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# APPLICATION OF THE METHOD OF AUXILIARY SOURCES FOR THE ANALYSIS OF PLANE-WAVE SCATTERING BY IMPEDANCE SPHERES

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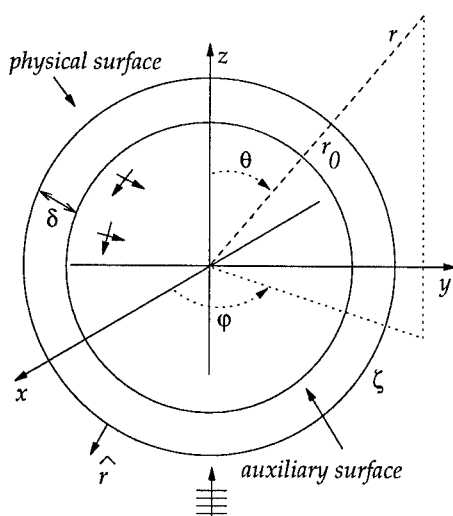
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## ABSTRACT

The Method of Auxiliary Sources (MAS) is applied to 3D scattering problems involving spherical impedance scatterers. The MAS results are compared with the reference spherical wave expansion (SWE) solution. It is demonstrated that good agreement is achieved between the MAS and SWE results.

## INTRODUCTION

The Method of Auxiliary Sources (MAS) is a numerical technique applicable to electromagnetic scattering problems. In the general case, a set of spatially impulsive electric and/or magnetic sources is introduced to radiate an approximation to the unknown scattered field. These so-called auxiliary sources are located on an auxiliary surface, typically conformal to, and enclosed within, the physical surface of the scatterer. Point matching of the boundary condition on the physical surface is enforced to determine the complex amplitudes of the auxiliary sources. The MAS originates from an application of a special case of the Method of Moments (MoM), utilising spatially impulsive expansion and testing functions, to a generalised surface integral equation formulation [1]. An overview of MAS is given in [2]. Utilisation of MAS for numerical solution of various 2D scattering problems and 3D PEC and dielectric scattering problems has been reported earlier [3], [4], [5]. The purpose of this work is to investigate the performance of MAS when the method is applied to scattering problems involving spherical impedance scatterers. The spherical wave expansion (SWE) solution is developed and used as a reference. The scattering problem under investigation is



illustrated in Figure 1, together with the introduced Cartesian and spherical co-ordinate systems. The spherical scatterer of radius  $r_0$  is illuminated by an  $x$ -polarised uniform plane wave ( $\mathbf{E}^i, \mathbf{H}^i$ ) of wavelength  $\lambda$  and propagating in the  $z$ -direction. The Standard Impedance Boundary Condition (SIBC) holds on the physical surface of the scatterer,  $\hat{\mathbf{r}} \times (\mathbf{E} \times \hat{\mathbf{r}}) = \zeta \hat{\mathbf{r}} \times \mathbf{H}$ . The chosen auxiliary sources are pairs of crossed  $\hat{\theta}$ - and  $\hat{\phi}$ -directed electric Hertzian dipoles located on a sphere of radius  $r_0 - \delta$ . The auxiliary source pairs, as well as the matching points, are placed equidistantly in the angular co-ordinates  $(\theta, \varphi)$ . The total number of source pairs is denoted by  $N$ .

Figure 1: The scattering problem geometry

The SWE solution is given by (see [6, Chapter 2] for definitions of symbols)

$$\mathbf{E}^s = k/\sqrt{\eta} \sum_{s,m,n} Q_{s,m,n}^{(3)} \mathbf{F}_{s,m,n}^{(3)}, \mathbf{H}^s = -ik\sqrt{\eta} \sum_{s,m,n} Q_{s,m,n}^{(3)} \mathbf{F}_{3-s,m,n}^{(3)}, Q_{s,m,n}^{(3)} = E^i a/b,$$

$$a = i^{n+1} \sqrt{\pi(2n+1)} (-1)^{4-s-m} R_{3-s,n}^{(3)} R_{s,n}^{(1)} (-\delta_{s,1}(\delta_{m,1} + \delta_{m,-1}) + \delta_{s,2}(\delta_{m,-1} - \delta_{m,1})) -$$

$$i^{n+2} \zeta \eta \sqrt{\pi(2n+1)} (-\delta_{s,1} R_{2,n}^{(1)} R_{2,n}^{(3)} (\delta_{m,1} + \delta_{m,-1}) + \delta_{s,2} R_{1,n}^{(1)} R_{1,n}^{(3)} (\delta_{m,-1} - \delta_{m,1})),$$

$$b = -k/\sqrt{\eta} (-1)^{3-m-s} R_{3-s,n}^{(3)} R_{s,n}^{(3)} + \zeta ik \sqrt{\eta} (-1)^m (R_{3-s,n}^{(3)})^2,$$

$$R_{1,n}^{(3)} \equiv h_n^{(1)}(kr_0), R_{2,n}^{(1)} \equiv j_n'(kr_0), R_{2,n}^{(3)} \equiv h_n^{(1)'}(kr_0).$$

### NUMERICAL RESULTS

We define the boundary condition error (BCE) by

$$BCE \equiv \frac{1}{MN} \sum_{m,n}^M \sum_{i=1}^N |\hat{\mathbf{r}}_{m,n} \times (\mathbf{E}_{m,n} \times \hat{\mathbf{r}}_{m,n}) - \zeta \hat{\mathbf{r}}_{m,n} \times \mathbf{H}_{m,n}| \quad (1)$$

The summation is performed over the scatterer physical surface. Figure 2 shows the BCE as a function of  $N$  for four different scatterer surface impedances  $\zeta$  and three values of  $\delta$  ( $\blacklozenge$ :  $\delta/\lambda=0.2$ ,  $\blacktriangle$ :  $\delta/\lambda=0.5$ ,  $\blacksquare$ :  $\delta/\lambda=0.8$ ). In all cases,  $r_0/\lambda=1$  is chosen. It is observed that the BCE attained for  $\delta/\lambda=0.5$  is the lowest, and that it starts with a rapid decrease, whereafter it attains a constant level.

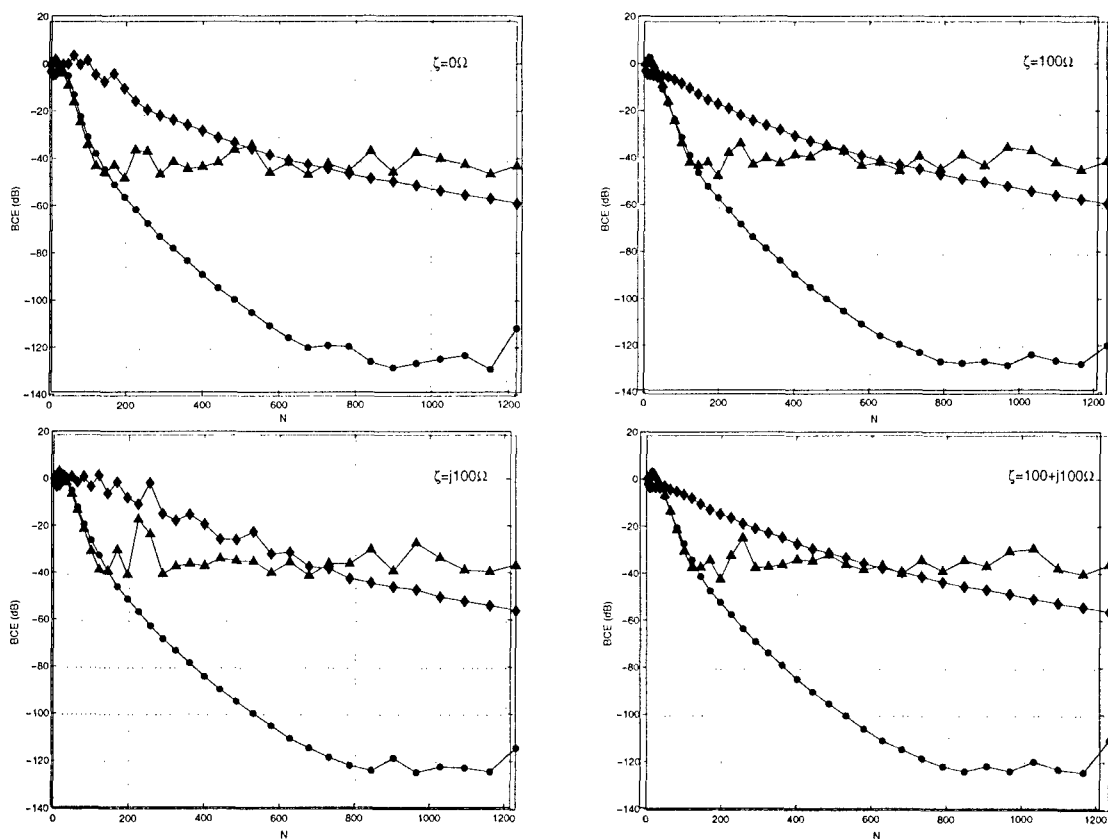


Figure 2

In Figure 3, the normalised bistatic radar cross section (BRCS) results obtained using MAS for four different scatterers and for two different planes of observation ( $\phi=0$  and  $\phi=\pi/2$ ) are compared with the corresponding SWE results. In all cases,  $\zeta=100+j100\Omega$  is chosen. There is a very good correspondence between the MAS and SWE results. The small discrepancies can be diminished further if  $N$  is increased.

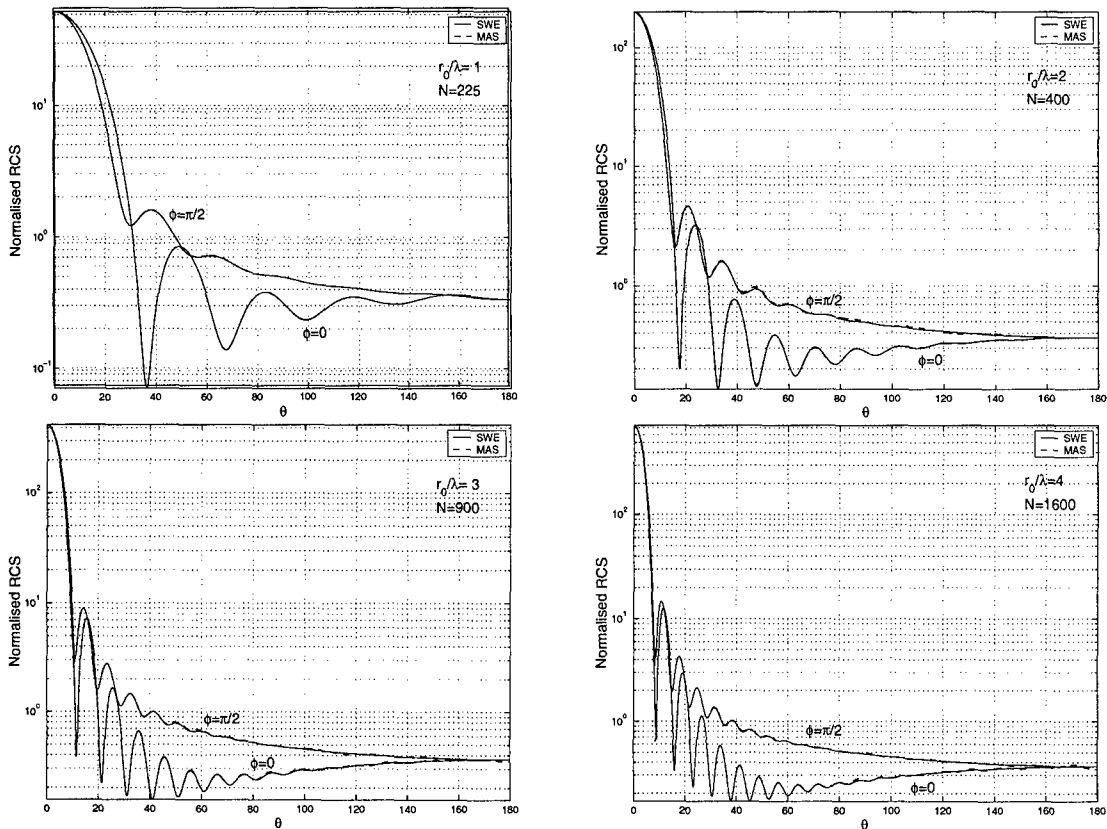


Figure 3

## CONCLUSION

It is seen that the initial decrease in the boundary condition error of the MAS numerical solution is relatively fast. Also, the limiting error level achieved is small enough for the MAS to be able to produce good approximations to exact results at a relatively small computational cost. However, the existence of a finite limit level for the MAS boundary condition error indicates that the present numerical implementation of the method is not convergent. Bistatic radar cross section results obtained by MAS are found to be in good agreement with the reference spherical wave expansion (SWE) solution.

## REFERENCES

- [1] Y. Leviatan et al., IEEE Trans. Antennas Propagat., vol. 36, 1722-1734, Dec. 1988
- [2] D. I. Kaklamani, European Congress on Computational Methods in Applied Sciences and Engineering, Barcelona 11-14 Sep. 2000
- [3] S. Eisler et al., IEE Proceedings, vol. 136, 431-438, Dec. 1989
- [4] H. T. Anastassiou et al., IEEE Trans. Antennas Propagat., vol. 50, 59-66, Jan. 2002
- [5] Y. Leviatan et al., IEEE Trans. Antennas Propagat., vol. 38, 1259-1263, Aug. 1990
- [6] J. E. Hansen, *Spherical Near-Field Antenna Measurements*, Peter Peregrinus 1988