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A METHODOLOGY FOR THE DEVELOPMENT OF FIRE CONTROL EQUATIONS FOR GUNS AND ROCKETS FIRED FROM AIRCRAFT

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ABSTRACT

The accurate firing of unguided projectiles (bullets or rockets) from aircraft leads to a requirement for rapid computation of the launch vector needed to assure the projectiles striking a given target. The computation of this laying vector and fuze time is the function of the on-board fire control system. The fire control system includes sensors which measure target range and velocity, aircraft attitude, position and velocity, and atmospheric conditions. These measurements are fed to an on-board fire control computer which in real time, typically at 50 Hz, must compute anew the laying vector appropriate for the rapidly varying variables which influence the ballistic trajectory. Six-degree-of-freedom models, which are normally used in laboratory ballistic modeling and simulation, are computationally too slow and otherwise cumbersome to be implemented for real time fire control. A methodology for developing an alternative—simplified, yet very accurate, model is described in detail.

1. INTRODUCTION

The fundamental objective of fire control ballistics is the provision of a means for aiming and fuze setting of a rocket or projectile so as to best assure placement of accurate and effective fire on a selected target. This objective was accomplished historically by the publication, and use in the field, of the classical "firing table", a book of numbers tabulated so as to provide an easy means for determining azimuth and elevation settings for guns and rockets. As computer technology has evolved, the firing table has been relegated to manual backup and the field computer computes gun and rocket settings in real time. Modern weapon systems such as computer controlled air defense guns, and helicopters and tanks which fire on the move encounter a fire control problem characterized by very rapidly changing variables which drive the ballistics. As a result, a wholly new level of difficulty and sophistication is placed on the problem of fire control ballistic prediction.

The fundamental "generic" models used extensively for prediction of launch and exterior ballistic performance have come to be known as Six-Degree-of-Freedom (SDF) models. Such models embody the equations of motion for both translational and rotational displacement of a projectile or rocket. The development, refinement, maintenance and modifications for new technology, of this type model, has long been performed in Defense Department Laboratories, and in particular, Army laboratories such as BRL. See, e.g., *Lieske* and *McCoy* [1] and *Barnett* [2]. The SDF model is a natural extension of the three-degree-of-freedom (TDF) model, long a mainstay of ballistic fire control prediction. This latter model represents only the translational aspects of projectile motion and is of an order of magnitude less difficult in computational labor. However, TDF models may, in some circumstances, be insufficiently accurate because they do not model the interaction of translational motion with the aerodynamic effects associated with yaw and pitch along with the yaw and pitch interaction with spin.

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[1] R. F. Lieske and R. L. Mc Coy, "Equations of Motion of a Rigid Projectile," BRL Report No. 1244, March 1964.

[2] D. Barnett, "Trajectory Equations for a Six-Degree-of-Freedom-Missile Using a Fixed-Plane Coordinate System," Technical Report 3391, Picatinny Arsenal, Dover, New Jersey, June 1966.

Artillery projectiles, like the classical spinning top in mechanics, have very high frequency motion associated with spin, precession and nutation. A fundamental computational requirement of numerical integration, (the technique used for solving these models) is that oscillatory variables need to be sampled at several points within each oscillation to maintain accuracy and stability. The result of this is that small steps in time are required in the forward marching process of integration. Accordingly, complete SDF calculations for spinning artillery or automatic cannon projectiles is time consuming, relatively expensive, and is done sparingly on laboratory computers. An alternative to the SDF and TDF models was developed by *Lieske* and *Reiter* [3] which has been found extremely useful for firing table computation and implementation in some ground based fire control computers. Their successful approach was to develop a modified TDF model, MTDF, which incorporated an explicit estimate of the "yaw of repose" and its effects into the translation equations without having to integrate the high frequency motion of the full SDF models. This model has resulted in greatly reducing the computational burden referred to above while incurring only a slight loss in accuracy. It is now a standard model in the repertoire of ballistic mathematical tools widely used in the United States and the NATO defense community.

Helicopter fire control requires the use of ballistic models as described above. The SDF model is needed for rockets and the MTDF (and SDF) model are needed for automatic cannon. Unfortunately, for reasons detailed herein, these models, can only serve an intermediate role, albeit an important one, in the process of developing an on board—real time ballistic prediction model. A fundamental problem in attack aircraft fire control is that of maintaining the "timeliness" of the ballistic solution. In a turning, climbing maneuver, the aircraft velocity components, the geometrical relations between target and cannon/rocket and other variables— all change rapidly. Note also that these models really solve the "inverse" of the fire control problem. In fire control one specifies terminal conditions such as target location with respect to the launch platform. The solution desired consists of the departure attitude needed to strike the target and the time of flight. Accordingly, one would need to solve the problem iteratively, i.e., guess at a trial solution, and continually readjust the departure angles until the desired terminal conditions are satisfied. Meanwhile, if the aircraft is in a maneuver, the problem has changed because the variables driving the ballistics have changed since the iterations were initiated. The consequence of this is that a ballistic solution that is not computed instantaneously, (or nearly so), is old and obsolete before the munition can be fired. Despite the recent and continuing revolution in computer technology, the embedded computers in aircraft fire control systems are small in memory capacity, and are not fast enough to iteratively solve the models described above at the required frequency.

Modern attack helicopters such as the Cobra and Apache are armed with an automatic cannon and a family of 2.75 inch or Hydra 70 rockets. By the flip of a switch, the pilot can select the munition he wishes to fire. Accordingly, the model(s) embedded in the on board fire control computer must be able to key on this switching process and compute the solution for the munition selected. In fact, it is possible, and sometimes desirable, to be able to fire both the gun and rockets simultaneously. The requirement for representing ten or more rocket types, each having differing weights, measures, aerodynamics, staging and fuzing characteristics— all add to the need for developing a common general model. Furthermore, Army aircraft such as the Cobra and Apache have differing modes of articulating rocket pods. It is highly desirable that the rocket ballistics be developed independently of the method of pod articulation. See Appendix F. The logical strategy evolves for developing a procedure which makes the general model applicable to a specific munition by selective retrieval of a pool of constants (based on switch position). Some constants of the model, however, may differ from one aircraft type to another. See Appendix E. This necessity for providing a capability for many munitions tends to reduce the storage capacity available for the collection of instructions related to the model itself. The requirement for an accurate ballistic solution, for such a family of munitions, varying types of aircraft, air defense guns, tanks, or other moving gun platforms, that can be computed cyclically, in real time, during an engagement, leads to the need for the methodology described herein.

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[3] R. F. Lieske and M. L. Reiter, "Equations of Motion for a Modified Point Mass," Ballistic Research Laboratories Report No. 1314, March 1966.

2. GENERAL DESCRIPTION OF THE MATHEMATICAL METHODOLOGY

A perspective for defining the mathematical problem can be obtained by examination of Figure (2-1). Therein an attack helicopter in flight is depicted engaging a ground target. While the scenario

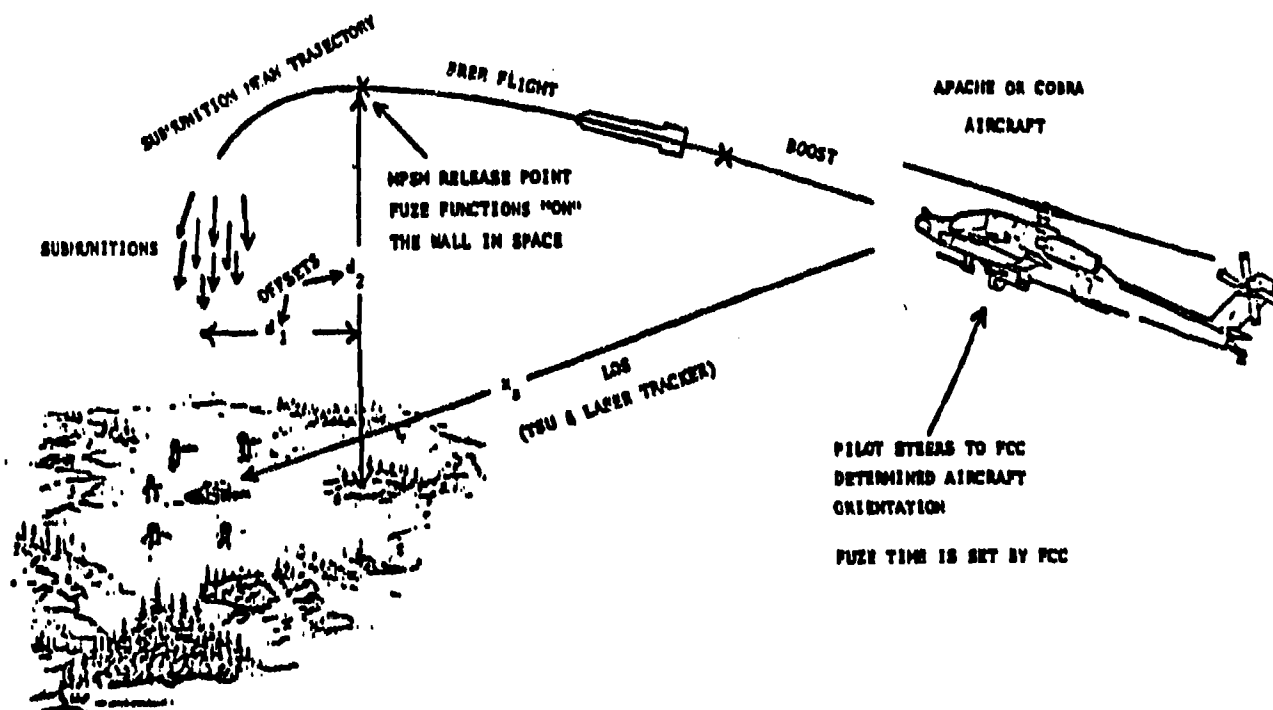## HYDRA 70 FIRE CONTROL (INDIRECT FIRE)

Figure (2-1). Typical attack helicopter scenario for rocket firing.

depicts the use of a Hydra-70 rocket, the employment of an automatic cannon would be similar. The telescopic sight unit, (TSU), is continually maintaining the line of sight, (LOS), which can be viewed as a vector connecting a reference point on the aircraft to a selected point on the target. On board sensors continually measure the vector velocity and attitude orientation of the aircraft along with the LOS range, z_s, between target and aircraft. For moving targets, sensors coupled with mathematical "filters" continually estimate the vector velocity of the target. Target motion, along with other considerations, determine components z_m and z_i which along with z_s define where the projectile should be in "one time of flight". Environmental factors such as wind, air density and temperature are also provided by sensors on the aircraft. The temperature of the munition is also known, either by use of a magazine thermometer or by assuming that the munition has the same temperature as the environment. Required information on downwash due to the rotation of the helicopter blades is obtained by means described in Appendix D. Components of the gravity vector are made available by reference to orientation of coordinate axes that are aligned with the local gravity vector.

The above described information is fed, at typically 50 Hz, to the Fire Control Computer, (FCC). The primary function of the FCC is to compute the angular settings (the attitude of departure) and for

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some munitions, fuze time, which best assures accurate placement of fire on the target. The objective is thus seen to be the development of a collection of formulae, to implement in the FCC, which accomplishes this task. Toward this end it is useful to review two procedures, used previously, that can serve as building blocks toward development of a more generalized procedure.

Global Fitting Approach

A perspective on the global fitting *approach*[*] can be obtained from the work of Chandler, Baker and Dinjar at the US Army Redstone Arsenal and C. Masaitis and H. Breaux at the Ballistic Research Laboratory. That work is reviewed (and references listed) by *Breaus*[4]. The objective of that work was to find an alternative to the SDF models for use in fire control with ground based missiles such as the Redstone, Jupiter, Pershing and Lance. For these stationary-at-launch, ground systems, certain complexities unique to the moving platform are not present and the computational speed factor is not as critical. A computational cycle time of seconds, or even tens of seconds, is tolerable. However, at the longer ranges of these systems, other complexities enter such as those associated with earth rotation and curvature. Nevertheless, a basic procedure employed in that work remains as the cornerstone of current methodology. That concept is depicted in Figure (2-2) below and consists of approximating one model
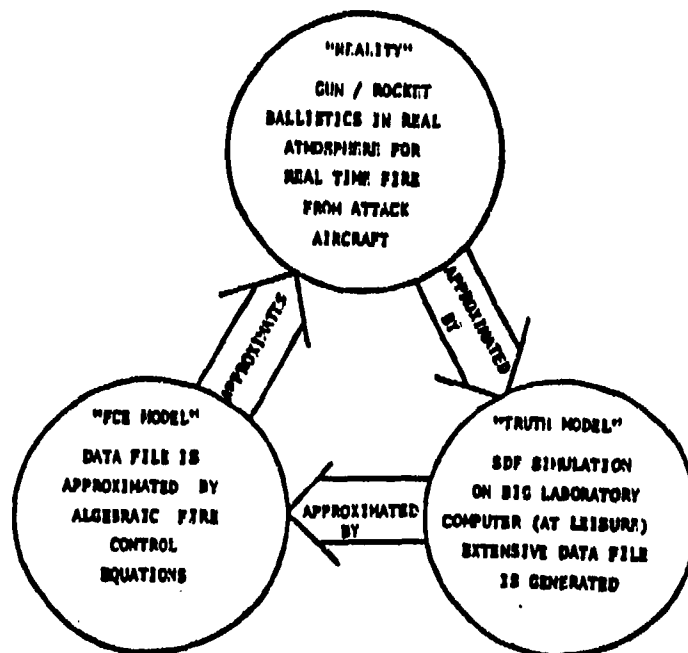

Figure (2-2). Triangle of Approximations for Development of Fire Control Ballistics.

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with another model. First a "truth" model must be developed—both as a means for assessing actual ballistic performance and then to make possible the development and accuracy assessment related to the fire control model. This model must represent all the munitions of interest— fired with launch conditions representing the aircraft environment. Speed of computation is not a significant factor here and this model is normally implemented in a laboratory computer.

The global fitting approach consists of basically five phases.

(A) Development of the SDF Model.
(B) Specification of a candidate fire control equation (FCE) model to be fitted.
(C) Development of a data base of trajectory calculations which span the expected range of all variables (and mixtures) in the planned deployment scenarios.
(D) Fitting of the model to obtain values for all coefficients.
(E) Computer validation of the model.

Phase (A) generally consists of obtaining all weights and measures, physical characteristics, and aerodynamic functions that are appropriate to the munition(s) of interest and processing the data into a form acceptable to the SDF program. Special features of the munition ballistics or launch dynamics may require modification of the SDF program. This is a very critical phase in that everything done hereafter hinges on the adequacy and accuracy of this model. Phase (B) is the most difficult aspect of the problem and is the core of the methodology to which a large part of this paper is devoted. Phase (C) consists of designing a matrix of conditions and use of the SDF program to compute the corresponding trajectories. The fitting process, Phase (D), is itself a part of the methodology. However, the procedure designed by *Breaux*[4], will be used herein to form a hybridized methodology which will be described. Phase (E) is designed to test the execution of the model in a form similar to it's field employment and simultaneously compare its predicted results with the SDF "truth model".

Phase (E) should not be confused with field validation. In field validation the results of the effort, namely the ballistic FCE's, are programmed into the Fire Control Computer and actual live tests conducted. Poor performance in this phase can frequently occur due to improper programming of the FCE's, poor sensor performance, or a poorly designed or specified SDF model. If the latter is found to be true, the SDF model must be fixed and the process described above must be *repeated*[6].

Closed Form Solution Approach

A solution approach has been employed by *Norwood*[5], for airborne gunfire, that makes use of the "Method of Siacci". In this approach, the computational problem is reduced to a collection of integrals. Norwood's approach was made more practical by *Benokratis*[6], who obtained closed form expressions for the Siacci integrals by approximating the projectile drag by three connected line segments. This approach has the benefit of "permanence" in that the approximate solution is there for all time—for all projectiles that satisfy the flat fire assumption and which can be adequately approximated by a TDF model. The drag curve must also permit adequate approximation by the three line segments. The disadvantage is that this technique does not incorporate ballistic effects that arise from factors that only the SDF model

[*] Now that the methodology and related software has been developed, reprocessing for a revised or new SDF data base can be done routinely and very quickly.

[5] Norwood, John M., "A Review of the Method of Siacci Trajectory Computation for Airborne Gunfire," ARL-TM-71-27, Applied Research Laboratories, The University of Texas at Austin, Austin, Texas, August 1971.

[6] Benokraitis, Vitalius, "A Closed Form Siacci Function Method for Trajectory Calculations," Journal of Ballistics, V 6, No.1, 1982, pp. 1326-1347.

could predict. Ballistic effects due to aerodynamic "drift" and "jump" and secondary drag effects related to initial yaw and yaw rate must be added to the model by extraneous methods which require essentially the same effort as the global fitting approach. This method can represent the free flight phase of a rocket but not the boost phase. Extension of such a model to include more complex ballistics generally leads to the need for some form of fitting and in turn necessitates all the stages of the global fitting approach.

The methodology developed herein can be viewed as a hybrid method which combines features of both the global fitting technique and the closed form approximation. It can also be viewed as a generalization and extension of the earlier work done by the *author*[4]. The key aspect of the work described therein was the use of step-wise regression (least squares) to facilitate the process of model building. This process can be be exhibited by simple example. Assume that a process, whose outcome is represented by w, is dependent on the variables x,y,z. Assume that a very accurate model exists that permits the specification of x,y and z and then computes w. However, the computation is so expensive and time consuming that it can only be done sparingly and at facilities remote from where the need exists. This suggests the need and possibility of developing an approximating model which is easier to evaluate. In attempting such an effort consider the following two approaches to global fitting.

The Empirical Approach to Global Fitting

Here no specific information on the underlying mathematical model is assumed known other than the dependence of w on x, y and z. A candidate linear model is specified by the formula

$$w = a_o + a_1 x + a_2 y + a_3 z + a_4 x y + a_5 x z + a_6 y z$$

$$+ a_7 x y^2 + a_8 x z^2 + a_9 z^2 y + \cdots \qquad (2\text{-}1)$$

Numerous types of linear models could be employed, however, a polynomial model is assumed for illustration. By providing a data base of w versus x,y and z for an appropriate mix and range of the variables and the above model to a stepwise regression procedure as described by *Breaux*[7,8], or *Hocking*[9] one might arrive at a suitable model. The result of such a regression procedure is the sequential listing of submodels, each differing from the previous submodel, by one term taken from the above collection of terms. The first submodel is the one term alone which best approximates w. The second model contains two terms, including the one term model plus a second term which when added to the first provides the best approximation. The process continues in this fashion but simultaneously seeks to weed out terms that are no longer useful due to cross correlations between clusters of variables that have been introduced into the model. Variations in strategy and computational sequencing are discussed at length by *Hocking*[9]. By examining the program output which includes the progression of the variance of residuals in w, the correlation coefficient, the "t" values on the regression coefficients, etc., and performing some experimentation on model definition, one can generally build a suitable model by this process.

The Physical Approach to Global Fitting

In the physical approach one first recognizes that the problem at hand embodies a field of knowledge to include a collection of literature. Such a literature indicates that the process leading to the outcome w has a "closed form" result, dependent on x,y, and z, if certain idealizing assumptions are made. One such outcome might be

[7] Breaux, H. J., "On Stepwise Multiple Linear Regression," BRL Report No. 1369, August 1967.
[8] Breaux, H. J., "A Modification of Efroymson's Technique for Stepwise Regression Analysis," Comm. of the ACM, 1968, Vol. 11, No. 8.
[9] Hocking, R. R., "The Analysis and Selection of Variables in Linear Regression," Biometrics, V. 32, March 1976, pp. 1-49.

$$w = A_o \, x + B_o \, exp(\alpha \, x \, y/z) . \qquad (2\text{-}2)$$

This might suggest that a more intelligently defined model would proceed from Eq. (2-2) as a base. For this simple model one might use non-linear least squares and try to fit A_o, B_o and α as free parameters. When the model is complex, containing many variables and terms, it is generally better to linearize the model and proceed as follows: In Eq. (2-2), for example, expand non linear terms in a Taylor series

$$w = a_o + a_1 \, x + a_2 \, x \, y/z + a_3 \, (x \, y/z)^2 + a_4 \, (x \, y/z)^3 + \cdots \qquad (2\text{-}3)$$

As before, this candidate model and the data base is processed by the stepwise regression process. The resulting model, developed by the latter procedure, will, in general, be superior to one developed by the empirical process. Any insight, born of experience and knowledge of the process, helps toward better defining the candidate model, used as a point of departure, when thereafter employing a fitting process.

The fire control problem addressed herein is analogous to the above simple example with one very important difference. The example concerned itself with one process and one approximating model was needed. Hence, either of the two approaches, if found to lead to an adequate model, could be viewed as successful. The attack helicopter fire control problem, by contrast, can be viewed as representing ten or more related processes. If the problem is addressed by the empirical approach described above, intuition and experience indicates that the result will be as many separate models as their are rocket/munition types [*].

The approach taken will be one of examining all components of the launch and flight process, the boost and free flight coupling, and practical idealizations which provide a physical basis for simplified closed form solutions leading to an intelligent definition of the candidate model. These components, will be processed by the stepwise regression procedure by use of a data base of calculations from an SDF program to determine their adequacy. The approach is thus seen to be a hybridized one which includes obtaining closed form approximations and then employs the features of the global fitting approach as described above.

Note: Due to its length, only Sections 1. and 2. of this paper have been included in the Proceedings. Information on the availability of the complete document can be obtained by writing to:

Director
USA Ballistic Research Laboratory
ATTN: DRXBR-TSD
Aberdeen Proving Ground, Md. 21005

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[*] This aspect was found to be critically important in the currently on-going Fire Control Systems Integration Program for Cobra. The magnitude of effort, and resulting time required, to provide for as many as nine different rocket types, forced au early "freeze" in the basic structure of the model long before work was completed. The basic model was then programmed, validated for specific rocket types, and then coefficients for the remaining members of the family were added as they were obtained. This permitted timely progress on the overall program.