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Nonlinear absorption of surface acoustic waves by composite fermions

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Abstract. Absorption of surface acoustic waves by a two-dimensional electron gas in a perpendicular magnetic field is considered. The structure of such system at the filling factor ν close to $1/2$ can be understood as a gas of *composite fermions*. It is shown that the absorption at $\nu = 1/2$ can be strongly nonlinear, while small deviation from $1/2$ will restore the linear absorption. Study of nonlinear absorption allows one to determine the force acting upon the composite fermions from the acoustic wave at turning points of their trajectories.

Introduction

Two-dimensional electron gases (2DEG) have been intensely studied in the last years. One of the experimental techniques used to investigate the properties of this system its interaction with a surface acoustic wave (SAW). We will consider the 2DEG in the fractional quantum Hall regime where a strong magnetic field is applied normal to the plane of the 2DEG. As well known, close to the half filling of the lowest Landau level the system will exhibit a metallic phase. This phase can be described in terms of a new type of quasiparticles, called composite fermions. The theory of composite fermions, as formulated in the language of Chern–Simons field theory in [1], has successfully explained the acoustic properties of the electron gas [2]. So far, however, greatest attention, both experimentally and theoretically, has been given to the linear response regime, which is appropriate for low intensity acoustic waves. From the study of ultrasonic absorption by electrons in metals it is known [3] that very interesting nonlinear effects can be observed. It is therefore natural to study nonlinear effects in the context of SAW absorption by the composite fermion metallic state. In particular are we interested in the fact that when nonlinear absorption occurs in a metal, a weak external magnetic field will restore the linear absorption [4].

First we need to understand how the acoustic wave interacts with the electron gas. Two mechanisms are usually considered, the deformational and the piezoelectric interactions. In typical experimental sets the deformational interaction can be neglected, and we consider only the piezoelectric field of the wave. Letting the x -axis point in the direction of sound propagation this is then given by $\mathbf{E}(x, t) = \mathbf{E}_0 \sin \xi = -\nabla\Phi$ with $\Phi = \Phi_0 \cos \xi$. Here Ω is the SAW frequency, and $\xi = qx - \omega t$ is the wave coordinate; $\mathbf{E}_0 \parallel \mathbf{q} \parallel \hat{\mathbf{x}}$. By Φ we mean the screened electrostatic potential.

The sensitivity to external magnetic fields was explained in the following way. If the wavelength $2\pi/q$ of the acoustic wave is much smaller than the electron mean free path ℓ , the electrons will traverse many periods of the wave before being scattered. If the electron moves in such a way that the component of its velocity in the direction in which the acoustic wave is propagating, it will experience a rapidly oscillating force. Consequently, the interaction between this electron and the acoustic wave will be weak. Since the electron (Fermi) velocity is much larger than the sound velocity, this will be the case for most of

the electrons. Only a small group, called the resonant group, will have their velocities in the sound propagation direction matched to the sound velocity, and thus interact strongly with the wave. The linear absorption, which is obtained in the limit of an acoustic wave of small (infinitesimal) amplitude, is determined by these resonant electrons. Consider a finite amplitude wave. Some of the resonant electrons will then be trapped in the valleys of the potential of the acoustic wave, see Fig. 1(a). The trapped electrons, moving in complete

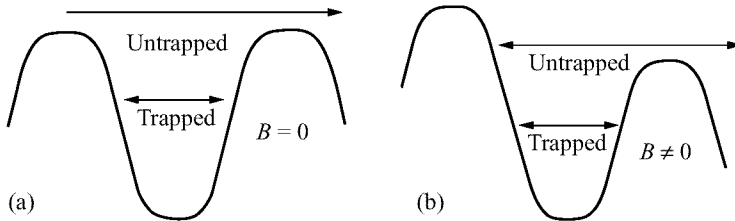


Fig. 1. On the trapping of resonant electrons in the absence (a) and in the presence (b) of an external magnetic field B .

synchrony with the wave, will not contribute to the absorption, and the absorption decreases. This leads to an amplitude dependent absorption coefficient, *i.e.* to nonlinear absorption. Consider now the effect of an externally applied magnetic field. The electrons will feel an extra force which will act to remove the electrons from the trapped group, see Fig. 1(b). Thus, we expect the linear absorption to be restored by the application of a magnetic field. The important point is that the magnetic force needed to restore the linear absorption gives a direct measure of the trapping force from the acoustic wave. That is, from the strength of the field needed to restore linear absorption we can infer the strength of the force from the acoustic wave on the electrons.

Theory of nonlinear acoustic absorption

Let us recall the main facts of the Chern-Simons theory for the composite fermions [1]. The Chern-Simons transformation mapping the electron system to an equivalent system of composite fermions can be described as attaching an even number of fictitious flux quanta of Chern-Simons magnetic field to each electron. In the mean field approximation the composite fermions will experience an effective field $B_0^* = B - m\phi_0 n_0$, where B is the external field, m is the number of attached flux quanta, $\phi_0 = h/e$ is the flux quantum and n_0 is the electron density. If $m = 2$, the effective field will vanish if the Landau level filling factor $\nu = 1/2$. The perturbation by the acoustic wave will induce a density modulation in the electron gas, $n = n_0 + \delta n$. This will lead to an oscillating component of the Chern-Simons magnetic field, so that the total effective field will be $B^* = B_0^* + b^{\text{ac}}$ with $b^{\text{ac}} = -2\phi_0 \delta n$. In addition, the motion of the electrons will drag with it the attached flux, and by Faraday law induce a Chern-Simons electric field, $\mathbf{e}^{\text{ac}} = (2\phi_0/e)[\hat{\mathbf{z}} \times \mathbf{j}]$. The y -component of this field is found from the x -component of the current, which can be related to the density modulation by the equation for conservation of charge. Assuming the density modulation to be harmonic, $\delta n = (\delta n)_0 \cos \xi$, we get $e_y^{\text{ac}} = 2\phi_0 v_s \delta n$. We will later see that this assumption is justified. As explained in [5] we can combine the x -component of \mathbf{e}^{ac} with the piezoelectric field of the acoustic wave. The corresponding total potential will be denoted Ψ . The main objective of this work is to determine the trapping of the electrons by the combined action of these fields, which are not present in the previously

studied electron problem.

Now we use the Boltzmann equation to calculate the nonequilibrium distribution function, f , of composite fermions, from which the absorption can be found. The Hamiltonian is

$$H = (\mathbf{P} + e\mathbf{A})^2/2m - e\Psi, \quad (1)$$

where \mathbf{P} is the canonical momentum (the kinematic momentum is $\mathbf{p} = \mathbf{P} + e\mathbf{A}$), \mathbf{A} is the vector potential. The vector potential consists of two parts. One emerges from the static external effective magnetic field B^* , and one from the AC Chern–Simons field that is created by the SAW-induced density modulation. the magnetic field is then $b^{\text{ac}} = 2\phi_0 [n_0 - (2\pi\hbar)^{-2} \int d^2P f]$.

It is convenient to split the distribution function as $f = f_0(H) + f_1$ where f_0 is the Fermi function. Then the Boltzmann equation for f_1 is

$$\partial f_1/\partial t + \nabla_{\mathbf{p}} H \nabla_{\mathbf{r}} f_1 - \nabla_{\mathbf{r}} H \nabla_{\mathbf{p}} f_1 + f_1/\tau = -(\partial H/\partial t)(\partial f_0/\partial H). \quad (2)$$

Here we use the relaxation time approximation $-f_1/\tau$ for the collision operator which significantly simplifies the calculations. It should be noted that the Hamiltonian (1) is written in terms of the AC Chern–Simons magnetic field b^{ac} . The latter must be expressed through the density modulation as an integral over the distribution function. The Boltzmann equation (2) is then in reality a complicated integro-differential equation for the non-equilibrium distribution function. It is easy to show, however, that the main contribution to the density modulation comes from the equilibrium part $f_0(H)$, so that in calculating f_1 we can approximate the density modulation with $\delta n^{(0)}$ coming from $f_0(H)$. Indeed, using the fact that in all the region of acoustic amplitudes $e\Psi \ll \epsilon_F$, where ϵ_F is the Fermi energy, we can then expand $f_0(H)$ around the point $H = p^2/2m$. The lowest-order term, $\delta n^{(0)}$, is estimated as

$$\delta n^{(0)} = -e\Psi(2\pi\hbar)^{-2} \int d^2p (\partial f_0/\partial H) |_{H=p^2/2m} = g e\Psi \quad (3)$$

Here $g = m/2\pi\hbar^2$ is the density of states per spin (as usual, we assume the 2DEG to be fully spin-polarized). Then we can solve Eq. (2) for f_1 with the assumption that $\delta n = \delta n^{(0)}$, and come back to show that the non-equilibrium correction coming from f_1 is small compared to $\delta n^{(0)}$. This will then justify our assumption of a harmonic density perturbation.

Solving the Boltzmann equation (2) by the method of characteristics we find the distribution function from which we calculate the absorption using the expression

$$P = \int \frac{d^2p}{(2\pi\hbar)^2} \langle \dot{H} f \rangle, \quad (4)$$

where $\langle \dots \rangle$ denotes average over the period of the acoustic wave. For the case $\nu = 1/2$, $B^* = 0$ we find, after rather tedious calculations, the result

$$P = C(e\Pi_0/2\pi)^2 (v_s/v_F) g\omega a, \quad (5)$$

where $C \sim 1$ is some numerical factor that can be found from numerical integration. Here $\Pi_0 = \Psi_0\sqrt{1+\alpha^2}$, $\alpha = 2mv_F/q\hbar$ and $a = (\omega_0\tau)^{-1}$. $\omega_0 = q\sqrt{e\Pi_0/m}$ is the typical oscillation frequency of the trapped electrons in the potential of the acoustic wave. Since each scattering event rotates the particle momentum and leads to its escape from the

resonant group, nonlinear behavior exists only if $\omega_0\tau \gg 1$, or $a \ll 1$. Thus a is the main parameter responsible for nonlinear behavior. The above result (5) is the first term in an expansion in powers of a . We see that the absorption decreases when there is pronounced nonlinear behavior.

By studying the solution of (2) in the presence of a nonzero effective magnetic field we find that as expected the absorption is restored to the value of the linear absorption. The field necessary for this is $B^* \geq B_c = q\Pi_0/v_F$.

Discussion

We can now see how the various fields contribute in the trapping of the composite fermions. For electrons we would have Φ_0 appearing in place of Π_0 in the expression for B_c . In the present case we can show using our solution of the Boltzmann equation that in the limit of strong nonlinearity, $a \ll 1$, $\Psi_0 = \Phi_0$. That is, the effect of the x -component of the Chern–Simons field vanishes. In that case we have $\Pi_0 = \Phi_0\sqrt{1 + \alpha^2}$, the factor $\sqrt{1 + \alpha^2}$ describing the trapping effect of the oscillating Chern–Simons magnetic field b^{ac} . Inserting reasonable values, $m = 10^{-30}$ kg, $v_F = 10^5$ m/s, $v_s = 3 \cdot 10^3$ m/s and $\Omega/2\pi = 3 \cdot 10^9$ GHz, we get $\alpha \approx 50$. We see that the effect of the Chern–Simons field is to considerably enhance the efficiency of the acoustic wave in trapping composite fermions, and consequently that the effective magnetic field necessary to restore linear absorption will be correspondingly larger. Consequently, a way to check the above concept is first to reach nonlinear behavior at $B = B_{1/2}$, then restore the linear behavior by changing magnetic field by a quantity $\geq B_c$, without changing the SAW intensity.

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