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Intrabeam Scattering and Halo Formation in Intense Ion Beams

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Abstract. An essential problem for the operation of high current linacs is the control of beam losses due to halo formation. Here we concentrate on an interplay between intrabeam scattering (IBS) and the incidence of particles which are driven to high amplitudes by resonances with the nonlinear space charge fields of a mismatched beam. For these studies, we developed a particle-core-molecular-dynamics (PCMD) method which suitably joins the time evolution in framework of the envelope equations and a calculation of IBS between pseudo-particles with a renormalized charge. With this technique we investigated continuous KV-beams in a FODO channel. Our first results strongly suggest, that IBS in fact supports the halo formation in a mismatched beam.

INTRODUCTION

One of the most critical aspects for a successful operation of a high current linac is the control of beam losses, since only a very small fraction of particles can be tolerated to collide with the accelerator structures. Such beam losses are associated with the formation of a halo of particles significantly outside the beam core. Substantial theoretical effort is presently addressed to the understanding of halo formation, e.g. to the stability of collective excitation modes which has been studied in the framework of the nonlinear Vlasov-Maxwell equations by both analytical and numerical approaches [1, 2, 3, 4]. Also the resonances between nonlinear space charge forces outside the core of a mismatched beam and the betatron oscillations have been identified as a source for a halo. This has been studied mainly by a frequency map analysis of the single particle motion in framework of a particle core (PC) model [3]. Hard collisions between the beam particles, which are neither included in the Vlasov-Maxwell equation nor the particle core approach, could also scatter particles into a halo. But for the typical beam parameters of high power linacs, it is very unlikely that this intrabeam scattering (IBS) is sufficiently effective to directly drive a beam halo [5]. IBS may, however, play a non-negligible role in amplifying the other mechanisms. We here focus on an interplay between intrabeam scattering (IBS) and the incidence of particles which are pushed to high amplitudes by nonlinear resonances driven by the space charge fields of a mismatched beam. As a fully microscopic numerical treatment including all mutual Coulomb interactions of the beam particles, and thus all space charge phenomena *and* IBS, considerably exceeds the capacity even of modern computers, we developed a alternative method. These so-called particle-core-molecular-dynamics (PCMD) simulations suitably join the time evolution of the beam core in framework of the envelope equations with a microscopic calculation of the Coulomb interactions between pseudo-particles with a renormalized charge.

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THE PARTICLE-CORE-MOLECULAR-DYNAMICS METHOD

The equations of motion for the position $\mathbf{r}_\alpha$ and the velocity $\mathbf{v}_\alpha$ of a particle with mass m and charge q in an accelerator structure or a storage ring can be written in a local center of mass frame of the beam as

$$\frac{\mathrm{d}\mathbf{r}_\alpha}{\mathrm{d}t} = \mathbf{v}_\alpha, \qquad \frac{\mathrm{d}\mathbf{v}_\alpha}{\mathrm{d}t} = \mathbf{K}(\mathbf{r}_\alpha,t) - \frac{1}{m}\nabla\Phi(\mathbf{r}_\alpha,t). \tag{1}$$

Here $\mathbf{K}$ denotes all the forces on the particle due to the beam manipulating elements of the structure and Φ represents the potential generated by all the other charged particles in the beam. In case of an ideal FODO-channel extending along the z-direction, the external forces are linear and $\mathbf{K}(\mathbf{r},t) = (K_1(t)x, K_2(t)y, 0)$, where the $K_i(t)$ only depend on time t or likewise on the position s along the channel. In general a numerical treatment of Eq. (1) is unavoidable. The most complete approach are molecular dynamics (MD) simulations which take completely into account all the mutual Coulomb interactions between the beam particles and thus include IBS without any approximation. They have been applied successfully for low current beams and studies of e.g. beam crystallization and related issues [6, 7, 8, 9, 10]. Because the numerical effort scales like N^2 the computational effort is, however, prohibitive for high current beams which require at least $N \sim 10^7$.

If one is, on the other hand, interested e.g. in the motion of a few particles in the nonlinear space charge fields outside the beam core, one can assume a given space charge field due to the large number of core particles with regular motion and neglect the charges of the actual test-particles. This approach becomes easily applicable for the Kapchinsky-Vladimirsky (KV) distribution, where a fairly simple self-consistent solution exists. The continuous KV-beam has the uniform charge density $\rho(s) = \lambda_z\,\Theta(1-x^2/a^2(s)-y^2/b^2(s))/\pi a(s)b(s)$ with envelopes $a(s)$ and $b(s)$ which vary with the position in the FODO channel $s = \gamma\beta ct$ according to

$$\frac{d^2a}{ds^2} + K_x(s)a - \frac{\xi}{a+b} - \frac{\varepsilon_x^2}{a^3} = 0 \quad \text{and} \quad x \leftrightarrow y, a \leftrightarrow b. \tag{2}$$

In Eq. (2) the constants ε_i are the emittances, $\xi = (qI)/(\pi\varepsilon_0 mc^3\gamma^3\beta^3)$ is the space charge parameter for a beam current I, $\lambda_z = I/c\gamma\beta$ is the line density and β,γ are the relativistic factors. Here the potential $\Phi(\mathbf{r},t)$ generated by the space charge of the beam can be obtained analytically from the known $\rho(t)$ via the Poisson equation. Combining now the time evolution of the core, given by a numerical solution of Eq. (2), and the particle motion, Eq. (1), establishes the particle core (PC) model, see e.g. [3]. As we are mainly interested in the influence of IBS on the small fraction of beam particles which may form a halo, we separate the collective selfconsistent space charge field from the individual particle-particle collisions. The basic idea is now to use the PC model for a KV-beam of density n and charge density qn as a starting point and to superimpose a MD treatment for a relatively small number ($\sim$ some hundred) N_t of test-particles in a small slice moving with the beam. Applying periodic boundary conditions along the beam extension, i.e. in z-direction, the test-particles then constitute an additional beam of density $n_t \ll n$ which is subject to the external forces $\mathbf{K}$, the space charge fields generated by the underlying KV-beam and the direct Coulomb collisions between the test-particles. But in the test-particle beam IBS is strongly reduced as compared to the real beam with the much higher density n. Because no length scale exists for the Coulomb interaction, we compensate the smaller test-particle density by increasing the test-particle charge to a renormalized value q_t. The N_t test-particles with mass m and charge q_t are now propagated according to Eq. (1) feeling the same external forces as the physical beam particles and the potential

$$\Phi(\mathbf{r}_\alpha,t) = \sum_{\beta\neq\alpha}^{N_t} \frac{q_t^2}{4\pi\varepsilon_0|\mathbf{r}_\alpha-\mathbf{r}_\beta|} + \int \frac{d^3r'\,(q^2n-q_t^2n_t)}{4\pi\varepsilon_0|\mathbf{r}_\alpha-\mathbf{r}'|}\,\frac{a_0b_0}{\gamma ab}\,\Theta\left(1-\frac{x'^2}{a^2}-\frac{y'^2}{b^2}\right), \quad (3)$$

with $n = I/q\beta c\pi a_0b_0$, $a_0 = a(s=0)$, $b_0 = b(s=0)$ and the step function $\Theta(z)$. Here the test-particles directly interact as charges q_t but couple with the original charge q to the space charge field of the underlying real beam. To avoid double counting, the space charge field ($\propto q^2n$) of the core beam is corrected by the test-particle contribution ($\propto q_t^2n_t$). This combination of MD simulations and the PC model, the particle-core-molecular-dynamics (PCMD) method, allows to study IBS also for intensive beams on a qualitative and, at least, semi-quantitative level, with a moderate computational effort. The crucial point is the appropriate choice of the test-particle charge q_t. Taking $q_t = 0$ we retrieve a pure PC description with a completely two dimensional dynamics in the transversal plane. It turns into a three dimensional, pure MD simulation of the test-particle beam if $q_t^2n_t$ approaches q^2n and the space charge term completely vanishes. But then we deal with a dilute beam of high charges with a much lower current $I_t = I(q_tn_t/qn)$. The disappearance of the space charge term $\propto (q^2n-q_t^2n_t)$ defines, for a given density n_t, a critical, upper limit q_c for the test-particle charge by $q_c = q(n/n_t)^{1/2}$. Meaningful applications of the PCMD description are restricted to values $q_t \ll q_c$, where the model is expected to reproduce the basic effects of interparticle correlations and IBS.

The coupling within a real beam of density n and temperature T is described by the plasma parameter $\Gamma = (q^2/4\pi\varepsilon_0k_BT)(4\pi n/3)^{1/3}$, which can be interpreted as the ratio between the potential energy of a pair of particles at their mean distance and the mean thermal energy. For a large plasma parameter intrabeam scattering (IBS) leads to a secular growth of emittance [6]. But here, we consider beams of protons, $q = e$, with a large temperature, so that the plasma parameter is small, with typical values around $\Gamma \sim 10^{-5}$. It is, however, a key quantity to characterize the beam properties, and the test-particle beam in the PCMD description should have a similar plasma parameter Γ_t as the real beam, where $\Gamma_t/\Gamma = (q_t/q)^2(n_t/n)^{1/3} = (q_t/q_c)^2(q_c/q)^{4/3}$ with $q_c = q(n/n_t)^{1/2}$. An analogous argument concerns the scattering rate $\nu = n\langle v\rangle\sigma(\langle v\rangle)$ for beam particles with an averaged relative velocity $\langle v\rangle$ and a scattering cross section $\sigma(\langle v\rangle)$. It can be estimated by taking the Coulomb cross section $\sigma \sim (q^2/4\pi\varepsilon_0m\langle v\rangle^2)^2$. This yields $\nu \propto nq^4$ and a ratio of the scattering rates in the test-particle beam and the real beam $\nu_t/\nu = (q_t/q)^4n_t/n = (q_t/q_c)^4(q_c/q)^2$. For the envisaged high current proton beams, test-particle numbers $N_t \sim 1000$ and typical settings in the PCMD simulations, we have $n_t \sim 10^{-5}n$ and $q_c \sim 10^2q$. Hence, a test-particle charge $q_t \sim 10q \sim 0.1\,q_c$ can be chosen which allows a matching of the physical relevant parameters, i.e. $\Gamma_t \approx \Gamma$ and $\nu_t \approx \nu$, while being fairly below the critical value q_c where the model breaks down.

PCMD SIMULATIONS OF A BEAM IN A FODO CHANNEL

With the outlined PCMD scheme we studied a continuous beam of 100 MeV protons in a FODO channel of 500 periods of 1 m and with a beam current I=0.8A, which may represent the peak current in the bunch for a linac with a beam current in the mA range. The geometry of the nominal FODO cell was taken from Ref. [3]. The external focusing is $K_x = -K_y = 12\,\mathrm{m}^{-2}$, the emittances are $\varepsilon_x = \varepsilon_y = 10^{-6}$ m and the space charge parameter is $\xi = (eI)/(\pi\varepsilon_0 mc^3\gamma^3\beta^3) = 9.6 \times 10^{-7}$. With these settings a matched solution of the envelope equations Eq. (2) is constructed numerically and yields $a_0 = 1.43 \times 10^{-3}$ m, $b_0 = 8.54 \times 10^{-4}$ m, $da/ds(0) = 0$ and $db/ds(0) = 0$ at $s = 0$ ($t = 0$). With these initial conditions, the envelopes $a(s), b(s)$ are propagated according to Eq. (2) throughout the structure and thereby define the time dependent core field contribution in expression (3). For the KV-distribution the corresponding beam density is $n = 1.0 \times 10^{10}\mathrm{cm}^{-3}$ and the plasma parameter is $\Gamma \approx 1.4 \times 10^{-5}$ when averaging over the temperatures $T_x(s)$ and $T_y(s)$ which are in the range $2.9 \times 10^5 \ldots 8.4 \times 10^5$ K and are connected to the envelopes via $k_B T_x/m = \varepsilon_x^2(\gamma\beta c)^2/4a^2$ and $k_B T_y/m = \varepsilon_y^2(\gamma\beta c)^2/4b^2$. The initial conditions $\{\mathbf{r}_\alpha(0), \mathbf{v}_\alpha(0)\}$ of $N_t = 800$ test-particles are generated according to a KV-distribution in the transversal degrees of freedom $f(x,y,v_x,v_y) \propto \delta(1 - x^2/a^2 - y^2/b^2 - mv_x^2/4k_BT_x - mv_y^2/4k_BT_y)$ and homogeneously in z-direction. The density of the test-particle beam is $n_t = 7.3 \times 10^4\mathrm{cm}^{-3}$ and $q_c = 3.7 \times 10^2 q$. The simultaneous propagation of the N_t test-particles, given by Eqs. (1) and (3), is treated within a usual MD scheme by standard numerical algorithms with a time step which guarantees a sufficient temporal resolution as required for both the rapid changes of the external focusing and close Coulomb collisions between the test-charges. In z-direction periodically continued slices of the beam are considered, while no boundaries exist in the radial direction. To study a mismatched beam, a 20% larger a_0 than for the matched solution is chosen and taken as input for the envelope propagation (2) with the same space charge parameter and emittances. Since the current is kept at $I = 0.8$ A, the beam density n and the test-particle density n_t are lower for the mismatched beam, but the ratio n_t/n and thus q_c are the same as for the matched beam.

In our first studies, we concentrate on the qualitative effects of IBS on the halo formation and are mainly interested in density profiles $\rho(\kappa, t)$ and particle numbers $N(\kappa, t)$ which are sampled by counting and normalizing the number of events in a given interval of the normalized transversal position $\kappa = (x_\alpha^2/a(t)^2 + y_\alpha^2/b(t)^2)^{1/2}$. Usually also some time average over a couple of time steps is performed.

Unfortunately, there is no clear cut definition of a halo. The sharp edges of a KV-beam provide, however, at least a explicit distinction between particles inside and outside the core. This is already anticipated in the normalized distance κ, where values $\kappa \leq 1$ indicate particles inside the core, regardless of its actual shape as given by $a(t)$ and $b(t)$. Fig. 1 presents PCMD-simulation results of the particle density $\rho(\kappa, \tau)$ as a function of κ for various q_t and different positions τ in the FODO channel, and for a matched and a mismatched beam. Clearly visible are the following general trends of the density at $\kappa > 1$: it increases with q_t, that is, the strength of the Coulomb collisions, i.e. IBS, and the number of passed structures, and, compared to the matched beam, substantial larger $\rho(\kappa > 1)$ are observed for the mismatched beam. These effects are specifically pronounced at the largest shown $q_t = 50$, where IBS is presumably unrealistic high. The more realistic scenario, with $\Gamma_t \approx \Gamma$ and $\nu_t \approx \nu$ is close to $q_t = 10$. Here, only moderate effects show up. The difference between matched and mismatched beam is, however, as well highly significant. The relative number $N_\kappa(\tau)/N_t$ of (test-) particles within a certain interval of $\kappa > 1$ (not plotted) shows the same trends as $\rho(\kappa, \tau)$. Here for realistic IBS rates, corresponding to $q_t = 10 \ldots 20$ a fraction of $\approx 10^{-4}$ of the beam particles are

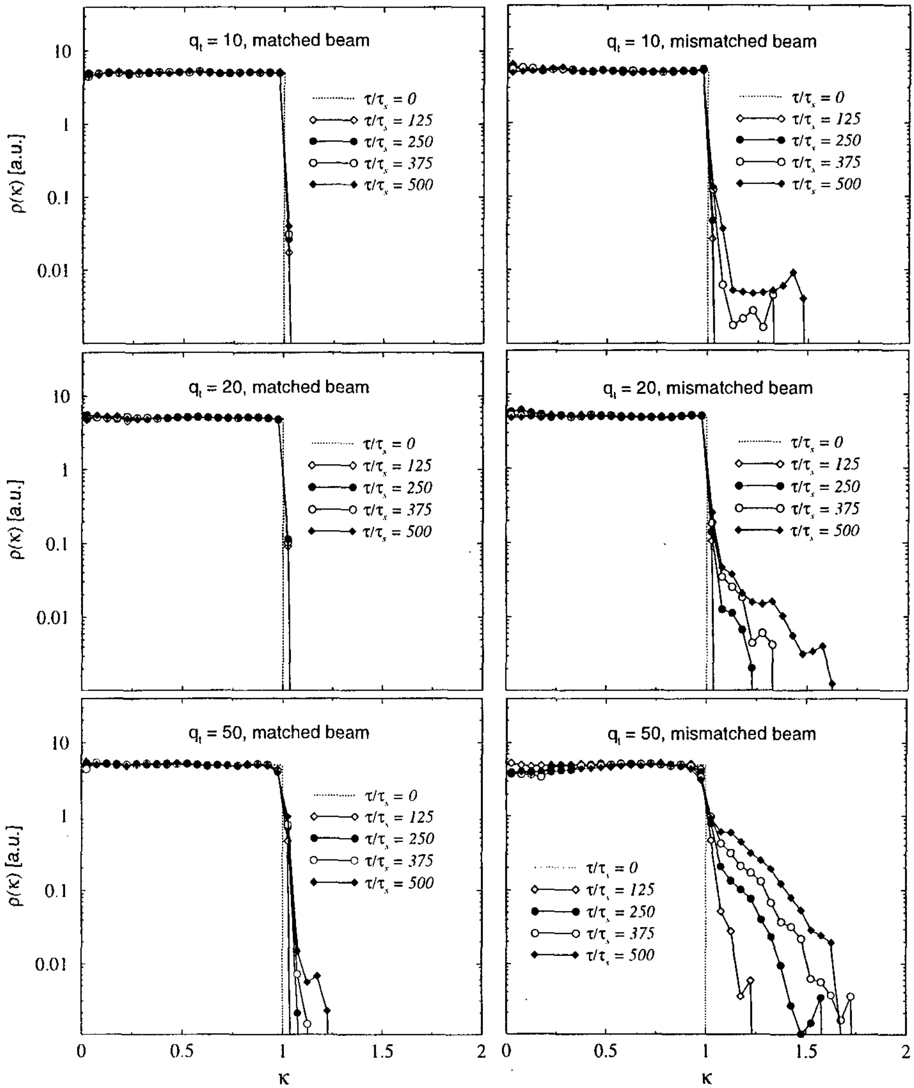

FIGURE 1. PCMD results for the test-particle density ρ as a function of the dimensionless distance $\kappa = (x_\alpha^2/a(t)^2 + y_\alpha^2/b(t)^2)^{1/2}$ for various test-particle charges q_t and different positions τ in the FODO channel in units of the length τ_s of one elementary structure. The matched KV-beams are shown on the left, the mismatched one on the right.

around $\kappa = 1.5$ at the end of the channel ($\tau \gtrsim 400\ \tau_s$). But we remark the problem of statistics for these PCMD-runs with $N_t = 800$ particles and the related strong fluctuations of the extracted quantities. When performing more independent simulation runs and/or using larger particle numbers we expect a comparable fraction of particles at even larger κ.

In summary, first results obtained with the PCMD-model for a KV-beam in a FODO channel demonstrate that even rather small IBS rates significantly enhances the number of particles outside the beam core in a mismatched high current beam. In the matched beam, yet much higher IBS, produces only a small number of particles slightly above the surface of the core. This strongly supports the existence of a scenario whereby already a small amount of IBS is sufficient to transport a non-negligible fraction of particles in the region of nonlinear space charge forces outside the beam core within the typical time scale of beam acceleration in a linac. In the case of a mismatched beam, resonances between these nonlinear space charge forces and the betatron oscillations push the particles away from the core to higher amplitudes. This mechanism has been already identified in a pure PC-picture [3] when studying particles which have been explicitely put in the nonlinear region outside the core. Including IBS the population of this region is provided by the hard collisions which thus serve as a trigger for the elevation of particles by the space charge force resonances. This supplies a strong evidence that IBS serves as an indirect source for the halo formation in a mismatched beam. To verify if this has real significance for the beam losses in a high current linac, subsequent and supplementary more quantitative investigations are needed.

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