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A STUDY OF THE EFFECT OF SMOOTH FILTERING IN LES

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Abstract. The large eddy simulation equations of turbulent flows are formally derived by applying a low-pass filter to the Navier-Stokes equations. As a result the subgrid-scale stress tensor strongly depends on the assumed filter shape, which causes a subgrid scales model to be filter dependent. Depending on the choice of the filter, the corresponding model should satisfy very different requirements in terms of large scale dynamics and kinetic energy budget. In this paper, it is demonstrated that the assumed filter shape can have a significant effect in terms of spectral content and physical interpretation of the solution.

1. Introduction

The large eddy simulation (LES) equations of turbulent flows are formally derived by applying a low-pass filter to the Navier-Stokes equations. The resulting equations have the same structure as the original ones plus additional terms, called subgrid scale (SGS) stresses. The success of the LES approach clearly depends on the ability of the SGS model to accurately represent the effect of the unresolved scales on the resolved ones. However, both the definition of resolved scales and the model for the SGS stress ten-

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sor, $\tau_{ij} = \overline{u_i u_j} - \bar{u}_i \bar{u}_j$, strongly depend on the assumed filter shape. If the low-pass filter is exactly the sharp cut-off or close to it, then there is a clear separation of scales into large (resolved) and small (unresolved) ones and SGS stresses represent the effect of small scales on large ones. However, if the filter is smooth (like Gaussian or top-hat filter) then the boundary between resolved and unresolved scales is not well defined and the resulting SGS stresses represent the effect of small as well as large scales interactions.

The present paper is mainly aimed at demonstrating the importance of looking at filtering and SGS modeling as one inseparable issue and to provide LES practitioners a reference for interpreting the results of their simulations. Moreover, the knowledge of how the filter shape affects LES solution is of great importance in constructing efficient and consistent turbulence models. Thus, the objective of the present study is to establish the general framework for looking at the effect of the filter shape on large scale dynamics and energy transfer.

2. Theoretical Accomplishments

In this section the effect of the filter shape is briefly studied from a theoretical point of view. For a deeper analysis one can see in the related paper by the authors (De Stefano & Vasilyev, 2001). For simplicity reasons we consider one-dimensional (1D) homogeneous flow governed by the viscous Burgers equation, originally proposed as a model equation for turbulence (Burgers, 1974),

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} = \nu \frac{\partial^2 u}{\partial x^2}. \tag{1}$$

Despite the simplicity of this model, the analysis and subsequent conclusions drawn from it are applicable for the general LES.

By filtering Eq.(1) one obtains the following LES equation describing the evolution of the filtered field:

$$\frac{\partial \bar{u}}{\partial t} + \bar{u} \frac{\partial \bar{u}}{\partial x} = \nu \frac{\partial^2 \bar{u}}{\partial x^2} - \frac{1}{2} \frac{\partial \tau}{\partial x}, \tag{2}$$

where $\tau = \overline{u^2} - \bar{u}^2$ stands for the SGS stress, that must be modeled in order to close the problem.

From a physical point of view, one would like to follow the dynamics of flow structures down to a given size, *i.e.* to solve the velocity field up to a certain characteristic wave-number. Thus, the most natural choice for the filter function is the sharp cut-off filter in Fourier space. This way, the velocity fluctuation does not contain resolved wave-numbers components and the resolved velocity field has a clear physical meaning. On the

contrary, smooth filters provide an overlapping between resolved and unresolved scales and the physical interpretation of the large-eddy field is more obscure. The influence of the filter shape on the SGS stress is very clear if one looks at its spectral content

$$\hat{\tau}(\kappa) = \int_{\kappa'+\kappa''=\kappa} \left[\hat{G}(\kappa) - \hat{G}(\kappa')\hat{G}(\kappa'')\right] \hat{u}(\kappa')\hat{u}(\kappa'')d\kappa', \qquad (3)$$

where $\hat{G}$ stands for the filter transfer function. When a sharp cut-off (say at wavenumber κ_c) is applied, the SGS stress spectrum exactly accounts for the effect of small scales ($|\kappa| \leq \kappa_c$) on large ones. For a smooth filter, one can define a characteristic wave-number $\overline{\kappa}$ so that scales at $|\kappa| \leq \overline{\kappa}$ are referred to as large ones. In this case the SGS stress accounts not only for the effect of small scales, but also for the effect of filtering on large ones. It is illustrative to consider a velocity field with no Fourier components beyond $\overline{\kappa} = \kappa_c$. In this case, according to Eq. (3), the SGS stress for the spectral cut-off vanishes while, for a smooth filter, the same equation provides a non-zero SGS stress, being $\hat{G}(\kappa' + \kappa'') \neq \hat{G}(\kappa')\hat{G}(\kappa'')$.

The effect of the smoothness of the filter appears also evident by considering the kinetic energy budget in wave-numbers space

$$\frac{\partial \mathcal{E}}{\partial t} = -2\nu\kappa^2\mathcal{E} + \mathcal{T} + \mathcal{P}, \qquad (4)$$

where $\mathcal{E}$ is the resolved field energy density, $\mathcal{T}$ the energy transfer among different resolved wave-numbers and $\mathcal{P}$ a source term due to the interaction between resolved and unresolved eddies. When the sharp cut-off filter is adopted, $\mathcal{P}$ exactly represents the energy transfer between large and small scales while, for smooth filters, it must also account for large scales interactions. In the illustrative example considered, the sharp cut-off filter does not alter the energy transfer, since $\mathcal{P} = 0$, while any smooth filtering results in a drain of energy, since $\mathcal{P}$ no longer vanishes.

3. Numerical Experiments

In this section the results from numerical experiments are presented confirming the strong effect of the filter shape on LES with explicit filtering. First, we report some results dealing with the numerical simulation of a 1D freely decaying turbulent flow, governed by the Burgers equation (1), in terms of temporal evolution of energy and dissipation; for a deeper analysis one can see in (De Stefano & Vasilyev, 2001). Then, some preliminary result about the numerical simulation of isotropic turbulence are shown. In particular, the effect of the filter shape on the energy spectrum of the flow is discussed.
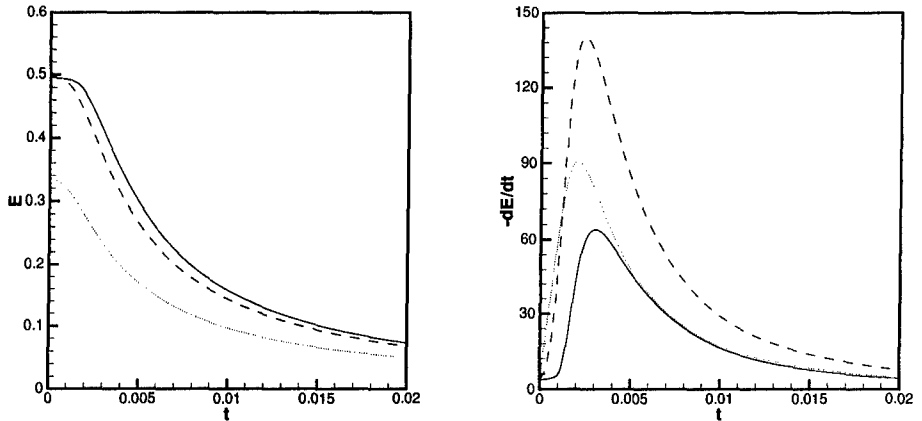
Figure 1.   Temporal evolution of total energy **E** and dissipation $d\mathrm{E}/dt$ for *perfect* LES, for both sharp cut-off (– – –) and top-hat (··········) filters, together with those for DNS (———).

Herein a top-hat filter is used as an example of smooth filters. It is worth noting that when comparing LES results for different filters, one needs to consider these latter corresponding to the same filter width. Thus, it is important to use consistent filter width definitions. In this paper, for the smooth filter, we adopt the definition according to which the filter width is taken to be proportional to the inverse wave-number where the filter transfer function falls to 0.5 (Lund, 1997), that is $\overline{\kappa} = \kappa_c = \pi/\Delta$, being Δ the common filter width. LES runs are performed with the aid of the so-called *perfect* SGS model, an ideal model constructed by definition upon the DNS data, assumed as the exact solution. Then, in order to mimic a real SGS model, in which it is hard to model large scales, this ideal model is modified by taking the effect of large scales out, but leaving the small scales one intact. This is achieved by treating the modeled stress with a high-pass filter, whose Fourier transform is $1 - \hat{G}(\kappa)$, where $\hat{G}(\kappa)$ corresponds to a sharp low-pass discrete filter (Vasilyev *et al.*, 1998) with filter width $\widetilde{\Delta}$. Note that with the increase of the ratio $\widetilde{\Delta}/\Delta \geq 1$ more scales are left intact and the *perfect* LES model is approached.

3.1. BURGERS SOLUTION

The numerical solution of the 1D homogeneous Burgers problem is obtained integrating in time Eq. (1) with periodic boundary conditions. In order to
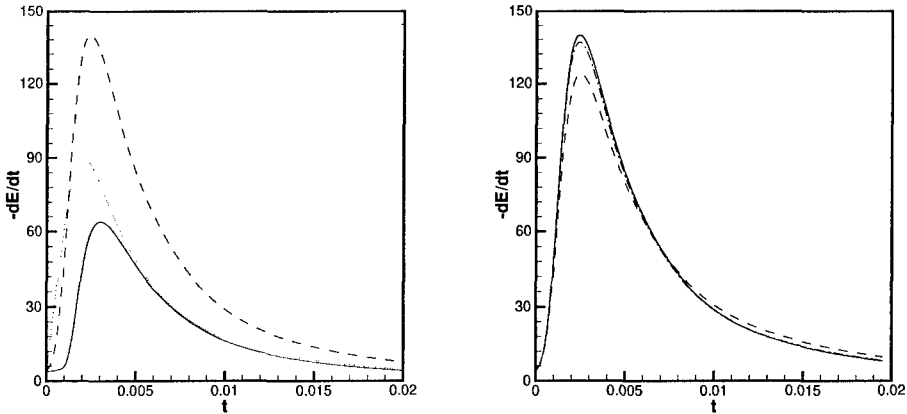
Figure 2. Sharp cut-off filtering: temporal evolution of total energy $\mathbf{E}$ and dissipation $d\mathbf{E}/dt$ for *perfect* LES (———) and altered LES with different ratios $\widetilde{\Delta}/\Delta$: 2 (- - -), 3 (— · —), and 4 (··········).

minimize the influence of truncation and aliasing errors, the numerical integration is carried out with the aid of a high-order non-dissipative numerical method and a sufficiently fine grid. LES simulation with *perfect* SGS stress results in the energy spectra identical to the filtered DNS solution regardless of the filter used (De Stefano & Vasilyev, 2001). The temporal evolution of the total flow energy and dissipation are shown in Fig. 1: in case of sharp cut-off filtering, the solution keeps a high fraction of the energy content of the flow, while, for smooth filtering, a large part of it is lost, even in the ideal case. Due to the perfect modeling and the very good numerics exploited this loss of energy exactly accounts for the effect of smooth filtering on LES solution.

Results for 1D LES with altered SGS model are presented in Fig. 2 and 3. Altering the *perfect* SGS model causes a wrong evolution of total energy: the dissipation provided by the model is not enough and kinetic energy decays less in time with respect to the ideal case. For LES with top-hat filtering this effect is more important and the energy content of the flow is clearly badly represented.

3.2. ISOTROPIC TURBULENCE SIMULATION

In order to extend the 1D results to a real turbulent flow, some numerical test dealing with three-dimensional isotropic forced turbulence at $Re_\lambda \cong 70$
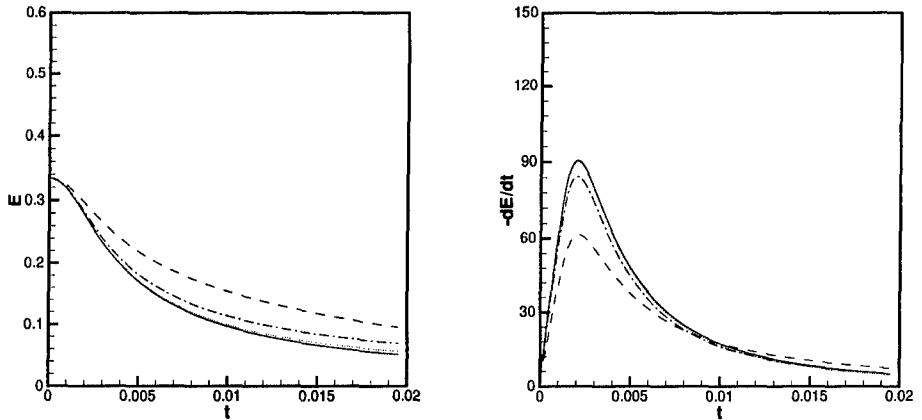
Figure 3.   Top-hat filtering: temporal evolution of total energy **E** and dissipation dE/dt for *perfect* LES (————) and altered LES with different ratios $\widetilde{\Delta}/\Delta$: 2 (– – –), 3 (— · —), and 4 (··········).

are presented. The Navier-Stokes equations are solved with the convective term in rotational form and numerical simulation, both DNS and LES, are performed using a de-aliased pseudo-spectral code (Ruetsch & Maxey, 1991).

The DNS is conducted in the wave-numbers range $|\kappa| \leq 36$, while time integration is carried out till a statistical stationary field is reached. The simulation is further advanced in time, storing each N time steps the *perfect* SGS stress, where N is the integer ratio of LES to DNS time steps.

LES with both sharp cut-off and top-hat explicit filtering are conducted in the range $|\kappa| \leq 16$. The filter width is chosen such that the characteristic wave-number $\overline{\kappa} = 12$ is in the inertial range. In order to avoid generation of frequencies beyond the characteristic frequency $\overline{\kappa}$ we adopted an alternative SGS stress definition $\tau_{ij} = \overline{u_i u_j} - \overline{\overline{u}_i \overline{u}_j}$ suggested by Vasilyev *et al.* (1998).

The energy spectra corresponding to LES with *perfect* SGS model agree quite well with the filtered DNS ones, as illustrated in Fig. 4. For comparison the spectra of LES without any model are also reported. This fact is not surprising, since we provide *perfect* SGS model which gives us the exact dynamics and energy transfer. Moreover, as it is expected, the application of a smooth filter strongly affects the shape of the spectrum. In other words, for this kind of filter, even when LES is conducted with the ideal SGS model, the resolved field loses some important features of the *real* field. In particular, the slope corresponding to the inertial range is clearly
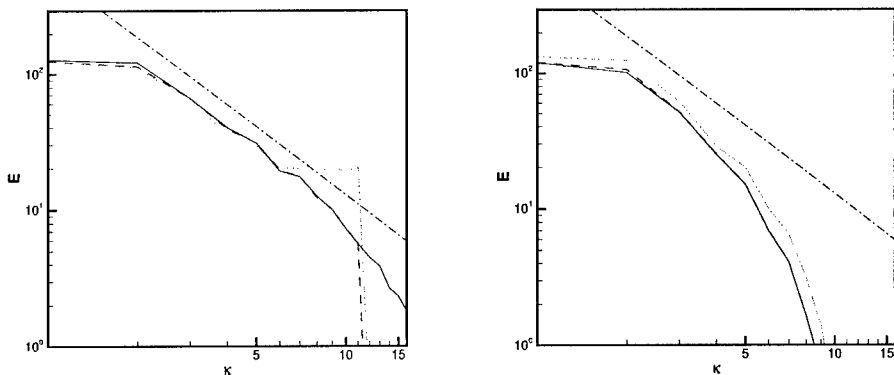
Figure 4. Energy spectra for filtered DNS (——), *perfect* LES (– – –) and LES without any model (·········) , for both sharp cut-off (left) and top-hat (right) filters. There is also shown the ideal slope of the inertial range −5/3 (— · —).

misrepresented.

The energy spectra corresponding to LES with altered SGS modeling are presented in Fig. 5. For sharp cut-off filtering, according to (3), the SGS stress has a very little contribution from large scales; thus, the altered *perfect* model appears to work well, even for $\widetilde{\Delta}/\Delta = 1$. The same is no longer true for smooth filtering, for which, even for high $\widetilde{\Delta}$ one cannot recover a good spectrum.

4. Concluding remarks

In this study we carried out some numerical experiments in order to address the influence of the filter shape in LES with explicit filtering. It is worth noting that the analysis was conducted not by means of *a priori* tests, as often made in similar studies, but performing actual LES with SGS models obtained by filtering DNS time series. Due to the good numerics adopted, the pure effect of filtering on actual LES solution was illustrated. In order to mimic a real simulation, the ideal SGS model was altered by filtering out the contribution of large scales on it. In the future, different SGS models used for LES simulations will be tested. Numerical results presented in this paper have clearly demonstrated the importance of considering SGS modeling and filtering as an inseparable issue. In particular, it was shown that if LES is based on smooth filter, then SGS model should also model the effect of the filter on large scales, *i.e.* forces (stresses) produced by interaction of large scales which are filtered out. The same is for energy cascade:
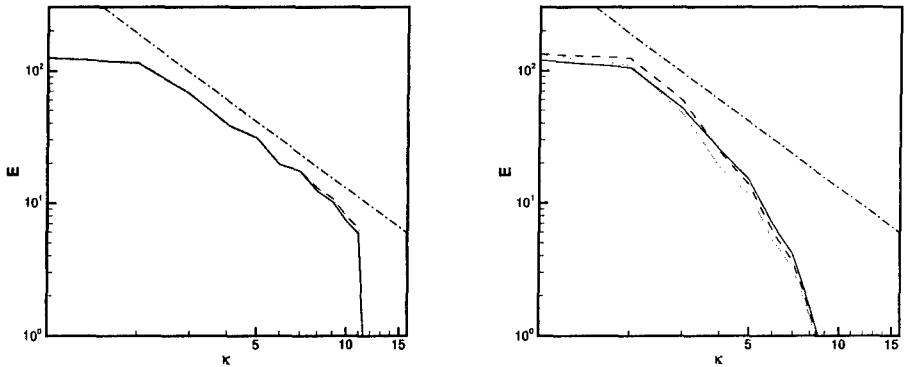
Figure 5. Energy spectra for LES with altered *perfect* SGS model. For sharp cut-off filtering (left), plots refer to $\widetilde{\Delta}/\Delta = 1$ (– – –) and 2 (⋯⋯) while, for top-hat filtering, they refer to $\widetilde{\Delta}/\Delta = 2$ (⋯⋯) and 4 (– – –). Perfect LES spectra are also shown (———).

SGS model should remove (or add) energy at the resolved scales due to the simple fact that filtering procedure removes them. Thus, depending on the smoothness of the filter, the corresponding SGS model should satisfy very different requirements in terms of large scale dynamics and kinetic energy budget. This contradicts the basic motivation behind LES: to resolve large scales and model unresolved ones. One should not model the interaction of resolved scales, otherwise it will be hard to see the difference between unsteady RANS and LES. However, unless one considers homogeneous turbulence, it is difficult, if not impossible, to adopt an implicit or explicit sharp cut-off filter. Then, the next best choice is to minimize the effect of filter on large scales dynamics and energy transfer. This can be achieved by making the filter as close to sharp cut-off as possible.

References

De Stefano, G. and Vasilyev, O.V. (2001) Sharp cut-off vs. smooth filtering in LES, To appear in *Phys. Fluids*.

Burgers, J.M (1974) The nonlinear diffusion equation, *Reidel Publishing Company*.

Lund, T.S. (1997) On the use of discrete filters for large eddy simulation, *Annual Research Briefs, Center for Turbulence Research, NASA Ames/Stanford Univ.*, 83-95.

Vasilyev, O.V., Lund, T.S. & Moin, P. (1998) A general class of commutative filters for LES in complex geometries, *J. Comp. Phys.*, **146**, 105-123.

Ruetsch G. R. & Maxey, M. R. (1991), Small-scale features of the vorticity and passive scalar fields in homogeneous isotropic turbulence. *Phys. Fluids.* **3** 1587-1597.