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Investigations into a method of shearing stress determinations from an airplane in flight

By

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Abstract

An anemometer, a vertical accelerometer and a gyroscope mounted in an airplane (Navy PBY-6A) yield simultaneous records of the total air speed, the vertical accelerations, and the attitude of the airplane. From these records, airplane lift theory and the characteristics of the PBY, the vertical and horizontal fluctuations of the wind, w' and u', are computed and hence the shearing stress, $\overline{-\rho w'u'}$. The effects of pilot control and instability of the aircraft upon the determinations of the stresses have been investigated and are found to be negligibly small if sufficient care is taken and supplementary instruments are used in the airplane. The airplane can respond completely only to gusts larger than 50 meters and fails to measure reliably horizontal gusts larger than 500 to 1000 meters, thus limiting the contributions to the shearing stress to gusts within this size range. Several measurements of the shearing stress, eddy viscosity, and the roughness coefficient in the atmosphere are presented and discussed from the point of view of measurement technique. Analysis of the atmospheric turbulence and transport will be attempted only after more measurements are available.

I ATMOSPHERIC SHEARING STRESSES AND EDDY VISCOSITIES

The present investigation describes a method of measuring the shearing stresses and eddy viscosities of the atmosphere in regions not accessible to ground-based instruments. An airplane is used both as a meteorological instrument and an observational base to study these regions allowing stress determinations to be made wherever an airplane may be flown with safety.

The method is based upon determining the average of the cross products of the horizontal and vertical components of the fluctuations of the wind. Reynolds (1883) first showed the importance of these cross products when he redeveloped the Navier-Stokes equation for a turbulent atmosphere. In his equations the products appeared as apparent stresses which have been shown to be orders of magnitude larger than the laminar stresses. Since Reynolds' time relatively few determinations of the turbulent stresses have been made, but in recent years superior instruments have been developed and many workers are engaged now in the measurement of this quantity.

The Newtonian friction law may be written as follows to describe a turbulent atmosphere:

$$\mu = -\tau / \frac{dU}{dz} = \overline{\rho u'w'} \Big/ \frac{dU}{dz} \qquad (1)$$

Here μ is the eddy viscosity, dU/dz the wind shear, u' and w' the horizontal and vertical components of the rapid fluctuations of the wind, and τ the shearing stress. The roughness coefficient c, defined by equation (2)

$$\tau_0 = c \rho u_a^2 \tag{2}$$

may be found if the shearing stress at the lowest determined level is assumed equal to τ_0, the surface stress, and, u_a, the horizontal velocity at anemometer height is known.

The problem of finding τ and μ resolves itself into obtaining a suitable series of simultaneous values of w' and u' and a corresponding value of dU/dz. This problem has been solved satisfactorily by analyzing and accounting for the flight and lift characteristics of the airplane, and by installing a pressure gage anemometer, a vertical accelerometer, a gyroscope, and a drift sight meter on the airplane. Reducing the observations made with these instruments in accordance with the theory of lift of airplanes and the characteristics of the PBY yields the desired velocity fluctuations. Numerous tests were conducted to prove that the values found were true shearing stresses and not meaningless values reflecting an instability in the flight of the aircraft.

II APPLICATION OF SIMPLIFIED AIRPLANE LIFT THEORY TO THE COMPUTATION OF SHEARING STRESSES

Whenever an aircraft flies through a turbulent air mass the airplane experiences a rapid succession of changes in angle of attack and speed of the apparent air flow. The effect of these changes is to modify the lift of the airplane thus producing irregular vertical motions of the airplane. Recorded vertical accelerations of the airplane and recorded changes in the indicated air speed can be used to compute the variations in the angle of attack. From these quantities, determinations of gust velocities can be made. Their significance and trustworthiness depends to a great degree on the simplifying assumptions made concerning the flight of the PBY and its reactions to gusts. A vast amount of work, both theoretical and observational, has been done by aerodynamicists (see Durand, 1935) on the performance of aircraft, and the consequences of many assumptions. Experiments performed to determine the flight characteristics of the PBY-6A will be described later.

The incremental lift equation applicable to the gust problem (see Duncan, 1952) derived from the definition of the lift coefficient,

$$L = \tfrac{1}{2}\rho V^2 S C_L \tag{3}$$

is,

$$\Delta L = \tfrac{1}{2}\rho S \left(V^2 \frac{dC_L}{d\alpha}\Delta\alpha + 2VC_L\Delta V\right) \tag{4}$$

where ΔL is the change in the lifting force of the airplane,

ρ the density of the air, S the effective wing area, V the true air speed, C_L the coefficient of lift, and α the angle of attack.

Conditions required for the correct application of this equation to the determination of gust velocities are as follows:

1. The airplane is a rigid body.
2. The attitude of the airplane does not change.
3. A steady flow has been established over the airfoils.
4. The airplane flight is level, rectilinear and properly trimmed.
5. The gust is sharp-fronted and symmetrical across the span of the wing.

The performance of the PBY has been investigated to see if it fulfills the necessary conditions. Whenever it failed to fulfill them and consequent large errors were made in the gust determination it was determined whether an additional instrument would rectify the situation. This instrument was designed to measure departures from the required conditions and to determine corrections. Changes in the attitude of the airplane have been treated in this manner by adding a recording gyroscope to the airplane's equipment. Other assumptions have been found to have a negligible effect on the velocity calculations. The rigidity of the PBY falls into this category.

The condition of a steady circulation around the airfoil can never be fulfilled in a turbulent air mass but the observations may be studied in such a way as to minimize this failure. A value of the lift coefficient consistent with the non-steady flow was determined from studies made by aerodynamicists. Donely (1949) in a study of gust loads on airplanes discusses the results of many workers in this field. He presents graphs of the ratio of non-steady flow circulations and lift coefficients to steady flow circulations and lift coefficients as functions of gust penetration in chord lengths. This ratio rises from zero outside the gust to about 0.85 at 5 chord lengths penetration and is nearly 1 at 10-15 chord lengths penetration.

To arrive at an average value of the transient lift ratio that can be applied to the measured accelerations the atmospheric turbulence may be considered to be composed of a complete spectrum of sharp fronted gusts. Each gust will contribute to the vertical acceleration by an amount proportional to the magnitude of the gust times the lift ratio corresponding to the penetration of the airfoil into the gust. As many of the different sized gusts will be affecting the lift simultaneously, it is obvious that all degrees of gust penetration are also present simultaneously. A satisfactory ratio has been determined by averaging the ratios given by Donely over 0 to 200 chord lengths penetration into a gust. Weights for each degree of gust penetration were found from the accelerometer record of a flight through an atmosphere with a turbulence typical of that desired for study. The weights used were the averages over the entire flight of the absolute values of the average accelerations for each degree of gust penetration. The weighted average determined was 0.88. This value would, of course, not necessarily be suitable for work in cumulus clouds as the major accelerations are of large scale.

If we correct the measured accelerational increment for this diminution due to unsteady flow, we get

$$\Delta n_{true} = 1.1\ \Delta n_{meas} \tag{5}$$

As the accelerometer records the accelerations normal to the flight of the airplane, equation (4) may be rewritten in terms of this normal acceleration, Δn, and the mass, M, of the airplane.

$$\Delta n = \frac{\frac{1}{2}\rho S}{M}\left(V^2\frac{dC_L}{d\alpha}\Delta\alpha + 2C_L V\Delta V\right) \tag{6}$$

The factor, $\Delta\alpha$, may be broken down into its three components: $\Delta\alpha_{turb}$, the change in angle of attack due to the turbulent gusts; $\Delta\alpha_{att}$, the change in angle of attack due to the varying attitude of the airplane; and $\Delta\alpha_{ap}$ the change in angle of attack due to the vertical motions of the airplane. Since $\sin\Delta\alpha \approx \Delta\alpha$ for small angles, $\Delta\alpha_{turb}$ may be written as w'/V_t where w' is the vertical velocity of a gust, V_t the instantaneous value of the true air speed. Likewise, $\Delta\alpha_{ap}$ may be written $W_{ap}/V_t = -(\Sigma\Delta nt_1 - w_o)/V_t$ where W_{ap} is the vertical velocity of the airplane and w_o is the initial vertical velocity of the airplane. With these substitutions equation (6) may be solved for w' giving,

$$w' = \frac{\Delta n\ V_t\ M}{\frac{1}{2}\rho V^2 S\ \frac{dC_L}{d\alpha}} - \frac{4M_g\ V_t\ \Delta V_t}{\rho V^3 S\ \frac{dC_L}{d\alpha}} - \Delta\alpha_{att}\ V_t + \Sigma\Delta nt_1 \qquad (7)$$

This equation gives the vertical velocity of the gust in terms of the vertical accelerations of the airplane and changes in air speed and attitude.

Applying the non-steady flow factor to the Δn of equation (7) and substituting the values of the constants for the PBY-6A, we get

$$w' = 4.98\ \frac{\Delta n\ V_t}{\rho V^2}\quad \frac{8.9\text{x}10^3\ V_t\ \Delta V_t}{\rho V^3} - \Delta\alpha_{att}\ V_t + \Sigma 1.1\Delta nt_1 \qquad (8)$$

The values entered into equation (8) are as follows:

$M = 1.34\text{x}10^7$ gm (varies with gas load, etc., $\pm$ $0.05\text{x}10^7$ gm)

$S = 1.30\text{x}10^6$ cm^2

$dC_L/d\alpha = 4.54$

The value of $dC_L/d\alpha$ was determined for the PBY-6A, loaded and equipped in a manner similar to the loading during observational runs, by flying at various attitudes and velocities and plotting the computed lift coefficient against angle of attack.

III OBSERVING PROCEDURES AND AIRCRAFT INSTRUMENTATION

Observations for the shearing stress computation were taken during short horizontal flights lasting 1 or 2 minutes. Many of these horizontal runs were made during vertical helical soundings to obtain temperatures and humidities. As the sounding progressed to a desired altitude, the airplane was levelled, trimmed for rectilinear flight, and flown upwind for a minute or more. During this upwind run complete records of the airspeed, normal accelerations and attitude were obtained. Also, if no pilot balloon observations of the wind were available, drift sight readings were made during the run and again after the airplane had turned 90°. After the run was completed the vertical sounding was continued on up to the next level at which a shearing stress was desired.

At the beginning of each shearing stress run care was taken in trimming the airplane for rectilinear, level flight and in damping oscillatory motions. After several runs with and without the auto-pilot in operation it was found that the pilot could control the attitude of the airplane sufficiently well for the work at hand. Considerable time is saved by not setting up the auto-pilot control. Since the attitude of the airplane is recorded by a gyroscope, the effect on the lift of small departures from the mean may be corrected by the $\Delta\alpha_{att} V_t$ term of equation (8). If departures become too extreme ($>2°$) in certain parts of the run, those parts of the run either must be discarded or, if used, accepted as an approximate determination.

The horizontal runs were flown upwind in all cases except a few downwind flights. The purpose of flying upwind was to fulfill or at least approximate fulfillment of condition number 5 concerning the symmetry of the gust across the wing span. From a consideration of the unsteady flow lift coefficient, it will be seen that failure to achieve complete symmetry will have only a secondary effect on the vertical velocity computation. This is true since small gusts of one or two chord length dimensions change the lift of the wing only slightly. As the size of the gusts increase to ten chord lengths and longer the full change in lift is experienced, and because of the larger size the probability of symmetry is greater.

Only a brief description of the equipment installed aboard the PBY will be given here as a very complete description of the PBY, its instrumentation, and its flight characteristics is to be written as a separate paper. All the instruments described here were not used simultaneously, but rather at some stage of the development of the method. Figure 1 shows the general arrangements of the airplane and its instruments. At the present moment the instrumentation consists of a pressure gage anemometer, an electric accelerometer, and a gyroscope. Many of the airplane's instruments were used such as air-speed indicator, altimeter, fluxgate compass, bubble level, and gyrostabilized drift meter. The recorders used consist of a multichannel oscillograph, Leeds and Northrup Speedomax potentiometers, converted Brown potentiometers, and an Esterline-Angus milliammeter. Although plans are being worked out to change permanently the recording system to a multichannel oscillograph and a differential pressure gage to replace the bead anemometer, a description of all equipment used will be given.

The original anemometer consisted of a heated thermistor bead mounted on the top and ahead of the forward gun turret of the PBY. Use of thermistor beads for anemometry have been discussed by Sanford (1951). A modified Brown self-balancing potentiometer with a full scale response of about one second recorded the air speed variations. Calibrations of the bead thermistor were made in flight against the airplane's air-speed indicator.

The vertical accelerations of the airplane were measured first by a water column accelerometer mounted rigidly near the center of gravity of the airplane and perpendicular to the plane of level flight. Conversion has been made to a Statham electric accelerometer. This records on a converted Brown recorder identical to the one used for the anemometer. The two recorders were mounted side by side and the two chart drives were connected so that irregularities would be diminished and synchronous.

The gyroscope unit of an automatic pilot system has

FIG. 1

been converted to record the attitude of the airplane on an Esterline-Angus recorder.

A Consolidated Engineering Company multichannel oscillograph was used during the later stages of the investigation as the measuring and recording unit. The signals from the electric accelerometer, differential pressure gage, and the gyroscope were amplified by a 1 kc amplifier and recorded by the oscillograph. In addition, a record was obtained from a strain gage altimeter.

A timer device was installed which made one-second side marks on all of the records to insure identification of the time. The earliest runs were made with only fifteen-second side marks punched by hand.

The type B-3 drift sight meter used for wind determinations was the standard Navy issue instrument equipped with a gyrostabilized reference grid. This meter has a rotatable prism at its lower extremity so the water surface ahead of the airplane may be viewed as desired. This enabled the observer to make good readings at altitudes of only 50 feet. The drift meter was used only when pilot balloons were unavailable or not applicable.

IV COMPUTATION OF SHEARING STRESSES

Two differing ways of computing shearing stresses have been developed to treat each of two sets of data. The original combination of observed quantities consisted of (1) normal accelerations recorded by a water column accelerometer, and (2) air speed variations as recorded by a hot bead anemometer. The second combination included (1) the attitude which was recorded by a gyroscope, (2) the air speed which was sensed by a differential pressure gage, and (3) the accelerations which were detected by an electric accelerometer. All elements of the second combination were recorded on a multichannel oscillograph.

The original anemometer and accelerometer were read with a device which allowed the direct conversion of the records into meters per second and centimeters per second, respectively. Records with the one-second side marks were reduced by reading one-second averages. From the earliest records with the fifteen-second side marks, distances along the trace of significant points of the record and of the time side marks were measured so that times of occurrence of points could be determined. Each significant point on the traces was measured. Two or three such points usually occurred each second. Values recorded on the multichannel oscillograph were averaged and read every one-fifth second. Scales were placed over the traces so meteorological quantities could be read directly. Two examples of the oscillograph records are presented here (Figure 2) to show the magnitude of the recorded fluctuations.

The averages of the measured vertical accelerations of the airplane cannot be used to compute vertical velocities

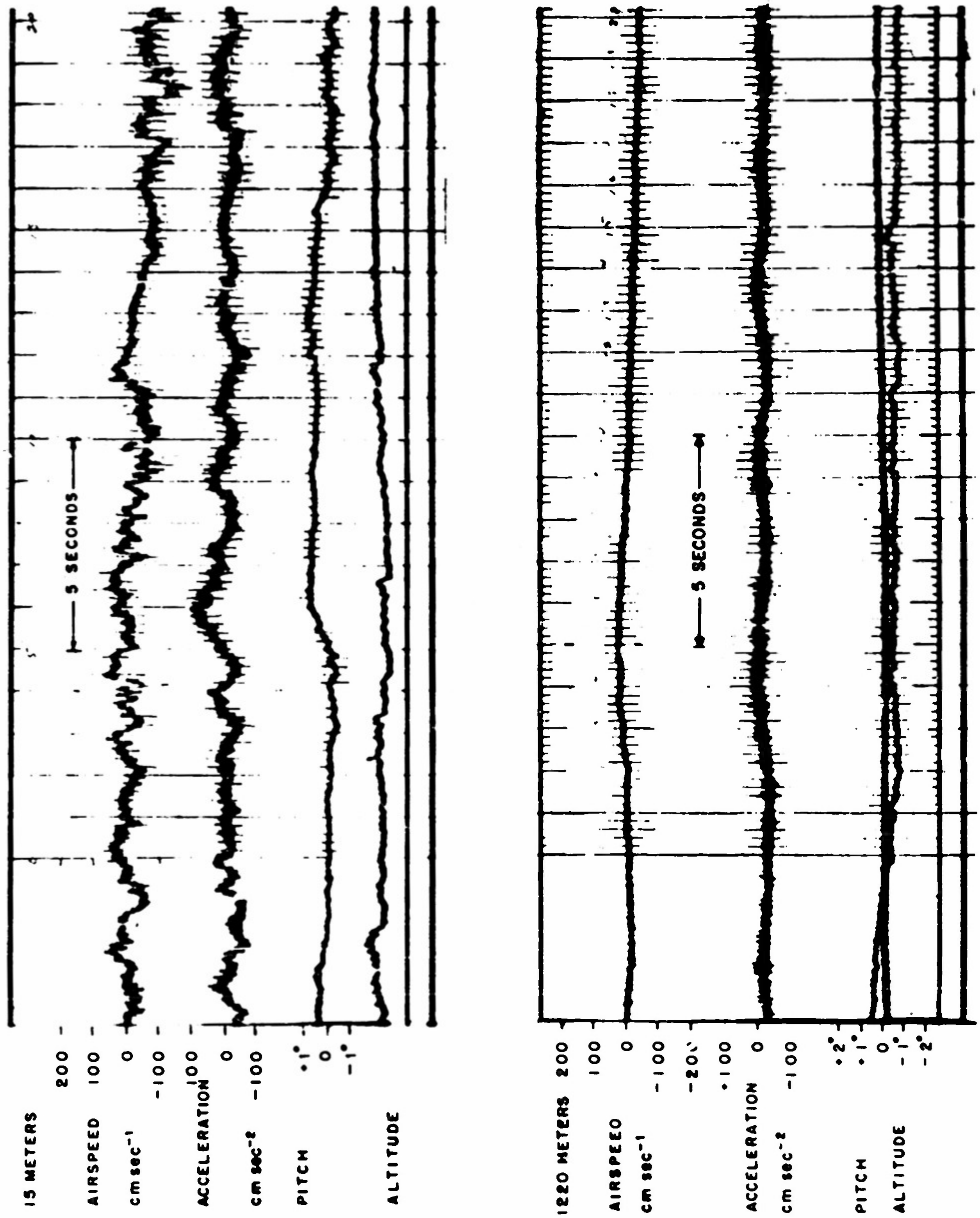
15 METERS
AIRSPEED
cm sec-1
ACCELERATION
cm sec-2
PITCH
ALTITUDE
5 SECONDS
1220 METERS
AIRSPEED
cm sec-1
ACCELERATION
cm sec-2
PITCH
ALTITUDE
5 SECONDS

FIG. 2

without first averaging and computing the deviations from the average. There are several contributing causes that make this step necessary. One important cause may be centrifugal accelerations produced by trimming the airplane so that it describes a long arc rather than a straight line. The addition of such accelerations to the accelerations due to turbulence cannot be detected except by an averaging process. Another more fundamental cause is the turbulent structure of the air. Eddies large in size compared to the distance travelled by the airplane can produce the same long-time wandering of the accelerometer zero. By setting the total accelerations over the observation time equal to zero, new values of the accelerations can be computed. Any contribution to the shear stress of the larger eddies will be lost by this step.

To find values of ΔV_t and $\Delta \alpha_{att}$ to enter into equation (8) that are as free as possible from effects of aircraft performance, ten-second running averages were found for both quantities and deviations computed from the averages. This procedure is required since an airplane may be flown with innumerable combinations of attitude and air speed. During a horizontal run with constant power setting a gust may change the attitude and air speed to a new equilibrium combination or the pilot may slowly change the attitude of the airplane. In selecting values to enter into equation (8) it is essential to find deviations from the proper attitude-air speed combination. Deviations from the averages of the preceding ten seconds give the desired variations of air speed and attitude and is satisfactory since the airplane will require about this amount of time to achieve a new equilibrium of attitude and air speed and will not revert to the original combination except by chance. The V_t entered in equation (8) is formed from the deviations from the average velocity plus a reference velocity indicated by the aircraft's pitot tube air-speed indicator.

In many cases the V_t appearing in the first two terms of equation (8) may be cancelled against the $\bar{V}$ appearing in the denominator, thus saving computation time. Inspection of the amplitude of variation of V_t in an individual case will show whether it is safe to make this approximation.

The coefficient of the $\Delta n\ V_t/\bar{V}^2$ term should be determined for each run according to the weight of the airplane at the time. Therefore, a constant record of the gas consumption of the airplane is necessary.

The errors introduced by measuring the air speed at a single point and coupling them to an acceleration integrated over the entire wing area are discussed in a later section, as will also the effect of the finite inertia of the airplane.

The value of u', the horizontal component of the wind, should be found from the determined value of w' and the value of V_t, the total indicated air speed. However, as even maximum values of w' are small (about 1/50) compared to V_t, u' is approximately equal to $V_t - \bar{V}$ rather than to the exact relation $u' = \sqrt{V_t^2 - w'^2} - \bar{V}$. It is apparent that $u' = \bar{V} - V_t$ for downwind runs rather than the relation $u' = V_t - \bar{V}$ which applies to upwind runs.

Since the vertical velocities were computed in a manner that accounted for changes in the attitude of the airplane, it is necessary to correct the horizontal velocities for changes due to changing pitch. Flight tests were made to determine the acceleration and deceleration of the PBY due to changes in attitude. The airplane was flown in smooth air and with constant power settings, the nose was depressed and elevated and the accelerations and decelerations noted. Once these values were determined, corrections were applied to the records of the horizontal runs to get the turbulent fluctuations of the air independent of the airplane performance. The initial acceleration and deceleration resulting from a one-degree lowering or raising of the nose is $\pm$ 13.5 cm/sec^2. Deviations from a twenty-second running average of the attitude are used for the correction computation. The average time of twenty seconds was found empirically from test pull-ups and dives to be the time required for the airplane to reach a new attitude-air speed equilibrium.

Deviations from the lines of regression of w' and u' upon the time were used in finding a value for $\overline{w'u'}$. This step is necessary to eliminate any effect upon the value of the shearing stress that a trend in either the horizontal or vertical velocities would produce. Such trends are always present to some degree as they may arise from many sources, ranging from instrumental drifts and airplane performance to large scale eddies in the atmosphere.

Another method that eliminates trends that vary during the run is to compute deviations from running means. The effect of atmospheric eddies or airplane motions larger than the range over which the running mean is taken is eliminated. Only a few runs have been reduced in this manner.

No attempt has been made to measure the fluctuations in the density of the air of the gusts. Therefore the shearing stress was computed by multiplying the average value of $w'u'$ by the average density of the air.

V DISCUSSION OF THE PROBLEMS CONNECTED WITH THE MEASUREMENT OF SHEARING STRESS FROM AN AIRPLANE

In the building, installing, and flying of the present equipment, and reducing and anlyzing the data obtained, numerous problems were encountered. These problems ranged from selection of instruments, placement of the instruments aboard the airplane, and exposure of sensing heads to the performance of the aircraft. All of these problems have been investigated and solved in a manner that allows reliable determinations of the stress. Problems mentioned earlier will be discussed here. An even more detailed discussion of all phases of the performance of the PBY in relation to these problems is planned as a separate report.

It was decided at the outset of this investigation to use one hot bead anemometer to obtain velocities to enter into the lift equation. It is obvious that this gives but an approximation of the desired average wind speed over the wing area. However, as averages are used there is little difference between the point measurement averaged over a linear distance of 50 meters along the line of flight and a measurement averaged over the area swept out by a 30 m line (the wing span) perpendicular to the line of flight. To test the importance of using more than one anemometer, a pressure gage was attached to the airplane's static and dynamic pressure lines. The pitot tube sensing the pressures is mounted about 7 m from the center line of the airplane about 1-1/2 m ahead of the leading edge of the wing. This position is about 5 m behind and 7 m to port of the bead anemometer. Records taken simultaneously by the two instruments show that there is very close agreement between the one second averages although the fluctuations of tenths of second duration vary considerably. As fifth-second averages are used in part of the shearing stress determinations it is planned to unite and balance the dynamic pressure lines of two pitot static tubes and record the average wind speed.

The previous experiment also settled the serious problem of whether the crowding of the streamlines around the nose of the airplane adversely affected the velocity measurements. The velocities in the vicinity were mapped out by probing with a hot bead anemometer from holes in the forward turret. This investigation showed that the bead was mounted in a stagnant region where the flow was but 65% of the airplane speed. Above and behind the bead the air reaches a speed of 140% of the airplane's air speed. As the stagnant region has a relatively flat velocity profile, little trouble arises from a varying streamline pattern provided the attitude of the airplane remains constant. At present, a pressure gage anemometer is attached to a pitot static tube mounted on the starboard wing a meter and a half ahead of the leading edge. This position is undoubtedly ideal although no tests have been made.

Reliable positions for the accelerometer were determined by Brewer (1953). Using strain gages attached to the wing struts of the PBY, it was shown that accelerometers mounted at the bulkheads at either end on the engineer's compartment gave a true measure of the mean aerodynamic load on the wing. This experiment not only proves that both of these positions are free from accelerations due to pitching of the aircraft, but that the accelerometer is a satisfactory measure of the changes in the lift of the wing. Proof that the measured accelerations are a measure of the changing lift supplants the assumption stated earlier that the airplane is a rigid body. We know that although the airplane is not a rigid body it reacts as such in so far as the present work is concerned. It may be noted here that the PBY is one of the very few modern airplanes that has such a rigid design, and the assumption of rigidity should not be applied to other airplanes without proof.

An additional check upon the rigidity of the airplane is the relation between the natural period of the wing bending and the time required for the airplane to reach the center of a gust. Kordes and Houbolt (1953) have studied this problem and devised the following empirical rule. An airplane may be treated as a rigid body if the quotient of the time to penetrate a sine gust to the point of maximum gust velocity divided by one quarter the natural period of the fundamental wing bending mode is of the order of 5 or greater. Applying this rule to the PBY which has a wing frequency of 3.5 cycles per second and an air speed of 50 meters per second, we get a quotient of 14 for a penetration of a gust of 50 meters radius. Only when gusts larger than 18 m are penetrated do we get a value of 5 and the airplane can be considered strictly rigid. However, as the contribution to the lift increment due to gusts smaller than 18 m is diminished greatly by the unsteady circulation around the airfoil relative to these small gusts, we may reduce the acceleration record as though the airplane were completely rigid.

One source of difficulty in the stress determination is that the airplane accelerates and decelerates with long period gusts. From the power requirements of the PBY listed in the Navy Pilot's Handbook the deceleration can be computed. A PBY flying 120 mph (TAS) at 1000 ft and weighing 29,000 lbs will decelerate 1 cm/sec/sec if it encounters a sudden increase of head wind speed of 2 m/sec. Varying power output of the automatic propeller adds a further uncertainty to the interpretation of the air speed. From this computation it will be seen that shearing stresses determined from short period fluctuations will be valid, but determinations based on longer period averages are subject to question. There seems to be no way out of this difficulty short of developing a means of measuring fluctuations in the ground speed. No plans for such a development are being considered at the present, although this problem is of the utmost importance as Panofsky has shown that the larger gusts contribute a large proportion of the total shearing stress.

The effect of phugoid motion (see Duncan, 1952), an oscillatory flight of an aircraft, on the shearing stress computation has been investigated and found to be negligibly small when equation (8) is used for the computation of w'. The $\Delta\alpha_{att} V_t$ term corrects for the changing lift due to changing pitch angle and hence little or no fictitious vertical velocity of the air is included in the computation. In cases where no attitude recorder is used only a slight fictitious shearing stress results. This is true since the computed vertical velocities are 120° out of phase with the resulting horizontal velocities. An unreal stress of +0.3 dynes/cm^2 results from an oscillation of 12 m in vertical extent and 1 m/sec horizontal velocity. An oscillation of this magnitude is larger than any known to exist during a shearing stress run. The June to November, 1952, observations were made without the gyroscope and are subject to this inaccuracy.

Before discussing computed values of the shearing stress it is necessary to find the probable errors of the shearing stress determination. This has been done by finding the errors of the individual observations and measurements and propogating these errors to the final computation. Reading errors of the measurer have been computed from deviations of his measurements of a single quantity from the mean. Instrumental, recorder, and calibration errors have been figured from the performance of the instruments and the accuracy of making the calibration scales. Errors of airplane performance have been estimated from our studies of the PBY and the studies by Kordes and Houbolt (1953). The use of a single pitot tube for the measurement of the gust velocity averaged across the wing span introduces an error estimated to be ± 12%. This value was found from an inspection of 1/5 second average velocities recorded, assuming the velocities very in the same manner perpendicular to the line of flight. The probable errors are presented in the following list.

<u>Probable Errors of the Shearing Stress Determination</u>

A. Acceleration Measurement Errors

Acceleration increment, single observation	± 1.8 cm sec^{-2}
Airplane deformation and location of accelerometers (Estimated at ±5%, $\overline{\Delta n}$ = 40 cm sec^{-1})	± 2.0 cm sec^{-2}
Accelerometer, oscillograph and calibration scale (2.5%)	± 1.0 cm sec^{-2}
Combined probable error	± 2.9 cm sec^{-2}

B. Velocity Measurement Error

Single observation	± 2.0 cm sec^{-1}
Pressure gage, oscillograph and calibration scale (2.5%)	± 2.0 cm sec^{-1}
Single point velocity determination (based on $\pm 12\%$ for 1/5 second averages and $\overline{\Delta V}$ = 80 cm sec-1)	$\pm$ 10 cm sec^{-1}
Combined probable error	± 10.4 cm sec^{-1}

C. Attitude Measurement Error

Single observation	$\pm$ 0.05°
Gyroscope, recorder and calibration scale	$\pm$ 0.1 °
Combined error	$\pm$ 0.11°
Combined error	$\pm$ 0.002 rad

D. Vertical Velocity Measurement Error

Acceleration term	± 2.6 cm sec^{-1}
Velocity term	± 3.4 cm sec-1
Attitude term	± 8.6 cm sec-1
Airplane motion term	± 13.0 cm sec^{-1}
Combined probable error	± 16.2 cm sec-1

E. Horizontal Velocity Fluctuation Error

Velocity error	± 10.4 cm sec^{-1}
Attitude correction ($\Sigma \Delta act$) based on 20-second summation	± 11.1 cm sec^{-1}
Combined probable error	± 15.6 cm sec^{-1}

F. w'u' Product Error

Rough air

Based on P.E. u' = ± 15.6,
u' = 50 cm sec^{-1}

P.E. w' = ± 16.2,
w' = 50 cm sec-1 ± 1010 cm sec-1

Smooth air	$u' = 5$ cm sec^{-1}	
	$w' = 5$ cm sec^{-1}	$\pm$ 110 cm sec^{-1}

G. $\overline{w'u'}$ Error (150 Observations)

Rough air	$\pm$ 81 cm^2 sec^{-2}
Smooth air	$\pm$ 9 cm^2 sec^{-2}

H. Shearing Stress Error (150 Observations)

Rough air	$\pm$0.09 dynes/cm^2
Smooth air	$\pm$0.01 dynes/cm^2

The computed errors show that in rough air with large shearing stresses the probable errors amount to only about 5% of the stresses. On the other hand, in very smooth air as is usually found at higher altitudes the error may be 100% of the stress.

The shearing stresses cannot be determined as accurately as the probable error indicates if the attitude of the airplane changes radically from the mean. It is hard to put an upper limit on the attitude deviation that, if exceeded, will change the probable error appreciably. Small slow changes of $\pm 1^\circ$ undoubtedly do not produce errors greater than the expressed value. Fluctuations of $\pm 2^\circ$ or $\pm 3^\circ$ seem to produce changes in the shear stress of $\pm$0.1 to $\pm$0.5 dynes/cm^2. These errors which are larger than the listed probable errors arise from uncertainties in establishing the proper mean velocity from which the fluctuations can be computed. The deviation of the computed mean velocity from the ideal or effective mean cannot be determined and is not accounted for in the tabulated probable errors. As a practical expedient it seems best to discard all observations taken during flights on which the attitude of the airplane was changing by more than $\pm 1°$. Only in cases of small slow changes can we be reasonably sure that we obtain values whose errors do not exceed those listed.

VI RESULTS OF SHEARING STRESS AND EDDY VISCOSITY DETERMINATIONS

In discussing the significance of the present stress determinations it is necessary to consider the size of eddies that the airplane responds to and can measure. Since one-second averages were taken of one set of data and the airplane travels about 50 m in that time, and since the wing span is roughly 30 m, any value found will be an average over an area of approximately 1500 m^2. Considering linear extents, an individual average over 50 m of air subtracted from an average over 2250 m corresponds roughly with the "small" eddies of Panofsky and Singer (1951) which are 70 m averages from 490 m averages if the wind speed is 7 m/sec. Since their discussions show that most of the stress is produced by medium sized eddies; i.e., 490 m averages from 3430 m averages, it must be assumed that 30 to 45 second runs do not yield the total stress. That is, provided that the wind in the trade wind and other regions has the same spectral characteristics as the wind near the Brookhaven, Long Island, tower. Reduction of the complete five minute trace would give nearly the entire stress but as has been pointed out, the long period horizontal wind variations cannot be detected.

One set of observations was made in the trade winds of the Atlantic Ocean, 40 km north of San Juan, Puerto Rico, on June 29, 1952. Computed stresses and viscosities are presented in Table I, together with the wind gradients observed with the drift sight meter. These observations were made before a gyroscope was installed in the aircraft and the computations are subject to the uncertainties of the phugoid motion mentioned earlier. It is also subject to the uncertainties of the variation of the air speed with pitch angle.

TABLE I

Shearing Stresses, Wind Gradients and Eddy Viscosities Observed June 29, 1952, in the Atlantic Trade Wind Region

Height m	Stress dynes cm^{-2}	Wind Gradient cm^{-1}	Eddy Viscosity gm $cm^{-1}sec^{-1}$
15	3.9 (1.1)	2×10^{-2}	190
70	1.6	1.5×10^{-2}	110
150	0.1	0.9×10^{-2}	10
300	0.2	0.6×10^{-2}	33
640	0.8	?	--
960	-0.4	?	--
1460	1.4 (.12)	?	--

The stress values appearing without brackets were computed from deviations from lines of regression of w' and u' upon time. This treatment of the data eliminates any single trend of the atmosphere or the airplane during the run, but allows any other atmospheric trends or instabilities of the airplane's motions to be included in the shearing stress. The data for the 15 m level and the 1460 m level have been reduced by finding deviations from ten second running averages. These values are enclosed in parentheses. It is seen that the shearing stress is reduced by a factor of three at the lower level and a factor of ten at the higher level. Since no gyroscope was available at the time it cannot be determined whether this method eliminates an effect of the airplane or the contribution of atmospheric eddies larger than 500 meters. The comparison serves to bring out the necessity of employing a recording gyroscope.

Eddy viscosities have been computed for the four lowest values since the wind gradient was sufficiently well determined in only these cases.

The roughness coefficient computed from $\tau_o/\rho u_a^2$ is found to be 1.4×10^{-2}. This value is determined when the τ_o is assumed equal to the shear stress at the 15 m level and the horizontal velocity (5 m/sec) is the drift sight measured at 15 m. A value of this magnitude is much larger than observed by other workers. The disagreement may arise from poor wind determinations or the lack of a gyroscope in the airplane to measure the attitude. If the shear of 1.1 dynes/cm^2, determined from ten-second running averages, is used a value of 4×10^{-3} is obtained. This agrees quite well with the findings of others using different measuring techniques.

The shearing stresses observed have been studied in relation to the vertical distribution of the potential temperature and mixing ratio. A shallow ground layer extending to 300 m was topped by a thin wafer of warmer drier air. Above this wafer the air was stable and progressively drier to the 1200 m level where a slightly stronger inversion existed. Above the 1200 m level the air continued to get warmer and drier. The average wind speed as observed by drift sight shows a rapid increase with height in the lowest 300 m and a more moderate shear above this level.

Another set of data obtained in the trade wind region but with a gyroscope installed in the airplane has been reduced and shearing stresses computed. A glance at Table II shows that the stresses fall into a more consistent pattern of decrease with height, diminishing to very small values within the first 150 m of the sea surface and remaining small in the relatively stable air above this height. The wind gradients were computed from the pilot balloon observations obtained on Anegada Island by the British meteorological expedition members. While these values may not be identical to the values over the open ocean where the stresses were measured, they do serve to allow the computation of the order of magnitude of the eddy viscosities. These also are seen to fall off rapidly in the lowest layer and become negligibly small in the stable air.

TABLE II

Shearing Stresses March 22, 1953, 20 Miles NE of Anegada Island in the Trade Wind Region

Altitude m	Shearing Stress dynes/cm^2	Wind Gradient cm-1	Eddy Viscosity gm cm^{-1}sec^{-1}
15	0.59	5.8x10-3	100
30	0.13	5.8x10-3	22
150	0.03	5.8x10-3	5
300	-0.001	4.5x10-3	-0.2
490	0.006	4.5x10-3	1
624	--	0	indeterminate
760	0.008	-2.5x10-3	-3
1220	0.003	$\sim$0	indeterminate
2610	0.001 (Est.)	--	--

Scatter diagrams of the fluctuations of the horizontal velocities, u', plotted against the vertical velocity fluctuations, w', are presented in Figure 3. The diagrams show both the decrease in the amplitude of the fluctuations with height and the decay of the correlation between the two components of the wind.

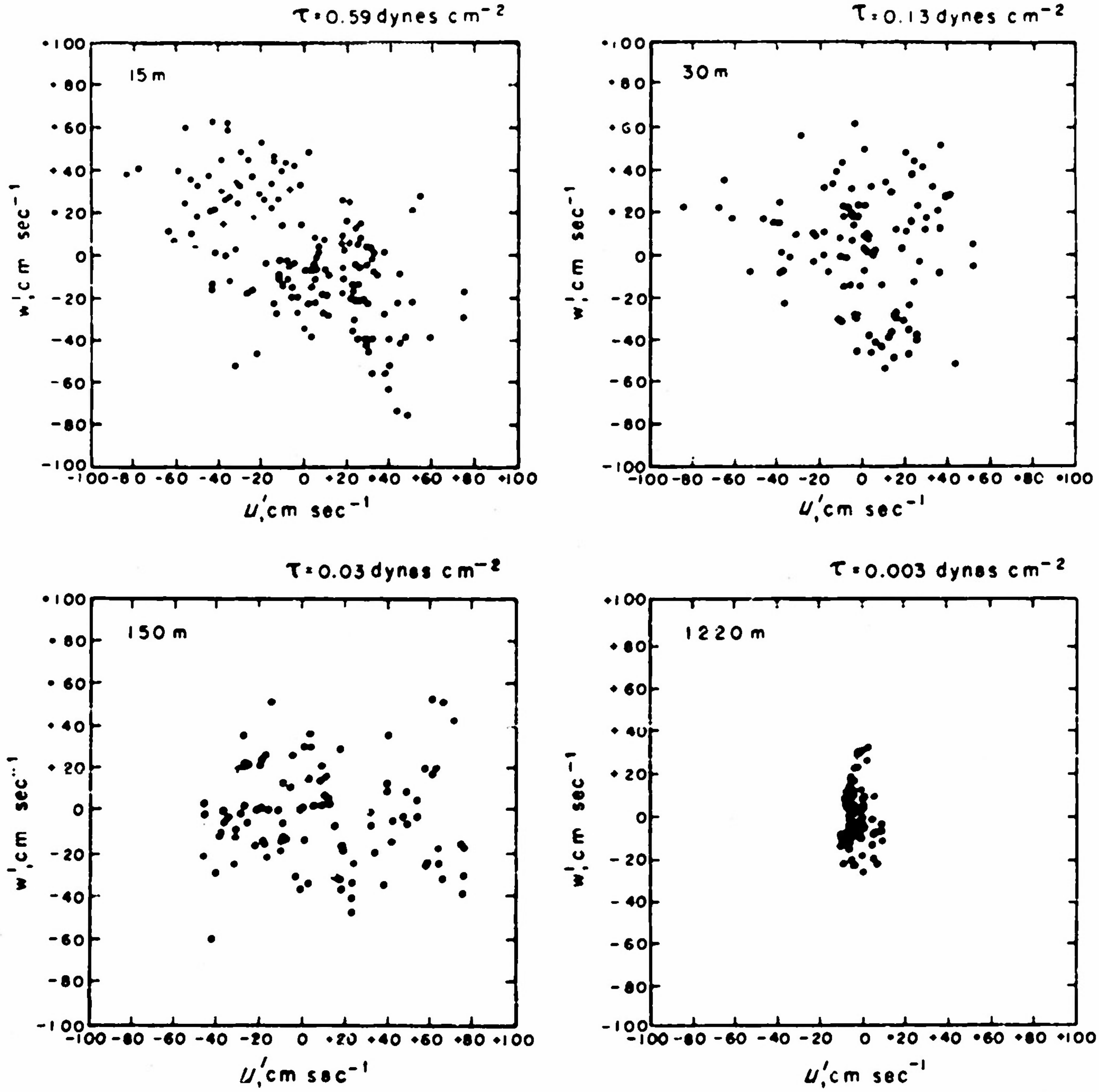
τ = 0.59 dynes cm⁻²
15 m
τ = 0.13 dynes cm⁻²
30 m
τ = 0.03 dynes cm⁻²
150 m
τ = 0.003 dynes cm⁻²
1220 m
w', cm sec⁻¹
U', cm sec⁻¹
+100
+80
+60
+40
+20
0
-20
-40
-60
-80
-100

FIG.3

The roughness coefficient, c, turns out to be 1.3×10^{-3} when τ_0 is 0.59 and a surface wind of 6.4 m/sec is used.

Further examples of the observed shearing stresses will not be given here as the emphasis of this paper is upon the observing and measuring techniques rather than upon the study of atmospheric turbulence and processes through the measurement of shearing stresses. Many horizontal runs have been flown under varying conditions of stability both in the trade wind region and at higher latitudes from which shearing stresses have yet to be computed. When this task is completed a detailed analysis of the relation of the stresses and their variations to turbulence, height, stability, heat and water vapor fluxes, and clouds can be given. It is hoped that more knowledge of the relation between the wind speed and the roughness coefficient can be obtained.

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Fig. 1. View of PBY aircraft showing sensing heads mounted on forward gun turret. Psychrograph is mounted on strut projecting upwards from the top of turret at a 45° angle. The hot bead anemometer is mounted on boom extending straight forward from top of turret. Ship's pitot tube shows on port wing. A second pitot tube was installed later on starboard wing in a similar position. The normal accelerometer was mounted in the navigator's compartment near the center of gravity of the aircraft which is near the front struts.

Fig. 2. Two oscillograph records of March 22, 1953, flights in the trade wind region at 15 m and 1220 m. Air speed, acceleration, attitude and altitude traces, and calibrations are labelled, as is also the time scale.

Fig. 3. Four scatter diagrams of the horizontal, u', and vertical, w', fluctuations of the wind observed on March 22, 1953, in the trade wind region.

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