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# EVALUATION AND COMPARISON OF METHODS OF FILTERING

by

B. W. Davis and J. F. Heyda

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# EVALUATION AND COMPARISON OF METHODS OF FILTERING

by

B. W. Davis and J. F. Heyda

Research and Test Department





SECURITY INFORMATION

## FOREWORD

This report has been prepared under Task Assignment NOPI-Re8e-4-1-53, Flexible Gunnery Computer Study, which is a continuing task.

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## ABSTRACT

In mechanizing a computer to solve the lead equations for an airborne flexible-gun fire-control system, accurate values are needed for the input quantities of range and sight-line angles-off and their derivatives. It is the purpose of this report to investigate the effectiveness of various electrical filters and the theories underlying them in simultaneously smoothing and differentiating various input signals provided by the radar. Emphasis has been placed upon evaluating RC-networks.

## CONCLUSIONS

A partial investigation of the effectiveness of several electrical filters and the theories underlying them has been made. In particular, it has been found that, for smoothing and differentiating the angular coordinates of the sight-line as provided by the tracking radar, a filter based upon the Zadeh-Ragazzini theory is no more effective than a single stage RC-filter in the case of linearly varying time signals and noise characterized by an exponentially decaying autocorrelation function. This conclusion is still valid for non-linear time signals if the single stage RC-filter is replaced by an RC-network characterized by a transfer function

$$H_{nj}(s) = \frac{s(1 + a_1 s + a_2 s^2 + \dots + a_{n-1} s^{n-1})}{1 + a_1 s + a_2 s^2 + \dots + a_{n-1} s^{n-1} + b_n s^n + \dots + b_j s^j},$$

with $j \geq n$. Because of this fact, the main effort in this report deals with transfer functions of this type.

The investigation has been partial in that the signal input functions were of two types only and the noise autocorrelation function a decaying exponential. However, the two signals were representative of the two most probable target paths: pursuit courses and straight line interception attacks. The noise autocorrelation function, selected on the basis of available radar tracking data, is fairly representative, much more so than the assumption of white noise implicit in the Blackman-Bode-Shannon theory. For this reason the latter theory has not been investigated further.

In general, it appears that RC-networks with transfer functions $H_{nj}(s)$ can be used to give results within required limits for major portions of nearly all realistic tactical courses. The design of a range rate filter, considered briefly in this report, must await definite knowledge concerning radar range noise characteristics. The problem here, however, is eased by the fact that higher order derivatives of range rate are small and as a result signal distortion can be kept within bounds more easily than in the case of an angular rate filter.

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# I. INTRODUCTION

## A. STATEMENT OF THE PROBLEM.

In mechanizing a computer to solve the lead equations for an airborne flexible-gun fire-control system, accurate values are needed for the input quantities of range and sight line angle-off and their derivatives. The range $r$ is furnished directly by the radar while the sight line angle-off is approximated by the azimuth and elevation angles, $A$ and $E$, respectively, of the axis of the radar antenna. The quantities $r$, $A$ and $E$ thus furnished are not smoothly varying, being contaminated with noise. The noise is generated as the sum of several different effects which may be noted. These are

a. Wandering of the radar beam over the target surface,

b. Fading of the radar echo as a result of changing target aspect and atmospheric variations,

c. Antenna and receiver noise,

d. Servo follow-up noise.

The derivative quantities $\dot{r}$, $\dot{A}$ and $\dot{E}$, obtained by differentiating the rough $r$, $A$ and $E$ values, will depart sharply from the desired derivative values unless steps are taken to smooth the original input data.

It is the purpose of this report to investigate the effectiveness of various electrical filters and the theories underlying them in simultaneously <u>smoothing</u> and <u>differentiating</u> the azimuth angle $A$ of the axis of the radar antenna as it tracks a target moving on a prescribed path relative to the ownship. The quantity, $A(t)$, free of noise, will be spoken of as the <u>signal</u>. The noise, superimposed upon the signal, will be denoted by $N(t)$.

## B. SIGNALS USED.

Two different types of time functions were used for $A(t)$, one associated with the target on a straight line path relative to the ownship and the other associated with the target on a pure pursuit course relative to straight line motion of the ownship. For the first of these, the space path of the target was assumed to form an angle $\beta$ with the path of the ownship, as shown in Figure 1.

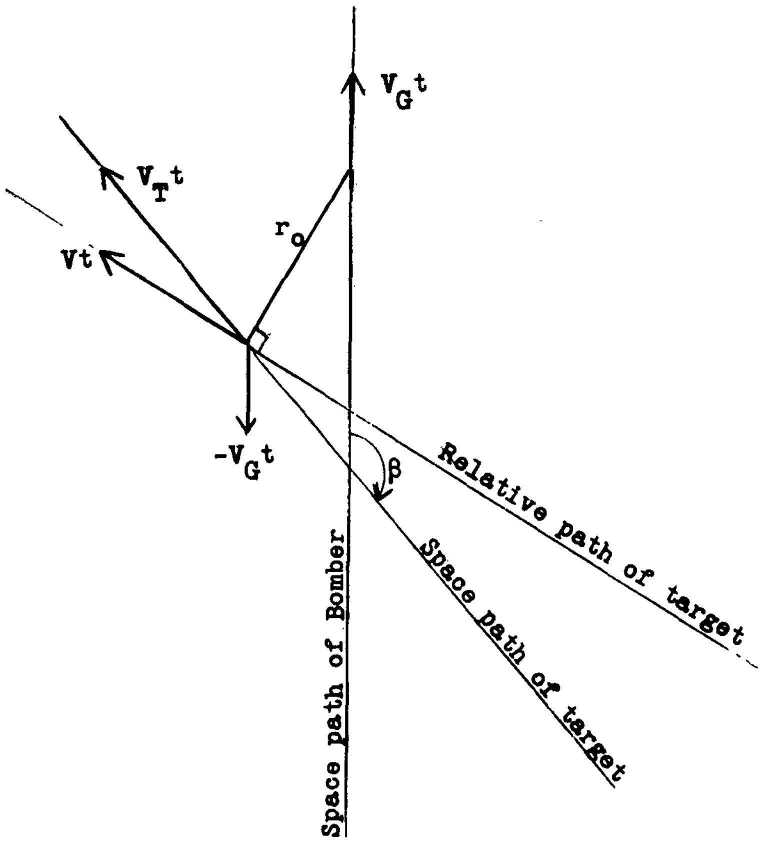


FIGURE 1. GUN-TARGET SPACE AND RELATIVE STRAIGHT LINE PATHS

Denoting the ownship and target speeds by $V_G$ and $V_T$, the relative speed by V, the cross-over range by $r_o$ and $r_o/V$ by k, we note that the functional form of the signal for azimuth angle, measuring time from cross-over, is

$$(1) \qquad A(t) = C - \tan^{-1} \frac{t}{k}$$

The time interval of interest for the rear turret of a bomber would be that preceding cross-over. The interval considered is $-8 \leq t \leq 0$. Data used were $V_G = 175$ yd/sec., $V_T = 225$ yd/sec., $r_o = 400$ yards, with C and k being determined by combining these with β.

Initial evaluations of filters were made assuming $\beta = 90^\circ$. Later evaluations were based upon a compromise assumption of $\beta = 135^\circ$.

The second signal used assumed a parabolic fit to $\dot{A}(t)$ for $A(t)$ associated with the target on a pure pursuit course. In particular, with units in radians and seconds,

$$(2) \qquad \dot{A}(t) = .0125\,t^2 - .1t - .2 \qquad (0 \leq t \leq 8)$$

The maximum angular rate here amounts to 200 mils/sec. at $t = 4$.

C. **NOISE ASSUMPTIONS.**

The noise function $N(t)$, associated with $A(t)$, is not capable of direct analytic formulation and hence must be dealt with statistically using the basic concepts of autocorrelation and spectral density. (For a good introduction to these techniques, the reader should consult Chap. VI of reference (1).) From numerous sources (see references (1), (2),(3)) a very typical form of the autocorrelation function $R(t)$ for noise in angle $A$, obtained experimentally in extensive radar tracking tests, is

$$(3) \qquad R(t) = \sigma^2 e^{-a|t|} \quad .$$

The quantity $\sigma^2 = R(o)$ gives the mean-square value of the noise amplitude $N(t)$,

$$(4) \qquad \sigma^2 = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} N^2(t)dt.$$

This follows directly from the "time-average" definition of $R(t)$, namely

$$(5) \qquad R(t) = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} N(\tau)\, N(t + \tau)\, d\tau \quad .$$

The spectral density function $G(f)$ for the input noise $N(t)$ is, by the Wiener-Khinchine theorem, the Fourier cosine transform of $R(t)$.

The function G(f) multiplied by df gives a direct measure of the amount of noise power in the frequency range f to f + df. With the representation (3) for R(t), we find that

$$(6) \qquad G(f) = 4 \int_0^\infty R(t) \cos 2\pi ft \, dt = \frac{4a\sigma^2}{a^2 + \omega^2} ,$$

where

$$\omega = 2\pi f.$$

The constants a and $\sigma^2$ appearing in (3) depend upon the radar type, the rate of change of target aspect and other factors. Values of a in the references cited show variation from 5 to 40 ($sec^{-1}$). In this report $a = 5$ and $a = 10$ are accepted as representative values. For $\sigma^2$ the value $7.1(mils)^2$ was taken in agreement with the value given in reference (2).

D. CRITERIA FOR FILTER EFFECTIVENESS.

A general outline will now be given of the criteria used in determining the effectiveness of a given filter in smoothing and differentiating the input A(t) + N(t). Considering the noise N(t) with spectral density G(f), we desire to compute the mean-square value of the output noise, which we designate by the symbol $\overline{\dot{N}_o^2}$. (The subscript o indicates "output", the dot refers to the fact that the input noise is differentiated, and the bar yields the <u>mean</u> of the quantity $\dot{N}_o^2$.) If we denote by $G_o(f)$ the spectral density of the output noise $\dot{N}_o(t)$, then by the fact that $\overline{\dot{N}_o^2} = R_o(0)$, where $R_o(t)$ is the autocorrelation function for $\dot{N}_o(t)$, and the Wiener-Khinchine relation,

$$R_o(t) = \int_0^\infty G_o(f) \cos 2\pi ft \, df ,$$

we find the all-important relation

$$(7) \qquad \overline{\dot{N}_o^2} = \int_0^\infty G_o(f) \, df .$$

To find $G_o(f)$ we need the theorem relating $G_o(f)$, $G(f)$ and the filter transfer function $H(s)$. [See reference (1), p. 288.] This is

$$(8) \qquad G_o(f) = G(f)|H(s)|^2 \quad \text{for } s = j\omega .$$

The transfer function $H(s)$ is in turn defined to be the ratio of the Laplace transforms of output and input functions to the filter. An equivalent definition describes the transfer function $H(s)$ as the Laplace transform of the filter weighting function, $W(t)$. Thus,

$$H(s) = \int_0^\infty W(t)\, e^{-st}\, dt .$$

The function $W(t)$, sometimes known as the smoothing function or memory function, is the response of the filter to the unit impulse function $\delta(t)$ as input. The latter, known also as the Dirac delta function, is defined by the relations

$$\delta(t) = 0 \quad \text{for} \quad t \neq 0 , \quad \delta(0) = \infty , \quad \int_{-\infty}^{\infty} \delta(t)dt = 1 .$$

Combining (6) and (8) we have

$$(9) \qquad \overline{\dot{N}_o^2} = \int_0^\infty G(f)|H(j\omega)|^2\, df .$$

Consider now the signal input to the filter, $A(t)$. The <u>desired</u> filter output is $\dot{A}(t)$, the actual output is $\dot{A}_o(t)$. We shall denote the signal distortion, $\dot{A}_o - \dot{A}$ by the symbol $\Delta\theta$. The quantity $\Delta\theta$ will be a function of $t$ and of filter parameters $F_1$, $F_2$, etc. (For a simple capacitance-resistance filter, e.g., $F_1 = RC$.) For reasons indicated later, all the F's are assumed equal. Thus we write:

$$(10) \qquad \text{Signal Distortion} = \Delta\theta(F,t).$$

It now remains to combine (9) and (10) to yield a criterion for filter effectiveness. To this end, the following two definitions are made.

(11) $\Delta\theta_M$ = Max. Value of $\Delta\theta(F,t)$ with respect to t

(12) $$\overline{\Delta\theta^2} = \frac{1}{T}\int_0^T \Delta\theta^2(F,t)\,dt = \text{Mean Square Value of } \Delta\theta(F,t)$$

(T is the time on the path, here taken to be 8 seconds.)

Relations (9), (11) and (12) are now combined as follows:

(13) $$Q = \sqrt{\Delta\theta_M^2 + \overline{\dot{N}_o^2}}$$

(14) $$\overline{Q} = \sqrt{\overline{\Delta\theta^2} + \overline{\dot{N}_o^2}}$$

It should be noted that $\Delta\theta_M$, $\overline{\Delta\theta^2}$ and $\overline{\dot{N}_o^2}$ are each functions of F alone. On the basis of the two criteria (13) and (14), filters are compared by noting how small their corresponding Q's or $\overline{Q}$'s become for a given type of signal input. The criterion using $\overline{Q}$ is somewhat more realistic than that using Q.

## II. SOME PARTICULAR THEORIES OF FILTERING

### A. GENERAL BACKGROUND.

Basing their work upon the fundamental work of Wiener (see reference (4)), numerous writers have put forth theories of filtering wherein the filter is characterized by a weighting function which is the best possible according to some definite mathematical criterion for selecting the "best" function out of a whole class of functions. In particular there should be mentioned the fundamental papers by Phillips and Weiss (reference (5)), Cunningham and Hynd (reference (6)), and Zadeh-Ragazzini (reference (7)). Since the last of these is more general, including the other papers as special cases, we shall include a very brief outline of it.

B. ZADEH-RAGAZZINI THEORY.

In the Z-R theory, the signal is assumed to be representable as a polynomial $P(t)$ of degree not higher than a specified number $n$. The noise function $N(t)$ is assumed to be a stationary function of time described by the autocorrelation function $R(t)$. [For a precise definition of "stationarity", see reference (1), p. 270.] Denoting the weighting function of the filter by $W(t)$, where $W(t)$ is defined for $0 \leq t \leq T^*$ and is assumed zero elsewhere, we may write for the filter output, $E_o(t)$, corresponding to the input $E_1(t) = P(t) + N(t)$:

$$(15) \qquad E_o(t) = \int_0^{T^*} W(\tau)[P(t-\tau) + N(t-\tau)]\, d\tau \, .$$

If the moments of $W(\tau)$,

$$(16) \qquad \mu_r = \int_0^{T^*} \tau^r W(\tau)\, d\tau \, , \qquad r = 0,1,2,\ldots,n \, ,$$

are introduced and $P(t - \tau)$ is expanded as a polynomial in $\tau$, we may rewrite (15) in the form:

$$(17) \qquad E_o(t) = \mu_o P(t) - \mu_1 \dot{P}(t) + \frac{\mu_2}{2!} \ddot{P}(t) + \ldots + (-1)^n \frac{\mu_n}{n!} P^{(n)}(t) + \int_0^{T^*} W(\tau)\, N(t-\tau)\, d\tau \, .$$

Consider now the error of the filter, namely,

$\varepsilon$ = Output - Desired Output;

or in symbols,

$$(18) \qquad \varepsilon(t) = E_o(t) - \dot{P}(t)$$

The Z-R theory now selects a "best possible" filter as one which is characterized by a $W(t)$ determined to satisfy the following two conditions:

(a) $$\lim_{L \to \infty} \frac{1}{L} \int_0^L \varepsilon(t)\, dt = 0 \quad ,$$

(b) $$\sigma_o^2 = \lim_{L \to \infty} \frac{1}{L} \int_0^L \varepsilon^2(t)\, dt \qquad \text{is a minimum.}$$

Noting (17) and (18), we may write condition (a) in the equivalent form:

(19) $$\dot{P}(t) \equiv \mu_o P(t) - \mu_1 \dot{P}(t) + \frac{\mu_2}{2!} \ddot{P}(t) + \ldots + (-1)^n \frac{\mu_n}{n!} P^{(n)}(t) \; .$$

(Equation (19) assumes that the noise N(t) has zero mean.)

Equation (19) is equivalent to imposing $n + 1$ constraints on the function $W(\tau)$:

(20) $$\mu_i = 0, \quad i = 0,2,3,\ldots,n; \quad (i \neq 1)$$
$$\mu_1 = -1 \; .$$

It is of interest to rewrite (18) in the light of (20).

(21) $$\varepsilon(t) = \int_0^{T^*} W(\tau)\, N(t - \tau)\, d\tau$$

Since the filter is linear and differentiating, we recognize the right side of (21) as $\dot{N}_o(t)$, and hence $\varepsilon(t) \equiv \dot{N}_o(t)$. Condition (b) is thus equivalent to minimizing $R_o(0)$, the mean square value of the output noise. Thus, $\sigma_o^2 \equiv R_o(0)$.

For the actual minimization of $R_o(0)$, the condition (b) can be rewritten as

(22) $$R_o(0) = \int_0^{T^*} \int_0^{T^*} W(t)\, W(\tau)\, R(t - \tau)\, dt\, d\tau \; .$$

The minimization procedure involves an application of the Calculus of Variations. The result will now be given for the autocorrelation assumption made in (3). It is found that

$$(23) \qquad W(t) = A_o + A_1 t + A_2 t^2 + \dots + A_n t^n + C_1 \delta(t) + D_1 \delta(t - T^*),$$

where the A's, $C_1$, $D_1$ are constants which depend on $T^*$. The acutal minimum value of $R_o(0) = \overline{\dot{N}_o^2}$ for the case $n = 1$ turns out to be

$$(24) \qquad \overline{\dot{N}_o^2} = \frac{24\, a\, \sigma^2}{T^*(a^2 T^{*2} + 6aT^* + 12)}$$

C. BLACKMAN, BODE AND SHANNON THEORY.

In addition to the work cited in paragraph A of this section, there should be mentioned the paper by Blackman, Bode and Shannon (reference (8)) which bases the optimum filter on a weighting function $W(t)$ which minimizes the mean square prediction error of the filter under the assumption of "white" noise, i.e. noise with a spectral density which is constant for all frequencies (flat spectrum). The reason for this assumption, according to B-B-S, is that actual noise spectra are "subject to variations due to factors which it is not desirable in practice to attempt to control". The B-B-S theory can be considered to be a particular case of the Z-R theory.

D. THE RC DIFFERENTIATING FILTER.

It is possible to compare directly, using the criterion developed in equation (4), the effectiveness of filtering achieved by a Z-R filter and that achieved by a network of constant resistances and capacitances, a so-called RC-filter. In particular it can be shown that the simple RC-filter shown in Figure 2 is, under fairly reasonable assumptions as to signal and noise, entirely equivalent to an $n = 1$, Z-R filter.

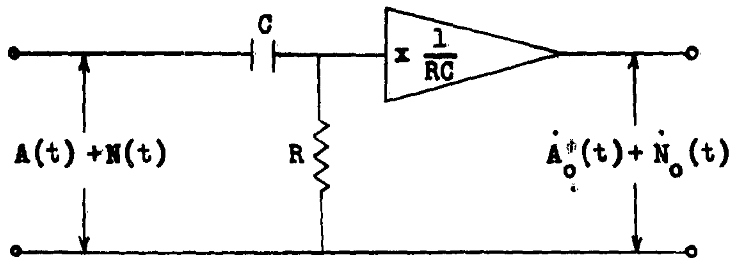


FIGURE 2

To see this, let us compare the minimum values of Q for the two filters (see eq. (13) for the definition of Q). The expression for $\overline{\dot{N}_o^2}$ for the Z-R filter is given by (24). The corresponding expression for the RC-filter is found, using (9), to be

$$(25) \qquad \overline{\dot{N}_o^2} = \frac{a\sigma^2}{F(1 + aF)} , \qquad (F = RC).$$

[ The noise is assumed to be described by the autocorrelation function given by (3).]

Now for the Z-R filter,

$$\overline{\dot{N}_o^2} = \frac{24\, a\, \sigma^2}{T^*(12 + 6aT^* + a^2T^{*2})} = \frac{a\, \sigma^2}{\left(\frac{T^*}{2}\right)\left[1 + a\left(\frac{T^*}{2}\right) + \frac{a^2T^{*2}}{12}\right]}$$

which, for small $T^*$ and moderate values of a, is approximately

$$(26) \qquad \frac{a\,\sigma^2}{\left(\frac{T^*}{2}\right)\left[1 + a\left(\frac{T^*}{2}\right)\right]}$$

Thus, we note that (26) is the same function of $\frac{T^*}{2}$ that (25) is of F.

Let us now compare the signal distortions produced by the two filters, assuming the signal to be non-linear in time. We find for the RC-filter:

$$\Delta\theta = \int_0^\infty \frac{e^{-\frac{\tau}{F}}}{F} \dot{A}(t - \tau)\, d\tau - \dot{A}(t) \quad ,$$

or, upon expanding $\dot{A}(t - \tau)$ in a Taylor series and integrating,

(27) $$\Delta\theta = -F\,\ddot{A}(t) + F^2\,\dddot{A}(t) - F^3\,A^{(4)}(t) + \dots \quad .$$

Since F is generally small and $\dddot{A}(t)$ and higher derivatives of not much importance, we see that

(28) $$\Delta\theta \doteq -F\,\ddot{A}(t) \; .$$

From (17) we note that for $n = 1$, Z-R,

$$\Delta\theta = \frac{\mu_2}{2!}\ddot{A}(t) - \frac{\mu_3}{3!}\dddot{A}(t) + \dots \quad .$$

A direct calculation shows that $\mu_2 = -T^*$, <u>independent of a</u>. Hence, for assumptions already made,

(29) $$\Delta\theta \doteq -\left(\frac{T^*}{2}\right) \ddot{A}(t) \; ,$$

which, again we note, is the same function of $\frac{T^*}{2}$ that (28) is of F.

We are thus able to conclude that the value of F which minimizes Q(F) is the same as the $\frac{T^*}{2}$ which minimizes $Q(T^*)$.

Computations were made using the signals described in section I.B. The results appear in Table 1 with $Q_M$ representing the minimum of Q with respect to F or $T^*$.

TABLE 1

COMPARISON OF Z-R AND RC $\dot{A}$-FILTERS

| | | | | | |
|---|---|---|---|---|---|
| a = 5 — Straight Line Case ($\beta = 90^\circ$) | | | | | |
| | n = 1, CR | n = 2, CR | | n = 1, Z-R | n = 2, Z-R |
| F | .054 sec. | .132 | T* | 0.11 sec. | 0.44 |
| $\Delta\theta_M$ | 17.8 mils/sec. | 12.6 | $\Delta\theta_M$ | 18 | 12.1 |
| $\overline{\dot{N}_o^2}$ | 517 $(m/s)^2$ | 502 | $\overline{\dot{N}_o^2}$ | 508 | 445 |
| $Q_M$ | 28.8 | 25.7 | $Q_M$ | 28 | 24.3 |
| a = 10 — Straight Line Case ($\beta = 90^\circ$) | | | | | |
| | n = 1, CR | n = 2, CR | | n = 1, Z-R | n = 2, Z-R |
| F | 0.065 | 0.147 | T* | 0.13 | 0.48 |
| $\Delta\theta_M$ | 21.4 | 15.6 | $\Delta\theta_M$ | 21.5 | 14.9 |
| $\overline{\dot{N}_o^2}$ | 662 | 663 | $\overline{\dot{N}_o^2}$ | 610 | 511 |
| $Q_M$ | 33.4 | 30 | $Q_M$ | 33 | 27.1 |
| a = 5 — Parabolic Case | | | | | |
| | n = 1, CR | n = 2, CR | | n = 1, Z-R | n = 2, Z-R |
| F | 0.11 | 0.48 | T* | .23 | 1.50 |
| $\Delta\theta_M$ | 11.3 | 5.6 | $\Delta\theta_M$ | 11.5 | 5.2 |
| $\overline{\dot{N}_o^2}$ | 208 | 79 | $\overline{\dot{N}_o^2}$ | 188 | 53.6 |
| $Q_M$ | 18.9 | 10.5 | $Q_M$ | 17.9 | 9.0 |

## III. EVALUATION OF RC ANGULAR-RATE FILTERS

In the process of investigating the synthesization of circuits designed to approximate the weighting functions for Z-R differentiating filters, Washington University Research Foundation (see reference (9)) found that nearly the same results could be achieved with proper combinations of resistances and capacitances without the complications encountered in including the required delay lines.

The transfer functions of the differentiating RC networks can, in general, be expressed by

$$(30) \qquad H_{nj}(s) = \frac{s(1 + a_1 s + a_2 s^2 + \dots + a_i s^i)}{1 + b_1 s + b_2 s^2 + b_3 s^3 + \dots + b_j s^j} \quad (n = i + 1,\ j \geq n)$$

The emphasis, in evaluating filters, has been placed upon the formulation of the transfer function and its effect upon the total errors Q and $\bar{Q}$ as defined by (13) and (14), respectively. The Laplace transform of the normal response, $E_o(t)$, to an input signal, $E_i(t)$, is given by

$$(31) \qquad L\{E_o(t)\} = H_{n,j}(s)\, L\{E_i(t)\}$$

or

$$(32) \qquad E_o(t) = \int_0^\infty W(\tau)\, E_i(t - \tau)\, d\tau$$

where

$$(33) \qquad W(\tau) = L^{-1}\{H_{nj}(s)\}.$$

If $H_{nj}(s)$ is expanded, "s" treated as a differentiating operator and the inverse Laplace transform found when the input signal is $A(t)$ and the output $\dot{A}_o(t)$, it is noted that

$$(34) \qquad \dot{A}_o(t) = \dot{A}(t) - k_1\ddot{A}(t) + k_2\dddot{A}(t) - \dots .$$

In this expression, the $k_r$ are determined by the network constants $a_r$ and $b_r$. One notes that setting $a_r = b_r$ for $r = 1,2,3,\ldots,i$ reduces this to

$$(35) \qquad \dot{A}_o(t) = \dot{A}(t) - b_{(i+1)}A^{(i+2)}(t) + C_{(i+2)}A^{(i+3)}(t) - \ldots.$$

where the coefficients of the higher order terms are still determined by the network constants.

Since the desired output is $\dot{A}(t)$, it is seen that the distortion is given by

$$(36) \qquad \Delta\theta_{nj} = - b_{(i+1)} A^{(i+2)}(t) + C_{(i+2)} A^{(i+3)}(t) - \ldots.$$

This reduces to the result given in (27) when $a_1 = 0$, $b_1 = F$ and $a_r = b_r = 0$ for $r = 2,3,\ldots$ .

If it happens that the input signal can be expressed as a polynomial of degree not greater than n, then (36) shows that the signal distortion will vanish identically. Thus, it is seen that a network whose transfer function is given by

$$(37) \qquad H_{nj}(s) = \frac{s(1 + b_1 s + b_2 s^2 + \ldots + b_{n-1}s^{n-1})}{1 + b_1 s + b_2 s^2 + \ldots + b_j s^j} \qquad (j \geq n)$$

will provide the undistorted derivative of a polynomial signal of degree n or less.

It is very unlikely that input signals could be represented entirely by a single polynomial of low degree; however, it is noted that, if the higher order derivatives do not become large, the distortion can be made small by proper adjustment of the network constants. For the major portion of realistic attack courses, these higher order terms are small; however, for interception courses (straight lines) the higher order terms become large near cross-over. This type of course provides the supreme test in evaluating a filter.

After the distortion is determined, it must be combined with noise, as found by using (9), to find Q and $\bar{Q}$. Adjusting the network constants to reduce distortion is found to increase the noise output and

TABLE 2

$\dot{A}$-FILTER - OUTPUT ERRORS - $H_{11}(s)$

STRAIGHT LINES CASE ($\beta = 90^{\circ}$)

| | $\Delta\theta_{11}(b,t)$ | | | | | |
|---|---|---|---|---|---|---|
| t \ b | 6 | 7 | 8 | 9 | 10 | 11 |
| -8.0 | .8124 | .7016 | .6178 | .5514 | .4980 | .4545 |
| -7.5 | .9741 | .8420 | .7416 | .6623 | .5984 | .5461 |
| -7.0 | 1.1817 | 1.0217 | .9004 | .8043 | .7271 | .6632 |
| -6.5 | 1.4511 | 1.2557 | 1.1070 | .9897 | .8948 | .8164 |
| -6.0 | 1.8087 | 1.5662 | 1.3799 | 1.2343 | 1.1117 | 1.0195 |
| -5.5 | 2.2907 | 1.9841 | 1.7499 | 1.5658 | 1.4162 | 1.2931 |
| -5.0 | 2.9546 | 2.5612 | 2.2600 | 2.0231 | 1.8307 | 1.6717 |
| -4.5 | 3.8894 | 3.3761 | 2.9817 | 2.6671 | 2.4171 | 2.2080 |
| -4.0 | 5.2449 | 4.5550 | 4.0281 | 3.6078 | 3.2698 | 2.9881 |
| -3.5 | 7.2665 | 6.3179 | 5.5898 | 5.0083 | 4.5409 | 4.1552 |
| -3.0 | 10.3727 | 9.0310 | 7.9936 | 7.1692 | 6.5035 | 5.9484 |
| -2.5 | 15.2648 | 13.3031 | 11.7865 | 10.5797 | 9.6032 | 8.7863 |
| -2.0 | 23.0355 | 20.0922 | 17.8116 | 15.9981 | 14.5237 | 13.2865 |
| -1.5 | 34.8118 | 30.3527 | 26.9036 | 24.1530 | 21.9205 | 20.0641 |
| -1.0 | 48.9302 | 42.4941 | 37.5412 | 33.6073 | 30.4295 | 27.7885 |
| -0.5 | 51.6345 | 44.1117 | 38.4539 | 34.0348 | 30.5251 | 27.6463 |
| 0 | 17.5577 | 13.2854 | 10.3814 | 8.3049 | 6.8093 | 5.6640 |
| $\overline{\dot{N}_o^2}$ | 159.750000 | 204.647058 | 252.444444 | 302.684210 | 355.000000 | 409.095238 |
| $\bar{Q}$ | 25.25999 | 23.64476 | 22.91930 | 22.77955 | 23.036202 | 23.547576 |
| Q | 53.1589 | 46.3734 | 41.6071 | 38.2237 | 35.8717 | 34.3700 |

TABLE 2 (Cont.)

$\dot{A}$-FILTER - OUTPUT ERRORS - $H_{11}(s)$

STRAIGHT LINES CASE ($\beta = 90^{o}$)

| | $\Delta\theta_{11}(b,t)$ | | | | |
|---|---|---|---|---|---|
| t \ b | 12 | 13 | 14 | 15 | 16 |
| -8.0 | .4177 | .3862 | .3595 | .3351 | .3152 |
| -7.5 | .5015 | .4645 | .4320 | .4035 | .3786 |
| -7.0 | .6099 | .5644 | .5257 | .4907 | .4608 |
| -6.5 | .7501 | .6945 | .6462 | .6036 | .5676 |
| -6.0 | .9375 | .8678 | .8079 | .7547 | .7089 |
| -5.5 | 1.1887 | 1.1012 | 1.0253 | .9588 | .9004 |
| -5.0 | 1.5369 | 1.4251 | 1.3274 | 1.2405 | 1.1657 |
| -4.5 | 2.0315 | 1.8833 | 1.7538 | 1.6405 | 1.5405 |
| -4.0 | 2.7490 | 2.5477 | 2.3750 | 2.2208 | 2.0857 |
| -3.5 | 3.8231 | 3.5461 | 3.3041 | 3.0916 | 2.9050 |
| -3.0 | 5.4784 | 5.0861 | 4.7402 | 4.4370 | 4.1723 |
| -2.5 | 8.0944 | 7.5172 | 7.0053 | 6.5594 | 6.1663 |
| -2.0 | 12.2509 | 11.3748 | 10.6027 | 9.9285 | 9.3392 |
| -1.5 | 18.4866 | 17.1517 | 15.9937 | 14.9706 | 14.0741 |
| -1.0 | 25.5609 | 23.6793 | 22.0423 | 20.6162 | 19.3627 |
| -0.5 | 25.2447 | 23.2431 | 21.5151 | 20.0165 | 18.7138 |
| 0 | 4.7690 | 4.0975 | 3.5330 | 3.0706 | 2.6939 |
| $\overline{\dot{N}_o^2}$ | 464.727272 | 521.695652 | 579.833333 | 639.000000 | 699.076923 |
| $\overline{Q}$ | 24.22639 | 25.01983 | 25.88040 | 26.78257 | 27.71061 |
| Q | 33.4378 | 32.8999 | 32.6450 | 32.6194 | 32.7718 |

TABLE 3

$\dot{A}$-FILTER - OUTPUT ERRORS - $H_{22}(s)$

STRAIGHT LINES CASE ($\beta = 90^{\circ}$)

| | $\Delta\theta_{22}(b,t)$ | | | | |
|---|---|---|---|---|---|
| t \ b | 1 | 2 | 3 | 4 | 5 |
| -8.0 | .912248 | .310607 | .155728 | .093441 | .062258 |
| -7.5 | 1.127073 | .388562 | .195894 | .117915 | .078726 |
| -7.0 | 1.408985 | .492338 | .249717 | .150835 | .100932 |
| -6.5 | 1.784659 | .632774 | .323075 | .195888 | .131406 |
| -6.0 | 2.293885 | .826311 | .424952 | .258740 | .174044 |
| -5.5 | 2.997351 | 1.098460 | .569411 | .348300 | .234998 |
| -5.0 | 3.989880 | 1.489805 | .779017 | .478941 | .324223 |
| -4.5 | 5.423436 | 2.054370 | 1.090929 | .674452 | .458256 |
| -4.0 | 7.547990 | 2.939529 | 1.567899 | .975208 | .665250 |
| -3.5 | 10.784821 | 4.299140 | 2.318112 | 1.451076 | .994038 |
| -3.0 | 15.855837 | 6.473597 | 3.529392 | 2.223427 | 1.529428 |
| -2.5 | 23.990711 | 10.020337 | 5.517305 | 3.494431 | 2.411611 |
| -2.0 | 37.132386 | 15.772961 | 8.730266 | 5.538639 | 3.823580 |
| -1.5 | 57.414426 | 24.340609 | 13.345563 | 8.382068 | 5.733537 |
| -1.0 | 82.405897 | 32.816067 | 16.922349 | 10.062919 | 6.561702 |
| -0.5 | 88.044928 | 24.661025 | 7.989237 | 2.321574 | .144286 |
| 0 | 20.526622 | -26.582836 | -26.994241 | -21.978739 | -17.305163 |
| $\overline{\ddot{N}_o^2}$ | 24.938 | 88.75 | 179.601 | 289.796 | 414.167 |
| $Q(b,t)$ | 88.146495 | 34.141532 | 30.137850 | 27.800377 | 26.713960 |
| $\bar{Q}(b)$ | 36.819997 | 16.124889 | 15.111159 | 17.621798 | 20.613682 |

TABLE 3 (Cont.)

$\dot{A}$-FILTER - OUTPUT ERRORS - $H_{22}(s)$

STRAIGHT LINES CASE ($\beta = 90^{\circ}$)

| | $\Delta\theta_{22}(b,t)$ | | | | |
|---|---|---|---|---|---|
| t \ b | 6 | 7 | 8 | 9 | 10 |
| -8.0 | .044435 | .033294 | .025865 | .020663 | .016878 |
| -7.5 | .056269 | .042206 | .032815 | .026233 | .021438 |
| -7.0 | .072254 | .054260 | .042226 | .033779 | .027621 |
| -6.5 | .094234 | .070857 | .055197 | .044191 | .036157 |
| -6.0 | .125053 | .094165 | .073435 | .058844 | .048181 |
| -5.5 | .169210 | .127619 | .099647 | .079926 | .065494 |
| -5.0 | .234010 | .176803 | .138239 | .111000 | .091038 |
| -4.5 | .331612 | .251032 | .196572 | .158027 | .129733 |
| -4.0 | .482763 | .366226 | .287242 | .231217 | .190018 |
| -3.5 | .723506 | .550067 | .432170 | .348347 | .286591 |
| -3.0 | 1.116402 | .850579 | .669356 | .540217 | .444901 |
| -2.5 | 1.764181 | 1.346167 | 1.060534 | .856645 | .705960 |
| -2.0 | 2.796381 | 2.132747 | 1.679245 | 1.355600 | 1.116501 |
| -1.5 | 4.159379 | 3.149964 | 2.464989 | 1.979301 | 1.622602 |
| -1.0 | 4.565552 | 3.333637 | 2.526479 | 1.972220 | 1.576946 |
| -0.5 | -.728013 | -1.060763 | -1.156894 | -1.146856 | -1.090192 |
| 0 | -13.674679 | -10.949430 | -8.901286 | -7.343346 | -6.140073 |
| $\overline{\dot{N}_o^2}$ | 545.141 | 692.189 | 841.481 | 995.672 | 1153.75 |
| Q(b,t) | 27.131860 | 28.497000 | 30.343268 | 32.397480 | 34.517394 |
| $\overline{Q}(b)$ | 23.566426 | 26.383172 | 29.052139 | 31.581781 | 33.984902 |

## TABLE 4

### Ȧ-FILTER - OUTPUT ERRORS - $H_{33}(s)$

### STRAIGHT LINES CASE ($\beta = 90^\circ$)

| | $\Delta\theta_{33}(b,t)$ | | | | |
|---|---|---|---|---|---|
| t \ b | 1 | 2 | 3 | 4 | 5 |
| -8.0 | .253406 | .034836 | .019657 | .009302 | .005120 |
| -7.5 | .324471 | .070168 | .025910 | .012325 | .006804 |
| -7.0 | .421308 | .093002 | .034679 | .016591 | .005039 |
| -6.5 | .554795 | .125263 | .047206 | .022725 | .012653 |
| -6.0 | .742444 | .171751 | .065472 | .031734 | .017748 |
| -5.5 | 1.011564 | .240201 | .092711 | .045269 | .025443 |
| -5.0 | 1.406135 | .343391 | .134332 | .066117 | .037358 |
| -4.5 | 1.998771 | .502969 | .199608 | .099089 | .056307 |
| -4.0 | 2.912466 | .756459 | .304785 | .152660 | .087265 |
| -3.5 | 4.359698 | 1.169149 | .478671 | .241901 | .139080 |
| -3.0 | 6.711427 | 1.859467 | .771651 | .392996 | .227028 |
| -2.5 | 10.600168 | 3.017301 | 1.264127 | .646233 | .373821 |
| -2.0 | 16.973926 | 4.885658 | 2.037656 | 1.032902 | .592012 |
| -1.5 | 26.506470 | 7.347010 | 2.910051 | 1.399083 | .762131 |
| -1.0 | 35.106452 | 7.808440 | 2.208448 | 6.446968 | 1.336843 |
| -0.5 | 24.608280 | -3.959481 | -5.619277 | -4.372891 | -3.198995 |
| 0 | -41.226517 | -36.154438 | -20.663597 | -11.740575 | -6.932616 |
| $\overline{\dot{N}_o^2}$ | 54.297 | 188.019 | 371.817 | 588.286 | 826.690 |
| Q | 41.879 | 38.667 | 28.263 | 26.947 | 29.576 |
| $\bar{Q}$ | 16.115 | 14.889 | 19.595 | 24.357 | 28.787 |

The tabular results indicate that, for these networks and this particular signal, the total error for most points would be far too large; however, a comparison of the results for the straight line and pursuit course in Table 1 indicates that the total error can be reduced considerably for other courses. It was decided that a compromise straight line course, with $\beta = 135^\circ$, would be more realistic in evaluating networks and that network transfer functions with extra terms in the denominator should be used to reduce the noise, since noise was the major contributor to the total error. It was also decided that the major portion of future investigations of this type of network should be done experimentally, since it is believed to be the more economical approach.

To indicate the regions to be investigated, some results are presented wherein the previously described method for approximating the maximum distortion was used.

TABLE 5

$\dot{A}$-FILTER - OUTPUT ERRORS - Max. $\Delta\theta$ Min. Q

STRAIGHT LINES CASE ($\beta = 135^\circ$)

| Transfer Function | Pole for Min. Q | $\Delta\theta_M$ mils/sec | $\overline{\dot{N}_o^2}$ mils$^2$/sec$^2$ | Q mils/sec |
|---|---|---|---|---|
| $H_{22}(s)$ | -3.5 | 10.5 | 232.7 | 18.5 |
| $H_{23}(s)$ | -6.3 | 9.7 | 182.2 | 16.6 |
| $H_{24}(s)$ | -8.2 | 11.4 | 193.7 | 18.0 |
| $H_{33}(s)$ | -2.4 | 8.7 | 256.7 | 18.2 |
| $H_{34}(s)$ | -3.8 | 8.75 | 170.4 | 15.7 |
| $H_{35}(s)$ | -5.1 | 9.05 | 216.3 | 17.3 |

The results of Table 5 indicate that Q still does not approach the 3 mil/sec total error which, at present, is considered as an upper limit. It may be found that $\overline{Q}$ can be brought into this neighborhood.

Since noise is always present, it seems more logical to reduce its RMS value to near the upper allowable limit on the total error and then examine the distortions for a number of realistic tactical courses. Perhaps the $\overline{\Delta\theta}$'s for a large number of these courses can be combined statistically or the expected distortion can be determined from a statistical representation of all realistic courses. Tables 6 and 7 present the maximum distortion and total error, Q, to be expected when the equal poles are fixed so as to make noise equal to 100 mils$^2$/sec$^2$ and 25 mils$^2$/sec$^2$, respectively. $\Delta\theta_M$ was computed, as before, from the expansion.

TABLE 6

$\dot{A}$-FILTER - OUTPUT ERRORS - $\overline{\dot{N}_o^2} = 100$ mils$^2$/sec$^2$

Max. $\Delta\theta$ - STRAIGHT LINES CASE ($\beta = 135°$)

| Transfer Function | Poles | $\overline{\dot{N}_o^2}$ | $\Delta\theta_M$ | Q |
|---|---|---|---|---|
| $H_{22}(s)$ | -2.14 | 100 | 28 | 29.7 |
| $H_{23}(s)$ | -4.75 | 100 | 17 | 19.7 |
| $H_{24}(s)$ | -6.11 | 100 | 20.6 | 22.9 |
| $H_{33}(s)$ | -1.4 | 100 | 43.7 | 44.8 |
| $H_{34}(s)$ | -3 | 100 | 17.8 | 20.4 |
| $H_{35}(s)$ | -3.7 | 100 | 23.7 | 25.7 |

TABLE 7

$\dot{A}$-FILTER - OUTPUT ERRORS - $\overline{\dot{N}_o^2}$ = 25 mils$^2$/sec$^2$

Max. $\Delta\theta$ - STRAIGHT LINES CASE ($\beta = 135^\circ$)

| Transfer Function | Poles | $\overline{\dot{N}_o^2}$ | $\Delta\theta_M$ | Q |
|---|---|---|---|---|
| $H_{22}(s)$ | -1 | 25 | 128 | 128.1 |
| $H_{23}(s)$ | -2.64 | 25 | 55 | 55.2 |
| $H_{24}(s)$ | -3.5 | 25 | 62.7 | 62.9 |
| $H_{33}(s)$ | -0.66 | 25 | 417.4 | 417.5 |
| $H_{34}(s)$ | -1.7 | 25 | 97.7 | 97.8 |
| $H_{35}(s)$ | -2.15 | 25 | 120.7 | 120.8 |

From the foregoing, it would appear that the networks described by transfer functions $H_{23}(s)$ and $H_{24}(s)$ would give better results; however, the evidence is not conclusive since this analysis is based upon necessary approximations.

As information of possible interest, we include the numerical results for two $H_{23}(s)$ networks, evaluated for the straight line case, $\beta = 135^\circ$, with unequal poles and no approximations made in determining signal distortion from (36).

TABLE 8

$\dot{A}$-FILTER - OUTPUT ERRORS - $H_{23}(s)$

STRAIGHT LINES CASE ($\beta = 135^\circ$)

| | $H_{23}(s) = \frac{s(81 + 58.5s)}{(s+3)(s+4.5)(s+6)}$ | $H'_{23}(s) = \frac{s(15 + 23s)}{(s+1)(s+3)(s+5)}$ |
|---|---|---|
| t | Δθ mils/sec | Δθ mils/sec |
| -8 | -.348 | -0.984 |
| -7.5 | -.427 | -1.201 |
| -7.0 | -.530 | -1.477 |
| -6.5 | -.663 | -1.832 |
| -6.0 | -.839 | -2.295 |
| -5.5 | -1.070 | -2.901 |
| -5.0 | -1.376 | -3.699 |
| -4.5 | -1.780 | -4.758 |
| -4.0 | -2.310 | -6.122 |
| -3.5 | -2.984 | -7.936 |
| -3.0 | -3.784 | -10.160 |
| -2.5 | -4.585 | -12.659 |
| -2.0 | -5.010 | -14.820 |
| -1.5 | -4.229 | -15.118 |
| -1.0 | -.962 | -10.787 |
| -0.5 | 5.601 | 1.113 |
| 0 | 13.658 | 20.001 |
| $\overline{\dot{N}_0^2}$ mil²/sec² | 80.8 | 23.1 |
| $\overline{Q}$ | 9.7 | 9.6 |
| Q | 16.6 | 20.6 |

## IV. EVALUATION OF RC RANGE-RATE FILTERS

In analyzing networks designed to provide smoothed range rate data, the emphasis has been placed on network transfer functions of types $H_{2j}(s)$. Since the third and higher order time-derivatives of range are small for reasonable tactical courses of interest in designing a rear turret defensive system, it was thought unnecessary, and probably not profitable, to resort to more complicated networks. For a more extensive investigation along these lines see reference (10).

As in the angular rate case, higher order time derivatives of range were found to be larger for interception (straight line) courses than for pursuit type courses. For this reason it was decided to place the emphasis upon the compromise straight lines course, with $\beta = 135^\circ$. If we measure time from cross-over, we have

$$(38) \qquad r = \sqrt{160{,}000 + 25{,}565\, t^2} \qquad (-8 \leq t \leq 0)$$

Since almost no information concerning the nature of noise corrupting radar range data was available, emphasis was placed upon distortion in the output signal. It was desirable that the total error in range rate be less than 5 yd/sec. For purposes of analysis, it was decided that networks, having half this total error as their maximum distortions, should be designed and tested for attenuation of an assumed noise. These approximate maximum distortions were found by the same methods used in the angular rate case. Transfer functions of networks, having 2.5 yd/sec maximum distortion with the prescribed signal input, are given by

$$(39) \quad \begin{cases} H_{22}(s) = \dfrac{s(8.75 + 6s)}{(s + 2.5)(s + 3.5)} \\[2ex] H_{23}(s) = \dfrac{s(146.545 + 88.07s)}{(s + 3.5)(s + 5.3)(s + 7.9)} \\[2ex] H_{24}(s) = \dfrac{s(3003.7392 + 1667.136s)}{(s + 5.4)(s + 6.6)(s + 8.6)(s + 9.8)} \end{cases}$$

Temporarily, an assumption was made for noise characteristics to facilitate a comparison of these three filters. This assumption is based on the findings reported in reference (12). From this we have

$$(40) \qquad G_i(f) = \frac{100\, G_i(0)}{10^2 + \omega^2} \qquad \text{where} \qquad \omega = 2\pi f$$

The above reference indicates that most of the noise in ranging is probably due to "glint". $G_i(0) = kL^2$ for glint noise where $k$ and $L$ are dependent upon the aspect and type of target. A compromise is made which seems to be in keeping with present day tactics. The attacking plane is assumed to be near head-on in aspect and both planes are assumed to be doing some maneuvering. We have assumed $G_i(0) = 7 \times 10^{-3} L^2\ ft^2/cps$ where $L$ is the length of the attacking plane. In referring to measurements of modern fighter planes, it was found that $L$ may be as great as 50 feet, but is usually about 37 feet. Two noise outputs are quoted corresponding to these measurements. The noise outputs for the above networks are shown in Table 9.

TABLE 9

$\dot{r}$-FILTER - NOISE OUTPUT

| Transfer Function | $\overline{\dot{N}_0^2}$ in $yd^2/sec^2$ | |
|---|---|---|
| | L = 37 ft. | L = 50 ft. |
| $H_{22}(s)$ | 67.1 | 122.5 |
| $H_{23}(s)$ | 46.4 | 84.8 |
| $H_{24}(s)$ | 55.8 | 101.9 |

Although the network with transfer function $H_{23}(s)$ gives better results than the other two, it does not approach the desired results. With the assumed noise input, it was necessary to allow the maximum distortion to increase in order that the noise output could be brought within bounds. It should be noted that the maximum distortion for interception type courses occurs at short ranges where given errors in range rate cause smaller errors in lead angle than that at long ranges. The latter may be established by examining the results of reference (11). Although the maximum distortions for pursuit courses would occur at longer ranges, these would never attain the magnitudes of those for most interception courses.

An $H_{23}(s)$ network, which makes the mean-square noise output less than 25 $yd^2/sec^2$, when $L = 37$ ft., is that with transfer function

$$H_{23}(s) = \frac{s(55 + 45.75s)}{(s + 2.5)(s + 4)(s + 5.5)}$$

Tabular results for this network are shown in Table 10. The distortions, $\Delta\theta$, were determined by numerical integration.

TABLE 10

$\dot{r}$-FILTER - OUTPUT ERRORS - $H_{23}(s)$

STRAIGHT LINES CASE ($\beta = 135^\circ$)

| t | $\Delta\theta$ in yd/sec |
|---|---|
| -8 | 1.032 |
| -7.5 | 0.943 |
| -7.0 | 0.847 |
| -6.5 | 0.742 |
| -6.0 | 0.625 |
| -5.5 | 0.490 |
| -5.0 | 0.269 |
| -4.5 | 0.134 |
| -4.0 | -0.116 |
| -3.5 | -0.445 |
| -3.0 | -0.881 |
| -2.5 | -1.464 |
| -2.0 | -2.210 |
| -1.5 | -3.074 |
| -1.0 | -3.827 |
| -0.5 | -3.981 |
| 0 | -2.966 |
| $\overline{\dot{N}_o^2}$ $yd^2/sec^2$ | 22.6 |
| Q | 6.2 |
| $\bar{Q}$ | 5.1 |

These results are being checked by the experimental group, and the filter will be checked for distortion on other types of signal. If it is found to be acceptable on other types of signal, this network will probably be designed into the present FGCS breadboard model.

It will be necessary to conduct a more thorough investigation of range rate filters when more definite information concerning radar ranging noise characteristics is available; however, there would be little value in carrying the present investigation any further.

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## NOTATIONS

| | |
|---|---|
| $A$ | Azimuth angle of the sight-line as measured from the ownship datum line |
| $E$ | Elevation angle of the sight-line as measured from the ownship azimuth plane |
| $r$ | Present range to the target |
| $\dot{A}, \dot{E}, \dot{r}$ | Time derivatives of A, E, r |
| $A(t)$ | The noise-free input signal |
| $N(t)$ | The noise superimposed upon the input signal |
| $V_G$ | Ownship linear velocity |
| $V_T$ | Target linear velocity |
| $V$ | Target-ownship relative velocity |
| $\beta$ | Angle between the target and ownship linear space paths |
| $\dot{A}(t)$ | Time-derivative of A(t) |
| $\dot{A}_o(t)$ | Actual noise-free output signal of the filter |
| $R(t)$ | Autocorrelation function of the input noise |
| $R_o(t)$ | Autocorrelation function of the output noise |
| $G(f)$ | Spectral density of the input noise |
| $G_o(f)$ | Spectral density of the output noise |
| $a$ | Empirical constant appearing in R(t) |
| $\sigma^2$ | Mean-square amplitude of the input noise |
| $\overline{\dot{N}_o^2}$ | Mean-square amplitude of the output noise |
| $W(t)$ | The weighting or memory function of a filter |
| $H(s)$ | Laplace transform of W(t) |
| $\Delta\theta$ | Distortion in the output signal |

| | |
|---|---|
| $\Delta\theta_M$ | Maximum value of $\Delta\theta$ with respect to time |
| $\overline{\Delta\theta^2}$ | Mean-square of $\Delta\theta$ for $0 \leq t \leq T$ |
| Q | Value of $\sqrt{\Delta\theta_M^2 + \overline{\dot{N}_o^2}}$ |
| $Q_M$ | Minimum value of Q with respect to filter parameters |
| $\overline{Q}$ | Value of $\sqrt{\overline{\Delta\theta^2} + \overline{\dot{N}_o^2}}$ |
| Z-R | Zadeh-Ragazzini |
| T* | Time constant of the Z-R Filter |

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