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STRESS WAVES AND COHESIVE FAILURE IN A FINITE STRIP SUBJECTED TO TRANSIENT LOADING

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New solutions are obtained herein for the dynamic mechanical response of a one-dimensional finite elastic strip that is cohesively bonded to a rigid substrate. Analytic solutions are obtained for a variety of cohesive zone constitutive behaviors including linear elastic, viscous, viscoelastic, and cohesive zones with evolving internal damage. Results are compared to numerically obtained predictions using the explicit finite element code DYNA3D [1]. It is shown that the analytic and numerical results are essentially indistinguishable, and validate the implementation of the damage dependent cohesive zone model [2] into DYNA3D.

INTRODUCTION

A considerable amount of research has been recently focused on the subject of dynamic crack propagation in solid continua, both experimentally [3], [4], [5], and theoretically [6],[7]. As is well known, in the case wherein the object is ductile, the application of the J-integral [8] may be erroneous due to path dependence in the energy released during crack growth. Indeed, while the Griffith criterion may still apply to crack propagation [9], it is possible that the energy required for crack propagation may not even be a material constant, but is more likely a material property that is both rate and history dependent [10]. Thus, it is quite likely that a new breed of fracture models will need to be developed in order to predict crack propagation in these complicated media.

While several models have been previously proposed for predicting fracture of viscoelastic media [10], [11], few have proposed a methodology whereby the fracture toughness may be measured a priori. One model that has been proposed utilizes a micromechanical analysis to build a cohesive zone (c.z.) model [12], [13], thereby resulting in a cogent methodology for determining the fracture toughness of viscoelastic media [14],[15]. This model results in a set of constitutive equations for the plane ahead of the crack tip that is of the following general form in one-dimension,

$$T=(1-\alpha)\int_{0}^{t}E^{c}(t-\tau)\frac{\partial\lambda}{\partial\tau}\partial\tau, \qquad \frac{\partial\alpha(t)}{\partial t}=f(\alpha(t),\lambda(t)), \tag{1}$$

where T is the traction vector across the cracks faces, $\lambda = u^c/\delta$ is the dimensionless Euclidean norm of the crack opening displacement vector, α is an internal variable representing microscale damage ahead of the crack tip, and $E^c(t)$ and δ are material properties that can be measured in laboratory experiments. Because this model is micromechanically based, it includes a methodology for obtaining the material parameters experimentally [16]. Predictions of energy release rates during crack growth have been made with this model [17], [18], [19], for the case of quasi-static crack propagation.

The authors have imbedded the fully three-dimensional nonlinear viscoelastic c.z. model described above into DYNA3D for the purpose of predicting the propagation of multiple cracks in viscoelastic media under dynamic loading conditions. In order to verify this new subprogram within the existing algorithm, it has been propitious to obtain analytic solutions for several problems that could be compared to computationally generated solutions. Analytic results are obtained herein for one-dimensional finite elastic media subjected to transient stress boundary conditions at one boundary, and with c.z. type boundary conditions at the other boundary. The most interesting cases involve c.z.'s that carry load up to some time, and then lose their ability to sustain additional loading, thus simulating, at least one dimensionally, crack propagation. The analytic results are compared to predictions made with DYNA3D, utilizing an algorithm previously developed [2], [20], and recently implemented in DYNA3D. In the following sections we present the analytic results and compare them to those predicted by DYNA3D.

ELASTIC COHESIVE ZONE

We now examine the transient behavior of a finite strip which is permitted to debond from the rigid boundary at $x = 0$. The debond 'layer', can also be viewed as a one-dimensional c.z., imbued with an arbitrary constitutive behavior which governs the relative motion between the strip and the rigid boundary. It is interesting to conjecture that a wide range of interesting behaviors can be predicted from such a model since in c.z.'s with infinite stiffness, i.e. $E^c = \infty$ the behavior of the strip will be that essentially of the fixed-free system. Alternatively, if $E^c = 0$, the strip will behave as a free-free system. We are interested in examining the behaviors for c.z.'s where $0 < E^c < \infty$, and $E^c(t) = const$.

For the solution to this boundary value problem, we write the wave equation using dimensionless variables, and boundary conditions,

$$\frac{\partial^2 u(x,t)}{\partial x^2} = \frac{\partial^2 u(x,t)}{\partial t^2}, \qquad E\frac{\partial u(0,t)}{\partial x} = ku(0,t), \qquad E\frac{\partial u(1,t)}{\partial x} = p(t) = p_o H(t), \tag{2}$$

where, $k = E^c / \delta$. The dimensionless displacements and stresses in the finite strip are,

$$u(x,t) = F_2(t-x) + F_1(t+x), \qquad \sigma(x,t) = E[-F_2'(t-x) + F_1'(t+x)] \tag{3}$$

The d'Alembert functions are,

$$F_1(t) = L^{-1}\{\overline{F}_1(s)\} = \frac{p_o}{E}\sum_{n=0}^{\infty} L^{-1}\{\frac{1}{s^2}(\frac{s-a}{s+a})^n\}H(t-2n-1), \tag{4}$$

$$F_2(t) = L^{-1}\{\overline{F}_2(s)\} = \frac{p_o}{E}\sum_{n=0}^{\infty} L^{-1}\{\frac{1}{s^2}(\frac{s-a}{s+a})^{n+1}\}H(t-2n-1), \tag{5}$$

where L^{-1} is the inverse Laplace transform operator, and $a = lk / E$. The analytical solution for the stress at the center $x = 0.5$ of an elastic strip with an elastic c.z., and subjected to a tensile Heaviside step loading is illustrated using dimensionless variables in Figure 2 which also compares well with the DYNA3D solution using 30 hexahedral elements through the thickness. The analytical and DYNA3D solutions are plotted using Maple 8 [21], with $n = 30$ terms, $k = 10^5\ lb_f/in$, $E = 30 \times 10^6\ psi$, $\rho = 7.33 \times 10^{-4}\ lb_f s^2/in^4$, $l = 30$ *inches*, or $a = 0.1$.

Generally speaking, the DYNA3D predictions of the transient stress at the center of the elastic strip are not as accurate as the displacements. Early in the stress history, stresses tend to be over predicted in the vicinity of the stress jump discontinuities, whereas as time progresses these discontinuities are artificially damped; this behavior is common in explicit codes that utilize bulk viscosity methods for shock capture. Precision of the DYNA3D stresses can be greatly enhanced by increasing the number of elements through the thickness of the finite strip [22].

VISCOUS COHESIVE ZONE

The boundary conditions for the viscous c.z. problem are written as,

$$E\frac{\partial u(0,t)}{\partial x} = \eta \dot{u}(0,t), \qquad E\frac{\partial u(1,t)}{\partial x} = p(t) = p_o H(t), \tag{6}$$

where $\eta = \eta^c / E^c \delta$. Space restrictions do not permit presentation of the d'Alembert functions here, but the dimensionless analytical solution for the stress at the center $x = 0.5$ of an elastic strip with a viscous c.z., subjected to a tensile Heaviside step loading is illustrated in Figure 3, which also compares well with the DYNA3D solution. The analytical and DYNA3D solutions are plotted using $\eta = 10\, lb_f s/in^2$.

VISCOELASTIC COHESIVE ZONE

The elastic and viscous solutions from the previous two sections can be combined to generate a viscoelastic (Voigt model) c.z. solution. The boundary conditions for this problem are thus,

$$E\frac{\partial u(0,t)}{\partial x} = ku(0,t) + \eta \dot{u}(0,t), \qquad E\frac{\partial u(1,t)}{\partial x} = p(t) = p_o H(t), \tag{7}$$

Space restrictions do not permit presentation of the d'Alembert functions here, but the dimensionless analytical solution for the stress at the center $x = 0.5$ of an elastic strip with a viscoelastic c.z., subjected to a tensile Heaviside step loading is illustrated in Figure 4, which also compares well with the DYNA3D solution.

COHESIVE ZONE WITH GROWING DAMAGE

Finally, we consider an example of a c.z. that has within it evolving microstructure that degrades the stiffness of the c.z. such that decohesion eventually results. In this case the boundary conditions are as follows,

$$E\frac{\partial u}{\partial x}(0,t)=(1-\alpha)\int_0^t E^c(t-\tau)\frac{\partial \lambda}{\partial \tau}\partial \tau, \quad E\frac{\partial u}{\partial x}(1,t)=p(t)=p_o H(t), \tag{8}$$

together with damage evolution law of the form,

$$\frac{\partial \alpha}{\partial t}=k_1^c+k_2^c\lambda^{n^c}, \tag{9}$$

where the superscripted variables are material constants. Note from equation (8) above that when α attains its maximum value of unity, the traction at the left end of the strip will become null, thus resulting in rigid body motion of the strip in the direction of the externally applied loading on the other boundary. Due to the nonlinearity in this problem, analytic solutions have been obtained only for a special case with the following conditions on the left boundary of the strip,

$$\alpha=\begin{Bmatrix} 0 & t\le t_f \\ 1 & t>t_f \end{Bmatrix}. \tag{10}$$

It is to be noted that (10) represents a special case of (9) that physically represents instantaneous crack growth at time $t = t_f$. This solution represents a limiting case that is nevertheless useful for validating the computational algorithm. In order to obtain a solution to the above problem for the elastic c.z. with failure, the following boundary conditions,

$$E\frac{\partial u(0,t)}{\partial x}=ku(0,t)H(t-t_f), \qquad E\frac{\partial u(1,t)}{\partial x}=0, \tag{11}$$

are adjoined with those in (4) to derive the following d'Alembert functions,

$$F_1(t)=-a\sum_{n=0}^{\infty}H(t-2n-2-t_f)\int_{t_f}^{t-2n-2}u(0,\tau)d\tau, \quad F_2(t)=-a\sum_{n=0}^{\infty}H(t-2n-t_f)\int_{t_f}^{t-2n}u(0,\tau)d\tau, \tag{12}$$

where, $a = lk/E = 0.1$, and $u(0,t)$ is the boundary displacement using the d'Alembert functions (4), and (5) that were derived from the problem for an elastic c.z. without failure.

The solution to the problem with the elastic c.z. with failure is illustrated in Figure 5, which compares favorably with the DYNA3D solution. This solution is obtained using $k = 10^5\ lb_f/in$, $E = 30 \times 10^6\ psi$, and $t_f = 10$. As expected, the stress solution at position $x = 0.5$ up to the failure time, $t = t_f$ is identical to that derived for the elastic c.z. without failure; compare for example, Figure 2 with Figure 5 for times $t < t_f$. After failure, $t > t_f$, the stress wave traverses the "failed" finite elastic strip and remains unaltered in form for all time. The solutions derived in this section are general in form and can predict the transient failure behavior of finite elastic strips with arbitrary values of the post-failure stiffness characterized by the constant $a = lk/E$.

CONCLUSIONS

The authors have utilized a previously developed c.z. model, reported by the second author and coworkers, to predict the dynamic response of one-dimensional solid continua subjected to transient stress loading. These predictions have been made both analytically and computationally, with essentially indistinguishable results for all problems considered.

Quasistatic and dynamic solutions to crack propagation boundary value problems do exist in the open literature. However, the analytical solutions obtained herein for the case of a c.z. in a finite strip are entirely new. As such, these results have been used to validate the implementation of our fully three-dimensional c.z. model into DYNA3D for predicting dynamic crack growth in complex three-dimensional structures. It remains for careful experimental investigations to determine whether this approach to the prediction of crack propagation in viscoelastic media is accurate.

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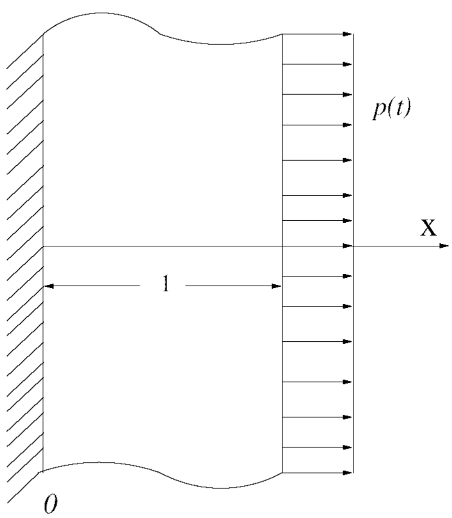


Figure 1. Finite elastic strip subjected to dynamical tractions with c.z. at $x = 0$.

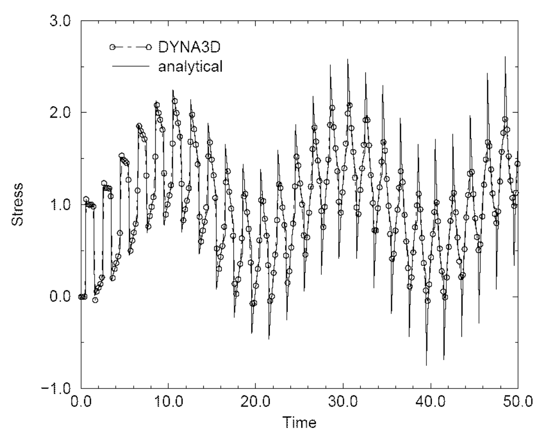
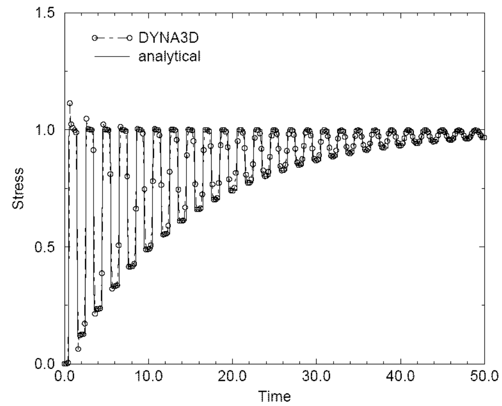


Figure 2. Stress at the center of an elastic strip with an elastic c.z.



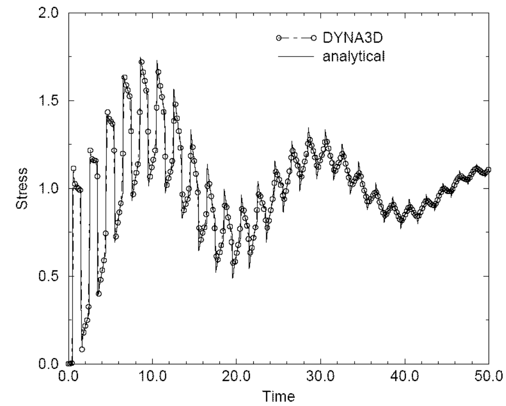


Figure 3. Stress at the center of an elastic strip with a viscous c.z.

Figure 4. Stress at the center of an elastic strip with a viscoelastic c.z.

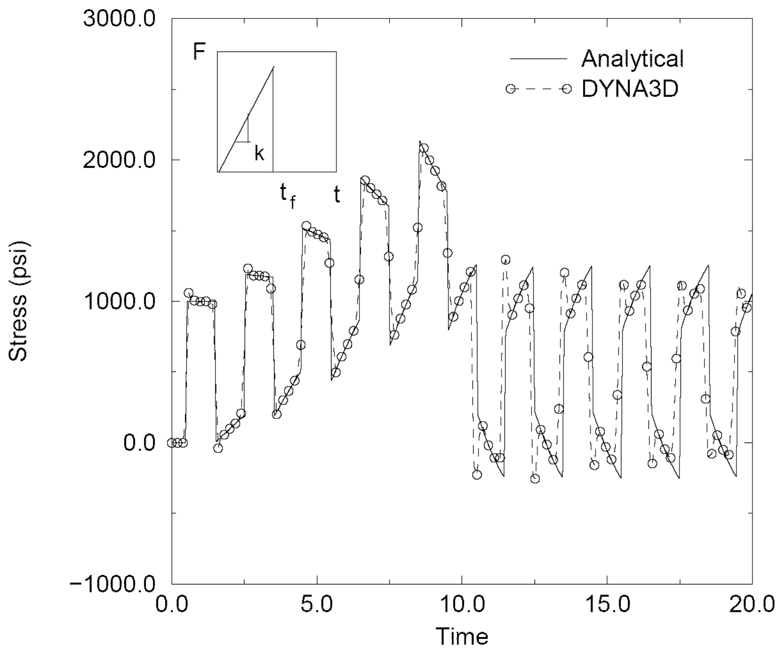


Figure 5. Stress at the center of an elastic strip with an elastic c.z. with failure.