Resonant Effective Properties of Plane Stratified Structures

O. V. Ivanov

Department of Quantum Electronics and Optoelectronics, Physical Faculty,
Ulyanovsk State University, L. Tolstoy str. 42, 432700 Ulyanovsk, Russia
Fax: +7-8422-412-340; e-mail: oliv@sv.uven.ru

Abstract

Effective properties of plain stratified bianisotropic structures are found on the basis of 4x4 matrix method without restriction on the ratio of the period of structure and the wavelength of incident radiation. The effective permittivity, magnetic permeability and magnetoelectric dyadics are obtained for isotropic two-layer and anisotropic helicoidal structures in resonant case.

1. Introduction

The problem of effective properties of stratified media is known in optics for a long time [1]. The effective properties of an inhomogeneous plane stratified structure are defined as the properties of a certain homogeneous layer which cannot be distinguished from the investigated inhomogeneous structure by optical means. As a rule, periodic structures with large number of periods are considered in long wave approximation, i.e. in the case when the length of electromagnetic wave is much greater than the period of the structure. Such structures exhibit anisotropic, chiral and, generally, bianisotropic properties [1-3].

In case when the wavelength of light is approximately equal to the period of structure, the resonant interaction of light with the structure is observed. This problem is solved by various methods, for example, by the method of coupling waves or Bloch functions formalism [4]. The method of effective properties was not earlier used in this case, being limited in application by media with short period.

The purpose of the present work is to find the effective properties of periodic bianisotropic structures having the period approximately equal to the wavelength of propagating light, and also non-periodic structures made of a few layers. For solution of the given problem we apply a modification of the Berreman 4x4 matrix method.

2. Method of 4x4 Matrices; Matrix of Material Properties

Consider a medium consisting of bianisotropic layers parallel to XY plane. A plane electromagnetic monochromatic wave propagates in the medium. Its wave vector $\mathbf{k}$ is parallel to XZ plane. Maxwell equation can be transformed to the following 4x4 differential matrix equation:

$$\frac{d\mathbf{g}}{dz}=-ik_0G\left(\hat{\varepsilon},\hat{\mu},\hat{\alpha},\hat{\beta}\right)\mathbf{g},. \tag{1}$$

where k_0 is the wave number in vacuum; the four dyadics of permittivity $\hat{\varepsilon}$, permeability $\hat{\mu}$ and magnetoelectric $\hat{\alpha}$, $\hat{\beta}$ describe the relations between electric and magnetic fields; a four-component vector $\mathbf{g}=\left(E_x,-E_y,H_x,H_z\right)$ is composed of tangential components of the fields. Here the 4x4 matrix of material properties (MMP) G is determined by local properties of the medium and the incidence angle. It is constructed on the basis of the four constitutive dyadics and allows in general way taking into account bianisotropic properties of the medium. MMP consists of the sum of three matrices which are proportional to zero, first and second degrees of the tangential component of the wave vector k_x:

$$\mathbf{G}=\mathbf{G}_0+\frac{k_x}{k_0}\mathbf{G}_1+\frac{k_x^2}{k_0^2}\mathbf{G}_2. \tag{2}$$

For a homogeneous medium MMP does not depend on z, and the solution of the matrix equation (1) in this case is a superposition of eigen waves:

$$\mathbf{g} = \sum c_j \mathbf{q}_j \exp\left(-ik_{zj} z\right), \quad j = 1,..,4 \tag{3}$$

where c_j is the amplitudes of the waves corresponding to the eigen vectors $\mathbf{q}_j$ of matrix $\mathbf{G}$. The eigen numbers of this matrix $n_{zj} = k_{zj}/k_0$ are the solutions of the dispersion equation $\det(\mathbf{G} - n_z\mathbf{I})$, where $\mathbf{I}$ is the unit matrix. As follows from (3), there are four eigen waves with various polarizations determined by the fourth order equation with respect to n_{zj}.

If the structure contains several homogeneous layers, the investigation of propagation of electromagnetic waves turns into the solution of a boundary problem. The continuity conditions for tangential electrical and magnetic field components must be satisfied on the boundaries of adjoining layers, that is identical to the continuity of the four-component vector $\mathbf{g}$. Let a wave described by vector $\mathbf{g}(z_m)$ is incident on m-th layer. The vector $\mathbf{g}(z_{m+1})$ of the wave transmitted through the layer is determined by the matrix relation:

$$\mathbf{g}(z_{m+1}) = \mathbf{S}^{(m)} \mathbf{g}(z_m), \tag{4}$$

where the transformation matrix for m-th layer of thickness Δz_m has the form:

$$\mathbf{S}^{(m)} = \exp\left[-ik_0 \mathbf{G}(z_m) \Delta z_m\right]. \tag{5}$$

Here the exponential function is applied to a 4x4matrix. It is known from the theory of matrices that for arbitrary matrix $\mathbf{A}$ and function f the following relation is satisfied:

$$f(\mathbf{Q}\mathbf{A}\tilde{\mathbf{Q}}) = \mathbf{Q} f(\mathbf{A}) \tilde{\mathbf{Q}}, \tag{6}$$

where $\tilde{\mathbf{Q}}$ is the matrix inverse for $\mathbf{Q}$. Let's take $\mathbf{Q}$ as a matrix which are formed by columns of eigen vectors of $\mathbf{G}$: $\mathbf{Q}_{jk} = (\mathbf{q}_k)_j$. Then the lines of $\tilde{\mathbf{Q}}$ are the vectors $\tilde{\mathbf{q}}_j$ complimentary to $\mathbf{q}_k$ $(\tilde{\mathbf{q}}_j, \mathbf{q}_k = \delta_{jk})$. We substitute MMP in (5) as: $\mathbf{G} = \mathbf{Q}\tilde{\mathbf{Q}}\mathbf{G}\mathbf{Q}\tilde{\mathbf{Q}}$. Taking into account that $\tilde{\mathbf{q}}_j \mathbf{G}\, \mathbf{q}_k = \tilde{\mathbf{q}}_j n_{zk} \mathbf{q}_k$ the transformation matrix of m-th layer can by presented as:

$$S_{ij}^{(m)} = \sum_{k,l} Q_{ik}^{(m)} \delta_{kl} \exp(-ik_0 n_{zl}^{(m)} \Delta z_m) \tilde{Q}_{lj}^{(m)}. \tag{7}$$

For a system consisting of $p-1$ layers (a wave is incident from medium $n = 0$, passes p boundaries and propagates to the medium $m = p$) the resulting transformation matrix is the product of transformation matrices of separate layers:

$$\mathbf{S} = \mathbf{S}^{(p)} \mathbf{S}^{(p-1)} \mathbf{L} \mathbf{S}^{(1)} \tag{8}$$

For inhomogeneous medium MMP is a continuous function of z, and the resulting transformation matrix can be found as the product of transformation matrices of indefinitesimal layers, into which the medium is divided. When the thickness of these layers tends to zero we obtain the transformation matrix as a multiplicative integral:

$$\mathbf{S} = \lim_{\Delta z \to 0} \prod_j e^{-ik_0 \mathbf{G}(z_j)\Delta z} = \int e^{-ik_0 \mathbf{G}(z) dz}, \tag{9}$$

where multiplicative integral is designated as:

$$\int \mathbf{A}(z)^{dz} = \lim_{\Delta z \to 0} \prod_j \left[\mathbf{I} + \mathbf{A}(z)\Delta z\right] = \lim_{\Delta z \to 0} \prod_j \mathbf{A}(z_j)^{\Delta z}.$$

The same result can be obtained by separating the variables and integrating the basic equation (1).

3. Effective Matrix of Material Properties

Arbitrary stratified bianisotropic structure accordingly to (8) can be described by a transformation matrix S, which contains the complete information on all optical characteristics of the structure as whole. This allows replacing such structure by a certain homogeneous layer with effective properties so that the transformation matrix does not changes. Thus, in order to find the effective properties of the structure it is necessary at first to find the transformation matrix from the properties of the inhomogeneous structure, then to solve the inverse problem and find effective MMP assuming the structure to be a homogeneous layer. From MMP it is possible to obtain permittivity, permeability and magnetoelectric dyadics.

Equating the transmission matrices of a homogeneous layer (5) and an inhomogeneous structure with arbitrary function $G(z)$ (9) and taking the logarithm we obtain the effective MMP:

$$G_{eff} = \frac{i}{k_0 L} \ln\left[\int_0^L e^{-ik_0 \mathbf{G}(z) dz} \right], \tag{10}$$

where L is the whole thickness of the structure. The obtained expression is exact. It allows calculating effective properties of structures both with large number of layers and few layers, for example, two or three-layer structures. For periodic medium it is enough to take integral over the period of the structure.

It is known that the logarithmic function is multivalued, in particular, for scalars it is determined up to $2\pi im$, where m is an integer number. The multyvaluedness of logarithmic function is the result of the periodicity of exponential function. For a matrix of dimension sxs there exist s independent integer numbers m_1, K, m_s, varying which it is possible to obtain new matrices so that the exponential function upon these matrices is a constant:

$$\mathbf{A}' = \mathbf{A} + 2\pi i \mathbf{Q}\mathbf{M}\tilde{\mathbf{Q}}, \quad \mathbf{M} = \begin{pmatrix} m_1 & 0 & 0 \\ 0 & \mathbf{O} & 0 \\ 0 & 0 & m_s \end{pmatrix} \tag{11}$$

where the matrix $\mathbf{Q}$ is made of the columns of eigen vectors of the matrix $\mathbf{A}$. Thus, the logarithm of a matrix is a set of counted number of matrices. Ambiguity of determination of MMP follows from this fact. Determination of MMP is ambiguous not only when we find effective MMP for inhomogeneous structure but also for usual homogeneous medium. Really, let we transform $\mathbf{G} \to \mathbf{G}'$ so that $\exp(-ik_0\mathbf{G}) = \exp(-ik_0\mathbf{G}')$. Then the matrix $\mathbf{S}$ (5) and, therefore, and reflection and transmission coefficients will not change. Hence, the determination of constitutive dyadics of medium is ambiguous at fixed optical characteristics.

The effective MMP for an inhomogeneous medium is a complex function of k_x, and, generally, it cannot be presented in the form of (2) when $\mathbf{G}_i$ is independent on k_x. Therefore the effective dyadics $\hat{\varepsilon}$, $\hat{\mu}$, $\hat{\alpha}$ and $\hat{\beta}$ at any splitting of G into three summands depend on k_x and on the direction of wave propagation. Hence, such medium exhibit spatial dispersion.

4. Examples of Stratified Structures

Consider two most typical examples demonstrating application of the general approach of calculation of effective properties of layered bianisotropic media.

4.1 Two-layer structure

We consider a structure which consists of two dielectric layers with permittivities $\varepsilon_{1,2} = \varepsilon \pm \Delta\varepsilon/2$ ($\Delta\varepsilon = \varepsilon$) and thickness $d_{1,2}$. Non-zero components of the effective dyadics of such structure can be written as:

$$\varepsilon_{xx,yy} = \varepsilon\left[\Delta l + \left(\gamma^{(p,s)}/2\right)\sin 2l_1\right]\left(\frac{i}{\nu^{(p,s)}} + \frac{1}{\pi}\right), \quad \mu_{yy,xx} = \left[\Delta l - \left(\gamma^{(p,s)}/2\right)\sin 2l_1\right]\left(\frac{i}{\nu^{(p,s)}} + \frac{1}{\pi}\right),$$
$$\varepsilon_{zz} = \varepsilon, \quad \mu_{zz} = 1, \quad \alpha_{xy,yx} = -\beta_{yx,xy} = -i\sigma\gamma^{(p,s)}\sqrt{\varepsilon}\sin^2 l_1\left(\frac{i}{\nu^{(p,s)}} + \frac{1}{\pi}\right). \tag{12}$$

where

$$\nu^{(r)} = \sqrt{\left(\gamma^{(r)}\sin l_1\right)^2 - \Delta l^2}, \ \gamma^{(p,s)} = \frac{\Delta\varepsilon}{2\varepsilon}\left(\frac{k_x^2}{\varepsilon k_0^2 - k_x^2}\mathrm{m}1\right), \ \Delta l = \pi - l_1 - l_2, \ l_i = k_0\sigma_i d_i\sqrt{\varepsilon_i}, \ \sigma_i^2 = 1 - \frac{k_x^2}{\varepsilon_i k_0^2}.$$

The propagation constants of p and s polarized waves in such medium $k_z^{(p,s)} = \pm\sigma\sqrt{\varepsilon}\left(1 - i\nu^{(p,s)}/\pi\right)k_0$ describe attenuating waves. The positive direction wave propagating in such medium attenuates and transfers its energy to the negative direction wave, i.e. Bragg reflection of light from periodic structure takes place. The penetration depth of the wave in the structure is determined by the parameter ν and equals $\pi/\left(2\sigma\nu^{(p,s)}k_0\sqrt{\varepsilon}\right)$. The effective medium exhibits the property of strong spatial dispersion, since constitutive dyadics are functions of k_x, i.e. functions of the wave vector. This is consisted with the fact that the contribution of spatial dispersion is essential when the relation between the period of structure and the wavelength is close to unit.

4.2 Helicoidal structure

A Helicoidal structure represents a twisted medium in which there is a rotation of anisotropy axes around Z axis at displacement along its direction. The effective properties of such medium correspond to a chiral medium with

$$\alpha_{xx} = \alpha_{yy} = -\beta_{xx} = -\beta_{yy} : \ i\lambda/L_0, \tag{13}$$

where L is the period of the structure; λ is the wavelength of the incident radiation.

5. Conclusion

On the basis of 4x4 matrix method the exact equation for effective MMP of arbitrary bianisotropic structure is obtained without restriction on the relation between the wavelength and the thicknesses of the layers constituting the structure.

The effective constitutive dyadics for a two-layer isotropic dielectric structure are found in the resonant case. It is shown that the effective magnetoelectric dyadics contain off-diagonal components which were absent for the isotropic layers of the structure. At this conditions the permittivity and permeability become complex quantities, that results in strong Bragg reflection of a wave from such structure. The effective constitutive dyadics for helicoidal medium are found. It is shown that such medium is chiral.

MMP and constitutive dyadics of any medium are determined ambiguously. The transformation for MMP which completely conserves the optical characteristic of the layered structure is found.

References

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