PROPAGATION OF LONGITUDINAL WAVES IN A GUN BARREL

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Very high stresses could occur in long gun barrel wall during firing. The stresses induced are due to the propagation of axisymmetric longitudinal waves along the barrel. To study this propagation, the tube was modelled using medium thickness shell elements. The equations of motion thus obtained were discretised using the centred finite difference method and resolved explicitly using a specifically designed computer programme. The latter outputs the displacements and stresses for all nodal points in the barrel, at any time during the firing cycle. Validation against experimental results gave very good agreement. A parametric study was then contucted to evaluate the influence of certain parameters. Finally, the limits of the model were studied. The study was completed by comparison with finite element modelling.

INTRODUCTION

Currently, the design and analysis of gun barrels are done on the basis of the statics of continuous environments, that is, the barrel is dimensioned in such a way that it withstands a constant pressure in each section of the barrel, corresponding to the maximum firing pressure at that point. Premature wear of the barrels of certain weapons and the existence of high stresses in their wall at the moment of firing have been found.

The technical literature attributes the excessive stresses in the barrel's wall to a longitudinal propagation of axisymmetric waves. This phenomenon is shown and described qualitatively by several documents [1], [2], [3] and [4] and analytical and experimental studies have been published about it.

Since the mid-50s, several analytical formulations have been published about the longitudinal vibrations of a barrel ([5], [6], [7], [8], [9] and [10]).Nevertheless, in these studies, very restrictive hypotheses are made, particularly on the geometry of the barrel and its loading. In particular, most of the studies consider an infinitely long or semi-infinite barrel and do not therefore manage any boundary condition. Moreover, the barrel is more often than not modelled by a thin shell, always of constant thickness whereas, in a weapon barrel, the thickness is never constant. Furthermore, a weapon barrel is very thick by the breech and thin at the muzzle. These geometric hypotheses are therefore not at all realistic in the case of a weapon barrel. Moreover, the velocity of the projectile and the pressure are not constant values : they are functions of time whereas the models reported in the literature assume them to be constant. Finally, contrary to what is represented in these models, the barrel is not free and open at its two ends : it is linked to the breech, whose mass is not inconsiderable and on which a pressure is applied.

From the numerical point of view, all the published studies ([9], [2], [11], [4], [12])are done on the basis of commercial finite element software programs, supplemented in order to be able to model a moving pressure front. In this way, it has been possible for simulations of actual cases to be carried out : this time, the pressure, the geometry of the barrel, the velocity of the projectile and the boundary conditions at the muzzle are realistic. Nevertheless, the munition itself is not taken into account. Only the moving pressure front is modelled on the basis of the actual, calculated or arbitrary curves (square wave). The friction and driving of the munition in the barrel do not seem to be taken into account.

Moreover, as in the analytical models, the breech is not modelled : the barrel is simply locked axially. In this way, neither the inertia due to the mass of the breech nor the pressure which is applied on it is modelled. Moreover, this type of modelling is troublesome to implement : finite element software programs such as ABAQUS require very powerful computers and very considerable computing times. Furthermore, a finite element model is difficult to modify : the slightest change of geometry means that the meshing has to be redone, which is not compatible with a dimensioning procedure or with a parametric study. Finally, several parameters of the computation need to be fixed precisely : for example, the fineness of the meshing or the minimum temporal increment. Indeed, if these two parameters are incorrectly evaluated, the high frequencies cannot be observed ; conversely, if they are too fine, the computing time and the size of the files may well be very considerable. However, in view of the computing times, it is impossible to do a scan of these parameters in order to find their optimum values. Finally, it is impossible to dissociate the forces so as to judge their respective influence.

The only way to overcome these disadvantages is to develop a tool specifically suited to weapon barrels, on the basis of the equations of motion of the barrel dynamically. This tool will make it possible to understand the phenomena, to do the initial simulations and to evaluate thus certain parameters like the fineness of the meshing. These computations could then possibly be supplemented and validated by well chosen finite element simulations.

MODELLING

So as to be able to quantify these phenomena, a model of the barrel was developed. To do this, various hypotheses were advanced.

The first is an axisymmetry hypothesis. Indeed, besides the axisymmetry of geometry, since flexion waves are propagated far more slowly than axisymmetric waves, it is possible to consider that the two motions are decoupled. Moreover, it has been shown numerically [10] that during the internal ballistics phase, the effects of flexion are relatively unimportant for high velocities of the projectile. Finally, Simkins [1] has shown by experiment that the phenomenon studied is perfectly axisymmetric.

Secondly, so as to take account of the internal pressure and of the phenomenon of friction of the munition in the barrel, it is necessary to model it either in shell form or on the basis of the complete three-dimensional theory. Various studies have been published about this, for various theories of shells, and the results have been compared with the three-dimensional theory. In view of the small difference obtained, because the distribution of the stresses in the thickness of the barrel does not interest us here, the Midlin-Reissner theory of shells, or complete theory, was chosen. It takes account of the transverse shear and of the rotational inertia of the straight section of the barrel, so as to model «medium thickness» shells. Moreover, a correction on the radial stress was added to the model, for a more faithful modelling of the thickest barrels.

Thirdly, the barrel is considered to be uniform, and its behaviour is assumed to be elastic.

The lining is not taken into account and the barrel is therefore assumed to be homogeneous.

Finally, American studies including Simkins's in 1978 [13],show that the boundary conditions which exist on a large-calibre weapon before firing are no longer valid during the ballistics phase. The links at the fixing parts seem far from being rigid. In fact, it was found that an excellent agreement between the calculated and experimental self-frequencies can be observed only if the barrel is considered to be free during its ballistics phase. This hypothesis remains valid so long as the displacements of the barrel are less than the plays of the fixing parts.

Various forces are applied on the barrel during the internal ballistics phase.

The friction forces of the projectile on the barrel are represented, in an initial approximation, by a surface distribution of forces which is assumed to be constant depending only on the material of the band, applied on the inner surface of the barrel in contact with the band.

The computation of the pressure forces is carried out on the basis of actual pressure curves at the base and at the breech, a linear interpolation being done at each moment between the two curves.

Furthermore, the breech is taken into account in this model in two ways : by the pressure which is applied on it and by the mass it represents.
In view of the preceeding hypotheses, equations were found for the problem using Hamilton's principle, based on a balance of the system's energy. Mathematical simplifications were carried out, in keeping with the previously formulated geometric hypothesis.
The barrel is assumed to be of length L, of mean radius R, and of thickness h, these two variables being capable of varying with the x-axis in the barrel. Because of the geometry of the problem, any point on the wall of the barrel is identified by its cylindrical co-ordinates r, θ, x. By designating by u and w the axial and radial components of the displacement vector of a point of the neutral axis of the wall of the barrel, and by ψ the angle of rotation of the straight section at the same point, applying Hamilton's principle leads to a system of equations of motion :

$$\begin{aligned}
\rho\cdot\left(R^2\cdot h-\frac{h^3}{12}\right)\cdot\ddot{u} = \frac{E}{1-\nu^2}\cdot & \left[\begin{array}{l}\left(R^2-\frac{h^2}{4}\right)\cdot h'\cdot u' + \left(R^2\cdot h-\frac{h^3}{12}\right)\cdot u'' + \nu\cdot(R\cdot h'-R'\cdot h)\cdot w \\ +\left(-R^2\cdot R''\cdot h - R^2\cdot R'\cdot h' + R''\cdot\frac{h^3}{12} + R'\cdot\frac{h^2}{4}\cdot h'\right)\cdot\psi \\ +\nu\cdot h\cdot R\cdot w' - R'\cdot\left(R^2\cdot h-\frac{h^3}{12}\right)\cdot\psi'\end{array}\right] \\
& -\frac{\nu}{1-\nu}\cdot\frac{(2R-h)^2(4R+h)}{64\cdot R^3}\cdot\left(R^2\cdot h-\frac{h^3}{12}\right)\cdot p' \\
& -\frac{\nu}{1-\nu}\cdot\frac{2R-h}{64\cdot R^4}\cdot\left(\begin{array}{l}6\cdot R^3\cdot R'\cdot h^2 - 8\cdot R^4\cdot h\cdot h' - 6\cdot R^3\cdot h^2\cdot h' \\ +8\cdot R^5\cdot h' + 3\cdot R^2\cdot R'\cdot h^3 + R^2\cdot h^3\cdot h' \\ +\frac{1}{2}\cdot R\cdot h^4\cdot h' - \frac{1}{2}\cdot R\cdot R'\cdot h^4 - \frac{1}{4}\cdot R'\cdot h^5\end{array}\right)\cdot p \\
& +h\cdot\kappa\cdot G\cdot R\cdot(w'+\psi) + \left(R+\frac{h}{2}\right)\cdot\left(R-\frac{h}{2}\right)\cdot q
\end{aligned}$$

(1)

$$\begin{aligned}
\rho\cdot R\cdot h\cdot\ddot{w} = & -\frac{E}{1-\nu^2}\cdot\left[\ln\left(\frac{2R+h}{2R-h}\right)\cdot w + \nu\cdot h\cdot u' - \nu\cdot R'\cdot h\cdot\psi\right] + \left(R-\frac{h}{2}\right)\cdot p \\
& +\kappa\cdot G\cdot\left((R'\cdot h+R\cdot h')\cdot w' + R\cdot h\cdot w'' + (R'\cdot h+R\cdot h')\cdot\psi + R\cdot h\cdot\psi'\right) \\
& +\frac{\nu}{1-\nu}\cdot\frac{(2R-h)^2(4R+h)}{64\cdot R^3}\cdot h\cdot p
\end{aligned} \qquad (2)$$

where E is the Young Modulus of the barrel material, ν its Poisson ratio and G its shear modulus.

$$
\rho\cdot\frac{h^3}{12}\cdot\left(\frac{h^3}{12}-R^2\cdot h\right)\cdot\ddot{\psi}=\frac{E}{1-\nu^2}\left[\begin{array}{l}
\left(\frac{h^3}{12}\cdot(R'\cdot h+R\cdot h')-R\cdot h^2\cdot\left(\frac{h\cdot h'}{4}+R\cdot R'\right)\right)\cdot u' \\
+\nu\cdot\left(h'\cdot\frac{h^3}{12}-R\cdot R'\cdot h^2\right)\cdot w+\nu\cdot h\cdot\frac{h^3}{12}\cdot w' \\
+\left(R'\cdot R\cdot h^2\left(R'\cdot R+\frac{h\cdot h'}{4}\right)-\frac{h^3}{12}\left(R'^2\cdot h+R\cdot R'\cdot h'\right)\right)\psi \\
\left(\frac{h^2}{4}\cdot\frac{h^3}{12}\cdot h'-2\cdot R\cdot R'\cdot h\cdot\frac{h^3}{12}-R^2\cdot\frac{h^3}{4}\cdot h'\right)\cdot\psi' \\
+\frac{h^3}{12}\cdot\left(\frac{h^3}{12}-R^2\cdot h\right)\cdot\psi''
\end{array}\right]
$$

$$
+h^2\cdot\kappa\cdot G\cdot R^2\cdot(w'+\psi)+\left(\frac{h^3}{12}+R\cdot h^2\right)\cdot\left(R-\frac{h}{2}\right)\cdot q
$$

$$
-\frac{\nu}{1-\nu}\cdot\frac{2R-h}{64\cdot R^3}\cdot\left[\begin{array}{l}
20\cdot R^3\cdot R'\cdot\frac{h^4}{12}-2\cdot R^2\cdot R'\cdot\frac{h^5}{12}+4\cdot R^3\cdot\frac{h^4}{12}\cdot h' \\
+2\cdot R^2\cdot\frac{h^5}{12}\cdot h'-16\cdot R^4\cdot\frac{h^3}{12}\cdot h'-R\cdot R'\cdot\frac{h^6}{12} \\
-8\cdot R^5\cdot R'\cdot h^2+2\cdot R^4\cdot R'\cdot h^3
\end{array}\right]\cdot p \tag{3}
$$

The equations of motion of the problem constitute a system of equations involving hyperbolic second-order linear partial derivatives. Knowing the state of the system at time t therefore makes it possible to predict its changes at time t+dt. The numerical method used here in order to arrive at the resolution of the problem is the centred finite difference method, combined with an explicit resolution, well suited to transient phenomena. After programming, the displacements, deformations and stresses are available at any node, and at any time during firing. The details of this finding of equations and programming are available in [14].

The software thus obtained runs in Windows, which makes for great user-friendliness and ease of use. The computing times were reduced to about twenty minutes on a Pentium 450 as against several weeks on an O_2 type Silicon Graphics work station, for a 6 metre barrel.

NUMERICAL VALIDATION

The numerical validation was done in several stages :

The first phase is a validation phase of the system of equations itself. This system was written in matrix form, and the diagonalisation of the matrix thus obtained supplied the frequencies and self-modes. These were then compared to the results of a modal analysis carried out by finite elements, using the ABAQUS software program. This comparison proved satisfactory. The maximum error on the frequency is only 5% at the 25th mode.

Subsequently, we checked, by programming the equations, that the self-modes and frequencies were indeed retained with time, which confirms the validity of the explicit scheme.
Computations by modal superposition were carried out, for simple cases, and the agreement proved to be good.
Finally, finite element computations made it possible to validate the modelling for more complex configurations. These computations were carried out with the ABAQUS software program. This software program was used with success in the technical literature, with a good correlation between experience and simulation. These results were the subject of an initial publication [15].
Fig 1 shows the changes in the axial displacement in a barrel of constant thickness as a function of time. This barrel is 400 mm long, with an internal radius of 60 and a constant thickness of 40. This mean thickness-to-radius ratio corresponds to the mean ratio of a large-calibre weapon barrel. A pressure front of 1 MPa, moving at a constant velocity of 1000 m/s travels through this barrel. This configuration corresponds to a test case, but permits very reduced computing times and therefore lends itself well to repeated calculations, necessary for validation. On this figure, the computation results for four different computation methods have been entered:
finite element method, using the ABAQUS software program,
finite difference method, with a correction on the radial stress which was nil, constant, or variable in the thickness.

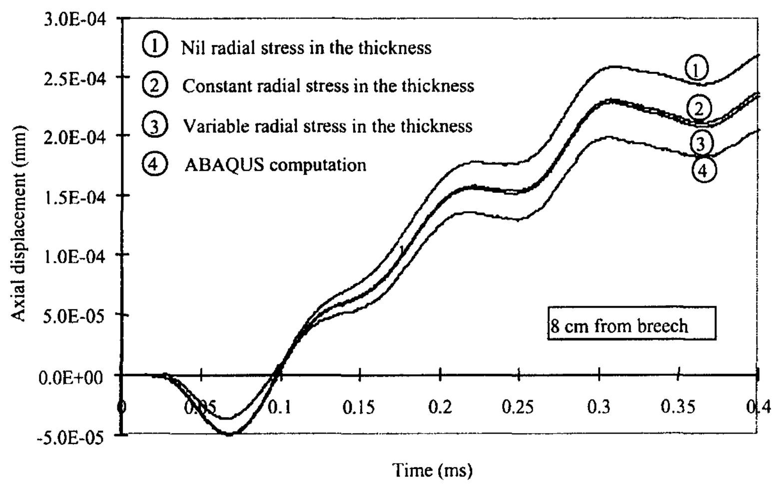

FIG 1 : AXIAL DISPLACEMENT 8 CM FROM THE BREECH OF THE 40 CM THICK BARREL

It is clear that, while the three methods give curves with the same trend, same frequency and same order of magnitude, the introduction of the radial stress permits a clear improvement in the results (the difference compared with the ABAQUS curve is halved). However, the fact of assuming this radial stress to be variable in the thickness does not change much.

Fig 2 shows the changes in the radial displacement with time of a point on the neutral axis of the same barrel. The agreement is clearly better than on the axial displacement, but the same findings can be made on the choice of computation method.

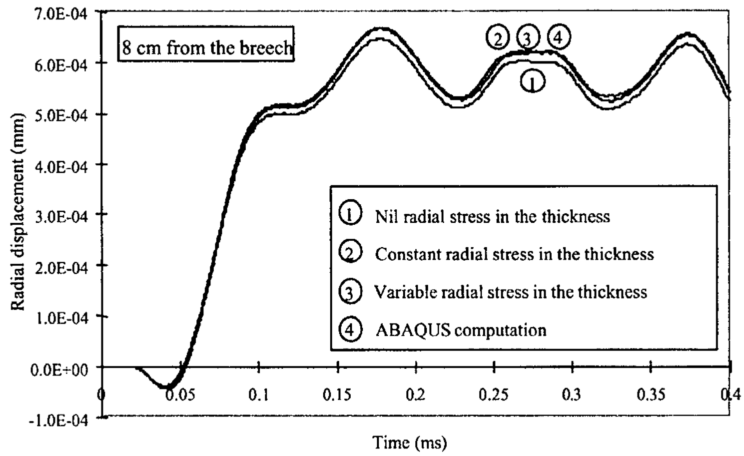

Fig 2 : RADIAL DISPLACEMENT 8 CM FROM THE BREECH OF THE 40 CM THICK BARREL

COMPARISON WITH EXPERIENCE

Once the numerical validation had been carried out, the results were compared with actual experience. To do this, results of firing with a 120 mm gun were used. The firing cycles were simulated for two different types of munitions, with for input data the pressure and kinematics curves of the projectile in the barrel obtained by experiment. The displacements at several points on the external skin of the barrel were obtained experimentally and compared to the numerical results. In the interest of confidentiality, the results in this paragraph will be given without any numerical values. Since the aim is to compare experience and simulation, this precaution does not affect the demonstration.
The curves in Fig 3 show the results obtained. The experimental curves were filtered after analysis, so as to eliminate the noise.

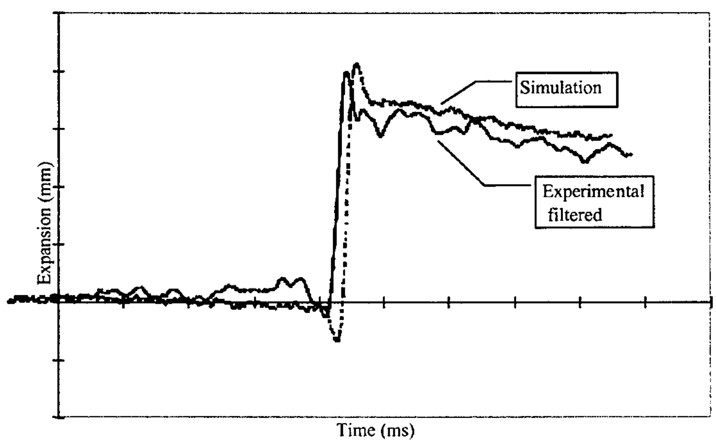

FIG 3 : COMPARISON BETWEEN EXPÉRIENCE AND SIMULATION

It is clear that the pressure rise moment, and the amplitudes and frequencies observed are completely similar numerically and experimentally.
The same agreement was found for all the rounds and for all the gauges, for both types of munitions.
Fig 4 shows the dynamic amplification of the expansion at several places on the barrel

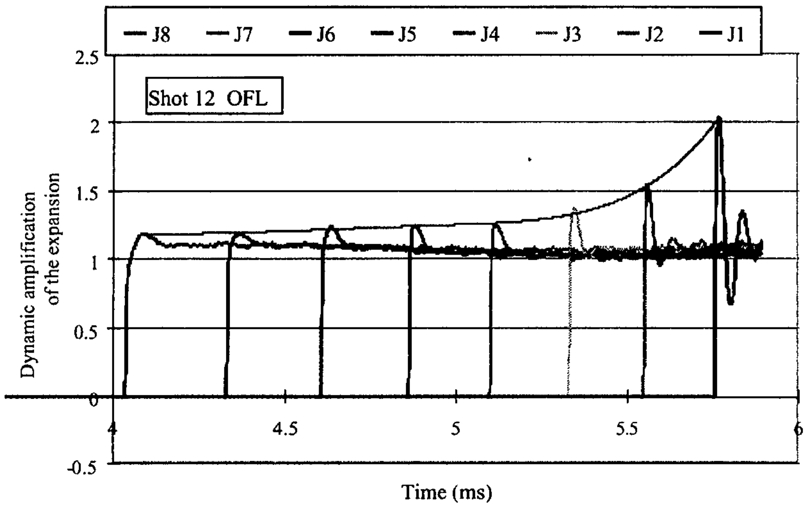

FIG 4 : DYNAMIC AMPLIFICATION OF EXPANSION AT SEVERAL PLACES ON THE BARREL

It can be seen that the closer one gets to the muzzle of the barrel, the greater the amplification.

The swell varies with the thinness of the barrel, the internal pressure and the velocity of the projectile. The first two points play a part in the static component of the swell. Their effects are therefore masked by the computation of the dynamic amplification. The increase in the amplification is therefore due only to the velocity of the projectile.
If one compares at each place the velocity of the projectile with the critical velocity, one notices that the ratio varies between 45 and 75%.

Now that the numerical simulation is validated numerically and experimentally, it can be used to evaluate the influence of certain parameters and to dissociate their effects.

SIMULATIONS AND PARAMETRIC STUDY

Qualitative analysis

So as to properly understand the phenomenon, it is useful to study qualitatively the dynamic response of a barrel in a simplified configuration. The influence of each parameter and each force can thus easily be determined. This analysis was also published in [15].
In the first instance, a barrel of constant thickness, through which a square wave type pressure front, moving at constant velocity, travels, is studied. Only the radial pressure is applied to the barrel: the forces due to the mass of the breech or to the pressure it undergoes are not taken into account ; neither are the forces produced by the interaction between the barrel and the band.

Fig 5 represents, in arbitrary co-ordinates, the changes in the profile of the barrel over its entire length as a function of time. It is clear that :
there is a vibratory phenomenon before and after the pressure front which superimposes itself on the swell of the barrel,
this precursor, with a damped sinusoid trend, is propagated more quickly than the pressure front, is reflected at the muzzle, and disrupts the initial signal,
at the rear of the pressure front, waves with an amplified sinusoid trend are constructed, disrupted on the one hand by the edge effect at the breech and on the other hand by the wave reflected at the muzzle,
the barrel is shortened at its two ends, because of the Poisson effect.

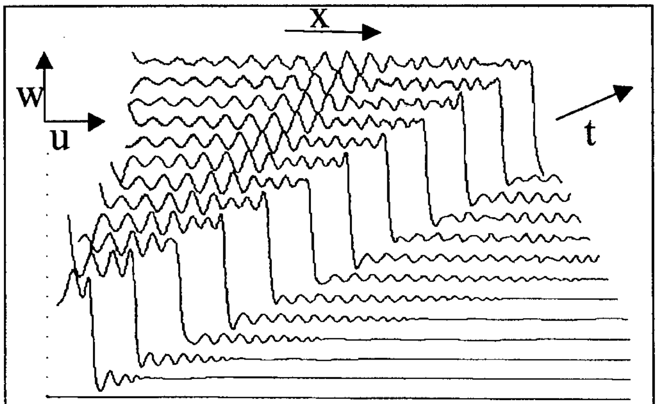

FIG 5 : TREND OF THE DEFORMATION OF BARREL WITH TIME

Fig 6 shows the axial (u) and radial (w) displacements as well as the angle of rotation of the straight section of the barrel (ψ) as a function of the x-axis (horizontal axis) and of time (vertical axis). The position of the pressure front is marked with a line.

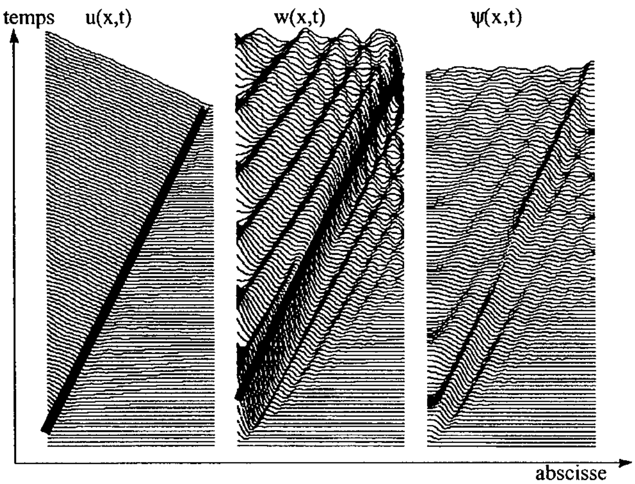

FIG 6 : DISPLACEMENTS AS A FUNCTION OF X-AXIS AND TIME

These graphs make it possible to show clearly the wave which is propagated ahead of the pressure front, as well as its damped sinusoid trend. Similarly, the amplified sinusoid present to the rear of the front is perfectly visible. These findings are present in the three graphs, even though the phenomena are more marked on radial displacement.

Model used for the simulations

In order to study the phenomenon more precisely, and to estimate the influence of the parameters, it is necessary to take realistic orders of magnitude. To do this, a 5 metre barrel with an internal radius of 60 mm and a thickness varying between 90 and 20 mm was chosen (Fig 7).

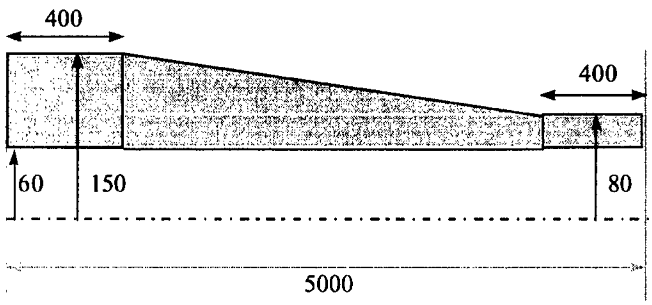

FIG 7 : PROFILE OF THE BARREL USED FOR THE PARAMETRIC STUDY

A square wave type pressure profile is modelled here.
The velocity of the projectile is assumed to vary linearly over the length of the barrel and is chosen in such a way that the muzzle velocity is between 1500 and 2700 m/s.
The friction forces between the projectile's band and the barrel are assumed to be constant and applied on the inner surface of the barrel in contact with the band. In some configurations, a contact pressure was added to the model.
Finally, the forces at the breech are modelled : the pressure and the inertial forces are taken into account.

Influence of the forces

Fig 8 shows the influence of the breech on the Von Mises equivalent stress. All these curves were produced under the same computation conditions. One notes that the most influential parameter is the breech pressure, which disrupts the signal fairly strongly, clearly increasing the amplitude of the oscillations. Mc is breech mass, and Pc represents pressure et breech.

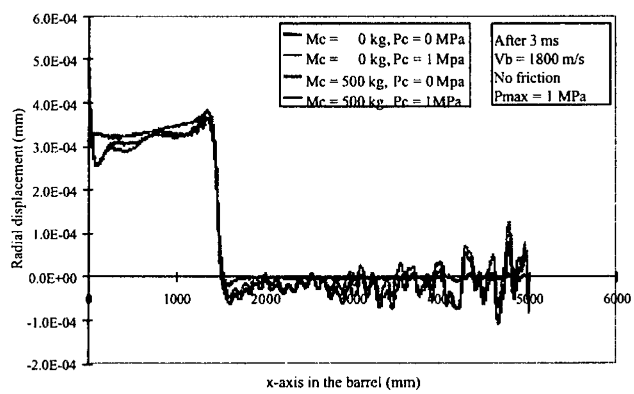

FIG 8 : INFLUENCE OF THE BREECH MASS ON THE RADIAL DISPLACEMENT

The mass of the breech changes only the axial displacement, limiting the rigid body motion associated with the weapon's recoil.

Influence of the velocity

Numerous bibliographical references ([6], [7], [8], [1] and [14]) show, using a quasi-stationary model, the existence of four particular velocities of the projectile in the barrel :

- the propagation velocity of the longitudinal waves in a thin plate,
- the propagation velocity of the longitudinal waves in a bar,
- the propagation velocity of the shear waves,
- a propagation velocity particular to each barrel.

The first three velocities depend only on the barrel's material; the last one depends in addition on its geometry. Of these four particular velocities, the second one and the last one lead to unlimited displacements and can therefore be qualified as critical. A quick calculation shows that the lowest critical velocity is the only one which can be approximated in the case of a weapon barrel. As this velocity depends on the geometry of the barrel, it is necessary to evaluate it at all points on the barrel. To do this, as the expression given by [8] was established in the case of a barrel with constant thickness, it is necessary to consider that the barrel is in fact a series of sections of constant thicknesses.

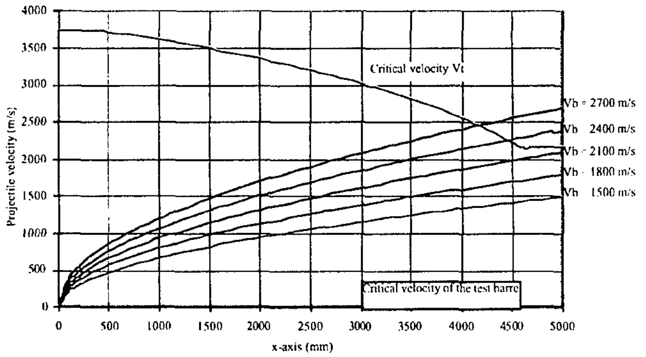

Fig 9 : LOWEST CRITICAL VELOCITY OF THE TEST BARREL

Such a calculation leads to the first curve of Fig 9. On this figure, the trend of the velocity of the munition as a function of the x-axis has been superimposed for several values of the muzzle velocity. The first two velocity values tested are therefore sub-critical throughout the internal ballistics phase. The value of 2100 m/s may well lead to critical conditions, whereas the two largest values of the velocity will be in turn sub-critical, critical and super-critical. The response of the barrel for all these projectile velocities will therefore make it possible to test the three velocity conditions.

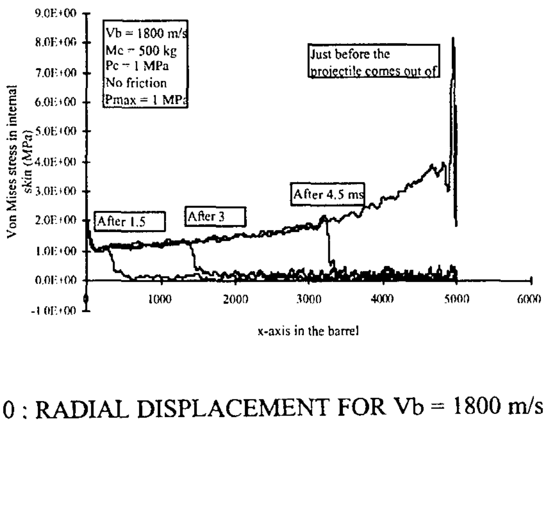

Fig 10 : RADIAL DISPLACEMENT FOR Vb = 1800 m/s

When the muzzle velocity is 1800 m/s, the radial displacement is clearly amplified at the muzzle, just before the projectile comes out of the barrel. The calculation of the dynamic amplifications shows at this point values of about 1.7 in radial displacement, and 2.5 in Von Mises stress. This amplification does not appear as an isolated peak, but indeed as an amplified sinusoidal oscillation. Even though, from Fig. 10, the velocity of 1800 m/s is still clearly sub-critical (about 83% of the critical velocity at the muzzle), the influence of the critical velocity begins to make itself felt, and the dynamic amplification phenomenon is constructing itself.

When the muzzle velocity reaches 2100 m/s (i.e. 96.8 % of the critical muzzle velocity), the dynamic amplification at the muzzle when the projectile comes out of the barrel exceeds 2.5 in radial displacement and 5 in Von Mises stress. This amplification (Fig 11) constructs itself in the same way as when the muzzle velocity is 1800 m/s, and always has the trend of an amplified sinusoid. The conditions in this case can be qualified as critical.

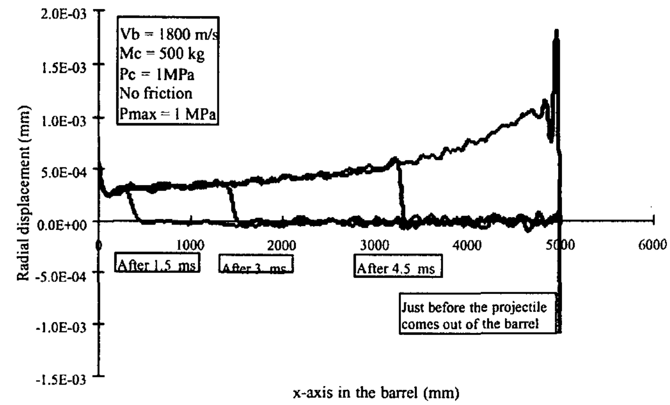

FIG 11 : VON MISES STRESS FOR A MUZZLE VELOCITY OF 2100 M/S

For a muzzle velocity of 2400 m/s, Fig 9 shows that the critical velocity is reached, then exceeded.

shows the changes in the radial displacement of the neutral axis of the barrel as a function of the x-axis, at various times. A numerical calculation shows that the critical velocity is reached by the munition at time t=3.94 ms, which means that the munition's band is at about 4470 mm. At 3.75 ms, the munition velocity is therefore still sub-critical, but the amplification is already very clear. The last curve of shows the deformation just before the munition comes out of the barrel. The muzzle velocity is here

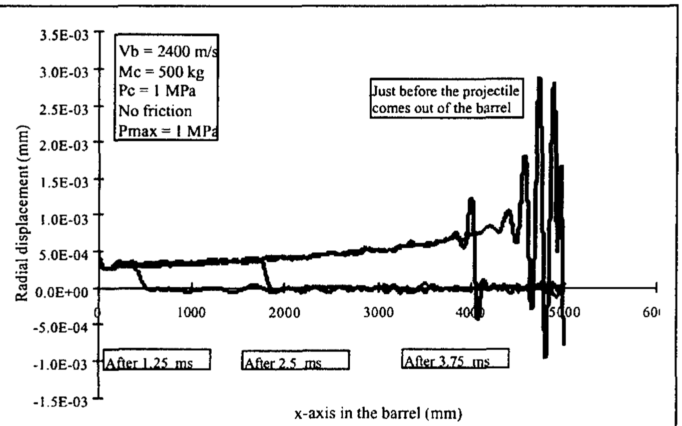

FIG 12 : RADIAL DISLPACEMENT FOR A MUZZLE VELOCITY OF 2400 M/S super-critical. Just before the munition comes out of the barrel, the dynamic amplification is 3 in radial displacement and 4 in Von Mises equivalent stress.

CONCLUSION

The aim of this study was twofold : on the one hand, to understand and study the longitudinal propagation of waves in a weapon barrel and, on the other hand, to produce a simulation tool suitable for a weapon barrel whose purpose, in time, is to help to dimension barrels dynamically. The results of validation and especially of comparison with actual experience have shown that the modelling implemented is true to reality, both from the points of view of amplitudes and of the frequential content of the barrel's response. The preponderant effects are therefore well represented. The simulation tool developed is therefore validated for studying dynamic amplifications in weapon barrels. Furthermore, this tool has been designed to be user-friendly, easy to use and fast. It is therefore well suited for a parametric study or for a dimensioning procedure.

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REFERENCES

1 **SIMKINS, T.E.** - *Travelling waves resonance in gun tubes.* - Watervliet (N.Y.) : U.S. Army armament research, development and engineering center, Benet Laboratories, 1987, 12 p - Final report *N°* RPT ARCCB-TR-87028

2 **HASENBEIN, R.G., GABRIELE, A., FINLAYSON, D., ARTUS, B., CUNNINGHAM, C., AND GAST, R.**- Dynamic strain waves : a development perspective.- *In proceedings of the sixth U.S. Army Symposium on gun dynamics*, 1990, p. 286-297

3 **SIMKINS, T.E.** -Dynamic strains in a 60 mm gun tube - an experimental study - *In proceedings of the sixth U.S. Army Symposium on gun dynamics*, 1990, p. 252 - 271

4 **HASENBEIN, R. AND HYLAND, A.** - Dynamic strain waves and bore enlargement.- *Proceedings of the seventh U.S. Army Symposium on Gun Dynamics*, 1993, p. 504-529

5 **HERMANN, G. AND MIRSKY, I.**- Three dimensional and shell theory analysis of axially symetric motions of cylinders - *Journal of Applied Mechanics* - Dec 1956, p.563-568

6 **JONES, J.P. AND BHUTTA, P.G.** - Response of cylindrical shells to moving pressure loads. - *Journal of Applied Mechanics*, 1964, Vol. 86, p. 105-111

7 **TANG, S.C.** - Dynamic response of a tube under moving pressure. - *Journal of the Engineering Mechanics Division*, 1965, Vol. 115, p. 97-122

8 **RENARD, J.** - *Réponse d'un confinement circulaire mince à une onde de pression (Response of a circular confinement to a pressure wave)* – State Thesis : University of Orléans, 1986, 157 p

9 **SIMKINS, T.E.** - *The influence of transient flexural waves on dynamic strains in gun tubes* - Watervliet (N.Y.) : U.S. Army armament research, development and engineering center, Benet Laboratories, 1989, 34 p - Final report *N°* RPT ARCCB-TR-89020

10 **TADJBAKHSH, I.G. AND YUAN-AN S.** - Transient vibrations and instability in flexible guns - *Proceedings of the sixth U.S. Army Symposium on Gun Dynamics*, 1990, p. 199-229

11 **HOPKINS, D.A.**.- *Predicting dynamic strain amplification by coupling a finite element structural analysis code with a gun interior ballistic code* - Aberdeen Proving Ground (M.D.) :. Army armament research and development Command center, Ballistic Research Lab-1991 Final report *N°* ARBRL-CR-3269, 25 p

12 **SIMKINS, T.E.** - Beating the critical velocity problem - *In proceedings of the eighth U.S. Army Symposium on gun dynamics*, 1996, p. 29-1 - 29-11

13 **SIMKINS, T.E., SCANLON, R., PFLEGL, G**- *Radial and transverse response of gun tubes to travelling ballistic pressure* - Watervliet (N.Y.) : U.S. Army armament research, development and engineering center, Benet Laboratories, 1978, 14 p - Report *N°* 780609086

14 **RANDRIANANGALY, I.** – *Propagation longitudinale des ondes dans un tube (Propagation of longitudinal waves in a gun barrel)* – Doctoral thesis – University of Orléans – 1999, 166 p

15 **RANDRIANANGALY, I., RENARD, J., CAMPION, B., PETITPAS, E.** – *Apports d'un schéma explicite pour l'étude de la propagation d'ondes dans un tube d'arme (Input of an explanatory sketch to the study of the propagation of waves in a weapon barrel)* – Mécanique industrielle et matériaux - Dec. 1998, Vol. 51, n°4, p173 – 175