A Simulation Process for Determining Reliability
of Cyclic Random Loaded Structures

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Abstract

A unique application of the Monte Carlo method was developed for determining reliability vs. cycles to failure of the M60 tank torsion bar. In applying the method, material torsional fatigue and spectrum loads were modelled such that variability in the functional parameters and operational loads were represented. Random torsional displacement values obtained from the amplitude displacement distributions applied to the fatigue equations resulted in an exponential distribution for cycles to failure of the in service bar. The number of simulations in the Monte Carlo process was determined from a convergence criteria involving stability of the third and fourth moments of the cycles to failure distribution.

Reliability vs. bar life computations indicated a negligible amount of life after flaw initiation. Assuming a design change involving a twenty percent reduction in bar stresses increased the life estimates by a factor of three. An increase in reliability can also be realized if computations are made by assuming a bar has been in operation for a specified number of cycles. A comparison of minimum life (ninety nine percent probability of survival) between predicted and in service results showed excellent agreements (less than eight percent difference).

Introduction

The current need for establishing reliabiity of various components and systems for U.S. Army weapon vehicles is being realized. The consequences of over-or under design are often reflected in either premature failure or excessive costs and poor performance due to excessive weight. The mean life estimates used as a criteria for defining acceptability of cyclic loaded component will often provide a false sense of security regarding its capability. The application of higher strength ferrous materials or the less conventional structural materials such as composites and ceramics will often result in premature failure because of the inability to recognize the inherent variability of the materials strength.

The objective of this paper is to determine a methodology which will circumvent the present deterministic approach used in establishing an acceptable design for cyclic random loaded structure. Instead of analyzing the worst case situation related to the spectrum loads, S/N curve, or crack propagation laws, the authors introduce a method which simulates the variability in loading and materials capability. Use of this methodology eliminates the over (worst case) or under design (mean life) situation by introducing a probabilistic design criteria. Recognition of the reliability values as a function of the life cycles of operation can provide the opportunity for selecting a specified life value corresponding to the probability estimate. The remaining component life can then be determined as related to its probability number.

The recommended ASTM procedure for determining acceptable design, involves establishing a lower confidence 3 Standard Deviation bound on the S/N Curve then selecting cycles to failure from the bounded curve consistent with predetermined maximum stress obtained from the spectrum load results. This procedure can often result in an over design situation since the maximum load may rarely occur in-addition to the fact there is a small chance that the lower S/N Curve bound is representive of the True S/N Curve.

The Monte Carlo process used in predicting life time versus reliability of the M60 torsion bars had a prior application in a report by (1). Conceptually, this method is quite simple, requiring modelling of the spectrum loads and the material fatigue life with respect to crack propagation or stress/cycles to failure.

Amplitude Displacement Model

In figure 1, a schematic of the torsion bar in the M60 Tanks is shown. The amplitude distributions of three bars from tests conducted at Aberdeen Proving Grounds (APG) is shown in figure 2 . Positive and negative angular displacements of the bars as function of tank travel are shown in figure 2a. In figure 2b the amplitude distributions are listed in a manner describing percent time less than by a plus sign (+) and percent time grater than by a minus sign (-), (eg. 25% level equals a -75% level. The + peak represents maximum angular displacement under load, the negative peak is maximum unloaded angular measure. In order to eliminate considering positive and negative peak values in figure 2a for determining angular displacements in the cyclic loading process, the angular displacement is defined as follows,

$$\Delta\theta = \theta + |\theta^-|$$

where $\theta^- =$ maximum negative angular displacement   (1)

$\theta =$ displacement from figure 2b

$\Delta\theta$ represents the adjusted angular displacement

The Beta distribution provided the best representation of the skewed amplitude distribution. The dampening effects that occured under load resulting from a stop used in preventing further angular twist of the bar producing a highly skewed discrete cumulative probability values. The Beta function is defined as:

$$f(\Delta\theta) = \frac{\Gamma(P + Q)}{\Gamma(P)\Gamma(Q)}(\Delta\theta)^{P-1}(1-\Delta\theta)^{Q-1}$$

and $0 \leq \Delta\theta \leq 1$ $P, Q > 0$ (2)

The P and Q values are selected in a manner that provides the best Probability Density Function (PDF) for representing the data. Figure 3 describes a typical distribution and Table 1 shows the excellent correlation between predicted (Beta representation) and actual test results. Angles less than $20°$ represent stresses sufficiently low that infinite torsion bar life could be expected, therefore, a good representation below this angle is not essential.

Crack Growth Law For Estimating Torsion Bar Life

Initial efforts in applying the Monte Carlo Method for determining reliability vs cycles to failure of the torsion bar involved using the crack propagation laws. The da/dN relationships for materials metallurgically similar to the specified material were obtained from (2), (3), and (4) and is shown in figure 4. The dry air results made available by Barsom (4) provided the most representative estimates of crack growth vs stress intensity (ΔK) described in figure 4 since the torsion bar is protected from the environment. From the basic da/dN relationship, N cycles to failure as a function of crack growth, angular displacement and the geometry of the region where the crack initiates in the bar, may be obtained from the following relationships:

$$N = \int_{c_i}^{c_f} \frac{dc}{.66 \times 10^{-8}\Delta K^{2.25}}$$

                                                                 (3)

$$\text{where } \Delta K = A_j \Delta \theta \sqrt{\pi C}$$

and $A_1 = 4.91$ (Key Way)

$A_2 = 3.29$ (Other Spline Regions)

$A_3 = 3.26$ (Shaft Section)

Note, a percent reduction in A_i's will provide a decrease in the stresses in the specific region of the torsion bar. The C_i and the C_f parameters are initial and critical crack size respectively. The C_f is obtained from critical stress intensity value K_{IC} for the material considered. The angular displacement of the bar can be also represented by the equivalent stress value τ as

$$\text{Max } \tau - rG (\Delta \theta)/L$$

r = radius of shaft

G = torsional modulas

$\Delta \theta$ = max. allowed angle

L = length of torsion bar

(4)

The Monte Carlo Process

(A) Crack Propagation Analysis

A schematic of the process is outlined in figure 5 for determination of frequency of occurence vs. cycles to failure of the torsion bar using the crack propagation law. An assumed normal distribution is used to represent variability in the A_j, C_i, and C_f parameters. A coefficient of variation (C.V.) defined as

$$\text{C.V.} = \frac{\text{S.D}}{\text{mean}}$$

(5)

establishes the standard deviation S.D. for the corresponding known mean value (eg C_i for initial crack size). C.V. values of 5, 10 and 15 percent were considered in developing the distributions in order to examine the effects of variability (inherent errors in measurements, flaw size assumption or the stress analysis) in the parameters. By selecting the above C.V.'S a sensitivity analysis can be developed, thereby providing a method for recognizing the importance of the parameters as related to cycles to failure number. The Beta distribution as shown in figure 5 has been previously defined in equation (2).

The random numbers used in the Monte Carlo process are obtained from solving for X in

$$\int_{-\infty}^{X} f_i \, dX = R$$

(6)

where R is a uniform random number and f_i corresponds to the desired type of frequency distribution for the parameter. A probability density function for the N cycles to failure can be obtained by randomly selecting from C_i, C_f, A_i, and $\Delta\theta$ distributions of discrete sets of numbers and substituting them into equation 3. Note, there should be an equal amount of random numbers for each parameter to have proper amount of numbers for the N distribution.

(B) S/N Curve Analysis

Torsional bar life expectancy was obtained using the Monte Carlo process applied to the S/N Curve relationship. The procedure provided a method for obtaining life time estimates of the bar by combining the effects of crack initiation and propagation. A description of the S/N Curve is shown in figure 6, where the base line data was obtained from a literature survey for material metallurgically similar to the torsion bar material. The survey provided a set of S/N Curves for torsional fatigue shown below for best representing the current materials used in the bar.

$$Log_{10}N = B + .068\Delta\theta \qquad (7)$$

$$\text{where } B = 7.70$$

The slope value of .068 was essentially the same for all curve in the set. The adjustment in B from 7.70 to 8.06 made on the basis of M60 torsion bar quality assurance tests at a single $\Delta\theta$ value performed at the Scranton manufacturing facility (See figure 6). A single load equivalent to a 42 degree angular displacement was applied during the quality assurance torsional fatigue test. Using the mean value and the cycles to failure in Figure 7 provided a more accurate estimate of (B). The curves representing a range of 10 and 20 percent reduction in bar stress are shown in figure 6.

The S/N Curve Monte Carlo process is similar to the previously outlined method for da/dN relationships. The primary difference involves using Models for (B) and $\Delta\theta$ from figure 6 and 2 respectively. A schematic of the basic S/N representation is shown in figure 8a and 8b. In figure 8a simulation of S/N curve variability is shown for a specific value. Figure 8b describes probability density function (PDF) for (B). A random selection of a discrete set of numbers from $\Delta\theta$ and (B) distributions is then applied to equation 7 in order to obtain $Log_{10}N$ value. The process is repeated until all values from the two distributions are selected. This process will then provide a PDF to represent $Log_{10}N$.

Torsion Bar System Reliability

By assuming a tank with a N torsion bar system the following procedures would be applied in order to establish reliability of the system. If any one bar could cause failure (independence) then reliability R will be

$$R = \prod_{j=1}^{N} P_j \qquad \begin{cases} P_j - \text{Prob. of Survival} \\ j - j^{th} \text{ Torsion Bar} \end{cases} \qquad (8)$$

if it is assumed that all torsion bars must fail for system failure (dependence) then,

$$R = P_1 \times P_2 / P_1 \times \ldots \times P_N / P_{N-1} / \ldots / P_1 \qquad (9)$$

where $P_N / P_{N-1} / \ldots / P_1$ is the reliability of Nth bar, given reliabilities of N-1 ----, 1 bars.

Reliability of Operation After Specific Number of Cycles

The reliability of operating an additional number of cycles when a specified number of cycles of operation has been completed is obtained in the following manner. Initially it is assumed that a specified distribution function say $f(N)$ is known. For example the distribution of $Log_{10}N$ from Monte Carlo method previously described. The reliability $R(n_1, n)$ is a conditional probability requiring the probability of operating for $n_1 + n$ cycles when n_1 cycles have been completed. That is

$$R(n_1 + n) = \frac{R(n_1 + n)}{R(n_1)} = \frac{\int_{n_1 + n}^{\infty} f(N)dN}{\int_{n_1}^{\infty} f(N)dN} \qquad (10)$$

where n is the additional mission in cycles after n_1, cycles of operation. The number $N_s(n_1, n)$ of components (torsion bars) that will survive an additional n cycles is given by

$$N_s(n_1 \; n) = N_s(n_1) \cdot R(n_1 \; n) \qquad (11)$$

where $N_s(n_1)$ = number of components starting the mission of n additional cycles.

Results and Discussion

The proper number of simulations for the Monte Carlo Method depended on the models under consideration. For example 5000 and 3000 were required for the da/dN and S/N curve models respectively. Using a convergence rate criteria for the calculated 1 percent values (see Ps in figure 9) and recognition of the third and fourth moment stability of the $Log_{10}N$ distribution provided an excellent method for determining required number of simulations. Differences in percentile values for C.V.'s of 10 and 15 percent were minimum. The 10 percent value was used for all $\frac{da}{dN}$ calculations.

The torsion bar reliability results from the da/dN relationship as shown in figure 10. The current design results were obtained from equation 3, with $A_2 = 3.29$. They indicated relative limited lifetime range of 14 to 500 miles, with a probability of survival values of .99 and .01 respectively. An appropriate increase in C_f from equation 3 represents the 40% increase in K_{IC} value. This represents an improvement in materials capability with respect to acceptance of larger flaw sizes prior to failure. The slight improvement in the bars capability indicates that an improvement in material will not significantly improve bar performance. The 25 and 50 % reduction in K_I (stress intensity) in figure 9 is obtained from reducing A_2 in equation 3 by the respective percentages. These reductions represent improvements in the design of spline section of the bar as shown in figure 1. The K_{III} failure in the shaft represents situations where failure occurs in shaft rather then spline region.

The maximum life of 70 miles at 25 mph achieved from 50 percent improvement in spline design with .99 probability of survivability indicates that there is a very limited life of the bar after crack initiation. Table 2 describes minimum life estimates (99 percent survivability) for the torsion bar with respect to various tank velocities and the design improvements. Tank travel at 5 mph (lowest speed) with a 50% reduction in K_I value shows propagation life expectancy of only 341 miles at .99 P_s.

In figure 11, the frequency distribution obtained from S/N curve – Monte Carlo application is shown. The resultant exponential form is consistent with that expected from the S/N modelled in the analysis.

A graphical display of P_s vs miles to failure is shown in figure 12 for the 25 mph tank velocity. The life expectancy of the bar is somewhat greater then that obtained from the da/dN analysis. The minimum life estimates ($.99P_s$) of 292 miles is 21 times greater than 14 miles determined from the da/dN results. This result indicates that most of bar life occurs prior to crack initiation. Therefore the torsion bar should be manufactured in such a manner that flaws are minimized. The current shot peening used in the manufacture of the bar indicates recognition of this fact by the manufacturer. The bar reliability estimate obtained after an assumed 741 miles of tank travel (see figure 12), was obtained from equation 10. The increase in Ps from .90 to .99 if the bar survives the initial 741 miles doe not provide a sufficient gain to warrant re-using bars since the minimum increase in expected life is reduced very rapidly. The results from a 20 percent reduction in design stress of 865 miles for a Ps of .99, is a considerable improvement when comparing that of 292 miles using in the current design. In table 3 the results from velocity ranging from 5 mph to 25 mph in increments of 5 mph are shown with respect to current 10 and 20 percent improvements in design. Reducing velocity of tank operation obviously improves reliability of the torsion bar. In this report, the experimental data and reliability calculations refer to failure of the first bar.

Examination of current design mileage capability of the bar for 20 and 25 indicates a range from 276 to 292 miles. These results agree with the 262 miles minimum life obtained from Aberdeen Proving Ground (APG) test results (Report MT-5376 of bar failure from 3 mile test course), (see figure 13). This course and tank velocity were similar to those used in obtaining the spectrum load results. The excellent agreement between the predicted and actual life expectancy of the bar indicates the desirability of Monte Carlo Process for modelling variability of spectrum loads (design stress) and S/N curve (material capability) results.

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Although excellent agreement has been obtained, the authors would have preferred representing the spectrum load consistent with an individual peak to peak angular displacement. The simplification applied using the negative peak as base and representing the displacement relative to this value was a good approximation to the available individual displacements. This approximation would provide a slightly conservative estimate in the reliability values. Using the ASTM recommended practice of representing lower 3 standard deviation band of the S/N curve as measure of material fatigue loading capability combined with maximum angular displacement (46 degrees) for 25 mph. The tank operation resulted in a minimum life estimate of 112 miles for the bar. Selecting this number as a design allowable could result in an overly conservative estimate. The chance that this maximum displacement could occur and the S/N curve was the actual lower band described above is extremely small.

A minimum life of 575 miles was obtained from using the maximum $\Delta\theta$ displacement value with original S/N curve where $B = 8.06$. This result is obviously wrong since the limited samples of 23 bar failures two of them failed at mileage less than 400 miles (See figure 13).

Conclusions

1. A methodology for obtaining reliability of the M60 tank torsion bar subjected to cyclic random loads has been developed where probability of survival is represented as function miles of tank travel.

2. The developed methodology could be applied to other structures with cyclic random loads.

3. The use of the method appears justified from recognition of the excellent agreement between predicted reliability estimates and those obtained from the actual bar life (miles to failure) experienced during the tank operation.

4. Determination of minimum bar life was 21 times greater from application of S/N curve model than that of the assumed da/dN model. This indicates most of the bar life exist prior to crack initiation.

5. Application of deterministic procedures, (use of lower 3 S.D bound for S/N curve (ASTM method) and mean S/N curve providing over and under design allowable estimates while Monte Carlo method outlined in the text values accurately described acceptable design values.

References

1. D.M. Neal and D.S. Mason, "Determination of Structural Reliability Using A Flaw Simulation Scheme", Army Materials and Mechanics Research Center TR81-53, 1981.

2. "Damage Tolerant Design Handbook", Metals and Ceramics Information Center, Battelle Columbus Laboratories, MCIC-HB-01, Page 8.2-D, 1975.

3. Ibid. Page 8.2-E.

4. J.M. Barsom, Transactions of the ASME, Journal of Engineers for Industry, Series B, 93, No. 4, Nov. 1971.

SHOT PEEN SERRATIONS & BODY
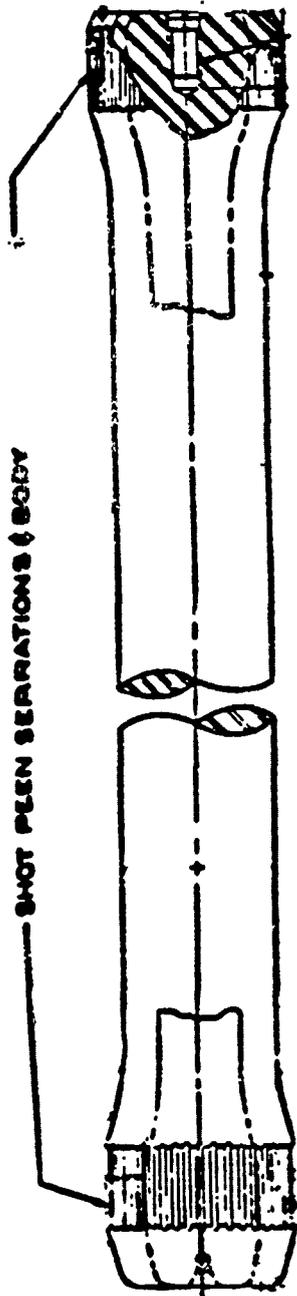
FIGURE 1 M60 TANK TORSION BAR

Load Spectrum (Run #48 - Speed 25 mph)

Bar 1

Bar 2

Bar 7 (equivalent 6)
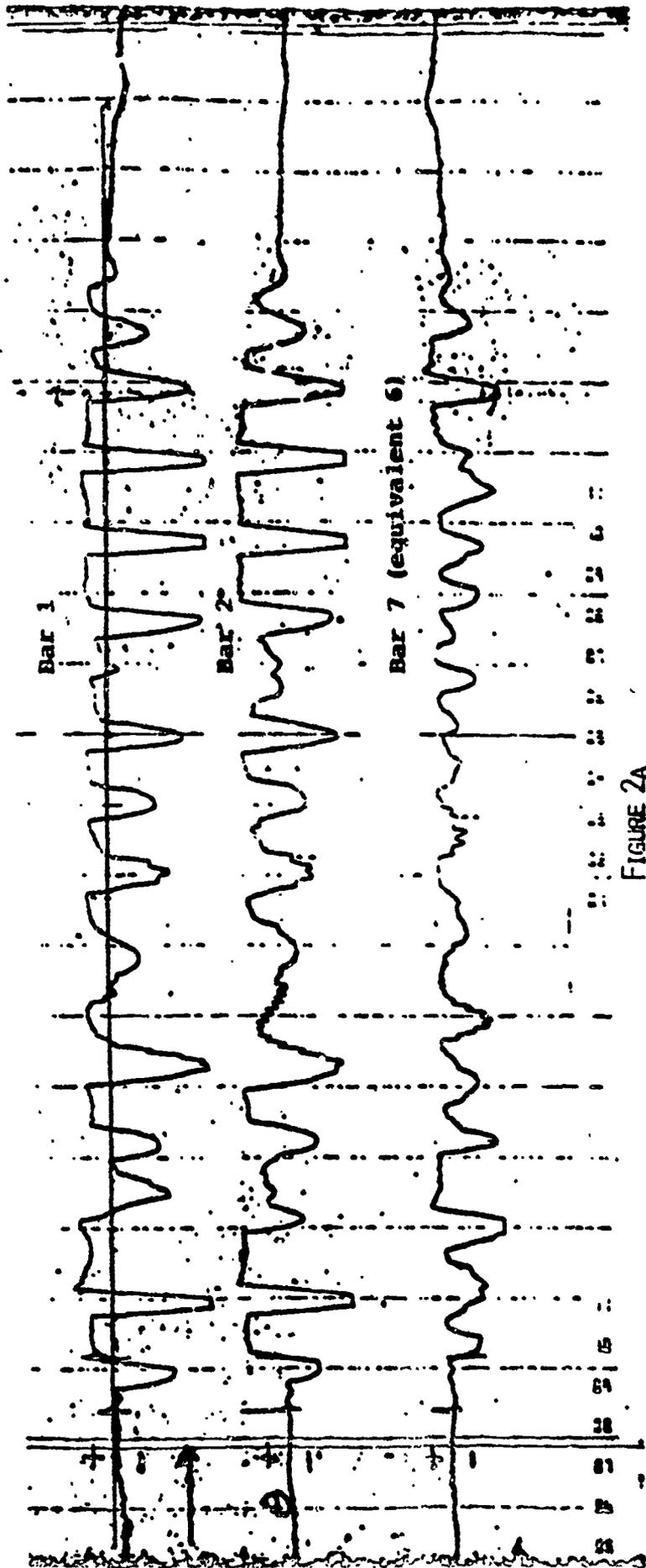
FIGURE 2A

Course Length 462 ft

AMPLITUDE DISTRIBUTION DATA RUN 48 (25 MPH)

	+ Peak	- Peak	+ 99%	- 99%	+ 66%	- 66%	+ 50%	+ 25%	- 25%
BAR 1	15.47	-30.71	15.33	-28.44	8.81	-3.51	3.00	-8.04	10.91
BAR 2	21.70	-28.92	21.42	-26.80	8.41	-7.43	-1.06	-11.53	13.92
BAR 7	13.91	-30.61	13.76	-30.47	6.63	-2.39	2.56	-7.19	8.35

FIGURE 2B

Spectrum Load Representation (Run #48 – 25 mph)

Beta Distribution

Probability Density

$\Delta\theta$ (degrees)

FIGURE 3

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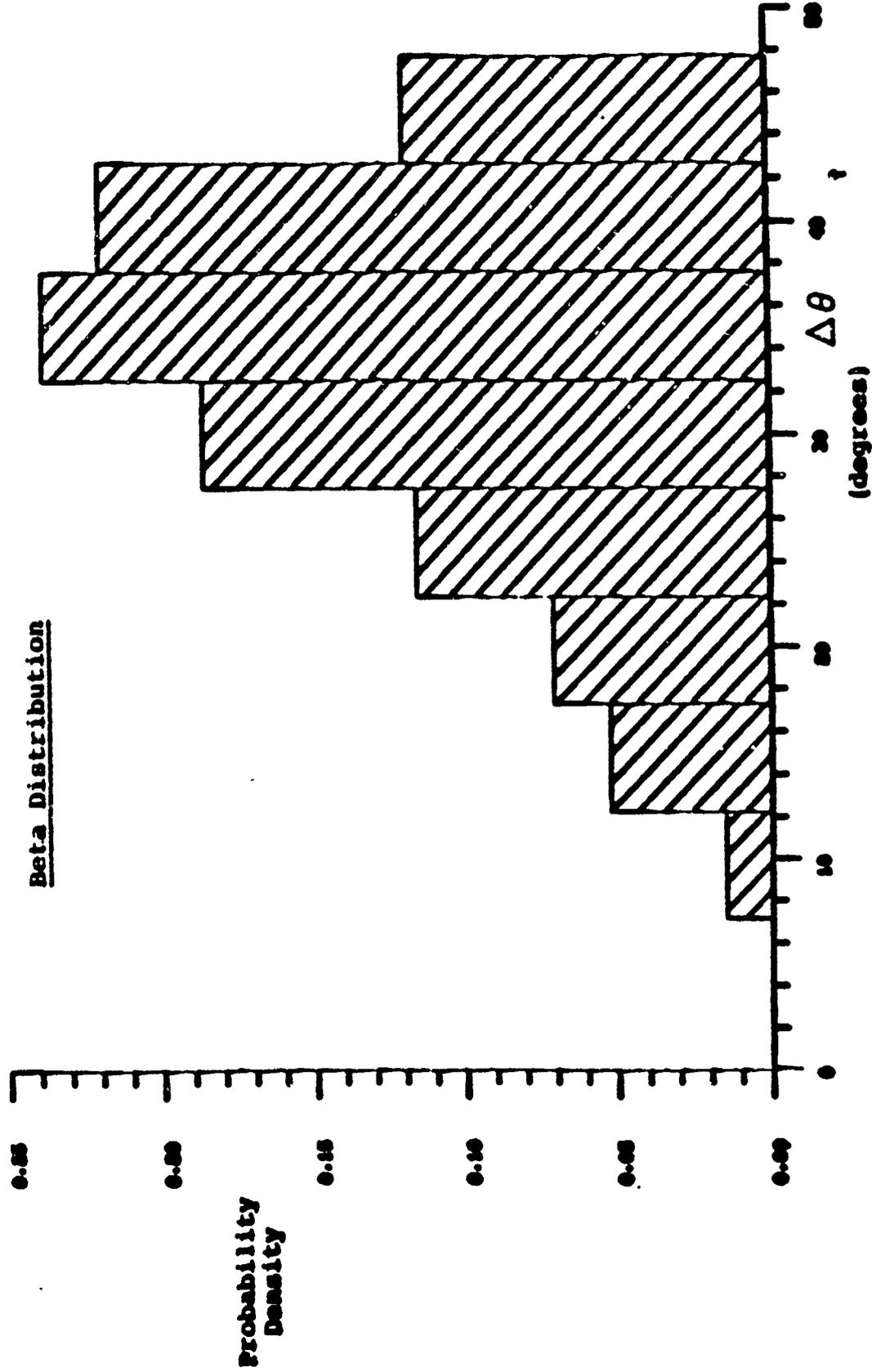
Determination of Cycles to Failure (LEFM Appr:)

Yield Stress 220 KSI
Distilled Water

$$\frac{da}{dN} = 3.5\times10^{-5}\Delta K^{.533}$$

ref. (2)

YS=220 KSI
Lab Air

$$\frac{da}{dN} = 9.1\times10^{-8}\Delta K^{1.71}$$

ref. (3)

$\dfrac{da}{dN}$

(in/cycle)

Dry Air (Barsom)
Frequency, YS Indep.

$$\frac{da}{dN} = .66\times10^{-8}\Delta K^{2.25}$$

ref. (4)

10^{-4}

10^{-5}

10^{-6}

10   20   30   40

ΔK (KSI in)

FIGURE 4

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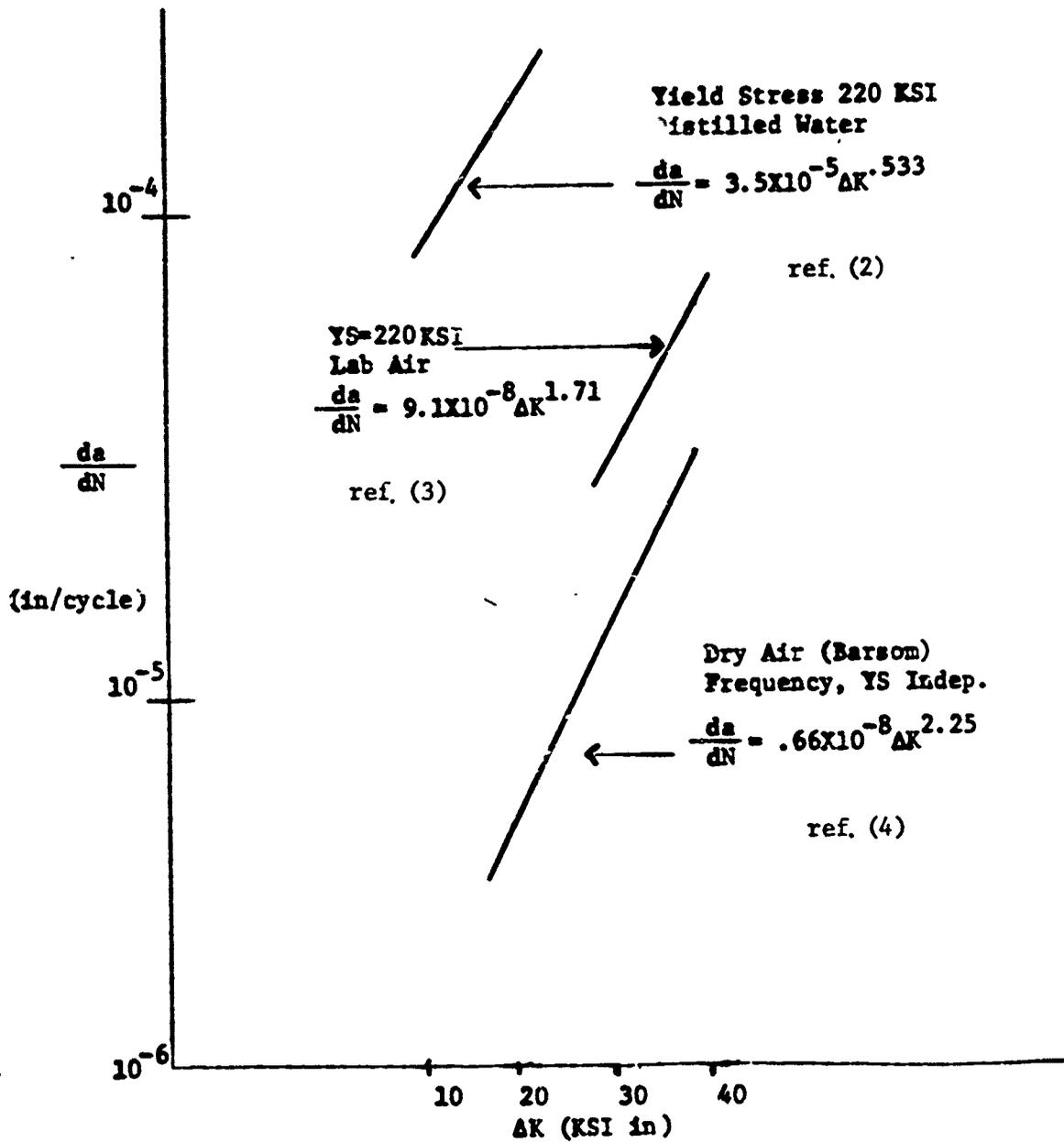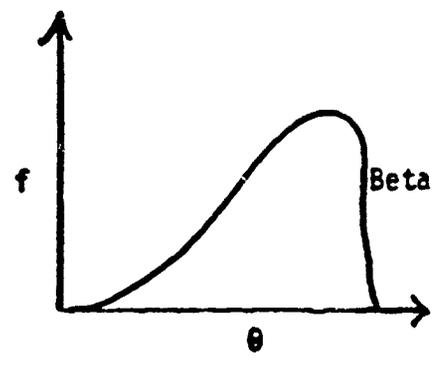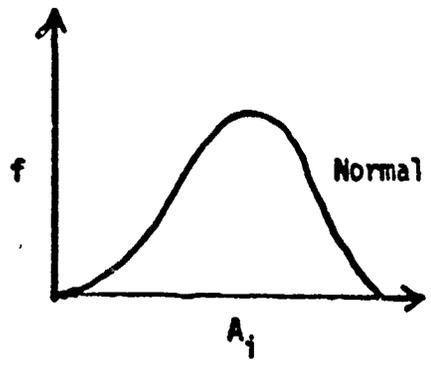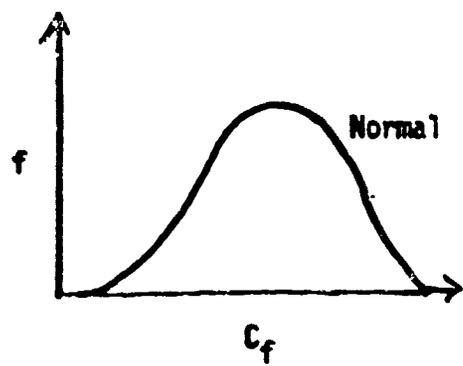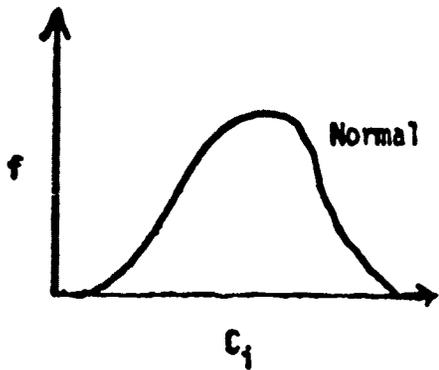
$$\int_{-\infty}^{X} f_i \, dX = R$$

R = UNIFORM RANDOM NUMBERS

f_i = FREQUENCY DISTRIBUTION

$\overline{C}_i$ = .001 in.

$\overline{C}_f$ = .0133 in.

$\overline{A}_2$ = 3.29 in. (Spline Region)

[C.V. = 5,10,15 Percent]
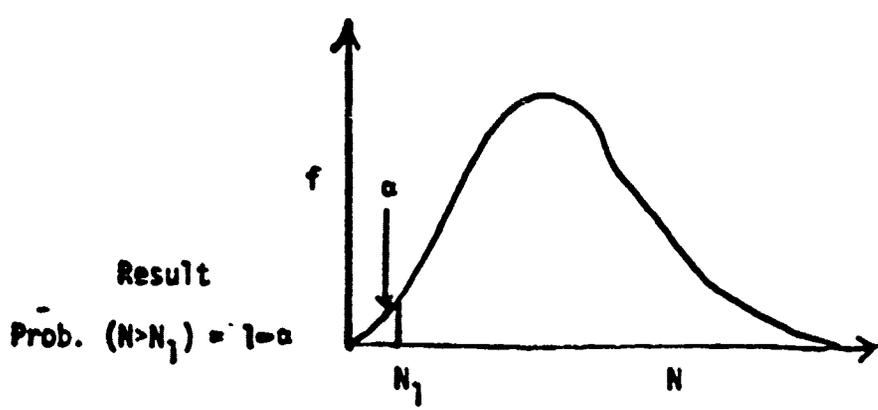
Result

Prob. $(N>N_1)$ = 1-α

FIGURE 5

Torsional Fatigue Life

Cycles to Failure Relation
$Log_{10}N = B - D \triangle \theta$

Current Design (B = 8.06, D = .068)
Adjusted From Base Line Data to
Agree with Scranton's Torsional Tests

B = 8.06, D = .0618 (10% Reduction in Bar Stress)

B = 8.06, D = .0567 (20% Reduction in Bar Stress)

Base Line Data
(B = 7.70 and D = .068)
4340 - 230 KSI Yield
Based on Smooth Spec. Data
Mean Stress = 0 (R = -1)
Room Temp. Lab. Air
Shoot Peened Surface

Angle Range (Deg.)

Cycles (N) to Failure

10^4 10^5 10^6 10^7 10^8

50 40 30 20

Figure 6

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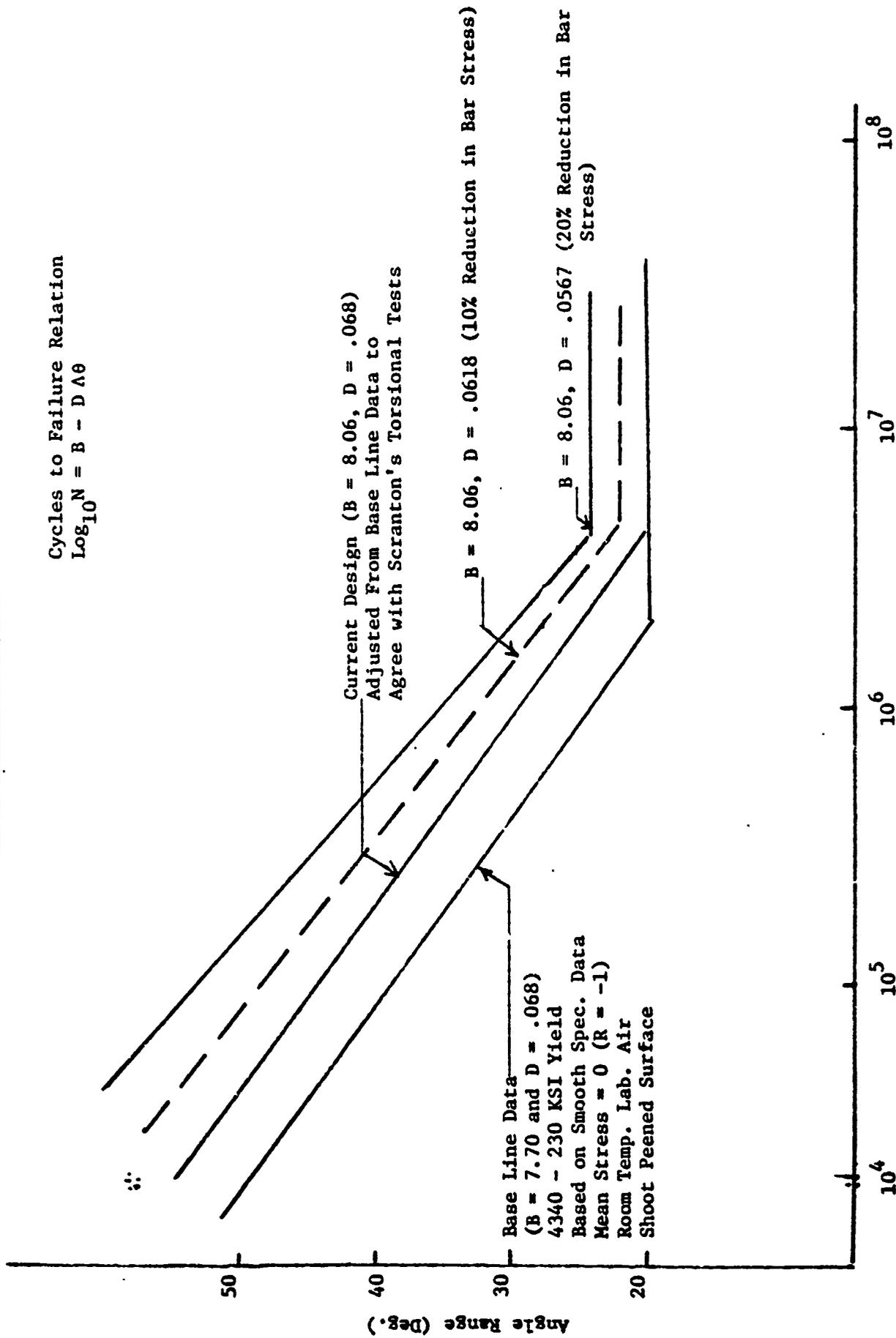
M60 Torsion Bar Fatigue Test Results (Pooled Data 1977 to 1982)

Mean = 5.206
Standard Deviation = .239

Design Allowable

A = 4.55
B = 4.83

STRESS DATA EVALUATION PROGRAM

Weibull
Normal
Lognormal
Medical

Log Base 10 Cycles to Failure

FIGURE 7

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Torsional Fatigue Life

Cycles to Failure Relation

$$\text{Log}_{10}N = B + .068\,\Delta\theta$$

B = 8.06

Angle Range (Deg)

50   40   30   20

Cycles (N) to Failure

10^4 10^5 10^6 10^7 10^8
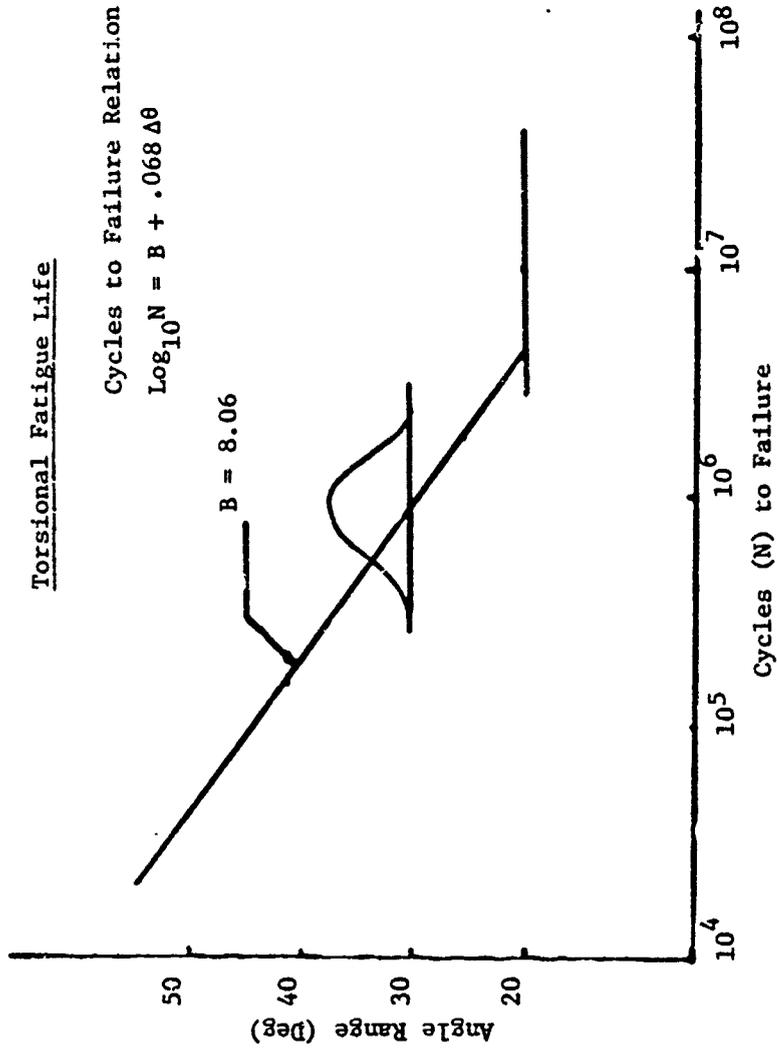
FIGURE 8A

Simulation of B from S/N Curve Representation

Probability Density

.25   .20   .15   .10   .05   .00

7.4   7.6   7.8   8.0   8.2   8.4   8.6

B

S/N Curve $\text{Log}_{10}N = B + D\Delta\theta$
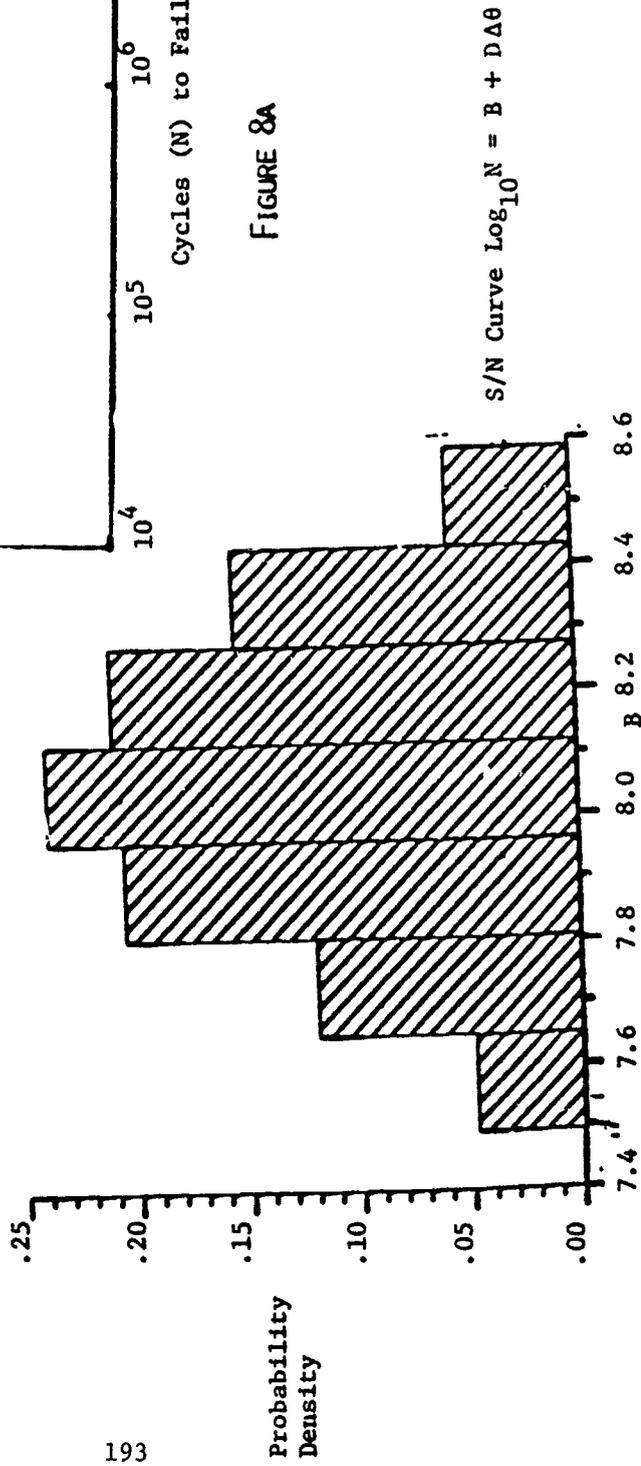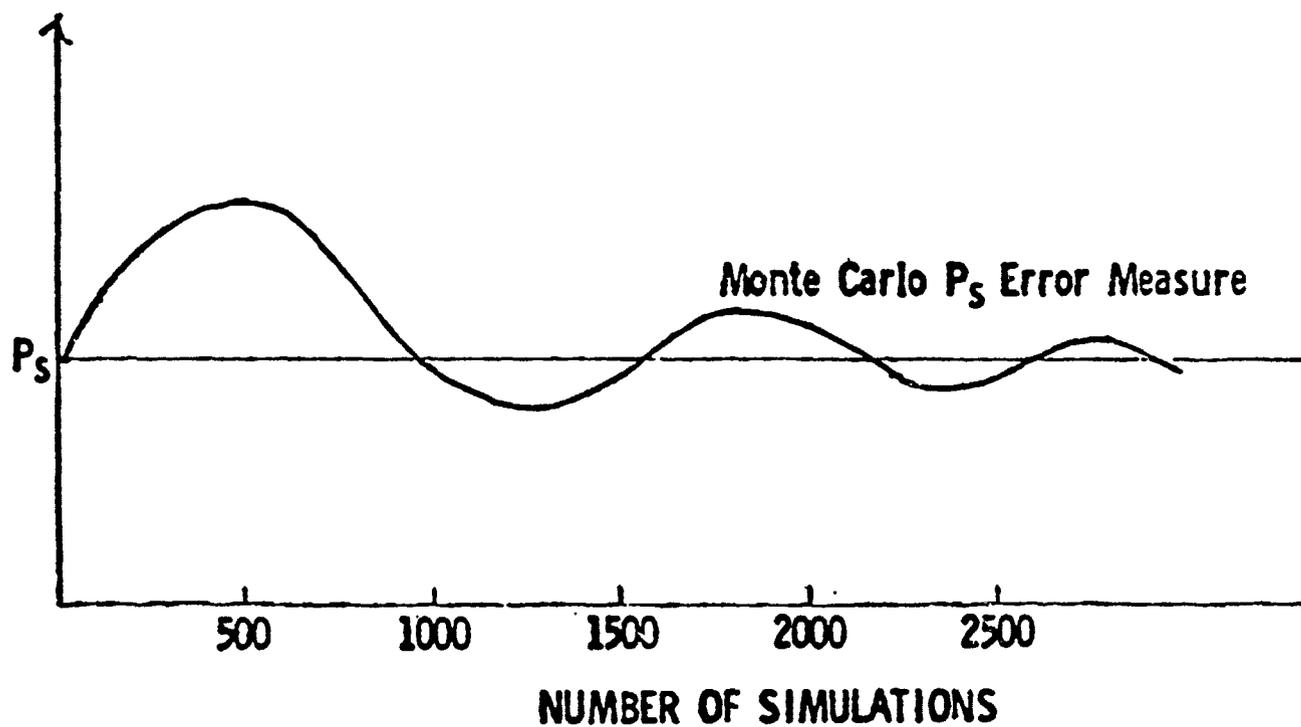
FIGURE 8B

Additional Criteria: Convergence of 3rd and 4th Moments

FIGURE 9

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Torsion Bar Reliability - Probability of Survival vs Miles da/dN Relationship

FIGURE 10

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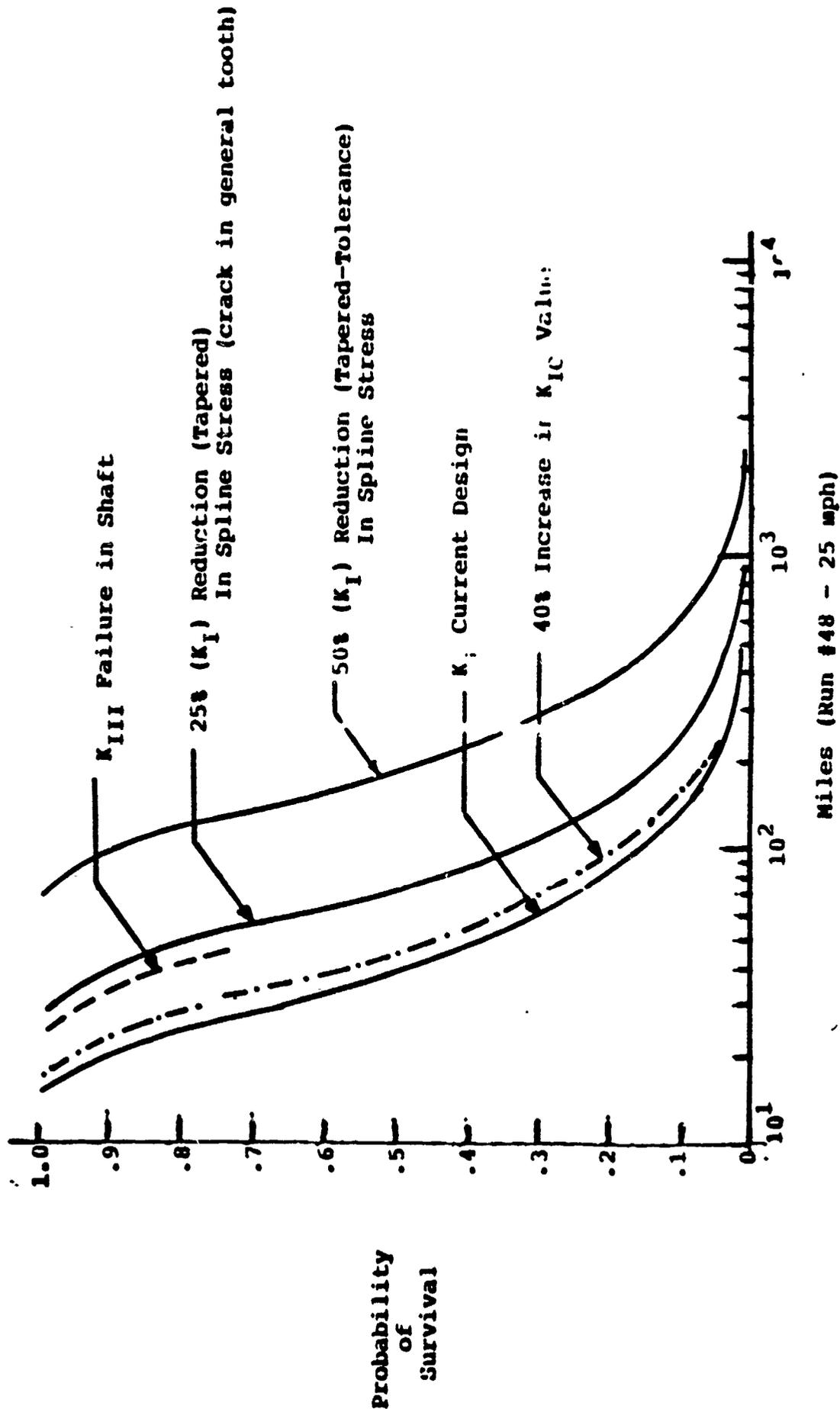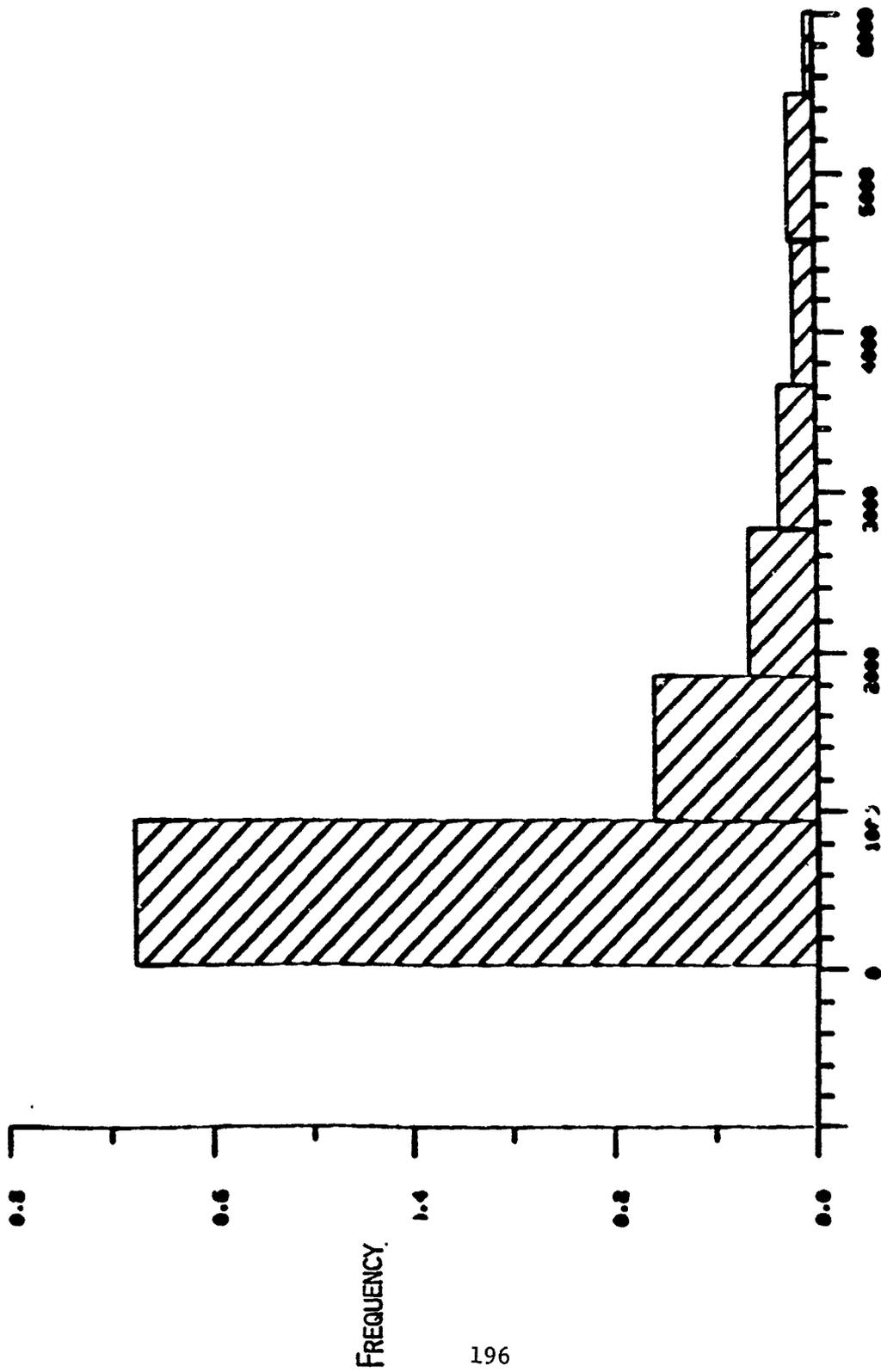
Miles to Failure #1 Torsion Bar (miles at 25 mph)

FREQUENCY

FIGURE 11

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Probability of Survival vs Miles
S/N Curve Results

Probability of remaining
life after 741 miles have
been completed.
$P_c = P_A/P_p$

20% Improvement in
Design (Stress
Reduction)

Current
Design

Miles (Run #48 - 25 mph)

Probability
of
Survival

FIGURE 12

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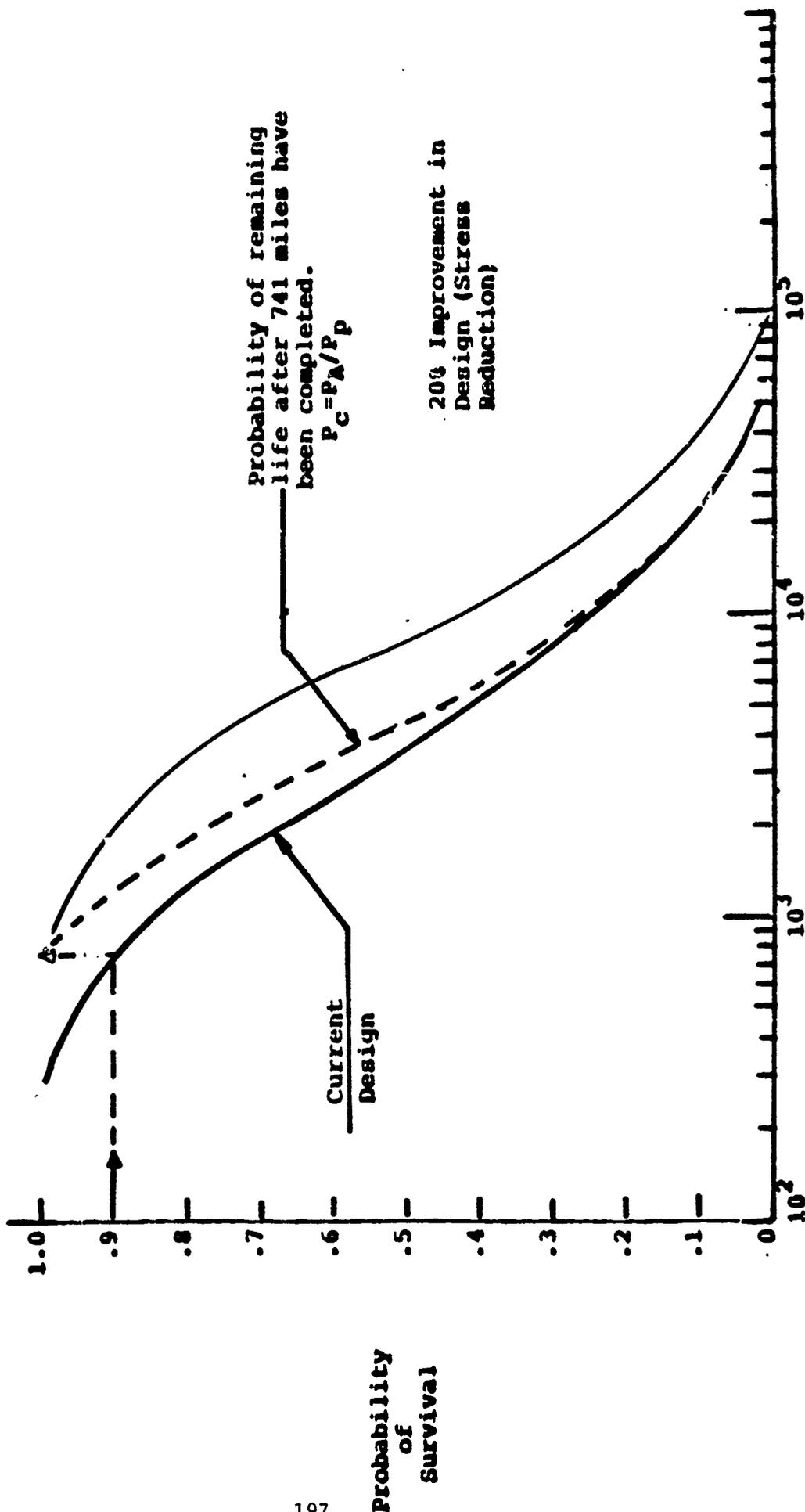
APG Reported Failure #1 Torsion Bars
(R and L together)

Mean=2.331
S Dev=1.176

99% Survivability =
262 miles from
Weibull
Distribution

Weibull
Normal
Lognormal
Radical
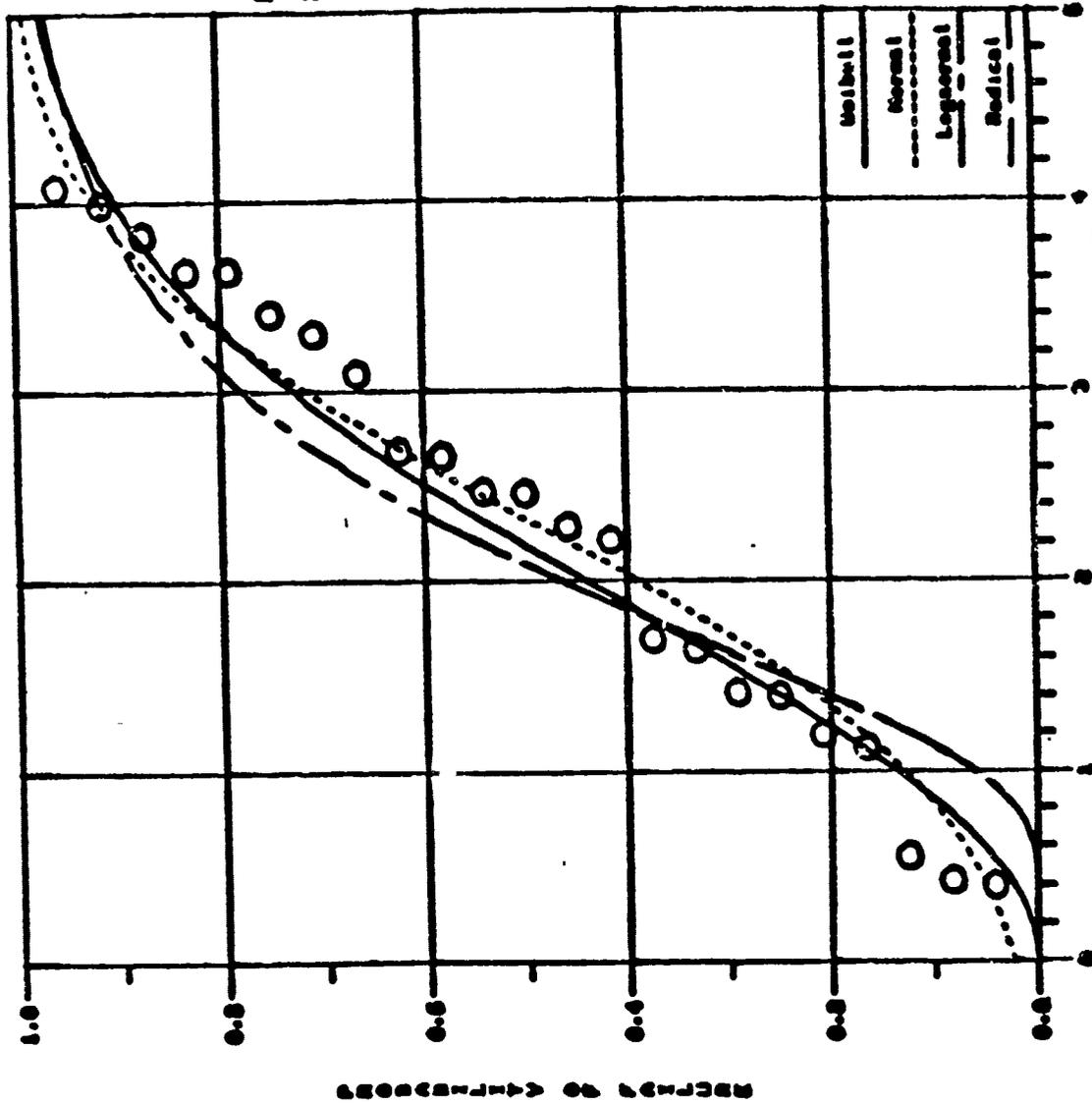
Miles to Failure (x10^{-3})

Figure 13

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Spectrum Load (Profile IV Course) – Beta Function Representation

Cummulative Probability	θ (Degrees) Test Results	θ (Degrees) Beta Representation
Run 40 (5 mph)		
.10	.14	.86
.25	4.4	5.8
.34	5.0	7.2
.50	8.6	9.3
.66	12.5	11.5
.75	14.2	12.7
.99	17.0	16.7
Run 42 (10 mph)		
.10	14.0	6.1
.25	16.0	19.2
.34	22.8	21.5
.50	25.8	24.8
.66	29.7	27.6
.75	30.6	29.0
.99	32.6	32.5
Run 48 (25 mph)		
.10	2.3	10.8
.25	22.7	26.2
.34	27.2	29.1
.50	33.7	33.3
.66	39.5	37.1
.75	41.6	39.3
.99	46.0	46.9

Cummulative Time Probabilities of Torsional Bar Angular Displacement θ adjusted to positive range by $\theta = \theta + |\theta^-|$ where $\theta^- = $ max. negative angular displacement.

TABLE 1

Minimum Life Estimates (99% Survivability)
da/dN Curve Results

Velocity	Mileage Expected (Function of Spline Stress)		
(MPH)	Current	25% Reduction	50% Reduction
5	71.0	138	341
10	29.9	51.9	143
15	15.2	29.3	72.3
20	14.0	26.9	66.3
25	14.2	28.4	70.2

TABLE 2

Monte Carlo Results for S/N Curve Minimum Life
Estimate (99% Probability of Survival) vs Velocity (MPH)

Velocity	Mileage	Expected	
(MPH)	Current Design	10% Design Improvement	20% Design Improvement
5	6,974	9474	12970
10	2,000	3138	4420
15	345	638	1089
20	276	515	860
25	292	557 ·	865

*Note: A 99% survivability estimate of <u>262</u> miles was obtained from cummulative APG mileage on vehicles at time of torsion bar failure. Velocity of vehicle during tests was approximately 15 to 25 mph.

TABLE 3