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PLASTIC STRESS-STRAIN RELATIONS BASED ON LINEAR LOADING FUNCTIONS

by

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PLASTIC STRESS-STRAIN RELATIONS BASED ON LINEAR LOADING FUNCTIONS[1]

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ABSTRACT

The stress-strain relations of incremental theories of plasticity have recently been generalized by Koiter who introduced the use of any number of loading functions instead of only one. The present paper is concerned with incremental theory based on any number of linear loading functions. In this case the resulting plastic strain increments are integrable in a restricted sense and the stress-strain relations can be given in a form partially resembling deformation theories. The slip theory of Batdorf and Budiansky falls within the class of theories which are discussed in this paper.

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Introduction. Within the last few years the development of plastic stress-strain relations for strain hardening materials has received increasing attention from workers in the field of applied mechanics [1], [2], [3], [4].[1] A knowledge of the stress-strain curve for a material in simple tension or compression is not in itself sufficient for predicting the behavior of a material under more complex types of loading. At least two problems immediatly present themselves when one considers loading which may involve all six components of stress. These are the generalization of the idea of a yield point to the idea of a yield surface in a stress space of six dimensions and the formulation of a plastic stress-strain relation for polyaxial loading.

During the history of the subject these problems have been attacked in two more or less distinct ways. One method of approach is to try to deduce the behavior of a bulk piece of metal from the behavior of the individual crystals making up the aggregate. A recent example of results obtained by pursuing this method is the slip theory of plasticity [2]. The other method of approach is to try to deduce from a few reasonable assumptions what the general form of the yield surface and the stress-strain relation must be without inquiring very deeply into the structure of metals but rather proceeding in a phenomenological manner [3].

Results obtained by one method often suggest research to be done by the other method which tends to unify the whole subject. The earliest known yield condition, the Tresca maximum shear

[1] Numbers in brackets refer to the bibliography at the end of the paper.

condition [5], when expressed in geometrical form as a yield surface in stress space possesses "corners". The initial yield surface according to slip theory is the Tresca yield surface which arises as an envelope of planes in stress space. The yield surface after work hardening according to slip theory is also an envelope of planes and naturally possesses corners. Drucker [3] allows for corners in his general discussion of yield surfaces from a phenomenological point of view. The relation between corners in the yield surface and linearity of the stress-strain relation was investigated experimentally by Stockton [6]. The yield surface is closely connected with a function called the loading function which appears in the stress-strain relation. The geometrical expression of the loading function, the loading surface, is identical to the yield surface if there is only one loading function in the stress-strain relation; if the stress-strain relation is based on many loading functions the yield surface is the boundary of those points in stress space which represent elastic behavior with respect to all the loading surfaces. Obviously corners in the yield surface may arise even though all the loading surfaces are smooth. Stress-strain relations based on many loading functions were suggested by Koiter who also investigated some of their properties [4]. Although slip theory was derived through physical reasoning Koiter states that it may be regarded as a theory of plasticity based on an infinite number of plane loading surfaces.

An interesting feature of slip theory is that the incremental form of its stress-strain relation is integrable in a

restricted sense which resembles deformation theories in this respect and is unlike most incremental theories. The present paper is concerned with the use of many linear loading functions in constructing stress-strain relations. It is shown that in some cases the resulting incremental stress-strain relation is integrable in a restricted sense. Means, suggested by the behavior of a single crystal, of including cross effect and Bauschinger effect in the stress-strain relation are also discussed.

A Single Plane Loading Surface. In the formulation of a plastic stress-strain relation it is commonly assumed that the strain may be separated into an elastic and a plastic part which are additive. This assumption is made in the following treatment and the plastic strain only is the subject of discussion. It is further assumed in the following that the plastic part of the strain may be separated into many parts each associated with a loading function.

It is convenient to phrase the discussion in geometrical terms and speak of loading surfaces, loading paths, strain increment vectors and so forth. Imagine a euclidean space with rectangular coordinates, one coordinate for each component of stress. Each stress state is represented by a point in this space. For convenience we also represent strain in the same space with each component of the strain acting as coordinate on the same axis as the corresponding component of stress. The loading history of a piece of strain-hardening material which has been subjected to a varying homogeneous state of stress is representable in this space

as a loading path. We now want to consider the contribution of a single plane loading surface to the plastic strain.

The stress-strain relation to be described in the following is consistent in that it satisfies all the mathematical conditions which have been discovered by phenomenological studies to apply to incremental stress theories of plasticity. The proof that in the particular case of a plane loading surface the incremental stress-strain relation is integrable is given in the appendix. The stress-strain relation in its integrated form is presented here.

Fig. 1 shows a symbolic representation of the loading plane and the loading paths of two hypothetical experiments. As loading proceeds from O to P the material behaves elastically. At P plastic straining begins and continues as long as the loading path is directed outward from the loading plane. Thus during the part Q'R of the loading path OPQ'R no plastic straining occurs. One may think of the loading plane being pushed out parallel to itself as loading proceeds. The total plastic strain suffered by this hypothetical material depends only on the distance the loading plane has been pushed out. Thus the total plastic strain is the same for either loading path OPQ or OPQ'R and is a function of d. The direction of the plastic strain vector is perpendicular to the loading plane.

Two Plane Loading Surfaces.

Several novel features of the use of many plane loading surfaces can be illustrated most simply by examining the case in

which there are only two plane loading surfaces f_1 and f_2. Fig. 2 is a schematic drawing which shows how stress space is divided into four regions by two plane loading surfaces. If the loading path remains in region I no plastic straining occurs. If the loading path only penetrates region II (or IV) then only one of the two planes is affected and we have the case treated in the last section. If however the stress path enters region III then both loading planes are affected as shown in Fig. 3.

Suppose the loading has reached a point such as P in Fig. 3 and now a small increment of stress $d\sigma_{ij}$ is added to the load. There are several cases to consider depending on the direction of the stress increment vector. Fig. 4 shows the region of stress space in the neighborhood of P. There are four regions around P into which the stress increment vector may extend. If $d\sigma_{ij}$ extends into region 1 there is no additional plastic strain, consequently region 1 may be called the elastic region. If $d\sigma_{ij}$ extends into either region 2 or 4 one of the two planes is affected and the other is not. If $d\sigma_{ij}$ extends into region 3 both loading planes are affected so region 3 may be called the region of total loading. The strain increment is taken to be the sum of the contribution from f_1 and f_2 as though each acts independently.

The boundary of the elastic region is the yield surface which is shown in heavy lines in Fig. 4. In the case under consideration a "corner" has developed in the yield surface at the load point P. If the load point were on any straight portion of the yield surface the strain increment vector would of course be

normal to the yield surface. At a corner such as in Fig. 4 there is no normal to the yield surface. However, the strain increment vector $d\varepsilon^p_{ij}$ is the vector sum of the contribution from both loading surfaces. Therefore at a corner the strain increment vector has some determinate direction between the normal to f_1 and the normal to f_2 (see Fig. 5).

The mathematical expression of the stress-strain relation which we are considering contains the differentials of stress $d\sigma_{ij}$ to the first power only and thus is linear in some respects. If the stress increment vector $d\sigma_{ij}$ causes a plastic strain $d\varepsilon^p_{ij}$ then $2d\sigma_{ij}$ causes twice as much plastic strain, $2d\varepsilon^p_{ij}$. However $-d\sigma_{ij}$ might not cause any plastic strain at all because if the vector $d\sigma_{ij}$ falls in the region of total loading then $-d\sigma_{ij}$ falls in the elastic region. Hooke's law for the elastic part of the strain is truly linear and does not possess this peculiarity. The plastic stress-strain relation is non-linear because the principle of superposition does not apply. A plastic stress-strain relation which involves even only one loading function is non-linear in this respect.

The non-linearity for multiple loading surfaces is of a less trivial sort. Suppose for instance that $d\sigma_{ij(C)} = d\sigma_{ij(A)} + d\sigma_{ij(B)}$ and that $d\sigma_{ij(A)}$, $d\sigma_{ij(B)}$ and $d\sigma_{ij(C)}$ each acting by itself causes non-zero plastic strain increments $d\varepsilon^p_{ij(A)}$, $d\varepsilon^p_{ij(B)}$ and $d\varepsilon^p_{ij(C)}$ respectively. In general it does not follow that $d\varepsilon^p_{ij(C)} = d\varepsilon^p_{ij(A)} + d\varepsilon^p_{ij(B)}$. Fig. 6 illustrates a case in which

superposition does not hold. The stress increment $d\sigma_{ij(A)}$ is applied followed by $d\sigma_{ij(B)}$. Clearly the resulting plastic strain is not the same as that produced by $d\sigma_{ij(C)}$ acting alone because the loading plane f_1 is moved out more in the first instance than in the second. Even the order in which $d\sigma_{ij(A)}$ and $d\sigma_{ij(B)}$ are applied makes a difference. Experimental evidence for the existence of corners in the yield surface has recently been found [6].

Many Plane Loading Surfaces. There is no difficulty in generalizing the previous results by introducing any finite or infinite number of plane loading surfaces. If there are an infinite number of loading surfaces the stress-strain relation may involve an integration. The set of all those points in stress space which are elastic with respect to all the loading planes constitutes the elastic region. The boundary of the elastic region is the yield surface. Figs. 7 and 8 show symbolically the elastic region and yield surface when the number of plane loading surfaces is finite or infinite respectively.

As loading proceeds the loading planes get pushed out more or less depending on the loading path, but at any instant of loading there is a well defined yield surface. The property of integrability for a single plane loading surface leads to the following simple rule in the case of many plane loading surfaces:

> The total plastic strain is the same
> for all loading paths which result in
> the same yield surface.

Fig. 9 illustrates a case in which two different loading paths A and B produce the same total plastic strain. If the loading had proceeded only as far as P the total strain for paths A and B would be different but continuing the loading to Q wipes out this difference.

The generalization of the notion "region of total loading" to the case of many plane loading surfaces can be done in the following way. Suppose loading has proceeded along a straight line loading path into the plastic range and reached a point P (Fig. 10). The yield surface is made up of part of the initial yield surface and of part of the envelope of the loading planes which pass through P and are tangent to the initial yield surface. The part of this conical envelope which extends outward from P bounds the region of total loading. All loading planes previously pushed out in reaching P are further pushed out by a stress increment $d\sigma_{ij}$ whose vector drawn from P falls in the region of total loading.

The straight line loading path (radial loading) is a special case of a more general loading path traced out by a load point P which always proceeds into the region of total loading. For such loading paths (and only for such loading paths) the interesting result has been obtained that J_2 deformation theory coincides with a particular incremental theory based on an infinite number of plane loading surfaces. The details are left to a later paper.

Further Remarks on Plane Loading Surfaces. In all the foregoing discussion it has been assumed that the loading planes act entirely

independently. Fig. 11 illustrates a case in which the loading planes act interdependently. The loading plane f_1 has been pushed out to the position indicated by the dotted line. The other planes also move according to some law as a consequence of the motion of f_1. This type of behavior is certainly more complicated than independent action but the phenomenon of latent strain hardening of a single crystal suggests such behavior. In a metal crystal there is a certain preferred set of planes called slip planes so oriented with respect to the crystal lattice that slip is most likely to occur in them rather than in any others. Within these planes there are several preferred directions of slip. Thus there is some definite number of slip-plane slip-direction combinations or slip systems within a given crystal. When a given slip system has been activated it becomes more resistant to further slipping, that is it becomes strain hardened. However the slip systems do not strain harden independently. When any one slip system is strain hardened so are the others. The condition that the shear stress on a given slip system remain constant is that a certain linear combination of the stress components remain constant. The locus of points in stress space satisfying this condition is of course a plane. Perhaps a theory describing the macroscopic plastic strains of a single crystal could be formulated based on a finite number of plane loading surfaces acting interdependently. Even though it is not at all obvious how to take the step from a single crystal to a crystal aggregate the behavior of a single crystal is very suggestive.

Fig. 12a shows a possible yield surface after an experiment performed on a material whose initial yield surface is

$J_2 = k^2$. The material has been stressed in simple tension in the x direction beyond the elastic limit. The yield stresses for tension and compression in the y direction have been altered (cross effect) and the yield stress for compression in the x direction has also been altered.(Bauschinger effect)[7]. Fig. 12b shows the results of the same experiment performed on a hypothetical material which obeys a stress-strain relation based on an infinite number of plane loading surfaces which act independently. The initial yield surface for this material is also $J_2 = k^2$. No cross effect or Bauschinger effect in the sense of a lowering of the yield point in compression following loading in tension is exhibited by such a material. Fig. 12c shows the result of the same experiment performed on another hypothetical material obeying the simple J_2 rule. The material of 12c could also be regarded as obeying a stress-strain relation based on an infinite number of plane loading surfaces in which the planes always move in such a way as to envelop a surface J_2 = constant. The property of integrability does not necessarily hold for stress-strain relations based on many plane loading surfaces which act interdependently. The properties of such stress-strain relations have only begun to be investigated, but the phenomenon of latent strain hardening exhibited by individual metal crystals as well as the evidence from experiments on polycrystalline materials suggests further investigations.

Many plate buckling experiments agree well with a theory in which the hypothetical material of Fig. 12b is taken as the mathematical abstraction of the real material. Unfortunately

other kinds of experiments show that this material is not acceptable as a good approximation to a real material. Recent investigations [8] on the stability of non-linear systems indicate that the classical buckling criterion is not applicable in the plate buckling problem. These investigations show that small and unavoidable initial imperfections can have a pronounced effect on the load carrying capacity of a plate and yet not cause much scatter in the test data. If these conclusions are correct plate buckling experiments are not decisive in the choice between various possible stress-strain relations.

APPENDIX

Consider the following stress-strain relation:

$$d\varepsilon^{p}_{ij} = G(f)\frac{\partial f}{\partial \sigma_{ij}}\,df \qquad df > o$$
$$= \quad o \qquad\qquad\qquad df \leq o \tag{1}$$

in which f, the loading function, is a function of stresses alone. Provided the surface f = constant be sufficiently smooth and non-convex toward the origin in stress space, this is the most general stress-strain relation involving only one loading function which belongs to the class of stress theories of plasticity. If f is a linear combination of the stresses then (1) may be integrated to give the components of total strain in terms of the stresses. The resulting relation is of the form:

$$\varepsilon^{p}_{ij} = H(f)\,\alpha_{ij} \tag{2}$$

where $dH = Gdf$ and α_{ij} are direction numbers of the normal to the loading plane. This may be verified by taking the differential of (2) to obtain (1). Conversely the only integrable stress-strain relations of the form (1) are those for which f = constant is a plane surface in stress space. This may be proved by the following considerations. If $d\varepsilon^{p}_{ij}$ is integrable then there exists a family of surfaces in stress space such that ε^{p}_{ij} is constant on each surface of the family. Suppose that a point moves on a surface f = constant; then according to (1) df = o and $d\varepsilon^{p}_{ij}$ = o so the surface f = constant must also be a surface ε^{p}_{ij} = constant, hence

ϵ^p_{ij} is a function of f alone. This implies that $\frac{\partial f}{\partial \sigma_{ij}}$ is a function of f alone, say:

$$\frac{\partial f}{\partial \sigma_{ij}} = F_{ij}(f). \tag{3}$$

Now $\frac{\partial f}{\partial \sigma_{ij}}$ are the direction numbers of the normal to the surface f = constant and (3) implies that the normal has a constant direction; hence the surface f = constant is a plane. All that has been proved is that f must be a function of a linear combination of the stresses, however there is no loss in generality in taking f itself to be linear in the stresses since any additional arbitrariness may be absorbed into the factor G which occurs in (1).

Bibliography

1. Handelman, G. H.; Lin, C. C.; Prager, W.; "On the Mechanical Behavior on Metals in the Strain-Hardening Range" Quar. Appl. Math. V.4(1947) pp. 397-407.

2. Batdorf, S. B.; Budiansky, B.; "A Mathematical Theory of Plasticity Based on the Concept of Slip". N.A.C.A. TN 1871 (1949).

3. Drucker, D. C.; "A More Fundamental Approach to Plastic Stress-Strain Relations." Proceedingsof the First National Congress of Applied Mechanics. ASME (1951) pp. 487-491.

4. Koiter, W. T.; "Stress-Strain Relations, Uniqueness and Variational Theorems for Elastic-Plastic Materials with a Singular Yield Surface." Quar. Appl. Math. V. 11 (Oct. 1953).

5. Tresca, H.; "Memoire sur l'ecoulement des corps solides soumis a de fortes pressions." Comptes rendus de l'academic des sciences Paris. V.59 (1864) pp. 754-758.

6. Stockton, F. D.; "Experimental Evidence of Non-Linearity in Plastic Stress-Strain Relations." Report A11-88 to ONR (1953). Graduate Division of Appl. Math., Brown University.

7. Edelman, F. and Drucker, D. C.; "Some Extensions of Elementary Plasticity Theory." Jour. Franklin Inst. V. 251 N.6 (Jun. 1951) pp. 581-605.

8. Onat, E. T. and Drucker, D. C. ; "Inelastic Instability and Incremental Theories of Plasticity." Jour. Aero. Sci. V. 20 (Mar. 1953) pp. 181-186.

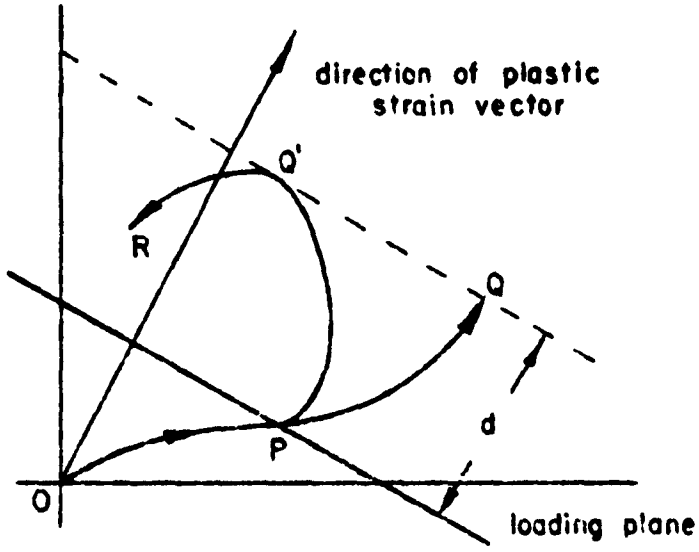

FIG. 1

Single plane loading surface

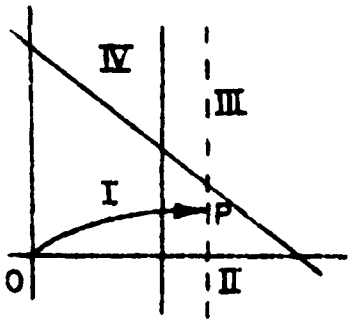

FIG. 2

Two plane loading surfaces
(one active)

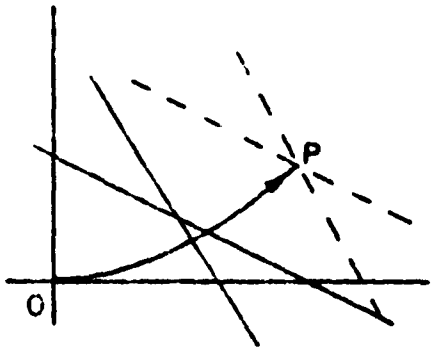

FIG. 3

Two plane loading surfaces
(both active)

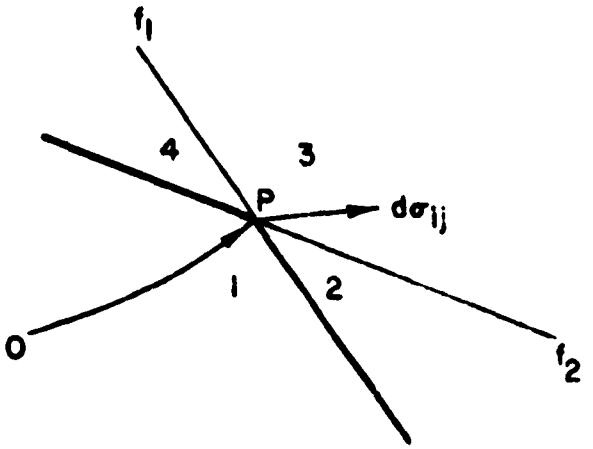

FIG. 4

Loading from a corner

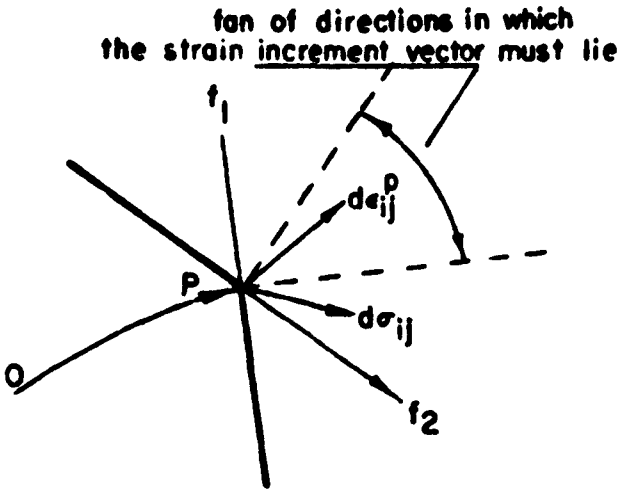

FIG. 5

Strain increment vector at a corner

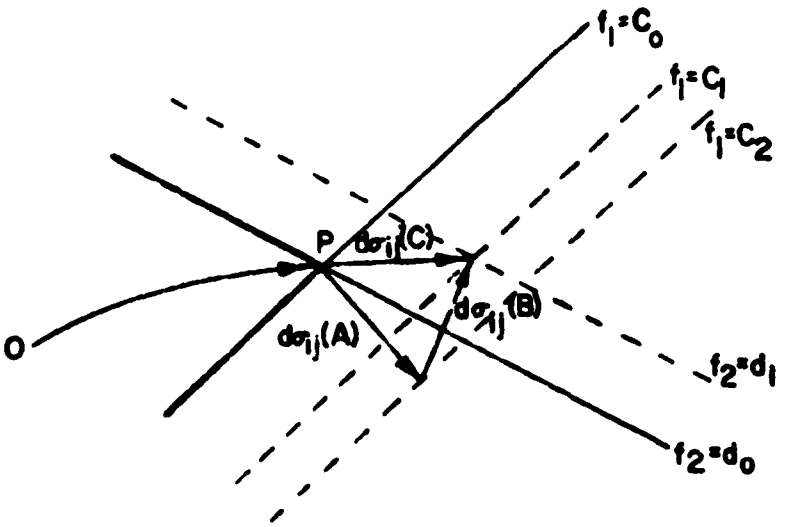

FIG. 6

Non-linear effects at a corner

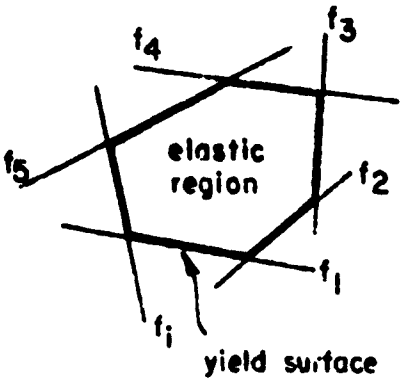

FIG. 7

Finite number of plane loading surfaces

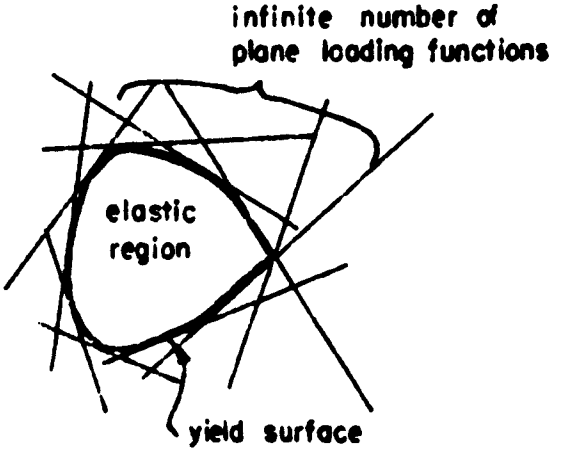

FIG. 8

Infinite number of plane loading surfaces

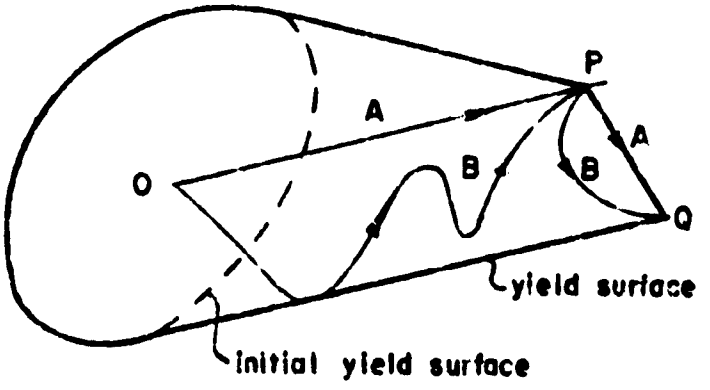

FIG. 9

Illustration of restricted path independence

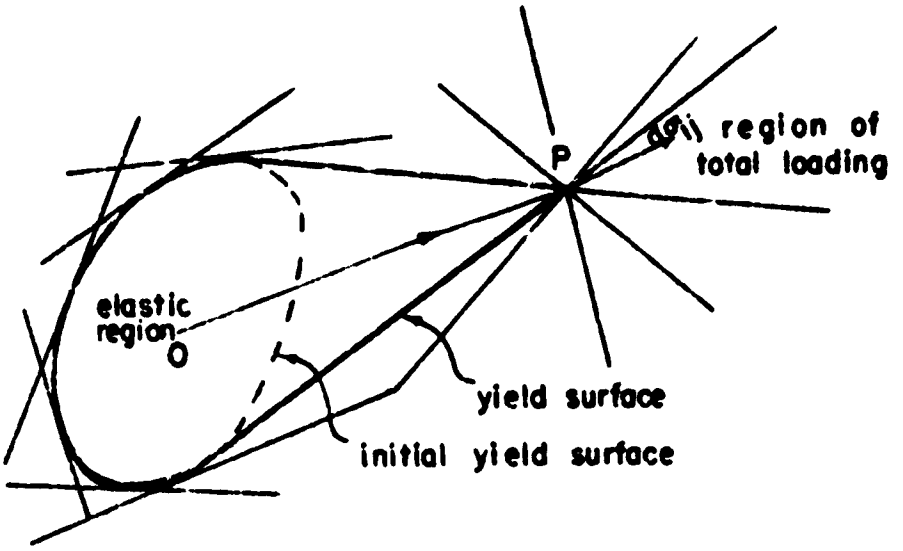

FIG. 10

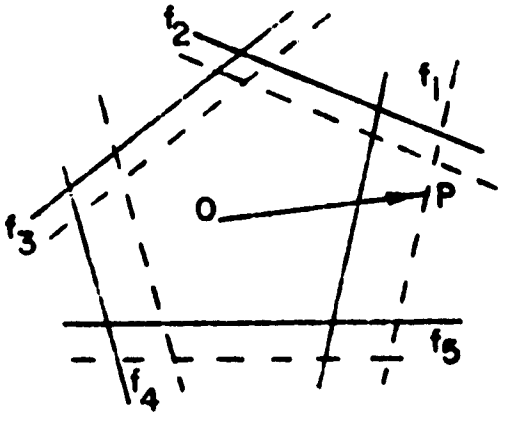

FIG. 11

Interdependent plane loading surfaces

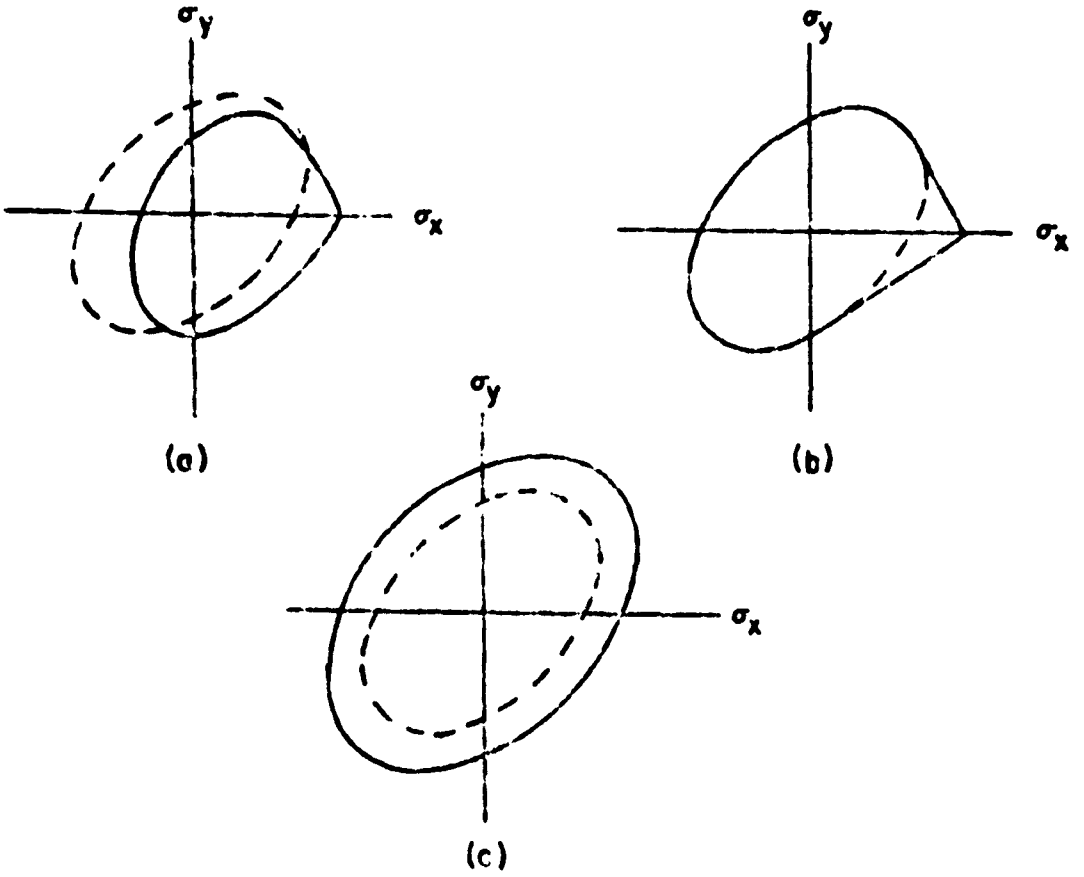

FIG. 12

Yield surfaces as envelopes of infinitely many plane loading surfaces.
a) cross effect and Bauschinger effect
b) no cross effect or Bauschinger effect
c) Isotropic work hardening

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