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LARGE EDDY SIMULATION OF ROTATING TURBULENT CONVECTION USING THE SUBGRID SCALE ESTIMATION MODEL

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Abstract. A large eddy simulation with the estimation subgrid scale model was used to simulate rotating convective flows up to a Rayleigh number of 8.4×10^6. The flow fields demonstrated expected qualitative properties of rotating flow including radially oriented convective plumes. The Nusselt number and mean temperature profiles show the decreased mixing due to rotation and agree well with DNS results. For higher Rayleigh numbers, including truncated Navier- Stokes dynamics resulted in better agreement with DNS.

1. Introduction

Turbulent convective flows under the influence of strong rotation are important in the study of geophysical flows. In the arctic regions, convection and other buoyancy effects drive large scale thermohaline circulations and convective processes which ultimately influence global oceanic circulations. The purpose of the current research is to apply turbulent simulations to rotating turbulent convection in order to better understand convective geophysical flows. The evolution of turbulent convective structures has been studied both experimentally (Fernando *et al.*, 1991; Coates *et al.*, 1995)

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and numerically, including DNS (Julien *et al.*, 1996) and LES with a constant eddy viscosity model (Jones & Marshall, 1993). Initially the growth of buoyant plumes are dominated by convection. When they reach a transition depth h_c (Fernando *et al.*, 1991), the plumes are constrained horizontally due to the rotation. In addition, high rotation causes the plumes to elongate in the radial direction and the distance between plumes to decrease.

The subgrid scale estimation model has been used successfully in both high Reynolds number flows (Domaradzki & Saiki, 1997) and turbulent convection (Kimmel & Domaradzki, 2000). In this study, we have examined the estimation model applied to convective, rotating flows. The incompressible Navier-Stokes equations are spatially filtered to yield the large eddy simulation (LES) equations for rotating turbulent convection. Spatial filtering applied to $f(x)$ is defined by the relation

$$\overline{f}(x) = \int f(x')G(x,x')dx', \tag{1}$$

where G is a given filter function. The LES equations to be solved are

$$\frac{\partial \overline{u}_j}{\partial x_j} = 0 \tag{2}$$

$$\frac{\partial \overline{u}_i}{\partial t} + \frac{\partial \overline{u}_i \overline{u}_j}{\partial x_j} + \sqrt{Ta}\,Pr\,\overline{u}_j \epsilon_{ij3} = -\frac{\partial P}{\partial x_i} + Pr\frac{\partial^2 \overline{u}_i}{\partial x_j \partial x_j} - \frac{\partial}{\partial x_j}\tau_{ij} + Pr\,Ra\overline{T}\delta_{i3} \tag{3}$$

$$\frac{\partial \overline{T}}{\partial t} + \frac{\partial \overline{u}_j \overline{T}}{\partial x_j} = \frac{\partial^2 \overline{T}}{\partial x_j \partial x_j} - \frac{\partial}{\partial x_j}\tau_{\theta j}. \tag{4}$$

In these equations, the flow is assumed to be incompressible, consistent with the Boussinesq approximation, and dependent only on the vertical component of rotation.

The Rayleigh number is defined as

$$Ra \equiv \frac{\alpha g \Delta T H^3}{\nu \kappa},$$

and the Prandtl number as

$$Pr \equiv \nu/\kappa,$$

where ΔT is the temperature difference between the upper and lower surfaces, α is the volumetric thermal expansion coefficient, g is the acceleration due to gravity, and ν and κ are the molecular viscosity and diffusivity,respectively. The equations are nondimensionalized using diffusivity κ and the depth of the flow H. The Taylor number (Ta), which represents the ratio of rotational forces to viscous forces, is defined as

$$Ta \equiv \frac{4\Omega^2 H^4}{\nu^2}.$$

In the estimation model, the subgrid scale stress tensor is computed directly from the definition using the approximated unfiltered velocity field (Domaradzki & Saiki, 1997). An estimate $\tilde{u}_i$ of the unfiltered velocity is obtained by expanding the resolved large scale velocity field, $\overline{u}_i$, to subgrid scales two times smaller than the grid scale. The estimation procedure consists of two steps. The first step utilizes properties of a top-hat filtering operation and the representation of the velocity field in terms of Fourier series. For the second step, the phases associated with the computed smaller scales are adjusted in order to correspond to the small scale phases generated by nonlinear interactions of the large scale field. Once the estimate $\tilde{u}_i$ of the full field u_i is known, the subgrid scale stress tensor is computed directly from the definition

$$\tau_{ij} = \overline{\tilde{u}_i \tilde{u}_j} - \overline{\tilde{u}}_i \overline{\tilde{u}}_j. \tag{5}$$

For convective flows the subgrid scale heat flux $\tau_{\theta i}$ is computed in a similar manner (Kimmel & Domaradzki, 2000).

Finally, to obtain a $\tilde{u}_i$ velocity field that obeys continuity, truncated Navier-Stokes dynamics are imposed by advancing the velocity field in time on the expanded mesh using the large eddy simulation equations without the terms that account for the interactions between the resolved scales and the subgrid scales (Domaradzki et al., 2000). The current study is the first application of the estimation subgrid scale model in an LES of rotating turbulent Rayleigh-Bénard convection. Horiuti (1999) shows that, unlike dynamic models, estimation subgrid scale model with truncated Navier-Stokes dynamics obeys transformation rules required for the subgrid scale stress tensor in a noninertial frame of reference, and the LES results agree well with DNS data for rotating, homogeneous turbulence.

2. Numerical Simulations

LES of rotating turbulent convection between two parallel plates were performed in a three-dimensional rectangular domain. The boundary conditions are periodic in the horizontal direction. No-slip velocity and constant temperature boundary conditions are imposed on the top and bottom boundaries. The convection is driven by an unstable temperature gradient in the vertical direction. The simulations and relevant parameters for this study are shown in Table 1. Case R3t includes the truncated Navier-Stokes dynamics. The LES results shown here were computed using a resolution of 32x32x64, the same vertical resolution as a DNS but one quarter of the horizontal resolution. Exact Nusselt numbers are from DNS results for rotating (Julien et al., 1996) and nonrotating (Kerr, 1996) turbulent convection. The

Rossby number, which is expressed as

$$Ro = \left(\frac{Ra}{PrTa}\right)^{1/2}, \tag{6}$$

measures the importance of rotation. These results were computed on a grid of 32x32x65 for a Rossby number of 0.75, and a Prandtl number of 1. At this Rossby number both buoyancy and rotational effects are important, and the LES correctly reproduces many qualitative features of experimental and DNS results for rotating convection.

TABLE 1. Parameters for LES simulations

case(LES)	Ra	Ta	Nu	Exact Nu
E2	2.5×10^6	0	13	11
R2	2.5×10^6	4.5×10^6	11	10
E3	1.0×10^7	0	19	16.6
R3	8.4×10^6	1.5×10^7	16	14.6
R3t	8.4×10^6	1.5×10^7	13.5	14.6

A measure of the amount of heat transfered between the two plates can be estimated using the Nusselt number which is the ratio of the total heat transfer to the conductive heat transfer. It is defined as

$$Nu = \frac{Q}{\kappa \Delta T / H}$$

where Q is heat flux between the upper and lower surfaces. Since the heat transfer depends on molecular transport and convective transport as a result of fluid motion, the Nusselt number has two components:

$$Nu = \frac{-\kappa d < T > /dz + < w'T' >}{\kappa \Delta T / H}, \tag{7}$$

the latter of which includes both the resolved and subgrid scale heat transfer. The fluctuating temperature T' and vertical velocity w' are variations from the mean

$$T' = T - <T>,$$

$$w' = w - <w>,$$

where $< ... >$ denotes a horizontally averaged quantity. The decreased mixing between the two plates is demonstrated by less heat transfer and

a smaller Nusselt number then the corresponding nonrotating flow. The Nusselt number from the simulation which includes the truncated Navier-Stokes dynamics has better agreement with the exact DNS results.

The influence of rotation on a turbulent convective flow is demonstrated for the temperature field by comparing LES data with results from an equivalent nonrotating flow (Kimmel & Domaradzki, 2000) in Figure 1 for vertical contours and in Figures 2 and 3 for horizontal planes. In both cases, the rotation causes the thermals to elongate in both the vertical direction and the horizontal direction perpendicular to the direction of rotation, as has been demonstrated experimentally (Rossby, 1969).
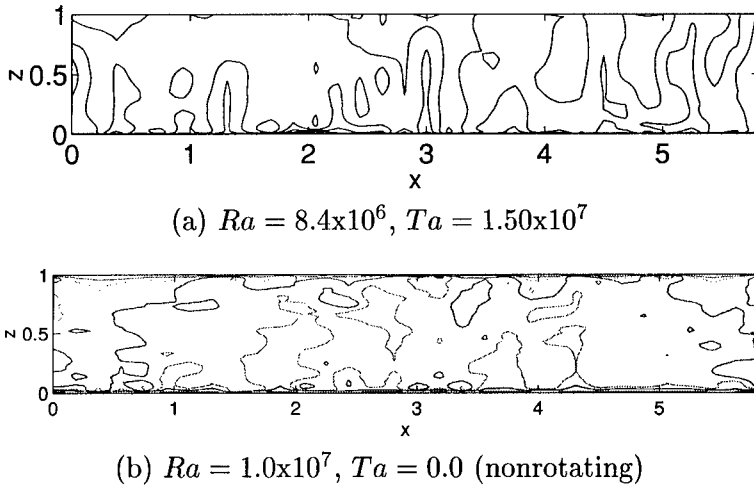

(a) $Ra = 8.4 \times 10^6$, $Ta = 1.50 \times 10^7$

(b) $Ra = 1.0 \times 10^7$, $Ta = 0.0$ (nonrotating)

Figure 1.   Temperature contours from LES data in a vertical plane through center of the domain for rotating and nonrotating turbulent convection.

The LES can correctly reproduce the steeper slope of the mean temperature gradient for nonrotating flow than for rotating flow. As shown in Figure 4, the agreement between the LES and DNS is very good for $Ra = 2.5 \times 10^6$. In Figure 5, mean temperature profiles for simulations at $Ra = 8.4 \times 10^6$ are shown. For the case, the slope of the mean temperature profile from the LES with truncated Navier-Stokes dynamics agrees better with the DNS results than the simulation without it. The difference between the results from these different simulations should be more significant at higher Rayleigh numbers.
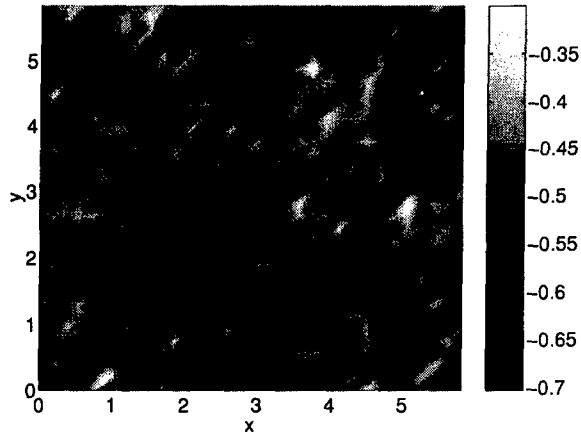
Figure 2. Gray scale plot of the fluctuating temperature in a horizontal plane through the center of the domain for $Ra = 8.4 \times 10^6$ and $Ta = 1.5 \times 10^7$.
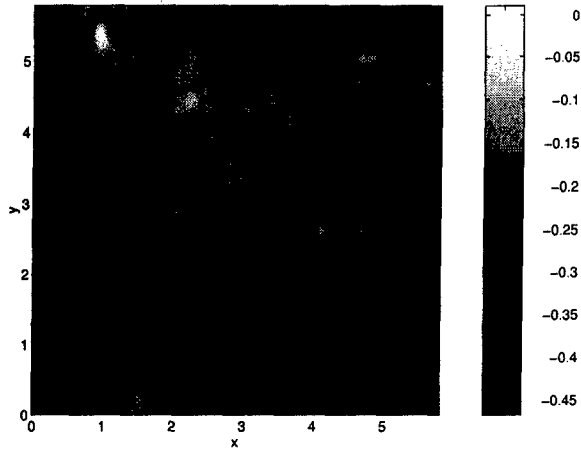

Figure 3. Gray scale plot of the fluctuating temperature in a horizontal plane through the center of the domain for $Ra = 8.4 \times 10^6$ and $Ta = 0$.
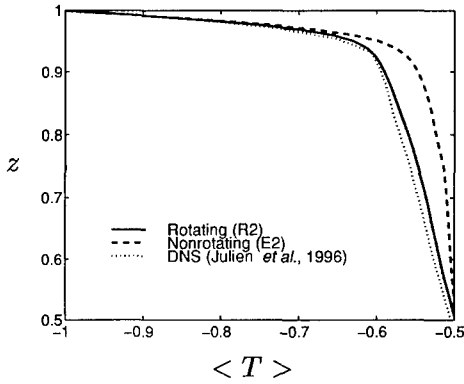
Figure 4.   Comparison of mean temperature profile for rotating and nonrotating convective flow.
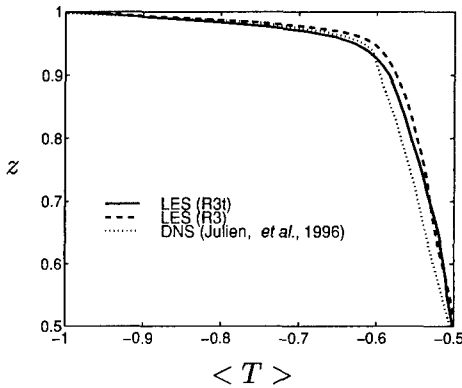

Figure 5.   Comparison of LES vertical mean temperature profile with DNS results for $Ra = 8.44 \times 10^6$, $Ta = 1.7 \times 10^7$ for LES with (R3t) and without (R3) truncated Navier-Stokes dynamics

3. Summary

The estimation model was used to simulate rotating turbulent convection, and good agreement with DNS results was seen. However, better agreement between the LES and DNS for $Ra = 8.4 \times 10^6$ and $Ta = 1.5 \times 10^7$ is seen for the mean temperature profiles and Nusselt number when truncated Navier-Stokes dynamics is also applied. This improved version of the estimation model should result in improved results for strongly rotating convective flows at higher Rayleigh numbers.

4. Acknowledgements

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