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CALCULATION OF LOWER CONFIDENCE BOUNDS ON SYSTEM RELIABILITY

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ABSTRACT. A general methodology, based on algorithms developed by the Ad-Hoc Methodology Working Group on Nuclear Weapons Reliability Assessment, is described for evaluating 90% lower confidence bounds on system reliability for configurations of series/parallel circuits. General configurations of non-repeated and repeated components are examined and a method for unpooling data is discussed. A technique is derived for representing "m out of n" decision logic gates. The methodology is applied to an example of the type of a sophisticated weapon fuzing system. Maximum likelihood estimates of reliability and 90% lower confidence bounds are calculated for the system and critical components are identified.

1. INTRODUCTION. For critical and expensive weapon systems, such as nuclear projectiles, highly reliable subsystems are required to produce a high probability of successful system performance. Not only must the reliability of these integral subsystems be very high, but, since often relatively few of such weapon systems will be used to attack an enemy target, there must also be a high degree of confidence that such reliability will be achieved. This report describes a general methodology for calculating maximum-likelihood estimates of reliability as well as 90-percent lower confidence bounds on the system reliability for general systems representable as configurations of series/parallel circuits.

In testing these weapon systems, because of the scarcity and cost of some of the components, the tester must be quite selective in the number and type of subsystems to be included in field tests. An important byproduct of the methodology to be presented is that it evinces those components that are critical, in that they constrain the lower confidence bounds, and those that are not. Therefore, it would be highly cost-effective to schematize the system in the format of this methodology before testing has begun, so that the test director can effectively allocate his test resources to the critical components.

The next section discusses the methodology for calculating confidence bounds on circuit system reliability in a completely general way. It is hoped that this section will serve as a handy reference to the analyst who desires to make confidence-bound determinations for many types of circuit systems. For example, the methodology should be readily applicable to various kinds of sensors, radars, and missile guidance systems. In later sections, the methodology is applied to a system of the type of an actual weapon fuzing system. Based on simulated test data, 90-percent lower confidence bounds on system reliability are calculated and critical components are identified.

2.1 EVALUATION OF CONFIDENCE BOUNDS. By a 90-percent lower confidence bound on system reliability is meant a statistic computed from the test data with the property that there is at least a probability of 0.90 that this statistic is lower than the unknown system reliability. Under the assumption that tests on a component are binomial, that is, the tests are independent with constant failure probability, the 90-percent lower confidence bound, LCB_{90}, on component reliability is computed as follows when the test data indicate N tests with F failures:

$$LCB_{90} = B_{90}(N,F) = 1 - p \quad , \tag{1}$$

where p satisfies the binomial relationship

$$\sum_{i=F+1}^{N} \binom{N}{i} p^{i}(1 - p)^{N-i} = 0.90$$

or, equivalently,

$$\sum_{i=0}^{F} \binom{N}{i} p^{i}(1 - p)^{N-i} = 0.10 \quad .$$

These formulas assume that N and F are integers; in calculating equivalent components later, there will be a need for evaluating 90-percent lower confidence bounds when N and/or F are not integral. In this case, the following linear interpolation formula is useful:

$$\begin{aligned} B_{90}(N,F) \approx\ & (1 - N_D)[(1 - F_D)B_{90}(N_I,F_I) + F_D B_{90}(N_I,F_I + 1)] \\ & + N_D[(1 - F_D)B_{90}(N_I + 1,F_I) + F_D B_{90}(N_I + 1,F_I + 1)] \end{aligned} \tag{2}$$

where

$N_I = [N]$, the integer part of N,
$N_D = N - [N]$,
$F_I = [F]$, the integer part of F, and
$F_D = F - [F]$.

Tables [1] are available from which binomial confidence bounds can be read for N = 1 to 150. In the calculation of equivalent-system lower confidence bounds later in this paper, we shall frequently encounter very large values of N together with very small values of F. To obtain such lower confidence bounds, the Poisson approximation to the binomial is used when N > 150 and F < 10. As long as F is an integer, regardless of whether or not N is an integer, the Poisson estimate is given by

$$P_{90}(F) \approx \frac{1}{2} \chi^2_{0.90}(2F + 2) \quad , \tag{3}$$

where $\chi^2_{0.90}(2F + 2)$ denotes the 90th percentile of a chi-square distribution with 2F + 2 degrees of freedom. Tables of the chi-square percentiles can be found in many statistics textbooks (see [2]). When F is not an integer, $P_{90}(F)$ may be calculated by linear interpolation:

$$P_{90}(F) \approx (1 - F_D)P_{90}(F_I) + F_D P_{90}(F_I + 1) \quad . \tag{4}$$

In either case, the 90-percent lower confidence bound is then estimated from the formula

$$LCB_{90} = B_{90}(N,F) = 1 - \frac{P_{90}(F)}{N} \quad . \tag{5}$$

Another useful formula for calculating component lower confidence bounds arises from the observation that, when F = 0 in equation (1), we have

$$[B_{90}(N,0)]^N = (1 - p)^N = 0.10$$

which leads to the exact solution

$$B_{90}(N,0) = (0.10)^{1/N} \quad . \tag{6}$$

It should be clear that all the preceding formulas can be readily extended to the computation of component lower confidence bounds at other than the 90-percent level.

The next several sections describe techniques for calculating lower confidence bounds for general series and parallel configurations of components. These procedures are taken from those recommended by a special Working Group chaired by the Army Materiel Systems Analysis Activity [3].

2.2 CALCULATION OF CONFIDENCE BOUNDS FOR A SERIES SYSTEM OF NONREPEATED COMPONENTS. The simplest case is a system whose configuration consists of a series arrangement of independent components, as exemplified in figure 1.

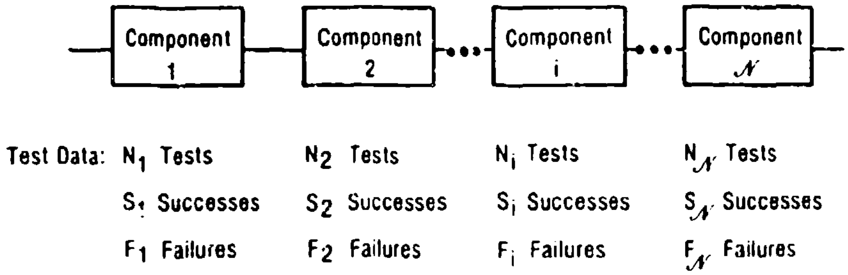


Figure 1. Series system of nonrepeated components.

None of the $\mathcal{N}$ components in this series are repeated; that is, all are independently functioning components which appear only once and have specific test data in terms of observed successes and failures. Note that $S_i + F_i = N_i$ for all values of i.

The lower confidence bound on the reliability of this series is obtained by reducing the combination to an equivalent component. This is done by means of the Lindstrom-Madden method [4] which calculates the maximum-likelihood estimate of the system reliability, R_s, by the formula

$$R_s = \prod_{i=1}^{\mathcal{N}} \frac{S_i}{N_i} \tag{7}$$

and takes the equivalent number of tests, N, for the system to be

$$N = \min_{1 \le i \le \mathcal{N}} N_i \ . \tag{8}$$

The equivalent number of successes and failures of the system, S and F respectively, are then given by

$$S = NR_s \ , \tag{9}$$

$$F = N(1 - R_s) \ . \tag{10}$$

Thus the series combination is now represented by a single equivalent component with S successes and F failures out of N tests. The 90-percent lower confidence bound for the series combination can now be computed by the methods of section 2.1.

For example, consider the three components in series in figure 2.

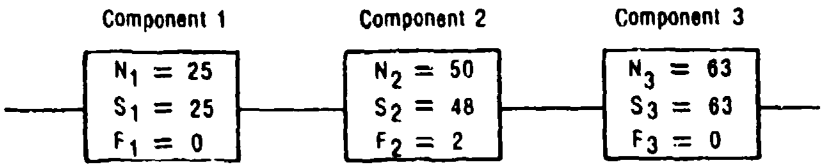


Figure 2. Example of series system.

The computational procedure gives the following:

Step 1. $R_s = 1 \times \frac{48}{50} \times 1 = 0.96$

Step 2. $N = \min(25, 50, 63) = 25$

Step 3. $S = 0.96(25) = 24$
$F = 0.04(25) = 1$

Step 4. $LCB_{90} = B_{90}(25, 1) = 0.853$ (from eq (1) and table lookup)

2.3 CALCULATION OF CONFIDENCE BOUNDS FOR A PARALLEL SYSTEM OF NONREPEATED COMPONENTS. For a system configured as in figure 3 with independent, nonrepeated components in parallel, an equivalent single component is again derived. The equivalent number of tests, N, is computed from the equation

$$N = \frac{1 - Q'}{Q' - Q} , \tag{11}$$

where

$$Q = \prod_{i=1}^{\mathcal{M}} \frac{F_i}{N_i} ,$$

$$Q' = \prod_{i=1}^{\mathcal{M}} \frac{F_i + 1}{N_i + 1} ,$$

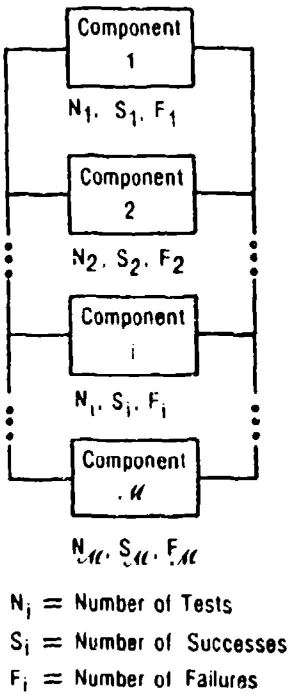


Figure 3. Parallel system of nonrepeated components.

and the maximum likelihood estimate of the system reliability is then given by:

$$R_s = 1 - Q \quad . \tag{12}$$

The equivalent numbers of successes and failures are then derived:

$$S = NR_s , \tag{13}$$

$$F = NQ \quad . \tag{14}$$

The 90-percent lower confidence bound for the system reliability can now be computed as that for the equivalent single component with F failures out of N tests.

An example of a parallel system is given in figure 4. The computational steps proceed as follows:

Step 1. $Q = 0 \times \frac{2}{20} \times \frac{1}{30} = 0$

Step 2. $Q' = \frac{1}{11} \times \frac{3}{21} \times \frac{2}{31} = 0.000838$

Step 3. $N = \frac{1 - Q'}{Q' - 0} = 1192.5$

Step 4. $R_s = 1$

Step 5. $S = 1192.5$

$F = 0$

Step 6. $LCB_{90} = B_{90}(1192.5,\ 0) = (0.10)^{1/1192.5} = 0.9981$ (from eq (6))

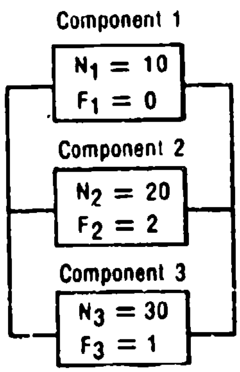


Figure 4. Example of parallel system.

2.4 CONSTRUCTION OF AN EQUIVALENT COMPONENT WITH SPECIFIED RELIABILITY AND CONFIDENCE BOUND. In reducing a complex combination of components to an equivalent single component, a sequence of substitutions may be required. It may occur, in the techniques to be developed in subsequent sections, that some reductions will calculate the maximum-likelihood estimate of reliability as well as the lower confidence bound for a subsystem without specifying the equivalent test data. Therefore, it will be useful in the sequel to have a technique for constructing the equivalent test data for a subsystem when given only the maximum-likelihood estimate of reliability, R, and the 90-percent lower confidence bound, B_{90}.

The technique for solving for the equivalent number of tests, N, and failures, F, given R and B_{90}, is actually just the solution of the following two equations in two unknowns:

$$R = 1 - \frac{F}{N} \ ,$$

$$B_{90} = B_{90}(N,F) \ .$$

However, the second of these equations cannot be solved explicitly, so an iterative approach to solution is used.

The iteration begins with an initial estimate of N, denoted by N_1, calculated from the formula

$$N_1 = \frac{\ln 0.10}{\ln B_{90}} \quad . \tag{15}$$

If the reliability estimate R is equal to 1, set $N = N_1$ and $F = 0$. If not, an estimate of F, denoted by F_1, is obtained from

$$F_1 = (1 - R)N_1 \quad , \tag{16}$$

and the confidence bound $B_{90}(N_1, F_1)$ is determined by the techniques in section 2.1. An adjustment factor given by

$$t = \frac{\ln B_{90}(N_1, F_1)}{\ln B_{90}} \tag{17}$$

is used to obtain the next estimate of N, denoted by N_2:

$$N_2 = tN_1 \quad . \tag{18}$$

If the adjustment factor is near enough to 1 (i.e., $|t - 1| < 0.01$), then use $N = N_2$ and $F = (1 - R)N_2$ as the equivalent test data. If not, N_2 is taken as the estimate of N and the above process (eq (16) through (18)) is repeated until the adjustment factor converges close enough to 1, resulting in the equivalent values of N and F. This procedure is illustrated in the next section.

2.5 CALCULATION OF CONFIDENCE BOUNDS FOR A SYSTEM CONSISTING OF ONLY A SINGLE COMPONENT REPEATED IN ANY CONFIGURATION. This section describes the methodology to be used for calculating confidence bounds for a system or subsystem which is a combination of series and/or parallel circuits composed solely of repetitions of the same component. More precisely, the components, although separate physical devices, are the same in the sense that they are of the same generic type and are described by the same test data.

Suppose the system to be analyzed is a series/parallel configuration consisting of repetitions of a component, C, characterized by test data indicating F_c failures in N_c tests. The maximum-likelihood estimate for the reliability of the component, C, is given by

$$R_c = 1 - \frac{F_c}{N_c} \quad . \tag{19}$$

Analysis of the system into its series and parallel branches of C components gives rise to a reliability estimate for the system which is a function of R_c:

$$R_s = f(R_c) \quad . \tag{20}$$

For example, if the configuration consisted of n components C in series, $f(R_c)$ would be R_c^n, whereas if the configuration were n components C in parallel, $f(R_c)$ would be $1 - (1 - R_c)^n$.

To calculate confidence bounds for the general series/parallel configuration of C components, the methodology begins by evaluating the 90-percent lower confidence bound for C:

$$LCB_c = B_{90}(N_c, F_c) \quad .$$

The 90-percent lower confidence bound for the system, LCB_s, is then calculated by means of the function in equation (20):

$$LCB_s = f(LCB_c) \quad . \tag{21}$$

Therefore, we have obtained the maximum-likelihood estimate of reliability (eq (20)) and the 90-percent lower confidence bound (eq (21)) for the system. Equivalent test data for the system (that is, N_s and F_s) can now be calculated by the method of section 2.4.

For the special case where the configuration of the system is just a series arrangement of n repeats of C and where F_c is small compared to N_c (that is, $F_c < N_c/10$), two simple but accurate approximations for N_s and F_s are available. Both of these approximations are conservative in that they tend to underestimate N_s:

Approximation 1:

$$N_s = \frac{N_c}{n} \tag{22}$$

$$F_s = (1 - R_s)N_s \tag{23}$$

Approximation 2:

$$N_s = \frac{F_c}{1 - R_s} \tag{24}$$

$$F_s = (1 - R_s)N_s = F_c \tag{25}$$

Note that the second approximation cannot be used when $R_s = 1$ (or, equivalently, $F_c = 0$), but in this case the first approximation yields exactly the same values as the general method in section 2.4, since

$$N_s = N_1 = \frac{\ln 0.10}{\ln LCB_s} \qquad \text{from equation (15)}$$

$$= \frac{\ln 0.10}{\ln LCB_c^n}$$

$$= \frac{\ln 0.10}{n \ln LCB_c}$$

$$= \frac{\ln 0.10}{n \ln (0.10)^{1/N_c}} \qquad \text{from equation (6)}$$

$$= \frac{N_c}{n} \ .$$

These approximations are often useful in the reduction of a complex system with series subsystems to an equivalent system.

As an example, consider the series/parallel configuration in figure 5, where $N_c = 15$ and $F_c = 1$. Computations proceed as follows:

Step 1. $R_c = 1 - \frac{1}{15} = 0.93333$

Step 2. $R_s = [1 - (1 - R_c)^2][1 - (1 - R_c)^3] = 0.99526$

Step 3. $LCB_c = B_{90}(15,\ 1) = 0.7643$

Step 4. $LCB_s = [1 - (1 - LCB_c)^2][1 - (1 - LCB_c)^3] = 0.93206$

Step 5. The iterative method of section 2.4 with $R = 0.99526$ and $B_{90} = 0.93206$ then gives the following table, where the $B_{90}(N_1,F_1)$ values are obtained by the interpolation formula (2).

Iteration	N_1	F_1	$B_{90}(N_1,F_1)$	t	N_2
1	32.73	0.155	0.9247	1.112	36.42
2	36.42	0.173	0.9314	1.011	36.81
3	36.81	0.174	0.9320	0.9997	36.80

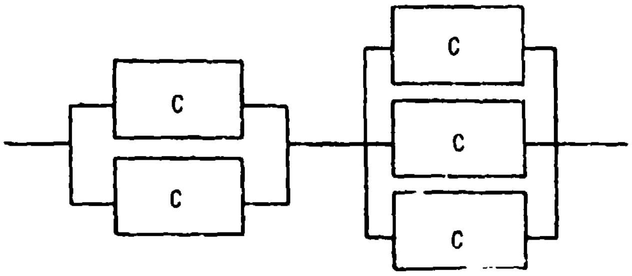


Figure 5. Series/parallel configuration.

Consequently, the equivalent test data for the system are given by

$$N_s = 36.80$$

$$F_s = (1 - R_s)N_s = 0.174 \quad .$$

2.6 CALCULATION OF CONFIDENCE BOUNDS FOR GENERAL CONFIGURATIONS--METHOD FOR UNPOOLING DATA. The techniques discussed so far permit the calculation of lower confidence bounds on system reliability for series/parallel systems of independent, nonrepeated components, as well as for systems which contain repeated component types, as long as each repeated component appears in only one subsystem. In order to handle configurations in which repeated components are distributed throughout several subsystems in combination with other repeated or nonrepeated components, a method will be described for unpooling the data for repeated components. This method divides the component test data into groups corresponding to the various subsystems in which the component appears, and then treats the component as distinct and independent within each subsystem. It has been found that such unpooling schemes provide somewhat conservative lower confidence bounds on reliability.

The basic idea behind the unpooling method is as follows. Suppose C is a component, with test data indicating F_C failures in N_C tests, which occurs in n subsystems, where the subsystems are chosen to each contain as many appearances of C as possible and still be analyzable by the techniques of sections 2.2 through 2.5. Thus each subsystem either contains just one appearance of C or, if it contains two or more appearances, that portion of the subsystem can be reduced to a configuration composed of repetitions of a single equivalent component. The component C will be relabeled as C_1, C_2, ..., C_n, respectively, for each of the n subsystems in which it appears. The test data for C is then allocated over the n subsystems in such a way as to keep the maximum-likelihood estimate of reliability for each C_i, i = 1, 2, ..., n, equal to that for C. That is, the constraints on the unpooling are

$$\sum_{i=1}^{n} F_{C_i} = F_C \quad ,$$

$$\sum_{i=1}^{n} N_{C_i} = N_C \quad ,$$

$$\frac{F_{C_i}}{N_{C_i}} = \frac{F_C}{N_C} \quad , \qquad \text{for } i = 1, 2, \ldots, n \quad .$$

There are many ways of unpooling which satisfy these constraints. The method used here unpools according to the following scheme:

(1) Unpool equally in a series direction.
(2) Then unpool equally in a parallel direction.
(3) Then unpool equally in a series direction.
etc

This sequence is best illustrated by an example, as shown in figure 6. In this system the component C appears in four subsystems and has been relabeled accordingly. The first step of the unpooling would allocate $N_c/2$ and $F_c/2$ to C_1 and the other $N_c/2$ and $F_c/2$ to the parallel combination. Since there are two branches in parallel, the second step of the unpooling would divide in half the equivalent test data for the parallel combination, thus allocating $N_c/4$ and $F_c/4$ to C_4 and the other $N_c/4$ and $F_c/4$ to the series combination containing C_2 and C_3. In turn the third step of the unpooling allocates $N_c/8$ and $F_c/8$ to each of C_2 and C_3. In summary, the unpooled test data for each appearance of C would be as follows.

Component	Test data	
	N	F
C_1	$N_c/2$	$F_c/2$
C_2	$N_c/8$	$F_c/8$
C_3	$N_c/8$	$F_c/8$
C_4	$N_c/4$	$F_c/4$
Total:	N_c	F_c

After unpooling, each of the C_i's is treated as a separate, independent component and the techniques in sections 2.2 through 2.5 are applied, as appropriate, to each of the subsystems.

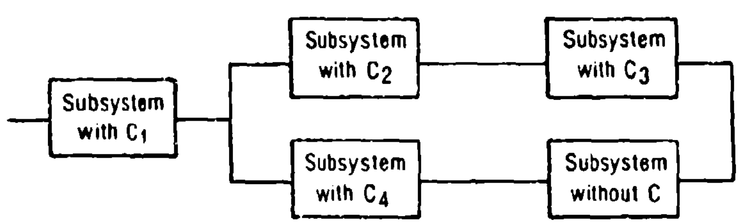


Figure 6. Example of unpooling scheme

2.7 REPRESENTATION OF "TWO OUT OF THREE" DECISION GATE. Synthesis of the techniques presented so far enables one to calculate lower confidence bounds on reliability for any series/parallel configuration. However, many of the sophisticated circuits of today contain other configurations, such as decision gates, to gain greater reliability and efficiency. A straightforward procedure will be formulated to handle decision gates by approximate equivalent combinations of series and parallel circuits.
The methodology will be illustrated for a "two out of three" decision gate; the extension to general "k out of m" decision logic gates should be clear.

First, observe that for a series combination of components C_1, C_2, ..., C_K, with component failure probabilities Q_{C_1}, Q_{C_2}, ..., Q_{C_K}, the failure probability of the combination, Q, is given by

$$Q = 1 - \left(1 - Q_{C_1}\right)\left(1 - Q_{C_2}\right) \cdots \left(1 - Q_{C_K}\right)$$
$$= Q_{C_1} + Q_{C_2} + \ldots + Q_{C_K} + \text{second and higher order terms} .$$

Mission reliability equations for modern weapon systems typically neglect the second and higher order terms and simply add together failure probabilities of components in series. On the other hand, if C_1, C_2, ..., C_K were in parallel, the failure probability for the system would be, simply,

$$Q = Q_{C_1} Q_{C_2} \cdots Q_{C_K} .$$

For a decision gate configuration which requires success in two (or more) of the three branches (with each branch consisting of the same component C) for a YES vote, the probability of failure, Q, of the gate (i.e., a NO vote) is given by

$$Q = \text{probability that 2 or 3 branches fail}$$
$$= 3Q_C^2 + Q_C^3 ,$$

where Q_C is the failure probability of the component C. In terms of failure probability, the decision gate is, therefore, approximately equivalent to the series/parallel combination shown in figure 7, which has a failure probability given by

$$Q = Q_C^2 + Q_C^2 + Q_C^2 + Q_C^3 + \text{fourth and higher order terms} ,$$

Since the terms omitted by introducing this approximation are two orders less than those already typically neglected in the mission reliability equation, this series/parallel combination should afford a sufficiently accurate representation of the decision gate.

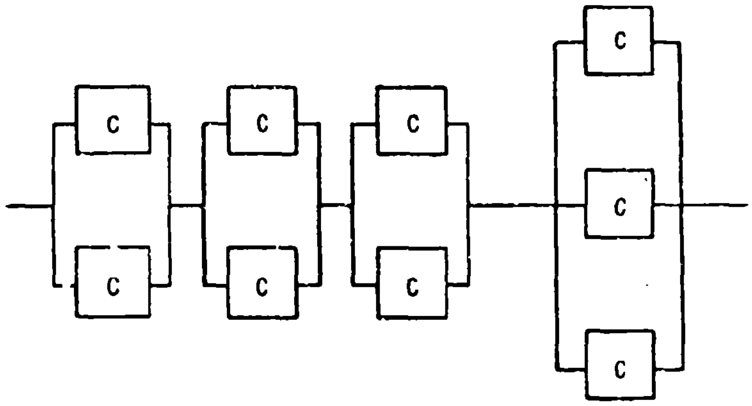


Figure 7. Decision gate approximate equivalent combination.

3. CASE STUDY - APPLICATION OF THE METHODOLOGY. The methodology described in the previous sections will now be applied to an example based on the actual fuzing system of a battlefield weapon. The system schematic, shown in figure 8, is at the same level of sophistication as the fuzing system. However, for purposes of keeping this report unclassified, a few modifications have been made to the actual schematic and simulated component test data is used. Note the "two out of three" decision gate equivalent in the upper right hand part of the system schematic in figure 8. The simulated component test data is displayed in table 1. For some of the components only a reliability value, R, is available, presumably based on a large number of tests by the manufacturer; such components are denoted by an asterisk in figure 8.

The lower-confidence-bound computation for this system will proceed through two reductions of the system, unpooling into subsystems and calculation of equivalent components, and then the calculation of the system lower confidence bound itself. In the process, components critical to the confidence-bound assessment will be evinced and pertinent observations made.

In the first reduction many of the series combinations which are repeated in a particular type of configuration throughout the system schematic are simplified. The computations are sketched in appendix A. Note that those components which have reliability estimates only are treated as having essentially an infinite number of trials; thus they do not affect the calculation of the equivalent component N (number of trials) but only the calculation of the equivalent component R (reliability). After the first reduction, the system schematic takes the form shown in figure 9.

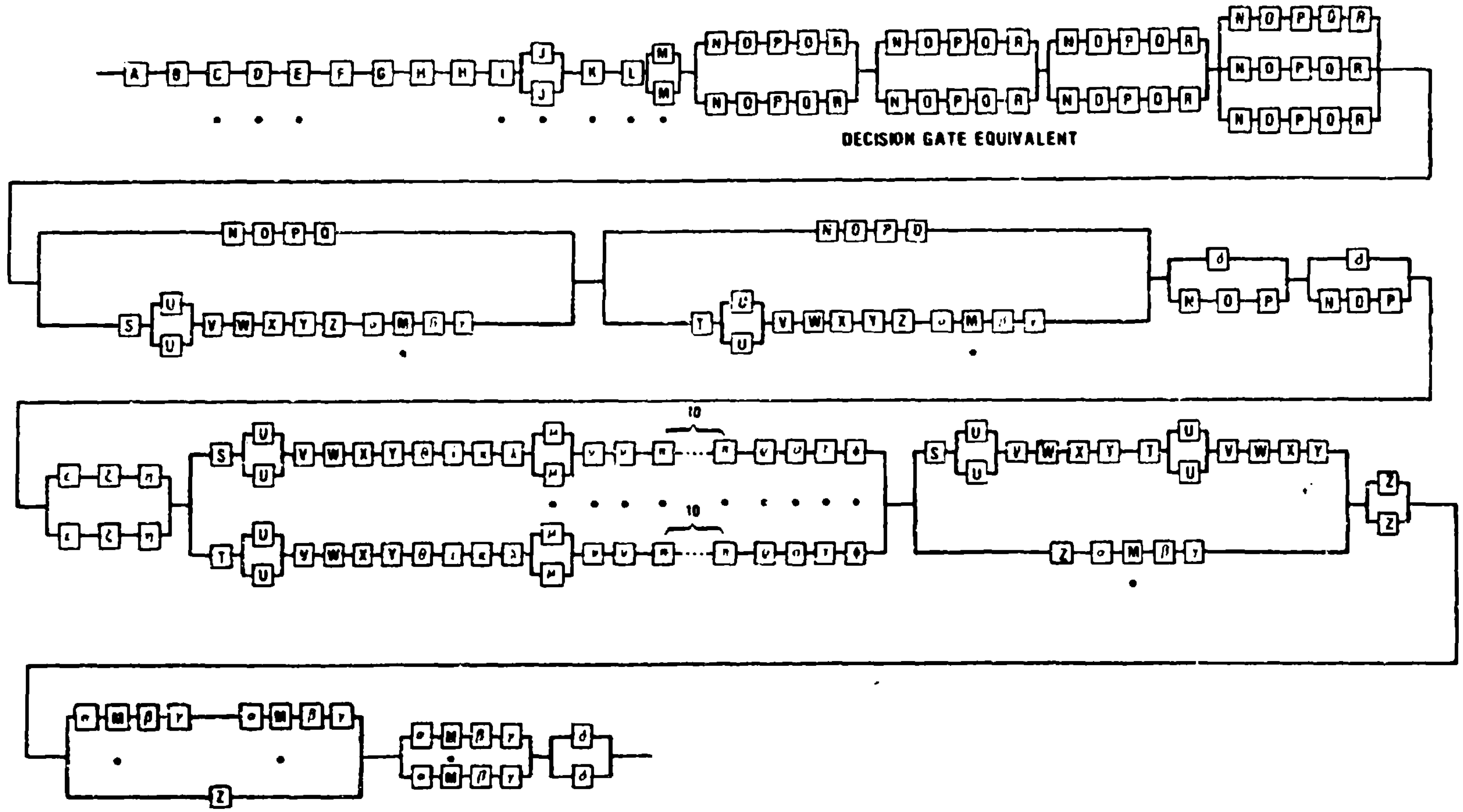


FIGURE 8. SYSTEM SCHEMATIC

TABLE 1. SIMULATED COMPONENT TEST DATA

Component	Number of Trials	Number of Failures
A	5000	0
B	201	0
C	R = .995	
D	R = .994	
E	R = .9997	
F	192	0
G	192	0
H	401	0
I	R = .9978	
J	R = .97	
K	R = .999	
L	R = .999	
M	R = .999	
N	573	0
O	573	0
P	573	0
Q	570	1
R	572	0
S	250	15
T	328	1
U	384*	0
V	384	0
W	383	0
X	383	0
Y	384	1
Z	92	0
α	401	0
β	1260	0
γ	1260	0
δ	384	0
ε	382	0
ζ	382	3
η	381	0
θ	381	0
ι	399	0
κ	371	0
λ	375	0
μ	R = .992	
ν	R = .998	
π	R = .9998	
ρ	R = .9999	
σ	R = .9995	
τ	R = .9995	
φ	R = .9982	

* For a parallel pair of U components

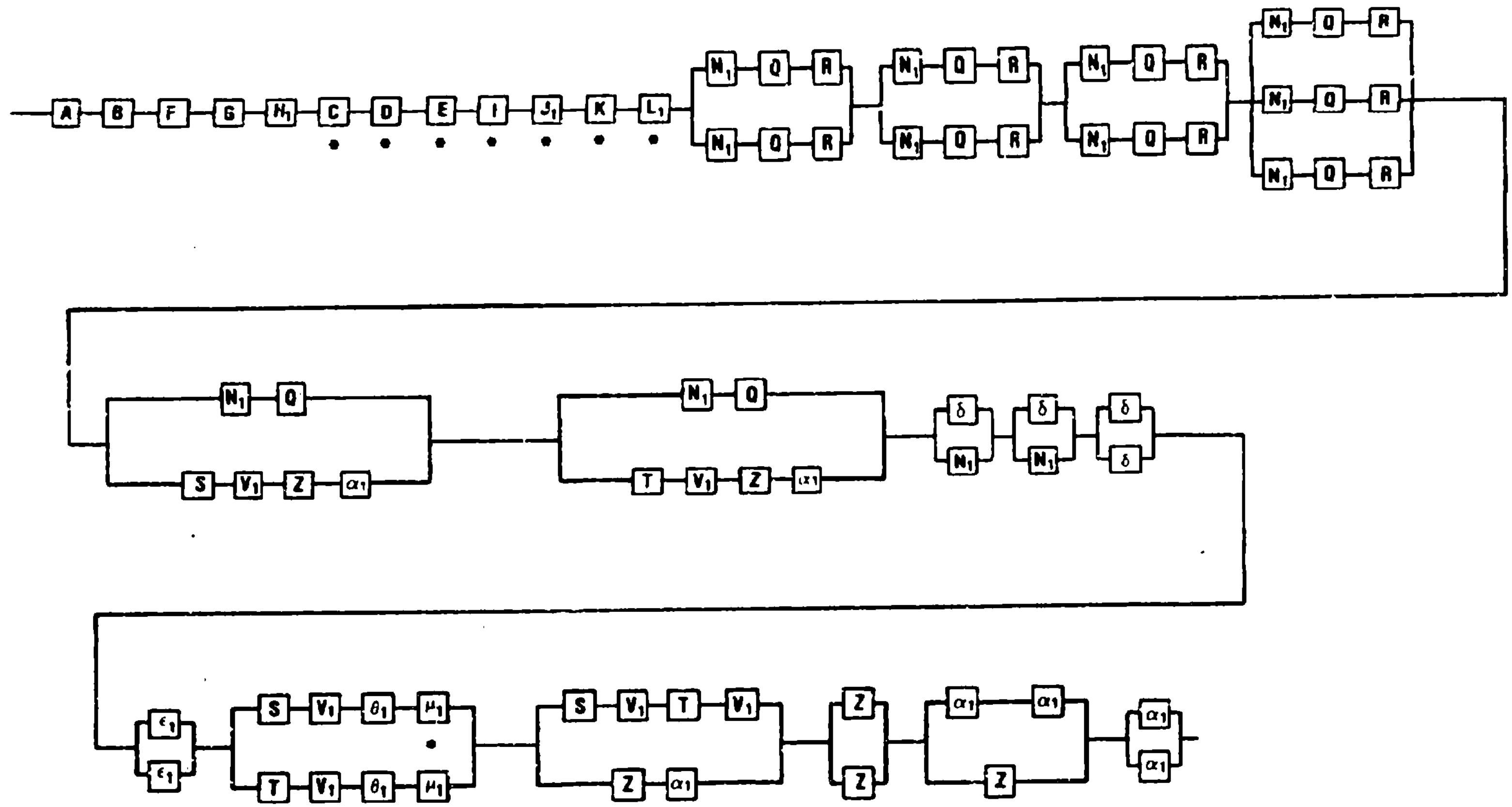


FIGURE 9. SYSTEM SCHEMATIC — AFTER FIRST REDUCTION

The primary function of the second reduction is to consolidate the long series of components at the beginning of the schematic in figure 9. After these substitutions are made, the reduced schematic assumes the tractable form shown in figure 10. Details of the reduction procedure are given in appendix B.

The reduced schematic, divided into subsystems as shown in figure 10, can now be treated by applying the methodology developed previously to each of the numbered subsystems and then determining the equivalent N and R for the overall series configuration of subsystems. However, the data must first be unpooled for components which appear in more than one subsystem. The components which appear in the reduced schematic, before unpooling, are listed in table 2 along with their equivalent test data. The equivalent test data after unpooling are shown in table 3. Note that those components which appear in more than one subsystem have had an extra subscript appended to indicate those repetitions. (For example, V_{13} refers to the third distinct appearance of V_1, in the top branch of subsystem 6.)

The equivalent number of trials, N, and the maximum-likelihood reliability estimate, R, are computed, subsystem by subsystem, in appendix C and tabulated in table 4. Since the overall system configuration is now represented as a series combination of these subsystems, the maximum-likelihood estimate of the overall system's reliability is just the product of the subsystem reliabilities (R = 0.9824), and the equivalent number of trials is the minimum of those for the subsystems (N = 165). This minimum number (indicated by an asterisk in table 4) corresponds to the critical subsystem--that which delimits the equivalent number of trials. Note how only a few subsystems, and thus only a few components, may determine the calculation of the confidence bound. Examination of the critical subsystem 8 identifies the critical component of the overall system (i.e., that component for which additional test data could increase the equivalent number of trials for the overall system and hence improve the resulting lower confidence bound), to be the Z component.

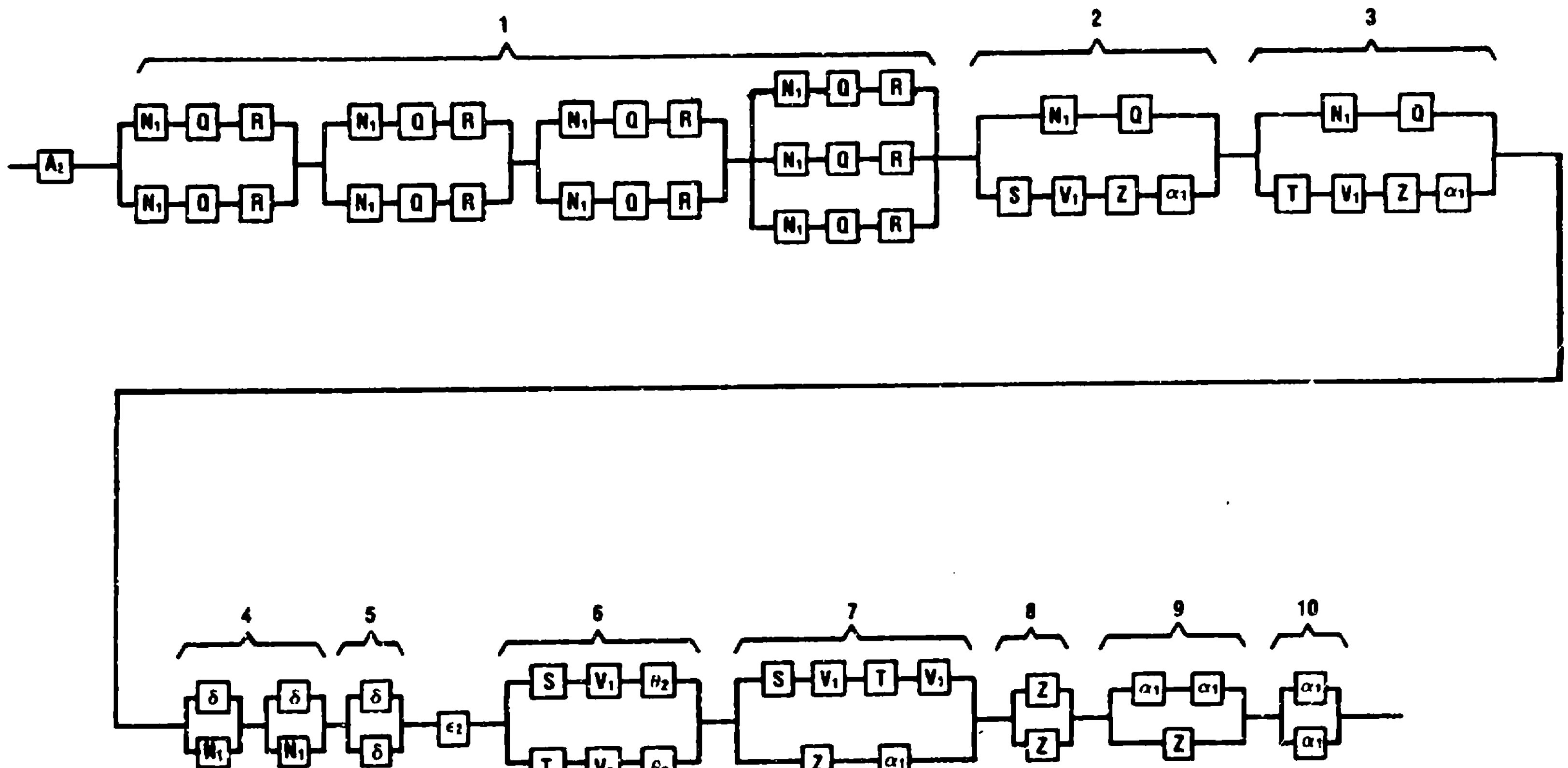


FIGURE 10. REDUCED SCHEMATIC

TABLE 2. EQUIVALENT TEST DATA — BEFORE UNPOOLING

Component	Number of Trials	Number of Failures
A_2	192	3.130
N_1	573	0
Q	570	1
R	572	0
S	250	15
T	328	1
V_1	383	0.997
Z	92	0
α_1	401	0.401
δ	384	0
ϵ_2	10,961	0.680
θ_2	371	3.313

TABLE 3. EQUIVALENT TEST DATA — AFTER UNPOOLING

Component	Number of Trials	Number of Failures
A_2	192	3.130
N_{11}	143.25	0
N_{12}	143.25	0
N_{13}	143.25	0
N_{14}	143.25	0
Q_1	190	0.3333
Q_2	190	0.3333
Q_3	190	0.3333
R	572	0
S_1	83.33	5
S_2	83.33	5
S_3	83.33	5
T_1	109.33	0.3333
T_2	109.33	0.3333
T_3	109.33	0.3333
V_{11}	95.75	0.24925
V_{12}	95.75	0.24925
V_{13}	47.875	0.124625
V_{14}	47.875	0.124625
V_{15}	95.75	0.24925
Z_1	18.4	0
Z_2	18.4	0
Z_3	18.4	0
Z_4	18.4	0
Z_5	18.4	0

TABLE 3. EQUIVALENT TEST DATA — AFTER UNPOOLING (CONT'D)

Component	Number of Trials	Number of Failures
α_{11}	80.2	0.0802
α_{12}	80.2	0.0802
α_{13}	80.2	0.0802
α_{14}	80.2	0.0802
α_{15}	80.2	0.0802
δ_1	192	0
δ_2	192	0
ε_2	10,961	0.680
θ_{21}	185.5	1.6565
θ_{22}	185.5	1.6565

TABLE 4. SUMMARY OF SUBSYSTEM DATA

Subsystem	R	Equivalent N
A_2	0.98370	192
ε_2	0.999938	10,961
1	0.999991	2,158
2	0.9998891	1,166
3	0.9999884	2,040
4	1	13,919
5	1	15,989
6	0.9989715	478
7	0.9999323	272
8	1	165
9	1	737
10	0.999999	2,586
System	0.9824	165

R = Maximum-likelihood estimate of reliability

N = Number of trials

The data are now in place to calculate the 90-percent lower confidence bound on the reliability of this example system. We have:

$$R = 0.9824$$

$$N = 165$$

$$F = (1-R)N = 2.904$$

The interpolation formula (2) and the Poisson estimate (3) give the 90-percent lower confidence bound:

$$LCB = 0.096\ B_{90}(165,\ 2) + 0.904\ B_{90}(165,\ 3)$$

$$= 0.096\left(1 - \frac{\frac{1}{2}\ \chi^2_{0.90}(6)}{165}\right) + 0.904\left(1 - \frac{\frac{1}{2}\ \chi^2_{0.90}(8)}{165}\right)$$

$$= 0.096(0.96788) + 0.904(0.95939)$$

$$= 0.9602$$

Note that R is a point estimate of the reliability of the system, whereas the lower confidence bound is a bound on the unknown actual system reliability, not on the point estimate.

In summary, the general methodology described in this report has been utilized to estimate the system reliability of a practical weapon system design, to obtain a 90-percent lower confidence bound on the system reliability, and to determine those system components which are prime candidates for further design tests.

REFERENCES

1. Engineering Design Handbook, Tables of Cumulative Binomial Probabilities, AMCP706-109, HQ U.S. Army Materiel Command (June 1972).

2. G. Hahn and S. Shapiro, Statistical Models in Engineering, J. Wiley & Sons, New York (1968).

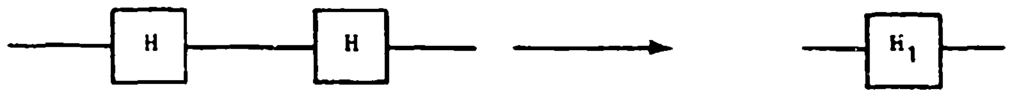
3. Handbook for the Calculation of Lower Statistical Confidence Bounds on System Reliability Assessment, Ad-Hoc Methodology Working Group on Nuclear Weapons Reliability Assessment (February 1980).

4. D. K. Lloyd and M. Lipow, Reliability: Management, Methods, and Mathematics, Prentice Hall, New Jersey (1962).

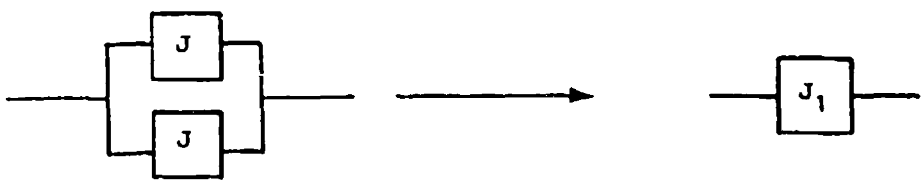
FIRST REDUCTION

In each replacement of a subsystem, shown in figure 8 in the body of the report, by an equivalent component, both the original subsystem and the new equivalent component will be depicted. The methodology used for the reduction will be referred to by the appropriate section in the body of the report. The symbols N, F, R, and Q will be used throughout to denote number of tests, number of failures, maximum likelihood reliability estimate, and failure probability, respectively.

Original subsystem — Equivalent component



$N_H = 401$, $F_H = 0$, $R_H = 1$ — (method of sect. 2.5, eq (22)) — $N_{H_1} = 200.5$, $F_{H_1} = 0$



$Q_J = 0.03$ — $Q_{J_1} = (0.03)^2 = 0.0009$

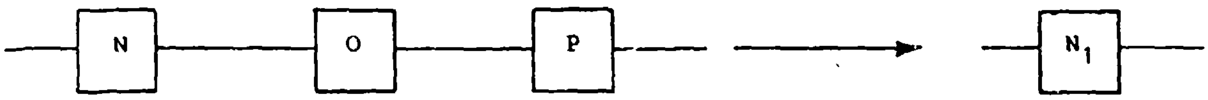
Original subsystem | Equivalent component

$Q_L = 0.001$

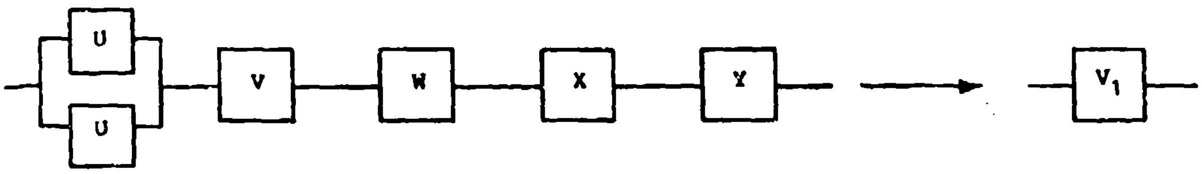
$Q_M = 0.001$

$R_{L_1} = (0.999999)\ (0.999)$

$Q_{L_1} = 0.001001$



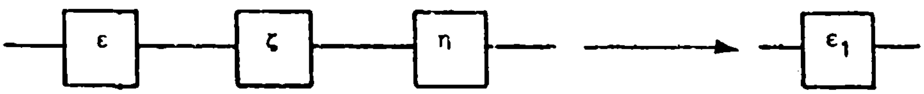
$N_N = 573$	$N_O = 573$	$N_P = 573$	(method of	$N_{N_1} = 573$
$F_N = 0$	$F_O = 0$	$F_P = 0$	sect. 2.2)	$F_{N_1} = 0$
$R_N = 1$	$R_O = 1$	$R_P = 1$	$R_{N_1} = 1$	



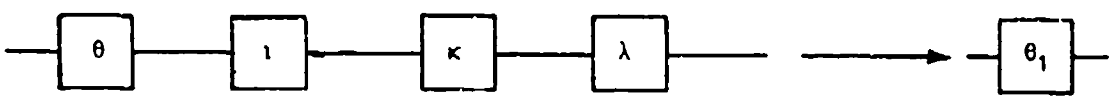
$N_U2 = 384$	$N_V = 384$	$N_W = 383$	$N_X = 383$	$N_Y = 384$	(method of	$N_{V_1} = 383$
$F_U2 = 0$	$F_V = 0$	$F_W = 0$	$F_X = 0$	$F_Y = 1$	sect. 2.2)	$F_{V_1} = 0.997$
					$R_{V_1} = \frac{383}{384}$	

Original subsystem | Equivalent component

$N_\alpha = 401$	$R_M = 0.999$	$N_\beta = 1260$	$N_\gamma = 1260$	(method of sect. 2.2)	$N_{\alpha_1} = 401$
$F_\alpha = 0$		$F_\beta = 0$	$F_\gamma = 0$	$R_{\alpha_1} = 0.999$	$F_{\alpha_1} = 0.401$



$N_\varepsilon = 382$	$N_\zeta = 382$	$N_\eta = 381$	(method of sect. 2.2)	$N_{\varepsilon_1} = 381$
$F_\varepsilon = 0$	$F_\zeta = 3$	$F_\eta = 0$	$R_{\varepsilon_1} = \frac{379}{382}$	$F_{\varepsilon_1} = 2.992$

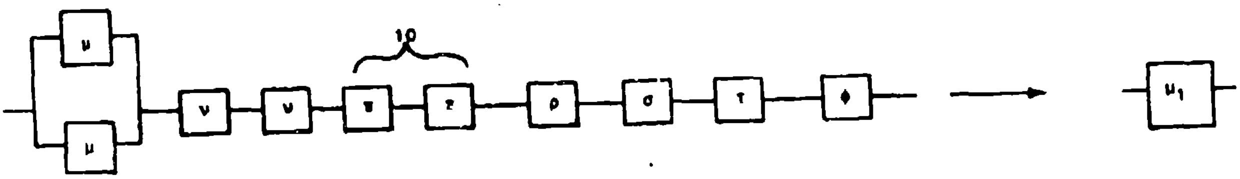


$N_\theta = 381$	$N_\iota = 399$	$N_\kappa = 371$	$N_\lambda = 375$	(method of sect. 2.2)	$N_{\theta_1} = 371$
$F_\theta = 0$	$F_\iota = 0$	$F_\kappa = 0$	$F_\lambda = 0$	$R_{\theta_1} = 1$	$F_{\theta_1} = 0$

This warhead component is repeated in other subsystems but, since it is being treated as having essentialy an infinite number of trials, it cannot affect calculation of the equivalent N and so it can be treated as independent, affecting only the calculation of R.

Original subsystem

Equivalent component



$Q_\mu = 0.008$

$Q_\nu = 0.002$

$Q_\pi = 0.0002$

$Q_\rho = 0.0001$

$Q_\sigma = 0.0005$

$Q_\tau = 0.0005$

$Q_\phi = 0.0018$

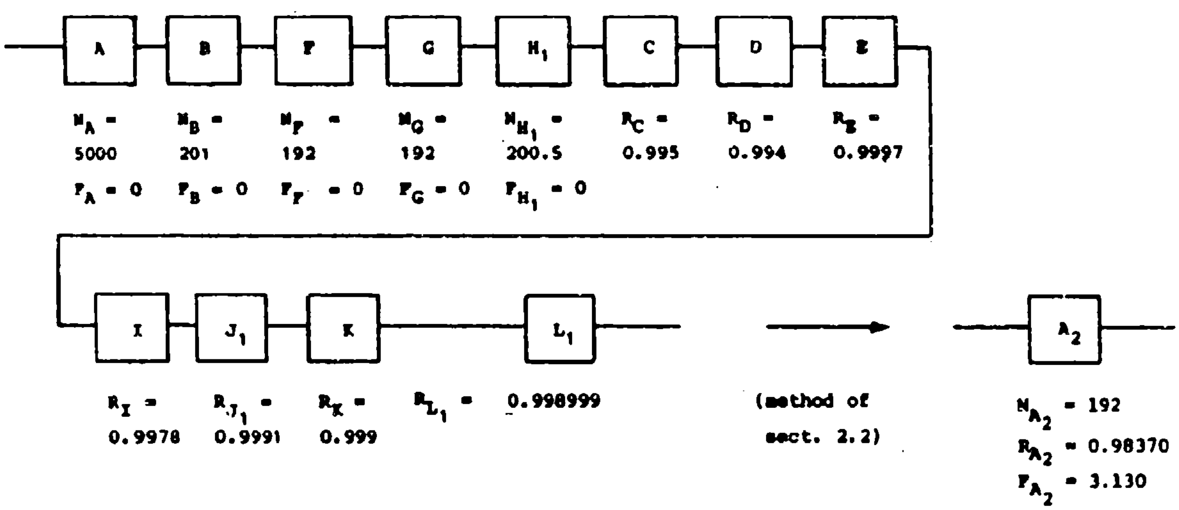
$R_{\mu_1} = (1 - 0.008)^2 \times 0.998^2 \times 0.9998^{10} \times 0.9999 \times 0.9995 \times 0.9995 \times 0.9982 = 0.99107$

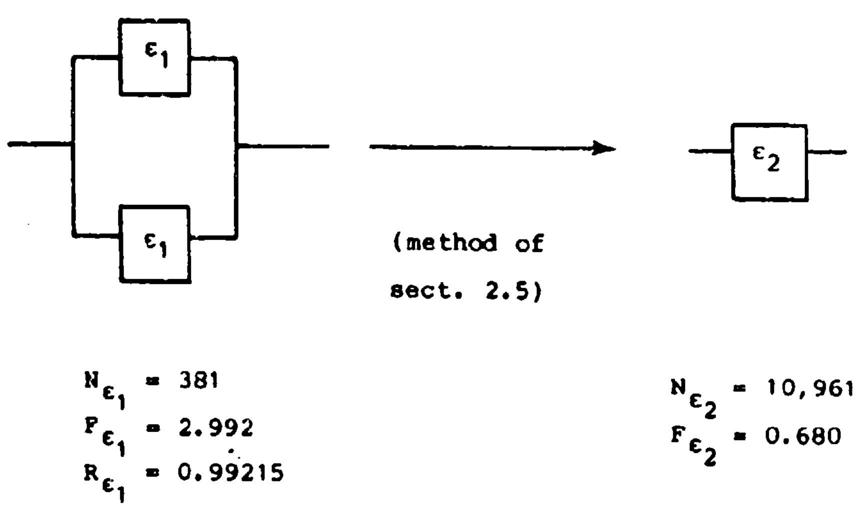
$Q_{\mu_1} = 0.00893$

SECOND REDUCTION

In each replacement of a subsystem, shown in figure 9 in the body of the report, by an equivalent component, both the original subsystem and the new equivalent component will be depicted. The methodology used for the reduction will be referred to by the appropriate section in the body of the report. The symbols N, F, R, and LCB will be used throughout to denote number of tests, number of failures, maximum likelihood reliability estimate, and 90-percent lower confidence bound, respectively.

Original subsystem | Equivalent component





$$LCB_{\varepsilon_1} = 0.98244 \text{ by the Poisson estimate (eq (5))}$$

$$R_{\varepsilon_2} = 1 - \left(1 - R_{\varepsilon_1}\right)^2 = 0.999938$$

$$LCB_{\varepsilon_2} = 1 - \left(1 - LCB_{\varepsilon_1}\right)^2 = 0.999692$$

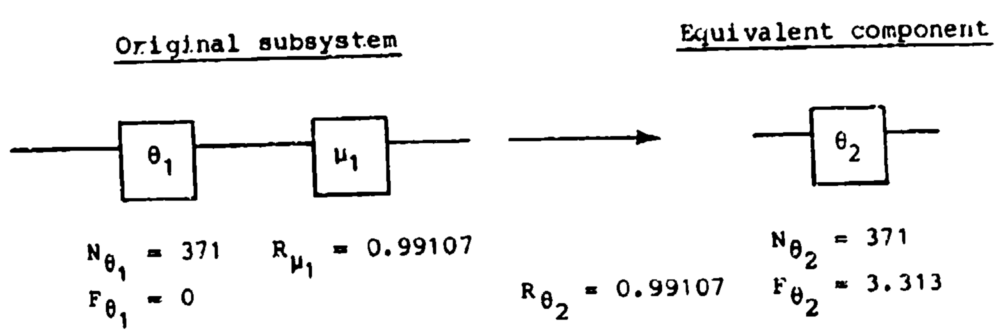
The iterative method of section 2.4 is used to find N_{ε_2} and F_{ε_2}, producing the results in the following table.

Iteration	N_1	F_1	$B_{90}\left(N_1, F_1\right)$	t	N_2
1	7,475	0.4635	0.999593	1.32	9,878
2	9,878	0.6124	0.999668	1.078	10,648
3	10,648	0.6602	0.999685	1.023	10,890
4	10,890	0.6752	0.999690	1.006	10,961

From these results,

$$N_{\varepsilon_2} = 10,961 \quad ,$$

$$F_{\varepsilon_2} = \left(1 - R_{\varepsilon_2}\right)N_{\varepsilon_2} = 0.680 \quad .$$

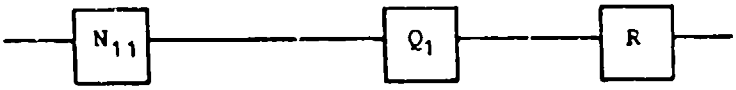


ANALYSIS OF SUBSYSTEMS

The equivalent number of trials and the maximum-likelihood reliability estimate will be calculated for each of the 10 subsystems in the reduced schematic in figure 10 in the body of the report. The methodology used for each subsystem will be referred to by the appropriate section in the body of the report. The symbols N, F, R, and LCB will be used throughout to denote number of tests, number of failures, maximum likelihood reliability estimate, and 90-percent lower confidence bound, respectively.

SUBSYSTEM 1

This subsystem is a series/parallel configuration consisting of repetitions of a single series combination:



$N_{N_{11}} = 143.25$ $\quad N_{Q_1} = 190$ $\quad N_R = 572$

$F_{N_{11}} = 0$ $\quad F_{Q_1} = 0.3333$ $\quad F_R = 0$

For this series,

$N = 143.25$

$R = 0.99825$

$LCB = 0.98107$ (by interpolation formula (2) in the body of the report

For subsystem 1 (using the method of sect. 2.5), we obtain

$$R_I = \left[1 - (1 - R)^2\right]^3\left[1 - (1 - R)^3\right] = 0.999991 \quad ,$$

$$LCB_I = \left[1 - (1 - LCB)^2\right]^3\left[1 - (1 - LCB)^3\right] = 0.998918 \quad ,$$

which leads to the results in the following table:

Iteration	N_1	F_1	$B_{90}(N_1, F_1)$	t	N_2
1	2127	0.0191	0.998905	1.010	2158

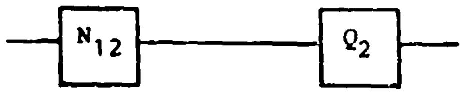
This gives the final data for subsystem 1:

$N_I = 2158 \quad ,$

$R_I = 0.999991 \quad .$

SUBSYSTEM 2

For the upper series:



$N_{N_{12}} = 143.25$ $\quad N_{Q_2} = 190$

$F_{N_{12}} = 0$ $\quad F_{Q_2} = 0.3333$

Equivalent N = 143.25

R = 0.99825

For the lower series:

$N_{S_1} = 83.33$ $\quad N_{V_{11}} = 95.75$ $\quad N_{Z_1} = 18.4$ $\quad N_{\alpha_{11}} = 80.2$

$F_{S_1} = 5$ $\quad F_{V_{11}} = 0.24925$ $\quad F_{Z_1} = 0$ $\quad F_{\alpha_{11}} = 0.0802$

Equivalent N = 18.4

R = 0.93662

For subsystem 2 (using the method of sect. 2.3), we obtain

$$Q = 0.00175 \times 0.06338 = 0.0001109 \quad ,$$

$$Q' = 0.000968 \quad ,$$

$$N_{II} = \frac{1 - Q'}{Q' - Q} = 1166 \quad ,$$

$$R_{II} = 0.9998891.$$

SUBSYSTEM 3

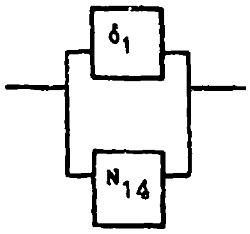
Subsystem 3 is the same as subsystem 2 except that S is replaced by T. A similar computation yields

$$N_{III} = 2040 \quad ,$$

$$R_{III} = 0.9999884 \quad .$$

SUBSYSTEM 4

For the parallel pair:



$N_{\delta_1} = 192$ $\qquad N_{N_{14}} = 143.25$

$F_{\delta_1} = 0$ $\qquad F_{N_{14}} = 0$

By the method of section 2.3, we obtain

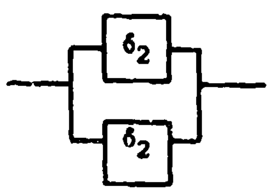
$$\text{Equivalent } N = 27838 \quad ,$$
$$R = 1 \quad .$$

Subsystem 4 is just this parallel pair repeated twice in series. By the approximation in equation (22), we have

$$N_{IV} = \frac{27,838}{2} = 13,919 \quad ,$$
$$R_{IV} = 1 \quad .$$

SUBSYSTEM 5

Subsystem 5 is just a single component repeated in parallel:



$N_{\delta_2} = 192$

$F_{\delta_2} = 0$

By the method of section 2.5 in the special case where R = 1, we have

$$LCB_{\delta_2} = 0.98799$$

$$R_V = 1 \quad ,$$

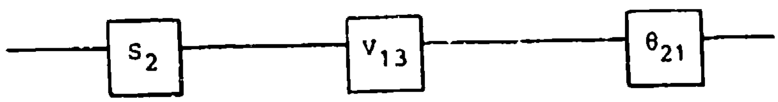
$$LCB_V = 1 - (0.01201)^2 = 0.999856 \quad ,$$

$$N_V = \frac{\ln 0.10}{\ln LCB_V} = 15{,}989 \quad ,$$

$$R_V = 1 \quad .$$

SUBSYSTEM 6

For the upper series:



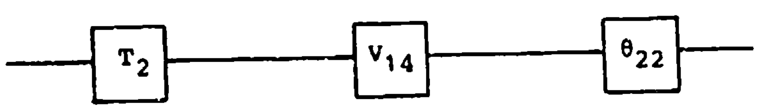
$N_{S_2} = 83.33$ $\quad N_{V_{13}} = 47.875$ $\quad N_{\theta_{21}} = 185.5$

$F_{S_2} = 5$ $\quad F_{V_{13}} = 0.124625$ $\quad F_{\theta_{21}} = 1.6565$

Equivalent N = 47.875

R = 0.92918

For the lower series:



$N_{T_2} = 109.33$ $\quad N_{V_{14}} = 47.875$ $\quad N_{\theta_{22}} = 185.5$

$F_{T_2} = 0.3333$ $\quad F_{V_{14}} = 0.124625$ $\quad F_{\theta_{22}} = 1.6565$

Equivalent N = 47.875

R = 0.98548

For subsystem 6 (using the method of sect. 2.3), we obtain

$$Q = 0.0010285 \quad ,$$

$$Q' = 0.0031159 \quad ,$$

$$N_{VI} = \frac{1 - Q'}{Q' - Q} = 478 \quad ,$$

$$R_{VI} = 0.9989715 \quad .$$

SUBSYSTEM 7

First the series repetition of V_1 is reduced:

$N_{V_{15}} = 95.75$

$F_{V_{15}} = 0.24925$

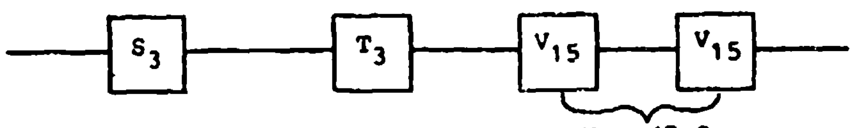
$R_{V_{15}} = 0.997397$

By the approximation in equation (24) in the body of the report, we have

$$\text{Equivalent } N = \frac{F_{V_{15}}}{1 - R^2_{V_{15}}} = 47.9 \quad ,$$

$$F = 0.24925 \quad .$$

For the upper series:



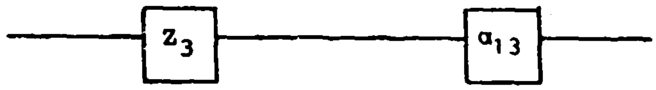
$N_{S_3} = 83.33$ $N_{T_3} = 109.33$ $N = 47.9$

$F_{S_3} = 5$ $F_{T_3} = 0.3333$ $F = 0.24925$

Equivalent $N = 47.9$

$R = 0.93226$

For the lower series:



$N_{Z_3} = 18.4$ $\quad$ $N_{\alpha_{13}} = 80.2$

$F_{Z_3} = 0$ $\quad$ $F_{\alpha_{13}} = 0.0802$

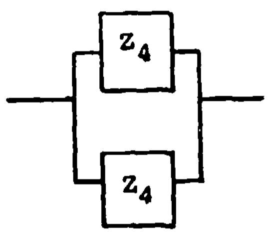
Equivalent $N = 18.4$

$R = 0.999$

For subsystem 7 (using the method of sect. 2.3), we obtain

$$Q = 0.00006774 \quad ,$$
$$Q' = 0.0045571 \quad ,$$
$$N_{VII} = 222 \quad ,$$
$$R_{VII} = 0.9999323 \quad .$$

SUBSYSTEM 8



$N_{Z_4} = 18.4$

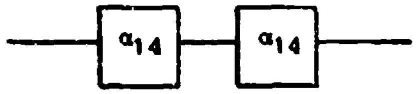
$F_{Z_4} = 0$

By the method of section 2.5 with R = 1, we have

$$LCB_{Z_4} = 0.88230 \text{ by interpolation,}$$
$$R_{VIII} = 1 \quad ,$$
$$LCB_{VIII} = 1 - \left(1 - LCB_{Z_4}\right)^2 = 0.98615,$$
$$N_{VIII} = 165 \quad ,$$
$$R_{VIII} = 1 \quad .$$

SUBSYSTEM 9

For the upper series:



$$N_{\alpha_{14}} = 80.2$$
$$F_{\alpha_{14}} = 0.0802$$
$$R_{\alpha_{14}} = 0.999$$

By the pproximation in equation (24) in the body of the report, we have

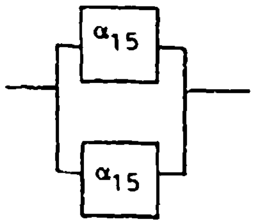
$$\text{Equivalent } N = \frac{F_{\alpha_{14}}}{1 - R^2_{\alpha_{14}}} = 40.1 \quad ,$$

$$F = 0.0802 \quad .$$

For subsystem 9 (using the method of sect. 2.3), we combine the upper series in parallel with Z_5, which has $N_{Z_5} = 18.4$ and $F_{Z_5} = 0$, and obtain

$$Q = 0 \quad ,$$
$$Q' = 0.0013548 \quad ,$$
$$N_{IX} = 737 \quad ,$$
$$R_{IX} = 1 \quad .$$

SUBSYSTEM 10



$$N_{\alpha_{15}} = 80.2$$

$$F_{\alpha_{15}} = 0.0802$$

By the method of section 2.5 we have

$$R_{\alpha_{15}} = 0.999$$

$$LCB_{\alpha_{15}} = 0.97011 \text{ by interpolation}$$

$$R_X = 1 - \left(1 - R_{\alpha_{15}}\right)^2 = 0.999999 \quad ,$$

$$LCB_X = 1 - \left(1 - LCB_{\alpha_{15}}\right)^2 = 0.999107 \quad .$$

The method of section 2.4 is then used to get equivalent test data, as follows.

Iteration	N_1	F_1	$B_{90}(N_1,F_1)$	t	N_2
1	2577	0.002577	0.999104	1.003	2586

These results lead to the following data for subsystem 10:

$$N_X = 2586 \quad ,$$

$$R_X = 0.999999 \quad .$$