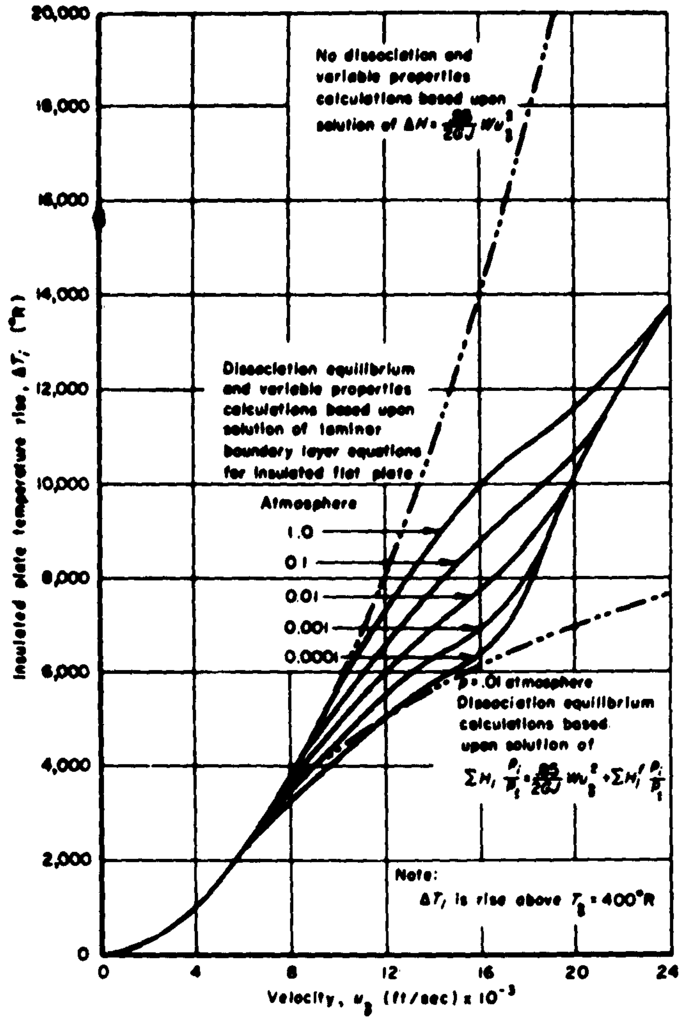


FIG. 13. Insulated plate temperature rise vs. velocity outside boundary layer.

Fig. C - Moore[2] - 1952

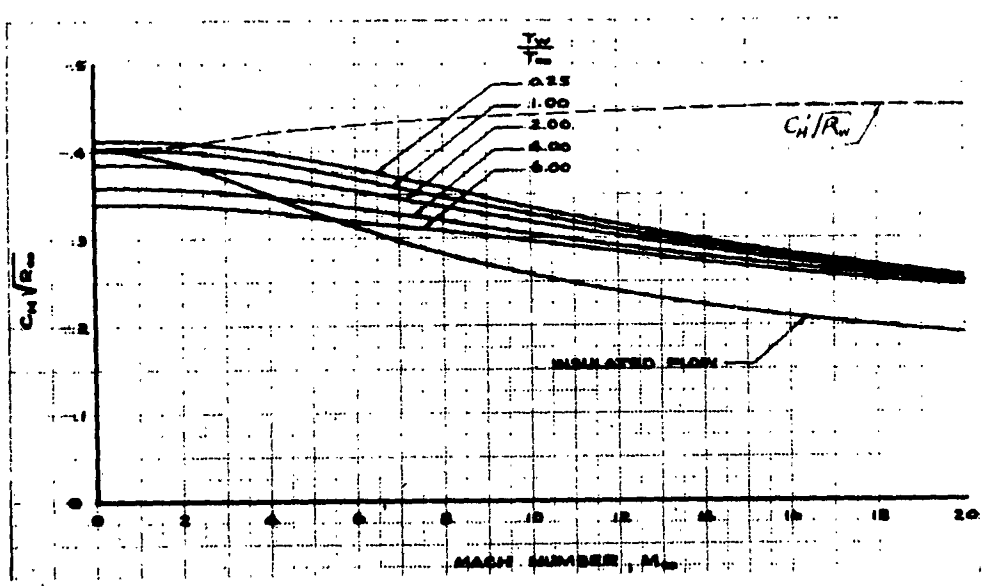


Figure 4.- Local heat-transfer coefficient for laminar boundary layer of a compressible fluid flowing along a flat plate. Prandtl number, 0.75; θ = 0.505.

Fig. D - Van Driest[3] - 1952

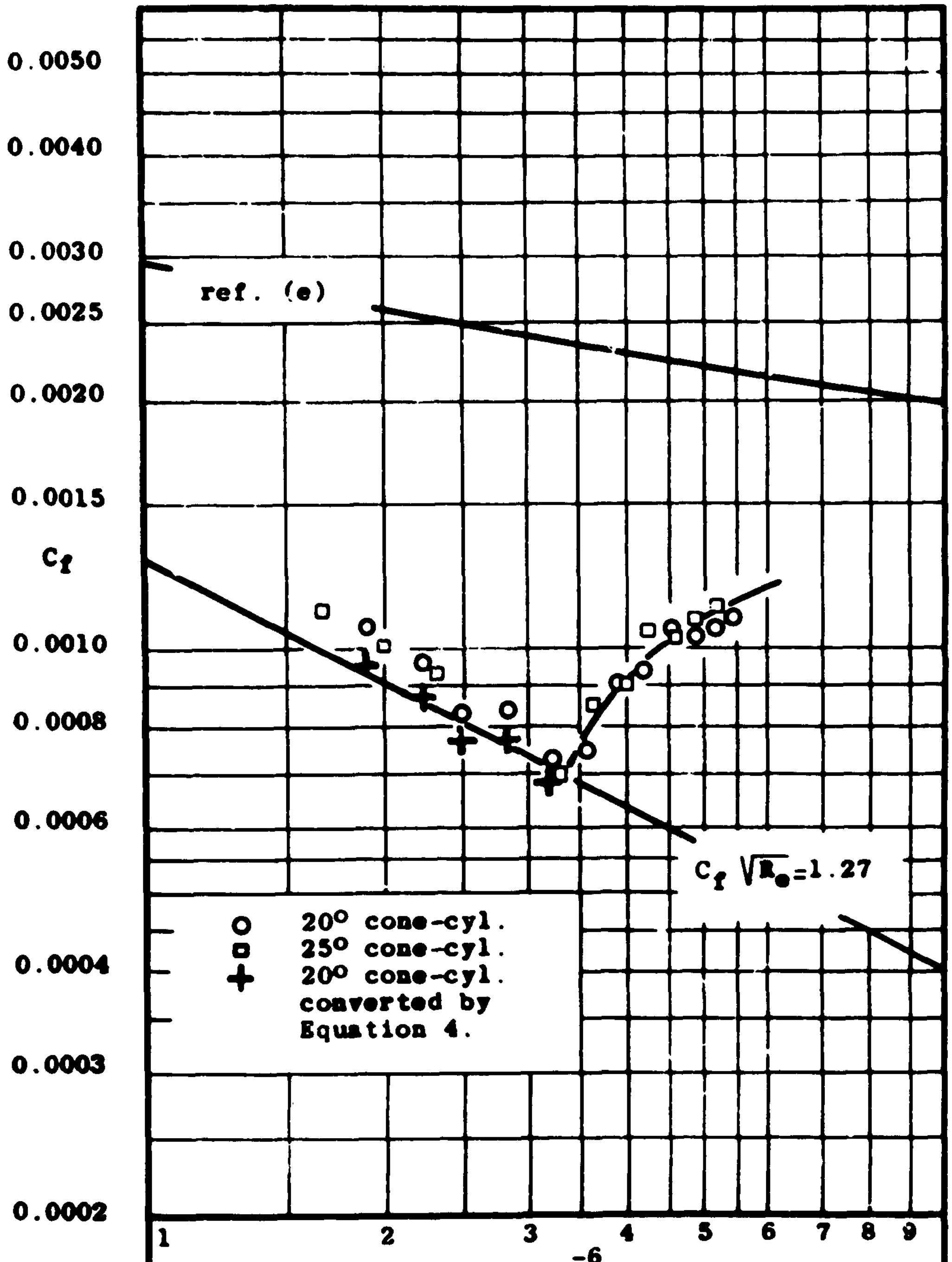


Figure 7 - Total Skin Friction Coefficients at M = 2.85 ($T_w/T \approx 2.5$)

Fig. E - Potter[7] - 1952

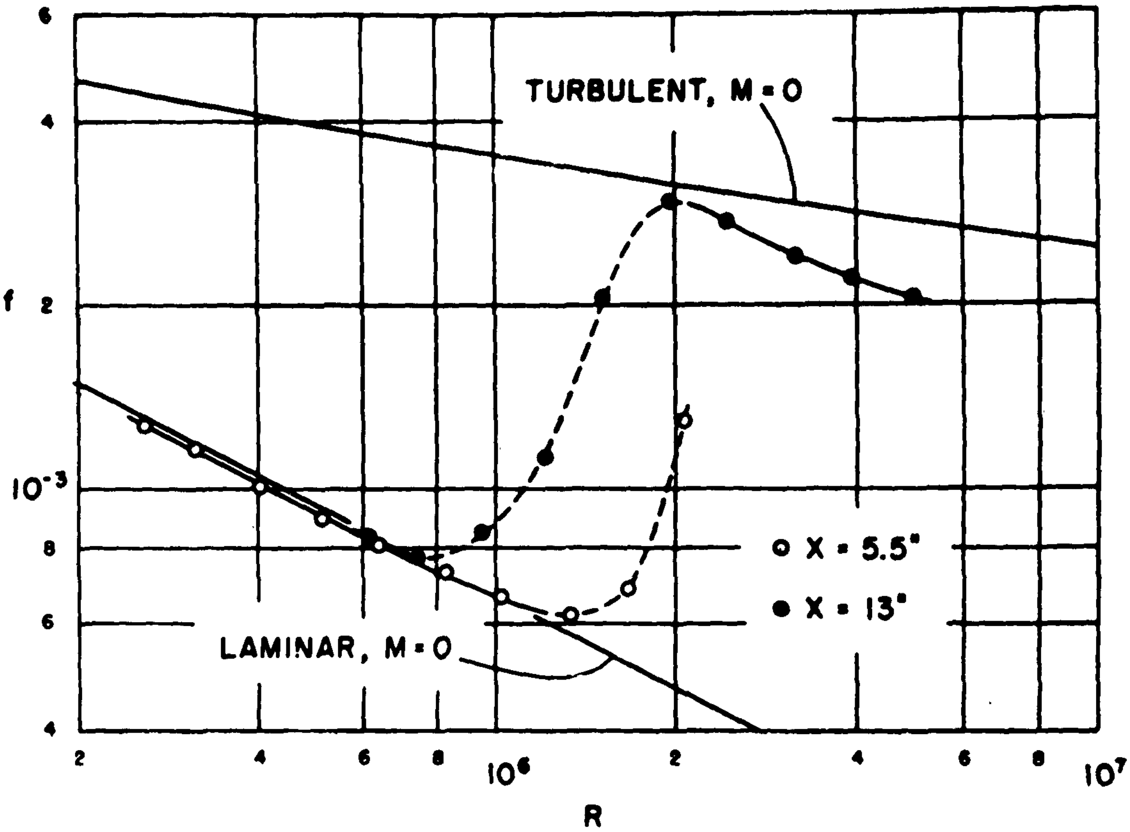


FIG. 1. Local friction on a smooth flat plate at a Mach Number of 2.6; no tripping device.

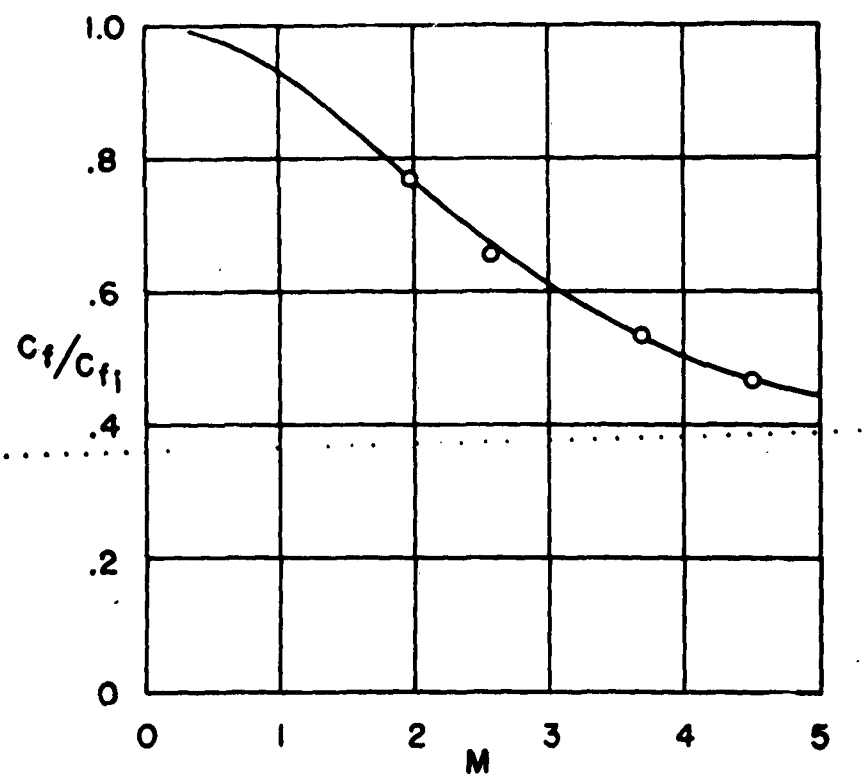


FIG. 3. Variation of turbulent local skin friction coefficient with Mach Number at a Reynolds Number of 8,000,000.

Fig. F - Coles[10] - 1952

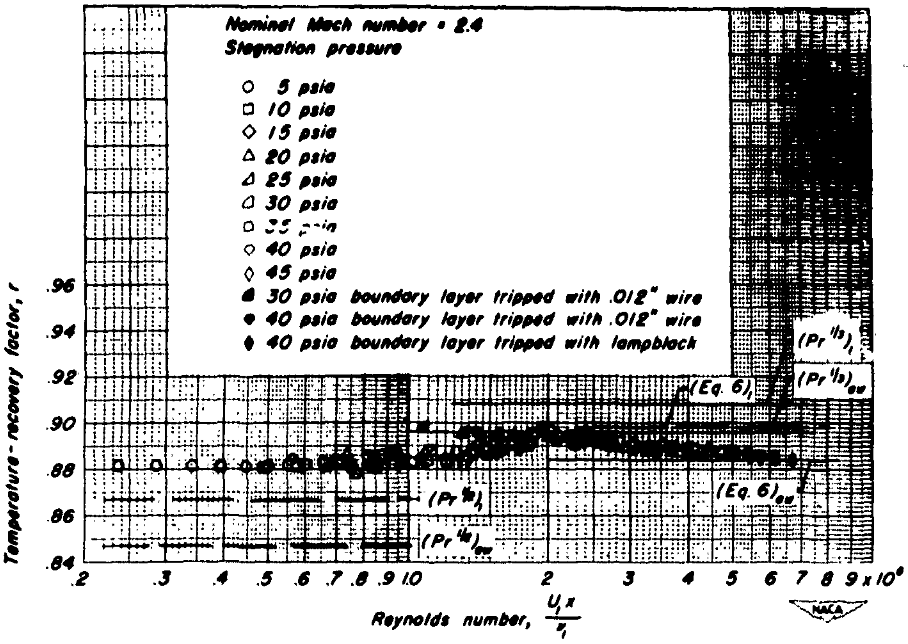


Figure 5.- Local temperature-recovery factor on flat plate.

Fig. G - Stalder, Rubesin, and Tendeland[13] - 1950

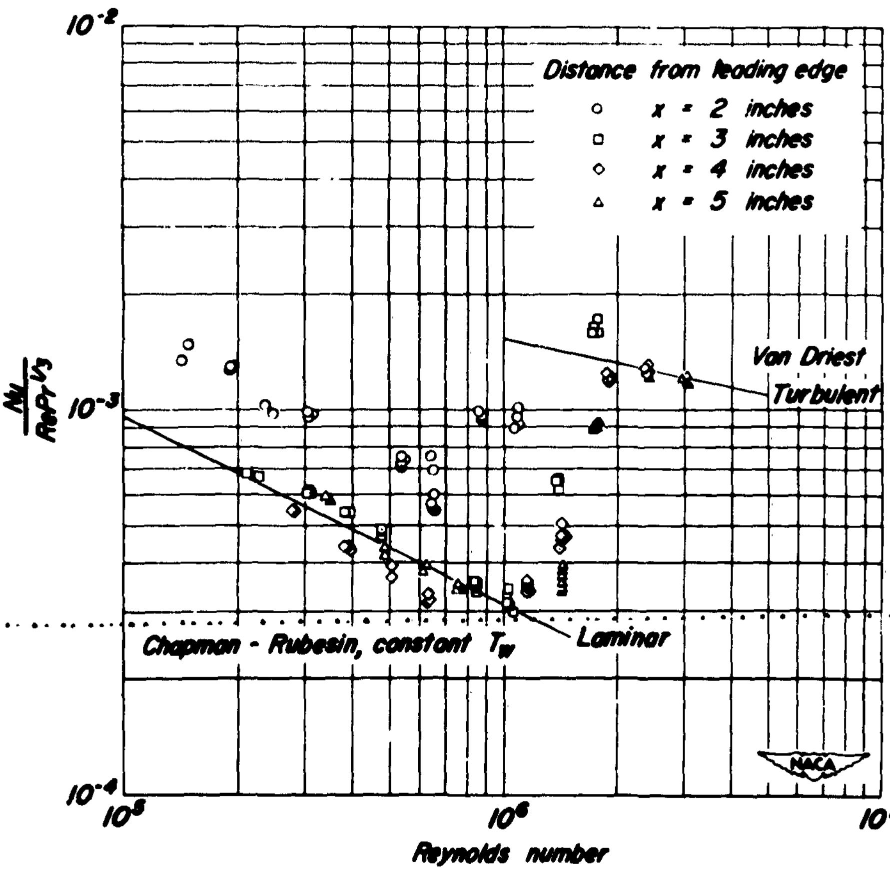


Figure 9. – Dimensionless representation of heat-transfer data referred to the case of constant-surface temperature.

Fig. I - Slack[16] - 1952

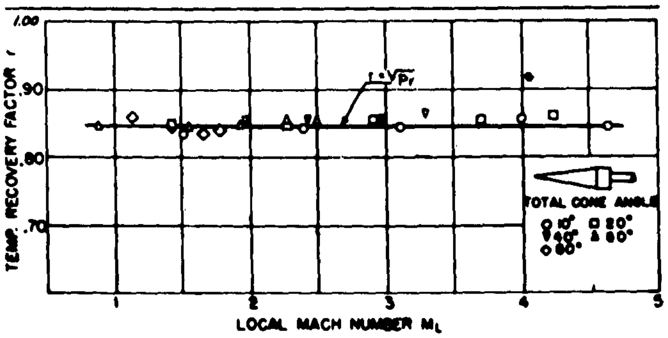


FIG. 3. Recovery factor for laminar boundary-layer flow vs. local Mach Number.

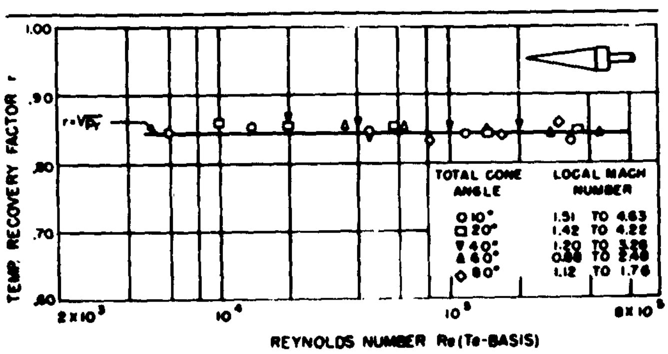


FIG. 4. Recovery factor for laminar boundary-layer flow vs. Reynolds Number.

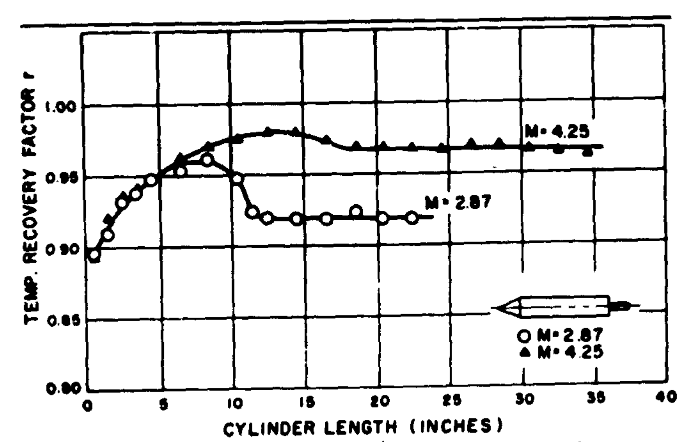


FIG. 5. Recovery factor along a 40° cone-cylinder.

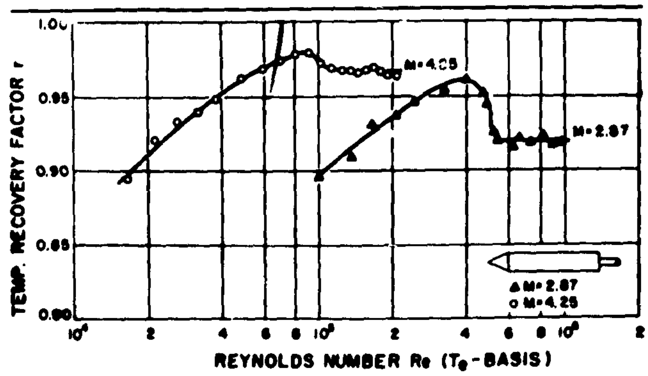


FIG. 7. Recovery factor for a cone-cylinder model vs. Reynolds Number.

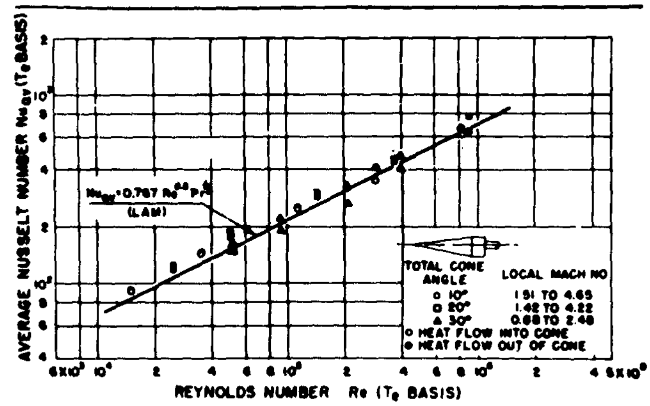


FIG. 8. Average heat transfer for cones with laminar boundary layer vs. Reynolds Number.

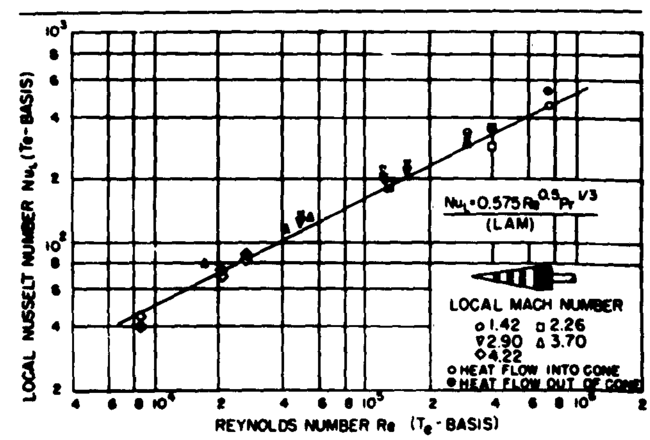


FIG. 9. Local heat transfer for cones with laminar boundary layer vs. Reynolds Number.

Fig. H - Eber[14] - 1952

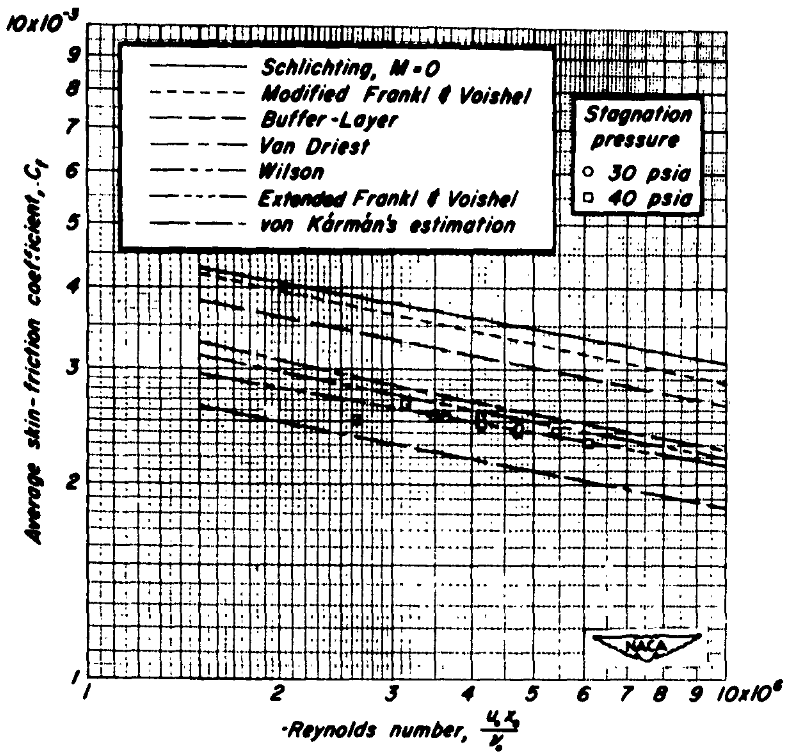


Figure 9.- Comparison of several analytical methods with experimental skin-friction coefficients along a flat plate, M = 2.5.

Fig. J - Rubesin, Maydew, and Varga[20] - 1951

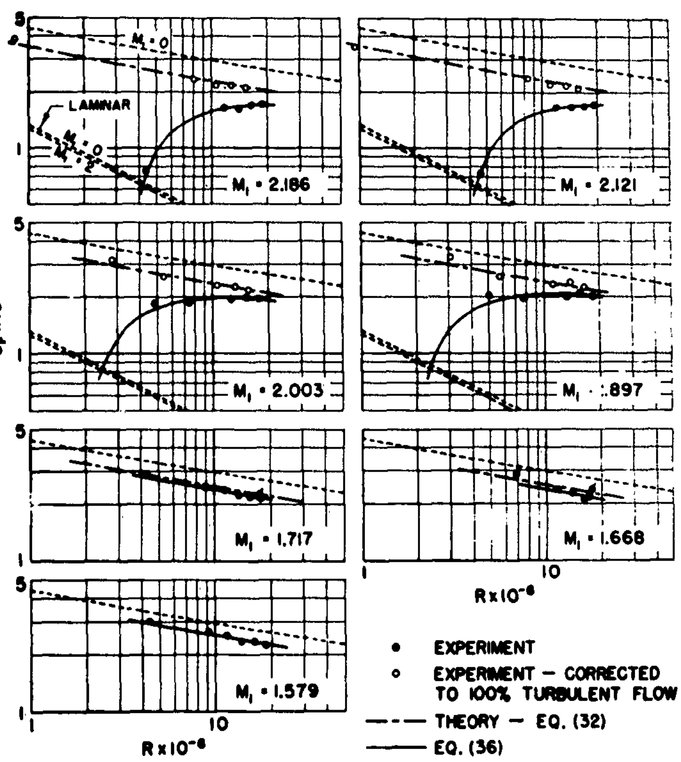


FIG. 6 MEAN TURBULENT SKIN FRICTION COEFFICIENT
— THEORY AND EXPERIMENT —

Fig. K - Wilson[21] - 1950

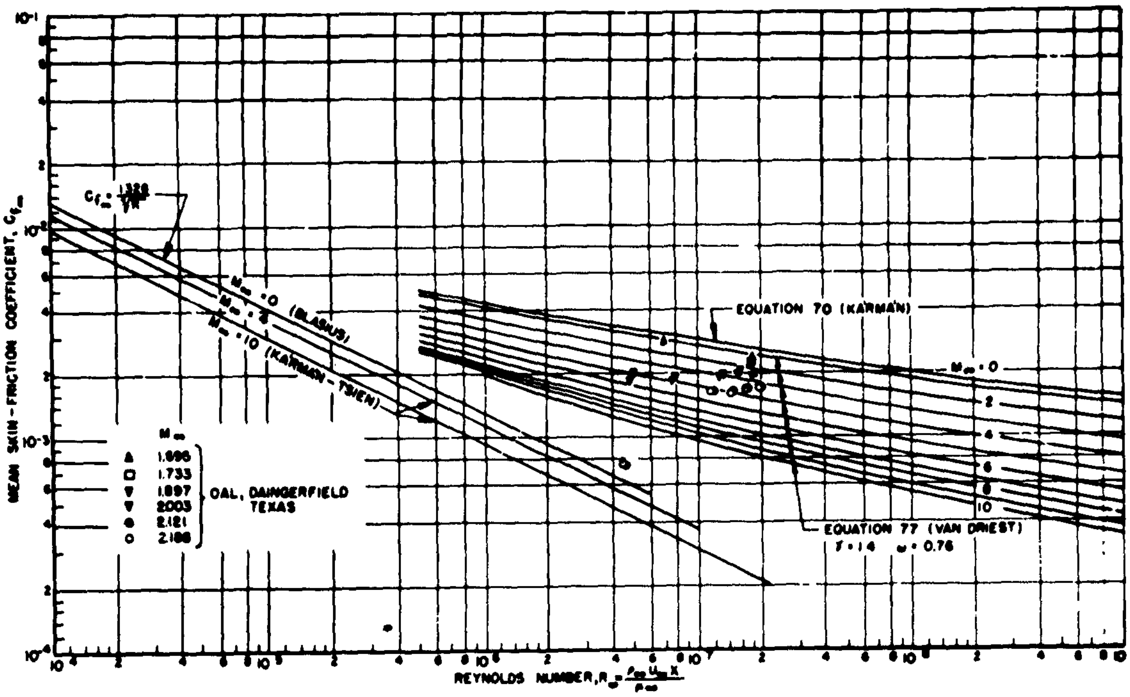


FIG. 18. Mean turbulent skin friction as a function of Reynolds Number and Mach Number for zero heat transfer

Fig. L - Van Driest[22] - 1951

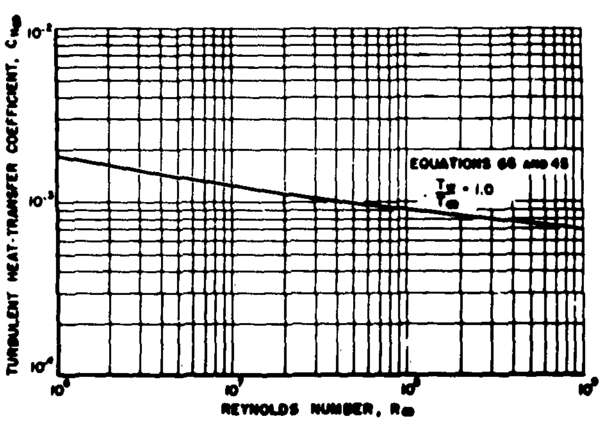


FIG. 1. Turbulent heat-transfer coefficient for air at $M_\infty = 0$.

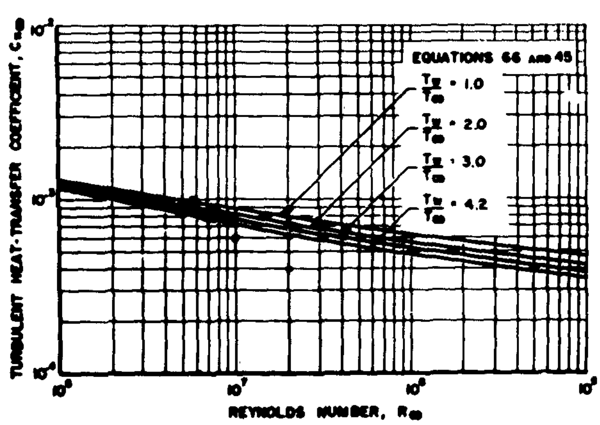


FIG. 3. Turbulent heat-transfer coefficient for air at $M_\infty = 4$

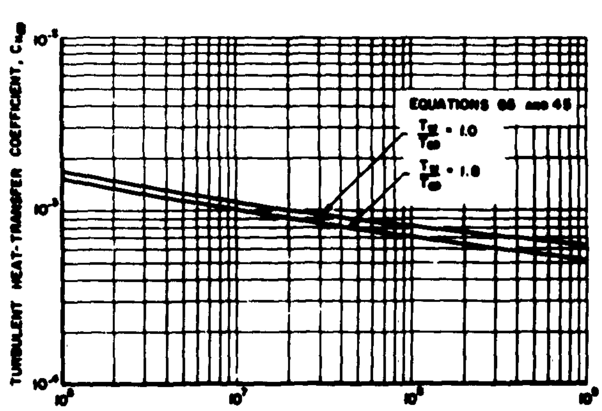


FIG. 2. Turbulent heat-transfer coefficient for air at $M_\infty = 2$.

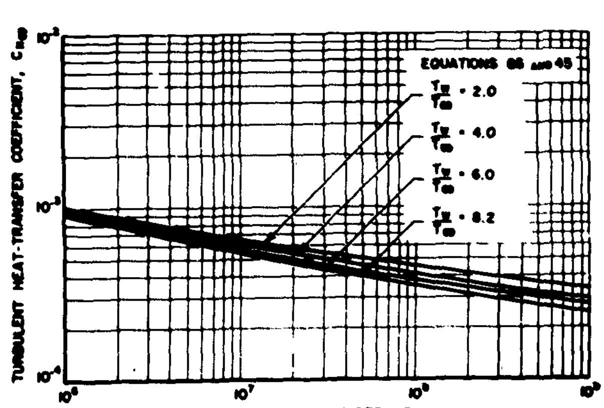


FIG. 5. Turbulent heat-transfer coefficient for air at $M_\infty = 6$.

Fig. M - Van Driest[22] - 1951

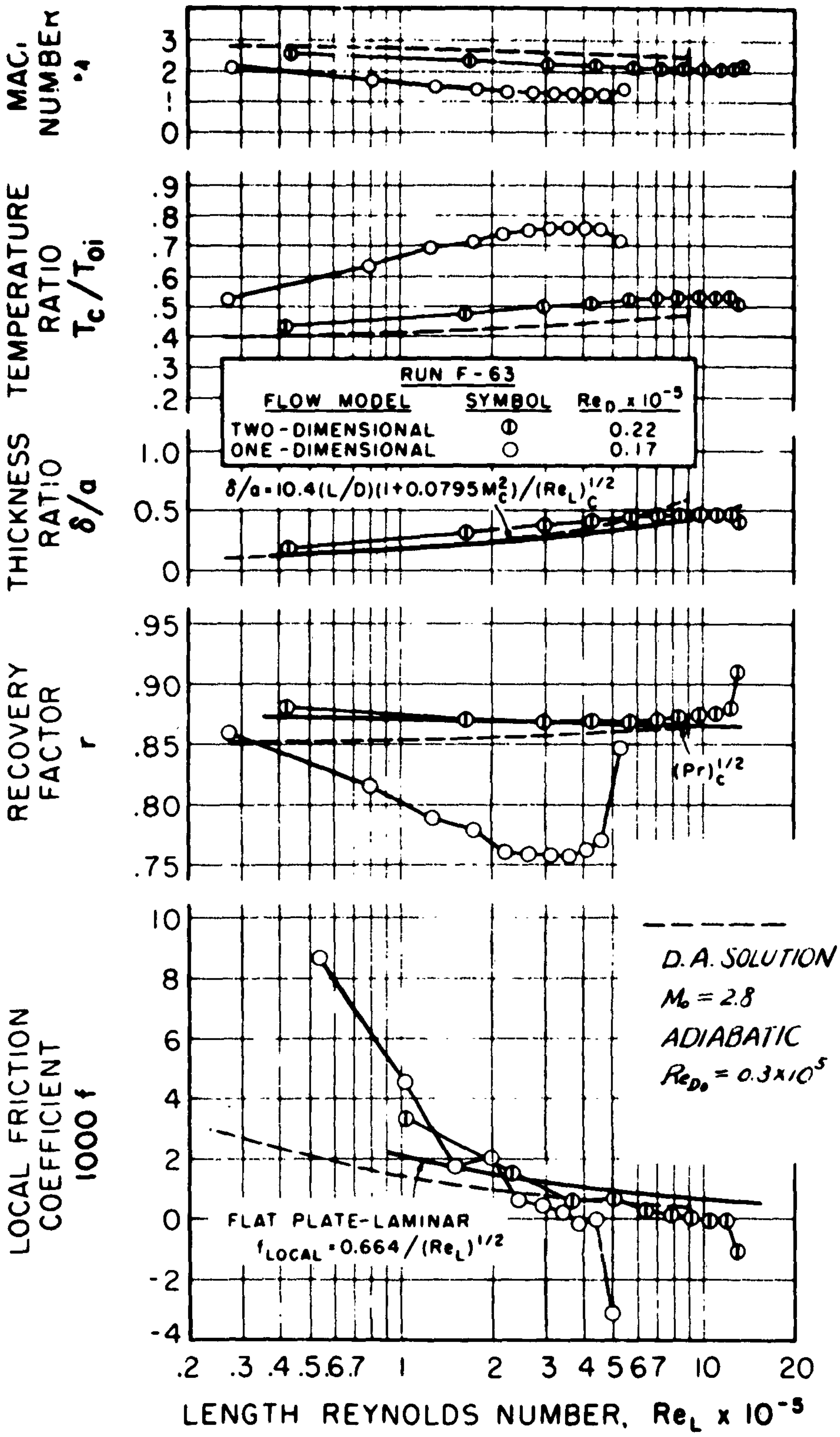

MACH NUMBER
TEMPERATURE RATIO T_c/T_{oi}
RUN F-63
FLOW MODEL SYMBOL $Re_D \times 10^{-5}$
TWO-DIMENSIONAL 0.22
ONE-DIMENSIONAL 0.17
THICKNESS RATIO δ/a
$\delta/a = 10.4(L/D)(1+0.0795 M_c^2)/(Re_L)_c^{1/2}$
RECOVERY FACTOR r
$(Pr)_c^{1/2}$
LOCAL FRICTION COEFFICIENT 1000 f
D. A. SOLUTION
$M_o = 2.8$
ADIABATIC
$Re_{D_o} = 0.3 \times 10^5$
FLAT PLATE-LAMINAR
$f_{LOCAL} = 0.664/(Re_L)^{1/2}$
LENGTH REYNOLDS NUMBER, $Re_L \times 10^{-5}$

Fig. N

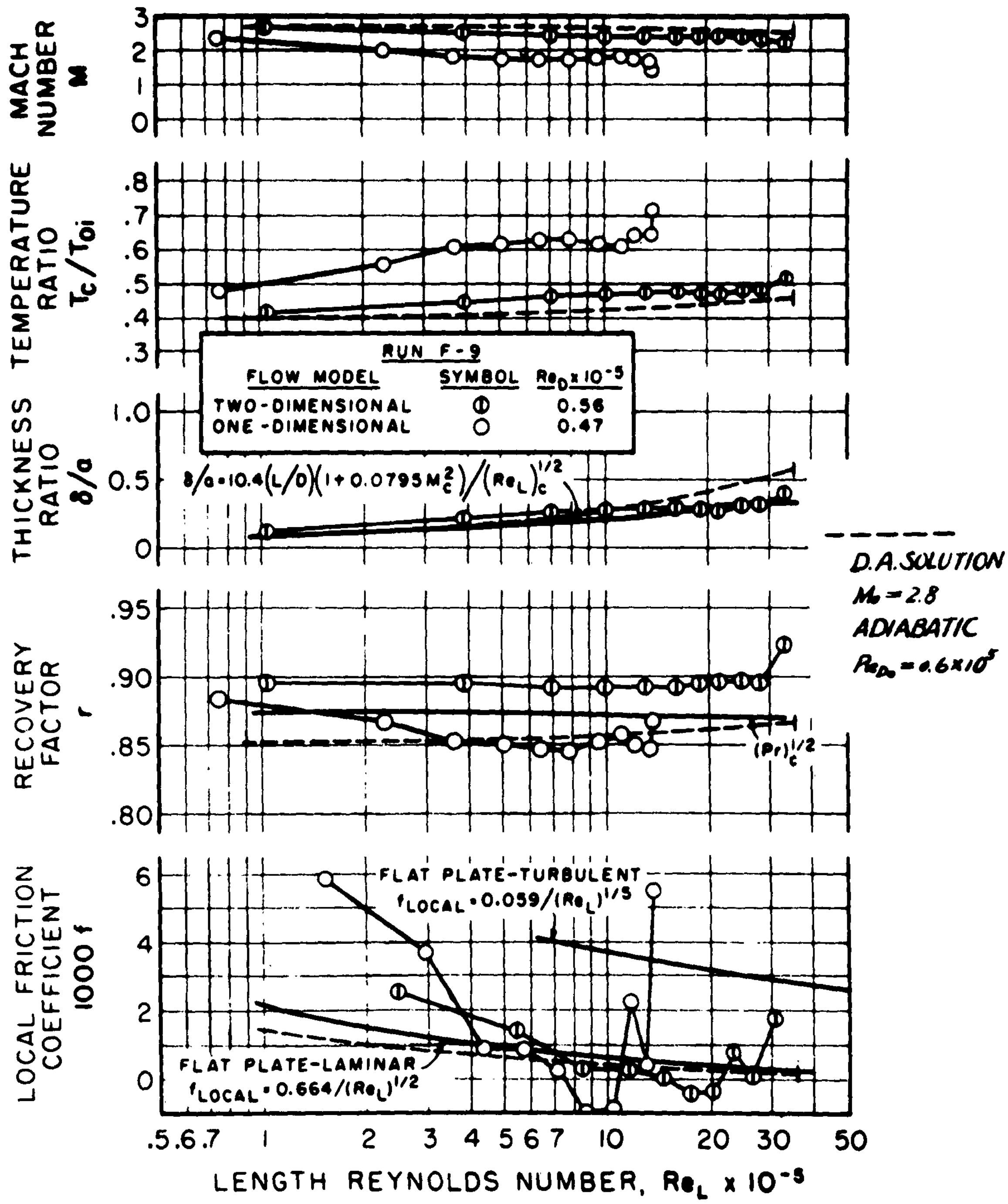
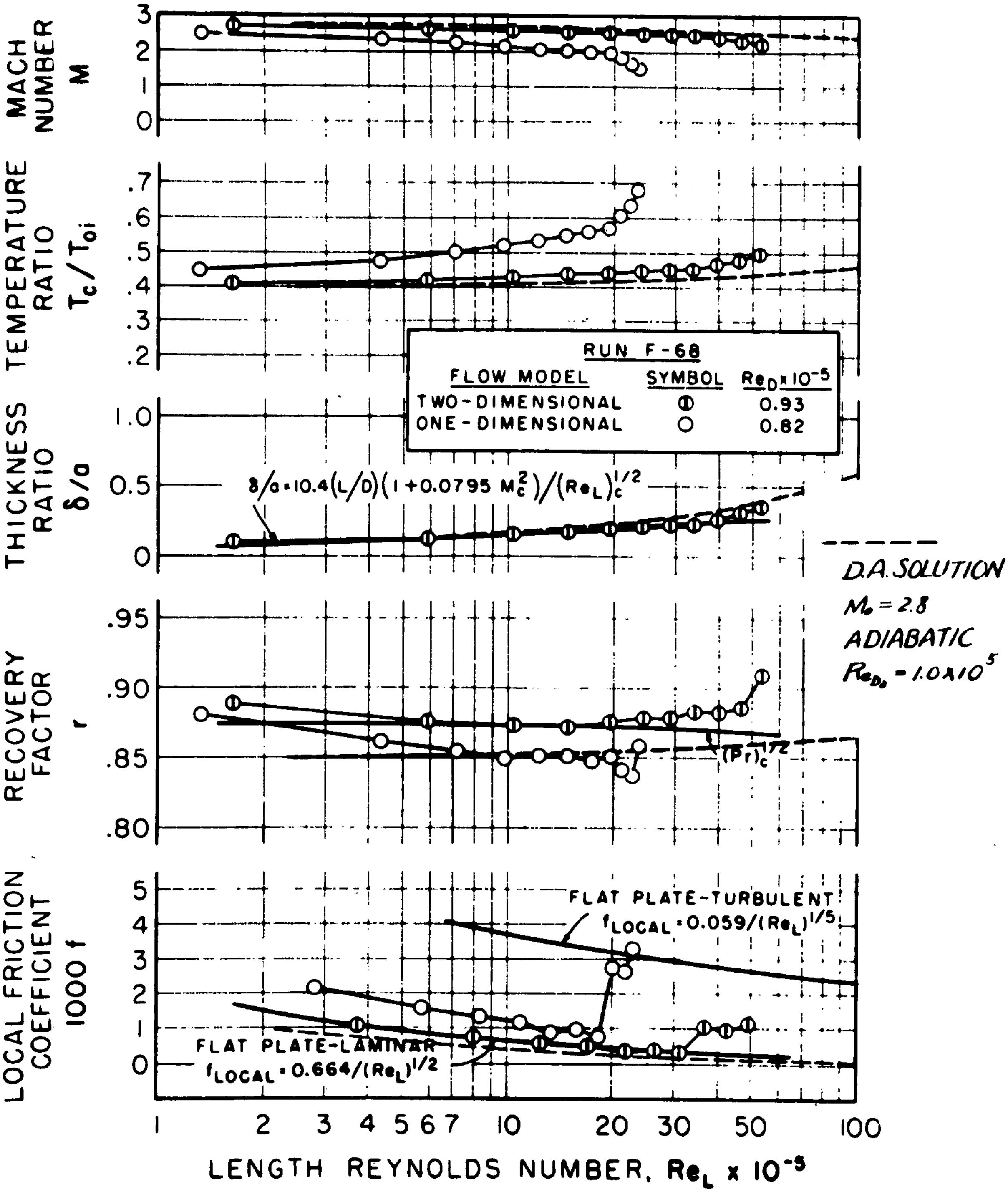

MACH NUMBER M
TEMPERATURE RATIO T_c/T_{0i}
THICKNESS RATIO δ/a
RECOVERY FACTOR r
LOCAL FRICTION COEFFICIENT 1000 f
RUN F-9
FLOW MODEL SYMBOL $Re_D \times 10^{-5}$
TWO-DIMENSIONAL 0.56
ONE-DIMENSIONAL 0.47
$\delta/a = 10.4(L/D)(1+0.0795M_c^2)/(Re_L)_c^{1/2}$
$(Pr)_c^{1/2}$
FLAT PLATE-TURBULENT
$f_{LOCAL} = 0.059/(Re_L)^{1/5}$
FLAT PLATE-LAMINAR
$f_{LOCAL} = 0.664/(Re_L)^{1/2}$
D.A. SOLUTION
$M_0 = 2.8$
ADIABATIC
$Re_{D_0} = 0.6 \times 10^5$
LENGTH REYNOLDS NUMBER, $Re_L \times 10^{-5}$

Fig. O



MACH NUMBER M
TEMPERATURE RATIO T_c/T_{oi}
THICKNESS RATIO δ/a
RECOVERY FACTOR r
LOCAL FRICTION COEFFICIENT 1000 f
RUN F-68
FLOW MODEL | SYMBOL | $Re_D \times 10^{-5}$
TWO-DIMENSIONAL | 0.93
ONE-DIMENSIONAL | 0.82
$\delta/a = 10.4(L/D)(1+0.0795\,M_c^2)/(Re_L)_c^{1/2}$
D.A. SOLUTION
$M_o = 2.8$
ADIABATIC
$Re_{D_o} = 1.0 \times 10^5$
$(Pr)_c^{1/2}$
FLAT PLATE-TURBULENT
$f_{LOCAL} = 0.059/(Re_L)^{1/5}$
FLAT PLATE-LAMINAR
$f_{LOCAL} = 0.664/(Re_L)^{1/2}$
LENGTH REYNOLDS NUMBER, $Re_L \times 10^{-5}$

Fig. P

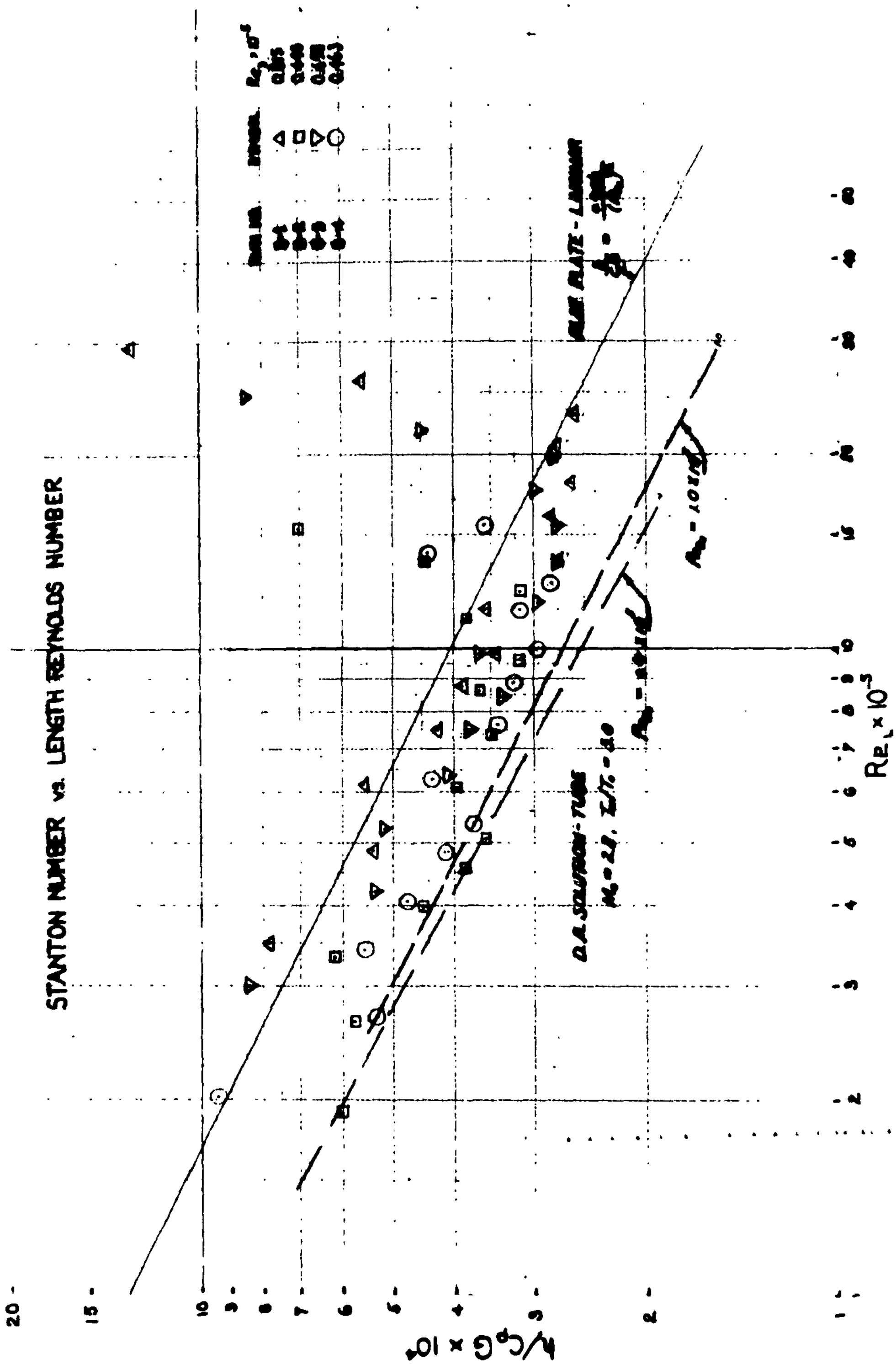


Fig. Q