Defense Technical Information Center Compilation Part Notice

ADP011632

TITLE: On the Constitutive Tensors for Bianisotropic Media

DISTRIBUTION: Approved for public release, distribution unlimited

This paper is part of the following report:

TITLE: International Conference on Electromagnetics of Complex Media [8th], Held in Lisbon, Portugal on 27-29 September 2000. Bianisotropics 2000

To order the complete compilation report, use: ADA398724

The component part is provided here to allow users access to individually authored sections of proceedings, annals, symposia, etc. However, the component should be considered within the context of the overall compilation report and not as a stand-alone technical report.

The following component part numbers comprise the compilation report:
ADP011588 thru ADP011680

On the Constitutive Tensors for Bianisotropic Media

V. Dmitriev

University Federal of Para, PO Box 8619
AGÊNCIA UFPA, CEP 66075-900
Belem-PA, Brazil
Tel/Fax: (91) 211-16-34, E-mail: victor@ufpa.br

Abstract

We consider one of the key problems of bianisotropic materials for electromagnetic applications-constitutive relations. The discussion will be restricted to macroscopic level of the tensor description. We will discuss the constitutive tensors under the following assumptions: the media under consideration are linear, homogeneous and in general lossy, possessing space and time dispersion and in general anisotropic. Description in terms of complex field variables and complex parameters of the constitutive tensors allows one to take into account noninstantaneous and nonlocal interaction between electromagnetic fields and matter. In the case of homogeneous media (long-wave approximation), macroscopic properties of the media are described by tensors which are not changed from point to point. This allows one to exclude from consideration translational symmetry and to characterize the properties of the media (and consequently the properties of the electromagnetic field) by tensors in an arbitrary space point. But the point symmetry of the medium in such a description is preserved. Material descriptors are tensor quantities of known symmetry but of unknown numerical values. The symmetry structure of them is frequency and model-independent. The numerical values can be calculated by some physical theories or by experimentation. We will use phenomenological approach based on the first physical principles: Onsager's relations and space-time symmetry.

1. Introduction

The functional dependence **D=D(E,B)**, **H=H(E,B)** in the constitutive relations may be involved and in general contains integral-differential operators. Our discussion will be restricted to macroscopic level of the tensor description. We will consider the constitutive tensors under the following assumptions: the media under consideration are linear, homogeneous and in general lossy. Description in terms of complex field variables and complex parameters of the tensors allows one to take into account non-instantaneous and nonlocal interaction between electromagnetic fields and matter. In the case of homogeneous media (long-wave approximation), macroscopic properties of the media are described by the tensors which are not changed from point to point. This allows one to exclude translational symmetry from consideration and to characterize the properties of the media (and consequently the properties of the electromagnetic field) by the tensors in an arbitrary space point. But the point symmetry of the medium in such a formulation is preserved. Therefore, material descriptors are tensors of known symmetry but of unknown numerical values. The symmetry structure of them is frequency and model-independent. The numerical values can be calculated by some physical theories or by experimentation.

The following discussion will be based on the Tables of the second-rank constitutive tensors for bianisotropic media presented in [1,2]. These tensors have been calculated using the first physical principles: Onsager's relations and space-time symmetry (the point magnetic groups). Notice that by virtue of the duality between the antisymmetric tensors of the third rank (polar or axial) and of the tensors of the second rank (axial or polar, respectively), we can use these Tables in the case of the media described by antisymmetric tensors of the third rank as well.

We will consider the following phenomenological form of the constitutive relations in the frequency domain:

$$\mathbf{D}=[\varepsilon]\mathbf{E}+[\alpha]\mathbf{B} \quad (1)$$

$$\mathbf{H}=[\beta]\mathbf{E}+[\mu]^{-1}\mathbf{B} \quad (2)$$

where the 3x3 tensors $[\alpha]$ and $[\beta]$ describe the cross-coupling between the electric and magnetic fields. We will call $[\alpha]$ and $[\beta]$ as crosscoupling tensors preserving the term "magnetoelectric tensor" to a special case of the magnetoelectric effect. Notice that the magnetoelectric tensors are a particular case of those published in [1,2]. The magnetoelectric tensors can be obtained using the Tables of [1,2] with an additional constraint $[\alpha]=-[\beta]^t$ where the superscript ^{t} means transposition.

The counterparts of (1) and (2) in the time-domain can be obtained by Fourier superposition. The relations (1), (2) describe a broad class of media with spatial and frequency dispersion.

2. Equivalence of Different Forms of the Constititutive Relations

Different forms of the constitutive relations can be met in literature [3]. We will use the **DH(EB)** presentation (1) and (2) where the fields **D** and **H** are written as linear functions of the fields **E** and **B**. Another form of the relations is the presentation **DB(EH)**:

$$\mathbf{D}=[\varepsilon]\mathbf{E}+[\xi]\mathbf{H} \quad (3)$$

$$\mathbf{B}=[\mu]\mathbf{E}+[\zeta]\mathbf{H} \quad (4)$$

It is not difficult to express the tensors of (1) and (2) in terms of (3) and (4) and vice versa. Moreover, from general properties of the tensors we know that any relation between the tensors expressed as a sum or a product of them, is invariant with respect to the group of the permissible coordinate transformations [4]. It means that if the tensors of the **DB(EH)**-system (3), (4) have been calculated by symmetry principles, the corresponding tensors of the system **DH(EB)** will have the same structure. Therefore the tensor for example, $[\alpha]$ of (1) expressed in terms of the tensors of (3) and (4) will have the same structure as the corresponding tensor $[\xi]$, the tensor $[\beta]$ will have the structure of the tensor $[\zeta]$, etc. Thus the tensor structure obtained by symmetry principles is invariant with respect to the presentations **DB(EH)** or **DH(EB)**. The same is valid for the presentation **EH(DB)**. Therefore, we can use the Tables of [1,2] for all these repersentations.

3. Decomposition of the Constitutive Tensors

Some of the medium properties can be deduced from the tensor decomposition. We can decompose a tensor into a sum of its symmetric and antisymmetric parts, then the symmetric part can be decompose into a sum of a spherical (scalar) one and a deviator, etc. The antisymmetric part of the tensor $[\mu]$ for example may describe an axial vector (dc magnetic field), the deviator of the tensor $[\varepsilon]$ may present the quadrupole electrical moment, etc. Thus we can evaluate the multipole contributions in the constitutive tensors and obtain additional information about the medium. One simple example of the tensor decomposition will be given in Section V.

4. Post's Constraint

During the last decade, we witnessed a strong controversy about interpretation and validity of the Post's constraint [5]. In terms of the tensors $[\alpha]$ and $[\beta]$ of (1) and (2), the Post's condition [6] can be written as follows:

$$\text{Trace}\,[\alpha] = \text{Trace}\,[\beta] \tag{5}$$

In order to discuss this constraint, we apply first to a well-recognized condition of reciprocity [3] for the crosscoupling tensors $[\alpha]$ and $[\beta]$:

$$[\alpha]=[\beta]^t \tag{6}$$

The tensors $[\alpha]$ and $[\beta]$ may always be decomposed into a sum of reciprocal and nonreciprocal parts as follows [7]:

$$[\alpha]= [\alpha]_r + [\alpha]_{nr} = ([\alpha] + [\beta]^t)/2 + ([\alpha] - [\beta]^t)/2 \tag{7}$$

$$[\beta]= [\beta]_r + [\beta]_{nr} = ([\beta] + [\alpha]^t)/2 + ([\beta] - [\alpha]^t)/2 \tag{8}$$

where the subscript $_r$ denotes reciprocal part and the subscript $_{nr}$ stands for nonreciprocal part of the tensor. Comparing (7) and (8) we see that the reciprocal parts of the crosscoupling tensors are coupled by the relation

$$[\alpha]_r = [\beta]^t_r \tag{9}$$

However for the nonreciprocal parts, we obtain another relation:

$$[\alpha]_{nr} = - [\beta]^t_{nr} \tag{10}$$

Analogous expressions obtained by quantum-mechanical calculations in the electric quadrupole-magnetic dipole approximation [8, p. 168,169, eqs. 58, 59] coincide structurally (in the sense of symmetry) with (9) and (10). Notice, that each of the magnetoelectric tensors $[\alpha]$ and $[\beta]$ may contain: 1) only reciprocal part, 2) only nonreciprocal part, 3) both reciprocal and nonreciprocal ones.

Now let us analyse the relations (9) and (10). First of all, there is no restriction on dispersion properties of $[\alpha]$ and $[\beta]$ in these identities. The relations (9) and (10) follow trivially from the condition of reciprocity (6) which is in its turn a consequence of time-reversal symmetry of the medium. Secondly, we did not make any restrictions on the losses in the medium, i.e. it may be lossy or lossless. Thirdly, we did not use in these relations any space rotation-reflection symmetry.

Taking into account that the diagonal elements of the tensors $[\beta]_r$ and $[\beta]_{nr}$ in the right-hand sides of (9) and (10) after transposition of the matrices remain in their positions, we obtain

$$(\alpha_{ii})_r = (\beta_{ii})_r \tag{11}$$

$$(\alpha_{ii})_{nr} = - (\beta_{ii})_{nr} \tag{12}$$

where the subscript i=1, 2, 3. The relation (11) means that the diagonal elements of the reciprocal parts of the tensors $[\alpha]$ and $[\beta]$, namely $(\alpha_{ii})_r$ and $(\beta_{ii})_r$ are equal in pairs. Analogously, the diagonal elements of the nonreciprocal parts of $[\alpha]$ and $[\beta]$ in (12), *i. e.*, $(\alpha_{ii})_{nr}$ and $(\beta_{ii})_{nr}$ are equal in pairs with opposite signs.

If the diagonal elements of two matrices are equal in pairs, the traces of these matrices (i.e. the sums of their diagonal elements) must be equal as well. Therefore from (11) and (12), it follows immediately

$$\text{Trace } [\alpha]_r = \text{Trace } [\beta]_r \quad (13)$$

$$\text{Trace } [\alpha]_{nr} = - \text{Trace } [\beta]_{nr} \quad (14)$$

Comparing (13) and (14) with (5) we see that the Post's constrant is fulfilled for reciprocal parts of [α] and [β]. But for nonreciprocal parts, this constraint is valid if only we extract from the tensors $[\alpha]_{nr}$ and $[\beta]_{nr}$ their spherical (isotropic or scalar) parts. It means that the tensors $[\alpha]_{nr}$ and $[\beta]_{nr}$ must be traceless, i.e.

$$\text{Trace } [\alpha]_{nr} = \text{Trace } [\beta]_{nr} = 0 \quad (15)$$

It corresponds to the obvious fact that the isotropic media cannot be nonreciprocal.

From the above discussion we see that the Post's constraint is a simple consequence of the more strong relations (9) and (10) between the elements of the tensors [α] and [β].

5. Examples of the Constitutive Tensors for Some Artificial Media

5.1 Omegaferrites

Recently, a new artificial material which is a combination of a ferrite and chiral particles has been proposed [9]. This material has been called chiroferrite. Earliear, media based on omega-particles embedded in a dielectric host material have been suggested in [10]. Here, we will discuss a new medium with omega elements in the form of a hat embedded periodically or randomly in a magnetized ferrite. The axes of the elements are oriented along the z-axis. One of such elements is shown in Fig. 1.

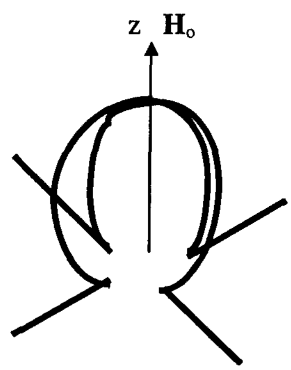

Fig. 1 Omega-element in the form of a hat

Let us first define the magnetic group of symmetry of the omegaferrite medium. The omega element has one four-fold axis of symmetry coinciding with the z-axis. This element possesses also the four plane of symmetry passing through the z-axis. If the omega elements are arranged in a three-dimensional square array with the axes of the elements along the z-axis, the resultant symmetry of the nonmagnetic medium will be $C_{\infty v}$ (in Shoenflies notations), because the four-fold axis is transformed in the axis of infinite order. A uniform dc magnetic field directed along the z-axis has the magnetic symmetry $C_{\infty v}(C_\infty)$. Using the Curie principle of symmetry superposition, we define the resultant symmetry of the omegaferrite medium as $C_{\infty v}(C_\infty)$. The constitutive tensors for media with such a symmetry calculated by the group-theoretical method of [1] are in Table 1.

Table 1

	$[\mu]$	$[\varepsilon]$	$[\alpha]$	$[\beta]$
Omegaferrite $C_{\infty v}(C_\infty)$	$\begin{vmatrix} \mu_{11} & \mu_{12} & 0 \\ -\mu_{12} & \mu_{11} & 0 \\ 0 & 0 & \mu_{33} \end{vmatrix}$	$\begin{vmatrix} \varepsilon_{11} & \varepsilon_{12} & 0 \\ -\varepsilon_{12} & \varepsilon_{11} & 0 \\ 0 & 0 & \varepsilon_{33} \end{vmatrix}$	$\begin{vmatrix} \alpha_{11} & \alpha_{12} & 0 \\ -\alpha_{12} & \alpha_{11} & 0 \\ 0 & 0 & \alpha_{33} \end{vmatrix}$	$\begin{vmatrix} -\alpha_{11} & -\alpha_{12} & 0 \\ \alpha_{12} & -\alpha_{11} & 0 \\ 0 & 0 & -\alpha_{33} \end{vmatrix}$

Any of the tensors $[\alpha]$ and $[\beta]$ in accord with (7) and (8) can be decomposed into reciprocal and nonreciprocal part. Consider for example the tensor $[\alpha]$:

$$[\alpha]=[\alpha]_r+[\alpha]_{nr}=\begin{vmatrix} 0 & \alpha_{12} & 0 \\ -\alpha_{12} & 0 & 0 \\ 0 & 0 & 0 \end{vmatrix}+\begin{vmatrix} \alpha_{11} & 0 & 0 \\ 0 & \alpha_{11} & 0 \\ 0 & 0 & \alpha_{33} \end{vmatrix} \tag{16}$$

This decomposition s easily obtained using the condition of reciprocity (6). Analogous decomposition is valid for the tensor $[\beta]$. Therefore the tensors $[\alpha]$ and $[\beta]$ for omegaferrites contain both reciprocal and nonreciprocal parts. We see from (16) that the nonreciprocity of omegaferrites is defined (along with the tensors $[\varepsilon]$ and $[\mu]$) by diagonal elements of the tensors $[\alpha]$ and $[\beta]$ while the nondiagonal elements of them stipulate reciprocal crosscoupling effect. Also we see from (16) and Table 1 that the reciprocal and nonreciprocal parts of the tensors $[\alpha]$ and $[\beta]$ satisfy the conditions (9) and (10).
As the next step in the tensor decomposition, we can calculate the spherical (scalar) parts of the diagonal tensors $[\alpha]_{nr}$ and $[\beta]_{nr}$. These parts defined by

$$\pm\frac{1}{3}\begin{vmatrix} (2\alpha_{11}+\alpha_{33}) & 0 & 0 \\ 0 & (2\alpha_{11}+\alpha_{33}) & 0 \\ 0 & 0 & (2\alpha_{11}+\alpha_{33}) \end{vmatrix} \tag{17}$$

correspond to an isotropic nonreciprocal medium which is nonphysical. Therefore, they can be extracted from the tensors. As a result we obtain a relation between the elements of the crosscoupling tensors for omegaferrites:

$$\alpha_{33}= -2\alpha_{11} \tag{18}$$

This condition means that both $[\alpha]$ and $[\beta]$ must be traceless.

5.2 Ziolkowskii's media

We will demonstrate here how to determine the structure of the constitutive tensors of the media formed by artificial Ziolkowski's moleculas [11]. These moleculas are linear electric and/or magnetic dipole antennas loaded with some combination of passive and/or active electronic elements. The electronic elements are assumed to be nonradiative and the host material is isotropic.

First, consider the moleculas which can be presented by two-ports, i.e. every molecula has only one antenna. The simplest variant is a random distribution of such moleculas in a dilectric matrix which leads to the group of the first category K. If such moleculas are oriented along a certain axis, the medium acquires the symmetry $C_{\infty v}$ of the first category in the case of the electric dipole antennas and the symmetry $D_{\infty h}(C_{\infty h})$ for the magnetic dipole antennas.

The moleculas can have two antennas and therefore can be considered as four-ports. Several variants of the antenna-type combinations with different orientations and the corresponding symmetry groups are presented in Fig. 2. The single arrow in Fig.2 denotes the electric dipole antenna and the double arrow stands for the magnetic dipole antenna. Notice that if the host material is anisotropic and/or magnetic, the resultant symmetry group can be defined by Curie's principle of symmetry superposition. The constitutive tensors for all the symmetries in Fig.2 are written in [1].

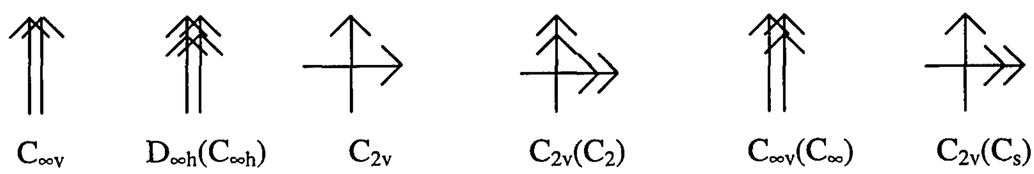

Fig. 2

6. Conclusions

The symmetry analysis presented here is essentially model-independent. Using some electromagnetic models, we can perhaps simplify the structure of the constitutive tensors. But in any case, the structure of them calculated by making use of physical models cannot be more complex that those obtained by symmetry methods.

References

[1] V. Dmitriev, "Tables of the second rank constitutive tensors for linear homogeneous media described by the point magnetic groups of symmetry," *Progress in Electromagnetic Research*, PIERS, vol. 28, 2000, pp.47-99.

[2] V. Dmitriev, "Some general electromagnetic properties of linear homogeneous bianisotropic media following from space and time-reversal symmetry of the second-rank and antisymmetric third-rank constitutive tensors", *The Eur. Phys. J.*, Appl. Phys., to be published.

[3] J. A. Kong, *Electromagnetic Wave Theory*. EMW Publishing, Cambridge, Massachusetts, USA, 1998.

[4] G. A. Korn and T.M. Korn, *Mathematical Handbook for Scientists and Engineers*. McGraw-Hill Book Company, New York, 1968, Ch.16.

[5] W. S. Weiglhofer and A. Lakhtakia, "The Post Constraint Revised", *Archiv fuer Ubertragungstechnik*, vol. 52, pp. 276-278, 1998.

[6] E. J. Post, *Formal Structure of Electromagnetics*. North-Holland, Amsterdam, 1962.

[7] A. H. Sihvola and I. V. Lindell, "Material Effects in Bi-anisotropic Electromagnetics," *IEICE Trans. on Electronics*, vol. E78-C, pp. 1383-1390, 1995.

[8] R. E. Raab and E. B. Graham, "Universal constitutive relations for optical effects in transmission and reflection in magnetic crystals," *Ferroelectrics*, vol. 204, pp. 157-171, 1997.

[9] W. S. Weiglhofer, A. Lakhtakia, and B. Michel, "On the constitutive parameters of a chiroferrite composite medium", *Microw. and Opt. Tech. Lett.*, vol. 5, N18, pp. 342-345, 1998.

[10] S. A. Tretyakov and A. A. Sochava, "Proposed composite material for nonreflecting shields and antenna radoms," *Electron. Lett.*, vol. 29, No. 12, pp. 1048-1049, 1993.

[11] R. W. Ziolkowski, "The design of Maxwellian absorbers for numerical boundary conditions and for practical applications using engineered artificial materials," *IEEE Trans.* Antennas Propagat., vol. AP-45, pp. 656-671, N4, 1997.