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NONLINEAR MESOSCOPIC ELASTICITY IN SOLIDS

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ABSTRACT

A variety of materials reside in the purported elasticity universality class characterized by "nonlinear mesoscopic elasticity". Rocks, the prototypical member of this class, have extreme nonlinearity (NL), hysteresis (H), and discrete memory (DM). The experimental manifestations of NL, H, and DM will be described with Berea sandstone as example. NL, H, and DM lead to unusual behavior in laboratory resonance measurements and in a variety of geophysical contexts. An effort will be made to identify the microscopic features of materials that might be placed in this elasticity universality class.

TRANSCRIPT

DR. GUYER: This talk was conceived by Paul Johnson -- at least he conceived the title, but then he went on vacation and asked me to give the talk in his stead, so I am going to give you a talk that is at least related to the title.

[Transparency 1]

This is going to be a sort of cultural interlude, because I am going to be telling you about nonlinearity in the elastic properties of materials and I am going to be paying attention to materials that are not the ones that you are necessarily used to talking about.

The materials will be something like sand, soil, rocks, and so forth, and I am going to try to suggest that there is a generalization about the elastic properties of these materials that might be helpful in providing some understanding of each of them based on some understanding you may have of the others. I am heading toward attempting to make a generalization but I am going to start by paying attention to rocks, using as a prototype.

Here is the outline. Somewhere along the way I am going to talk about nonlinearity, hysteresis, and something called discrete memory, so you will see NL-H-DM come back again and again, and I will be trying to emphasize the notion that there are certain experimental signatures of the quality that I am looking for in the behavior of these materials.

[Transparency 2]

If I had to discuss quasi-static measurements, I might, if I were in a freshman class, be telling you about the fact that if you pull on a spring that has a spring constant, you discuss the equation of state of the spring by talking about the relationship between the force and the length. Also, in the freshman class, you put the spring into an oscillator and make it oscillate and you discover that it has a frequency related to the same spring constant. This is really the relationship between the quasi-static equation of state of the material and the dynamics of the material.

I am going to be talking about that relationship but not for simple-minded materials like this, but for real pieces of material.

[Transparency 3]

Just to make a step in the direction of real materials, I am going to be talking about, typically, experiments on rocks. The quasi-static equation-of-state measurements are often made on a rock that might be about the size of a Coke can (7" this way and 2-3" that way). You put a pressure on it, you measure the strain, and discuss the behavior of the strain as a function of the stress.

In dynamics, as all of you know, you essentially resonate a piece of material. When you are doing this on a rock, it might be about that big in diameter and, roughly speaking, that long. You tickle at one end and detect at the other end. We are going to talk about what you see when you do this.

[Transparency 4]

Not everybody here knows how things behave then they resonate. Here is a quasi-static equation of state for a relatively nice material, a piece of Pyrex glass. Another material that is very similar to that in the qualitative sense of being a nice material is a piece of that "other" lucite. It has resonance curves that you are very familiar with. The reason that there are several of them, of course, is that the resonance is driven at several amplitudes, really voltage, v_1, v_2 .... Of course, the resonance curves look exactly the same; you do not go around paying attention to the voltage.

You take a Berea sandstone and take it through a force protocol, nobody even bothered to pay attention to the fact that you might have increased the pressure and decreased the pressure (it just runs up and down this curve, upper left). If you do that to a Berea sandstone, it will run up this side and come back down that side.

If you are fancy about the pressure protocol, there will be events in the interior of what is happening here, which we will pay attention to in a minute, which represent the so-called property of discrete memory.

If you take a Berea sandstone through a resonance curve -- and to show the fact that it is quite exotic (this is a logarithmic scale), you simply (it is the resonance of that little piece of material I described to you) -- you are increasing the voltage and the resonance curve looks relatively nice for a while, begins to drift over, and then whatever this is. These are actually modest accelerations compared to the ones up there on the nice piece of material, and the resonance curve is bent over like crazy and has all kinds of other complicated properties.

[Transparency 5]

Just to get you in the mood -- of course, that was a rock and not a mineral, it is a rock, which is to say it is a piece of material that might look something like that.

[Transparency 6]

Or it might look like this.

Or like that one.

The 2 outside pictures are what are called pore casts, in which you have intruded epoxy into the pore space of the rock, then removed the rock, and you are actually seeing a picture of the pore space.

The important qualitative thing to remember is that in these systems it is not the grains of the rock that cause the elasticity, it is the space between the grains where the important elastic properties occur.

[Transparency 7]

So think this way. You might be inclined to think of a rock as a bunch of pieces of material, but skip the pieces of material and think more, in fact, about what is between them. Because for the purposes of the kinds of elastic properties we are talking about here when you squeeze on the rock, the grains are more or less rigid entities and they simply move relative to one another and it is the elastic properties of what is between them that counts. So that is, in some sense, the elastic system.

PARTICIPANT: Is that well established?

DR. GUYER: Not at all. (Laughter)

So here is the abstraction, that the grains are here and that there is some elastic entity that works between the grains, and that is what we are mostly going to pay attention to. It is these elastic entities, whatever they happen to be, that are responsible for those relatively unusual elastic properties that you saw in both the quasi-static measurement and in the dynamic measurement.

I always remind my friends that there are at least 10^6 of these bonds per cubic centimeter, so you might think you ought to pay careful attention to the details of how the bonds reside in the system, but it makes no sense. It is perfectly adequate to think of it as a cubic lattice of unusual elastic elements.

What we really would like to do is to understand what those elastic elements are, or at least what properties they must have to describe what has been observed experimentally.

That is the so-called deep background, I guess.

[return to Transparency 4 – lower left]

Here it is now in a little bit more detail, that quasi-static measurement. The pressure is increased. These little curves here, which are not really very neat, occur when you increase the pressure, stop for a moment, decrease the pressure a little bit, then increase it, again. I said I was going to call this discrete memory and I will say that in a minute, but before I get to that. This is not at all linear.

Linearity, of course, is some straight line. This is extremely nonlinear and if you look at the scale of the pressures here, it is extremely nonlinear. That is hysteresis and it has this fancy feature, which I will now call a little bit more attention to.

[Transparency 8]

The kinds of materials I am talking about typically will have hysteresis in their quasi-static properties. They will be very nonlinear and they will have this property. Now let me point more carefully to that discrete memory. It is the fact that the rock sometimes can "remember" what is happening to it from the point of view of pressure.

[Transparency 9]

You might just decide to increase the pressure on the rock. If you do, it will track a trajectory that looks like this. You might decide to increase the pressure to a certain point, reduce it briefly, and come back, you put a little pressure loop in. The strain field will go to the point where you decided to change the pressure, then cut into the interior, return to this point, and it does not return asymptotically to where it was going. In fact, if you did not know where it had been, you might think it was going to go that way, but it stops at that point and tracks the trajectory it was originally on, so it knew where it wanted to go.

Just to be sure, put a little loop inside a little loop, and it will do that. There will be a small hysteresis loop here, it will return to this point and take off on that trajectory, and you can make this as complex as you like.

That is an interesting quality, and it is one of the qualities that the modeling men ought, in principle, to be trying to deal with. Some of the work I am going to talk about tries to deal with the fact that the elasticity of rocks has this relatively large nonlinearity, hysteretic behavior, and this property of discrete memory. One of the things we do is to try to model that and then talk about the consequences of the modeling.

What I just showed you in the way of properties are, actually, not particularly unusual. A magnet, random-field Ising model, has the same properties, it is just that it is the magnetic field that is driving the magnetization.

If you are familiar with the fluid configurations in the pore space, the chemical potential drives those fluid configurations and chemical potential protocols will produce fluid configurations that have the same discrete memory I was talking about.

If I am going to explain to you why rocks have those properties, I ought to make the explanation relatively generic (I do not want to make it specific, that various sandstones have this or that interior properties, because, in fact, these are generic properties of a whole variety of materials). That is, again, what we have tried to do. One of the things that I will remark on is that the description you make of these materials is not made at the microscopic detailed level but, in fact, is relatively macroscopic.

[Transparency 10]

What is the common element available among the things I have said that are similar to rocks in having these properties? The magnet, of course, has domains. It has pieces of material within itself, which, from a magnetic point of view, have 2 magnetic states.

What is it about fluid configurations and pore spaces? Well, they are relatively complicated but, at least in some simple situations, the pore is full of fluid or it is empty (or has fluid only on its walls, so there are 2 fluid configurations per pore in some sense: full and empty.

In the case of rocks, essentially the elastic elements have, roughly speaking, 2 states, and you can model that in a variety of ways, but the thing that is important qualitatively is the notion that many of the elastic elements between the grains somehow go in a hysteretic way as the domains go in a hysteretic way between being up and down and the fluid configurations in the pore space go hysteretically between being filled and empty.

Let me give you what I will call summary one. The properties that a rock has, nonlinearity, hysteresis, and discrete memory -- I actually pointed only to the quasi-static measurements, and I talked about those, and when I described the modeling that has been done of this system, it began with modeling of the quasi-static measurements (this is work I did with Katherine McCall).

We developed a picture of how you think about the elastic elements in a rock from the point of view of quasi-static behavior. I have to give a second talk in about 20 minutes, so I will elaborate a little bit of this in a couple of minutes. It was very important in a lot of work we did to have some nice experimental help from Greg Boitnott.

When you get done and you have understood the quasi-static measurements by making up your little phenomenology, what are you going to do with it and can you do anything with it? One of the things you can do is you can ask, well, how does this object behave dynamically? In that second experiment, where there was a long rod and it had all that crazy behavior, the dynamic response, the phenomenology that describes the quasi-static measurements here leads to certain very definite predictions about dynamic behavior.

Those predictions are here and that is what my second talk was really intended to be about, the experiment that confirms the predictions. The predictions are, essentially, that for relatively small strain fields or small driving forces the resonance will shift, it will shift to lower frequencies, it will shift proportionally to the magnitude of the strain field. This result is not available to you from traditional theory of nonlinear elastic systems.

Furthermore, it actually produces a prediction of not only the qualitative behavior of the frequency shift, but it provides you with a number that you can deduce from the study and compare to quasi-static data like the data of Greg Boitnott.

[Transparency 11]

Let me show you the answer. I am not going to try to explain to you how all that works or anything, but just to show you those data and to say that when one studies -- this complicated curve and there is a lot of action taking place over here -- if you study it, in particular, as it begins, with the smallest strain fields, the resonance drifts down proportional to the amplitude of the strain in the rock.

You almost cannot see the frequency shift on this scale and that frequency shift is linear in the magnitude of the strain field, so this amounts to the confirmation that the ideas we have about the quasi-static behavior of the rock can be transferred to a discussion of the dynamic behavior.

DR. ISAAK: Is there any idea that that frequency shift could be due to temperature change with the amplitude?

DR. GUYER: No, you know how much that temperature is and you know the thermal expansion and things like that, so no, not at all.

DR. ISAAK: Because you are driving it at a higher amplitude, you are going to be heating up.

DR. GUYER: Yes, no question, it is a big rock. This is a big rock. Yes, we have checked that very carefully.

There is another property that these systems have that is just beginning to dawn upon us. It is also in dynamics. There is a slow dynamics.

[Transparency 12]

Now I am going to go back and show you this picture again (lower right of Transprency 4). I pointed to the lower part of it and I showed you that curve and I said we sort of understand that now. I did not point to the upper part and you might ask, why fuss with this little tiny diddly business down here?

We paid careful attention to the resonance curve at strains up to about 10^{-6}. There is a lot of action out there, above 10^{-6}. Less attention is paid to that, because at about the same time one began to notice these data and pay attention to them, we also found that there is, in this region, what we called initially just slow dynamics. In fact, you can track a resonance curve, come back through it and it is not the same. Track it and come back through a little slower and it is not the same by a different amount. In fact, it is essentially a slow time evolution to the elastic properties of this material, so if you are going to sweep back and forth through a resonance, the answer you get will depend on how fast you sweep, and that tends to occur up in here, so we paid attention to these data for the purpose of talking about confirmation of the relationship between quasi-static and dynamic measurements.

But then, paying a little more attention to the data up at high amplitudes, we found there is the resonance curve having behavior that depends on how fast you sweep through it and there is a phenomenon -- and it is rather complicated, so I am not going to try to describe it in detail -- where you can drive the system at a fixed frequency for 10 or 15 minutes and then turn that frequency off and just test with a very delicate small probe where the resonance is located, and you will find that when it was being driven, the resonance might have been at 3000 Hz. When you turn the drive off, the resonance will snap up by, let's say, by 7 Hz, and then there will be a remaining 1 Hz frequency shift that will be a recovery back to where it might have been logarithmically in time.

When I first was told that this happened, I did not believe it. Remarkably, if you are willing to wait at least a day tracking the location of the resonance, it recovers logarithmically.

This is work done by Jim Tencate at Los Alamos, a lot of tests of this quality, and it seems to be part and parcel of the nonlinear dynamics I was talking about, and of the quasi-statics, so that we have come to think of rock as having three special properties, the nonlinear, hysteretic, and discrete memory properties in quasi-static measurements, a frequency shift proportional to the amplitude of the strain field, and a logarithmic time dependence in the recovery of some aspects of the modulus or, if you like, the elasticity, as a function of time, and these are the signatures.

[Transparency 13]

We wanted to make the generalization that this property is not just the property of a Berea sandstone but of a class of materials, and here is an illustration of one of the experimental evidences of that, 3 different types of material. This is Lavoux limestone. This is damaged concrete, probably (it just means it was in somebody's sidewalk for 10 or 15 years before the sample was picked up), and a synthetic slate.

All of these have a frequency shift of very modest acceleration that is essentially linear in the amplitude of the strain field to the degree that you can make that decision from a curve that looks like this, so the properties observed for rocks actually seem to be possessed by a few other things.

[Transparency 14]

Our notion is to wonder -- the phrase we use is an "elasticity universality class," saying that all these materials, while very different, one from another, have elasticity properties that are essentially similar, and trying to decide what might belong in that elasticity universality class we can use that notion to help us understand various kinds of materials.

Of course, the candidates are sand, soil, rocks, concrete, ceramics, things like that. Soils, certainly, with relatively low strain fields, actually have the quasi-static behavior, e.g. work of Veucetic.

We ourselves have looked at these frequency shifts. Koen van den Abeele, who actually was here about five or six years ago, has done the work on slate that is described there, so that is the box you put something in if it belongs in the universality class. Fontainebleau sandstone belongs in there, Berea, Lavoux -- that is a slate. Concrete, fresh concrete or damaged concrete, both, seemingly, have some of the properties we are talking about, and so does soil.

We found that not just Berea sandstone had this logarithmic time dependence to its recovery from certain disturbances, but so does Lavoux and Fontainebleau and the 2 types of concrete. This is actually, in some sense, some of the work that we are currently involved in. We are paying great attention to Berea and, at the same time, looking at these other materials.

The reason one does this, of course, is that if you are going to have an understanding of the behavior of these systems that is not specific to their details, then you transfer it to other systems, and so the principle behind trying to suggest that and to explore whether this makes sense is essentially what is illustrated here.

[Transparency 15]

This is the viewgraph that explains why we pay attention to these things. They are -- the phrase I use is are that they are the "blue collar materials of daily life" -- sand, concrete, stuff like that. We all cross bridges, if you live in the Los Angeles basin you sit on a resonant piece of material that is one of these materials, if you look for oil, the phenomenon of sanding, which has to do with things falling in on your drill bit, have to do with the elastic properties of these kinds of materials, so there is a great deal of interest in them and that is one of the reasons we pay attention to them.

Thank you.

DR. LEVY: An off-line question: Is there any absorption in these systems, attenuation coefficients?

DR. GUYER: Yes, in my second talk I will go into detail about measurements of the attenuation coefficients. This was Paul Johnson's talk. I am going to give the second talk after Katherine.

DR. MARSTON: The question I have is an understanding of these connecting springs in one of your earlier models. Is there a good model developed based on Hertzian contact forces?

DR. GUYER: No, a Herzine model simply will not work, it does not have the properties in a quantitative way that are called for. You can make a Hertzian model have the qualitative properties. That is one of the things we are working on. It requires more and more understanding of what is happening experimentally, you get a better idea of how to narrow yourself as to what kinds of models are involved.

We have also tried deliberately to make the description model-independent. The phenomenon is essentially model-independent. It calls for a 2-state character to the elastic bond and that is all, in some sense, that is called for to get most of what I described. You do not have to say what it is quantitatively, only what it is qualitatively.

Thank you.