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DIFFRACTION ON THE EIGEN WAVES ON AN INCLINED MEDIUM INTERFACE IN THE WAVEGUIDES WITH METALLIC BOUNDS

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ABSTRACTS

The electromagnetic wave diffraction problems on an inclined medium interface with a metallic plate and without it in the plane waveguide and in the rectangular waveguide are considered. It is shown that these problems can be reduced to boundary value problems for the Helmholtz equation or for the Maxwell system in a bounded rectangular domain.

INTRODUCTION

Let the infinite cylindrical waveguide with metallic bounds be separated by some surface into two parts filled up by the dielectric with different dielectric indexes. Let the eigen electromagnetic wave run on the medium interface (from the left, for example). It is necessary to calculate the scattered field.

The main idea of our method is to isolate some bounded domain containing the medium interface and to replace the rejected semi-infinity parts of waveguide by boundary conditions of special form. These conditions can be obtained by solving over-determined Cauchy type problems [1] for the Helmholtz equation or for the Maxwell system.

SEMI-INFINITE PLANE WAVEGUIDES

Consider the auxiliary boundary value problems. It is necessary to seek in the semi-strips $S_1 : -\infty < x < 0,\ 0 < z < h$ and $S_2 : g < x < +\infty,\ 0 < z < h$ of the plane (x,z) solutions of the Helmholtz equation

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial z^2} + k_j^2\, u(x,z) = 0 \qquad (1)$$

belonging to the classes of outgoing into infinity solutions satisfying the conditions

$$u(x,0+0) = 0, \quad u(x,h-0) = 0. \qquad (2)$$

It is shown in [2] (see also [1]) that

Lemma 1 *The solvability conditions for these problems can be written down in the non-local integral form*

$$u(0-0,z) = -i\int_0^h \frac{\partial u}{\partial x}(0-0,t)\ K_1(t,z)\ dt, \quad 0 < z < h, \qquad (3)$$

$$u(g+0,z) = i\int_0^h \frac{\partial u}{\partial x}(g+0,t)\ K_2(t,z)\ dt, \quad 0 < z < h, \qquad (4)$$

$$K_j(t,z)=\frac{2}{h}\sum_{m=1}^{\infty}\frac{1}{\gamma_{j,m}}\sin\frac{m\pi t}{h}\sin\frac{m\pi z}{h},\quad \gamma_{j,m}=\sqrt{k_0^2\varepsilon_j-(m\pi/h)^2}.$$

The boundary value problems (1), (2) and

$$u(0-0,z)=u_{00}(z),\ \frac{\partial u}{\partial x}(0-0,z)=u_{01}(z)\quad or\quad u(g+0,z)=u_{g0}(z),\ \frac{\partial u}{\partial x}(g+0,z)=u_{g1}(z)$$

define all eigen waves of semi-limited waveguides outgoing into infinity.

INCLINED BARRIER IN THE PLANE WAVEGUIDE

Let $I: z=x\tan\theta,\ \tan\theta=g/h,\ 0<x<g$ be the inclined medium interface in the plane waveguide. We give the typical conjunction conditions on the I

$$u(x+0,hx/g)=u(x-0,hx/g),\quad \frac{\partial u}{\partial n}(x+0,hx/g)=\frac{\partial u}{\partial n}(x-0,hx/g),\quad 0<x<g, \tag{5}$$

here $\partial/\partial n=\partial/\partial x\sin\theta-\partial/\partial z\cos\theta$. We denote $u^0(x,z)$ the potential function of the external wave and $u(x,z)$ the unknown potential function.

Theorem 1 *The diffraction problem for TE-wave on the inclined interface medium is equivalent to the boundary value problem for the Helmholtz equation (1) in the classes of outgoing into infinity solutions with boundary conditions (2), (5) and*

$$u(0+0,z)=-i\int_0^h\frac{\partial u}{\partial x}(0+0,t)\ K_1(t,z)\ dt+2u^0(0-0,z),\quad 0<z<h, \tag{6}$$

$$u(g-0,z)=i\int_0^h\frac{\partial u}{\partial x}(g-0,t)\ K_2(t,z)\ dt,\quad 0<z<h. \tag{7}$$

In [3] the numerical method for solving this boundary value problem is constructed and is investigated by abstract approximate scheme [4].

SEMI-INFINITE RECTANGLE WAVEGUIDES

Let $W_1: 0<x<a,\ 0<y<b,\ -\infty<z<0$ and $W_2: 0<x<a,\ 0<y<b,\ 0<z<+\infty$ be two semi-infinity rectangular domains (semi-beams). We consider the Cauchy type problem for the Maxwell system

$$rotH=i\omega\varepsilon_0\varepsilon E,\quad rotE=i\omega\mu_0\mu H \tag{8}$$

in the domain W_1 with boundary conditions

$$E_x=0,\ E_z=0\ \ for\ \ x=0,\ x=a;\quad E_y=0,\ E_z=0\ \ for\ \ y=0,\ y=b; \tag{9}$$

$$E_\tau(x,y,0)=e(x,y),\quad H_\tau(x,y,0)=h(x,y), \tag{10}$$

here E_τ, H_τ are the tangential components of vectors E, H.

Lemma 2 *The Cauchy type problem (8) – (10) has a solution in the class of outgoing into infinity solutions if and only if*

$$e(x,y) = \int_0^a \int_0^b K(s,t;x,y)\, h(s,t)\, ds\, dt, \qquad (11)$$

here $K(s,t;x,y)$ *is the functional matrix* 2×2 *with elements of the form*

$$const \sum_n \sum_m \frac{nm}{\gamma_{nm}} \varphi_{nm}(s,t)\, \psi_{nm}(x,y),$$

$$\varphi_{nm}(s,t),\ \psi_{nm}(x,y) = \sin\frac{\pi ns}{a}\cos\frac{\pi mt}{b} \quad or \quad \cos\frac{\pi ns}{a}\sin\frac{\pi mt}{b},$$

$$\gamma_{nm} = \sqrt{k_0^2 \varepsilon_l - (\pi n/a)^2 - (\pi m/b)^2}.$$

INCLINED MEDIUM INTERFACE IN THE RECTANGULAR WAVEGUIDE

Let the planes $x = 0$, $x = a$, $y = 0$, $y = b$ be the walls of rectangular waveguides and let the rectangle $P: z = x\tan\theta_1 + y\tan\theta_2$ separate the waveguide into two parts with different dielectric indexes.

Theorem 2 *The diffraction problem of electromagnetic wave on the inclined interface medium is equivalent to the boundary value problem for the Maxwell system (8) with boundary conditions (9), non-homogeneous boundary condition of the form (11) on the sides* $z = 0$, $z = c$ *and conjunction conditions on the rectangle* P.

METALLIC PLATE ON THE MEDIUM INTERFACE

If the metallic plate M is placed on the medium interface, then the conjunction conditions are to be replaced by the following ones: the tangential components of vector E are equal to zero on M and the tangential components of vectors E and H are continuous on the other part of the barrier. This fact is insignificant for the formulations of Theorem 1 and Theorem 2, but the calculation scheme will be more complicate. New unknown variables are added to the set of variables during the direct action of calculation for every node placed on the metallic plate, and size of linear algebraic system will increase.

The numerical method for solving this boundary value problem is constructed and is investigated by abstract approximate scheme also.

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