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A TECHNIQUE TO APPROXIMATE COMPLEX COMPUTER MODELS

An Approximation of the Teisberg Model

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Abstract

This paper presents a technique which was used to produce an approximation of a complex computer model, the Teisberg Model. The technique employs a complete 2^4 factorial design and uses the statistically significant effects as coefficients of the estimating equation.

Disclaimer

An Approximation of the Teisberg Model

Background

An approximation was constructed of the Teisberg model which estimates the economic benefit of constructing and maintaining a Strategic Petroleum Reserve. Four input factors, which replicated significant independent economic assumptions were identified as candidates for inclusion within the simplified model. The four variables were:

1. p = Annual probability of a major oil disruption
2. e = The short run price elasticity of demand for oil
3. b = The BAU price of crude oil
4. d = The discount rate

Using a one variable at a time approach three of these variables were set at the center, of their range of interest, and the Teisberg Model estimated the net economic benefit (Y) for a low, medium, and high value of the remaining variable.

This was done for the four candidate variables. An estimate of the rate of percent of change of the economic benefit Y to the percent change of the input factor X was calculated i.e., $\frac{dY/Y}{dX/X}$.

The results of this effort were:

Input factor	$\frac{dY/Y}{dX/X}$
Probability of a major disruption	0.543
Short run price elasticity	-2.196
BAU price of crude oil	0.330
Discount rate	-0.864

It was determined that only the short run price elasticity for demand need be considered when estimating the results of the Teisburg Model.

A linear regression was then performed on the three observations of the Teisberg Model with the low, medium, and high values for the elasticities and the three remaining variables set at the center of their range of interest.

The resulting equation was $Y = 275.85\ e + 85.67$ where e is the elasticity of demand, $-0.3 \leq e \leq -0.1$ and Y is the estimate of net economic benefit. The R^2 value was $\overline{0.86}$ which seems to indicate a good approximation. However, only three observations were used and two are required to determine a straight line, leaving only one degree of freedom, and thus a high R^2.

The Alternate Estimate

At the request of the principal investigator the sound principles of experimental design were applied to the same problem with the hope that an improvement might be made in the estimating equation. The remainder of this paper and the appendixes are the result of that request.

The estimate of the net economic benefit using the techniques of experimental design is:

$$y = 119.57 - 1137.87d + 398.00\,e + 2148.00\,p$$

$$- 4216.00de - 7488.00\,dp + 5246.00\,ep \quad \text{where}$$

d = discount rate $0.025 \leq d \leq 0.1$
e = elasticity of demand $-0.3 \leq e \leq -0.1$
p = annual probability of a major disruption $0 \leq p \leq 0.1$.

Details of the theory and construction of this estimate appear in the appendixes. The relative merits of the two estimates may be established by examining the estimates of both equations using the observations used in this study.

Observation	Teisberg Value	Original Estimate	Estimate Residual	Alternate Estimate	Estimate Residual
1	3.48	0.92	2.56	12.86	-9.38
2	14.69	0.92	13.77	3.34	11.35
3	2.56	0.92	1.64	12.86	-10.30
4	15.72	0.92	14.80	3.34	12.38
5	18.71	56.09	-37.38	8.14	10.57
6	50.06	56.09	-6.00	61.86	-11.80
7	17.28	56.09	-38.83	8.14	9.14
8	49.97	56.09	-6.12	61.86	-11.89
9	7.95	0.92	7.03	0.40	7.55
10	27.01	0.92	28.09	47.04	-20.03
11	27.01	0.92	11.60	0.40	12.12
12	43.39	0.92	42.47	47.04	-3.65
13	67.62	56.09	11.53	100.60	-32.98
14	169.16	56.09	113.07	210.48	-41.32
15	113.90	56.09	57.81	100.60	13.30
16	275.48	56.09	219.37	210.48	65.00
Sum of squared residuals $\sum(Y - Y)^2$			70,564.85		8,767.37
mean square error $\sum(Y - Y)^2/16$ (unadjusted for degrees of freedom)			4410.30		547.96

Table 1

Caveat

This estimate or approximation of the Teisberg Model was based on assumptions for several input factors which were not varied during this exercise. Changes in the values for these input factors may alter the quality of this estimate.

Next Steps

There are some promising techniques that may lead to additional improvements in an estimate of the Teisberg Model. The first is the application of response surface analysis to estimate the coefficient of higher ordered terms. The second involves various transformations, of the data, as the first step of the analysis. Thirdly, additional input factors might be included in the analysis. These techniques used independently, or in conjunction with each other, should improve the quality of the estimate.

Appendix A
METHODOLOGY

A1 Factorial Design Methodology

An experiment was performed to measure the effect of four sets of input factors on the average net economic benefit associated with four SPR alternatives, as represented by the Teisberg model. Two levels, for each set of input factors, were chosen and all 16 possible combinations of these input factors, were used as model input to the Teisberg model. This procedure, a 2^4 factorial design was chosen since it is economical, easy to use and provides a great deal of valuable information. Specifically a two (2) level factorial design has the following advantages:

1. If sets of input factors are varied one set at a time, with the remaining factors held constant, it is necessary to assume that the effect would be the same at other settings of the other sets of input factors. Factorial designs avoid this assumption.

2. If the effects of input factors act additively, a factorial design estimates those effects with more precision. If the effects of the input factors do not act additively, factorial designs can detect and estimate the interactions which measures the non-additivity.

3. Factorial designs require relatively few runs per set of input factors studied and can indicate major trends and determine promising direction for further investigation. To obtain the same precision of the estimate of the effects measured, in this effort, forty runs would have had to be run, using the traditional, one factor at a time approach, rather than the sixteen used in the experiment.

4. If a more thorough local exploration is needed, it can be suitably augmented to form composite designs.

5. These designs and their corresponding fractional designs may be used as building blocks so that the degree of complexity of the finally constructed design can match the sophistication of the problem.

To perform a 2^4 factorial design the two extreme levels (or versions), as defined by the principale investigator, were selected for the four (4) sets of input factors and all sixteen (16) possible combinations were run, which created sixteen observations. The four sets of input factors and their levels (or versions) are listed in Table A-1 on the following page.

Input Factor	Levels
1, Probability of a major oil disruption	1a, 0.0, no chance of a major oil disruption during any year of the study.
	1b, 0.1 A ten percent chance in any given year of a major oil disruption
2, The short run price elasticity	2a, - 0.3 a low short run elasticity of demand for oil
	2b, - 0.1 a high short run elasticity of demand for oil
3, The business as usual price for crude oil	3a, $52.00 per barrel, a low price
	3b, $90.00 per barrel, a a high price
4, The discount rate	4a, 10.0% the conventional government discount rate
	4b, 2.5% a low discount rate

TABLE A-1

The selection of the above levels were determined by the parent study and do not represent the policy of the Department of Energy. These levels were used solely to evaluate the reaction of the Teisberg Model to changes in the input factors.

These input factors combine to produce the following design matrix:

Design Matrix

OBS. NUMBER	PROB. DISRUPT	ELAS.	PRICE CRUDE	DISCOUNT RATE	TEISBERG NET BEN.
1	1a	2a	3a	4a	3.48
2	1a	2a	3a	4b	14.69
3	1a	2a	3b	4a	2.56
4	1a	2a	3b	4b	15.72
5	1a	2b	3a	4a	18.71
6	1a	2b	3a	4b	50.00
7	1a	2b	3b	4a	17.28
8	1a	2b	3b	4b	49.97
9	1b	2a	3a	4a	7.95
10	1b	2a	3a	4b	27.01
11	1b	2a	3b	4a	12.52
12	1b	2a	3b	4b	43.39
13	1b	2b	3a	4a	67.62
14	1b	2b	3a	4b	169.16
15	1b	2b	3b	4a	113.90
16	1b	2b	3b	4b	275.48

Table A-2

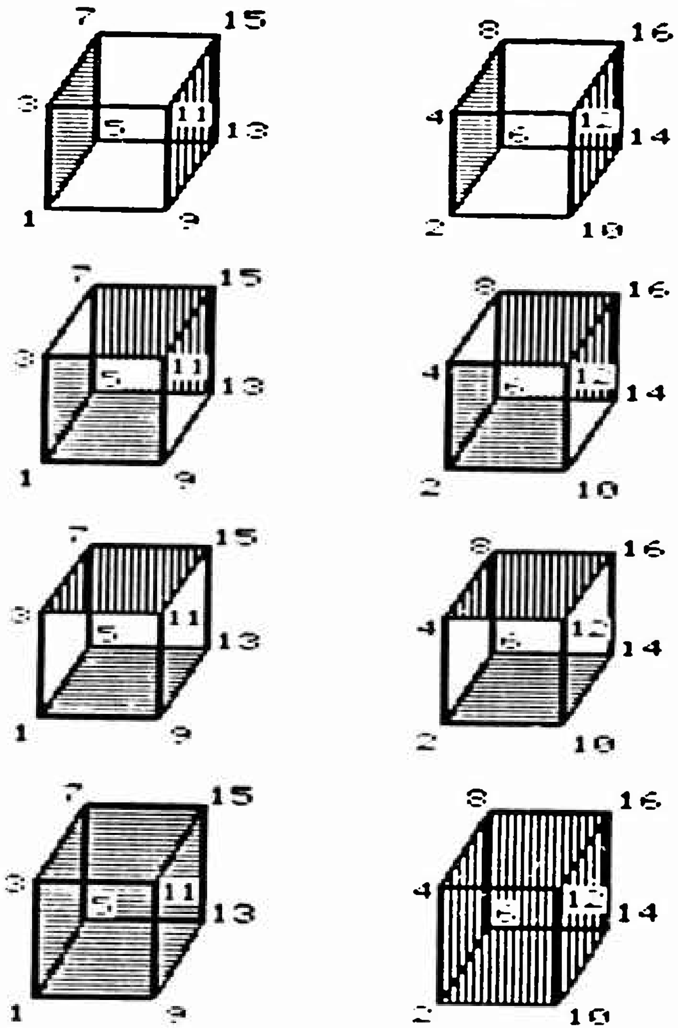
The interpertation of the observations in Table A-2 is easily illustrated by observation number 6 which assumes that the annual probaility of a major oil disruption is 0.0 i.e. there will not be a major disruption during this study. There is a high elasticity of demand for curde oil of -0.1 with a business as usual price for crude oil of $52.00 per barrel. Finally a low discount rate of 2.5% is assumed.

The sixteen observations of the design matrix, may be visualized geometrically as two cubes. One possible visualization appears in figure A-1 on the following page. The observation number is at each vertex.

THE TEISBERG MODEL

figure A-1

GEOMETRICAL REPRESENTATION OF THE 2^4 FACTORIAL DESIGN MAIN EFFECTS



PROBABILITY OF A MAJOR OIL DISRUPTION

DEMAND ELASTICITIES

B.A.U. CRUDE PRICE

DISCOUNT RATE

A2 Calculation of Main Effects

The "main effect" of a set of input factors is the change in the response i.e., the net economic benefit, y, as we move from the "a" case to the "b" case version of that set of input factors. To examine the effect of each of the selected input factors a table of four column vectors was constructed (see table A-3). Each column contrasts eight pairs of estimates of the net economic benefit. Aside from experimental error, the difference between the upper number of a pair and the lower number of the same pair is due to the change of the input factor that heads the column. For each column the average of these eight differences is the main effect due to the associated input factor that heads the column. Note that the only difference between the four columns is the order in which the observations appear.

Geometrically speaking, using Figure A-1 the main effects are calculated from the corresponding vertices of the two cubes as described below.

Input factor	
Probability of a major oil disruption	Left side of both cubes vs. the right side of both cubes
Demand elasticities	The front of both cubes vs. the backs of both cubes
Business as usual crude price	The bottom of both cubes vs. the tops of both cubes.
Discount rate	The left cube vs. the right cube.

Main Effects
Table of Contracts

Prob. of major oil disruption		Demand Elasticities		BAU Crude Price		Discount Rate	
Obs. Number	Net Econ. Benefit	Obs. Number	Net Econ. Benefit	Obs. Number	Net Econ. Benefit	Obs. Number	Net Econ. Benefit
1	3.48	1	3.48	1	3.48	1	3.48
9	7.97	5	18.71	3	2.56	2	14.69
2	14.69	2	14.69	2	14.69	3	2.56
10	27.01	6	50.06	4	15.72	4	15.72
3	2.56	3	2.56	5	18.71	5	18.71
11	12.52	7	17.28	7	17.28	6	50.06
4	15.72	4	15.72	6	50.06	7	17.28
12	43.39	8	49.97	8	49.97	8	49.97
5	18.71	9	7.97	9	7.97	9	7.97
13	67.62	13	67.62	11	12.52	10	27.01
6	50.06	10	27.01	10	27.01	11	12.52
14	169.16	14	169.16	12	43.39	12	43.39
7	17.28	11	12.52	13	67.62	13	67.62
15	113.90	15	113.90	15	113.90	14	169.16
8	49.97	12	43.39	14	169.16	15	113.90
16	275.48	16	275.48	16	275.48	16	275.48

TABLE A-3

A3 2nd-Order Interaction Effects

Suppose that one is interested in examining the effects of two sets of input factors; for example, the probability of a major interruption and the discount rate. Then the sixteen runs of the factorial design can be grouped into four sets of four runs each. Each run in the group would have the same value for the input factors studied, although other input factors would vary within each group. Assume that if there is no chance for a major oil disruption and the discount rate is 10%, that the average value for the output variable being studied is 100. This will be the base point. Also assume that the main effects for the probability of a major interruption and the discount rate are 25 and 10 respectively. This means that, on the average, changing from no chance of a major interruption to an annual probability of an interuption of 0.10 will increase the output variable under study by 25. Likewise a change in the discount rate from 10% to 2.5%, will on the average, increase this same output variable by 10. If the input factors act additively, then the average value of the output variable with 0.10 chance of an interruption and a 2.5% discount rate would be 100 + 25 + 10 = 135.

This artificial case is represented by the upper diagram in figure A-2. Note that the quantity

$$(b + c - a - d)/2 = (110 + 125 - 100 - 135)/2 = 0$$

i.e., there is no interaction.

Suppose that the input factors do not act additively, and the base point of 100 and main effects are the same. Then the resulting measurements could be described by the lower diagram in figure A-2. The input factors are now said to interact. By convention a measure of this interaction is

$$(b + c - a - d)/2 = (145 + 160 - 100 - 135)/2 = 35$$

This is a second order interaction and is called the probability of a major oil interruption X discount rate interaction.

Like a main effect, a 2nd order interaction is the difference between two averages, eight of the sixteen results being included in one average and eight in the other. Analogous explanations are easily constructed for all other 2nd order interaction effects.

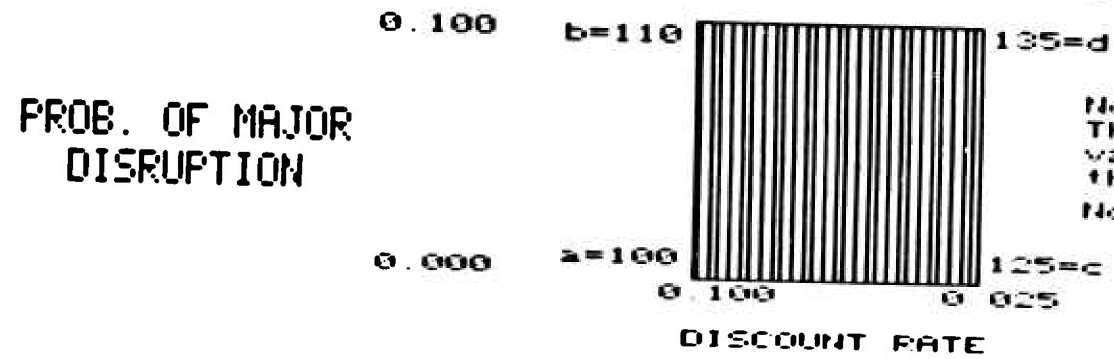
A4 Higher-Order Interaction Effects and the Standard Error.

Similar procedures to those above can be given for deriving the third and fourth-order interactions. Due to the similarity of response functions it is reasonable to assume that higher-ordered interactions are negligible and measure differences arising principally from experimental error. Thus the mean, of the sum of squares, of these interactions give an estimated value for the variance of an effect, having five degrees of freedom. The square root of this value is an estimate of the standard error.

THE TEISBERG MODEL

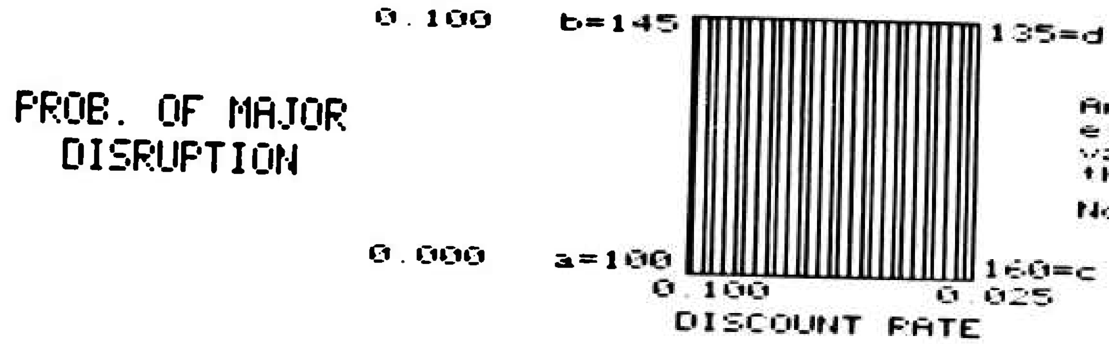
figure A-2

Interpretation of 2-Way Diagrams



No 2nd order interaction. The behavior of the output variable is explained by the main effects.

Note: (110+125-100-135)/2 = 0



An important 2nd order interaction effect. The behavior of the output variable is not fully explained by the main effects.

Note: (145+160-100-135)/2 = 35

The level of statistical significance chosen for this study was $p=0.10$. In order to select the statistically significant main effects and second order interactions multiply the standard error by $t_{1-p/2}=2.015$. Any main effect or interaction with absolute value greater than this product is considered statistically significant.

A5 The Plot of Effects

If the output from the model had simply occurred by chance, the observations would be normally distributed about some fixed mean, and the changes in the input factors would not have a real effect on the estimate of the net economic benefit. The fifteen effects, main effects plus all interactions, could then be ploted on normal probability paper as straight line. One may conclude that the effects that are not roughly on this straight line, are due to changes in the input factors and have a significant effect on the output variable being studied.

A6 The Binary Estimates

$$\text{Define } X_i = \begin{cases} -1 \text{ if } ia \text{ is the value of the } i \text{ th input factor (see table A-1).} \\ 1 \text{ if } ib \text{ is the value of the } i \text{ th input factor (see table A-1).} \end{cases}$$

Let a_i be the main effect of the i th input factor

Let a_{ij} be the 2nd order interaction of the i th and j th input factors.

Let I index the set of significant main effects at a fixed level of significance p.

Let IJ index the set of significant 2nd order interactions at the same fixed level of significance. The binary 2nd order estimates of the process is

$$Y = Y + \sum_{i \in I} (a_i/2)\, X_i + \sum_{ij \in IJ} (a_{ij}/2)\, X_i X_j$$

A7 The Residual Plot

If the number of significant effects is small compared to the total number of residuals then one can interpert the plot of residuals on normal probability paper. If the residual points lie more or less on a straight line then one may conclude that the unexplained variation is due to random noise and that the identified significant effects explain the process. If this does not happen then the proposed binary estimate does not fully capture the underlying process and more work needs to be done.

A8 The Continuous Estimate

If an input factor, is in fact a continuous variable, with an interval or ratio scale, then the binary estimate may be transformed to a continuous estimate. Let z_i be the continous input factor such that:

$$z_i = \begin{cases} ia \text{ in the a case} \\ ib \text{ in the b case} \end{cases}$$

Note that $X_i = (2z_i - ia - ib)/(ib - ia)$

has the following property:

a, if $z = ia$ then $X_i = -1$

b, if $z = ib$ then $X_i = 1$

To construct the continuous estimate replace X_i in the binary estimate with $(2z_i - ia - ib)/(ib - ia)$.

Appendix B
APPLICATION

B1 Analysis of the Net Economic Benefit

The main effects of three of the input factors, the discount rate, the demand elasticities and the probability of a major disruption are statistically significant at the $p \leq .10$ level. In addition there are perceptible 2nd order interactions between each pair of the input factors which had statistically significant main effects. Therefore each pair of these input factors must be evaluated jointly. The two way diagram of figure B-1 depicts the nature of these interactions.

Assuming a conventional discount rate of 10% the Teisberg Model estimates that an increase of the BAU price of crude oil from $52.00 per barrel to $90.00 per barrel will increase the net economic benefit from $6.63 billion to $54.38 billion. If a discount rate of 2.5% is assumed, the identical change in the price of crude oil will increase the net economic benefit from $25.20 billion to $136.17 billion.

Given the assumption that their is virtually no chance of a major disruption the Teisberg Model estimates that a change of the discount rate from 10.0% to 2.5% will increase the net economic benefit from $10.51 billion to $32.61 billion. If the annual probability of major disruption is 0.10 then the identical change in the discount rate increase the probability of a major disruption from $50.50 billion to $128.76 billion.

If one assumes that there is virtually no chance of a major disruption the Teisberg Model estimates that a change in the BAU price of oil, from $52.00 per barrel to $90.00 per barrel will increase the net economic benefit from $9.11 billion to $34.01 billion. An increase in the annual probability of a major interruption to 0.10 causes the Teisberg Model to estimates that a change in the price of crude oil from $52.00 per barrel to $90.00 per barrel will increase the net economic benefit from $22.72 billion to $156.54 billion.

Figure B-2 is the normal probability plot of the effects which appear in Table B-1 and represented by Figure B-1. If the fifteen effects from the model were not due to changes of the input factors then the effects are due to some random variation which is assumed to be normal. If this is the case the normal probability plot of effects should appear more or less as a straight line. Figure B-2 suggests that effects 3, 4, 10, 1, and possibly 6 and 7 are not on the same "straight" line formed by the remaining effects. This plot tends to confirm the identification of significant effects by the method outlined in paragraph A4.

The Teisberg Model
Average Net Economic Benefit

Mean	Estimate
	55.59
Main Effects	
1. Discount rate	50.18*
2. BAU crude price	21.52
3. Demand elasticities	79.36*
4. Probability of a major disruption	68.07*
2nd Order Interactions	
5. Discount rate X BAU crude price	9.39
6. Discount rate X Demand elasticities	31.61*
7. Discount rate X Probability of a major disruption	28.08*
8. BAU crude price X Demand elasticities	16.25
9. BAU crude price X Probability of a major disruption	21.87
10. Demand elasticities X Probability of a major disruption	54.46*
3rd Order Interactions	
11. Discount rate X BAU crude price X Demand elasticities	5.95
12. Discount rate X BAU crude price X Probability major disruption	8.57
13. Discount rate X Demand elasticticies X Probability of a major disruption	21.69
14. BAU crude price X Demand elasticities X Probability of a major disruption	16.66
4th Order Interaction	
15. Discount rate X BAU crude price X Demand elasticities X Probability of a major disruption	6.10
Estimated standard error	13.37
Level of statistical significance at $p < 0.10$	26.95

* Significant effects at $p < 0.10$

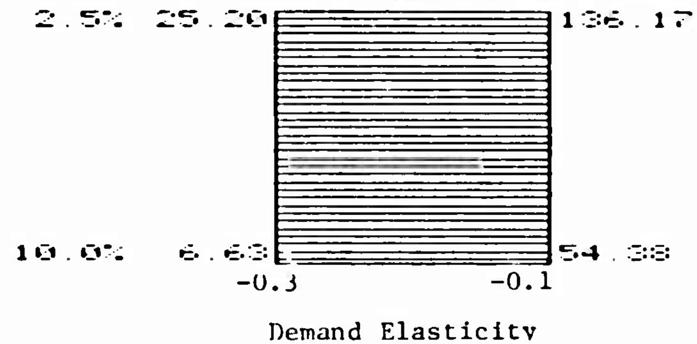
Table B-1

THE TEISBERG MODEL

figure B-1

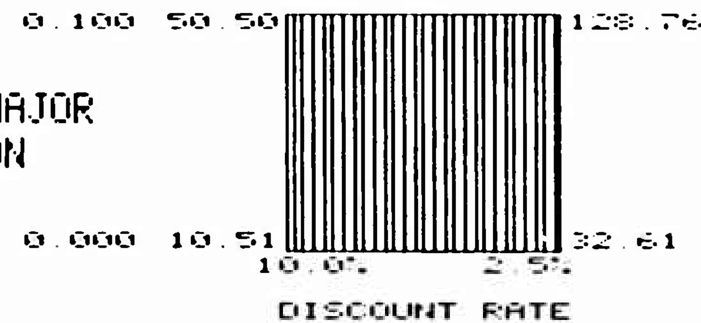
Total U.S. Consumption of Products in MMBD
2nd Order Interactions

DISCOUNT RATE



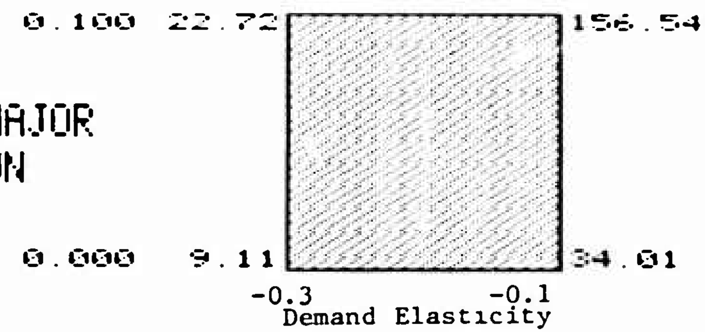
The net economic benefit as a function of the discount rate and the Demand Elasticity

PROB. OF A MAJOR DISRUPTION



The net economic benefit as a function of the probability of a major disruption and the discount rate.

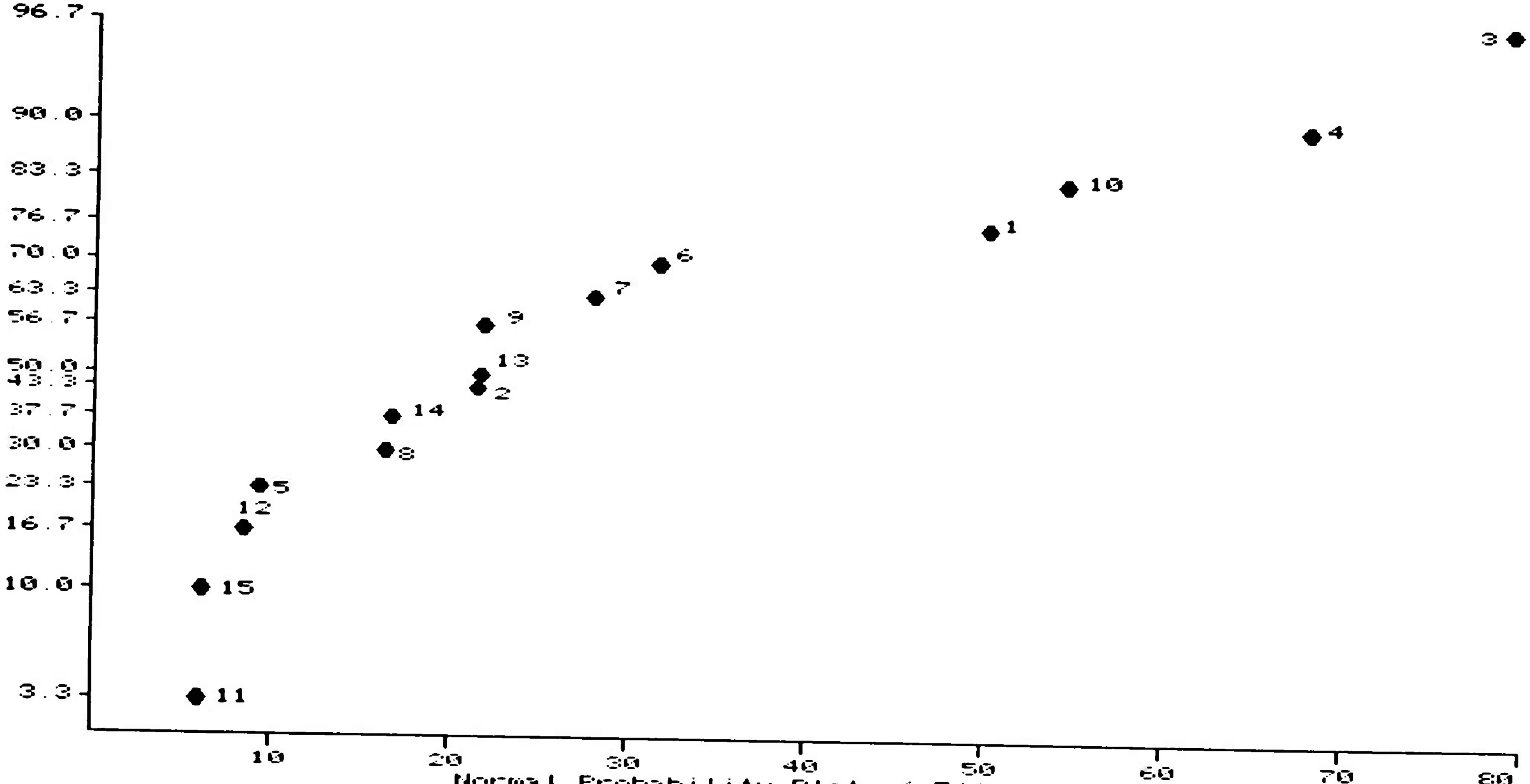
PROB. OF A MAJOR DISRUPTION



The net economic benefit as a function of the probability of a major disruption and the Demand Elasticity

THE TEISBERG MODEL

figure B-2



Normal Probability Plot of Effects
Numerals Indicate Effect Number

B2 The Binary Estimate

Define:

$$x_d = \begin{cases} -1 \text{ if } d = 10.0\% \\ 1 \text{ if } d = 2.5\% \end{cases}$$

$$x_e = \begin{cases} -1 \text{ if } e = -0.3 \\ 1 \text{ if } e = -0.1 \end{cases}$$

$$x_p = \begin{cases} -1 \text{ if } p = 0.000 \\ 1 \text{ if } p = 0.100 \end{cases}$$

Where d is the discount rate, e is the elasticity of demand, and p is the probability of a major oil disruption.

With the definitions above and the information contained within the analysis of the net economic benefit (section B1) one can construct the following binary estimate:

$$Y = 55.59 + (50.18)/2\ x_d + (79.36)/2\ x_e + (68.07)/2\ x_p +$$
$$(31.61)/2\ x_d x_e + (28.08)/2\ x_d x_p + (54.46)/2\ x_e x_p$$

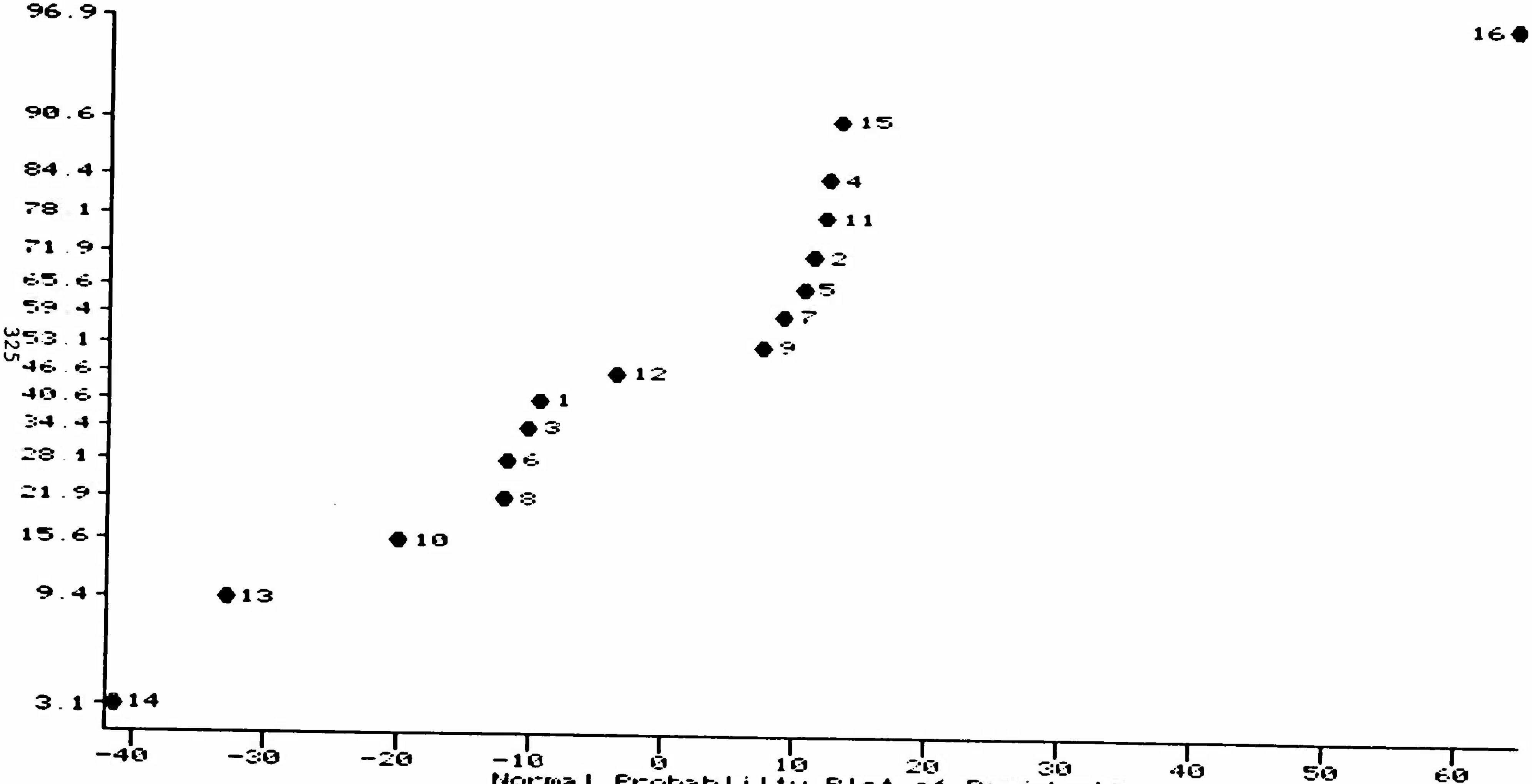
or

$$Y = 55.59 + 25.09\ x_d + 39.68\ x_e + 34.04\ x_p +$$
$$15.81\ x_d x_e + 14.04\ x_d x_p + 26.23\ x_e\ x x_p$$

A normal probability plot of the residuals, figure B-3 can be used to examine the adequacy of this estimate of the Teisberg Model. The residuals for this estimate, are found in Table 1. If all of the variation is expalained by the proposed estimating equation then the normal probability plot of residuals will lie more or less on a straight line. Clearly the residual from observation 16 and most likely observations 14 and 13 do not lie on the "straight" line formed by the remaining observations. This suggests that although an improvement in the original estimate has been accomplished, more work remains to be done. Promising avenues of investigation include transforming the data before the application of a factorial design as proposed by Daniel and/or the use of response surface analysis.

THE TEISBERG MODEL

figure B-3



Normal Probability Plot of Residuals
Numerals Indicate Observation Number

B3 The Continuous Estimate

To construct the continuous estimate from the binary estimate replace:

$$X_d \text{ with } \frac{2d - 0.025-0.1}{0.025 - 0.1} = \frac{2d - 0.125}{-0.075}$$

$$X_e \text{ with } \frac{2e + 0.1 + 0.3}{-0.1 + 0.3} = \frac{2e + 0.4}{0.2}$$

$$X_p \text{ with } \frac{2p - 0.1}{0.1 - 0} = \frac{2p - 0.1}{0.1}$$

to obtain:

$$\begin{aligned} Y = 55.59 &+ 25.098\ ((2d - 0.125)/-.075) \\ &+ 39.68\ ((2e + 0.4))/0.2) + 34.04\ ((2p-0.1)/0.1) \\ &+ 15.81\ ((2d - 0.125)/-0.75))((2e + 0.4)/2) \\ &+ 14.04\ ((2d - 0.125)/-0.75)((2p-0.1)/0.1) \\ &+ 26.23\ ((2e + 0.4)/0.2)((2p-0.1)/0.1) \end{aligned}$$

which simplifies to:

$$\begin{aligned} Y = 119.57 &- 1{,}137.87\, d + 398.00\, e + 2198.00\, p \\ &-4216.00\, de - 7488.00\, dp + 5246.00\, ep \end{aligned}$$

B4 The Differential Estimate

If c(w) denotes the change in the variable w, then the estimate of the change of the net benefit is:

$$\begin{aligned} c(y) = &-1137.87\, c(d) + 398.00\, c(e) + 2198.00\, c(p) \\ &-4216.00\, d\, c(e) - 4216.00\, c(d)\, e \\ &-7888.00\, d\, c(p) - 788.00\, c(d)\, p \\ &+5246.00\, e\, c(p) + 5246.00\, c(e)\, p \end{aligned}$$

Although this was developed as a global estimate it can be used for local approximations. If the model has been evaluated for a set of input factors (d,e,p) and one wishes to estimate the net economic benefit for a point (d',e',p') which is close to (d,e,p) then calculate the c(y), the change in the net economic benefit and add that value to the model's estimate for the point (d,e,p).

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