# SEISMIC SOURCES AND WAVE PROPAGATION IN 3-D: RADIATION FROM CYLINDRICAL CAVITIES

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## ABSTRACT

We apply the boundary element method (BEM) to study seismic wave radiation from explosions in cylindrical cavities. This approach simulates wave propagation by replacing the boundary between the cavity wall and the surrounding formation by a distribution of fictitious sources that produce the same composite wavefield as the original model. The cavity wall is discretized into elements in the numerical implementation, and the fictitious source amplitudes on each element are determined by solving boundary condition equations. In order to accomplish this, the interactions of all elements must be computed as well as the wavefields radiating from the sources to the receiver positions. A parallel computer implementation of the algorithm is therefore advantageous, since the frequency domain computations are easily parallelized for rapid solution of the problem. We also compute discrete wavenumber synthetic seismograms at regional distances from the cavity source, using an equivalent moment tensor model of the explosion in the cylindrical cavity.

Results for the radiation patterns show that at large aspect ratios (the ratio of cavity length to cavity diameter), the source has a strong variation in signal strength at high frequency. However, at lower frequencies typical of regional wave propagation, the P-wave radiation pattern is more nearly isotropic, and the S-wave amplitude is not so large. In addition, we find the interesting result that the radiation pattern of a large aspect ratio cavity in a transversely isotropic medium can be more like a classical point source than an a similar isotropic example. Incorporation of the cavity source into simulations of regional wave propagation using discrete wavenumber techniques shows that even though the shear wave is relatively weak at the source, it can still lead to the generation of a significant transverse component *Lg* wave at regional distances.

# OBJECTIVE

The primary goal of the work reported in this paper is to examine the influence of source cavity geometry on radiation of seismic waves at regional distances. Specifically, by examining radiation patterns of these sources, we seek to determine whether or not these sources can explain variations in S and P-wave energy observed in field data, and whether or not they can generate significant SH-waves. We therefore simulate the radiation patterns of explosion sources located in cylindrical cavities that model tunnels using the boundary element method (BEM). In addition, we examine the behavior of such sources in representative models of the earth's crust using the discrete wavenumber method and have computed results for these sources in 3-D earth models using finite difference methods. Results from these numerical experiments show that the radiation patterns from explosion sources located in tunnels may not produce significantly strong shear waves. If the rock formations surrounding the borehole are transversely isotropic (axis of symmetry parallel to the cavity axis), the BEM results show that the quasi-shear wave produced in this case can actually be weaker than in the isotropic case. Discrete wavenumber seismograms show that even though the shear wave is somewhat weaker than the compressional wave at the source, it still produces a strong *Lg* wave at regional distances with some amplitude variations as a function of observation azimuth.

# RESEARCH ACCOMPLISHED

## Introduction

The radiation patterns of seismic sources are very important in utilizing seismic signals to distinguish between nuclear explosions and natural earthquake events. The standard model of the nuclear explosion source, the spherically symmetric pressure field corresponding to a moment tensor model with three equal, mutually perpendicular dipoles, generates only an isotropic compressional wave disturbance. However, any deviation from this ideal configuration can significantly change the radiation pattern of the source. For example, Zhao and Harkrider (1992) show that when the source is located off-center in a spherical cavity it will generate significant shear wave energy. Their work considers a source located in a spherical solid, which is in turn embedded within another infinite solid, representing a fully tamped explosion.

Another way of altering the radiation of the source is to change the shape of the cavity. Glenn et al. (1985, 1986), Rial and Moran (1986) and Glenn and Rial (1987) considered the radiation patterns of sources located in ellipsoidal and cylindrical cavities. Using a combination of analytical and numerical methods, they obtained results suggesting that the P-wave radiation pattern would be strongest perpendicular to the cavity axis and that a strong shear wave would be generated as cavity aspect ratio (ratio of length to diameter) became large. However, Stevens *et al.* (1991) performed non-linear numerical simulations that suggested that at lower frequencies more typical of regional wave propagation, the compressional wave radiation pattern from a cylindrical cavity would be relatively isotropic and only a small S-wave would be generated. More recently, Ben-Menahem and Mikhailov (1995) derived approximate analytic solutions for radiation from cylindrical cavities. In contrast to the results of Stevens *et al.* (1991), they predict a comparatively large S-wave amplitude for low frequencies. Hence, there are significantly different results for cylindrical cavity radiation patterns available in the literature.

We therefore continue this line of investigation by applying the boundary element method

(BEM) to simulate the radiation of seismic waves from a cylindrical cavity representing a source located in a vertical shaft or a horizontal tunnel. The BEM is well suited to problems such as this, where the model combines small features (the cavity) with large features (propagation distances on the order of 10 km or more) (Bouchon, 1993; Dong, 1993; Dong *et al.*, 1995; Dong and Toksöz, 1995). After a brief description of this method, we show radiation patterns obtained with the BEM that are similar to those obtained by Stevens *et al.* (1991). They show a weak variation of P-wave amplitude with direction and a comparatively small shear wave at frequencies characteristic of regional wave propagation. After presenting these results and demonstrating the effects of locating the cavity in a transversely isotropic medium, we consider the effects of such a source in a plane layered model of the earth. The source is applied in a discrete wavenumber code by developing an equivalent moment tensor representation of the source. Synthetic seismograms at regional distances show that the effect of the cavity, though subtle, will generated SH-wave motion and may still be distinguishable in terms of an azimuthal variation in amplitudes.

## Method

The BEM has recently been applied to synthesize borehole wave propagation in experimental configurations typical of exploration geophysics applications (Bouchon, 1993; Dong, 1993; Dong *et al.*, 1995; Dong and Toksöz, 1995). One model considered in this earlier work is a semi-infinite borehole, where the seismic source is applied at or near the end of the borehole. A straightforward, logical extension of this approach is to model a cylindrical cavity of finite length, such as the source cavities that might be utilized in testing nuclear devices. Like the exploration simulations, we model the interior of the cavity as an acoustic medium and also assume symmetry about the cavity axis.

The essence of the method is that the effects of the diffracting boundary between the source cavity and the surrounding medium can be reproduced by a discrete set of fictitious secondary sources (Bouchon, 1993). Hence, implementation of the algorithm begins by discretizing the wall, both the cylindrical portion and the flat ends capping the cavity. The magnitudes of the fictitious sources are determined by enforcing boundary conditions at the cavity wall (continuity of radial displacement and stress, vanishing of shear stress). These three boundary conditions are sufficient to determine the sources, since for each element there is a volume injection source radiating energy into the acoustic material in the cavity, and a vertical and radial force generating elastic waves in the rock formation outside the source region. The determination of source magnitudes is accomplished by computing the displacement field generated by each element incident on the other elements using a discrete wavenumber method to evaluate the integrals over horizontal wavenumber. Given the intense computation that must be performed in order to obtain these quantities, it is advantageous to implement the algorithm on a parallel computer. The computations for each frequency are independent, so we assign each processor on an nCUBE-2 computer the work for a different frequency.

In order to solve the resulting matrix of boundary condition equations, we must also specify the initial field generated by the true, primary source. A straightforward model of this field, when there is complete decoupling of the explosion, is to assume that the explosion instantaneously pressurizes the cavity. In computing our frequency domain results, we therefore model the source condition as a step function of constant pressure applied to the cylindrical cavity wall. Earlier versions of this work (Gibson *et al.*, 1994) instead used a volume injection point

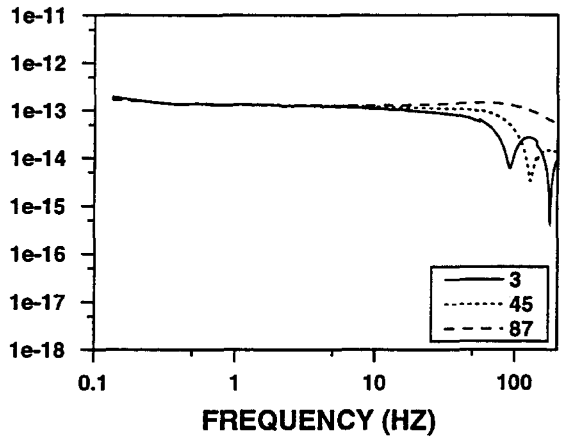

Figure 1: P-wave amplitude spectra of the seismic wavefields radiated by a source in a cylindrical cavity of aspect ratio 5. Spectra are presented for three different propagation directions relative to the cavity axis, where 0° is parallel to the axis. The directions corresponding to 3° and 87° were used instead of 0 and 90° to avoid some numerical problems associated with computations for these directions.

source located on the axis of the cavity. The differences from the results we show below are caused by the different definition of the initial wavefields. Although for relatively small cavity, the incident wavefield generated by the point source will strike the cavity wall essentially simultaneously, the amplitude of the incident field will decay with distance. This change in amplitude alters radiation patterns somewhat and makes the patterns sensitive to the location of the source within the cavity.

## Results

### Isotropic medium

We applied the BEM to a cavity with aspect ratio 5 located in a medium with a P-wave velocity of 4000 m/s, S-wave velocity 2200 m/s and density 2.2 gm/cm$^3$. These parameters will show the behavior of the source in a hard rock site. Spectra for observation directions of 3°, 45°, and 87° (0° is along the axis of the cylindrical cavity, 90° is perpendicular to it) show that the corner frequency of the wavefields depends on this direction (Figure 1). Specifically, the corner frequency is higher for observation directions perpendicular to the cavity. These results are similar to those of Glenn *et al.* (1985, 1986) and Stevens *et al.* (1991).

However, most of the amplitude difference in these wavefields does occur at high frequencies. To test the effects that might be seen in regional seismograms, we computed radiation patterns at lower frequencies. Radiation patterns of both P and S-waves were obtained by first computing synthetic seismograms for a set of observation points at a constant distance of 10000 m from the source and convolving the frequency domain results with a Ricker wavelet of 1 Hz.

In Figure 2, we show the resulting radiation patterns for cavities 80 m long with varying aspect ratio. In general, the changes in these radiation patterns are relatively subtle, but some variations are evident. As aspect ratio decreases from 5 to values near 1, the amplitude of the

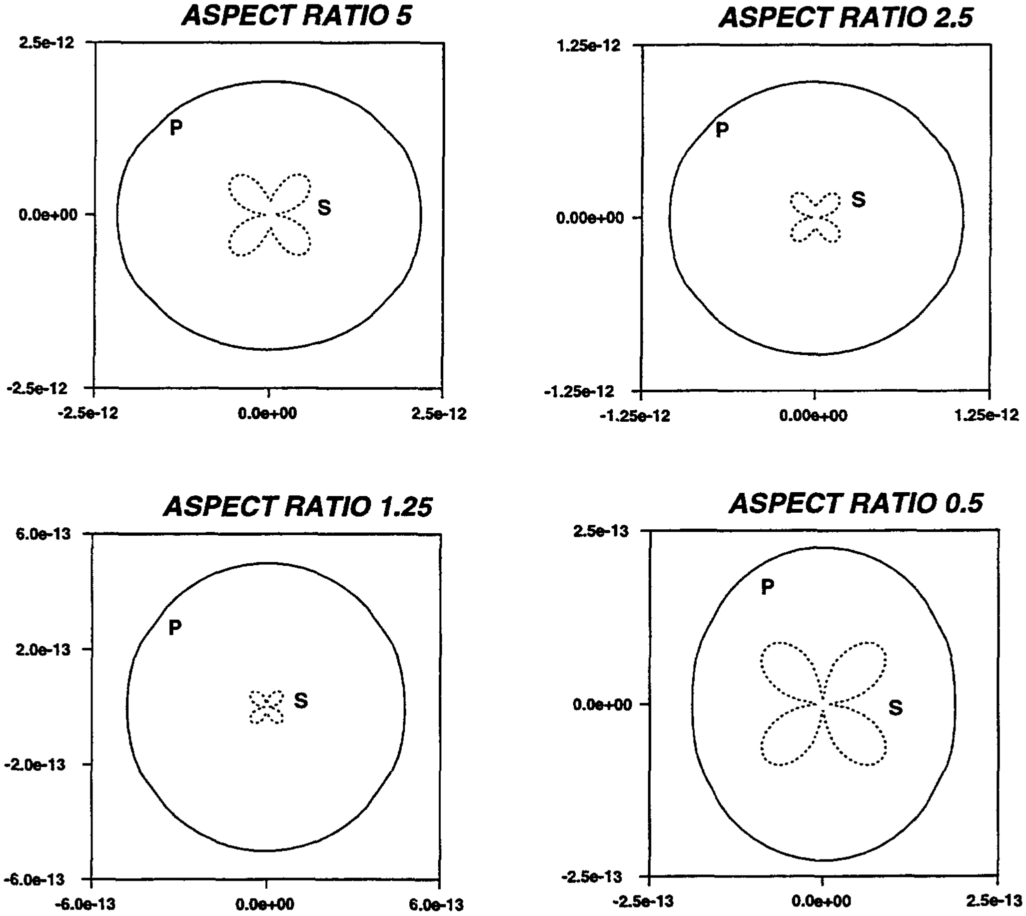

Figure 2: Radiation patterns for explosion sources in a hard rock formation (velocities specified in the text). Results are shown for several values of the cavity aspect ratio to demonstrate the effect of variations in cavity shape on far-field seismic wave radiation.

shear wave decreases relative to the compressional wave, and the compressional wave pattern becomes more isotropic. When the aspect ratio is less than one, the shear wave amplitude increases once again. In the end however, the changes suggest that the shear wave amplitude will be less than the P-wave at frequencies representative of regional seismic wave propagation, a conclusion also reached by Stevens *et al.* (1991). Ben-Menahem and Mikhailov (1995), on the other hand, find a much larger S-wave amplitude using an approximate analytic solution that maps the cylindrical cavity surface onto an equivalent spherical cavity. The differences in results may be caused by this mapping for a cylindrical cavity, which is not very sphere-like.

### Transversely isotropic formation

The implementation of the BEM simulation assumes axial symmetry of both the cavity model and the wave propagation. A transversely isotropic medium can be incorporated into the modeling as long as the axis of symmetry of the formation is parallel to the axis of symmetry of the cylindrical cavity. An example of this situation would be a medium containing aligned

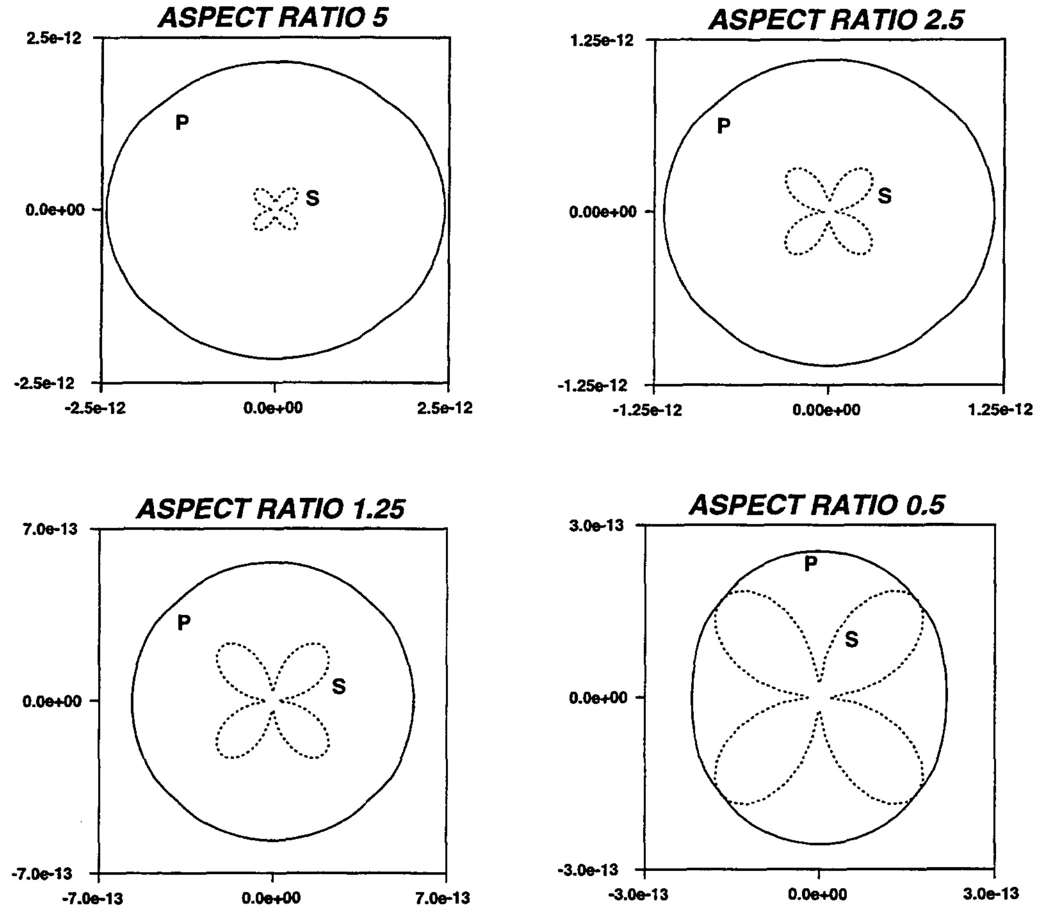

Figure 3: Radiation patterns for explosion sources in a fractured hard rock formation. This medium, with elastic constants shown in Table 1, contains cracks aligned perpendicular to the cavity axis. It is therefore transversely isotropic. At large aspect ratio, the S-wave radiation is weaker than in the isotropic formation (Figure 2).

vertical cracks, a material that is effectively anisotropic with a horizontal axis of symmetry (e.g., Crampin, 1981). If the axis of a tunnel is horizontal and is oriented perpendicular to the vertical cracks in such a transversely isotropic medium, we can model the radiation patterns with the current implementation of the BEM. The elastic moduli of the medium can be computed using the theory of Hudson (1980, 1981). We take the elastic properties of the uncracked rock to be the same as the isotropic formation considered above and set the crack density to $\xi = na^3 = 0.05$ ($n$=number density of cracks, $a$=radius of the penny-shaped cracks). We also assume that the cracks are dry (empty). Table 1 displays the elastic constants of both the isotropic material and the composite medium with aligned cracks.

Radiation patterns for the cylindrical cavities in this formation are displayed in Figure 3. The results are somewhat surprising in that the shear waves are somewhat *weaker* than in the isotropic medium for cavities with large aspect ratio. Theoretical results show that a point source in an anisotropic material can have generate a very large shear wave (e.g., Ben-Menahem *et al.*, 1991). Since the ideal point source explosion in an isotropic medium will generate no shear waves at all, this is a dramatic change, and our initial intuition might suggest that shear waves should also be large for the cavity sources. However, our results show that when the explosion is located in the cylindrical cavity, the shape of the cavity alters the effective moment tensor corresponding to the source in such a way that the resulting wave field resembles the classic isotropic explosion more than the cylindrical cavity source in an isotropic medium!

### Regional synthetic seismograms

Although the radiation patterns show that the amplitude of the shear wave is not too large (Figures 2, 3), the effects of these sources at regional distances is not necessarily obvious. In order to more directly predict the effects of such source within layered models of the crust, we computed discrete wavenumber synthetic seismograms (e.g., Bouchon, 1982). The layer thickness and velocities are shown in Table 2. The receivers were located on the free surface at a distance of 500 km from the explosion source, and the full range of azimuths was considered to see how displacement fields would depend on propagation direction relative to the cavity axis. The explosion source itself was incorporated within the modeling by estimating the moment tensor producing the radiation pattern for the aspect ratio 5 cavity in Figure 2 (tests show that the moment tensor representation does not depend too much on the velocity outside the cavity).

The seismograms show that the vertical component seismograms do not change much with azimuth, and they look essentially the same as the seismograms computed for an explosion point source. However, unlike the classical explosion model, this cylindrical cavity source does generate SH-wave energy that has a strong azimuthal amplitude variation (Figure 4). Hence, even though the S-wave is not too strong at the source, the development of the *Lg* wavetrain is still efficient enough that the cavity shape has a detectable influence at regional distances. We have also computed 3-D finite difference synthetic seismograms for an earth model essentially the same as the one described in Table 2, except with a randomly irregular Moho. In this case, the effect of the cavity shape still is more effective in generated a transverse component signal than is the Moho roughness.

## CONCLUSIONS AND RECOMMENDATIONS

The BEM provides a useful means of simulating the seismic wavefields generated by explosion sources in cylindrical cavities. Unlike the finite difference method, which requires a discretization of the entire earth model, an implementation of the BEM requires only that the boundary and the cavity be divided into elements, allowing an efficient simulation of the elastic waves at large distances from the source cavity. Results for cavities in a hard rock situation show that though there are significant variations in amplitudes of P-waves at very high frequencies (e.g., 150 Hz), at frequencies more typical of regional wave propagation, only a modest change in amplitude is predicted. Likewise, only a relatively small shear wave is predicted. An explosion source located in a cylindrical cavity in a transversely isotropic formation can have shear wave amplitudes even lower than in similar isotropic media. Incorporation of these explosion sources into discrete wavenumber models of regional propagation suggest that there still will be some azimuthal variation of *Lg* wave amplitude depending on propagation direction relative to the cavity axis (when the cavity is horizontal).

Future work should include further testing of these theoretical predictions against observed field data. The best scenario would be tests against data collected where a source cavity

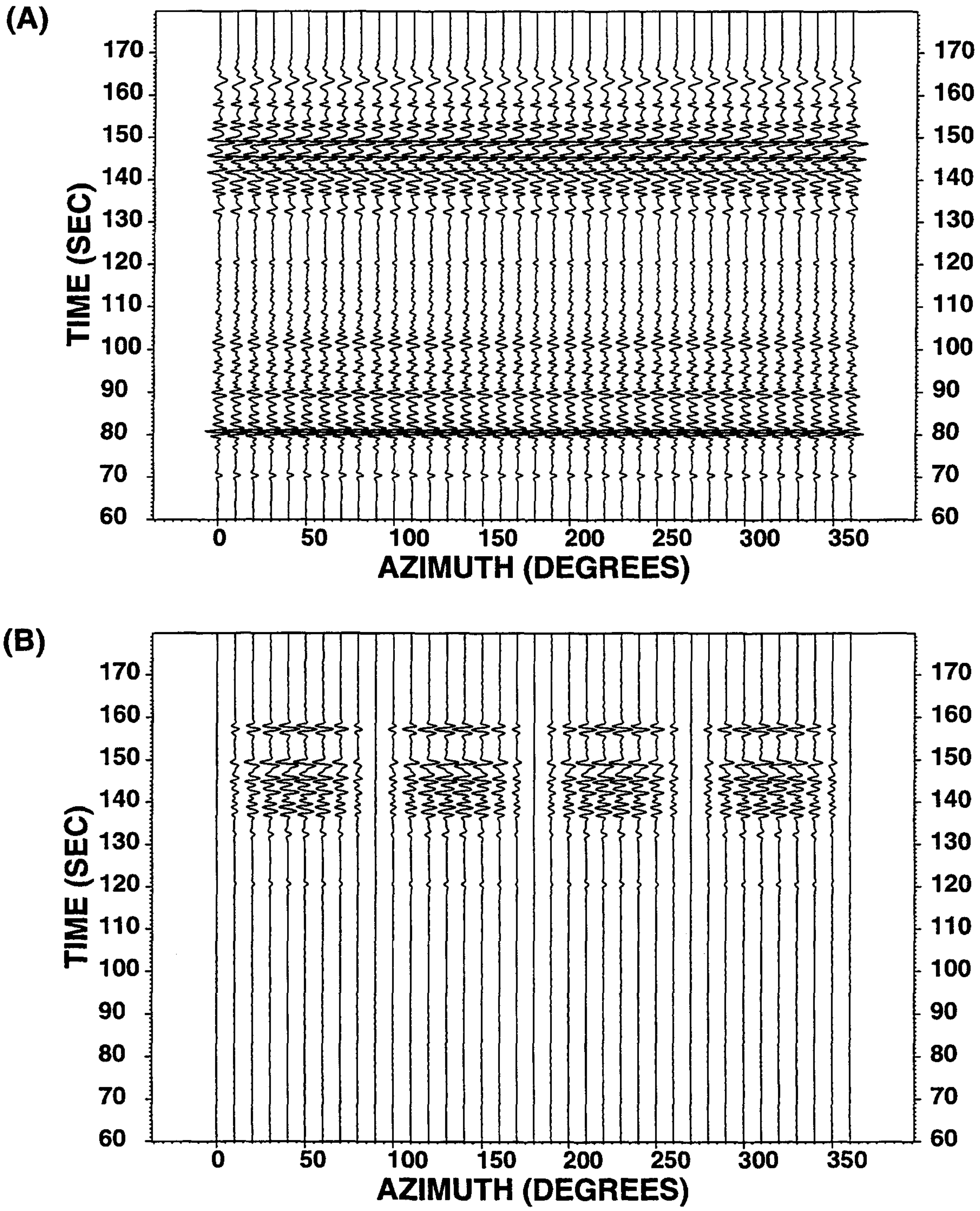

Figure 4: Synthetic seismograms computed for the earth model described in Table 2. Receivers were located at a constant offset of 500 km and cover the full range of azimuth to show the effect of a source located in a cylindrical cavity on regional wave propagation. Both component plots have the same plotting gain applied for direct comparison of amplitudes. The cavity axis direction corresponds to 0°. (A) Vertical component. (B) Transverse component.

orientation and approximate shape is known, as well as the location of the source within the cavity. Another valuable task is further consideration of simulations where we include equivalent moment tensors in simulation of regional wave propagation in order to explore the effects of the explosion sources on such features as *Lg* wave amplitude. The overall goal is to better develop our understanding of near source effects in order to better understand the influence on regional seismic wave propagation.

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| Parameter | Background Value (GPa) | Perturbed Value (GPa) |
|---|---|---|
| $C_{11}$ | 35.2 GPa | 33.8 GPa |
| $C_{33}$ | 35.2 | 26.0 |
| $C_{13}$ | 13.9 | 10.3 |
| $C_{44}$ | 10.6 | 9.53 |
| $C_{66}$ | 10.6 | 10.6 |
| $\rho$ | 2.2 g/cm$^3$ | 2.2 |

Table 1: Elastic constants for the background medium and a medium with aligned cracks computed using the theory of Hudson (1980, 1981). The background parameters correspond to P and S-wave velocities of 4000 m/s and 2200 m/s, respectively, and the perturbed values were obtained using a crack density of 0.05.

| Layer thickness | $V_p$ | $V_s$ | Density |
|---|---|---|---|
| 1 km | 5.3 km/sec | 3.1 km/sec | 2.4 g/cm$^3$ |
| 12 | 6.1 | 3.5 | 2.6 |
| 22 | 6.6 | 3.8 | 2.7 |
| *(half space)* | 8.1 | 4.7 | 2.8 |

Table 2: Crustal velocity model used to compute the synthetic seismograms in Figure 4.