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# LARGE EDDY SIMULATION USING COMPLETE FOURTH ORDER DIFFERENCE METHOD AND VORTICITY-VECTOR POTENTIAL FORMULATION IN GENERALIZED COORDINATES

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**Abstract.** The vorticity-vector potential formulation and the fourth-order difference method are combined with LES with dynamic subgrid scale model. The new sharp cut off filtering is devised, which has the fourth order accuracy. The present computational method is validated for the simulation of transition in channel. The channel flow with two rows of transverse riblet is computed by the present method and is shown that the drag reduction is attained after the turbulent field develops fully on the riblet surface. It is also found that the transverse riblet has the same effect as the suction.

## 1. Introduction

Recently, the large eddy simulation is devoted to study the turbulence phenomena with complex geometric configurations such as turbulence over the airfoil (Weber et al 2000) and the turbulence flow in motored piston-cylinder assembly (Verzicco et al 2000). In conducting these simulations accurately, it is crucially important to use the accurate large eddy simulation technique in the generalized coordinate system. The accurate filtering technique is also important as well as accurate computational method. The dynamic subgrid scale model (Germano et al 1991) should be adopted in these simulations. It has also to be considered the importance of the formulation method in dealing with incompressible turbulent flow. The fractional step method has been widely used as the formulation method since the regular grid is applicable and the continuity equation of the velocity field is satisfied sufficiently.

Although, the vorticity-vector potential method has been used in the early stage of the large eddy simulations (Mansour *et al.*, 1979), this method was replaced to the fractional step method due to the difficulty on applying the wall boundary conditions adequately. However, recently this problem has been dramatically solved by the research works of the present author (Tokunaga, 1992; Shimada *et al.*, 1991; Tokunaga *et al.*, 1991; Tokunaga, 1999). The fourth order accurate difference method has been also combined to this formulation in the frame work of the dynamic subgrid scale model.

The present author conducts the research works on the drag reduction of the transverse riblet by making use of the large eddy simulation technique which consists of the fourth order difference method, fourth order accurate sharp-cut-off filter, dynamic subgrid scale model and the vorticity-vector potential formulation in the curved coordinate system.

It has to be pointed out at first the direct numerical simulation over riblet (Choi *et al.*, 1994) about research works of turbulent drag reduction. However, another studies exist on numerical simulations of the turbulent drag reduction, such as the linear feedback control (Cortelezzi *et al.*, 1999) as well as the transverse riblet (Tokunaga and Yamauchi, 1997; Tokunaga, 1999; Manuilovich, 1994; Liu *et al.*, 1994; Mochizuki *et al.*, 1994). The present author showed that the concave transverse riblet has the significant effect for drag reduction since the streak structure of wall bounded turbulence is blocked by the concave transverse riblet and the turbulence is weakened as the suction (Tokunaga, 1999). The suction is used as an effective means for laminarizing the flow along airfoil and reducing the drag, in which the slotted surface is accompanied for delaying the transition and reducing the drag, although this apparatus includes the complex mechanical arrangements (Bobbit *et al.*, 1991).

In the present paper, we intend to verify more precisely the drag reduction effect by conducting the large eddy simulation of the turbulence over the two rows of transverse riblet in the periodic domain of the streamwise direction. The computational grid consists of  $129 \times 85 \times 65$  where the curved coordinate is used.

## 2. Governing Equations

We adopt the fourth order accurate sharp-cut-off filter which is able to be applied to the generalized body fitted coordinate system since the constant width  $\delta_{min}/2$  is used as shown in Fig. 1, where  $\delta_{min}/2$  denotes the minimum grid spacing in the generalized coordinate, and therefore the filtering and differentiation process is commutable each other with respect to the order. The dynamic subgrid scale is used and the integration in the box filtering

is performed analytically by approximating the turbulent field with the second order Lagrange interpolation both on the basic and test filtering. The integration is performed by the Simpson's rule, so that the fourth order accuracy is assured.

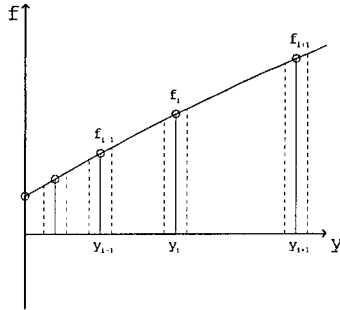


Figure 1. Filtering process

The vorticity-vector potential formulation is applied to the equation of LES with dynamic subgrid scale model. The vorticity  $\omega$  is defined as

$$\omega = \nabla \times \mathbf{u}. \tag{1}$$

The vorticity transport equation is derived by taking the rotation of the Navier-Stokes equation with the SGS model. After introducing the vector potential

$$\mathbf{u} = \nabla \times \psi, \tag{2}$$

the Poisson equations are also derived as

$$\Delta \psi_i = -\omega_i, \tag{3}$$

where the solenoidal condition is assumed for the vector potential

$$\frac{\partial \psi_j}{\partial x_j} = 0. \tag{4}$$

These equations are transformed to the generalized coordinate  $\xi$  and  $\eta$  from  $x_1$  and  $x_2$ .

The vorticity-vector potential method was validated for the numerical simulation of  $180^\circ$  curved duct with the circular section and the transition simulation in a plane channel (Tokunaga et al 1991).

### 3. Computational Methods

In order to compute accurately the shear flow turbulence, the fourth order difference method is used. In this case, the first and second derivative, for example, are discretized in the  $\xi$ -direction as

$$\frac{\partial \omega}{\partial \xi} \Big|_{ijk} = (-\omega_{i-2,j,k} - 8\omega_{i+1,j,k} - 8\omega_{i+1,j,k} + \omega_{i+2,j,k}) / (12 \Delta \xi), \quad (5)$$

$$\frac{\partial^2 \omega}{\partial \xi^2} \Big|_{ijk} = (-\omega_{i-2,j,k} + 16\omega_{i-1,j,k} - 30\omega_{i,j,k} + 16\omega_{i+1,j,k} - \omega_{i+2,j,k}) / (12 \Delta \xi^2),$$

where  $\Delta \xi$  represents the grid interval in the  $\xi$ -direction. The derivative in  $\eta$ - and  $z$ -direction as well as the cross derivative are also differenced in the same way, and finally the set of the ordinary differential equations (ODEs) is led.

$$\frac{d\vec{\omega}}{dt} = \vec{F}(\vec{\omega}), \quad (6)$$

$$\vec{\omega} = (\omega_{1,2,1}, \omega_{2,2,1}, \dots, \omega_{I,J-1,K})^T, \quad (7)$$

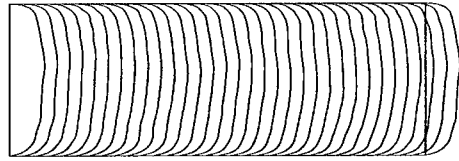
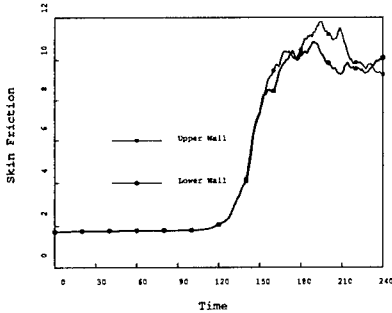
where  $I$ ,  $J$  and  $K$  denote the number of grid points in  $\xi$ ,  $\eta$  and  $z$  direction. For time-stepping in the set of ODEs, we apply the TVD Runge-Kutta method with the third order accuracy.

The same discretization procedure is adopted for the Poisson equation for the vector potential, where the point Jacobi is used in  $\xi$  and  $z$  direction while the SOR method is only applied in  $\eta$  direction, since the computation is performed by Fujitsu FACOM VPP-800/63 of the architecture of pipeline with the huge length. The convergence of the Poisson equation gives a significant influence on the turbulent field and insufficient relaxation weakens the turbulence, so that we adopt the five hundreds relaxations as maximum.

### 4. Transition Simulation in Channel Flow

In order to validate the present computational method, the transition process in channel flow is simulated. The Reynolds number is  $Re = 8,000$ , and the initial condition consists of the Poiseuille flow with 2% amplitude 2-D and .02% amplitude 3-D T-S waves, which is the standard case (Germano et al 1991).

The time evolution of the wall shear stress is depicted in Fig. 2 (a), where the wall shear is depicted both on the lower and upper wall. The transition occurs at  $t = 180$  due to the impurity of the initial condition. The figure 2 (b) shows the streamwise velocity distribution average in the



(b) Averaged streamwise velocity distribution

(a) Time evolution of skin friction  
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Figure 2. Time evolution of skin friction on channel wall surfaces (a) and streamwise velocity distribution averaged in spanwise direction (b)

spanwise direction, which shows that the turbulent flow fully develops.

## 5. Numerical Simulation of Channel Flow with Two Rows of Transverse Riblet

Since the possibility of drag reduction has been already suggested in the single row of transverse riblet in the periodic domain (Tokunaga, 1999), we investigate the drag reduction in two rows of transverse riblet which is shown in Fig. 3.

The Reynolds number  $Re = U_{max} 2\delta/\nu$  is chosen as 8,000 and the height of the transverse riblet  $\kappa = -0.03$  which are the same in LES of the single row of transverse riblet.

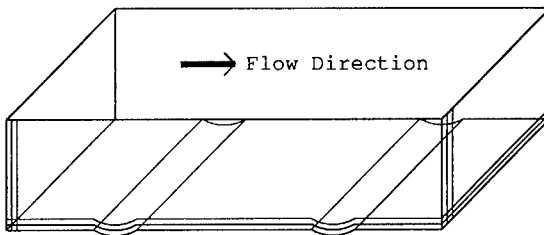
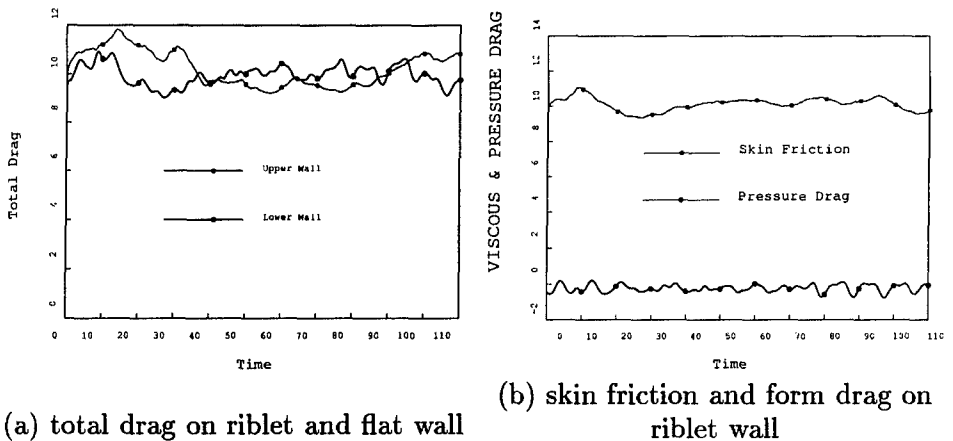


Figure 3. Transverse riblet

The initial turbulent field is initiated by adopting incidently the fully developed channel turbulent flow, which is obtained in the previous chapter, to the channel with two transverse riblets in order to elucidate the robustness of the present computational method. The computational time is 17 hours to proceed 10 non-dimensional time by making use of single vector processor of Fujitsu VPP-800/63 which is the parallel super computer with 63 processors at the maximum performance 504 GFlops . Figure 4 (a) and (b) depict the time history of the total drag at the lower and upper wall, as well as the skin friction and pressure drag at the lower wall. In the present paper, the total drag denotes the sum of the skin friction and pressure drag.

It is shown that the darg on the flat upper wall exceeds the total drag on the lower riblet surface after the initial channel turbulent field fits to the geometry of two rows of transverse riblet. It is also shown that the form drag is too small compared with the wall shear. From this point of view, it is concluded that the transverse riblet has potential as the device of drag reduction.



(a) total drag on riblet and flat wall  
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(b) skin friction and form drag on riblet wall

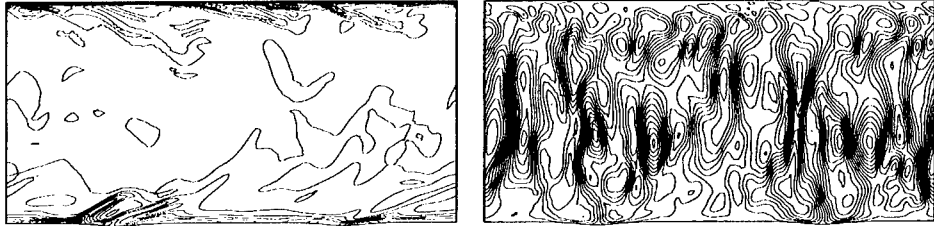
Figure 4. Time history of total drag (a), skin friction and form drag (b)

## 6. Turbulent Field on Riblet Surface

In this section, we investigate the mechanism of drag reduction from visualizing the turbulet field on the riblet surface. Figure 5 (a) shows the contours of the spanwise vorticity, where it is found that the turbulet vortices are

attracted to the transverse riblet and weakened. The effect is the same as the suction used in the drag reduction device.

In Fig. 5 (b), we show the contours of the velocity normal to the wall which is averaged in the spanwise direction. It is depicted the average velocity directs to the riblet surface. Therefore, it is thought that the transverse riblet has the same effect as the suction.



(a) Contours of spanwise vorticity (b) Contours of the normal velocity  
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Figure 5. Contours of spanwise vorticity on  $x - y$  plane ( $k = 9$ ) at  $t = 110$  (a) and velocity normal to the wall average in spanwise direction (b)

The iso-surface of the spanwise vorticity is depicted in Fig. 6. It is found that the streak structure on the upper flat surface is smooth and strong, while the one on the riblet surface is blocked on the two rows of riblets and weakened.

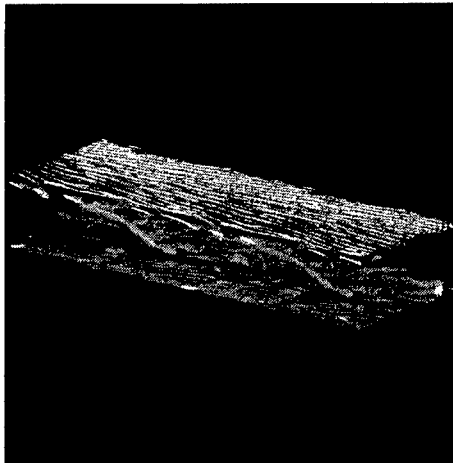


Figure 6. Iso surfaces of streamwise vorticity



## 7. Conclusions

The complete fourth order accurate method is presented for LES in the vorticity-vector potential formulation and LES is performed for channel flow with two rows of transverse riblets. As results, the following conclusions are obtained.

- (1) It is shown that the channel transition simulation is successfully carried out by the present computational method.
- (2) It is found that the total drag is reduced after the turbulent field develops fully on the riblet surface.

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