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### Nonuniform Quantization for Diffractive Optical Elements design

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### ABSTRACT

Recently, the applications of diffractive dptical element (DOE) for commerce and research have become more and more popular. DOE itself has a lot of advantages like small volume, low weight, ease of reproduce and low cost. A DOE actually can be considered as a wavefront modulator, and its performance can also be described as a complex amplitude transmittance. In the past, we usually design a DOE by quantizing the phase of DOE from Gerchberg-Saxton algorithm or other methods with equal etching-depth and etching-width because of the ease of process. In this paper, we present a novel approach for design DOE. We change the element's etching-depth and etchingwidth nonuniformly. The reason why we strike out this approach is that one who can control the timing within the etching process can make any depth and width after all. Therefore, we have more components of etching-depth and etchingwidth that can be produced to reach the better diffractive efficiency on output diffraction plane than the conventional etching method. In terms of our proposed method, the conventional method of DOE design will become a special case of our new approach. According to the minimum etching-depth, we try all possible combinations to find a set of DOEs phases that have better diffractive efficiency than the conventional method can achieve. The DOE design with the proposed method has higher efficiency on output diffraction plane than those achieved by the conventional method

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#### 1. INTRODUCTION

In recent years, optical element for diffracting its input light to any desired output position, named as diffraction optical element (DOE), has a lot of advantages like small volume, low weight, ease of reproduce and low cost. The applications of Diffractive Optical Element (DOE) have become more and more popular in optical communications, optical interconnect, integrated optics and optical sensing.

Various methods for DOE design consider only the problem of the DOE with arbitrary phase and use Fourier transform to calculate the approximate light distribution of the DOE. Because the optical system for the DOE design has enormous complexity, there exist a number of mathematical problem that do not yield to analytical solutions. In order to design DOE with high efficiency most methods for DOE design are based on the theory of projection onto convex set (POCS)<sup>1</sup> and far field assumption. In addition, the amplitude transmittance of the designed DOE is set to be a constant. The most-known technique is the iterative Gerchberg-Saxton (G-S) algorithm.<sup>2</sup> In G-S algorithm, the Fourier transform is used to calculate the light field distribution at the output plane of the DOE. The transmittance of a DOE and its output light distribution must satisfy both the input and output domain constraints.

In order to fabricate the DOE with binary grating the constraints of input domain in the G-S algorithm are the corresponding phase of the DOE that procedure of the fabrication can provide. Conventional procedure of DOE fabrication are using uniform etching depth to match the phase function of designed DOE. By the advance technology etching depth of the DOE can be accurate fabricated and more corresponding phase of DOE can be implemented. For this advantage we proposed a new method for DOE design that quantized the phase of DOE nonuniformly. The reason why we strike out this approach is that one who can control the timing within the etching process can make

any depth and width after all. Therefore, we have more components of etching-depth and etching-width that can be produced to reach the better diffractive efficiency on output diffraction plane than the conventional etching method.

The organization of this paper is as follows: Section 2 reviews the theory of G-S algorithm firstly and determines the phase using nonuniform quantization method. Section 3 shows the results (from simulation) of the proposed nonuniform method and compares the results with uniform case. Finally, a conclusion is given in Section 4.

#### 2. THEORY

The schematic of a typical optical system being considered is shown in Figure 1. This optical system is composed of one DOE, an input plane  $P_1$ , and an output plane  $P_2$  is placed behind the DOE at distance L. The wave functions at input plane  $P_1$  and output plane  $P_2$  are denoted by  $U_1$  and  $U_2$ , respectively, and expressed as

$$U_1(x,y) = A_1(x,y) \exp[j\phi_1(x,y)]$$
(1)

$$U_2(f_x, f_y) = A_2(f_x, f_y) \exp[j\phi_2(f_x, f_y)]$$
(2)

The z axis is chosen as the optical propagation axis of the system, and the coordinates of the input and output planes are (x, y) and  $(f_x, f_y)$ , respectively.

We consider the Fourier transformation as the mathematical model for the light propagation and the relation between  $U_1$  and  $U_2$  are

$$U_2(f_x, f_y) = \int \int U_1(x, y) \exp[\frac{j2\pi}{\lambda z} (xf_x + yf_y)] dxdy$$
(3)

$$U_1(x,y) = \int \int U_2(f_x, f_y) \exp\left[\frac{-j2\pi}{\lambda z} (xf_x + yf_y)\right] df_x df_y \tag{4}$$

where z is the propagation distance, and  $\lambda$  is the wavelength.

The synthesis problem arise when the Fourier transform<sup>3</sup> of a DOE has certain desirable properties as its output diffraction pattern itself must satisfy certaion constraints. However, it is not easy to find a Fourier transform pair<sup>4</sup> (or the pair does not exist) that satisfies all the constraints in both input and output planes. The iteration method is shown to be very powerful in solving the problems above mentioned. There are many variations in the iterative method and not limited to single fixed algorithm. The most known algorithm is the G-S algorithm. The block diagram of the G-S algorithm is shown in Figure 2. The G-S algorithm is an effective method for recovering the phase. It involves iterative Fourier transformations back and forward between input and output domains and is performed as follows:

- 1. Given the desired output diffraction pattern and found the phase function of DOE in input plane.
- 2. Match the constraints of the input domain and get the new phase function of DOE in input plane.
- 3. Fourier transform the phase function of DOE in step 2 and get the output diffraction pattern in output plane.
- 4. Match the constraints of output domain.
- 5. Repeat the Steps 1 to 4 until satisfies all constraints in the input and output domains.

The quantized phase method for DOE design is a significant problem. It determines the performance of DOE. Figures 3 and 4 show the uniform and nonuniform quantization cases. We find that the nonuniform case that can provide the phase function that is similar to the phase function without quantization.

#### 3. SIMULATION

In this section, we first compare the results of our proposed method with those in the past. Then we show the far-field patterns of each methods and compare the difference. Summaries of the optical system parameter are shown below:

- 1. Light beam : uniform beam
- 2. Index of reflection : 1.5
- 3. Incident wavelength : 0.85nm
- 4. DOE size :  $64 \times 64$  pixel

The simulation result shows in Table 1. It's well known that G-S algorithm is very sensitive to the initial guess, so we experimented with ten different random phases in order to find the global optimum. We see that the improvements between proposed and conventional under 8 phase levels are relatively small and the average improvement is 0.77%. On the other hand, we can get much more improvements in 4 phase levels and the improvement is 10.30%.

Figure 5 shows the far-field patterns. (a) far-field pattern after 4 uniform phase levels. (b) far-field pattern after 8 uniform phase levels. (c) far-field pattern after 4 non-uniform phase levels. (d) far-field pattern after 8 non-uniform phase levels. We see that (b) and (d) are almost the same because their efficiency are close, while in (a) and (c), the differences are so obvious especially in the four corners and sidelobs.

Figure 6 shows the difference of phase functions. (a) difference of phase function between uniform and nonuniform quantized phase in 4 level case. (b) difference of phase function between uniform and non-uniform quantized phase in 8 level case. The phases in many pixels are changed in (a), while in (b), there are just several pixel changes. It implies that the ranges we can improve the DOE efficiency under 4 phase levels using non-uniform quantization are larger than that under 8 phase levels using non-uniform quantization.

#### 4. CONCLUSION

Based on the POCS technique and Fourier transformation, the G-S algorithm is proposed to design DOE. We use the nonuniform quantized method to quantize the phase of DOE and the design DOE that can better achieve the desired pattern compared to the uniform quantized method. For microlens design, 10.3% light energy increase is obtained at the desired output position by our method compared with the uniform quantized method.

#### REFERENCES

- 1. G. Strang, Introduction to Applied Mathematics, Wellesley, 1986
- R. W. Gerchberg and W. O. Saxton, "A Particaular algorithm for the determination of phase from image plane picture," Optik 35, pp. 237-246, 1972
- 3. F. Wyrowski and O. Bryngdahl, "Iterative Fourier-Transform Algorithm Applied to Computer Holograms," J. Opt. Soc. Am. A 5, pp. 1058–1065, 1988
- 4. J. W. Goodman, Introduction to Fourier Optics, McGraw-Hill, New York, 1968

[	G-S Algorithm	Proposed	Conventional	Improved	Proposed	Conventional	Improved
	without quantization	4 Level	4 Level	Efficiency	8 Level	8 Level	Efficiency
	Efficiency	Efficiency	Efficiency	%	Efficiency	Efficiency	%
Phase1	0.8927	0.7016	0.6421	9.27	0.8383	0.8322	0.73
Phase2	0.8929	0.7093	0.6514	8.89	0.8386	0.8309	0.93
Phase3	0.8924	0.7016	0.6421	9.27	0.8383	0.8322	0.73
Phase4	0.8863	0.7148	0.6355	12.48	0.8321	0.8281	0.48
Phase5	0.8929	0.7093	0.6514	8.89	0.8368	0.8309	0.71
Phase6	0.8929	0.7093	0.6514	8.89	0.8368	0.8309	0.71
Phase7	0.8863	0.7148	0.6355	12.48	0.8321	0.8281	0.48
Phase8	0.8861	0.7107	0.6347	11.97	0.8302	0.8275	0.33
Phase9	0.8861	0.7107	0.6347	11.97	0.8302	0.8275	0.33
Phase10	0.8902	0.7011	0.6437	8.92	0.8418	0.8228	2.31
Average	0.8899	0.7083	0.6423	10.30	0.8355	0.8291	0.77
Maximum	0.8929	0.7148	0.6514	12.48	0.8418	0.8322	0.93

Table 1. simulation results.



Figure 1. optical system



- $U_2(f_x f_y)$ : wave function at output plane  $U'_2(f_x f_y)$ : wave function at output plane with constraints





Figure 3. uniform quantization



Figure 4. nonuniform quantization





Figure 5. (a) far-field pattern after 4 uniform phase levels. (b) far-field pattern after 8 uniform phase levels. (c) far-field pattern after 4 nonuniform phase levels. (d) far-field pattern after 8 nonuniform phase levels



Figure 6. (a) difference of phase function between uniform and nonuniform quantized phase in 4 levels case. (b) difference of phase function between uniform and nonuniform quantized phase in 8 levels case.