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SLIDING MODE OBSERVERS VERSUS KALMAN FILTER IN THE HOMING LOOP[n]

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ABSTRACT

A comparison via Monte-Carlo simulations is made of Sliding Mode Observers versus Kalman Filter in the homing missile guidance system using different guidance laws. It's shown that the sliding mode observer contributes to less miss distance due to noise than Kalman filter. Application of sliding mode estimators in the homing loop with Proportional Navigation guidance and with a phase-lead compensation based on sliding mode estimators for the flight control system phase-lag is demonstrated. It is shown that this system gives performance comparable to the optimal guidance law with Kalman filter that has perfect knowledge of all additional information it requires for estimation.

INTRODUCTION

An ultimate performance criterion of a homing missile guidance system, the miss distance, is crucially dependent on each element of guidance, navigation, and control of a homing interceptor. In particular, performance of the estimator (filter or observer) of data necessary to compute a guidance command is an important factor to affect the miss distance. This performance is difficult to analyze under the influence of the following factors: different effects caused by parasitic intercoupling of navigation and flight control systems, the signal phase lag due to bandwidth limits in navigation and flight control systems, guidance command saturation in presence of evasive target maneuvers, and higher measurement noise due to other target countermeasures. Estimator contributes to each of the aforementioned effects in a peculiar way. In this case numerical Monte-Carlo simulations using even a simplified mathematical model of the homing loop may give a true relative picture of miss distance projections to different engagement conditions under different estimation procedures.

In this paper numerical Monte-Carlo simulations are selected as a primary instrument to compare the performance of the widely used Kalman filters in the homing loop, presented by linearized state-model for the relative separation between missile and target perpendicular to the fixed reference[1], versus performance of Sliding Mode observers based on an emerging technique in the Sliding Mode Control (SMC) theory – higher order sliding modes[2,3]. A unique feature of these new observers is that they separate signal from noise (i.e., reconstruct a signal and its derivative from noisy measurements) on the basis of a specified limit for the signal second time-derivative.

The main goal of this experiment is to study robustness to noise (miss distance standard deviation) and to the target step-constant maneuver at different times-to-go (mean miss distance) of the guidance system with Kalman filter (KF) and with SMC filter/differentiator. These two criterions ought to show how good two estimators track given signal and attenuate noise relatively to each other. Separation principle for estimation process in this nonlinear, overly constrained problem is not valid, so evaluating performance of two algorithms in terms of estimation error without considering homing loop does not make a good projection.

The main expectation of the intended experiment is that, having less input information (line-of-site (LOS) only), a SMC robust-to-noise filter/differentiator will demonstrate the overall guidance system performance comparable to that with KF, which require in addition

to LOS measurements the following data: range to target, time-to-go, missile acceleration, maximum expected target acceleration, and noise standard deviation. In simulations, we assume noise in LOS measurements only and perfect information of other parameters necessary for KF, as well as perfect information of closing velocity. In this case, KF performance in the homing loop will be more optimistic than that of SMC filter as far as real world situation is concerned. Thus, demonstrating comparable performance in this idealized test will indicate clear advantage for SMC algorithm.

The main reason to carry out this experiment is in the following. Quite a few works has appeared recently in the literature[4-8] studying application of SMC methods to homing missile guidance and flight control systems design. Theoretical and numerical results look promising. There were shown clear advantages in robustness and the missile/target acceleration ratio. The SMC-based navigation data processing can be made the last step towards designing a truly integrated homing missile guidance, navigation, and control system within a unified framework of SMC design methods including recent advances in the field of higher order sliding modes. Thus, new principles in estimation and control decisions for missile guidance, navigation, and control are really worth studying and evaluating.

PRELIMINARIES

Guidance System Model

As known[1], the linearized guidance system model serves as a good approximation of nonlinear engagement kinematics for the purpose of comparative analysis of different guidance laws. True performance projections are obtained even under nonlinear constraints (such as acceleration command and command rate saturation) if the engagement happens in the vicinity of the collision course and the closing velocity is relatively constant during the flight (excluding the last moment, when the rapid reverse in the sign of the closing velocity occurs).

The structure of a linearized guidance system presented in the work[1] will be used to investigate the performance of the system in presence of measurement noise, and using different estimates of Line-of-Sight (LOS) rate. The following constraints on missile acceleration and acceleration jerk will be used $|n_c| \le 30G$, $|\dot{n}_L| \le 100G/S.$, and implemented as the "Performance Limits" block given in Fig. 1 where the flight control system is modeled as a single-lag block with the time-constant $T = 0.5S$.
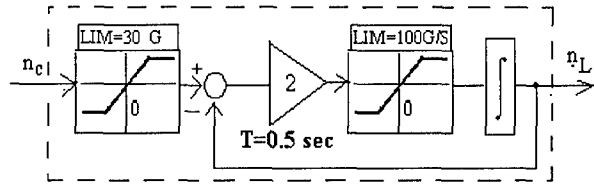

Fig. 1 "Performance Limits" Block

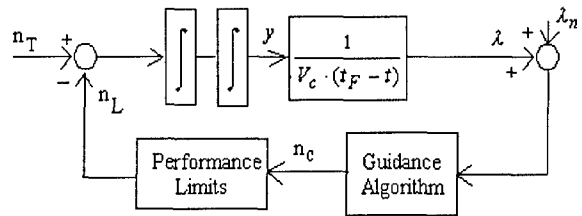Then, the guidance system will have the structure

Fig. 2 Guidance System

The linearized state equation for the relative separation between missile and target perpendicular to the fixed reference is

$$\ddot{y} = n_T - n_L \qquad (1)$$

where the target acceleration, n_T, is modeled as a constant maneuver with 6G amplitude, such that acceleration ratio is relatively large but finite,

$$\frac{n_{c\,LIM}}{n_{T\max}} = 5.$$

The relative separation along the fixed reference is supposed to obey

$$x = V_c(t_F - t) \qquad (2)$$

where the closing velocity $V_c = 9000\,Ft/S.$ is held constant, and the homing time $t_F = 10S.$ if not posted otherwise. It's assumed that the missile initially is on a collision course, and that the target evasive step-constant maneuver will occur at different times-to-go before the collision. The expected peak miss distance will be at the time which is large enough to escape safely but too small for the tracking filter to accommodate a sudden change in the signal behavior.

The measurement, LOS angle λ, is sensed with noise

$$\lambda_s = \lambda + \lambda_n \qquad (3)$$

where λ_n is a Gaussian white noise with zero mean and standard deviation of $10^{-3}\,Rad\,(1\,mR)$.

Line-of-sight rate (LOS), ω_λ, is estimated from measurements λ_s by the three-state discrete Kalman filter presented on p.166 in the reference[1] and by a SMC-based robust-to-noise filter/differentiator presented in Section 2.3.

Different guidance laws will be simulated starting with Proportional Navigation law,

$$n_c = N'V_c\hat{\omega}_\lambda, \quad \omega_\lambda \equiv \dot{\lambda}. \tag{4}$$

The expectations are that, having less information requirements, SMC observers can keep good balance between phase lag in the LOS rate estimate and noise attenuation, and achieve performance comparable to an idealized KF (with perfect information about the range-to-target, R_{TM}; time-to-go, t_{go}; missile acceleration, n_L; maximum expected target acceleration $n_{T\,max}$, and noise standard deviation.)

Kalman Filter in the Homing Loop

The tree-state $\left(\hat{y}, \hat{\dot{y}}, \hat{n}_T\right)$ digital Kalman filter is used to estimate LOS rate $\hat{\omega}_\lambda$,

$$\hat{\omega}_\lambda = \frac{\hat{y} + \hat{\dot{y}}t_{go}}{V_c t_{go}^2},$$

and target acceleration, $\hat{n}_T$, which is identical to the one shown in Fig. 9.2 in the reference[1], except that the achieved missile acceleration, n_L, rather than the commanded acceleration, n_c, is fed back into the filter. Different modifications of the program code "C9L3.cpp", developed in the reference[1], Chapter 9, are used to simulate the guidance system in Fig. 3 with KF and different guidance laws.

SMC Filter/Differentiator in the Homing Loop

Developed recently[3], an exact differentiator, based on the second order SMC method, can reconstruct a given signal $f(t)$ and its derivative $\dot{f}(t)$ provided certain

$$\begin{cases} \hat{x}_1 = \int \hat{x}_2 d\tau, \\ \hat{x}_2 = \rho_1 \dfrac{(f(t) - \hat{x}_1)}{|f(t) - \hat{x}_1|^{0.5}} + \rho_0 \int \mathrm{sgn}(f(t) - \hat{x}_1)d\tau. \end{cases} \tag{5}$$

If $\rho_0 \geq 4L$, $\rho_1 \geq 0.5L^{0.5}$,

then $\hat{x}_1 \to f(t)$, $\hat{x}_2 \to \dot{f}(t)$ in a finite time, as proved in the work[3]. Since the structure (5) uses the signal state-model

$$\dot{x}_1 = x_2,$$

$$\dot{x}_2 = \ddot{f}(t), \quad \left|\ddot{f}(t)\right| \leq L,$$

to estimate the states x_1, x_2 of a signal, it can be called the state-observer.

In case of the violation of condition $\left|\ddot{f}(t)\right| \leq L$, say,

$$f(t) = f_o(t) + v(t), \quad \left|\ddot{f}_o(t)\right| \leq L, \left|\ddot{v}(t)\right| \gg L, \tag{6}$$

the observer (5) will reconstruct the actual signal $f_o(t)$ with good accuracy, $\hat{x}_1 \to f_o(t)$; however, $\hat{x}_2(t)$ will have significant high-frequency component due to noise, $v(t)$, amplification on the first "high-gain" term in the expression for $\hat{x}_2(t)$ (Eq. (5)). Moreover, because $\hat{x}_2(t)$ cannot follow too aggressive $\dot{f}(t)$, in digital implementations of the observer (5), there will appear so-called "chattering" of term $\hat{x}_2(t)$ due to a finite time-step of the algorithm. However, $\hat{x}_1(t)$ will be smoothening out by an integrator. To estimate $\dot{f}_o(t)$, one can feed $\hat{x}_2(t)$, which contains information about $\dot{f}_o(t)$, to the input of another
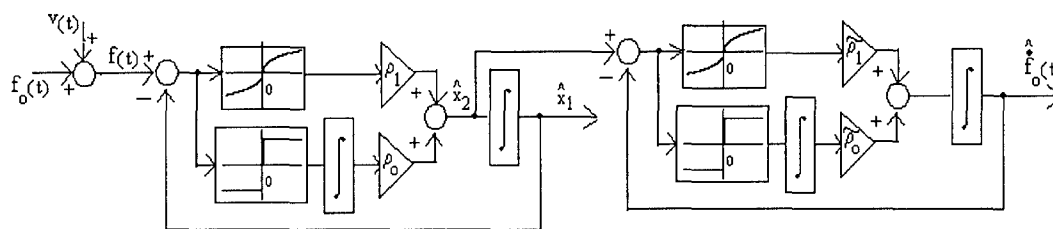

Fig.3 Robust-to-Noise Differentiator

constraint on the signal second derivative $\left|\ddot{f}(t)\right| \leq L$, for known $L > 0$. It has the following structure

observer of the form (5), and reconstruct $\dot{f}_o(t)$, applying certain constraint L_1, $\left|\dddot{f}_o(t)\right| \leq L_1$. The value of L_1 should be a result of a tradeoff between the ripple magnitude and the phase lag in $\dot{f}_o(t)$ estimate.
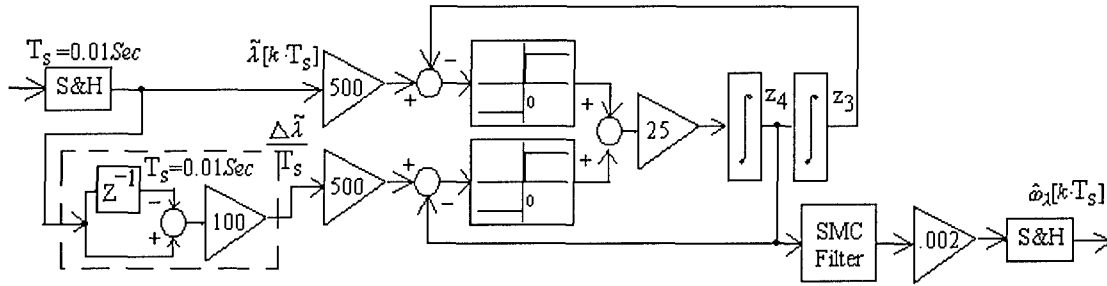
Fig.4 SMC Estimator: Twisting Algorithm

Thus, for noisy signals satisfying (6), two observers connected in series (such that the output $\hat{x}_2(t)$ of the first one is fed into the input of another, and the state $\hat{x}_1(t)$ of the second one is the designated output, as shown in Fig. 3, become a robust-to-noise differentiator. This system will separate a desired signal from noise by the factor of the actual signal second derivative. This important time-domain property of a signal makes this observer/differentiator to be unique among various filters and state-observers, which use statistical or spectral signal properties.

The second order sliding mode scheme presented in (5) is known as the Super Twisting Algorithm developed by Levant[3]. Another SMC estimator, based on Twisting Algorithm by Yemelyanov, Korovin, Levantovsky[2,3], is working as a filter producing estimates of a given signal, $x(t)$, and its derivative, $\dot{x}(t)$, using their noisy measurements

$$y_1(t) = x(t) + v_1(t),$$
$$y_2(t) = \dot{x}(t) + v_2(t). \tag{7}$$

A robust 2-sliding estimator based on Twisting Algorithm is given as

$$\ddot{\hat{x}} = \rho_1 \, \text{sgn}(y_2 - \dot{\hat{x}}) + \rho_0 \, \text{sgn}(y_1 - \hat{x}), \tag{8}$$

In absence of noise, 2-sliding mode in the system (8) provides for $\hat{x} \to x(t)$, $\dot{\hat{x}} \to \dot{x}(t)$ in a finite time if $|\ddot{x}(t)| \le L$. Given noisy signals $y_1(t), y_2(t)$; the high frequency switching of $\text{sgn}(e)$ and $\text{sgn}(\dot{e})$ will reconstruct $x(t)$ and $\dot{x}(t)$ to the extent where $|\ddot{x}(t)| \le L$. So, if $|\ddot{x}(t)| \le L$ and $|\ddot{v}_{1,2}(t)| \gg L$. Then we have a good $x(t), \dot{x}(t)$ recovery in a practical filter of the form (8).

The structure of the LOS rate estimator based on the observer (8) is presented in Fig. 4.

The last structure will be used for LOS rate estimation, and the Super Twisting Algorithm will be employed further for the purpose of compensation for flight control system dynamics via a phase-lead block based on the structure in Fig. 3.

Thus, the following equations of Twisting SMC estimator together with guidance algorithm and flight control system dynamics are used in simulations

$$\dot{z}_1 = 60(z_2 - z_1),$$
$$\dot{z}_2 = 60(\lambda_s - z_2), \quad \text{Analog Prefilter,}$$

$$\tilde{\lambda}[k \cdot T_s] = 500 \, z_1[k \cdot T_s], \quad \text{Sampled LOS,}$$

$$\begin{cases} \dot{z}_3 = z_4, \\ \dot{z}_4 = 25 \cdot \left(\text{sgn}(\tilde{\lambda}[k \cdot T_s] - z_3) + \right. \\ \left. \text{sgn}(\dfrac{\tilde{\lambda}[k \cdot T_s] - \tilde{\lambda}[(k-1) \cdot T_s]}{T_s} - z_4 \right) \end{cases}$$

SMC-differentiator

$$\begin{cases} \dot{z}_5 = 50\left(3|z_4 - z_5|^{\frac{1}{2}} \cdot \text{sgn}(z_4 - z_5) + 4z_6 \right), \\ \dot{z}_6 = \text{sgn}(z_4 - z_5), \end{cases}$$

SMC-filter

$$\hat{\omega}_\lambda = 0.002 \cdot z_5, \quad \text{LOS rate estimate,}$$

$$n_c = N'V_c \hat{\omega}_\lambda, \quad N' = 3; 4,$$

Proportional Navigation Guidance Law,

$$\bar{n}_c[k] = SAT_{966}(n_c[k \cdot T_s]),$$
$$\dot{n}_L = -SAT_{3220}(2 \cdot (n_L - \bar{n}_c[k]))$$

Performance Limits Block ($T = 0.5 \sec$)

$$\text{Function } SAT_L(x) = \begin{cases} L \cdot \text{sgn}(x), & |x| \ge L, \\ x, & |x| < L. \end{cases},$$

$$966 \frac{ft}{\sec^2} \text{ equivalent to 30G,}$$

$$3220 \frac{ft}{\sec^3} \text{ equivalent to 100 G/S.} \tag{9}$$

A digital computer in the homing loop processing measurement information and calculating the guidance command is simulated as given in Fig.5.

flight times comparable to the effective guidance system time-constant. Optimal guidance law to compensate for a single phase lag in flight control system has been derived[1],
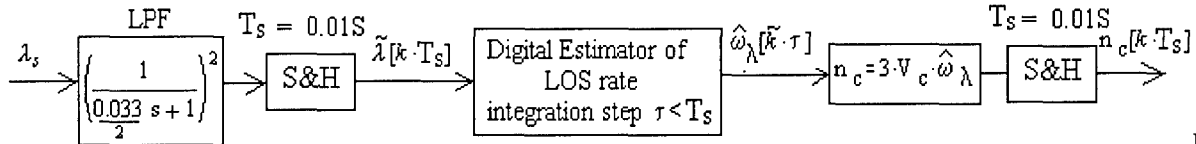
Fig.5 Discrete Part of Guidance System

The sensed continuous signal λ_s being low-pass pre-filtered is coming to the A/D converter represented by S&H (Sample & Hold) element with the sampling time $t_S = 0.01 Sec$. Then, the stepwise constant digital signal $\tilde{\lambda}[k \cdot T_s]$ is used by a discrete guidance algorithm to calculate n_c. The guidance command is stepwise constant, $n_c[k \cdot T_s]$, and it is updated with the same rate on another S&H element representing D/A converter. When a discrete Kalman filter is used to produce necessary estimates, the step of the digital computer algorithm for a discrete Kalman Filter is equal to the sampling time. In case of SMC-based differentiators and filters calulating $\hat{\omega}_\lambda$, the rate of the digital computer algorithm for discrete implementation of SMC-estimators is higher than the sampling rate (SMC-estimators are dynamic between samples).

Advanced Guidance Laws based on Kalman Filter Estimation and Proportional Navigation Guidance with Phase-Lead Network based on SMC Differentiators

One significant difference between Kalman filter and SMC differentiator is that Kalman filter is a three-state estimator, while SMC filter/differentiator estimates only two states: LOS and LOS rate. The third estimates in Kalman filter, target acceleration, is used to implement more advanced guidance laws. Therefore, fair comparison should be made of any available guidance law using Kalman filter with any guidance law that can be implemented using SMC differentiator only. It's known[1] that Proportional Navigation (PN) is optimal for zero-lag guidance system in absence of target maneuvers, while Augmented Proportional Navigation (APN) is optimal for zero-lag guidance system in presence of step-constant target maneuver

$$n_c = N' \left(V_c \omega_\lambda + \frac{1}{2} n_T \right).$$ (10)

However, phase lag in flight control system and the estimator itself is a source of significant miss at short

$$n_c = N' \left(V_c \omega_\lambda + 0.5 n_T - n_L \cdot \frac{(e^{-x} + x - 1)}{x^2} \right),$$

$$N' = \frac{6x^2(e^{-x} + x - 1)}{2x^3 + 3 + 6x - 6x^2 - 12xe^{-x} - 3e^{-2x}}, \quad x = \frac{t_{go}}{T}.$$ (11)

Thus, having additional information about n_L, t_{go} and using estimates $\hat{\omega}_\lambda, \hat{n}_T$ from Kalman filter, one can implement guidance laws (10) and (11).

The main concern of the optimal guidance law (OGL) (11) is to compensate for parasitic single lag dynamics $n_L = \frac{1}{1 + sT} n_c$. Knowing about a phase lag in flight control system, one can design a lead block, $1 + sT$, to compensate for known phase lag, using SMC robust-to-noise differentiator presented in Fig. 3. Using the lead block $1 + sT$, we can expect significant reduction of mean miss at short flight times, although, for the expense of increase in miss standard deviation since differentiation leads to amplification of noisy component. So, an alternative to OGL (11) may be the PN guidance law with a phase lead based on two cascades of SMC differentiators. The idea is to use SMC-differentiator one more time to produce LOS second derivative estimate and create LOS rate estimate with a phase lead as $\hat{\omega}_{\lambda\,lead} = \hat{\omega}_\lambda + 0.5\hat{\omega}_\lambda$. Then, $\hat{\omega}_{\lambda\,lead}$ will be used to compute the PN-guidance command.

Thus, PN guidance law with a phase lead is implemented as follows. Equations (9) are amended with

$$\begin{cases} \dot{z}_7 = \eta_0, \\ \dot{z}_8 = \mathrm{sgn}(z_5 - z_7), \\ \eta_0 = 2\left(8|z_5 - z_7|^{\frac{1}{2}} \cdot \mathrm{sgn}(z_5 - z_7) + 15 z_8\right), \end{cases}$$

$$\begin{cases} \dot{z}_9 = 2\left(8|\eta_0 - z_9|^{\frac{1}{2}} \cdot \mathrm{sgn}(\eta_0 - z_9) + 15 z_{10}\right), \\ \dot{z}_{10} = \mathrm{sgn}(\eta_0 - z_9), \end{cases}$$

$$\hat{\omega}_\lambda = 0.002 \cdot z_5,$$

$$\hat{\dot{\omega}}_\lambda = 0.002 \cdot z_9,$$

$$\hat{\omega}_{\lambda \text{ lead}} = \hat{\omega}_\lambda + 0.5 \cdot \hat{\dot{\omega}}_\lambda,$$

$$n_c = N' V_c \hat{\omega}_{\lambda \text{ lead}}, \quad N' = 4.$$

$$(12)$$

Thus, another important round of numerical tests is to compare performance of APN (10) and OGL (11) based on Kalman filter estimates with the work of PN with a phase lead based on two cascades of SMC robust-to-noise differentiators (9), (12).

EXPERIMENT OBJECTIVE AND SETUP

A flight control system phase lag plus a filter phase lag, plus a phase lag introduced by a sampler altogether with the commanded acceleration limit $|n_c|_{max} = 30g$ produce a peculiar miss distribution pattern under the presence of target maneuvers and measurement noise. We believe that the homing loop model and the selected constraints will highlight performance of both estimators the way we are looking for.

The main objective of the current experiment is to obtain via 50-runs Monte-Carlo statistics for each particular flight time within range $[0.5;10.0]Sec$, for a total of 1000 times, mean miss distance and miss distance standard deviation, using with both estimators and different guidance laws discussed in Section 2.

Certainly, we can consider that the mean miss is mostly due to the target maneuver, while the standard deviation of the miss is due to noise[1]. As far as the initial heading error is zero, we can interpret the flight time t_F as time-to-go at which a constant 6-g target maneuver occur[1]. We can expect that a large miss happens at short flight times due to overall time lag in the guidance system and actual instability of the homing loop.

Computer simulations were accomplished using Microsoft Visual C++ 5.0 and the program code "C9L3.cpp" developed by P. Zarchan[1] for a PN guidance system with Kalman Filter. There were made modifications of this program code for the guidance system with SMC estimator amd different guidance laws.

A case was considered in which there was a constant 6-g target maneuver and 1 miliradian of measurement noise $\sigma_{noise} = 0.001 Rad$. Initial Heading Error is zero. Kalman filter has absolute knowledge of $\max(n_T), t_{go}, R_{TM}, n_L$ and noise standard deviation. The sampling time of measurement and command update is $T_s = 0.01S$ (Kalman filter works at this sampling rate). The integration time step is $\tau = 0.0001S$ (the continuous part of the homing loop and the SMC filter work at this sampling rate). The program "C9L3.cpp" uses Euler integration method.

MONTE-CARLO SIMULATIONS

First, we simulate PN guidance law with the effective navigation gains $N' = 3$ and $N' = 4$ and with LOS rate estimate by Kalman filter and by SMC differentiator. The goal is to observe the sensitivity of the guidance system to a step constant target maneuver with acceleration ratio $\dfrac{n_{c\ LIM}}{n_{T_{max}}} = 5$ and measurement noise in the presence of a single-lag flight control system. The results of simulations are given in Figs. 6-9.

The next set of simulations is to identify what possible advantage SMC differentiatiors can contribute to compensation of a single-lag flight control system as compared to advanced guidance laws using Kalman filter estimates. Performances of APN (10) and OGL (11) based on Kalman filter estimates and PN with a phase lead based on two cascades of SMC robust-to-noise differentiators (9), (12) are compared. The results of simulations are given in Figs.10,11.

Short summary of important benchmarks is presented in Table 1.

Table 1

Guidance Law	Estimator	Peak Mean Miss (Ft)	Steady-state Mean Miss (Ft)	Peak Miss Std. Dev. (Ft)	Steady-state Std. Dev. (Ft)
PN, N'=3	KF	51	0.3	11	1.0
PN, N'=3	SMC	54	0.012	0.7	0.1
PN, N'=4	KF	34.5	0.3	10.8	1.16
PN, N'=4	SMC	32.7	0.01	0.82	0.11
APN, N'=4	KF	29.3	0.25	10.4	1.98
OGL	KF	16.6	0.06	7.7	1.05
PN, N'=4 with Phase Lead	SMC (9) and SMC (12)	14.4	0.001	2.5	0.8

Summary

The guidance system with SMC estimator is more robust to noise than with the optimally tuned Kalman filter. However, it is known that the detuning of Kalman filter may improve performance. On the other hand, performance of Kalman filter will degrade from the values obtained in this experiment if there is noise in the additional information (range to target, time-to-go, missile acceleration). Particular benefits in using SMC methods for missile guidance are

1. The SMC estimators are proved to be effective as robust-to-noise differentiators,

2. SMC estimators require less input information than KF to produce a necessary estimate.

Application of SMC estimators in the homing loop with PN guidance and with a phase-lead compensation for the flight control system phase lag gives performance comparable and actually very close to the optimal guidance law with Kalman filter and perfect knowledge of all additional information it requires for estimation.

CONCLUSIONS

In this work we have compared the performance of Proportional Navigation, Augmented Proportional Navigation, and Optimal Guidance Law with a three-state Kalman filter versus Proportional Navigation with and without a phase-lead compensator employing Higher Order Sliding Mode Observers. It was shown that one could expect the good work of SMC estimators in a missile guidance system. GIven less information requirements for the SMC-based estimators than for Kalman filters, one can expect benefits in applying this technology to a homing interceptor. The next step in this research effort is to create an integrated GNC system for an advanced interceptor using SMC theory, and derive the advanced guidance laws which guarantee robust to target maneuvers intercept under less acceleration ratio requirements.
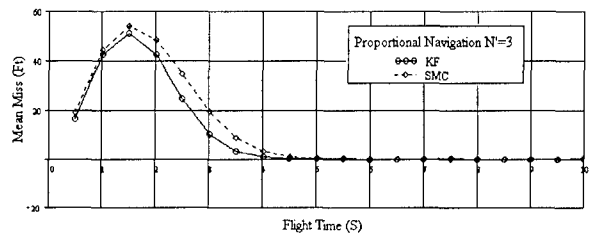

Fig.6 Mean Miss versus Flight Time for PN guidance with KF and SMC observer
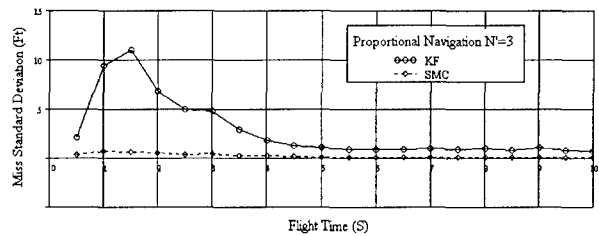

Fig.7 Miss Standard Deviation versus Flight Time for PN guidance with KF and SMC
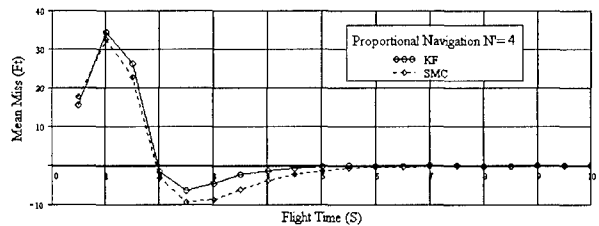

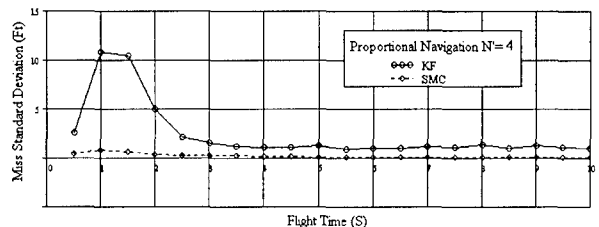Fig.8 Mean Miss versus Flight Time for PN guidance with KF and SMC observer

Fig.9 Miss Standard Deviation versus Flight Time for PN guidance with KF and SMC
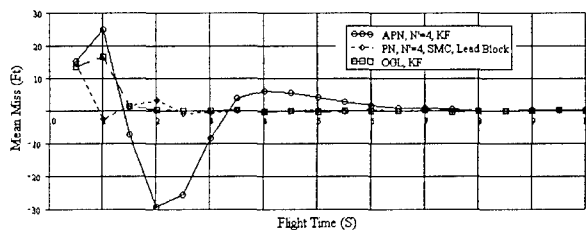
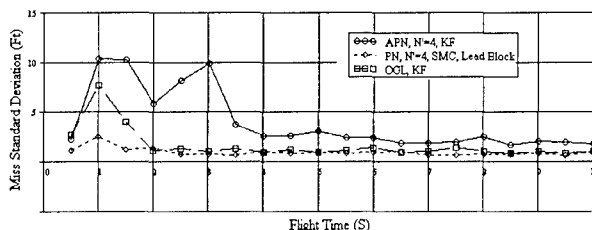Fig.10 Mean Miss versus Flight Time for APN, OGL with KF, and PN with SMC observer and a phase-lead compensator

Fig.11 Miss Standard Deviation versus Flight Time for APN, OGL with KF, and PN with SMC observer and a phase-lead compensator

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