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# THE ANALYTICAL METHOD OF INVESTIGATION OF FARADAY CHIRAL MEDIA

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## ABSTRACT

A plane-parallel homogeneous bianisotropic plate under an oblique incidence of a plane harmonic wave is considered. The bianisotropy axis is not coincide with a normal to a plate. The propagation and reflection coefficients are found in the analytical form.

## INTRODUCTION

Electromagnetic of chiral and bianisotropic media is developing very extensively.. Wave behavior within infinite bianisotropic media have been analyzed in [1]. The phenomena at an interface between isotropic achiral and isotropic chiral materials have been studied in [2]. In that paper reflection and transmission by a chiral slab have been considered. Waves refracted by interface between an isotropic achiral and a bianisotropic materials have been investigated in [3]. The problem of reflection and propagation for an omega-slab has been considered in [4]. The problem of realization of soft-and-hard surfaces have been studied in [5]. Analogous problem has been studied in [6]. A bianisotropic slab under a normal incidence of a plane harmonic wave is investigated analytically in [7]. In this paper a bianisotropic slab under an oblique incidence of a plane harmonic wave is studied analytically. It is assumed also that the bianisotropy axis is not co-inside with a normal to a slab.

## STATEMENT OF THE PROBLEM

The homogeneous lossless bianisotropic medium described by the constitutive relations

$$\bar{\mathbf{D}} = \bar{\epsilon}\bar{\mathbf{E}} + \bar{\zeta}\bar{\mathbf{B}} \quad \bar{\mathbf{H}} = \bar{\xi}\bar{\mathbf{E}} + \bar{\mu}^{-1}\bar{\mathbf{B}} \quad (1)$$

is considered in this paper. The constitutive relations (eqn.1) contain four constitutive dyadics,  $\bar{\epsilon}$ ,  $\bar{\xi}$ ,  $\bar{\zeta}$ ,  $\bar{\mu}$ , in gyrotropic form:

$$a = \begin{vmatrix} a_{xx} & -ja_{xy} & 0 \\ ja_{xy} & a_{xx} & 0 \\ 0 & 0 & a_{zz} \end{vmatrix} \quad (2)$$

Our purpose here is to find the propagation and reflection coefficients of the homogeneous plane-parallel plate under an oblique incidence of a plane harmonic wave. It is assumed also that the bianisotropy axis is not coincide with a normal to the plate. For this it is necessary to obtained the wavenumbers of the refracted waves, to write the translation matrix for a plate, and to express the reflected and propagating fields as functions of the incident field.

## METHOD

The wavenumbers of the refracted waves are obtained by using the dispersion relation for the infinite bianisotropic medium and Snell's law taking into account the geometry

of the problem:

$$\begin{cases} a_4 k_{\text{refr } i}^4 + a_3 k_{\text{refr } i}^3 + a_2 k_{\text{refr } i}^2 + a_1 k_{\text{refr } i} + a_0 = 0 \\ \cos \alpha = \frac{1}{k_{\text{refr } i}} \left( \cos \beta \sqrt{k_{\text{refr } i}^2 - k_{\text{inc}}^2} \sin^2 \alpha_{\text{inc}} + k_{\text{inc}} \sin \beta \sin \alpha_{\text{inc}} \cos \psi \right) \end{cases} \quad (3)$$

where  $\beta$  is the angle between the bianisotropy axis and a normal to the interface,  $\psi$  is the angle between the incidence plane and the plane including the bianisotropy axis and a normal to the interface. the coefficients in the first equation (3) are:

$$\begin{aligned} a_4 &= A_{44} \cos^4 \alpha + A_{42} \cos^2 \alpha + A_{40}; \quad a_3 = A_{33} \cos^3 \alpha + A_{31} \cos \alpha; \\ a_2 &= A_{22} \cos^2 \alpha + A_{20}; \quad a_1 = A_{11} \cos \alpha; \quad a_0 = A_{40}. \end{aligned} \quad (4)$$

$A_{kl}$  are the coefficients expressed in terms of the constitutive dyadic.

After algebraic transformations we obtain the dispersion relation for refracted waves in the form:

$$c_8 k^8 + c_6 k^6 + c_4 k^4 + c_2 k^2 + c_0 = 0. \quad (5)$$

The translation matrix can be written using the well-known technique [8]:

$$\mathbf{L}(d) = \sum_{i=1}^4 \begin{pmatrix} \gamma_{11}^i \mathbf{M}_i & \gamma_{12}^i \mathbf{M}_i & \gamma_{13}^i \mathbf{M}_i & \gamma_{14}^i \mathbf{M}_i \\ \gamma_{21}^i \mathbf{M}_i & \gamma_{22}^i \mathbf{M}_i & \gamma_{23}^i \mathbf{M}_i & \gamma_{24}^i \mathbf{M}_i \\ \gamma_{31}^i \mathbf{M}_i & \gamma_{32}^i \mathbf{M}_i & \gamma_{33}^i \mathbf{M}_i & \gamma_{34}^i \mathbf{M}_i \\ \gamma_{41}^i \mathbf{M}_i & \gamma_{42}^i \mathbf{M}_i & \gamma_{43}^i \mathbf{M}_i & \gamma_{44}^i \mathbf{M}_i \end{pmatrix}, \quad (6)$$

where

$$\mathbf{M}_i = \begin{pmatrix} \cos(k_z i d) & -j \sin(k_z i d) \\ -jk_z i \sin(k_z i d) & \cos(k_z i d) \end{pmatrix}; \quad \gamma_{m,l}^i = 2h_{2m+1,2i+1} \frac{\det(\mathbf{B}_{2l+1,2i+1})}{\det(\mathbf{B})}. \quad (7)$$

$\mathbf{B}$  is the  $8 \times 8$  matrix obtained directly from Maxwell's equations.  $\mathbf{B}_{2l+1,2i+1}$  is the minor of the element with the indices  $2l+1, 2i+1$  of the matrix  $\mathbf{B}$ .  $\mathbf{M}_i$  is the  $2 \times 2$  matrix analogous to the one for an isotropic medium.

$\mathbf{L}(d)$  is  $8 \times 8$  in general case, but considering the particular cases it is possible to obtain  $4 \times 4$  translation matrix. This matrix (6) relates the field components and them derivations at both surfaces of the bianisotropic plate. The tangential field components are continuous at an interface, but for them derivations it is necessary to write the surface matrix. One must relate the field components and them derivations at both sites of an interface. Such matrix can be found directly from Maxwell's equations

$$\mathbf{U}_{\text{bianis}} = \mathbf{I} \mathbf{U}_{\text{isotr}} \quad (8)$$

$\mathbf{U}$  is the column-matrix including the tangential field components and them derivations. Therefore it is possible to write

$$\begin{pmatrix} E_{xm}^{prop}(d) \\ -j\sigma_2 E_{xm}^{prop}(d) \\ E_{ym}^{prop}(d) \\ -j\sigma_2 E_{ym}^{prop}(d) \\ -E_{ym}^{prop}(d)/\rho_2 \\ j\sigma_2 E_{ym}^{prop}(d)/\rho_2 \\ E_{xm}^{prop}(d)/\rho_2 \\ -j\sigma_2 E_{xm}^{prop}(d)/\rho_2 \end{pmatrix} = \mathbf{L}_1(d) \begin{pmatrix} E_{xm}^{inc}(0) + E_{xm}^{refl}(0) \\ -j\sigma_1 (E_{xm}^{inc}(0) - E_{xm}^{refl}(0)) \\ E_{ym}^{inc}(0) + E_{ym}^{refl}(0) \\ -j\sigma_1 (E_{ym}^{inc}(0) - E_{ym}^{refl}(0)) \\ -(E_{xm}^{inc}(0) - E_{xm}^{refl}(0))/\rho_1 \\ j\sigma_1 (E_{xm}^{inc}(0) + E_{xm}^{refl}(0))/\rho_1 \\ (E_{xm}^{inc}(0) - E_{xm}^{refl}(0))/\rho_1 \\ -j\sigma_1 (E_{xm}^{inc}(0) + E_{xm}^{refl}(0))/\rho_1 \end{pmatrix} \tag{9}$$

where  $\mathbf{L}_1(d) = \mathbf{I}^{-1} \mathbf{L} \mathbf{I}$ . After algebraic transformations the  $2 \times 2$  matrices relating incident, refracted and propagating fields are written as following:

$$\begin{pmatrix} E_x^{refl} \\ E_y^{refl} \end{pmatrix} = \mathbf{R} \begin{pmatrix} E_x^{inc} \\ E_y^{inc} \end{pmatrix}, \quad \begin{pmatrix} E_x^{prop} \\ E_y^{prop} \end{pmatrix} = \mathbf{T} \begin{pmatrix} E_x^{inc} \\ E_y^{inc} \end{pmatrix} \tag{10}$$

**CONCLUSIONS**

The bianisotropic plate described by the constitutive relations (1) with all dyadic in gyrotropic under an oblique incidence of a plane harmonic wave is studied. The translation matrix of a homogeneous bianisotropic slab is written in analytical form. The reflection and propagation matrix of plane-parallel plate under an oblique incidence of a plane harmonic wave is found in analytical form.

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