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AXIAL SYMMETRIC WAVE DIFFRACTION BY A CIRCULAR WAVEGUIDE CAVITY

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INTRODUCTION

In this paper, we shall consider a three-dimensional (3-D) cavity formed by a finite circular waveguide with a planar termination at the open end, and analyze the axial symmetric diffraction problem by means of the Wiener-Hopf technique. The method of solution is similar to that we have developed for the analysis of parallel-plate waveguide cavities [1], but is more complicated because of the cylindrical geometry. The time factor is assumed to be $e^{-i\omega t}$ and suppressed throughout this paper.

WIENER-HOPF ANALYSIS OF THE PROBLEM

We consider a 3-D cavity formed by a finite circular waveguide with a planar termination, as shown in Fig. 1, where the cavity surface is perfectly conducting and of zero thickness. The cavity is assumed to be excited by a hypothetical generator with voltage of unit amplitude across an infinitesimally small gap at $z=d(<|L|)$. Thus the applied electric field becomes a uniform ring source given by $e_z^i(\rho=b-0,z)=\delta(z-d)$, where $\delta(\cdot)$ is the Dirac delta function. Let the total field $\phi^t(\rho,z)$ be

$$\phi^t(\rho,z) = \begin{cases} \phi^i(\rho,z)+\phi(\rho,z) \text{ for } 0<\rho<b, \\ \phi(\rho,z) \text{ for } \rho>b, \end{cases} \qquad (1)$$

where $\phi^i(\rho,z)$ is the field excited in an infinitely long circular waveguide due to the ring source, and $\phi(\rho,z)$ is the unknown scattered field satisfying the scalar Helmholtz equation. In the following analysis, we shall assume that the medium is slightly lossy. Applying the method established in our previous papers [1, 2], we derive the transformed wave equations as in

$$\hat{T}\Phi(\rho,\alpha)=0 \text{ in } \rho>b \text{ for } |\tau|<k_2, \qquad (2a)$$

$$\hat{T}\Psi_-(\rho,\alpha)=\alpha f(\rho) \text{ in } 0<\rho<b \text{ for} \tau<k_2, \qquad (2b)$$

$$\hat{T}\left[\Phi_1(\rho,\alpha)+e^{i\alpha L}\Psi_+(\rho,\alpha)\right]=-\alpha e^{-i\alpha L}g(\rho) \text{ in } 0<\rho<b \text{ for } \tau>-k_2, \qquad (2c)$$

where $\hat{T}=d^2/d\rho^2+\rho^{-1}d/d\rho-\gamma^2$, and $\gamma=(\alpha^2-k^2)^{1/2}$ with $\text{Re}\gamma>0$. In (2b,c), $f(\rho)$ and $g(\rho)$ are unknown inhomogeneous terms. The terms on the left-hand sides of (2a-c) are the Fourier transforms of the functions appearing in (1), and are defined by

$$\Phi(\rho,\alpha)=(2\pi)^{-1/2}\int_{-\infty}^{+\infty}\phi(\rho,z)e^{i\alpha z}dz \qquad (3)$$

with $\alpha=\text{Re}\alpha+i\text{Im}\alpha(\equiv\sigma+i\tau)$ and

$$\Phi(\rho,\alpha) = \Psi(\rho,\alpha) + \Phi_1(\rho,\alpha) - \Phi^i(\rho,\alpha), \tag{4}$$

$$\Psi(\rho,\alpha) = e^{-i\alpha L}\Psi_-(\rho,\alpha) + e^{+i\alpha L}\Psi_+(\rho,\alpha), \tag{5}$$

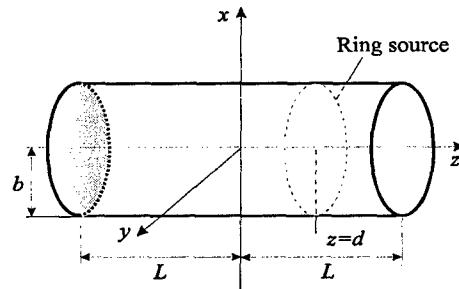
Fig. 1. Geometry of the problem.

where $\Psi_-(\rho,\alpha) = \Phi_-(\rho,\alpha) + Q_-(\rho,\alpha)$, $\Psi_+(\rho,\alpha) = \Phi_+(\rho,\alpha) + Q_+(\rho,\alpha)$, $\qquad$ (6)

$$\Phi_\pm(\rho,\alpha) = \pm\frac{1}{\sqrt{2\pi}}\int_{\pm L}^{\pm\infty}\phi(\rho,z)e^{i\alpha(z\mp L)}dz, \quad \Phi_1(\rho,\alpha) = \frac{1}{\sqrt{2\pi}}\int_{-L}^{+L}\phi^i(\rho,z)e^{i\alpha z}dz. \tag{7}$$

Here $\Phi^i(\rho,\alpha)$ and $Q_\pm(\rho,\alpha)$ are known functions. In (4)-(7), the subscripts ' $\pm$ ' imply that the functions are regular in the half-planes $\tau \gtrless \mp k_2$, whereas the subscript '1' implies an entire function. In addition, the function $\Phi(\rho,\alpha)$ defined by (3) is regular for $|\tau| < k_2$.

Solving (2a-c) for the unknown functions on the left-hand sides with the aid of the radiation condition and the boundary condition on the termination, we may derive a scattered field representation in the Fourier transform domain. Taking into account the boundary conditions at $\rho = b$, we derive the desired Wiener-Hopf equation. Applying the factorization and decomposition procedure, we finally obtain the exact solution with the result that

$$E_-(b,\alpha) + M_-(\alpha)\left[J_E^{(1)}(\alpha) + \sum_{n=1}^{\infty}\frac{M_+(i\gamma_n)E_-(b,-i\gamma_n)}{i\gamma_n(\alpha - i\gamma_n)}\right] = M_-(\alpha)R_-(\alpha), \tag{8}$$

$$E_+(b,\alpha) - M_+(\alpha)\left[J_E^{(2)}(\alpha) + \sum_{n=1}^{\infty}\frac{e^{-4\gamma_n L}M_+(i\gamma_n)E_+(b,i\gamma_n)}{i\gamma_n(\alpha + i\gamma_n)}\right] = M_+(\alpha)R_+(\alpha) \tag{9}$$

with

$$J_E^{(1,2)}(\alpha) = \frac{1}{2}\int_{\pm k}^{\pm i\infty \pm k}\frac{e^{\pm 2i\nu L}M_\pm(\nu)E_\pm(b,\nu)}{\gamma_\nu^2 K_0(\gamma_\nu b)[K_0(\gamma_\nu b) - i\pi I_0(\gamma_\nu b)]}\frac{d\nu}{\nu - \alpha}, \tag{10}$$

where $R_\pm(\alpha)$ and $M_\pm(\alpha)$ are known functions, and $E_\pm(b,\alpha)$ are unknown functions denoting the Fourier transform of the z-component of the electric field at $\rho = b$. In (10), $I_0(\cdot)$ and $K_0(\cdot)$ are the modified Bessel functions of the first and second kinds, respectively. Equations (8) and (9) provide the exact solution of the Wiener-Hopf equation, but are formal since they contain the branch-cut integrals $J_E^{(1,2)}(\alpha)$ with unknown integrands as well as infinite series with the unknown coefficients $E_\pm(b,\pm i\gamma_n)$ for $n = 1,2,3,\cdots$. Applying the approximation procedure developed in [1, 2], we can derive an approximate solution convenient for numerical computation, but the details are omitted here.

NUMERICAL RESULTS AND DISCUSSION

We shall now present numerical examples of the far field pattern for various physical parameters to discuss the scattering characteristics of the cavity. We have computed electric field components $|e_z^*| = |e_z(\rho,z)R|$ and $|e_\rho^*| = |e_\rho(\rho,z)R|$ as $R \to \infty$, where (R,θ) is the cylindrical coordinates defined by $z = R\cos\theta, \rho = R\sin\theta$ for $0 < \theta < \pi$.

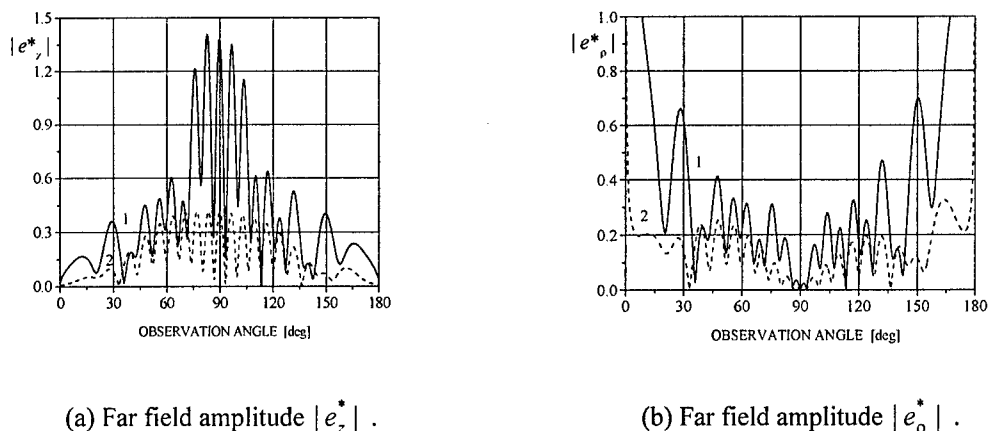(a) Far field amplitude $|e_z^*|$.   (b) Far field amplitude $|e_\rho^*|$.

Fig. 2. Radiation pattern of electric field components e_z^* and e_ρ^* for $d/L = 0$.
Line 1: $2b = 10\lambda, L/b = 1$. Line 2: $2b = 2\lambda, L/b = 5$.

Figure 2 shows the far field amplitude of e_z^* and e_ρ^* as a function of observation angle. It is seen from the figure that the radiated field oscillates rapidly with an increase of the cavity dimension. This sharp oscillation for larger cavities is due to the effect of the multiple diffraction between the aperture and the back corner. Next we evaluate the power of TM waves radiated from the cavity through the elementary surface $dS = \sin\theta d\theta d\varphi$. The radiated power P is found to be

$$P(\theta) \sim 0.5(\varepsilon/\mu)^{1/2} |e_z(\rho,z)/\sin\theta|^2 R^2 .$$

We investigate the power radiated from the cavities as a function of the observation angle and cavity parameters. We also show that, with an increase of the cross section of the cavity, dominant peaks of oscillations of the radiated power are formed in the region $75° < \theta < 105°$. The focusing effect of the radiated power in the direction $\theta = 90°$ is found for short cavities.

ACKNOWLEDGMENTS

REFERENCES
[1] Kobayashi, K. and S. Koshikawa, "Wiener-Hopf analysis of the radar cross section of parallel-plate waveguide cavities," *Tech. Rep.*, no. KK96-3-8, Chuo Univ., Tokyo, March 1996.

[2] Kuryliak, D. B., S. Koshikawa, K. Kobayashi, and Z. T. Nazarchuk, "Wiener-Hopf analysis of the axial symmetric wave diffraction problem for a circular waveguide cavity (*invited paper*)," *Proc. 2000 International Workshop on Direct and Inverse Wave Scattering* (September 25-29, 2000, Gebze, Turkey), pp. 2.67-2.81.