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DNS OF A STRAINED TURBULENT AXIAL VORTEX

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Abstract.

Direct numerical simulations (DNS) of a strained turbulent axial vortex are performed using a b-spline/Fourier spectral DNS code. Extending previous DNS further in time reveals that a global measure of the turbulent kinetic energy (GTKE) decays after growing to a certain extent, contrary to the earlier picture of unbounded growth. This observation leads to a re-investigation of the strained vortex simulations performed by Qin (Qin 1998). It is observed that the GTKE shows unbounded growth after a long period of decay. The Widnall instability seems to have been captured. Large eddy simulations (LES) shall be performed in the near future to study the vortex at higher Reynolds numbers.

1. Introduction

The work presented includes direct numerical simulations (DNS) of a strained turbulent axial vortex. This effort is part of an on-going project on the DNS and LES of turbulent axial vortices with and without the effect of an external strain field. The overall project is motivated by the wake hazard problem for commercial aircraft where the wing-tip vortices behind large aircraft present a serious safety concern for the following aircraft. In order for progress to be made in the prediction and control of these vortices, it will be helpful to have a better understanding of how turbulence within these vortices behaves and how the turbulence affects the distribution of vorticity. Also, current turbulence models do not perform well for strongly rotating flows and the current study provides needed data for the improvement of turbulence models for such flows.

There have been several experimental studies of turbulent vortices, which are discussed in the papers by Devenport *et al.* (1996) and Leweke and Williamson (1998). Numerical simulations of an isolated turbulent axial vortex have been presented in a series of papers by Ragab *et al.* (1995). More recently, DNS have been done by Coppens (1998), Qin (1998) and Blaisdell and Qin (1999), who considered the effects of an external strain field. The effect of strain was included because each vortex of a trailing vortex pair induces a strain field on the opposite vortex. The external strain field causes the streamlines to become elliptical (see figure 1) and introduces an instability (the short wavelength instability is known as the Widnall instability) which affects the development of turbulence within the vortex. A DNS has been done by Orlandi *et al.* (1998) of a trailing vortex pair corresponding to the experiment of Leweke and Williamson.

The objectives of the current study are to re-investigate the simulations of Qin by continuing them for larger times. It is found that the turbulence growth seen by Qin is a transient phenomenon, which is followed by decay. However, ultimately the instability due to strain does take over and the turbulence grows. The appropriate Reynolds number is $Re_\Gamma = \Gamma/\nu$, where Γ is the circulation of the vortex and ν is the kinematic viscosity.

2. Numerical Method

The vortex under consideration is time developing and, therefore, homogeneous in the axial (z) direction. This corresponds to a vortex far downstream, under the approximation that the flow changes slowly in the streamwise direction. Also, the azimuthal direction is naturally periodic. This enables the use of a Fourier spectral method in the streamwise and azimuthal directions. Basis spline polynomials (b-splines) are used in the radial direction. These provide spectral-like accuracy and are C^{k-2} continuous, where k is the order of the splines being used. The current study uses 4^{th} order b-splines. Also, since b-splines have local support on a given interval (see figure 2), they lead to sparse matrices that can be efficiently stored and solved.

Loulou (1996) developed a computer program to study turbulent pipe flow that solves the incompressible Navier-Stokes equations in cylindrical coordinates using a Galerkin formulation with a Fourier spectral method in z and θ and a b-spline method in r. The program was modified by Qin (Qin 1998) to solve the vortex problem and to run on an IBM SP2 parallel computer. This code has been further modified to perform large eddy simulations of a turbulent axial vortex using the dynamic SGS model which is implemented using the approach described in Eshpuniyani and Blaisdell (1999). LES will be performed in the near future, while the current study focuses on DNS results. The grid used for the DNS presented here contains 96 θ-modes, 160 z-modes and 128 basis splines in the radial direction.

3. Results and discussion

Qin (1998) presented four direct numerical simulations of a strained turbulent axial vortex (see table 1). STRN1 and STRN2 included a wake like axial velocity profile, while STRN3 and STRN4 considered a vortex without any axial flow. Qin observed an unbounded monotonic growth of the global turbulent kinetic energy (GTKE) in STRN2. Here GTKE is the non dimensionalized volume averaged turbulent kinetic energy defined as

$$\mathcal{K} = \frac{1}{\pi R^2 L_z {V_0}^2} \int_0^R \int_0^{2\pi} \int_0^{L_z} k r dz d\theta dr \,, \tag{1}$$

where $V_0 = V_z(r)|_{r=\infty} - V_z(r)\Big|_{r=0}$ is the initial mean axial velocity deficit, R is the radius of the computational domain, L_z is the length of the computational domain in the streamwise direction, and k is the local turbulent kinetic energy per unit mass.

Upon continuing the simulation STRN2, it was discovered that the GTKE eventually decays in a manner similar to the DNS at a lower Reynolds number (STRN1). Thus it was realized that the Widnall instability had not been captured (see figure 3), contrary to the conclusion drawn earlier based on a DNS that was not extended far enough in time. This led to a fresh look at the DNS results by Qin and an inquiry into the possible reasons behind the lack of success in capturing the Widnall instability. As this instability is introduced by the external strain field, the simulations of the strained vortex without an axial wake (STRN3 and STRN4) were revisited in order to study the effect of the strain field without any possible interference from the axial flowfield during the simulation.

The fact that these DNS were carried out at low Reynolds numbers was considered as a possible reason for not capturing the Widnall instability. Hence, DNS of a strained vortex at higher Reynolds numbers were performed. However, eventual decay of GTKE was observed in all these simulations (see figure 4). Here $Re = V_0 R/\nu$ is the computational Reynolds number.

Shariff *et al.*(1994) present the evolution of modal energies for a three-dimensional vortex ring simulation. The evolution of all but a few unstable modes show a striking resemblance to the GTKE behavior in Figure 3. However, after a long period of time the initially stable modes also show unbounded growth. Since a strained vortex has the same short wavelength instability as a vortex ring (Widnall *et al.* 1974), the possibility of a similar phenomenon occurring in the current DNS was explored by extending the

DNS at the largest Reynolds number considered to a longer period of time. It is observed that the GTKE begins to grow exponentially after a long period of decay (see figure 5).

Upon this observation, STRN4 was extended in time to see if the Widnall instability can indeed be captured at the Reynolds numbers considered by Qin. It is observed that the GTKE behaves in a manner similar to the high Reynolds number simulation (see figure 6). The GTKE grows exponentially after a long period of decay. The Widnall instability seems to have been captured.

STRN2 is currently being extended in time to see if the decay of the wake like axial velocity profile eventually stabilizes the flow, as in the case of an isolated vortex (Qin 1998), or whether the Widnall instability dominates.

4. Conclusions

Eventual unbounded growth of global turbulent kinetic energy seems to indicate that the Widnall instability is captured for a strained vortex without axial flow. An interesting feature revealed in these simulations is that the GTKE has a transient growth phase followed by a long period of decay before it grows unbounded. Thus if the simulations are not continued for a long enough time, one may reach a wrong conclusion of eventual decay of turbulence, and misunderstand the flow physics. A previous DNS of a strained vortex with a wake-like initial axial velocity profile is currently being extended in time.

In future work spectra shall be computed to ensure that the simulations are well resolved. Flow visualization shall be carried out to understand the transition process better. Isolated vortex simulations (Qin 1998) show the presence of helical waves during the transition to turbulence, which break down during the re-laminarization process. A similar study shall be performed for the strained vortex simulations.

LES shall be performed to study the vortex at higher Reynolds numbers. The DNS of Qin have a Reynolds number $Re_\Gamma = \Gamma/\nu$ of about $50,000$ which is about three orders of magnitude smaller than that found in the wakes of large commercial aircraft. While LES still cannot reach full scale Reynolds numbers, it will allow the effects of varying Reynolds number to be studied. Also, Reynolds number is particularly important for the case with strain, because viscosity creates a high wave number cut-off for unstable modes. The current DNS are barely able to capture the instability due to strain because the smallest unstable mode has a length scale comparable to the vortex core size. It is desirable to have a wider range of unstable length scales, and this should be achievable with LES.

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TABLE 1. Primary case parameters for the strained vortex (see Qin 1998 for a complete description).

Case	Re_Γ	Axial flow
STRN1	19268	wake
STRN2	29428	wake
STRN3	29428	none
STRN4	58836	none

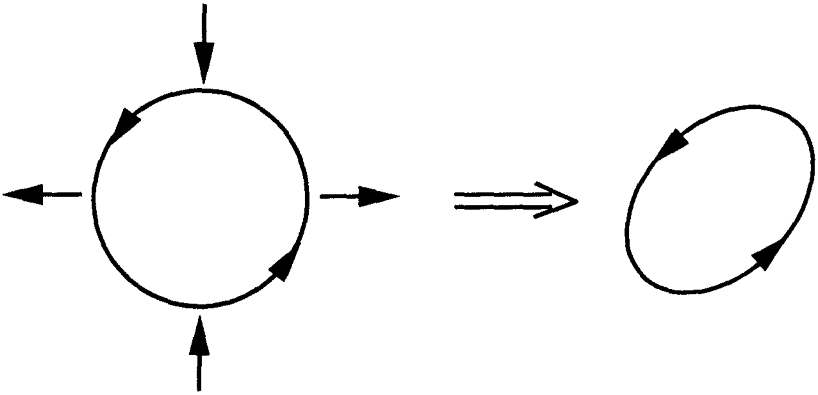

Figure 1. A strained vortex leads to a mean flow with elliptical streamlines.

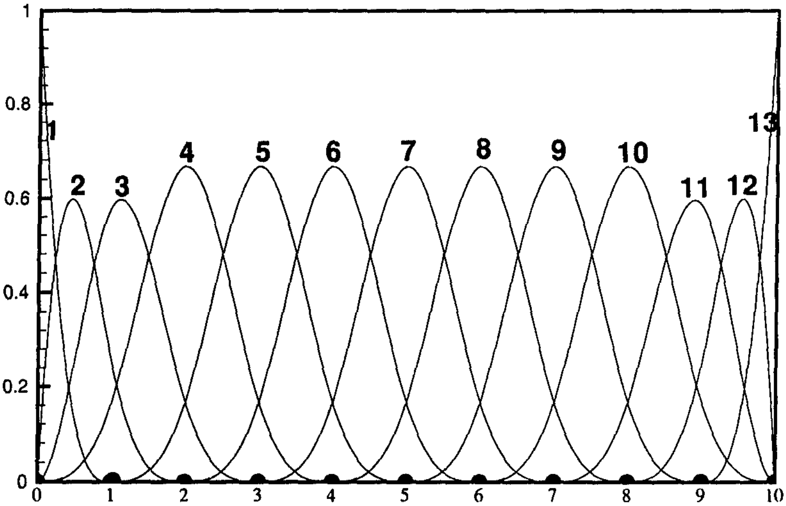

Figure 2. Fourth order B-splines on an 11-knot uniform grid.

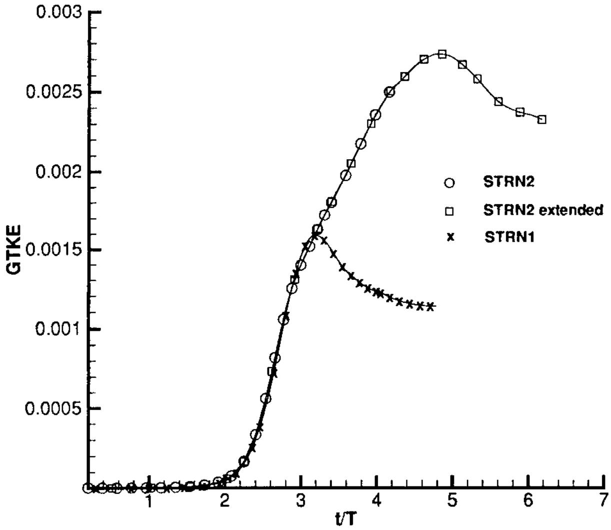

Figure 3. Global turbulent kinetic energy history for STRN 2 by Qin(1998) extended in time.

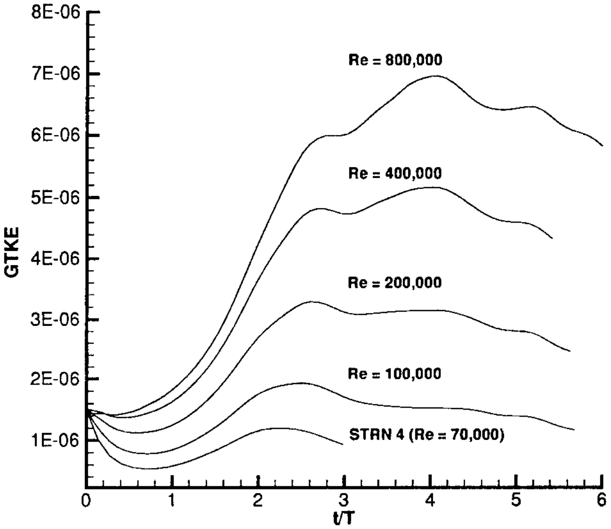

Figure 4. Global turbulent kinetic energy history for a strained vortex without axial flow at higher Reynolds numbers.

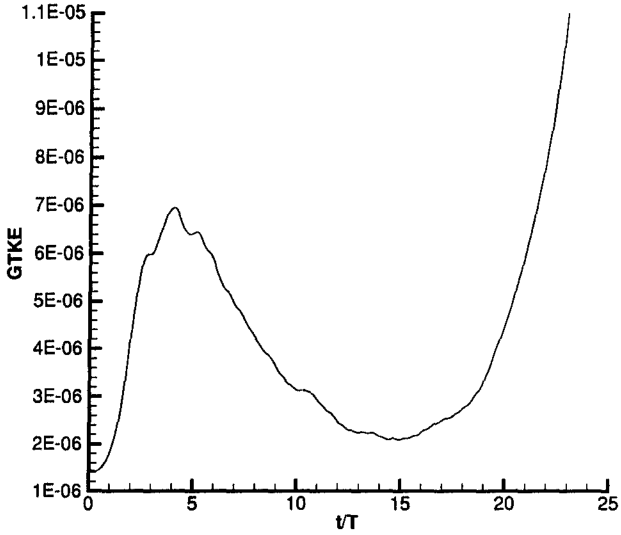

Figure 5. Global turbulent kinetic energy history for a strained vortex without axial flow at Re = 800,000 extended in time.

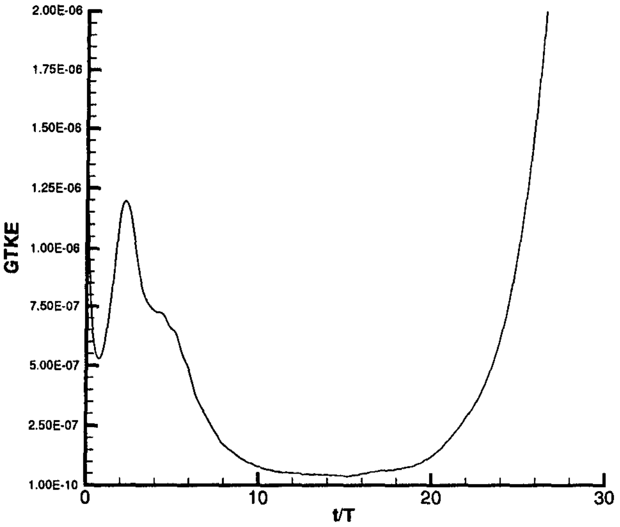

Figure 6. Global turbulent kinetic energy history for STRN4 extended in time.