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THE PARTIAL REGION METHOD IN 2-D ELECTROMAGNETIC AND ACOUSTIC PROBLEMS

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ABSTRACT

The mathematical methods, used for solving acoustic and electromagnetic wave problems, defined in two dimensions, are analogous in many cases. The analytical and numerical results of a field in the outside domain of curvilinear rectangle are presented in this paper. Based on the wave equation, boundary conditions and radiation condition the structure of the field is determined. The method of the partial regions is used. For the acceptability criterion of the quantity of the members by which the field is calculated, the fulfillment of boundary conditions on the radiating surfaces and the fulfillment of conjunction conditions between the partial regions were observed. The results are applicable to the optimum design of acoustic and electromagnetic antennas.

INTRODUCTION

It is known that in the 2-D case the Maxwell equations can be transformed in two independent equations for the vectors of electrical and magnetic fields [1]. By this reason in 2-D the solutions of acoustic and electromagnetic problems coincide. Using the technique of partial region [2,3], many interesting problems can be solved and results can be implemented in the two areas.

BOUNDARY-VALUE ANALYSIS AND ANALYTICAL RESULTS

The geometry of the problem is shown in **Fig. 1**. This is an outside boundary value problem, i.e. a problem in the infinite domain. The curvilinear rectangle is limited by the arcs with radii r_1 and $r_2 = a$, and the segments AD and BC. It is assumed that the surfaces $\theta = \pm\theta_0$, $r_1 \le r \le a$ are acoustically rigid. Sound field is generated by the surfaces $r = r_1$ and $r = r_2$, $\theta_0 < |\theta| \le \pi$, on which the distribution of particle velocity is assigned:

$$-\frac{\partial F}{\partial r} = f_1(\theta), r = r_1 \tag{1}$$

$$-\frac{\partial F}{\partial r} = f_2(\theta), r = r_2 \tag{2}$$

where F - velocity potential. In order to analyze a sound field with time dependence factor ***exp*** $(-i\omega t)$, which is created by the radiating body, shown in Fig., the boundary value problem for the Helmholtz equation in cylindrical coordinates must be solved:

$$\Delta F(r,\theta) + k^2 F(r,\theta) = 0 \tag{3}$$

where: $F(r,\theta)$ - velocity potential; $\Delta \equiv \nabla^2$ - Laplace's operator; k - wave number.

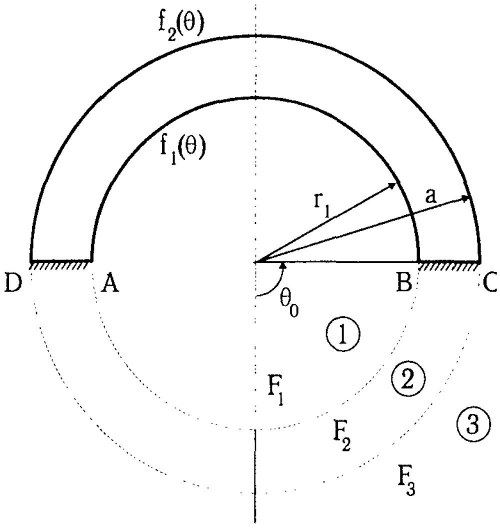

Fig. 1. Geometry of the problem

The whole field is divided in parts. In each partial region there must exist a solution of the Helmholtz equation (3), which satisfies the boundary conditions of some part of the whole surface and the conjunction conditions of the boundaries of the neighboring partial regions [2]. Using the orthogonal properties on the corresponding segments of the functions, which describe the field in regions 1, 2 and 3, the functional equations can be transformed into the following infinite simultaneous linear algebraic set of equations for the complex coefficients, A_n, B_m, C_m, and D_l :

$$\begin{cases} \delta_n J'_n(k\rho a)A_n - \sum\limits_{m=0}^{\infty}\gamma_{\alpha_m n} J'_{\alpha_m}(k\rho a)B_m - \sum\limits_{m=0}^{\infty}\gamma_{\alpha_m n} N'_{\alpha_m}(k\rho a)C_m = -\dfrac{\beta_n}{k} \\ -\sum\limits_{n=0}^{\infty}\gamma_{\alpha_m n} J_n(k\rho a)A_n + \delta_m J_{\alpha_m}(k\rho a)B_m + \delta_m N_{\alpha_m}(k\rho a)C_m = 0 \\ \delta_m J_{\alpha_m}(ka)B_m + \delta_m N_{\alpha_m}(ka)C_m - \sum\limits_{l=0}^{\infty}\gamma_{\alpha_m l} H_l^{(1)}(ka)D_l = 0 \\ \sum\limits_{m=0}^{\infty}\gamma_{\alpha_m l} J'_{\alpha_m}(ka)B_m + \sum\limits_{m=0}^{\infty}\gamma_{\alpha_m l} N'_{\alpha_m}(ka)C_m - \delta_l H_l^{(1)\prime}(ka)D_l = \dfrac{\beta_l}{k} \end{cases} \qquad (4)$$

where $n = m = l = 0, 1, 2, ..., N, ...$; $\alpha_m = \dfrac{m\pi}{\theta_0}$;

$$\delta_n = \begin{cases} 2\pi, & n = 0 \\ \pi, & n > 0 \end{cases}; \quad \delta_m = \begin{cases} 2\theta_0, & m = 0 \\ \theta_0, & m > 0 \end{cases}; \quad \delta_l = \begin{cases} 2\pi, & l = 0 \\ \pi, & l > 0 \end{cases}$$

$$\beta_n = -\frac{2\sin(n\theta_0)}{n}; \quad \beta_l = -\frac{2\sin(l\theta_0)}{l}$$

$$\gamma_{\alpha_m n} = \frac{2\alpha_m}{\alpha_m^2 - n^2}\sin(\alpha_m\theta_0)\cos(n\theta_0) - \frac{2n}{\alpha_m^2 - n^2}\sin(\alpha_m\theta_0)\cos(n\theta_0)$$

$$\gamma_{\alpha_m l} = \frac{2\alpha_m}{\alpha_m^2 - l^2}\sin(\alpha_m\theta_0)\cos(l\theta_0) - \frac{2l}{\alpha_m^2 - l^2}\sin(\alpha_m\theta_0)\cos(l\theta_0)$$

The prime (') means a derivative of the whole argument. Except this, it is accepted that $f_1(\theta) = f_2(\theta) = 1$. This simplification has not meaning of a principle [4].

Solving the set, the complex coefficients A_n , B_m , C_m , and D_l can be obtained. The velocity potential of the field $F(r, \theta)$ in each point of the outside domain of the curvilinear rectangle can be found.

NUMERICAL RESULTS

In **Fig. 2** the modulus of the potential F_1 for determined wave parameters near the geometric focus is shown. As can be seen, the structure of the field is complex and optimization of the main parameters (θ_0,ρ,a) is recommended to obtain the necessary intensity and focus spot. In **Fig. 3** the equipotential curves are drawn. It is clear that real focus spot is shifted ($\theta = 0$, $r = 0,3$). This effect can be explained with edge points diffraction and radiation of the surface *CD*.

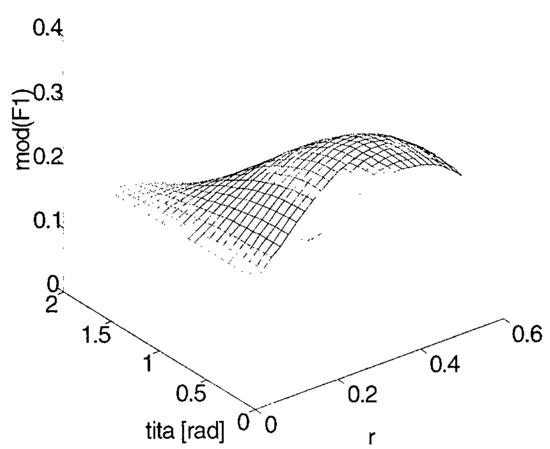

Fig.2.Modulus of potential F_1

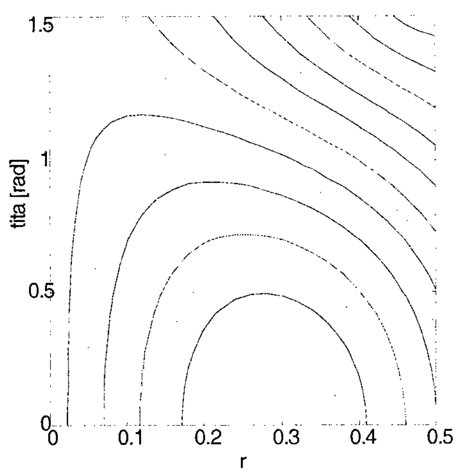

Fig.3.Equipotential curves

CONCLUSION

The results, obtained above, can be used to the optimum design of acoustic and electromagnetic antennas in 2-D space.

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