Defense Technical Information Center
Compilation Part Notice

ADP011660

TITLE: Bandwidth Control for Photonic Bandgap Waveguides

DISTRIBUTION: Approved for public release, distribution unlimited

This paper is part of the following report:

TITLE: International Conference on Electromagnetics of Complex Media
[8th], Held in Lisbon, Portugal on 27-29 September 2000. Bianisotropics
2000

To order the complete compilation report, use: ADA398724

The component part is provided here to allow users access to individually authored sections
of proceedings, annals, symposia, etc. However, the component should be considered within
the context of the overall compilation report and not as a stand-alone technical report.

The following component part numbers comprise the compilation report:
ADP011588 thru ADP011680

Bandwidth Control for Photonic Bandgap Waveguides

A. Boag[1], M. Gafni[2], and B. Z. Steinberg[2]

[1] Faculty of Engineering, Department of Physical Electronics, Tel Aviv University,
Tel Aviv 69978, Israel
Fax: + 972-3-6423508; E-mail: boag@eng.tau.ac.il

[2] Faculty of Engineering, Department of Interdisciplinary Studies, Tel Aviv University,
Tel Aviv 69978, Israel
E-mail: steinber@eng.tau.ac.il

Abstract

A widely spaced periodic array of defects in the photonic band gap crystal is studied with the goal of designing a waveguide with a prescribed narrow bandwidth. Tunnelling of radiation between the defect sites allows wave propagation along the line of the defects. A design procedure based on the weakly coupled cavity model is proposed. The frequency shift and the band structure of the periodic defect waveguide are linked by an analytic relationship to the distance between the defect sites and therefore can be tuned by varying the latter. Sections of such waveguides can be employed as ultra narrow band filters in optical routing devices.

1. Introduction

Photonic band gap materials attracted much attention in the context of designing optical and microwave devices. Recently numerical experiments have shown that line defects in photonic crystals can be used not only to guide but also to multiplex and demultiplex optical signals [1]. Most researchers studying the wave guiding by line defects employ photonic band waveguides obtained by removal or modification of consecutive posts in the periodic structure. The strong coupling between the adjacent defects produces relatively wideband waveguides.

In this paper, we address the issue of designing photonic bandgap waveguides with a prescribed narrow bandwidth. Specifically, we concentrate on a problem of a waveguide formed by widely spaced periodic defects in the photonic band gap crystal. Tunnelling of radiation between the defect sites allows wave propagation along the line of defects. Sections of such waveguides can be employed as ultra narrow band filters in optical routing devices. Here, we propose a design procedure based on the weakly coupled cavity model. This approach resembles the tight binding perturbation theory of the solid-state physics. A single defect mode with a resonant frequency in the band gap is analyzed first. Coupling between the periodic defects causes a discrete spectral line to turn into a narrow band of guided frequencies shifted from the original frequency of a single defect. The perturbation theory facilitates an approximate calculation of both the frequency shift and the band structure of the periodic defect waveguide. Furthermore, these parameters are linked by an analytic relationship to the distance between the defect sites. Consequently, the latter distance can be directly tuned to achieve the desirable waveguide properties. The design results are verified by a comparison with numerically rigorous computations employing the current model technique [2].

2. Formulation

Consider a problem of designing a narrow band waveguide formed by widely spaced periodic defects in the photonic band gap crystal. The time harmonic electromagnetic problem in an inhomogeneous dielectric can be cast in an eigenvalue form for the magnetic field H [3]:

$$\Theta H(r) = \left(\frac{\omega}{c}\right)^2 H(r) \tag{1}$$

where ω is the frequency, c is the free space speed of light, and Θ denotes a Hermitian operator defined by

$$\Theta H = \nabla \times \left(\frac{1}{\varepsilon(r)} \nabla \times H\right) \tag{2}$$

In (2), $\varepsilon(r)$ denotes the relative permittivity. Alternatively, the eigen-frequencies can be expressed in the variational form:

$$\left(\frac{\omega}{c}\right)^2 = \frac{\langle H, \Theta H \rangle}{\langle H, H \rangle} \tag{3}$$

where $\langle \cdot, \cdot \rangle$ denote the inner product defined by

$$\langle F, G \rangle = \int F^* \cdot G \, dr \tag{4}$$

The unperturbed crystal is characterized by a periodic relative permittivity $\varepsilon_p(r)$. First, consider a single defect within the periodic structure arbitrarily centered at the origin of the coordinate system. The localized defect can be characterized by the change in the reciprocal permittivity $d(r) = 1/\varepsilon_d(r) - 1/\varepsilon_p(r)$, where $\varepsilon_d(r)$ denotes the permittivity of the photonic crystal with a single defect. We assume that this defect allows for a localized mode $H_o(r)$ with a frequency ω_o falling in the band gap of the unperturbed crystal. Specifically by analogy to (1),

$$\left(\Theta_p + \Theta_o\right)H_o(r) = \left(\frac{\omega_o}{c}\right)^2 H_o(r) \tag{5}$$

Θ_p is the operator of the periodic structure and Θ_o is the defect operator. The operators Θ_p and Θ_o are defined by analogy to (2) via replacing $1/\varepsilon(r)$ by $1/\varepsilon_p(r)$ and $d(r)$, respectively.

Now, we turn to the case of a linear array of defects obtained by a periodic repetition of the defect. The reciprocal permittivity of the photonic crystal with the linear array of non-overlapping defects is given by

$$\frac{1}{\varepsilon(r)} = \frac{1}{\varepsilon_p(r)} + \sum_{n=-\infty}^{\infty} d(r - nb) \tag{6}$$

where the vector b is assumed to be an integral multiple of the lattice vector a, $b = \ell a$, $\ell \in N$. The operator Θ,

$$\Theta = \Theta_p + \sum_{n=-\infty}^{\infty} \Theta_n \qquad (7)$$

which comprises a superposition of Θ_p and shifted operators Θ_n defined by analogy with (2) via replacing $1/\varepsilon(r)$ by $d(r-nb)$ for $n \in \mathbf{Z}$.

Following the strong binding perturbation theory [4] for the linear array, we seek a modal solution of the form

$$H(r) = \sum_{n=-\infty}^{\infty} A_n H_n(r) \qquad (8)$$

where $H_n(r) = H_o(r-nb)$ and $\{A_n\}$ is a set of yet to be determined coefficients. Substitution of (7) and (8) in (3) yields

$$\left(\frac{\omega}{c}\right)^2 = \frac{\displaystyle\sum_{n=-\infty}^{\infty}\sum_{m=-\infty}^{\infty} A_n^* A_m T_{n-m}}{\displaystyle\sum_{n=-\infty}^{\infty}\sum_{m=-\infty}^{\infty} A_n^* A_m H_{n-m}} \qquad (9)$$

where

$$H_{n-m} = \langle H_n, H_m \rangle \qquad (10)$$

and

$$T_{n-m} = \langle H_n, \Theta H_m \rangle \qquad (11)$$

Note that due to the periodicity of the array the integrals in (10) and (11) depend only on $n-m$. According to variational principle the frequency expressed by (9) is stationary with respect to the coefficients $\{A_n\}$. We have

$$\forall n \quad \frac{\partial \left(\frac{\omega}{c}\right)^2}{\partial A_n} = 0 \quad \Rightarrow \quad \sum_{m=-\infty}^{\infty}\left[T_{n-m} - \left(\frac{\omega}{c}\right)^2 H_{n-m}\right] A_m = 0 \qquad (12)$$

Discrete shift invariant nature of (12) suggest that modal solution of the form $A_m = Ae^{jkm}$ whose use in (12) yields

$$\sum_{m=-\infty}^{\infty}\left[T_m - \left(\frac{\omega}{c}\right)^2 H_m\right] e^{jkm} = 0 \qquad (13)$$

Using (5), we have

$$T_m = \left(\frac{\omega_o}{c}\right)^2 H_m + T_m' \qquad (14)$$

where

$$T'_m = \int H^*_m \cdot \sum_{\substack{n=-\infty \\ n \neq 0}}^{\infty} \Theta_n H_o \, dr = \sum_{\substack{n=-\infty \\ n \neq 0}}^{\infty} \langle H_m, \Theta_n H_o \rangle \tag{15}$$

Substituting (14) in (13) we obtain

$$\left(\frac{\omega}{c}\right)^2 - \left(\frac{\omega_o}{c}\right)^2 = \frac{\sum_{m=-\infty}^{\infty} T'_m e^{jkm}}{\sum_{m=-\infty}^{\infty} H_m e^{jkm}} \tag{16}$$

namely, an expression for the frequency shift of the linear array mode with wavenumber k/b relative to the frequency of the localized mode. For sufficiently spaced defects we can make the first order approximation by retaining only the nearest neighbor interactions. Specifically, we have $\forall m \neq 0 \quad |H_m| \ll |H_0|$ and $\forall |m| > 1 \quad |T'_m| \ll |T'_0|$. On the other hand, T'_1 and T'_0 can be comparable in magnitude. With this approximations and assuming $|\omega - \omega_o| \ll \omega_o$ we obtain

$$\omega - \omega_o = \frac{c^2}{2\omega_o H_0} \left[T'_0 + 2T'_1 \cos(k) \right] \tag{17}$$

The total bandwidth of the waveguide formed by the periodic array for propagating modes $0 < k < \pi$ is given by

$$\Delta\omega = \frac{2c^2}{\omega_o H_0} T'_1 \tag{18}$$

Furthermore due to weak coupling and Hermitian property of Θ_1,

$$T'_1 \approx 2 \langle H_1, \Theta_1 H_o \rangle = 2 \langle \Theta_1 H_1, H_o \rangle = 2 \int (\Theta_1 H_1)^* H_o dr \tag{19}$$

Once the localized modal solution is known, equations (18)-(19) allow for straightforward estimation of the waveguide bandwidth for various spacings between defects, $b = \ell a$. Note that in (19), $H_1(r) = H_o(r - b)$ and Θ_1 is a local operator. For sufficiently large r, the localized magnetic field $H_o(r)$ is decaying exponentially away from the origin. Thus, (19) indicates that increasing ℓ can reduce the waveguide bandwidth. The bandwidth control by varying defect spacing will be demonstrated via numerical examples.

References

[1] E. Centeno, B. Guizal, and D. Felbacq, *J. Opt. A.: Pure Appl. Opt.*, No. 1, pp. L10-L13, 1999.

[2] A. Boag, Y. Leviatan, and A. Boag, *Radio Science*, Vol. 23, pp. 612-624, 1988.

[3] J. D. Joannopoulos, R. D. Meade, and J. N. Winn, *Photonic Crystals: Molding the Flow of Light*. Princeton University Press, 1995.

[4] R. E. Peierls, *Quantum Theory of Solids*. Oxford Clarendon Press.