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Critical Velocity of Electromagnetic Gun in Response to Projectile Movement

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ABSTRACT

A model is developed to investigate the dynamic response of an electromagnetic (EM) rail gun, induced by a moving magnetic pressure during launch of projectiles. As the projectile velocity approaches a critical value, resonance can occur and cause high amplitude stress and strain in the rail at the instant and location of projectile's passage. In this study, governing equations of a railgun under dynamic loading conditions are derived that illustrate a lower-bound critical velocity in terms of material properties, geometry, and barrel cross-section. That represents the worst case or a lower bound solution for the structure under a dynamic loading condition. A study is then performed to show the effect of these parameters on the critical velocity of the barrel. Accordingly, the model that accounts for projectile velocity and gun construction can be used to guide and improve barrel design.

INTRODUCTION

A strain of very high amplitude and frequency, commonly referred to as dynamic strain amplification, develops in a conventional gun tube due to the passage of the projectile. The phenomenon is caused by the resonance of flexural waves when the moving pressure approaches the velocity of wave propagation in the gun tube. The resonance response in an isotropic cylinder attributed to a moving pressure load has been investigated by Taylor [1], Jones and Bhuta[2], Tang [3], and Reismann [4]. Simkins [5] investigated the dynamic response of flexural waves in steel gun tubes, as very large strains have been observed in a 120mm tank gun barrel. Hopkins [6] applied finite element analysis to obtain a solution in a more complex taper geometry. Tzeng and Hopkins[7] investigated the dynamic strain effect in cylinders made of fiber-reinforced composite materials overwrap with a metal liner. Tzeng [8] extended the research to study fracture in the composite gun tube due to the dynamic response.

In this paper, an analytical solution was developed to obtain the critical velocity of an EM rail gun barrel attributed to dynamic loading conditions. Dynamic response could be a concern particularly since a fieldable EM barrel has to be a lightweight construction with hypervelocity launch capability. Figure 1 shows a schematic of an EM rail gun cross section

and loading condition [9]. The rail and insulator (typically ceramic or polymer composite) were contained and supported by a containment structure. The rails are in compression due to the EM force acting on the rail and reaction force resulting from the containment structure. Furthermore, the magnetic force in the rail is discontinuous at the location of projectile armature where the electrical current passes through. The discontinuity of the force causes local bending moment and shear stress in the rails near the armature location. The pressure front will move along the rails as the projectile moves down through the barrel. Accordingly, dynamic stress and strain occur as the projectile movement approaches the critical velocity of the railgun.

ANALYSIS

Consider a railgun cross section as shown in Figure 1. The rail has a rectangular cross section and is mechanically supported by a rigid material, a containment structure, and insulation material [10]. The structural response of the rail can be modeled as a beam sitting on an elastic foundation as shown in Figure 2. Accordingly, the rail is the beam and the support from the insulation material and containment is modeled as an elastic foundation. It is assumed that structural interaction between the rail and the containment is modeled through the elastic constant. The magnetic pressure traveling at the speed of the projectile on the rail can be expressed as a Heaviside step function. The governing equation for the rail gun subjected to a moving pressure can then be derived as follows:

$$m\frac{\partial^2 w}{\partial t^2} + EI\frac{\partial^4 w}{\partial x^4} + kw = q[1 - H(x - Vt)], \tag{1}$$

Here, w is the lateral displacement, dependent upon time, t, and axial position coordinate, x, m is the mass per unit length and is equal to ρBh, ρ is the density of rail material, and B and h are the width and thickness of the rail, respectively. E is the modulus of rail material and I is the moment of inertia of the rail cross section. The elastic constant, k, due to the elastic foundation will be derived in a later section. The loading function, $q(1 - H(x-Vt))$ in Equation (1), represents the magnetic pressure front traveling along the rail with a constant velocity V, represented by a Heaviside step function, $H(x-Vt)$. The magnetic pressure q, is assumed to be constant also. Accordingly,

$$\begin{aligned} q(1 - H(x - Vt) &= 0 &\quad \text{when } x > Vt \\ &= q &\quad \text{when } x \le Vt \end{aligned} \tag{2}$$

Eq.(1) can be solved using separation of variables with the assumption of

$$w(x,t) = \phi(t)\theta(x) \tag{3}$$

Accordingly, the left-hand side of Equation (1) can be rewritten to solve the homogeneous solution as follows:

$$m\ddot{\phi}\theta + EI\phi\theta''' + k\phi\theta = 0 \qquad (4)$$

The critical velocity of the beam (rail) can be derived from the characteristic function and the particular solution from Equation (3) as

$$V_{cr}^2 = \frac{1}{\sqrt{3}}\frac{1}{\rho}\sqrt{\frac{h}{B}}\sqrt{E}\sqrt{k} \qquad (5)$$

Equation (5) shows that the critical velocity of a railgun subjected to a moving pressure front is a function of the rail geometry, density, and elastic modulus. In addition, the support from the containment structure has great influence on the dynamic behavior of the rail. Critical velocity increases with the elastic modulus of the rails and the stiffness of containment structures. From a design point of view, a launcher constructed with high stiffness, lightweight, and a large moment of inertia is preferred for dynamic loading conditions.

The elastic constant of foundation, k, can be calculated from the containment structure if the coupling effects of the insulation material (ceramic in general) are neglected. We consider a circular containment of a unit length subjected to concentrated loads at the inner surface of a cylinder as shown in Figure 3(a). Accordingly, the concentrated loads are calculated from the summation of resulting magnetic pressure. Since both the containment geometry and loading conditions are symmetrical, the structural response can be calculated from the free body shown in Figure 3(b). The stiffness of the containment at the location of the concentrated load can then be obtained from the strain energy of the curved beam shown in Figure 3(b). Neglecting the shear contribution, the strain energy can be expressed as follows:

$$U = \int_0^{\pi/2}\frac{N^2 R}{2A_c E_c}d\theta + \int_0^{\pi/2}\frac{M_a^2 R}{2I_c E_c}d\theta \quad , \qquad (6)$$

Where N is the normal force, M_a is the moment resulting from the concentrated load, and R is the mean radius of the containment. A_c, I_c, and E_c are the cross sectional area, the moment of inertia, and elastic modulus of containment, respectively. N and M_a can then be derived as follows:

$$N = \frac{P}{2}\cos\theta \qquad (7)$$

and
$$M_a = \frac{PR}{2}(\cos\theta - \frac{2}{\pi}) \qquad (8)$$

Therefore, the strain energy of the quarter containment can be calculated in terms of concentrated load P, material properties, and geometry. Based on Castigliano's theorem, the

displacement at the location of the loading "P" can then be derived from the derivative of the strain energy with respect to the "P" as follows:

$$\delta_p = \frac{\partial U}{\partial P} = \frac{\pi PR}{16 A_c E_c} + \frac{\pi PR^3}{16 I_c E_c}\left(1 - \frac{8}{\pi^2}\right) \qquad (9)$$

Where I_c is the bending moment of inertia calculated from a unit length of containment (curved beam) shown in Figure 3, which is equal to $\frac{1}{12} bt^3$ (b=1); where b and t are the length and thickness of the containment, respectively. E_c is Young's modulus of the containment. The stiffness constant of the containment can then be defined as follows

$$k = \frac{1}{\delta_p} \quad , \text{and} \quad P = 1 \qquad (10)$$

NUMERICAL RESULTS

A baseline test case is used to obtain critical velocity for a parametric study. The gun is composed of a pair of aluminum rails, ceramic insulation, and a steel containment. The aluminum rail has a cross section of 12.5mm (0.5 inch) thick by 76.2mm (3.0) inch high. The containment is 12.5mm (0.5 inch) thick. The parameters required for the simulation are listed in Table 1:

Table 1: Mechanical properties and geometric parameters of the rail and containment

Rail (Aluminum)

Modulus (E)	Thickness (h)	Height (B)	Density (ρ)
68.95 GPa	$12.5x10^{-3}$ m	$76.2x10^{-3}$ m	2750 kg/m3

Containment (Steel)

Modulus(Ec)	Thickness (t)	Mean Radius (R)
206.85 GPa	$12.5x10^{-3}$ m	$63.5x10^{-3}$ m

The stiffness of the containment can be obtained from Equations (9) and (10) by application of a unit concentrated pressure load. The inverse of deflection at the center of rail (δ_p) yields the stiffness, k, as follows:

$$k = 1 / \delta_p = 3.48 \times 10^9 \, Pa \, .$$

The k represents the spring constant of foundation for the entire rail height of 76.2 mm (3.0 inch). The critical velocity of the barrel can then be calculated from Equation (5) as follows:

$$V_{cr} = 1148 \, m/sec$$

The critical velocity is strongly dependent on the cross section geometry and the mechanical properties of rail and containment. The model is based on the assumption of no structural coupling effects from insulation materials. It is a reasonable assumption since the bonding between rail and insulation is friction. Parametric studies are performed to compare the baseline case that is constructed with aluminum rails and a steel containment as listed in the Table1.

Figure 4 shows the effects of rail thickness on the critical velocity of railguns. The geometry of containment and all material properties are identical to the baseline case. The increase in the moment of inertia of the rail will enhance the bending stiffness of the rail. Accordingly, the critical velocity increases with the thickness of the rail. However, the critical velocity does not increase linearly. It varies with only a power of 0.25. The stiffness of containment also has strong effects on the critical velocity of the railgun. Figure 5 shows the effect of containment thickness on the critical velocity of the railgun. A thicker containment provides higher stiffness and structural support for the rail. Accordingly, the deflection of containment decreases as it is subjected to magnetic pressure from the rails. Mathematically, it is modeled as the stiffness of foundation, k, which increases as illustrated in Equation (5). The effect of the containment stiffness on the critical velocity is not linear either. It varies with a power of 0.75.

The effect of rail material properties on the critical velocity is illustrated in Figure 6. A combined effect on the dynamic behavior due to the density and elastic modulus of rail is illustrated using some potential material choices. The baseline case is 7075 aluminum alloys. Three different conductor materials are examined. GIGAS24 is an advanced aluminum alloy with a higher modulus of 88.25 GPa (12.8 Msi). The density is about the same as the 7075 aluminum. Accordingly, a higher critical velocity is obtained due to the increase of modulus. Glidcop is aluminum oxide dispersion strengthened cooper. It has a high modulus of 172.4 GPa and a high density of 8900 kg/m^3 material. The critical velocity turns out to be lower than the 7075 aluminum due to the high density. Finally, aluminum reinforced with aluminum oxide (Al$_2$O$_3$) fiber (45% volume fraction of fiber content) is used for comparison. The modulus and density of this material are 165.5 GPa (24 Msi) and 3400 kg/m^3, respectively. The combination of high modulus and low density gives a high critical velocity.

CONCLUSIONS

The dynamic behavior of an EM barrel can be modeled with reasonable assumptions as a rail sitting on an elastic containment. The derived solution illustrates effects of important design parameters and material properties on the critical velocity of barrel, which can be applied for barrel design under dynamic conditions. A high magnitude of cyclic stress can occur that might cause damage in the rail, accelerate growth of defect, and eventually shorten the rail life significantly. The dynamic phenomenon is particularly crucial if gun barrels are

designed to be a lightweight and fieldable system with hypervelocity capability. The developed model provides a meaningful tool to guide barrel design that accounts for dynamic response due to a moving projectile.

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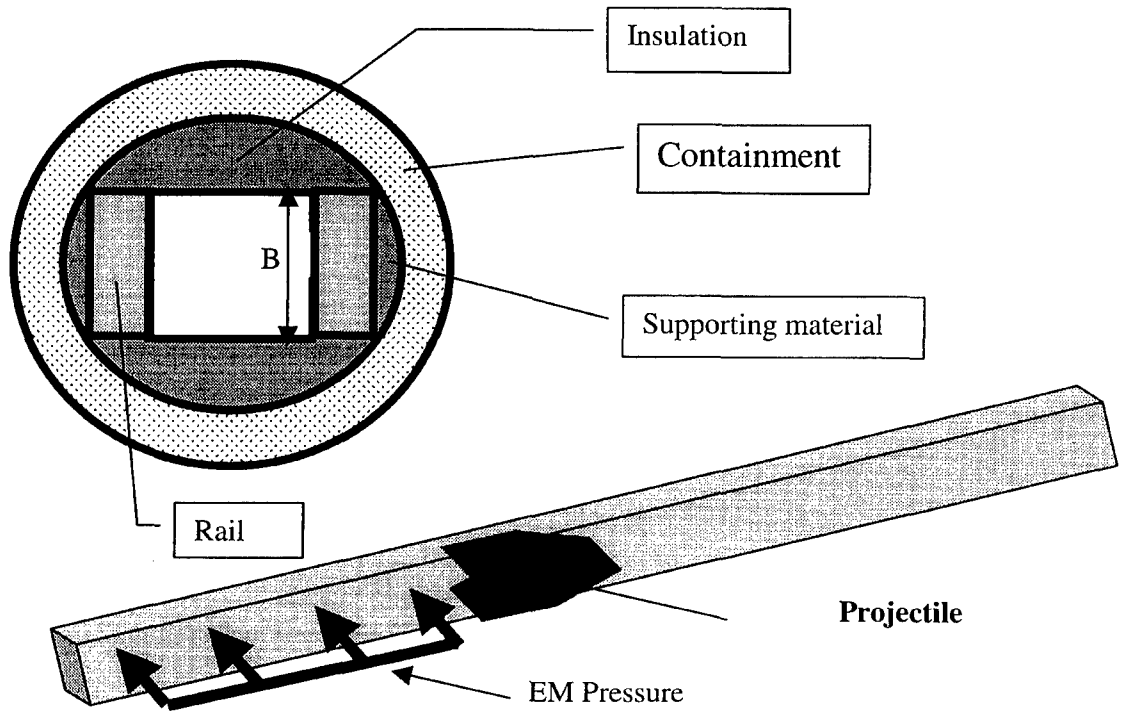
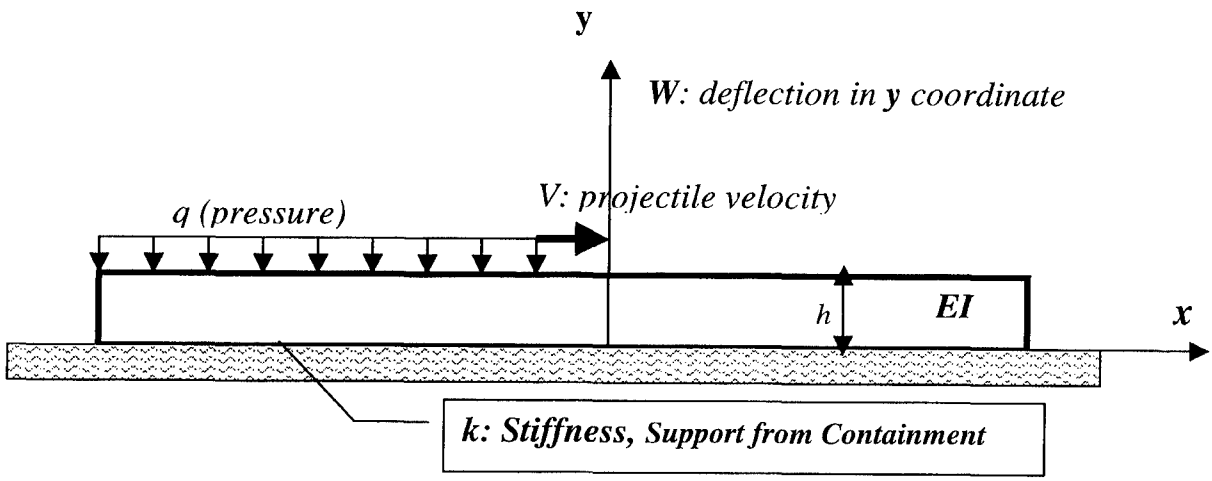Figure 1: A schematic of EM Gun Cross Section, Rail, and Loading Conditions

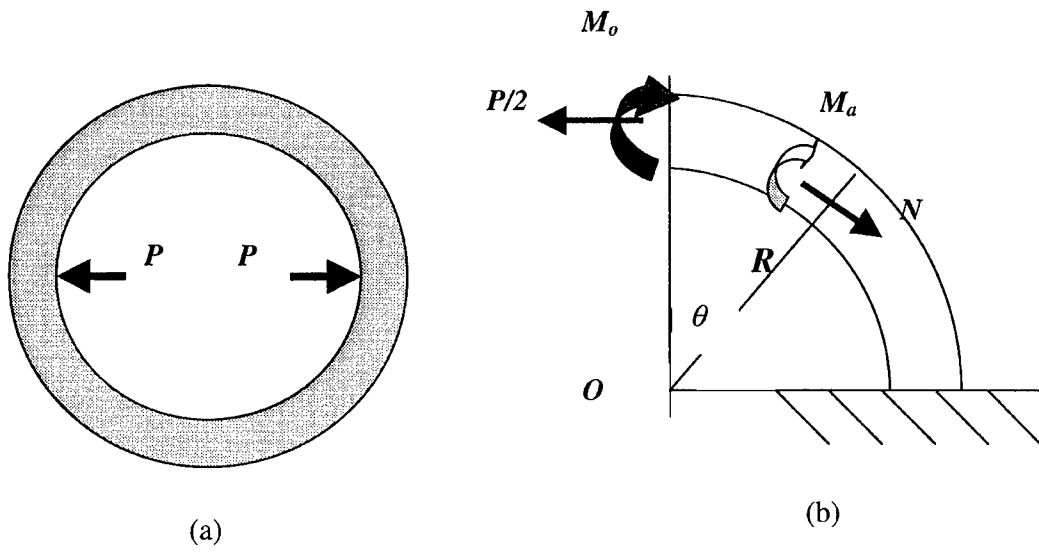Figure 2: Coordinate System and Model Simulation
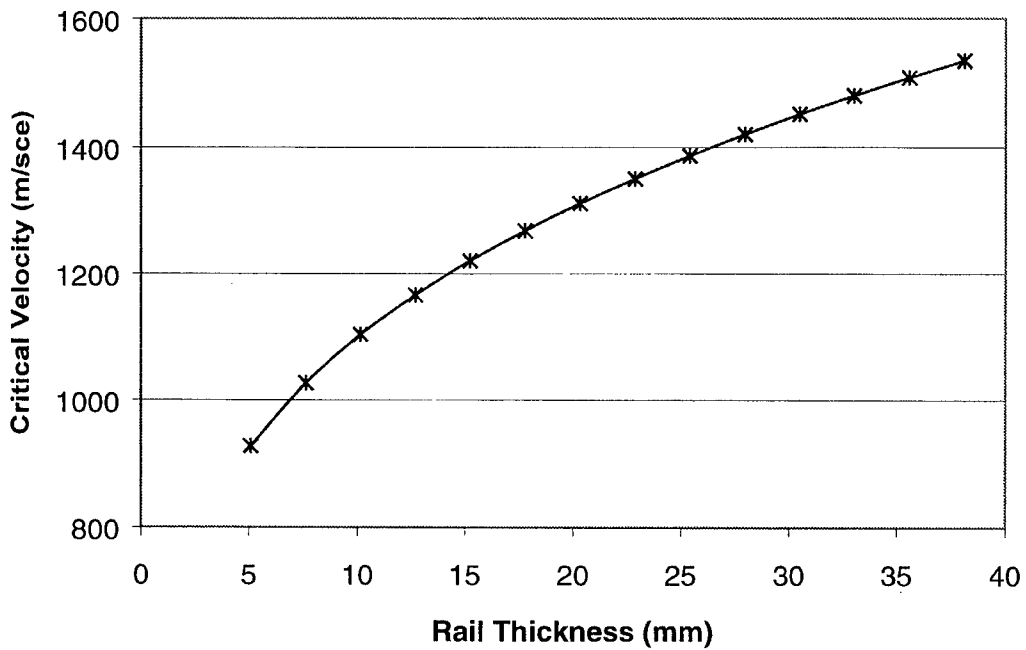
Figure 3: Modeling of Containment Stiffness

Figure 4: Effects of the rail thickness on the critical velocity
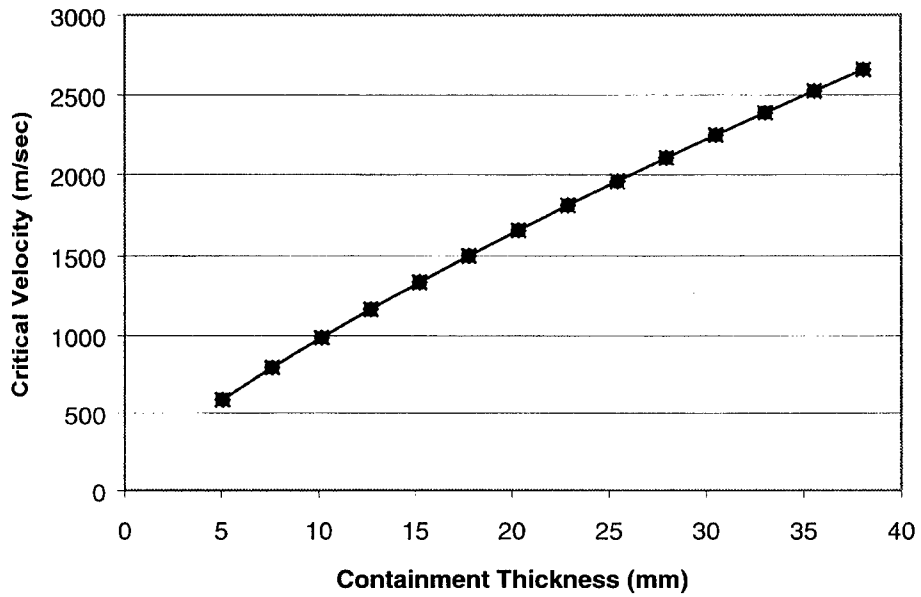
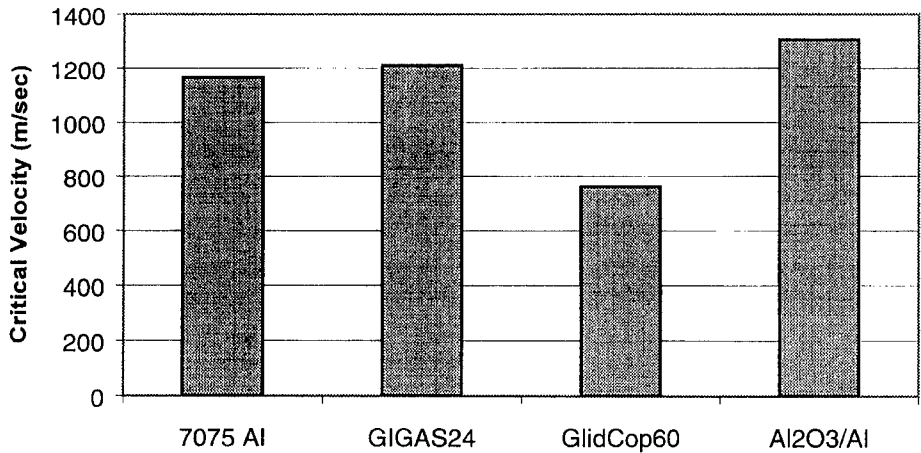Figure 5: Effects of the containment thickness on the critical velocity.

Figure 6: Effects of the rail material properties on the critical velocity.