A METHOD FOR CALCULATING THE NATURAL FREQUENCIES OF CONTINUOUS BEAMS

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ABSTRACT

This report describes a method for calculating the undamped natural frequencies of flexural vibration of elastic beams which are continuous over non-deflecting supports. Numerical values of the quantities of "dynamic flexural stiffness" and of "the product of dynamic flexural stiffness and dynamic flexural carry-over factor" which are necessary in the analysis by this method are tabulated in the Appendix. The method is illustrated by a numerical example.

A METHOD FOR CALCULATING THE NATURAL FREQUENCIES OF CONTINUOUS BEAMS

INTRODUCTION

This report describes a method for calculating the undamped natural frequencies of flexural vibration of elastic beams which are continuous over non-deflecting supports. The method is strictly analogous to Holzer's method[1] of determining the natural frequencies of torsional vibration of shafts, and like Holzer's method it is reduced to a routine tabular scheme of computation which when repeated a sufficient number of times will give the natural frequencies of the system to any desired degree of accuracy. In the available methods of Myklestad[2], Prohl[3], Rankin[4], and Bellin[5], which are likewise similar to the basic Holzer method, the mass is assumed to be lumped at a number of discrete stations along the length of the beam and the portion of the beam between these stations is assumed to be massless. In the method presented herein, the mass is considered to be uniformly distributed between consecutive supports of the beam.

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1. "Die Berechnung der Drechschwingungen," by H. Holzer, J. Springer, Berlin, 1921.

2. "A New Method of Calculating Natural Modes of Uncoupled Bending Vibration of Airplane Wings and Other Types of Beams," by N.O. Myklestad, Journal of the Aeronautical Sciences, Vol. 11, April, 1944, pp. 153-162.

3. "A General Method for Calculating Critical Speeds of Flexible Rotors," by M.A. Prohl, Journal of Applied Mechanics, Trans. ASME, Vol. 67, 1945, pp. 142-148.

4. "Calculation of the Multiple-Span Critical Speeds of Flexible Shafts by Means of Punched-Card Machines," by A.W. Rankin, Journal of Applied Mechanics, Vol. 13, 1946, pp. 117-126.

5. "Determination of the Natural Frequencies of the Bending Vibrations of Beams," by A.I. Bellin, Journal of Applied Mechanics, Vol. 14, 1947, pp. 1-6.

CHARACTERISTICS OF THE BEAMS CONSIDERED

The beams are assumed to be straight but may have any number of spans of arbitrary length. At their extreme ends they may be hinged, fixed, or only partially fixed by means of rotational restraints which are assumed to be proportional to the end rotations. The cross section and the mass per unit of length of the beam may vary from one span to the other, but in any one span these quantities are assumed to remain constant. It is further considered that vibration is restricted to one of the principal planes of flexure of the beam, and that the cross sectional dimensions of each span are small in comparison to its length so that the effects of shearing deformation and rotatory inertia can be disregarded.

OUTLINE OF THE METHOD

An undamped continuous beam vibrating at a natural frequency should satisfy the following conditions: the beam should execute harmonic oscillations of constant amplitude without any exciting force or exciting couple acting anywhere along its length; the deflection configuration of the beam should be a continuous smooth curve; and the imposed conditions of restraint at the two boundaries should be satisfied identically.

As formulated above, the condition regarding the external excitation is more restrictive than is actually necessary. In reality, the beam may be acted upon by an exciting force applied at a point which does not deflect during vibration, or it may be subjected to an exciting couple applied at a section which does not rotate during vibration. Since in either case the generalized force acts through zero generalized displacement, no energy is imparted to the beam and consequently the natural frequencies of the beam and its modes of vibration remain unaffected.

For any arbitrarily chosen frequency it is, in general, possible to determine a mode of vibration which satisfies all but one of the conditions referred to previously. The assumed frequency will represent a natural frequency of the system investigated only if the remaining condition is also satisfied. As presented in this report the method consists of (a) assuming a frequency of vibration (b) determining a configuration which satisfies all of the aforementioned conditions with the possible exception of the condition of restraint at (for example) the right hand boundary of the beam, and (c) determining the magnitude of the discrepancy between the actual condition of restraint at the right end and the condition corresponding to the computed configuration. These steps are carried out for a number of assumed frequencies and the magnitude of the discrepancy is plotted against the frequency of vibration. The points at which the resulting curve crosses the line of zero discrepancy represent the natural frequencies of the system.

DEFINITION OF TERMS

The following terms refer to a beam which is simply supported at one end and clamped at the other and which is undergoing steady-state oscillations under the action of a harmonically varying bending moment applied at the simply supported end. The frequency of the oscillations is the same as the frequency of the exciting moment. The rotation at the end is either in phase or 180 degrees out of phase with the exciting moment.

The moment necessary to produce a steady-state forced rotation of unit amplitude at the end of the beam at which the moment is applied is defined as the "dynamic flexural stiffness".

The ratio of the periodic moment at the fixed end of the beam to the moment at the end at which the exciting moment is applied is defined as the "dynamic flexural carry-over factor".

The foregoing terms are generalizations of those originally introduced by Cross[6] for the analysis of frames subjected to static loads and they were first used by Gaskell[7] in applying the method of moment distribution to the determination of the steady-state response of continuous beams and frames subjected to pulsating loads.

In this paper discussion will be restricted to members having constant flexural rigidity of cross section and constant mass per unit of length; the stiffness and the carry-over factor for both ends of such members are equal. The following letter symbols are used: $\underline{K}$ for the stiffness and $\underline{k}$ for the carry-over factor.

NUMERICAL VALUES OF DYNAMIC STIFFNESS AND OF DYNAMIC CARRY-OVER FACTOR

The pertinent analytical expressions for dynamic stiffness and dynamic carry-over factor are to be found in reference 7. As was to be expected from purely physical considerations, these expressions involve not only the flexural rigidity of the cross section and the length of the member, as is the case with the static problem, but also the mass of the member and the frequency of vibration. The influence of these factors is expressible in terms of a single dimensionless parameter

$$\lambda = \sqrt[4]{\frac{m\omega^2}{EI}}\cdot L \qquad [1]$$

6. "Analysis of Continuous Frames by Distributing Fixed-End Moments", by Hardy Cross, Transactions A.S.C.E., Vol. 96, 1932, pp. 1-10.

7. "On Moment Balancing in Structural Dynamics", by R. E. Gaskell, Quarterly of Applied Mathematics, Vol. 1, 1943, pp. 237-249.

in which m = the mass per unit of length of the beam

ω = the circular frequency of vibration

E = the modulus of elasticity of the material in the beam

I = the moment of inertia of the cross section of the beam about its centroidal axis, and

L = the span length of the beam.

A graphical representation of the variation of the stiffness, the carry-over factor, and of the product of these quantities as a function of λ is given in Figures 1 through 3. For $\omega = 0$ ($\lambda = 0$) the three quantities have the well known static values of $K = 4\frac{EI}{L}$, $k = 0.5$, $Kk = 2\frac{EI}{L}$. Values of λ equal to 3.927, 7.069, and 10.210 correspond respectively to the first, the second, and the third natural frequencies of a hinged-clamped beam. At these frequencies no exciting moment is required to maintain the vibration, consequently the value for dynamic stiffness is equal to zero. Furthermore, since the moment at the clamped end of the beam has a finite magnitude, the carry-over factor for the member becomes infinite at these frequencies. Values of λ equal to 4.730 and 7.853 correspond respectively to the first and the second natural frequencies of a beam clamped at both ends. At these frequencies the end moments have a finite value while the corresponding rotations are equal to zero, accordingly the stiffness of the member has a infinite value.

In the method to be presented only the stiffness and the product of the stiffness and the carry-over factor are needed. A detailed tabulation of the coefficients of these quantities is given in Table I of the Appendix for values of λ ranging from zero to 10.22.

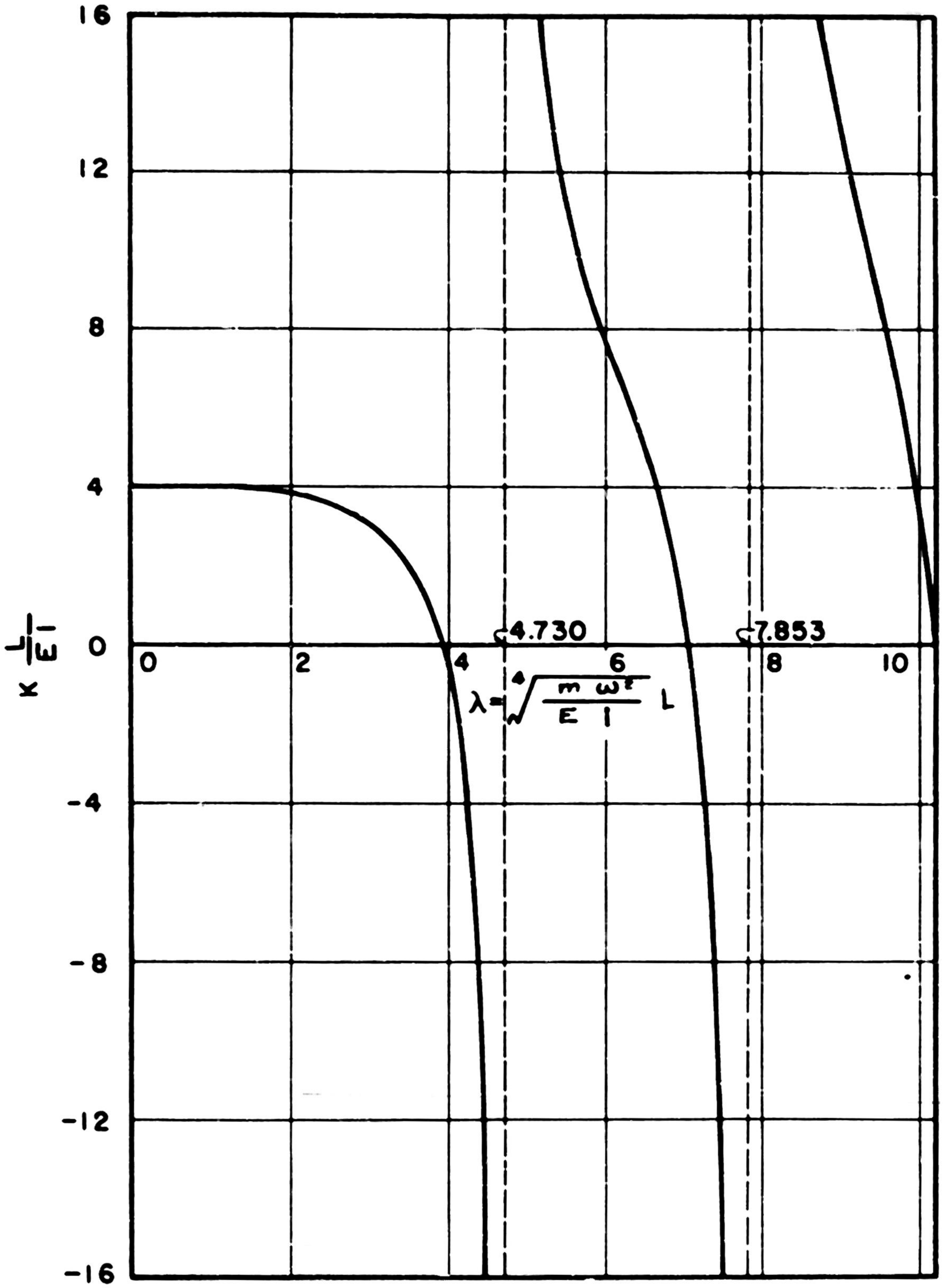

FIG. I COEFFICIENTS OF DYNAMIC FLEXURAL STIFFNESS FOR A BAR CLAMPED AT THE FAR END

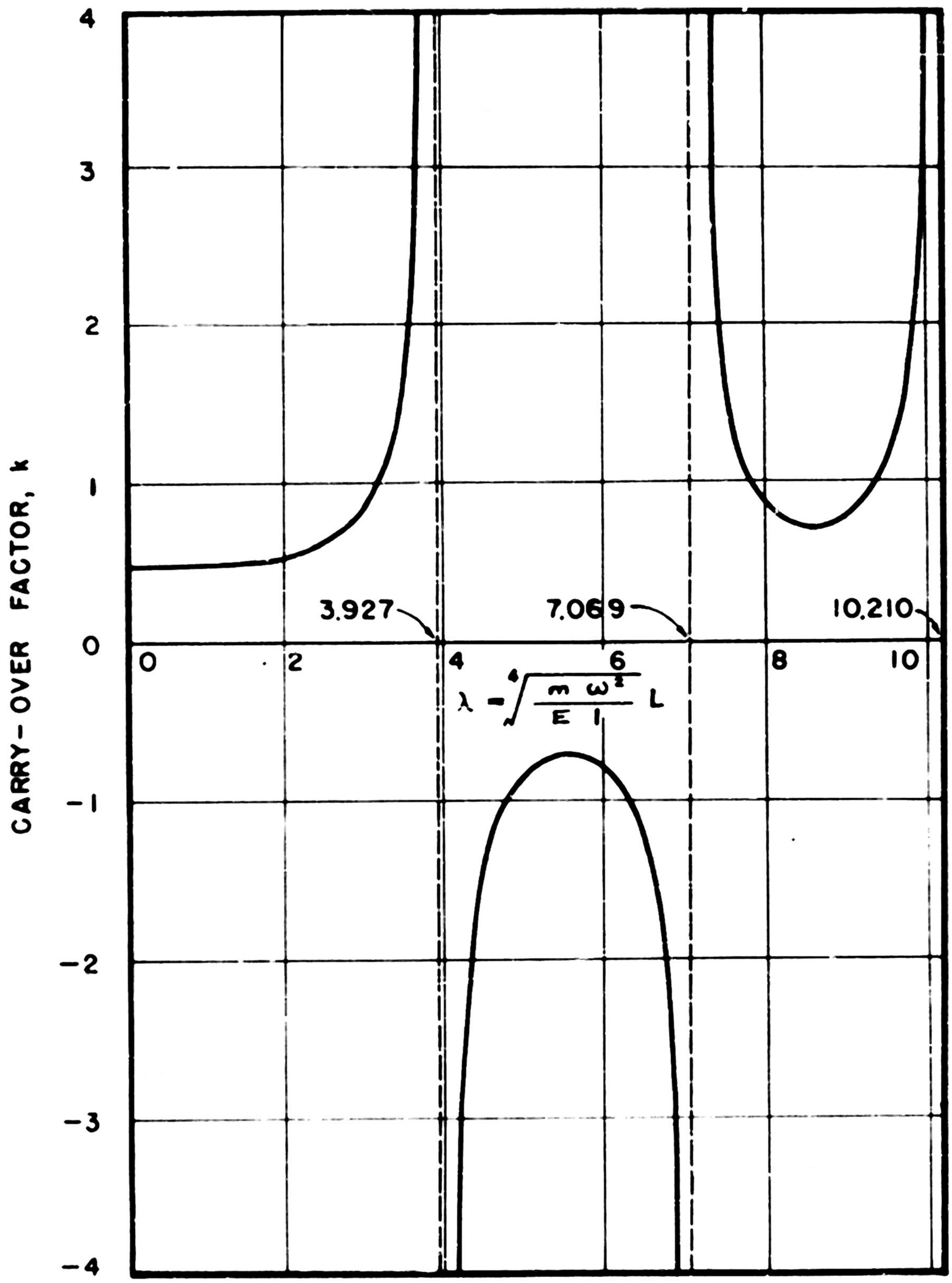

FIG. 2 DYNAMIC FLEXURAL CARRY-OVER FACTOR, k, FOR A BAR CLAMPED AT THE FAR END

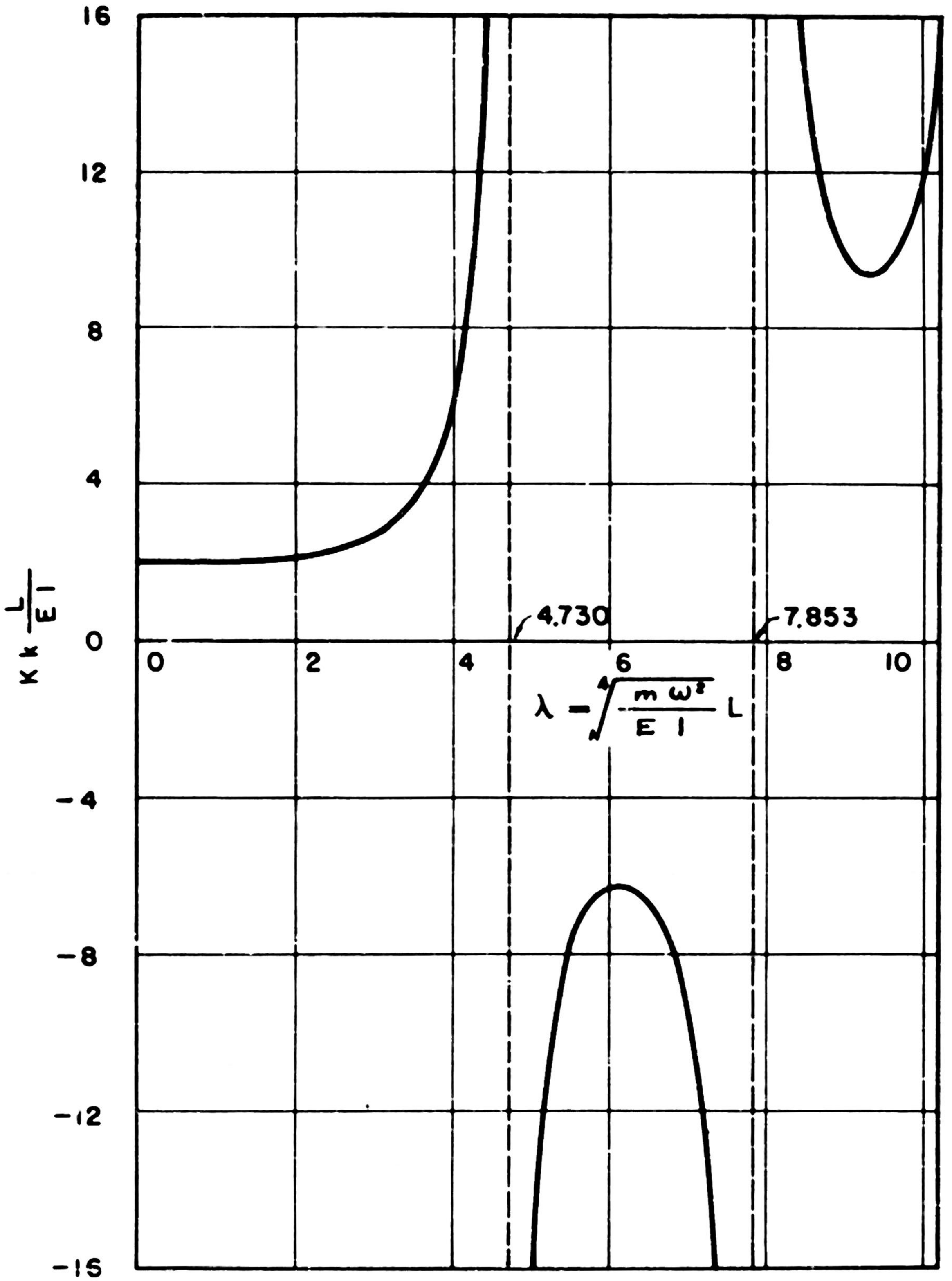

FIG. 3 COEFFICIENTS OF THE PRODUCT OF DYNAMIC FLEXURAL STIFFNESS AND DYNAMIC FLEXURAL CARRY - OVER FACTOR FOR A BAR CLAMPED AT THE FAR END

SIGN CONVENTION AND NOTATION

A clockwise rotation is taken as positive. A moment at the end of a member is taken as positive when it tends to rotate the member (not the joint) on which it acts in a clockwise direction.

The supports of the continuous beam are numbered successively from left to right starting with 1 at the extreme left hand end and terminating with n at the extreme right hand end.

The portion of the beam between the two consecutive supports j and j+1 is referred to as the j-th span. The quantities L_j, E_j, I_j, λ_j, K_j, and k_j refer to the j-th span.

θ_j denotes the amplitude of rotation of the deflected beam over the j-th support and M_j denotes the amplitude of bending moment across a section at the same support. The subscripts L and R designate respectively sections just to the left and just to the right of a support.

DEVELOPMENT OF THE BASIC EQUATIONS OF THE METHOD

Figure 4 shows spans j-1 and j of a continuous beam undergoing undamped harmonic oscillations. It is assumed that there is no exciting force or exciting couple acting on the beam.

Fig. 4

In the figure , the rotations and bending moments at the ends of each span are indicated in their positive directions. The slope and the bending moment at a time t for support j are

$$\theta_j(t) = \theta_j \cos\omega t \qquad M_j(t) = M_j \cos\omega t \qquad [2]$$

In the equations to be used the $\cos\omega t$ appears as a common factor; for convenience this will be omitted, and in the remainder of this discussion the terms "amplitude of slope" and "slope" and the terms "amplitude of moment" and "moment" will be used interchangeably.

To insure continuity and equilibrium of the beam over the interior support j it is required that

$$(\theta_j)_L = (\theta_j)_R = \theta_j \qquad [3]$$

and

$$(M_j)_L + (M_j)_R = 0 \qquad [4]$$

The moments $(M_j)_L$ and $(M_j)_R$ can now be expressed as functions of the rotations at the ends of the two spans as follows. We start by considering the j-th span. First, assume that the right end of the span is kept fixed while the left end is rotated through an angle θ_j; the moment at the end being rotated necessary to produce the rotation is equal to the product of the rotation θ_j and the stiffness of the member K_j. Next, imagine that the left end of the span is kept fixed while the right end is rotated through θ_{j+1}; the moment induced at the fixed left end is equal to the product of the rotation θ_{j+1} and the product of the stiffness and the carry-over factor of the member, $K_j k_j$. Because the principle of superposition holds true, the moment $(M_j)_R$ corresponding to the rotations θ_j and θ_{j+1} is the sum of the partial moments determined above.

$$(M_j)_R = K_j\theta_j + K_j k_j \theta_{j+1} \qquad [5a]$$

Considering span j-1, we obtain in a similar manner:

$$(M_j)_L = K_{j-1}\theta_j + K_{j-1}k_{j-1}\theta_{j-1} \qquad [5b]$$

Substituting Equations [5a] and [5b] in Equation [4] and solving for θ_{j+1} we obtain the following equation relating the slopes over three consecutive supports of a continuous beam:

$$\theta_{j+1} = - \frac{(K_{j-1} + K_j)\ \theta_j + K_{j-1}k_{j-1}\ \theta_{j-1}}{K_j k_j} \qquad [6a]$$

Equation [6a] is applicable only to interior supports; for the end supports the appropriate relations are given below.

It is assumed that the extreme ends of the beam are elastically restrained against rotation. The relationships between the moments and the rotations at these ends are

$$M_1 = -K_L\theta_1 \qquad [7]$$

$$M_n = -K_R\theta_n \qquad [8]$$

where, K_L and K_R are the known stiffnesses of the restraints at the left and the right ends, respectively. For a hinged end K = 0 and for a clamped end K = infinity. The negative signs in these expressions follow from the sign convention used and indicate that for a positive restraint, the moment exerted on the beam by the restraint acts in a direction opposite to the direction of rotation of the beam.

The moments M_1 and M_n can also be expressed by the following equations obtained respectively from Equations [5a] and [5b].

$$M_1 = K_1\theta_1 + K_1k_1\theta_2 \qquad [5a']$$

$$M_n = K_{n-1}\theta_n + K_{n-1}k_{n-1}\theta_{n-1} \qquad [5b']$$

Eliminating M_1 between [5a'] and [7] and M_n between [5b'] and [8], we obtain

$$(K_L + K_1)\theta_1 + K_1k_1\theta_2 = 0 \qquad [9a]$$

$$(K_R + K_{n-1})\theta_n + K_{n-1}k_{n-1}\theta_{n-1} = 0 \qquad [10a]$$

At a natural frequency both of these equations should be satisfied identically.

Equations [9a] and [10a] apply only to hinged and to partially fixed ends. For the special case of clamped ends, the relations to be used are

$$M_1 = K_1 k_1 \theta_2 \qquad [9b]$$

$$\theta_n = 0 \qquad [10b]$$

DETAILS OF THE PROCEDURE

The procedure for arriving at the natural frequencies of a continuous beam may be outlined now as follows.

1. A fixed value is assigned to the amplitude of slope or bending moment at the first support of the beam. Since the natural frequencies of a system depend only on the relative values of the deflection, any arbitrary amplitude consistent with the actual boundary conditions may be chosen. For a hinged or for a partially fixed end, θ_1 is taken, for convenience, equal to unity; for a clamped end, θ_1 being zero, M_1 is taken equal to unity instead.
2. A trial frequency of vibration, ω, is chosen and the values for each span are evaluated. These calculations are carried out conveniently in a tabular form as illustrated in the next section.
3. With the λ values available, the stiffness and the product of the stiffness and the carry-over factor for each span of the beam are found from Table 1 in the Appendix.
4. The rotation of the beam over the second support is determined from Equation [9a] or [9b] .
5. By successive applications of Equation [6] the rotations θ_3 to θ_n are evaluated. A convenient tabular

scheme for arranging the computations is described in the next section.

6. If support n is clamped, the determination of the rotation θ_n completes one cycle of the procedure (see Equation [10b]). However, if the support is hinged or is only partially fixed, it is necessary to carry out the additional step of evaluating the left hand side of Equation [10a].

7. Steps 1 through 6 are repeated for different assumed frequencies, and the calculated values for the left hand side of Equations [10b] or [10a] are plotted as a function of the assumed frequencies, or what is usually more convenient, as a function of the corresponding λ value for some one span. The zero intercepts of the resulting curve, the general shape of which resembles that shown in Figure 6, correspond to the natural frequencies of the system.

Since for each trial frequency, the rotations of the deflected beam over the supports are evaluated in this procedure, the deflection configuration of the beam for any desired frequency can ordinarily be sketched. If it is desired to evaluate the modes of vibration more precisely, it will be necessary to use appropriate influence coefficients relating the deflection of each span to the rotations at the ends of the span. Such coefficients are now being evaluated and it is expected that they will be made available in the near future.

ILLUSTRATIVE EXAMPLE

The details of the procedure and a convenient tabular scheme for arranging the computations are illustrated by considering the problem of determining the first five natural frequencies of a four-span continuous beam which is elastically restrained against rotation at one end and simply supported at the other, as shown in Figure 5. The stiffness of the end restraint and the characteristics of the various spans are shown on the figure.

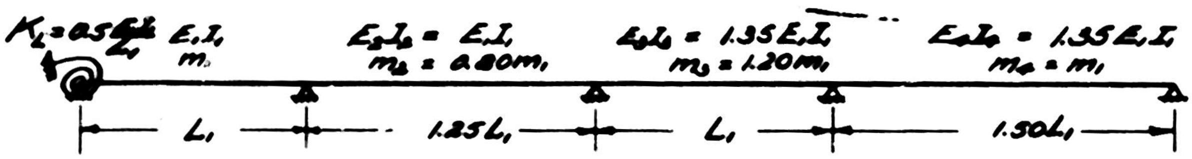

Fig. 5 Characteristics of the Beam

For convenience in carrying out the calculations the natural frequencies of the system are expressed in terms of the pertinent properties of some one span, say span g. In this particular example we take g = 1. In terms of the λ value of the g-th span, the λ value for any span j is

$$\lambda_j = \sqrt[4]{\frac{m_j}{m_g} \times \frac{E_g\, I_g}{E_j\, I_j}} \times \frac{L_j}{L_g} \times \lambda_g \qquad [11]$$

In terms of the EI/L of the g-th span, the stiffness and the product of the stiffness and of the carry-over factor for any span j are equal to the values obtained from Table I multiplied by the dimensionless factor

$$c_j = \frac{E_j\, I_j}{E_g\, I_g} \times \frac{L_g}{L_j} \qquad [12]$$

Equations [11] and [12] can be verified readily.

The quantities λ_j/λ_g and c_j are evaluated in Table A. It should be noted that the calculations in this table are independent of the frequency of vibration.

TABLE A

Span	(1)	(2)	(3)	(4)	(5)	(6)
	$\frac{m_i}{m_g}$	$\frac{E_i I_i}{E_g I_g}$	$\sqrt[4]{\frac{(1)}{(2)}}$	$\frac{L_i}{L_g}$	$\frac{\lambda_i}{\lambda_g} = (3)(4)$	$c_j = \frac{(2)}{(4)}$
1=g	1.00	1.00	1.00	1.00	1.00	1.00
2	0.80	1.00	0.9457	1.25	1.182	0.80
3	1.20	1.35	0.9710	1.00	0.9710	1.35
4	1.00	1.35	0.9277	1.50	1.392	0.90

TABLE B

Span or Support	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	$\frac{\lambda_j}{\lambda_g}$	λ_j	c_j	$K_j \frac{L_j}{E_j I_j}$	$(K_{j-1} + K_j)\frac{L_g}{E_g I_g}$	θ_j	$K_j k_j \frac{L_g}{E_g I_g}$	$K_j k_j \frac{L_j}{E_j I_j}$
				from Table I	$(3)_{j-1}(4)_{j-1} + (3)_j(4)_j$	Eq'n [6]	$(3)_j (8)_j$	from Table I
1=g	1.00	2.40	1.00	3.6649		1.0000	2.2555	2.2555
2	1.182	2.84	0.80	3.3015	6.3061	-1.8466	2.0330	2.5412
3	0.9710	2.33	1.35	3.7043	7.6420	4.6185	3.0038	2.2250
4	1.392	3.34	0.90	2.4810	7.2337	-10.500	2.8924	3.2138
5						21.463		

$$\text{Eq'n } [10a] = \left[(0 + 2.4810 \times 0.90)\, 21.463 + 2.8924\, (-10.500)\right] \frac{E_1 I_1}{L_1}$$

$$= 17.55 \frac{E_1 I_1}{L_1}$$

The trial-and-error procedure for determining the natural frequencies of the system is carried out in Table B. As an example of the use of this table a complete cycle of calculations is carried out for a trial value of $\lambda_g = \lambda_1 = 2.40$. This value, shown encircled in the g-th line of Column (2), corresponds to a circular frequency of vibration $\omega = \frac{(2.40)^2}{L_1^2}\sqrt{\frac{E_1 I_1}{m_1}}$. The arrangement of the various quantities in this table is believed to facilitate the computational work and to reduce substantially the probability for errors. The order in which the columns in this table are filled in is indicated by the following sequence of column numbers: (1), (3), (2), (4 and 8), (5), (7), and (6). Columns (1) and (3) are reproduced respectively from Columns (5) and (6) of Table A. The λ values for the various spans in Column (2) are obtained by multiplying the assumed λ_g by the values in Column (1). Columns (4) and (8) give respectively values of the stiffness and of the product of the stiffness and the carry-over factor for each span of the beam, in terms of $E_j I_j/L_j$; these quantities are obtained directly from Table 1 using the λ values computed in Column (2). Column (5) gives the total stiffness of the spans adjoining each support in terms of $E_g I_g/L_g$. The value for the j-th line in this column is determined by taking the sum of the products of the values in Columns (3) and (4) for lines j-1 and j. Column (7) gives the product of the stiffness and of the carry-over factor for each span of the beam, in terms of $E_g I_g/L_g$; the entries in this column are obtained by multiplying the entries in Column (8) by those in Column (3). Column (6) gives the rotation of the beam over the supports. The first value in this column is unity. (If the left support were clamped, this value would have been zero instead). The second value in the column, θ_2, is evaluated from Equation [9a].

$$\theta_2 = -\frac{(0.5000 + 3.6649)\ 1.0000}{2.2555} = -1.8466$$

This operation is not indicated in the Table. (If support 1 were clamped, Equation [9b] would have been used instead.) The values of θ_3 to θ_n are determined from the values in Columns (5) and (7) using Equation [6a] which in terms of column numbers takes the form:

$$\theta_{j+1} = - \frac{(5)_j\,(6)_j + (6)_{j-1}\,(7)_{j-1}}{(7)_j} \quad (\text{for } j \geq 2) \qquad [6b]$$

Thus,

$$\theta_3 = - \frac{6.3061\,(-1.8466) + 1.0000\,(2.2555)}{2.0330} = 4.6185$$

The left hand side of Equation [10a] is evaluated at the bottom of the table, and it is found to be equal to $17.55\, E_1 I_1/L_1$.

Since for the assumed values of $\lambda_1 = 2.40$, Equation [10a] was not satisfied identically, this value does not correspond to a natural frequency of the system. The physical significance of the computed value of $17.55 \frac{E_1 I_1}{L_1}$ is as follows: the negative of this value divided by the rotation θ_2

$$- \frac{17.55}{21.463} \frac{E_1 I_1}{L_1} = -0.8179 \frac{E_1 I_1}{L_1}$$

represents the stiffness of a rotational constraint which if it were imposed at the right end of the beam would have made the assumed frequency of vibration correspond to a natural frequency of the system.

By repeating similar cycles of computation for different values of λ_1 the curve in Figure 6 was obtained. The first five critical values of λ_1 as read off this curve are

$$(\lambda_1)_1 = 2.504$$

$$(\lambda_1)_2 = 3.07$$

$$(\lambda_1)_3 = 3.70$$

$$(\lambda_1)_4 = 4.11$$

$$(\lambda_1)_5 = 4.90$$

The corresponding natural frequencies, in radians per second, are

$$\omega_1 = \frac{6.27}{L_1^2} \sqrt{\frac{E_1 I_1}{m_1}}$$

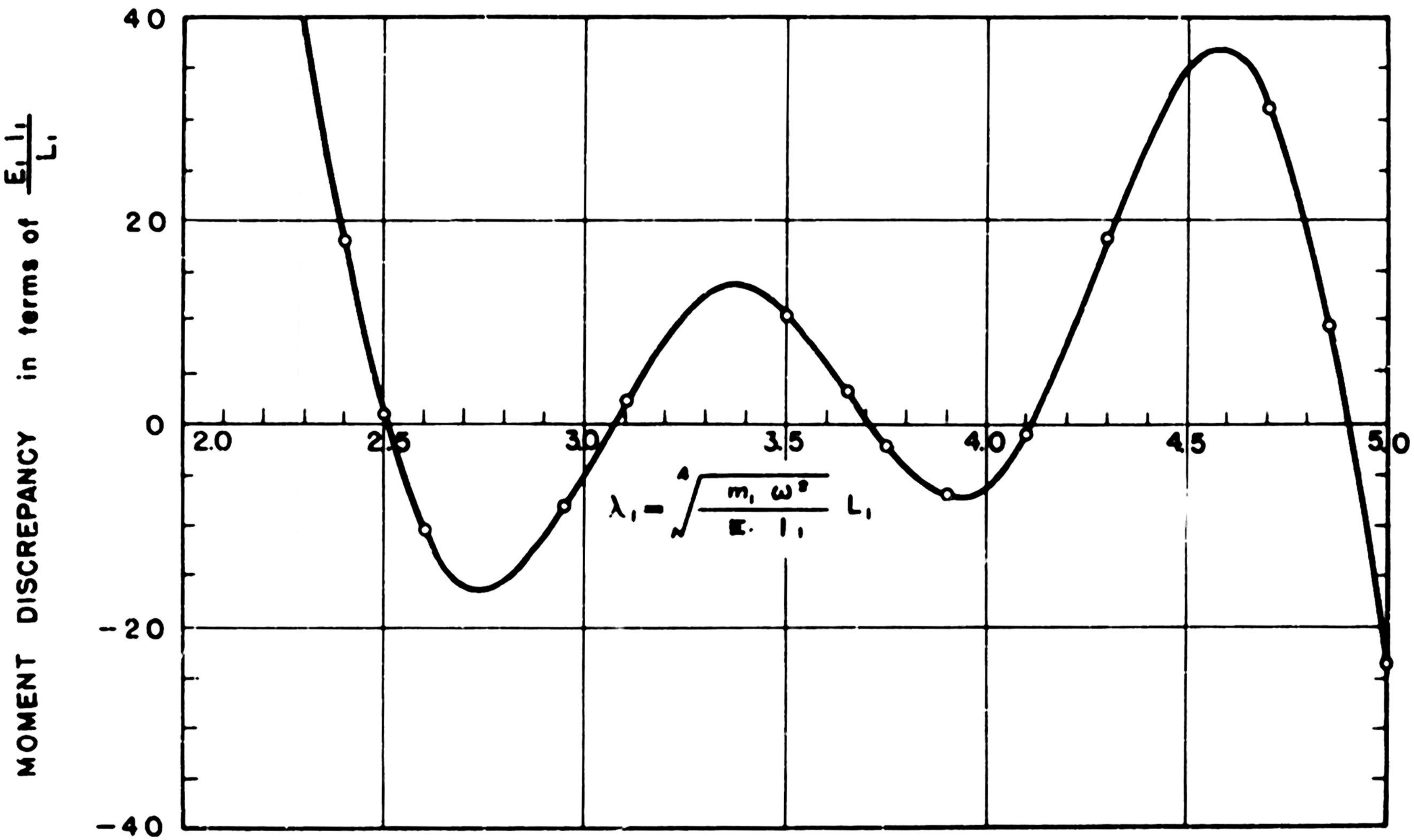

FIG. 6 DISCREPANCY BETWEEN COMPUTED AND ACTUAL BOUNDARY MOMENTS FOR THE RIGHT END OF THE BEAM AS A FUNCTION OF λ_1

$$\omega_2 = \frac{9.42}{L_1^2}\sqrt{\frac{E_1 I_1}{m_1}}$$

$$\omega_3 = \frac{13.7}{L_1^2}\sqrt{\frac{E_1 I_1}{m_1}}$$

$$\omega_4 = \frac{16.9}{L_1^2}\sqrt{\frac{E_1 I_1}{m_1}}$$

$$\omega_5 = \frac{24.0}{L_1^2}\sqrt{\frac{E_1 I_1}{m_1}}$$

If it is desired to evaluate these quantities more precisely, the computations should be repeated for several additional values of λ_1 in the neighborhood of the critical values and the results should be plotted on a larger scale.

The natural modes of the beam can be sketched from the rotations over the supports as determined in Column (6) of Table B for values of λ_1 corresponding to each of the natural frequencies. It should be stated that, in general, for the fundamental or lowest natural frequency the rotations of the beam over the supports are not very sensitive to the magnitude of the frequency of vibration. For some of the higher frequencies of vibration, however, a slight variation in the value of the frequency may affect the rotations materially. Accordingly the accurate evaluation of the rotations in these latter cases may become somewhat cumbersome.

CONCLUDING REMARKS

When properly extended, the method presented in this report can also be used to calculate the natural frequencies of beams having variable cross section or variable mass per unit of length. The natural frequencies of beams which are acted upon by fixed axial forces can be evaluated in a similar manner. To accomplish this, it is necessary however to tabulate values of the pertinent quantities of stiffness and of the product of stiffness and carry-over factor.

As presented herein, the method employs the three-slope equation for dynamic loading (Equation [6]). Actually any of the other basic equations of indeterminate stress analysis, if extended to account for the inertia effects, could have been used instead. In particular, one could have used the dynamic three-moment equation originally developed by W. Prager[8]. Values of the coefficients of this equation for different frequencies of vibration are available in tabular form in references[9] and [10]. It is probably true that the choice between any one of these two alternate methods of analysis is largely a question of personal preference and of one's familiarity with the particular method. In this paper the three-slope equation was adopted principaly because it is believed that, for the particular problem treated, it has the following definite advantages:

(a) As developed in this paper, the three-slope equation deals with concepts and quantities which are widely known among structural engineers.

(b) Because it involves rotations, the three-slope equation gives the clearest possible picture of the distortions which the structure undergoes during vibration. This feature is particularly important because in practice it is frequently desirable to have a rapid means of sketching the configuration of vibration corresponding to a given frequency.

(c) Probably the greatest advantage of the three-slope equation lies in the fact that, when properly extended, it is remarkably better suited than the three-moment equation for the analysis of continuous frames. The

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8. "Die Beanspruchung von Tragwerken durch schwingende Lasten", by W. Prager, Ingenieur-Archiv, Vol. I, 1930, pp. 527-532.

9. "Beanspruchung und Formanderung von Stabwerken bei erzwungenen Schwingungen", by S. Gradstein and W. Prager, Ingenieur-Archiv, 1932, Vol. II, pp. 622-650.

10. "Dynamic der Stabwerke", by K. Hohenemser and W. Prager, Julius Springer, Berlin, 1933.

application of the present method to the determination of the natural frequencies of continuous frames which are free from sidesway will be described in a forthcoming paper.

In general, for the determination of the lower natural frequencies of continuous beams slide-rule accuracy will prove satisfactory. However, for the higher natural frequencies the computations may involve small differences between large quantities and it may become necessary to retain a larger number of significant figures.

ACKNOWLEDGMENTS

The method described herein was developed as part of a dissertation by A. S. Veletsos submitted to the Graduate College of the University of Illinois in partial fulfilment of the requirements for the degree of Doctor of Philosophy in Engineering. The dissertation was prepared under the direction of Professor N. M. Newmark in the Department of Civil Engineering. The details of this investigation were carried out as part of a research program on "Numerical and Approximate Methods of Stress Analysis" sponsored by the Office of Naval Research (Mechanics Branch) in the Structural Research Laboratory, Department of Civil Engineering, of the University of Illinois. The numerical values of the elastic constants reported in Table I of the Appendix were calculated using the Electronic Digital Computer of the University of Illinois. The governing expressions for these constants were coded for machine solution by Mr. A. J. Carlson, Research Associate in Civil Engineering, whose valuable contribution to the completion of this work the writers acknowledge gratefully.

TABLE 1

DIMENSIONLESS COEFFICIENTS OF "DYNAMIC FLEXURAL STIFFNESS" AND OF "THE PRODUCT OF DYNAMIC FLEXURAL STIFFNESS AND DYNAMIC FLEXURAL CARRY-OVER FACTOR" FOR A BAR FIXED AT THE FAR END

The coefficients are given as a function of the dimensionless parameter

$$\lambda = \sqrt[4]{\frac{m\omega^2}{EI}} \cdot L$$

in which m is the mass per unit of length of the bar; ω is the circular frequency of vibration; E is the modulus of elasticity of the material in the bar; I is the moment of inertia of the cross section of the bar about its centroidal axis; and L is the span length of the bar. The m and EI are assumed to be constant along the length of the member.

λ	$K \frac{L}{EI}$	$Kk \frac{L}{EI}$
0	4.000000	2.000000
0.10	3.999999	2.000001
0.20	3.999985	2.000011
0.30	3.999923	2.000058
0.40	3.999756	2.000183
0.50	3.999405	2.000447
0.55	3.999128	2.000654
0.60	3.998766	2.000926
0.65	3.998299	2.001276
0.70	3.997712	2.001716
0.75	3.996985	2.002262
0.76	3.996821	2.002385
0.77	3.996650	2.002513
0.78	3.996473	2.002646
0.79	3.996288	2.002785
0.80	3.996096	2.002928
0.81	3.995897	2.003078
0.82	3.995691	2.003233
0.83	3.995476	2.003393
0.84	3.995254	2.003560
0.85	3.995024	2.003733
0.86	3.994785	2.003912
0.87	3.994538	2.004097
0.88	3.994283	2.004289
0.89	3.994018	2.004488
0.90	3.993744	2.004693
0.91	3.993461	2.004906
0.92	3.993169	2.005125
0.93	3.992867	2.005352
0.94	3.992554	2.005586
0.95	3.992232	2.005828
0.96	3.991899	2.006078
0.97	3.991556	2.006336
0.98	3.991202	2.006602
0.99	3.990836	2.006876
1.00	3.990460	2.007159
1.01	3.990072	2.007450
1.02	3.989672	2.007750
1.03	3.989260	2.008059
1.04	3.988836	2.008378
1.05	3.988400	2.008705
1.06	3.987950	2.009043
1.07	3.987488	2.009390
1.08	3.987013	2.009747
1.09	3.986524	2.010114
1.10	3.986021	2.010492
1.11	3.985505	2.010880
1.12	3.984974	2.011278
1.13	3.984428	2.011688
1.14	3.983868	2.012109
1.15	3.983293	2.012541
1.16	3.982702	2.012985
1.17	3.982096	2.013440
1.18	3.981474	2.013908
1.19	3.980836	2.014387
1.20	3.980181	2.014879
1.21	3.979510	2.015384
1.22	3.978821	2.015901
1.23	3.978116	2.016432
1.24	3.977392	2.016975

TABLE 1 (cont'd)

λ	$K \frac{L}{EI}$	$Kk \frac{L}{EI}$
1.25	3.976651	2.017533
1.26	3.975892	2.018104
1.27	3.975114	2.018689
1.28	3.974317	2.019288
1.29	3.973501	2.019901
1.30	3.972666	2.020530
1.31	3.971811	2.021173
1.32	3.970935	2.021831
1.33	3.970040	2.022505
1.34	3.969123	2.023194
1.35	3.968186	2.023899
1.36	3.967227	2.024621
1.37	3.966247	2.025359
1.38	3.965244	2.026113
1.39	3.964219	2.026885
1.40	3.963172	2.027674
1.41	3.962101	2.028480
1.42	3.961006	2.029304
1.43	3.959888	2.030146
1.44	3.958746	2.031006
1.45	3.957579	2.031885
1.46	3.956388	2.032782
1.47	3.955171	2.033699
1.48	3.953928	2.034635
1.49	3.952660	2.035591
1.50	3.951365	2.036567
1.51	3.950043	2.037564
1.52	3.948694	2.038581
1.53	3.947318	2.039618
1.54	3.945913	2.040677
1.55	3.944480	2.041758
1.56	3.943019	2.042860
1.57	3.941528	2.043985
1.58	3.940008	2.045132
1.59	3.938458	2.046302
1.60	3.936877	2.047495
1.61	3.935266	2.048711
1.62	3.933623	2.049952
1.63	3.931949	2.051216
1.64	3.930242	2.052505
1.65	3.928503	2.053819
1.66	3.926731	2.055158
1.67	3.924925	2.056522
1.68	3.923086	2.057912
1.69	3.921212	2.059329
1.70	3.919303	2.060772
1.71	3.917359	2.062242
1.72	3.915379	2.063740
1.73	3.913363	2.065265
1.74	3.911310	2.066818
1.75	3.909220	2.068400
1.76	3.907092	2.070011
1.77	3.904926	2.071651
1.78	3.902721	2.073321
1.79	3.900477	2.075021
1.80	3.898193	2.076751
1.81	3.895869	2.078512
1.82	3.893504	2.080305
1.83	3.891098	2.082129
1.84	3.888649	2.083986
1.85	3.886159	2.085875
1.86	3.883625	2.087797
1.87	3.881048	2.089753
1.88	3.878427	2.091743
1.89	3.875761	2.093767
1.90	3.873050	2.095826
1.91	3.870293	2.097920
1.92	3.867490	2.100050
1.93	3.864640	2.102217
1.94	3.861742	2.104420
1.95	3.858796	2.106661
1.96	3.855801	2.108939
1.97	3.852757	2.111256
1.98	3.849662	2.113611
1.99	3.846517	2.116006
2.00	3.843321	2.118441
2.01	3.840072	2.120916
2.02	3.836771	2.123431
2.03	3.833417	2.125989
2.04	3.830008	2.128588

TABLE 1 (cont'd)

λ	$K \frac{L}{EI}$	$Kk \frac{L}{EI}$	λ	$K \frac{L}{EI}$	$Kk \frac{L}{EI}$
2.05	3.826545	2.131230	2.45	3.634135	2.279348
2.06	3.823026	2.133915	2.46	3.627716	2.284333
2.07	3.819452	2.136644	2.47	3.621204	2.289394
2.08	3.815820	2.139417	2.48	3.614597	2.294531
2.09	3.812132	2.142235	2.49	3.607895	2.299745
2.10	3.808384	2.145098	2.50	3.601095	2.305038
2.11	3.804578	2.148008	2.51	3.594198	2.310411
2.12	3.800713	2.150964	2.52	3.587200	2.315864
2.13	3.796786	2.153968	2.53	3.580102	2.321400
2.14	3.792799	2.157019	2.54	3.572901	2.327018
2.15	3.788749	2.160119	2.55	3.565596	2.332722
2.16	3.784637	2.163269	2.56	3.558187	2.338510
2.17	3.780461	2.166468	2.57	3.550671	2.344386
2.18	3.776221	2.169718	2.58	3.543046	2.350350
2.19	3.771915	2.173019	2.59	3.535313	2.356504
2.20	3.767544	2.176373	2.60	3.527468	2.362548
2.21	3.763106	2.179779	2.61	3.519511	2.368785
2.22	3.758599	2.183238	2.62	3.511440	2.375115
2.23	3.754025	2.186751	2.63	3.503253	2.381540
2.24	3.749381	2.190320	2.64	3.494949	2.388061
2.25	3.744666	2.193943	2.65	3.486526	2.394681
2.26	3.739881	2.197624	2.66	3.477983	2.401399
2.27	3.735023	2.201361	2.67	3.469317	2.408219
2.28	3.730092	2.205156	2.68	3.460527	2.415141
2.29	3.725087	2.209010	2.69	3.451612	2.422167
2.30	3.720008	2.212923	2.70	3.442569	2.429298
2.31	3.714852	2.216897	2.71	3.433397	2.436537
2.32	3.709620	2.220932	2.72	3.424094	2.443885
2.33	3.704309	2.225028	2.73	3.414658	2.451343
2.34	3.698920	2.229188	2.74	3.405087	2.458913
2.35	3.693451	2.233411	2.75	3.395378	2.466598
2.36	3.687901	2.237699	2.76	3.385532	2.474398
2.37	3.682270	2.242052	2.77	3.375544	2.482316
2.38	3.676555	2.246472	2.78	3.365413	2.490354
2.39	3.670756	2.250959	2.79	3.355137	2.498513
2.40	3.664872	2.255514	2.80	3.344714	2.506796
2.41	3.658902	2.260138	2.81	3.334141	2.515204
2.42	3.652844	2.264833	2.82	3.323417	2.523740
2.43	3.646698	2.269599	2.83	3.312538	2.532405
2.44	3.640462	2.274437	2.84	3.301504	2.541202

TABLE 1 (cont'd)

λ	$K \frac{L}{EI}$	$Kk \frac{L}{EI}$
2.85	3.290311	2.550133
2.86	3.278957	2.559200
2.87	3.267439	2.568405
2.88	3.255755	2.577751
2.89	3.243903	2.587240
2.90	3.231880	2.596875
2.91	3.219682	2.606657
2.92	3.207309	2.616590
2.93	3.194755	2.626675
2.94	3.182020	2.636917
2.95	3.169099	2.647316
2.96	3.155990	2.657877
2.97	3.142690	2.668602
2.98	3.129196	2.679493
2.99	3.115505	2.690554
3.00	3.101613	2.701788
3.01	3.087517	2.713198
3.02	3.073214	2.724786
3.03	3.058700	2.736557
3.04	3.043971	2.748513
3.05	3.029025	2.760658
3.06	3.013857	2.772996
3.07	2.998464	2.785529
3.08	2.982841	2.798262
3.09	2.966986	2.811199
3.10	2.950893	2.824342
3.11	2.934558	2.837697
3.12	2.917978	2.851266
3.13	2.901148	2.865055
3.14	2.884064	2.879067
3.15	2.866720	2.893306
3.16	2.849113	2.907778
3.17	2.831237	2.922486
3.18	2.813088	2.937436
3.19	2.794660	2.952632
3.20	2.775949	2.968079
3.21	2.756949	2.983782
3.22	2.737655	2.999746
3.23	2.718060	3.015977
3.24	2.698160	3.032479
3.25	2.677948	3.049260
3.26	2.657419	3.066324
3.27	2.636566	3.083677
3.28	2.615383	3.101326
3.29	2.593864	3.119276
3.30	2.572001	3.137534
3.31	2.549788	3.156107
3.32	2.527218	3.175002
3.33	2.504283	3.194225
3.34	2.480976	3.213784
3.35	2.457290	3.233686
3.36	2.433216	3.253938
3.37	2.408746	3.274550
3.38	2.383872	3.295528
3.39	2.358586	3.316881
3.40	2.332878	3.338617
3.41	2.306739	3.360746
3.42	2.280160	3.383277
3.43	2.253131	3.406218
3.44	2.225642	3.429581
3.45	2.197683	3.453374
3.46	2.169243	3.477607
3.47	2.140312	3.502293
3.48	2.110878	3.527441
3.49	2.080929	3.553063
3.50	2.050454	3.579170
3.51	2.019439	3.605776
3.52	1.987873	3.632891
3.53	1.955741	3.660530
3.54	1.923031	3.688705
3.55	1.889728	3.717430
3.56	1.855817	3.746720
3.57	1.821284	3.776590
3.58	1.786113	3.807054
3.59	1.750287	3.838128
3.60	1.713790	3.869830
3.61	1.676604	3.902175
3.62	1.638712	3.935182
3.63	1.600096	3.968868
3.64	1.560734	4.003253

TABLE 1 (cont'd)

λ	$K \frac{L}{EI}$	$Kk \frac{L}{EI}$
3.65	1.520609	4.038357
3.66	1.479699	4.074199
3.67	1.437982	4.110800
3.68	1.395437	4.148183
3.69	1.352040	4.186371
3.70	1.307767	4.225387
3.71	1.262593	4.265255
3.72	1.216492	4.306002
3.73	1.169437	4.347654
3.74	1.121401	4.390238
3.75	1.072354	4.433783
3.76	1.022265	4.478320
3.77	0.9711032	4.523880
3.78	0.9188353	4.570494
3.79	0.8654270	4.618198
3.80	0.8108425	4.667027
3.81	0.7550443	4.717017
3.82	0.6979935	4.768208
3.83	0.6396492	4.820639
3.84	0.5799690	4.874353
3.85	0.5189083	4.929393
3.86	0.4564206	4.985807
3.87	0.3924572	5.043642
3.88	0.3269673	5.102948
3.89	0.2598972	5.163779
3.90	0.1911912	5.226191
3.91	0.1207906	5.290240
3.92	0.04863390	5.355990
3.93	-0.02534362	5.423503
3.94	-0.1012098	5.492847
3.95	-0.1790359	5.564093
3.96	-0.2588970	5.637315
3.97	-0.3408721	5.712593
3.98	-0.4250443	5.790008
3.99	-0.5115012	5.869648
4.00	-0.6003354	5.951605
4.01	-0.6916444	6.035976
4.02	-0.7855511	6.122864
4.03	-0.8821045	6.212376
4.04	-0.9814799	6.304628

λ	$K \frac{L}{EI}$	$Kk \frac{L}{EI}$
4.05	-1.083779	6.399742
4.06	-1.189131	6.497845
4.07	-1.297674	6.599074
4.08	-1.409552	6.703575
4.09	-1.524921	6.811501
4.10	-1.643944	6.923017
4.11	-1.766796	7.038296
4.12	-1.893664	7.157524
4.13	-2.024747	7.280900
4.14	-2.160257	7.408636
4.15	-2.300420	7.540957
4.16	-2.445480	7.678106
4.17	-2.595696	7.820343
4.18	-2.751348	7.967945
4.19	-2.912735	8.121212
4.20	-3.080179	8.280466
4.21	-3.254028	8.446053
4.22	-3.434655	8.618346
4.23	-3.622464	8.797749
4.24	-3.817893	8.984698
4.25	-4.021415	9.179666
4.26	-4.233543	9.383166
4.27	-4.454836	9.595756
4.28	-4.685901	9.818043
4.29	-4.927402	10.05069
4.30	-5.180064	10.29442
4.31	-5.444680	10.55003
4.32	-5.722121	10.81838
4.33	-6.013346	11.10044
4.34	-6.319410	11.39726
4.35	-6.641483	11.71000
4.36	-6.980856	12.03997
4.37	-7.338968	12.38859
4.38	-7.717421	12.75748
4.39	-8.118004	13.14841
4.40	-8.542724	13.56539
4.41	-8.993839	14.00468
4.42	-9.473899	14.47484
4.43	-9.985791	14.97674
4.44	-10.53280	15.51367

TABLE 1 (cont'd)

λ	$K \frac{L}{EI}$	$Kk \frac{L}{EI}$
4.45	-11.11869	16.08939
4.46	-11.74777	16.60822
4.47	-12.42501	17.37512
4.48	-13.15619	18.09587
4.49	-13.94805	18.87720
4.50	-14.80848	19.72701
4.51	-15.74682	20.65466
4.52	-16.77421	21.67124
4.53	-17.90398	22.79012
4.54	-19.15229	24.02745
4.55	-20.53888	25.40295
4.56	-22.08813	26.94103
4.57	-23.83053	28.67215
4.58	-25.80468	30.63493
4.59	-28.06021	32.87899
4.60	-30.66203	35.46924
4.61	-33.69666	38.49220
4.62	-37.28208	42.06585
4.63	-41.58344	46.35533
4.64	-46.83925	51.59916
4.65	-53.40723	58.15507
4.66	-61.84939	66.58504
4.67	-73.10214	77.82550
4.68	-88.85041	93.56137
4.69	-112.4624	117.1608
4.70	-151.7910	156.4769
4.71	-230.3633	235.0364
4.72	-465.4320	470.0923
4.73	-116083.6	116088.3
4.74	480.5811	-475.9468
4.75	+242.6212	-238.0001
4.76	+163.5132	-158.9054
4.77	+123.9967	-119.4023
4.78	+100.2974	- 95.71650
4.79	84.50119	- 79.93396
4.80	73.21909	- 68.66562
4.81	64.75741	- 60.21782
4.82	58.17555	- 53.64996
4.83	52.90931	- 48.39785
4.84	48.59974	- 44.10253

λ	$K \frac{L}{EI}$	$Kk \frac{L}{EI}$
4.85	45.00756	-40.52473
4.86	41.96715	-37.49882
4.87	39.36023	-34.90653
4.88	37.10005	-32.66111
4.89	35.12157	-30.69752
4.90	33.37505	-28.96602
4.91	31.82181	-27.42792
4.92	30.43130	-26.05271
4.93	29.17910	-24.81593
4.94	28.04544	-23.69783
4.95	27.01414	-22.68222
4.96	26.07182	-21.75574
4.97	25.20736	-20.90726
4.98	24.41139	-20.12741
4.99	23.67601	-19.40829
5.00	22.99447	-18.74315
5.01	22.36098	-18.12622
5.02	21.77057	-17.55252
5.03	21.21892	-17.01772
5.04	20.70227	-16.51807
5.05	20.21732	-16.05028
5.06	19.76119	-15.61146
5.07	19.33132	-15.19905
5.08	18.92545	-14.81081
5.09	18.54158	-14.44472
5.10	18.17790	-14.09898
5.11	17.83282	-13.77201
5.12	17.50490	-13.46236
5.13	17.19284	-13.16873
5.14	16.89548	-12.88997
5.15	16.61174	-12.62501
5.16	16.34069	-12.37290
5.17	16.08144	-12.13276
5.18	15.83319	-11.90381
5.19	15.59523	-11.68532
5.20	15.36689	-11.47662
5.21	15.14755	-11.27711
5.22	14.93667	-11.08624
5.23	14.73372	-10.90348
5.24	14.53823	-10.72837

TABLE 1 (cont'd)

λ	$K \frac{L}{EI}$	$Kk \frac{L}{EI}$
5.25	14.34976	-10.56047
5.26	14.16791	-10.39939
5.27	13.99230	-10.24473
5.28	13.82259	-10.09617
5.29	13.65845	- 9.953372
5.30	13.49958	-9.816046
5.31	13.34570	-9.683912
5.32	13.19655	-9.556712
5.33	13.05188	-9.434204
5.34	12.91148	-9.316162
5.35	12.77511	-9.202375
5.36	12.64259	-9.092647
5.37	12.51373	-8.986792
5.38	12.38835	-8.884636
5.39	12.26629	-8.786017
5.40	12.14738	-8.690782
5.41	12.03149	-8.598786
5.42	11.91847	-8.509894
5.43	11.80820	-8.423979
5.44	11.70055	-8.340919
5.45	11.59541	-8.260601
5.46	11.49265	-8.182919
5.47	11.39219	-8.107770
5.48	11.29392	-8.035059
5.49	11.19775	-7.964694
5.50	11.10359	-7.896590
5.51	11.01135	-7.830666
5.52	10.92096	-7.766844
5.53	10.83233	-7.705051
5.54	10.74540	-7.645218
5.55	10.66009	-7.587276
5.56	10.57635	-7.531168
5.57	10.49409	-7.476829
5.58	10.41328	-7.424205
5.59	10.33384	-7.373241
5.60	10.25573	-7.323886
5.61	10.17888	-7.276090
5.62	10.10326	-7.229808
5.63	10.02881	-7.184994
5.64	9.955479	-7.141607

λ	$K \frac{L}{EI}$	$Kk \frac{L}{EI}$
5.65	9.883234	-7.099606
5.66	9.812028	-7.058952
5.67	9.741819	-7.019609
5.68	9.672568	-6.981541
5.69	9.604238	-6.944714
5.70	9.536793	-6.909098
5.71	9.470196	-6.874660
5.72	9.404415	-6.841373
5.73	9.339416	-6.809207
5.74	9.275169	-6.778136
5.75	9.211643	-6.748135
5.76	9.148809	-6.719179
5.77	9.086639	-6.691245
5.78	9.025105	-6.664310
5.79	8.964181	-6.638353
5.80	8.903841	-6.613354
5.81	8.844060	-6.589294
5.82	8.784815	-6.566153
5.83	8.726082	-6.543913
5.84	8.667838	-6.522558
5.85	8.610061	-6.502072
5.86	8.552730	-6.482439
5.87	8.495825	-6.463643
5.88	8.439324	-6.445672
5.89	8.383208	-6.428511
5.90	8.327458	-6.412147
5.91	8.272055	-6.396569
5.92	8.216982	-6.381764
5.93	8.162219	-6.367722
5.94	8.107749	-6.354431
5.95	8.053556	-6.341882
5.96	7.999623	-6.330065
5.97	7.945933	-6.318972
5.98	7.892471	-6.308593
5.99	7.839219	-6.298920
6.00	7.786164	-6.289946
6.01	7.733290	-6.281664
6.02	7.680582	-6.274067
6.03	7.628025	-6.267148
6.04	7.575605	-6.260902

TABLE 1 (cont'd)

λ	$K \frac{L}{EI}$	$Kk \frac{L}{EI}$
6.05	7.523307	-6.255322
6.06	7.471119	-6.250404
6.07	7.419024	-6.246144
6.08	7.367011	-6.242535
6.09	7.315065	-6.239576
6.10	7.263173	-6.237261
6.11	7.211322	-6.235587
6.12	7.159499	-6.234552
6.13	7.107690	-6.234153
6.14	7.055883	-6.234388
6.15	7.004064	-6.235254
6.16	6.952222	-6.236750
6.17	6.900343	-6.238875
6.18	6.848415	-6.241627
6.19	6.796425	-6.245006
6.20	6.744360	-6.249012
6.21	6.692208	-6.253645
6.22	6.639957	-6.258904
6.23	6.587593	-6.264791
6.24	6.535105	-6.271306
6.25	6.482480	-6.278451
6.26	6.429704	-6.286228
6.27	6.376766	-6.294637
6.28	6.323653	-6.303681
6.29	6.270352	-6.313364
6.30	6.216850	-6.323687
6.31	6.163135	-6.334654
6.32	6.109192	-6.346268
6.33	6.055010	-6.358533
6.34	6.000575	-6.371453
6.35	5.945874	-6.385033
6.36	5.890893	-6.399277
6.37	5.835618	-6.414192
6.38	5.780036	-6.429781
6.39	5.724134	-6.446051
6.40	5.667896	-6.463009
6.41	5.611308	-6.480661
6.42	5.554357	-6.499013
6.43	5.497027	-6.518074
6.44	5.439303	-6.537851
6.45	5.381170	-6.558352
6.46	5.322613	-6.579586
6.47	5.263616	-6.601562
6.48	5.204162	-6.624289
6.49	5.144237	-6.647777
6.50	5.083822	-6.672036
6.51	5.022901	-6.697078
6.52	4.961457	-6.722913
6.53	4.899471	-6.749554
6.54	4.836927	-6.777011
6.55	4.773805	-6.805300
6.56	4.710086	-6.834431
6.57	4.645751	-6.864421
6.58	4.580781	-6.895282
6.59	4.515154	-6.927031
6.60	4.448851	-6.959682
6.61	4.381849	-6.993252
6.62	4.314127	-7.027759
6.63	4.245662	-7.063219
6.64	4.176432	-7.099651
6.65	4.106412	-7.137074
6.66	4.035577	-7.175508
6.67	3.963904	-7.214974
6.68	3.891365	-7.255493
6.69	3.817934	-7.297087
6.70	3.743584	-7.339780
6.71	3.668286	-7.383596
6.72	3.592012	-7.428559
6.73	3.514730	-7.474697
6.74	3.436410	-7.522035
6.75	3.357020	-7.570603
6.76	3.276526	-7.620429
6.77	3.194895	-7.671544
6.78	3.112090	-7.723981
6.79	3.028075	-7.777771
6.80	2.942811	-7.832950
6.81	2.856260	-7.889553
6.82	2.768381	-7.947617
6.83	2.679131	-8.007183
6.84	2.588466	-8.068289

TABLE 1 (cont'd)

λ	$K\frac{L}{EI}$	$Kk\frac{L}{EI}$
6.85	2.496341	-8.130978
6.86	2.402709	-8.195295
6.87	2.307519	-8.261284
6.88	2.210722	-8.328995
6.89	2.112265	-8.398476
6.90	2.012091	-8.469781
6.91	1.910143	-8.542963
6.92	1.806362	-8.618079
6.93	1.700686	-8.695189
6.94	1.593049	-8.774354
6.95	1.483383	-8.855641
6.96	1.371617	-8.939115
6.97	1.257679	-9.024850
6.98	1.141490	-9.112918
6.99	1.022970	-9.203398
7.00	0.9020346	-9.296371
7.01	0.7785956	-9.391924
7.02	0.6525604	-9.490145
7.03	0.5238323	-9.591130
7.04	0.3923097	-9.694976
7.05	0.2578861	-9.801789
7.06	0.1204498	-9.911677
7.07	-0.02011682	-10.02475
7.08	-0.1639370	-10.14114
7.09	-0.3111406	-10.26097
7.10	-0.4618643	-10.38437
7.11	-0.6162518	-10.51148
7.12	-0.7744550	-10.64246
7.13	-0.9366336	-10.77746
7.14	-1.102957	-10.91664
7.15	-1.273603	-11.06019
7.16	-1.448760	-11.20829
7.17	-1.628628	-11.36114
7.18	-1.813418	-11.51894
7.19	-2.003355	-11.68192
7.20	-2.198675	-11.85031
7.21	-2.399632	-12.02437
7.22	-2.606493	-12.20436
7.23	-2.819544	-12.39056
7.24	-3.039089	-12.58327

λ	$K\frac{L}{EI}$	$Kk\frac{L}{EI}$
7.25	-3.265451	-12.78282
7.26	-3.498976	-12.98955
7.27	-3.740034	-13.20383
7.28	-3.989019	-13.42605
7.29	-4.246354	-13.65662
7.30	-4.512494	-13.89601
7.31	-4.787924	-14.14470
7.32	-5.073169	-14.40321
7.33	-5.368790	-14.67209
7.34	-5.675395	-14.95196
7.35	-5.993640	-15.24347
7.36	-6.324231	-15.54732
7.37	-6.667936	-15.86427
7.38	-7.025586	-16.19517
7.39	-7.398085	-16.54090
7.40	-7.786414	-16.90246
7.41	-8.191645	-17.28090
7.42	-8.614949	-17.67740
7.43	-9.057605	-18.09323
7.44	-9.521018	-18.52981
7.45	-10.00673	-18.98866
7.46	-10.51645	-19.47349
7.47	-11.05204	-19.98017
7.48	-11.61560	-20.51679
7.49	-12.20942	-21.08364
7.50	-12.83607	-21.68330
7.51	-13.49843	-22.31864
7.52	-14.19971	-22.99285
7.53	-14.94351	-23.70956
7.54	-15.73391	-24.47282
7.55	-16.57552	-25.28726
7.56	-17.47358	-26.15809
7.57	-18.43405	-27.09131
7.58	-19.46380	-28.09375
7.59	-20.57071	-29.17330
7.60	-21.76392	-30.33911
7.61	-23.05408	-31.60181
7.62	-24.45364	-32.97387
7.63	-25.97732	-34.46999
7.64	-27.64257	-36.10762

TABLE 1 (cont'd)

λ	$K \frac{L}{EI}$	$Kk \frac{L}{EI}$
7.65	-29.47029	-37.90766
7.66	-31.48569	-39.89533
7.67	-33.71951	-42.10135
7.68	-36.20957	-44.56354
7.69	-39.00296	-47.32900
7.70	-42.15907	-50.45712
7.71	-45.75387	-54.02386
7.72	-49.88617	-58.12802
7.73	-54.68682	-62.90046
7.74	-60.33294	-68.51828
7.75	-67.07026	-75.22725
7.76	-75.25001	-83.37856
7.77	-85.39224	-93.49226
7.78	-98.30117	-106.3726
7.79	-115.2900	-123.3327
7.80	-138.6592	-146.6731
7.81	-172.8389	-180.8240
7.82	-227.5966	-235.5527
7.83	-329.5360	-337.4631
7.84	-585.8507	-593.7487
7.85	-2441.721	-2449.590
7.86	1164.517	1156.678
7.87	476.4133	468.6034
7.88	301.8962	294.1158
7.89	222.2283	214.4776
7.90	176.6028	168.8818
7.91	147.0380	139.3469
7.92	126.3206	118.6594
7.93	110.9942	103.3631
7.94	99.19553	91.59465
7.95	89.83116	82.26059
7.96	82.21726	74.67712
7.97	75.90419	68.39459
7.98	70.58414	63.10522
7.99	66.03932	58.59120
8.00	62.11128	54.69409
8.01	58.68201	51.29587
8.02	55.66176	48.30681
8.03	52.98113	45.65750
8.04	50.58554	43.29337

λ	$K \frac{L}{EI}$	$Kk \frac{L}{EI}$
8.05	48.43154	41.17097
8.06	46.48407	39.25523
8.07	44.71453	37.51756
8.08	43.09937	35.93442
8.09	41.61902	34.48624
8.10	40.25708	33.15662
8.11	38.99970	31.93171
8.12	37.83512	30.79976
8.13	36.75327	29.75069
8.14	35.74548	28.77584
8.15	34.80426	27.86771
8.16	33.92307	27.01980
8.17	33.09624	26.22639
8.18	32.31875	25.48251
8.19	31.58621	24.78373
8.20	30.89470	24.12617
8.21	30.24078	23.50637
8.22	29.62136	22.92125
8.23	29.03368	22.36805
8.24	28.47527	21.84431
8.25	27.94390	21.34779
8.26	27.43758	20.87651
8.27	26.95449	20.42865
8.28	26.49298	20.00258
8.29	26.05158	19.59680
8.30	25.62890	19.20995
8.31	25.22373	18.84081
8.32	24.83491	18.48824
8.33	24.46142	18.15119
8.34	24.10231	17.82874
8.35	23.75668	17.51999
8.36	23.42374	17.22414
8.37	23.10274	16.94046
8.38	22.79299	16.66825
8.39	22.49385	16.40688
8.40	22.20474	16.15576
8.41	21.92510	15.91436
8.42	21.65442	15.68215
8.43	21.39222	15.45867
8.44	21.13807	15.24347

TABLE 1 (cont'd)

λ	$K \frac{L}{EI}$	$Kk \frac{L}{EI}$	λ	$K \frac{L}{EI}$	$Kk \frac{L}{EI}$
8.45	20.89156	15.03616	8.85	14.57743	10.53909
8.46	20.65228	14.83634	8.86	14.46906	10.48372
8.47	20.41989	14.64366	8.87	14.36198	10.43009
8.48	20.19405	14.45778	8.88	14.25614	10.37815
8.49	19.97443	14.27839	8.89	14.15149	10.32786
8.50	19.76074	14.10520	8.90	14.04799	10.27920
8.51	19.55270	13.93792	8.91	13.94560	10.23211
8.52	19.35005	13.77631	8.92	13.84427	10.18657
8.53	19.15253	13.62011	8.93	13.74397	10.14255
8.54	18.95993	13.46909	8.94	13.64465	10.10000
8.55	18.77200	13.32305	8.95	13.54627	10.05890
8.56	18.58856	13.18177	8.96	13.44880	10.01922
8.57	18.40939	13.04506	8.97	13.35221	9.980935
8.58	18.23432	12.91275	8.98	13.25645	9.944014
8.59	18.06317	12.78465	8.99	13.16149	9.908450
8.60	17.89577	12.66061	9.00	13.06730	9.874161
8.61	17.73196	12.54048	9.01	12.97385	9.841183
8.62	17.57161	12.42410	9.02	12.88111	9.809472
8.63	17.41455	12.31135	9.03	12.78904	9.779007
8.64	17.26066	12.20208	9.04	12.69762	9.749769
8.65	17.10981	12.09618	9.05	12.60682	9.721736
8.66	16.96187	11.99352	9.06	12.51661	9.694890
8.67	16.81674	11.89401	9.07	12.42696	9.669213
8.68	16.67429	11.79752	9.08	12.33784	9.644688
8.69	16.53443	11.70396	9.09	12.24923	9.621299
8.70	16.39705	11.61323	9.10	12.16111	9.599029
8.71	16.26205	11.52525	9.11	12.07344	9.577865
8.72	16.12935	11.43992	9.12	11.98621	9.557793
8.73	15.99885	11.35716	9.13	11.89939	9.538798
8.74	15.87048	11.27689	9.14	11.81295	9.520868
8.75	15.74414	11.19903	9.15	11.72688	9.503991
8.76	15.61977	11.12353	9.16	11.64114	9.488156
8.77	15.49728	11.05029	9.17	11.55573	9.473353
8.78	15.37662	10.97927	9.18	11.47060	9.459570
8.79	15.25770	10.91040	9.19	11.38576	9.446799
8.80	15.14046	10.84361	9.20	11.30116	9.435030
8.81	15.02485	10.77886	9.21	11.21680	9.424256
8.82	14.91080	10.71608	9.22	11.13264	9.414468
8.83	14.79825	10.65523	9.23	11.04868	9.405660
8.84	14.68714	10.59625	9.24	10.96488	9.397824

TABLE 1 (cont'd)

λ	$K \frac{L}{EI}$	$Kk \frac{L}{EI}$
9.25	10.88124	9.390955
9.26	10.79772	9.385047
9.27	10.71432	9.380095
9.28	10.63100	9.376094
9.29	10.54776	9.373041
9.30	10.46457	9.370931
9.31	10.38141	9.369761
9.32	10.29826	9.369530
9.33	10.21511	9.370235
9.34	10.13194	9.371874
9.35	10.04873	9.374445
9.36	9.965451	9.377950
9.37	9.882096	9.382386
9.38	9.798643	9.387754
9.39	9.715072	9.394056
9.40	9.631366	9.401292
9.41	9.547507	9.409463
9.42	9.463474	9.418573
9.43	9.379250	9.428623
9.44	9.294815	9.439616
9.45	9.210151	9.451556
9.46	9.125239	9.464447
9.47	9.040059	9.478294
9.48	8.954592	9.493101
9.49	8.868820	9.508873
9.50	8.782721	9.525618
9.51	8.696277	9.543340
9.52	8.609467	9.562048
9.53	8.522272	9.581747
9.54	8.434670	9.602448
9.55	8.346641	9.624157
9.56	8.258165	9.646884
9.57	8.169220	9.670638
9.58	8.079784	9.695431
9.59	7.989835	9.721271
9.60	7.899352	9.748172
9.61	7.808312	9.776143
9.62	7.716692	9.805199
9.63	7.624468	9.835352
9.64	7.531617	9.866616

λ	$K \frac{L}{EI}$	$Kk \frac{L}{EI}$
9.65	7.438114	9.899006
9.66	7.343936	9.932536
9.67	7.249055	9.967223
9.68	7.153448	10.00308
9.69	7.057087	10.04013
9.70	6.959947	10.07839
9.71	6.861998	10.11787
9.72	6.763213	10.15861
9.73	6.663564	10.20060
9.74	6.563021	10.24389
9.75	6.461553	10.28849
9.76	6.359131	10.33442
9.77	6.255721	10.38171
9.78	6.151291	10.43038
9.79	6.045809	10.48046
9.80	5.939238	10.53198
9.81	5.831545	10.58496
9.82	5.722692	10.63944
9.83	5.612642	10.69544
9.84	5.501357	10.75299
9.85	5.388796	10.81214
9.86	5.274920	10.87291
9.87	5.159684	10.93534
9.88	5.043047	10.99947
9.89	4.924962	11.06533
9.90	4.805384	11.13297
9.91	4.684264	11.20243
9.92	4.561552	11.27375
9.93	4.437198	11.34697
9.94	4.311149	11.42216
9.95	4.183348	11.49935
9.96	4.053739	11.57859
9.97	3.922263	11.65994
9.98	3.788860	11.74346
9.99	3.653464	11.82921
10.00	3.516011	11.91723
10.01	3.376432	12.00761
10.02	3.234656	12.10040
10.03	3.090608	12.19568
10.04	2.944213	12.29350

TABLE 1 (cont'd)

λ	$K \frac{L}{EI}$	$Kk \frac{L}{EI}$
10.05	2.795390	12.39396
10.06	2.644055	12.49713
10.07	2.490123	12.60309
10.08	2.333502	12.71193
10.09	2.174099	12.82373
10.10	2.011816	12.93860
10.11	1.846549	13.05662
10.12	1.678192	13.17790
10.13	1.506632	13.30255
10.14	1.331752	13.43069

λ	$K \frac{L}{EI}$	$Kk \frac{L}{EI}$
10.15	1.153431	13.56242
10.16	0.9715397	13.69788
10.17	0.7859436	13.83719
10.18	0.5965024	13.98050
10.19	0.4030684	14.12793
10.20	0.2054871	14.27966
10.21	0.003595060	14.43583
10.22	-0.2027782	14.59661

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