UNCLASSIFIED

Defense Technical Information Center
Compilation Part Notice

ADP011968

TITLE: A Segmentation Method under Geometric Constraints after Pre-processing

DISTRIBUTION: Approved for public release, distribution unlimited

This paper is part of the following report:

TITLE: International Conference on Curves and Surfaces [4th], Saint-Malo, France, 1-7 July 1999. Proceedings, Volume 2. Curve and Surface Fitting

To order the complete compilation report, use: ADA399401

The component part is provided here to allow users access to individually authored sections of proceedings, annals, symposia, etc. However, the component should be considered within the context of the overall compilation report and not as a stand-alone technical report.

The following component part numbers comprise the compilation report:
ADP011967 thru ADP012009

UNCLASSIFIED

A Segmentation Method under Geometric Constraints after Pre-processing

D. Apprato, J. B. Betbeder, C. Gout, and S. Vieira-Teste

Abstract. For a geophysical image with homogeneous grey levels, we propose a method of segmentation that could be subdivided into two parts: the first one concerns a pre-processing of the image which provides an enhancement of some features present on the image. The originality of the method consists in using a scale transformation applied to the pixel values of the image. The second part presents a segmentation method using deformable surfaces. The originality of this segmentation method is that it considers the active contour model as a set of articulated curves, which corresponds to the interfaces between different layers and faults. Moreover, the a priori knowledge of well data allows us to make some geometric constraints on the model. The solution is obtained by minimization of a nonlinear functional under constraints in a suitable convex set. Solving the minimization problem consists in particular in a k-order Taylor formula applied to linearize the nonlinear term.

§1. Segmentation Pre-processing

Image segmentation is one of the most important steps leading to the analysis of processed image data. Its main goal is to divide an image into parts that have strong correlation with objects or areas of the real world contained in the image. The image is divided into separate regions that are homogeneous with respect to a chosen property such as brightness, color, reflectivity, context, etc. However, in certain cases, the grey levels of an image could be homogeneous and make the segmentation more difficult to realize. This is particularly true in the case of geophysical and medical images (cf. [14,15]). In the first part of this work, we propose a method to solve this problem using families of scale transformations. The use of scale transformations is common in imaging. The aim of this pre-processing is an improvement of the image function data that suppresses unwilling distortions, or enhances some image features important for further processing. It provides improvement of the contrasts, and it represents a tool to pre-process images used in most computer algorithms today.

Curve and Surface Fitting: Saint-Malo 1999

Albert Cohen, Christophe Rabut, and Larry L. Schumaker (eds.), pp. 9–18.
Copyright © 2000 by Vanderbilt University Press, Nashville, TN.
ISBN 0-8265-1357-3.
All rights of reproduction in any form reserved.

According to Sonka, Haclav and Boyle [15], the pre-processing of images may be classified into four categories (pixel brightness transformations, geometric transformations, pre-processing methods that use a local neighborhood of the processed pixel, and image restoration that requires knowledge about the entire image) according to the size of the pixel neighborhood that is used for the calculation of a new pixel brightness. The transformation of the brightness and of the contrast of an image allows us to focus on phenomena that are hard to see in the plain image.

For a given image, we are going to consider the pixel values as a topographic map: the brightness value of each pixel is the height of the (hyper-) surface at that point. For a data set of pixels $(x_i, y_i, z_i, A(x_i, y_i, z_i))_i$, we apply the following functions:

- ζ_d: $A(x_i, y_i, z_i)_i \subset [0,255] \longrightarrow [0,255]$,
- $T^d(\varphi_d \circ (\varsigma_d o A)) \in H^m(\Omega, \mathbb{R})$,
- $\psi_d\left(T^d(\varphi_d \circ (\varsigma_d o A))\right)$ converges to $\zeta \circ A$ when d converges to 0,

where A is an attribute function introduced in Section 2.1, ς_d (resp. φ_d and ψ_d) are scale transformations converging to ς (resp. φ and ψ), and T^d is a D^m*spline* operator (see Arcangéli [2]).

The scale transformation ς_d converges to a usual brightness transformations ς (see Apprato and Gout [1]): for instance, ζ could be a scale transformation which enhances the image contrast between brightness values p_1 and p_2.

Let us consider the subdivision $\left\{u_1, u_2, ..., u_i, ..., u_{p(d)}\right\}_{i=1,...,p(d)}$ of the interval $[0,255]$ satisfying $\zeta(A(x_i, y_i, z_i)) = u_i$, $p(d)$ being the number of different pixel values of the image (≤ 255 for a grey scale image). The function ζ_d is defined, for any $x \in [A(x_i, y_i, z_i) = w_i, A(x_{i+1}, y_{i+1}, z_{i+1}) = w_{i+1}]$, and for an integer $1 \leq i \leq p(d) - 1$, by

$$\begin{aligned}\zeta_d(x) = {} & u_i q_{0m}^0 \left[(x - w_i) / (w_{i+1} - w_i)\right] + u_{i+1} q_{0m}^1 \left[(x - w_i) / (w_{i+1} - w_i)\right] \\ & + \alpha_1(w_i)(w_{i+1} - w_i) q_{1m}^0 \left[(x - w_i) / (w_{i+1} - w_i)\right] \\ & + \alpha_1(w_{i+1})(w_{i+1} - w_i) q_{1m}^1 \left[(x - w_i) / (w_{i+1} - w_i)\right],\end{aligned}$$

where the q_{jm}^l, for $l = 0,1$, and $j = 1, ..., m$, are the Hermite finite element basis functions, and where $\alpha_1(w_i) = (u_{i+1} - u_i) / (w_{i+1} - w_i)$ and $\alpha_1\left(w_{p(d)}\right) = \left(w_{p(d)} - w_{p(d)-1}\right) / \left(u_{p(d)} - u_{p(d)-1}\right)$. Then, Gout [9] showed that for any $d \in D$, for an integer i, $1 \leq i \leq p(d) - 1$, $\zeta_d(w_i) = u_i$ and $\zeta_d \in C^m([0,255])$.

Likewise, in order to recover a finer image, it is useful to apply the "large variations" algorithm introduced in [9]. In fact, after having applied the function ζ_d to improve the contrast of the image (and thus increasing the variations of the corresponding pixel values), it is very useful to use a method that takes into account these rapidly varying data. Let us note that even without using the scale transformations ζ_d, an image often has large variations (this occurs for example when a dark zone is near a brighter one). That is why we propose to use the "Large variations" algorithm. This algorithm uses two-scale transformations, namely φ_d for the pre-processing, and ψ_d for the post-processing.

The first one, φ_d, is used to suppress the oscillations of the data. The preprocessing function φ_d is such that the data do not present large variations, and therefore a usual spline operator T^d (e.g. [2]) can subsequently be applied without generating significant oscillations. The second scale transformation ψ_d is then applied to the approximated values to map them back and obtain the initial approximated pixel values. It is important to underline that the proposed scale transformations do not create parasitic oscillations. Moreover, this method is applied without any particular knowledge of the location of the large variations in the dataset.

So, for pre-processing, we propose two algorithms: in the first one, we just apply a scale transformations ζ_d as a brightness transformation for contrast enhancement, in the second one, we also apply the "large variations" algorithm in order to obtain a finer represention of the image which represents the main advantage of this approach.

The reader is referred to [1,8,10] for a complete study of this method, including its convergence and numerical results. Let us note that this method is also efficient for noise removal as shown in [1].

§2. Segmentation Method

We use deformable models (external forces, evolution term, see Kass, Witkin and Terzopoulos [16,17]) and classical approximation techniques such as spline theory (see de Boor [3], Laurent [12], Schumaker [13]) and the finite element method [4].

We propose an analytic approach which uses deformable models instead of a geometrical one as done for instance in Sethian [14]. We recall that the principle of the deformable model method lies in attracting the representation towards the structure using forces:

- Internal forces describing properties of elasticity and rigidity of the representation, connected to its derivatives (e.g., the energy of thin plates);
- External forces coming from potentials which characterize the elements of the structure with respect to the attributes data.

Geometrical constraints are associated with well interpolation conditions (case of geophysical images with well data). Deformable models provide a way of interactively acting on the representation by adding a dynamic term in the minimization problem (see for instance Cohen and Cohen [5], Cohen, Cohen and Ayache [6], and Cohen, Bardinet and Ayache [7]), that permits upgrading the models to the solution of the minimization problem introduced.

In this section, we first give the geophysical data and then the minimization problem is studied. The nonlinear problem and its discretization are given in the subsequent sections.

2.1. The data on the structure

Two types of data are available: attribute data and well data. For each attribute A, the attribute data are $(x_i, y_i, z_i, A(x_i, y_i, z_i))$, where (x_i, y_i, z_i) are

the coordinates of the barycentre of a voxel, and $A(x_i, y_i, z_i)$ is the attribute value A in this voxel. The well data are depth data:
$(x_j, y_j, z_j)_{j=1,\ldots,N} = a_j$ where N is the number of interpolation points. This model allows a conceptual representation of the structure by identification of its various elements, and permits topological connections between those elements. This model induces the parameterization of the structure. Each element of the structure (layer, fault, etc.) is identified with a connection of four points with a label Σ. Furthermore, each quadruplet is connected by two points (which can be thought as a common side of a "quadrilateral" represented by the four points) with another quadruplet. Practically, the a priori model can be constructed by introducing a 3D block and a regular grid of this block. The aim is to find a space of admissible representations consistent with the a priori model and the criteria connected with the data. Therefore, it is necessary to choose a space of functions characterized by a domain of definition connected with the a priori model and regularity conditions connected to the data. The idea is to transform the a priori model into a normalized model called the model of reference (denoted by M'). For example, we can choose $M' \subset \overline{\Omega} = [0,1] \times [0,1] \times [0,1]$. The model M' is then the image by transformations of the set of vertical and horizontal closed sides of the a priori model as done in [18]. Let γ be the union of the common edges of any two sides of M', we define by M the interior of $M' \setminus \gamma$. All the functional spaces needed in this work are given in Vieira-Testé [18].

2.2. Minimization criterion

2.2.1. Internal forces: The criterion associated with the internal forces is a classical one. Modelling this criterion bring us to the following energy functional: for any $v \in V = H^2\left(M, \mathbb{R}^3\right) \cap C^0\left(M', \mathbb{R}^3\right)$,

$$E_1(v) = [v]_{1,M}^2 + [v]_{2,M}^2 ,$$

where

$$[v]_{1,M} = \left(\sum_{\Sigma \subset M} \varepsilon_1(\Sigma) \int_M \left[\left\langle \frac{\partial v}{\partial s} \right\rangle_3^2 + \left\langle \frac{\partial v}{\partial r} \right\rangle_3^2 \right] dsdr \right)^{1/2}$$

and

$$[v]_{2,M} = \left(\sum_{\Sigma \subset M} \varepsilon_2(\Sigma) \int_M \left[\left\langle \frac{\partial^2 v}{\partial s^2} \right\rangle_3^2 + \left\langle \frac{\partial^2 v}{\partial r \partial s} \right\rangle_3^2 + \left\langle \frac{\partial^2 v}{\partial r^2} \right\rangle_3^2 \right] dsdr \right)^{1/2}$$

with $\varepsilon_i(\Sigma) \geq 0, \quad \forall i = 1, 2, \forall \Sigma \in M$. The term $[v]_{1,M}$ corresponds to an approximation of the elastic deformation of the model while the term $[v]_{2,M}$ corresponds to an approximation of the rigid deformation of the model (cf. Cohen, Cohen and Ayache [6]).

2.2.2. External forces: External forces are issued from potentials connected with attributes. We introduce the following energy, for any $v \in V$,

$$E_2(v) = \sum_{\Sigma \subset M} \int_{\Sigma} P_{\Sigma}\left(v_{/\Sigma}(s,r)\right) dsdr,$$

where P_{Σ} is the potential associated with the element parameterized by Σ.

The modelling we propose consists in minimizing the previous energies E_1 and E_2 as we will see in subsection 2.2.4. In the case of the velocity attribute, we use the following potential to define the layers:

$$P(x,y,z) = -\alpha \left\| \overrightarrow{\nabla A}(x,y,z) \right\|^2, \qquad \alpha \geq 0$$

where A is the attribute "velocity of propagation of the seismic wave".

2.2.3. Interpolation data: If we suppose some parameterization $(s_j, r_j) \in M$ of each interpolating point $a_j = (x_j, y_j, z_j)$ is known, then we require that $v \in V$ satisfies $v(s_j, r_j) = a_j$ for any $j = 1, ..., N$.

2.2.4. Minimization criterion: Using the notation and definitions introduced above, we consider the functional E defined on V by

$$E(v) = [v]_{1,M}^2 + [v]_{2,M}^2 + \sum_{\Sigma \subset M} \int_{\Sigma} P_{\Sigma}\left(v_{/\Sigma}(s,r)\right) dsdr$$

for any $v \in V$. We consider the set K associated with the interpolation constraints, and defined by

$$K = \{v \in V, \quad \forall j = 1, ..., N, \quad v(s_j, r_j) = a_j\}.$$

This set is convex and closed in V. We also introduce the following linear mapping (continuous on V with the norm $\|\cdot\|_{2,M}$)

$$\rho_0 : v \in V \mapsto \rho_0 v = \left(v(s_j, r_j)\right)_{j=1,\ldots,N} \in \left(\mathbb{R}^3\right)^N.$$

We consider the following minimization problem: find $\sigma \in K$ satisfying

$$\forall v \in K, \quad E(\sigma) \leq E(v).$$

We note that this problem is nonlinear on the convex set K with respect to σ. There are two techniques to treat this problem. The first one consists in linearizing the nonlinear term (linked to the potentials) in the functional E. The second one consists in using the deformable models technique as done in the following subsection: we suppose that the solution is a function of time, which leads to a new evolution problem that will be discretized both in time and space.

2.3. The nonlinear problem

In this subsection, we give the nonlinear minimization problem and its discretization. Let us recall that the deformable models technique consists in assuming that σ depends on time, and so consists in adding a dynamic term to the functional $E(\sigma)$

$$\frac{1}{2}\frac{\partial}{\partial t}\int_M \varepsilon(M)\,\sigma^2(t,s,r)\,dsdr,$$

where $(\varepsilon(M))_{/\Sigma} = \varepsilon(\Sigma) > 0$. This term allows the control at each time of the deformation of the surfaces.

2.3.1. Evolution problem: Let $T > 0$. We note

$$W(0,T,V) = \left\{ w \in L^2(]0,T],V),\ \frac{\partial w}{\partial t} \in L^2(]0,T],V') \right\}.$$

We then consider the following evolution problem defined on $[0,T]$. For any $t \in]0,T]$ and any $\omega \in W(0,T,V)$, find $\sigma \in W(0,T,V), \sigma(t) \in K$, satisfying $(\mathbf{P_t})$:

$$E(\sigma)+\frac{1}{2}\frac{\partial}{\partial t}\int_M \varepsilon(M)\,\sigma^2(t,s,r)\,dsdr \leq E(\omega)+\frac{1}{2}\frac{\partial}{\partial t}\int_M \varepsilon(M)\,\omega^2(t,s,r)\,dsdr,$$

with

$$\sigma(0) = \sigma_0 \in L^2\left(M,\mathbb{R}^3\right).$$

We are currently studying existence and uniqueness of $(\mathbf{P_t})$ using a Lipschitz approximation of the sign function.

Likewise, for any $t \in]0,T]$, we consider the term

$$L_{\sigma(t)}(v) = -\sum_{\Sigma\subset M}\int_\Sigma P_\Sigma\left(v_{/\Sigma}(s,r)\right) dsdr.$$

The variational formulation of the problem $(\mathbf{P_t})$ with Kuhn and Tucker's relation is, taking as test function v on the stationary space V (necessary condition without uniqueness), for any $t \in]0,T]$ and any $v \in V$, find $(\sigma,\lambda) \in W(0,T,V) \times C^0\left([0,T],\left(\mathbb{R}^3\right)^N\right), \sigma(t) \in K$, satisfying $(\widetilde{\mathbf{P}})$:

$$\int_M \varepsilon(M)\,\frac{\partial\sigma(t,s,r)}{\partial t}v(s,r)dsdr + a(\sigma(t),v) + \langle\lambda(t),\rho_0 v\rangle_{N,3} = L_{\sigma(t)}(v)$$

under conditions

$$\sigma(0) = \sigma_0 \in L^2\left(\mathbb{R}^3\right)$$

and

$$\lambda(0) = \lambda_0 \in \left(\mathbb{R}^3\right)^N,$$

where

$$\frac{1}{2}a(u,v) = [v]_{1,M}^2 + [v]_{2,M}^2 .$$

2.3.2. Discretization in time: In this subsection, we discretize $(\widetilde{\mathbf{P}})$ both in time and space. The originality of this discretization consists in using a k-order Taylor development which allows us to take into account many more voxels and so to improve the convergence of the method (see Vieira-Teste [18] for more details). We cut the interval $]0,T]$ into sub intervals with length Δt. Consider

$$t_m = m \,\Delta\, t, \quad m = 1, ..., D_T.$$

We use the following approximation of the time derivative:

$$\frac{\partial \sigma}{\partial t}(t_m) \simeq \frac{\sigma(t_m) - \sigma(t_{m-1})}{\Delta t}.$$

Assuming that $\sigma^m = \sigma(t_m)$ and $\lambda^m = \lambda(t_m)$, we approximate the variational problem as follows: For any $m = 1, ..., D_T$ and any $v \in V$, find $(\sigma^m, \lambda^m) \in V \times (\mathbb{R}^3)^N, \sigma^m \in K$, satisfying $(\mathbf{P_m})$:

$$\int_M \varepsilon(M)\,\sigma^m v ds dr + \Delta t \left[a(\sigma^m, v) + \langle \lambda^m, \rho_0 v\rangle_{N,3}\right]$$

$$= \int_M \varepsilon(M)\,\sigma^{m-1} v ds dr + \Delta t L_{\sigma^m}(v)$$

with $\sigma^0 = \sigma_0 \in L^2(M, \mathbb{R}^3)$ and $\lambda^0 = \lambda_0 \in (\mathbb{R}^3)^N$.

The previous problem is implicit and nonlinear with respect to the solution σ^m. We propose to replace $L_{\sigma^m}(v) = L_{\sigma,v}(t_m)$ by a Taylor series expansion of order $k \geq 0$ about the time t_m. We suppose that σ is in $C^k([0,T], L^2(M, \mathbb{R}^3))$. We have

$$L_{\sigma^m}(v) = L_{\sigma,v}(t_m)$$

and $L_{\sigma,v}(t_m) \simeq L_{\sigma,v}(t_{m-1}) + \Delta t D L_{\sigma,v}(t_{m-1}) + \frac{(\Delta t)^2}{2} D^2 L_{\sigma,v}(t_{m-1}) + \cdots + \frac{(\Delta t)^k}{k!} D^k L_{\sigma,v}(t_{m-1})$. We note that the problem $(\mathbf{P_m})$ is linear and explicit with respect to σ^m. The following result is based on the Lax-Milgram Lemma.

Theorem. *The problem* $(\mathbf{P_m})$ *has a unique solution* (σ^m, λ^m).

2.3.3. Discretization in space: Let H be a nonempty bounded subset in $\mathbb{R}_+^*$ for which 0 is an accumulation point. For any $h \in H$, we solve the minimization problem $(\mathbf{P_m})$ in the finite element space $(V_h)^3 \subset V$. The generic finite element are the Hermite finite element of class C^1 for snakes and the Bogner-Fox-Schmit finite element rectangle of class C^1 (see [4]) for deformable surfaces. To have $(V_h)^3 \subset V$, it is necessary to have a C^0 connection on γ. To do that, it is sufficient to divide some degrees of liberty connected to derivatives as done in [18].

We denote by $\left(\alpha_1^m, ..., \alpha_{M_h}^m\right)$ the coordinates of σ^m in the basis of V_h and by $(\lambda_1^m, ..., \lambda_N^m)$ the coordinates of λ^m $\quad (M_h = dim(V_h))$. If σ^m is a solution of the discretized problem (P_m) in $(V_h)^3$, we can write σ^m in the basis of $(V_h)^3$: $\forall m = 1, ..., D_T, \quad \forall q = 1, 2, 3,$

$$(\sigma^m)^q = \sum_{j=1}^{M_h} \left(\alpha_j^m\right)^q \varphi_j$$

with $\left(\alpha_j^m\right)^q \in \mathbb{R}$ and where the $(\varphi_j)_{j=1,..,M_h,}$ are the basis functions of V_h.

In the following, we miss out q and h. Taking $v = \varphi_l$ in $(\widetilde{\mathbf{P}})$, we have to solve (for $q = 1, 2, 3$, in the linear problem $(\widetilde{\mathbf{P}})$) a system of $(M_h + N)$ equations with $(M_h + N)$ unknowns. We easily show that this system has a unique solution, and that the matrix $R = [C, B,^t B, 0]$ (first line : C, B; second line : ${}^tB, 0)$ of the system is symmetrical and sparse with

$$C_{j,l} = [\varphi_j, \varphi_l]_{0,M} + \Delta ta\left(\varphi_j, \varphi_l\right), \qquad i, j, l = 1, ..., M_h$$

$$B_{j,i} = \Delta t \cdot \varphi_j\left(s_i, r_i\right), \qquad j = 1, ..., M_h, \quad i = 1, ..., N,$$

where for any $u, v \in (V_h)^2$,

$$[u, v]_{0,M} = \int_M \varepsilon\left(M\right) \cdot u(s, r) \cdot v(s, r) ds dr,$$

and where $T = \left(\alpha_1^m, ..., \alpha_{M_h}^m, \lambda_1, ..., \lambda_N\right)$ is the unknown vector. We obtain a linear system $RT = L$, where the lines of L are

$$\int_M \varepsilon\left(M\right) \sigma^{m-1} \varphi_1 ds dr + \Delta t L_{\sigma^m}\left(\varphi_1\right),$$

$$\vdots$$

$$\int_M \varepsilon\left(M\right) \sigma^{m-1} \varphi_{M_h} ds dr + \Delta t L_{\sigma^m}\left(\varphi_{M_h}.\right),$$

$$\Delta t a_1,$$

$$\vdots$$

$$\Delta t a_N.$$

This method has been implemented in fortran, C and C++. Numerical examples on real data are given in [11,18].

Acknowledgments. The authors are very grateful to the Région Aquitaine and ELF which have supported this work.

References

1. Apprato, D., and C. Gout, Noise removal in Medical Imaging, preprint, PAM 1000, University of California at Berkeley, 1998.
2. Arcangéli, R., Some applications of discrete D^m-splines, in *Mathematical Methods in Computer Aided Geometric Design*, T. Lyche and L. L. Schumaker (eds.), Academic Press, New York, 1989, 35–44.
3. de Boor, C., *A Practical Guide to Splines*, Springer Verlag, Berlin - Heidelberg, 1978.
4. Ciarlet, P. G., *The finite element method for elliptic problems*, North Holland, Amsterdam, 1979.
5. Cohen, I. and L. D. Cohen, A finite element method applied to new active contour models and 3D reconstruction from cross sections, Technical report 1245, INRIA, 1990.
6. Cohen, I., L. D. Cohen, and N. Ayache, Using deformable surfaces to segment 3D images and infer differential strutures, CVGIP : Image Understading, vol.56, n°2, September, 1992, 242–263.
7. Cohen, L. D., E. Bardinet, and N. Ayache, Surface reconstruction using active contour models, INRIA report 1824, 1992.
8. Gout, C., Segmentation in Medical Imaging from images with homogeneous level of colors, preprint, UPRES A 5033 98/05, Université de Pau, 1998.
9. Gout, C., Approximation using scale transformations, in *Approximation Theory IX, Vol. 1*, Charles K. Chui and Larry L. Schumaker (eds.), Vanderbilt Univ. Press, 1998, 151–158.
10. Gout, C., and A. Guessab, A new family of extended Gauss quadratures with an interior constraint, Journal of Computational and Applied Mathematics, to appear.
11. Gout, C. and S. Vieira-Teste, 3D surfaces modeling using deformable models techniques, in preparation.
12. Laurent, P. J., *Approximation et optimisation*, Hermann, Paris, 1972.
13. Schumaker, L. L., Fitting surfaces to scattered data, in *Approximation Theory II*, G. G. Lorentz, C. K. Chui, and L. L. Schumaker (eds.), Academic Press, New York, 1976, 203–269.
14. Sethian, J. A., *Level Set Methods: Evolving interfaces in Geometry, Fluid Mechanics, Computer vision, and Material Science*, Cambridge University Press, 1996.
15. Sonka, M., V. Haclav, and R. Boyle, *Image, Processing, Analysis and Machine Vision*, Chapman & Hall, 1993.

16. Terzopoulos, D., A. Witkin, and M. Kass, Constraints on deformable models: Recovering 3D shape and nonrigid motions, Artificial Intelligence **36** (1988), 91–123.
17. Terzopoulos, D., A. Witkin and M. Kass, Symmetryseeking models for 3D object reconstruction, in Proceedings, First International Conference on Computer Vision, London, England, IEEE, Piscataway, NJ, 1987, 269–276.
18. Vieira-Teste, S., Représentations de structures géologiques à l'aide de modèles déformables sous contraintes géométriques, Thèse de Doctorat, Université de Pau, 1997.

D.A., J.B.B., C.G.
Université de Pau - Dpt de Mathématiques
IPRA - Av.de l'Université - 64000 Pau, France
`chris_gout@alum.calberkeley.org`

S.V.T.
TOPCAD SA - BP 521 - 31674 Labege, France