Scattering Attenuation in Fractally Homogeneous Random Media

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Abstract

Much of seismic nuclear monitoring depends on a proper understanding of attenuation and scattering of regional seismic waves. This report seeks to characterize wave scattering from both an observational and theoretical approach. A theory of wave propagation in fractally homogeneous random media is developed and applied to southern Tibet in order to quantify the analytical and statistical properties of crust and upper lithosphere heterogeneity. Seismic velocity heterogeneity is modeled by a self-affine band limited random fractal. An approximation to multiple scattering theory yields a relationship between the exponent of the frequency dependent scattering quality factor Q and the fractal dimension of velocity iso-surfaces. In this study, the attenuation quality factor of P-waves, Q_P, is calculated for southern Tibet and the implications for the fractal dimension of the velocity field are discussed. In addition, the topic of intermittent heterogeneity is introduced. The generalization of the model to fractally homogeneous media implies that scattering attenuation measurements are characterized by two fractal dimensions: one related to the self-affinity of the velocity field and the other related to the metric of the space which the field occupies.

Within a fractal random media model, the attenuation calculation for southern Tibet yields a fractal dimension of velocity iso-surfaces of $D = 9/4$ which implies the existence of complex large-scale structure in the upper lithosphere. For the more general case of fractally homogeneous heterogeneity, the frequency dependence of the scattering quality factor is a function of two parameters D and another fractal dimension $\bar{D}$. The predicted frequency exponent can differ from the homogeneous case by as much as 2/3 for reasonable choices of the parameters D and $\bar{D}$. However, the relative contributions of topological disorder and intermittency are inextricably linked and cannot be isolated by scattering attenuation measurements. Continued investigation of wave propagation in fractally homogeneous media may further quantify the statistical properties of crustal heterogeneity.

keywords: scattering attenuation, quality factor, crustal heterogeneity, fractal

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Objectives

Much of seismic nuclear monitoring depends on an understanding of the attenuation and scattering characteristics of regional seismic phases. The objective of this report is to quantify the nature of heterogeneity in the crust and upper lithosphere in southern Tibet and to show that for wave propagation in general fractally homogeneous media, the frequency dependence of the scattering attenuation quality factor Q is a function of two parameters related to the geometry of the velocity field. Following Wu and Aki, seismic velocity heterogeneity is modeled by a band limited random fractal (Wu and Aki, 1985). An approximation to multiple scattering theory in such media yields a relationship between the exponent of the frequency dependent quality factor and the fractal dimension of velocity iso-surfaces (Wu, 1982). In this study, the attenuation quality factor of P-waves, Q_P, is calculated for southern Tibet and the implications for the fractal dimension of the velocity field are discussed. In addition, the topic of intermittent heterogeneity is introduced. The generalization of the model to fractally homogeneous media implies that scattering attenuation measurements are characterized by two fractal dimensions: one related to the self-affinity of the velocity field and the other related to the metric of the space which the field occupies.

Research Accomplished

Model Random Media

The model random media is defined by a position dependent velocity $c(\vec{r}) = c_0 + \delta c(\vec{r})$ where the fluctuating part $\delta c(\vec{r})$ is a zero-mean, isotropic, random funtion. For the purposes of this study the fluctuation is taken to be a fractional Brownian function $\delta c(\vec{r}) = B_H(\vec{r})$ (Mandelbrot, 1977). The fractional Brownian function $B_H(\vec{r})$ is defined through a variance or structure function,

$$F(r) =< (B_H(\vec{r'} + \vec{r}) - B_H(\vec{r'}))^2 >= r^{2H}. \quad (1)$$

For the value of the parameter $H = 1/2$, the function $B_H(r)$ along any line in space is a Brownian walk whose variance scales linearly with sample size. Mandelbrot has shown that the fractal dimension of the iso-surfaces of such a field is $D = 3 - H$ (Mandelbrot, 1975). Thus, if the fluctuations are characterized by a root mean square variation γ, the structure function can be written as $F(r) = \gamma^2 r^{2(3-D)}$. The value of $F(r)$ characterizes the intensity of fluctuations with length scale smaller than or on the order of r. The one-dimensional spectral density is related to the structure function through an integral transform (Tatarski, 1960). One finds that the spectral density is given by,

$$\Phi(k) = \frac{\Gamma(1 + 2(3 - D)) \sin(\pi(3 - D))}{2\pi} \gamma^2 k^{-(2(3-D)+1)}. \quad (2)$$

Of course any real medium posseses some outer scale of heterogeneity such that the power law spectrum is truncated at some point. A spectrum that corresponds to Eq. (2) in the high frequency limit and includes a characteristic length is the von Karman spectrum,

$$\Phi(k) = \frac{\gamma^2 \Gamma(3 - D + 1/2)}{\sqrt{\pi}\Gamma(3 - D)} \frac{a}{(1 + k^2 a^2)^{(D-3+1/2)}}, \quad (3)$$

where a is the outer length scale of the inhomogeneity. Wu (Wu, 1982) has shown that within a cumulative-forward, single-backscatter approximation to multiple scattering theory, the inverse of the scattering quality factor

$$Q^{-1} = 2k_0\{\Phi(\sqrt{2}k_0) - \Phi(2k_0)\}, \quad (4)$$

where k_0 is the wavenumber in a homogeneous medium of velocity c_0. Thus, substituting for the spectral density Φ, in the high frequency regime the model yields a frequency dependent quality factor,

$$Q^{-1} \sim k^{-2(3-D)}. \quad (5)$$

If the fractal dimension of constant velocity surfaces D lies between $2 < D < 3$, then $Q^{-1} \sim k^{-\beta}$ where $0 < \beta < 2$. Thus, observations of smaller β correspond to larger amplitudes of high frequency spectral harmonics and smaller amplitudes of low frequency components. This implies the existence of small scale field structure and the absence of large scale structure. This is illustrated in Figure 1 for two approximations to two-dimensional fractional Brownian functions. As the value of D increases from two to three the surfaces become more erratic and small scale detail is increased. This fact is the underlying geometrical basis for the observed correlation between degree of frequency dependence of Q and crustal stability (Aki, 1980).

Southern Tibet

A three-component broadband array in southern Tibet was deployed from May 2, 1994 to October 23, 1994. The INDEPTH-II passive source experiment used both short period and broadband seismometers to record about 120 teleseismic and 80 local and regional earthquakes. The broadband seismometers used in the deployment were Guralp CMG-3T's. The data were continuously recorded by Reftek 72-06 24 bit digital recorders using 50 samples per second.

The quality factor Q_P was estimated for southern Tibet by computing spectral ratios of thirty seconds of P-wave train in narrow frequency bands for five local earthquakes. Figure 2 shows seismograms for a local event recorded at two epicentral distances along with the correpsonding spectra for the P-wave coda. The results of the attenuation calculation are shown in Figure 3. The theoretical curve correspds to a model random medium with $\gamma = 0.2$, background velocity $c_0 = 7.5$ km/sec, correlation length $a = 1.0$ km, and fractal dimension $D = 9/4$. Accordingly the high frequency dependence of Q_P^{-1} is

$$Q_P^{-1} \sim k^{-3/2}, \tag{6}$$

The seemingly large value of γ could be reduced by including intrinsic attenuation in the calculation.

Intermittent Inhomogeneity

Intermittency, as defined by Mandelbrot, is the fact that in natural turbulence dissipation is not distributed uniformly in space (Mandelbrot, 1977). Likewise, there is no *a priori* reason to believe that seismic velocity heterogeneity is homogeneously distributed in three dimensional Euclidean space. Instead, some crustal regions may be very heterogeneous while others are devoid of velocity impedence. Furthermore, the region where the scalar field quantity concentrates may sufficiently tortured and convoluted such that the geometry is appropriately described by a fractal. The case of homogeneous turbulence concentrated in a space of dimension $\bar{D} < 3$ has been called fractally homogeneous turbulence by Mandelbrot (Mandelbrot, 1976). Likewise the case of seismic velocity fluctuations concentrated on a fractal may be called fractally homogeneous inhomogeneity. Thus there are two dimensions which describe general fractally homogeneous fields. The dimension $\bar{D}$ describes the way the field is spread around while the dimension D describes the way the field is connected. An important result of intermittency is that the spectra of fluctuations is altered by a factor related to the dimension of the support of the fluctuations. The support of a function is the set of values of the argument for which the function is non-zero. For example, in the case of fluid turbulence where the structure function goes like $r^{2/3}$, the spectrum is proportional to $k^{-(5/3+B)}$ where B is related to a function that describes the process of 'curdling' which concentrates the inhomogeneity (Mandelbrot,1976). Likewise for fields with structure functions like $r^{2(3-D)}$, the spectral exponent is $-(2(3-D)+B+1)$ so that within an intermittent version of the model described above the scattering attenuation quality factor,

$$Q^{-1} \sim k^{-(2(3-D)+B)}. \tag{7}$$

In the case of absolute curdling, the support of the field is a closed set and the parameter

$$B = -(3-\bar{D})(2(3-D)-1). \tag{8}$$

If the dimension of the support $\bar{D}$ lies between $2 < \bar{D} < 3$ then $0 < B < 2/3(2(3-D)-1)$. As an example, consider a random fractal medium with $D = 2.25$ which has curdled into a support with fractal dimension $\bar{D} = 2.5$. One finds that $B = -1/4$ and

$$Q^{-1} \sim k^{-5/4}. \tag{9}$$

In general, if the model velocity field is indeed intermittent ($\bar{D} < 3$) and and the iso-surfaces have dimension $D > 2.5$, the factor B is positive. This implies that if the observed frequency dependence of scattering Q is attributed totally to homogeneous inhomogeneity, the amount by which the iso-surface dimension exceeds two would be overestimated. On the other hand, if $D < 2.5$ the factor B is negative and homogeneous theory yields an underestimate of the dimension D of constant velocity surfaces.

Several studies of coda wave attenuation have yielded frequency dependent quality factors Q_c. Some results are summarized below as a function of tectonic region. The value of the iso-surface dimension D is given for three different dimensions of support $\bar{D}$.

Region	β	$D(\bar{D}=3)$	$D(\bar{D}=2.75)$	$D(\bar{D}=2.5)$
Western U.S. (Singh and Hermann, 1983)	0.7	2.65	2.70	2.80
Garwhal Himalaya (Gupta *et. al.*, 1995)	0.9	2.55	2.57	2.60
Western Anatolia (Akinci *et. al.*, 1994)	1.0	2.5	2.5	2.5
Southern Tibet (Q_P) (This study)	1.5	2.25	2.17	2.0

Interestingly, when $\beta = 1$ intermittency has no effect on the fractal dimension D.

Conclusions and Recommendations

Scattering attenuation measurements provide an effective means to constrain the analytical and geometrical properties of seismic media with random velocity perturbations. Within a fractal random media model, the attenuation calculation for southern Tibet yields a fractal dimension of velocity iso-surfaces of $D = 9/4$ which implies the existence of complex large-scale structure in the upper lithosphere. For the more general case of fractally homogeneous heterogeneity, the frequency dependence of the scattering quality factor is a function of two parameters D and $\bar{D}$. The predicted frequency exponent can differ from the homogeneous case by as much as 2/3 for reasonable choices of the parameters D and $\bar{D}$. However, the relative contributions of topological disorder and intermittency are inextricably linked and cannot be isolated by scattering attenuation measurements. Continued investigation of wave propagation in fractally homogeneous media may further quantify the statistical properties of crustal heterogeneity.

References

Aki, K., Scattering and Attenuation of Shear Waves in the Lithosphere, *J. Geophys. Res., 85*, 6496-6504 (1980).

Akinci, A., Taktak, A.G., and Ergintav, S., Attenuation of coda waves in Western Anatolia, *Phys. Earth Planet. Int., 121*, 155-165 (1994).

Gupta, S.C., Singh, V.N., and Kumar, A., Attenuation of coda waves in the Garwhal Himalaya, India, *Phys. Earth Planet. Int., 87*, 247-253 (1995).

Mandelbrot, B.B., On the geometry of homogeneous turbulence, with stress on the fractal dimension of the iso-surfaces of scalars, *J. Fluid Mech., 72*, 401-416, (1975).

Mandelbrot, B.B., *Proceedings: Turbulence and Navier Stokes Equations* ed. R. Temam (Springer-Verlag, Berlin, 1976).

Mandelbrot, B.B., *Fractals, Form, Chance, and Dimension.* (W.H. Freeman and Company, San Francisco, 1977).

Singh, S. and Herrmann, R.B., Regionalization of Crustal Coda Q in the Continental United States, *J. Geophys. Res., 88*, 527-538, (1983).

Tatarski, V.I., *Wave Propagation in a Turbulent Medium* (McGraw-Hill, NY, 1960).

Wu, R.S., Attenuation of Short Period Seismic Waves Due to Scattering, *Geophys. Res. Lett., 9*, 9-12, (1982).

Wu, R.S. and Aki, K., The Fractal Nature of the Inhomogeneities in the Lithosphere Evidenced from Seismic Wave Scattering, *Pure Appl. Geophys., 88*, 805-818, (1985).

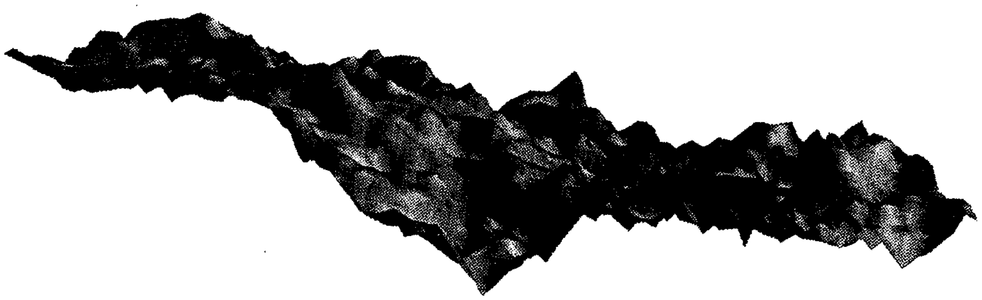

Figure 1. Two dimensional approximations to fractional Brownian functions. The fractal dimension of the top figure is D=9/4=2.25 while the fractal dimension of the bottom figure is D=8/3=2.66. In general, as D increases from two to three the small scale structure of the surfaces is enhanced.

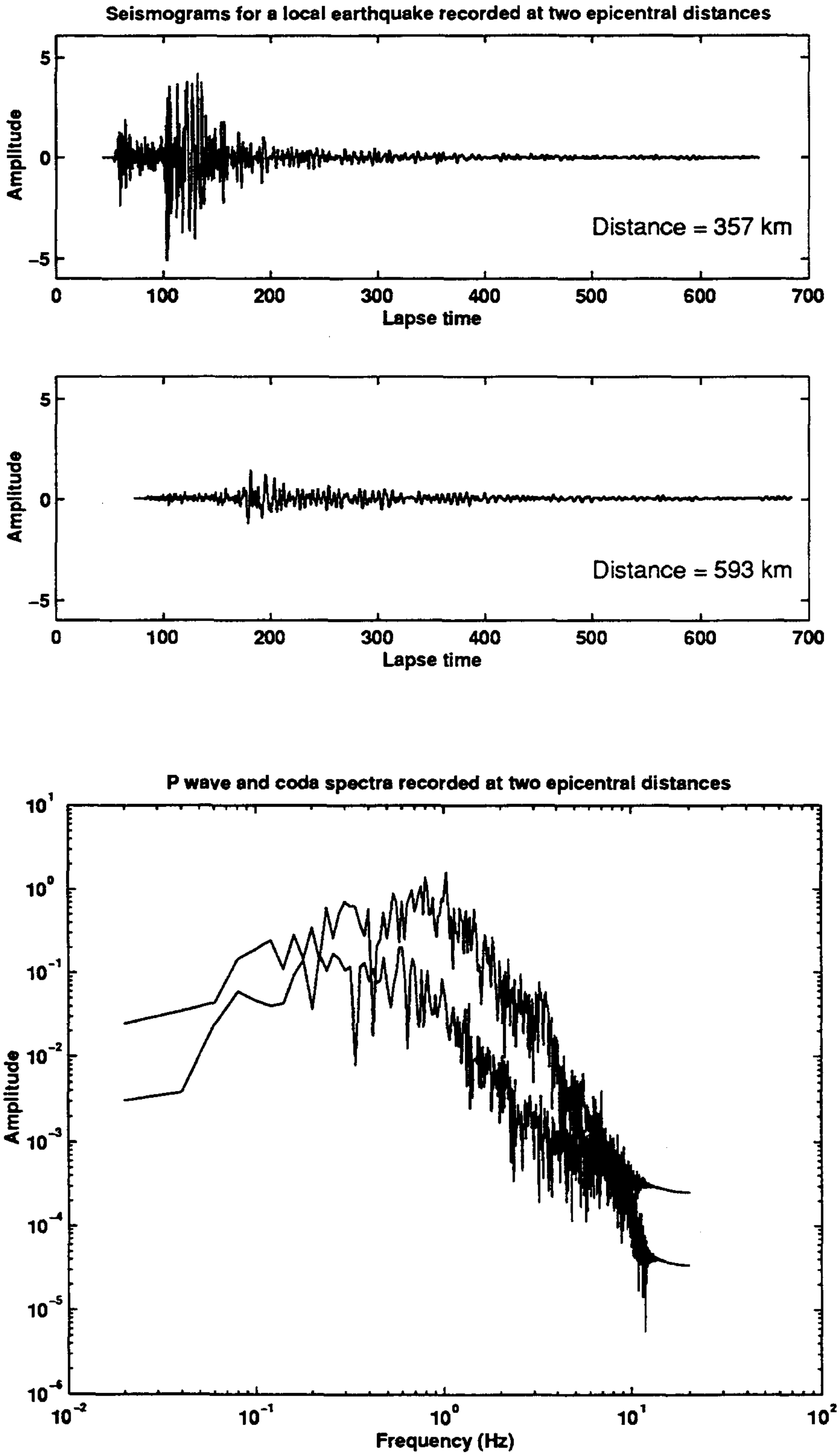

Figure 2. Vertical traces for a local event of magnitude 4.7 recorded at two epicentral distances. The corresponding P-wave coda spectra are plotted on the bottom demonstrating the frequency dependent attenuation.

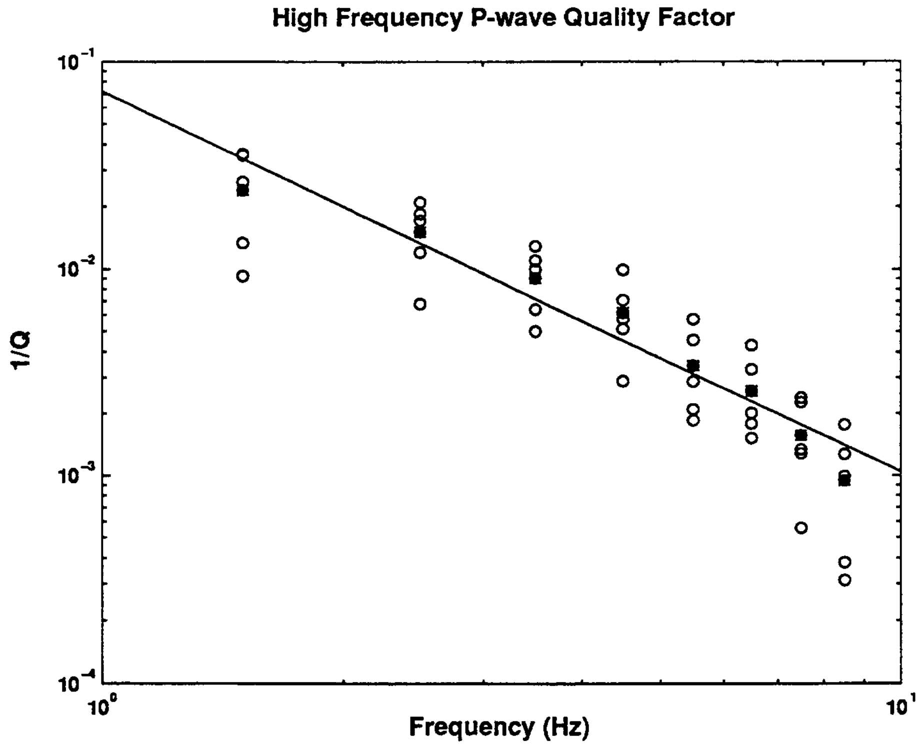

Figure 3. Observed quality factor Q for P-wave coda. All the data for five local earthquakes in Tibet are plotted as open circles. The mean in each frequency band is plotted as an asterisk and the straight line is fit in the least squares sense. The frequency dependence is $Q \sim f^{1.5}$