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WORK PERFORMED UNDER OFFICE OF NAVAL RESEARCH CONTRACTS
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Synopsis: Recent reports (references 1 and 2) have stressed the importance of the static hysteresis loop of the core as information needed to predict the transfer characteristics of certain kinds of magnetic amplifiers. An attempt is made to express the static properties of the core in terms of the separate effects of core configuration and core material.

INTRODUCTION

In a recent paper (1), Batdorf and Johnson discussed the correlation between the static properties of certain magnetic core materials and the unstable behavior known as "triggering" when such a core material is used in the totally constrained single core magnetic amplifier. In another recent publication (2), Dieterly has discussed the effect of core dimensions upon the static and dynamic properties of the core. It was shown that a core made of so-called "rectangular" core material actually has a more nearly rectangular hysteresis loop if the ratio of inside diameter to outside diameter is made as nearly unity as possible. The purpose of this paper is to establish a correlation between the work of Dieterly and that of Batdorf and Johnson.

INFLUENCE OF RADIAL DIMENSIONS OF THE CORE

First it will be demonstrated that the hysteresis loop with parallel but inclined sides, advanced by Batdorf and Johnson as a property of the material alone, can be predicted using the idea of Dieterly. It will be assumed that the magnetic properties of a small sample of the core material, laid out flat, follow the elementary rectangular hysteresis loop with flat top and vertical sides. A long strip of this material is wound into a toroid whose mean radius is R centimeters and whose thickness in the radial direction is 2X centimeters. The problem is to find the static hysteresis loop of this core.

This static loop is really a plot of average inductions versus average magnetic field intensity. This implies that the ordinate will be the net flux in the core divided by the cross section, while the abscissa will be $(4\pi/10)$ times the ampere-turns of the exciting winding divided by the mean length of magnetic path.

Now a more specific meaning for the elementary rectangular loop will be assigned. It is assumed that the dipole axis of every magnetic element in the material is aligned perfectly with the rolling direction in the strip. By symmetry, this is also the direction of the magnetomotive force applied by the exciting winding. Now it is further assumed that every dipole is oriented either parallel or anti-parallel with the rolling direction, there being no other stable magnetic state. The field intensity necessary to re-orient the dipole from parallel to anti-parallel, or vice versa, is the coercive force H_c oersteds.

The magnetic material is arranged in the form of concentric circles (neglecting the spiral effect and resultant air gaps). If there is one region in the core whose magnetic state is parallel oriented and another whose state is anti-parallel oriented, then by symmetry the surface of separation between the two regions must be a circular cylinder. Each region is magnetized to saturation but in opposite directions.

A change in the net flux of the core is accomplished by moving this surface of separation from one location to another. To illustrate the process, let the entire core be initially magnetized to negative saturation, and let a positive magnetomotive force be applied. The innermost layer of the toroid will re-orient itself when the field intensity at that radius is H_c. The surface of separation between negative saturation and positive saturation migrates outward as the field intensity is increased until finally the complete core is re-oriented when the field intensity at the outer layer of the toroid has reached H_c.

The slope of the static loop may be expressed simply in terms of the saturation induction, B_s, the coercive force, H_c, and the dimensions R and X (Figure 1). It will be assumed that the initial state is $-B_s$ throughout.

Let the surface of separation between the states $-B_s$ and $+B_s$ be located at x. Since the field intensity is necessarily H_c at that radius,

$$H_c = \frac{4\pi}{10} \frac{(ni)}{2\pi(R+x)} \qquad (1)$$

The field intensity we wish to plot on a static hysteresis loop is the mean intensity

$$\bar{H} = \frac{4\pi}{10} \frac{(ni)}{2\pi R} \qquad (2)$$

Therefore, when the core is being altered magnetically,

$$\bar{H} = H_c \left(\frac{R+x}{R} \right) \qquad (3)$$

The average induction will be

$$\bar{B} = \frac{B_s(X+x) - B_s(X-x)}{2X} = B_s \frac{x}{X} \qquad (4)$$

Eliminating the parameter x yields

$$\bar{B} = B_s \frac{R}{X} \left(\frac{\bar{H}}{H_c} - 1 \right) \qquad (5)$$

The slope of the hysteresis loop of the entire core is

$$\frac{d\bar{B}}{d\bar{H}} = \frac{B_s}{H_c} \frac{R}{X} \qquad (6)$$

Equation (6) shows that the slope of the static hysteresis loop depends only upon the saturation induction, the coercive force, and the ratio of mean diameter to core thickness. The validity of this expression has been checked experimentally by numerous tests on ten cores made of 0.004 by 0.5 inch Orthonol strip, wound into toroids. This test data is summarized in Table 1, where the permeability μ_d is the "dimensional" permeability computed according to equation (6), using in each case the measured values of B_s and H_c. In computing B_s, the stacking factor of 85% as recommended by the manufacturer was used. The permeability μ_m is the maximum slope of the static hysteresis loop, as measured point by point using a flux meter. In every case, the measured permeability μ_m is smaller than the value μ_d predicted on the basis of the dimensions, as would be expected since the core material cannot be idealized to the extent shown in Figure 1.

Equation (6) gives a result identical to the hypothesis used in reference (1) to explain the triggering phenomenon. The implication is that the static hysteresis loop with inclined parallel sides may be explained in terms of core configuration, as well as in terms of the magnetic material. That the core configuration has an important bearing on the performance of magnetic amplifiers has already been recognized by Dornhoefer (3).

The process of changing the average induction by means of shifting boundary surfaces also explains the existence of alternate regions oppositely magnetized to saturation as proposed in Figure 10 of reference (1). The change of magnetic state always proceeds from the inside surface of the toroid to the outside. How far the change of state extends depends (for currents) upon the highest net magnetomotive force, or (for voltages) the volt-seconds absorbed by the core. (By applying positive and negative voltage pulses of decreasing length but constant amplitude, it would be possible to set up a number of these alternately magnetized regions.) Experimental evidence that the change of magnetic state does in fact travel from inside to outside in a toroidal core is offered by Mita (reference 4).

The results so far obtained indicate that some attempt should be made to separate the effects of core configurations from those of the core material. If this can be done, it will be possible to predict the static hysteresis loop of a proposed core in advance of manufacture. Also, it will be possible to more accurately check on the quality of the material during manufacture (2).

EFFECTS OF NON-IDEAL CORE MATERIAL

Experimental observation of hysteresis loops reveals that the sides of the loops are not quite straight lines, but have a curvature, while the corners are rounded. This indicates the need for a more careful assumption concerning the behavior of the dipoles than is given in Figure 1. The suggestion made in Figure 2(c) of reference 1 will be accepted, and it will be assumed that a small flat section of the magnetic material contains dipoles which re-orient themselves according to a normal probability curve with respect to increasing field intensity. (It will still be assumed, however, that the magnetic state is irreversible - that is, there are no stable states other than parallel and anti-parallel alignment.) Such a functional behavior is shown in Figure 2. The constant h could be considered as a measure of quality since it expresses the "sharpness" of the probability function.

Consider now a flat sheet of the material initially magnetized to negative saturation, to which a field intensity H is applied in the positive direction.

If H is small, a further increment ΔH will cause relatively few dipoles to be re-oriented; at higher values of H, the same increment ΔH causes a larger number of dipoles to be re-oriented. At $H = H_c$ this same increment causes the greatest number of dipole reversals. Also at $H = H_c$, due to the symmetry of the probability curve, the average induction will be zero since half the total number of dipoles have been reversed.

Let M be the total number of dipoles in a unit volume of the material. Assuming all of the dipoles to be initially oriented anti-parallel, the number m which can be reversed by applying a field intensity H will be

$$m = \frac{M}{\sqrt{\pi}} \int_{-\infty}^{Z} e^{-z^2} dz \tag{7}$$

where the upper limit is

$$Z = \frac{1}{h}(H - H_c) \tag{8}$$

The number remaining with anti-parallel orientation will be (M - m). In the closed magnetic circuit the induction will be the saturation induction multiplied by the ratio of excess dipoles of one orientation to the total number of dipoles; or

$$B = -B_s \frac{M-2m}{M} = -B_s\left(1 - 2\frac{m}{M}\right) \tag{9}$$

Then the induction is

$$B = -B_s + B_s \frac{2}{\sqrt{\pi}} \int_{-\infty}^{Z} e^{-z^2} dz \tag{10}$$

In equation (7) the limit $(-\infty)$ was used as the beginning point for integration, while the hypothesis was set by assuming the integration to start at zero. In those materials having "sharp" hysteresis loops this apparent inconsistency will not lead to any difficulty because the probability integral takes the value 1.0 correct to 5 significant figures at $z = 3.08$. However, a "smooth" material could be defined qualitatively at least by allowing h to become larger. The integration from $z = -\infty$ takes on the meaning that if substantial numbers of dipoles assume parallel orientation before the field intensity has become positive, then there is a portion of the re-orientation process that is reversible.

If a material having the properties of equation (10) is wound into a toroidal core, the upper limit Z takes the value

$$Z(x) = \frac{1}{h}\left(\frac{4\pi}{10} \frac{ni}{2\pi(R+x)} - H_c\right) \tag{11}$$

Here the coordinate x is merely the location in the toroid where calculations are to be made. However, if h = o, x becomes the location of the boundary between regions having $-B_s$ and regions having $+B_s$ inductions.

The average induction in the toroidal core will be

$$\bar{B} = \frac{1}{2X}\int_{-X}^{X} B\,dx \qquad (12)$$

Substitution of equation (10) yields

$$\bar{B} = \frac{1}{2X}\int_{-X}^{X} B_s\left[-1 + \frac{2}{\sqrt{\pi}}\int_{-\infty}^{Z} e^{-z^2}dz\right]dx \qquad (13)$$

or

$$\bar{B} = -B_s + \frac{B_s}{2X}\cdot\frac{2}{\sqrt{\pi}}\int_{-X}^{X}\int_{-\infty}^{Z} e^{-z^2}dz\,dx \qquad (14)$$

The object is to determine $d\bar{B} / d\bar{H}$ where

$$\bar{H} = \frac{4\pi}{10}\;\frac{(ni)}{2\pi R} \qquad (15)$$

First, $\bar{H}$ is substituted into the function Z, and the limits Z_1 and Z_2 are defined:

$$\left.\begin{aligned} Z(x) &= \frac{1}{h}\left[\bar{H}\left(\frac{R}{R+x}\right) - H_c\right] \\ Z_1(X) &= \frac{\bar{H}}{h}\left(\frac{R}{R+X}\right) - \frac{H_c}{h} \\ Z_2(X) &= \frac{\bar{H}}{h}\left(\frac{R}{R-X}\right) - \frac{H_c}{h} \end{aligned}\right\} \qquad (16)$$

The double integration in (14) is performed with respect to x first, then $(d\bar{B} / d\bar{H})$ is found by differentiating under the remaining integral sign. First equation (14) is re-written in the form

$$\bar{B} = -B_s + \frac{B_s}{2X}\cdot\frac{2}{\sqrt{\pi}}\left\{\int_{-X}^{X}\int_{-\infty}^{Z_1(X)} e^{-z^2}dz\,dx + \int_{-X}^{X}\int_{Z_1(X)}^{Z(x)} e^{-z^2}dz\,dx\right\} \qquad (17)$$

or

$$\bar{B} = -B_s + B_s\frac{2}{\sqrt{\pi}}\int_{-\infty}^{Z_1(X)} e^{-z^2}dz + \frac{B_s}{2X}\cdot\frac{2}{\sqrt{\pi}}\int_{-X}^{X}\int_{Z_1(X)}^{Z(x)} e^{-z^2}dz\,dx \qquad (18)$$

Interchanging the order of integration in the double integral and performing the first integration over x yields

$$\bar{B} = -B_s + B_s \frac{2}{\sqrt{\pi}} \int_{-\infty}^{Z_1(X)} e^{-z^2} dz + \frac{B_s}{2X} \cdot \frac{2}{\sqrt{\pi}} \int_{Z_1(X)}^{Z_2(X)} e^{-z^2} \left[X - R + \frac{\bar{H} R}{hz + H_c} \right] dz \quad (19)$$

Now the derivative $d\bar{B} \,/\, d\bar{H}$ may be formed by differentiating under the integral sign. The details of this process will be omitted. The result is, assuming $X \ll R$ and $h \ll H_c$:

$$\left. \frac{d\bar{B}}{d\bar{H}} \right|_{\bar{H} = H_c} = \frac{B_s}{H_c} \cdot \frac{R}{X} \operatorname{erf}\left(\frac{H_c}{h} \frac{X}{R} \right) \quad (20)$$

The maximum value of the error function is unity, which occurs as $h \to 0$. Thus the result in equation (20) reduces to the result of equation (6) for such a "perfectly sharp" core material.

In order to better interpret equation (20) it may be re-written in the form

$$\left. \frac{d\bar{B}}{d\bar{H}} \right|_{\bar{H} = H_c} = \frac{B_s}{h} \frac{\operatorname{erf}\left(\frac{H_c}{h} \frac{X}{R} \right)}{\left(\frac{H_c}{h} \frac{X}{R} \right)} \quad (21)$$

If h can be determined experimentally as a constant of the material, then equation (21) is expressed in a form convenient for comparing different cores made of the same material. Unfortunately, the experiments performed so far have not confirmed this equation. Rather, the results shown in Table 1 indicate that the slope of the hysteresis loop depends primarily upon the effect of the radial dimensions as predicted from equation (6). That equation shows the maximum slope which can be obtained in a given core in the case of an "ideal" core material. The actual slope will be smaller. (The experiments reported here show the actual slope to be about 70% of the ideal in every case).

The difficulty met in the conduct of experiments to measure the quantity h may be a reason for the present inability to properly demonstrate the statistical behavior of the dipoles assumed in the second part of this paper.

Table 1

Core Number	R Inches	X Inches	B_s Gausses	H_c Oersteds	μ_m x10^{-6}	μ_d x10^{-6}	$\frac{\mu_m}{\mu_d}$
1	1.625	0.125	12,630	0.0701	1.643	2.34	0.702
2	1.625	0.125	12,630	0.0750	1.292	2.18	0.592
3	1.375	0.125	11,880	0.0588	1.611	2.22	0.726
4	1.375	0.125	13,090	0.0567	2.180	2.53	0.861
5	1.125	0.125	13,210	0.0770	1.042	1.544	0.674
6	1.125	0.125	13,900	0.0711	1.250	1.758	0.711
7	0.875	0.125	13,330	0.0773	0.721	1.205	0.598
8	0.875	0.125	13,330	0.0688	0.810	1.355	0.597
9	0.625	0.125	13,140	0.0643	0.773	1.041	0.742
10	0.625	0.125	12,800	0.0604	0.732	1.060	0.691

REFERENCES

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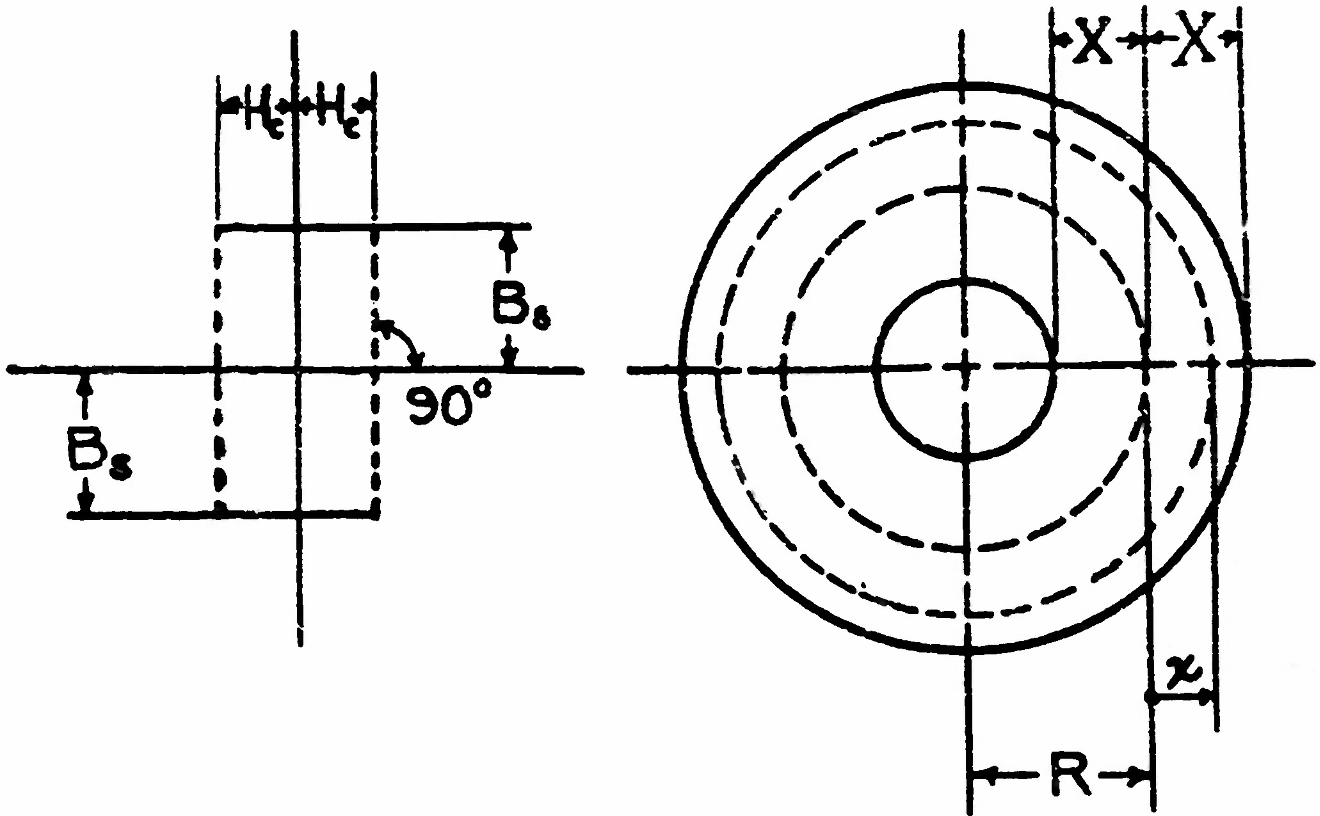

FIGURE 1

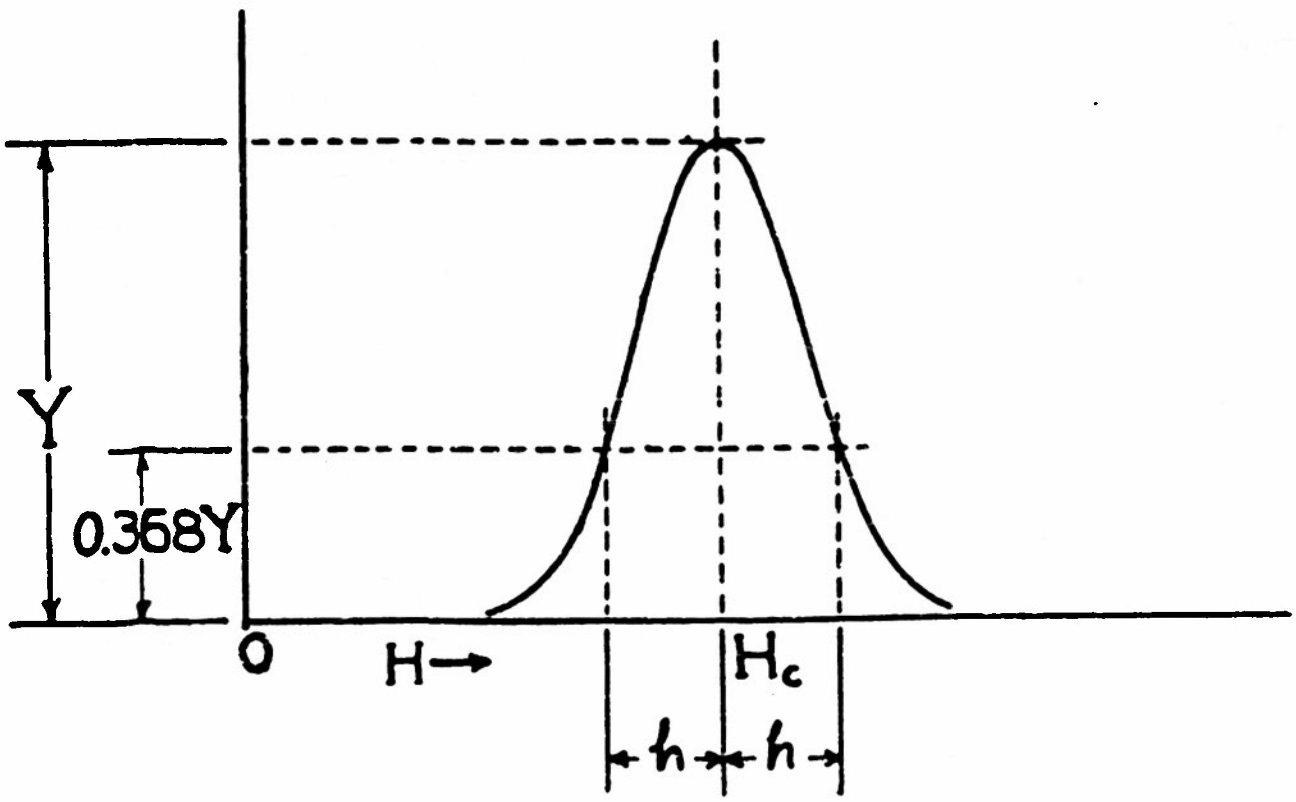

FIGURE 2