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THRUST AUGMENTING EJECTORS

by

Morton Alperin and Jiunn-Jenq Wu

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Flight Dynamics Research Corporation
15809 Stagg Street, Van Nuys, California

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Abstract

A discussion of the development of the compact jet-diffuser ejectors utilized for hovering and low speed flight propulsion has been presented. This is followed by a description of ideal ejector performance as derived from a compressible flow theory, over the range of flight speeds from zero to supersonic speed. These analyses introduced the concepts of ejector configuration optimization and the validity of the so-called "second solution" to the mixing problem, wherein the flow after complete mixing is supersonic. The ideal performance of thrust augmenting ejectors designed under this "second solution" has been shown to be far superior to those designed by conventional methods. The ability of properly designed ejectors to utilize the thermal energy of injected gas for the production of useful energy has also been described. Finally, the influence of major losses has been discussed, including means for avoiding excessive performance degradation by proper optimization of the geometry of the ejector in view of these losses.

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List of Symbols

A	duct area
a	primary jet area
C	coefficient
M	Mach number
NPR	nozzle pressure ratio $(= P_{or}/p_{\infty})$
P	stagnation pressure
p	pressure
T	temperature
U	secondary or mixed flow velocity
V	jet velocity
X	duct width
α	inlet area ratio $(\alpha_{\infty} = A_2/a_{\infty};\ \alpha_* = A_2/a_*)$
δ	geometric diffuser or outlet area ratio $(= X_3/X_2)$
η	efficiency factor
ΔS	change of total entropy due to mixing
ΔP	primary jet pressure rise $(= P_{or} - P_{o\infty})$
ΔT	primary jet temperature rise $(= T_{op} - T_{o\infty})$
ϕ	thrust augmentation

Subscripts

di	inlet drag
dj	diffuser jet or jet diffuser
i	induced flow
N	primary nozzle thrust
p	primary jet
r	reservoir
o	stagnation
1, 2, 3	ejector stations
∞	undisturbed or ambient condition
$*$	sonic throat

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Introduction

Early theoretical and experimental work in ejector technology was related primarily to the jet pump application. This work reported in References 1 and 2 for example, emphasized the inlet flow and assumed that a large subsonic (diverging) diffuser was required at the outlet to return the flow to small velocity at the ambient static pressure. The existence of two different resultant flows after complete mixing of compressible fluids in a constant area duct was observed in Reference 1, but the second solution, which represented supersonic flow after complete mixing was not considered as being of practical importance. These may have been proper conclusion for the jet pump application since the objective was the achievement of large secondary flow compression and entrainment ratios, rather than high momentum flux increment as is required in the case of thrust augmenting ejectors.

The use of jet propulsion for aircraft created interest in the ejector as a thrust augmenter. Unfortunately, the early work in thrust augmenting ejectors borrowed the jet pump concept of large divergent diffusing outlets as being desirable for high performance. This concept was reinforced by the use of incompressible flow theory (References 3 and 4 for example), in the analysis of the flow and performance of thrust augmenting ejectors, leading to a limited understanding of the optimal capability of ejector thrusters.

Analyses using compressible flow theory also presented a bleak picture of thrust augmenting ejector performance when the ejector was translating at even small velocities in the thrust direction, and when the primary injected gas was heated. Nagaraja, Hammond and Graetch (Reference 5) for example, indicate a very rapid decrease of thrust augmentation with increasing velocity and primary fluid stagnation temperature. It was noted in that document however, that "as speed increased above about 400 ft/sec, the downward trend of thrust augmentation begins to abate and indeed turns upward".

Very small performance improvement with increasing primary jet stagnation temperatures at speeds in excess of 400 ft/sec. is also illustrated in Reference 5, but the data terminated at speeds of about 600 ft/sec, and general conclusions appear to indicate a degradation of performance with increasing primary jet stagnation temperature.

As a result of the poor performance predicted by the incompressible flow analyses, and by compressible flow analyses as utilized in Reference 5 and others, very little effort was devoted to experimental work aimed at the application of ejectors as primary thrusters during high speed flight. Instead, the Air Force Aerospace Research Laboratory established a research program called Cold Thrust Augmentation (CTA), aimed at the development of ejectors using bleed or fan air for thrust vectoring and augmentation during hover and low speed flight. Under the assumption that ejectors should utilize cold gas injection, this program devoted itself to the investigation of methods for acceleration of the mixing process (References 4 and 6 for example). Other programs were sponsored by the Navy, Marine Corps, NASA, and the Air Force Flight Dynamics Laboratory in attempts to examine the fundamentals of ejector phenomena and to study the problems associated with integration of ejectors into realistic aircraft designs.

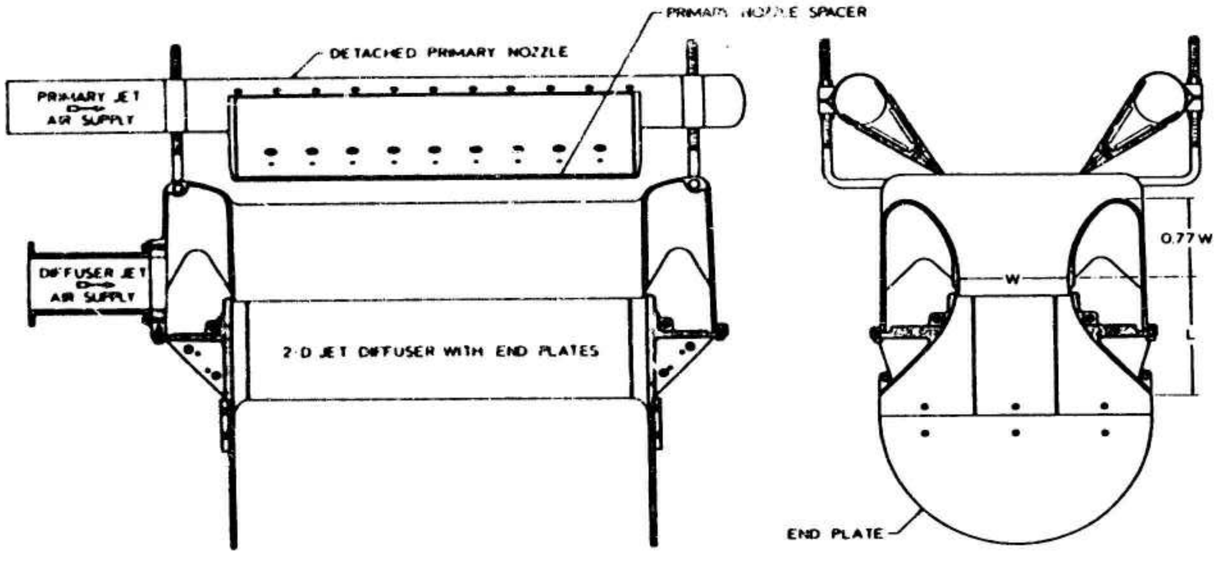
During that period of time other investigators have made valuable contributions to ejector technology and have in some measure overcome some of the objections to ejector utilization for V/STOL capability, however, the present remarks will be restricted primarily to work with which we are most familiar. This consists of efforts by Flight Dynamics Research Corporation to demonstrate the feasibility of designing very compact, high performance ejectors and to describe the realistic effects of injected gas characteristics and of translational velocities in the thrust direction, upon ejector performance.

Early investigations by FDRC resulted in demonstrating the feasibility of elimination of a discrete mixing section and of a very short wide-angle diffuser which diffused a fully attached flow to an area ratio of more than 3.0. This ejector developed under a Navy/Marine Corps program called STAMP (Small Tactical Aerial Mobility Platform), developed a thrust augmentation in excess of 2.0 with a configuration described in Reference 7 and shown on Figure 1a.

In this ejector, mixing was achieved by injection of the primary fluid upstream of the inlet of the ejector. The short 45 degree half-angle diffuser contained a diffuser jet which completely circumscribed the periphery of the diffuser, to prevent separation and to provide additional diffusion and mixing length beyond the exit of the solid surfaces of the diffuser. End plates extending beyond the diffuser exit were used to prevent collapse of the jet diffuser flow pattern thereby providing additional effective diffuser area ratio. This jet-diffuser ejector produced a thrust augmentation of 2.13 under stationary conditions and a net thrust augmentation of 2.68 at a tunnel speed of 66 ft/sec (perpendicular to the thrust), in the FDRC wind tunnel. This program is reported in detail in Reference 7.

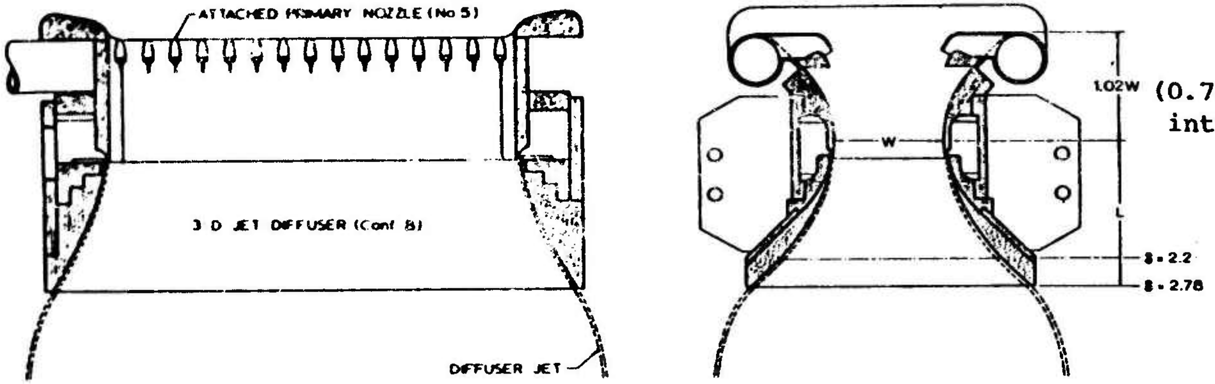
The removal of the diffuser end plates from the STAMP ejector resulted in a decrease of thrust augmentation from 2.13 to 1.82. To avoid this deterioration of performance, a more sophisticated diffuser was designed by FDRC under the sponsorship of NADC. This diffuser, having a shape derived from the potential flow theory is shown on Figure 1b. It produced a thrust augmentation of 2.13 with a length (measured from the diffuser jet exit) of only 1.25 times the throat width of the ejector without the end plates required by the STAMP design. The curvature and divergence of the ends of the diffuser provided a means for avoiding the collapse of the jet diffuser flow downstream of the solid surfaces and provided a means for thrust vectoring in the longitudinal axis of the ejector as will be discussed later. Methods were developed for the design of this type of diffuser, which can conceptually be utilized for design of even shorter diffusers. Details of this effort are described in References 8 and 9.

Protruding primary nozzles were considered as undesirable by aircraft designers. Therefore a modification of the STAMP ejector injection system was carried out with joint NASA and NADC support. Many interesting observations were made during this effort, but briefly a set of nozzles was designed for attachment to the inlet of the ejector in a non-protruding manner, as illustrated on Figure 1b. Thrust augmentation of 2.02 was measured with the new nozzles and the diffuser described above. The entire ejector from inlet to end of the diffuser had a length equal to 2.4 times its throat width, and later modification reduced this to less than 2 times its throat width for aircraft integration. This work is reported in References 9 to 11.



Diffuser	Area Ratio	L/W
2-D	2.2 2.5 3.0	1.01 1.16 1.41
3-D	2.2 2.3 2.78	1.16 1.21 1.41

a) STAMP Jet-Diffuser Ejector



(0.76W for aircraft integration)

b) Ejector with Three-Dimensional Diffuser and Attached Nozzles

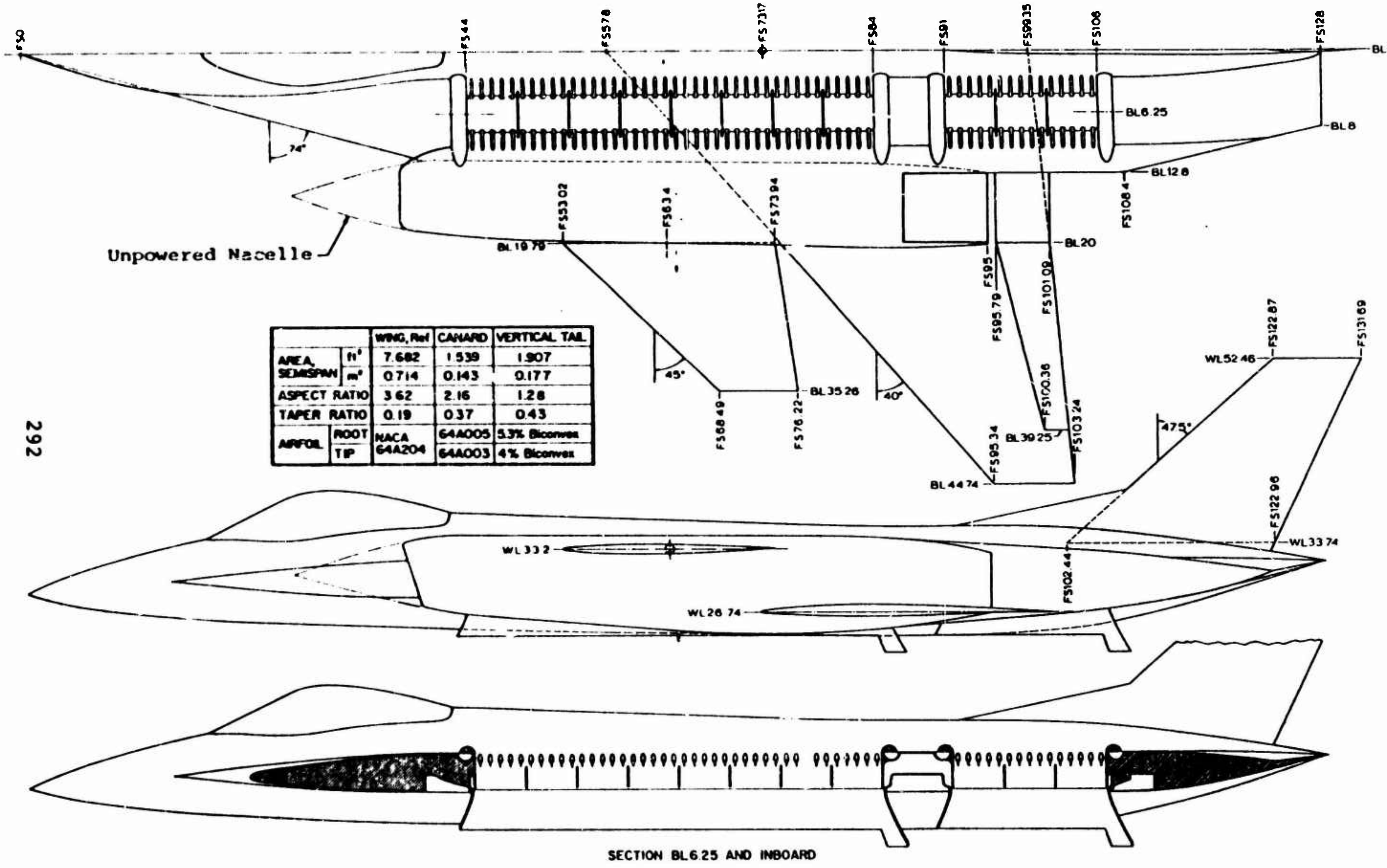
Figure 1. Jet-Diffuser Ejectors

Having developed this extremely small, high performance ejector, FDRC was given the opportunity to integrate the ejector into a supersonic fighter/attack aircraft designed by General Dynamics and designated E205 (Figure 2). This effort was sponsored jointly by NADC and NASA. The achievement of the required ejector lift force corresponding to a wing loading of 118.3 psf at a nozzle pressure ratio of 3.0 while limiting the total size of the ejectors to fit within the structural limitations of the strake of the small supersonic fighter/attack aircraft necessitated a large injected momentum per unit throat area and a large thrust augmentation.

In this design, the ejectors are required to provide a thrust force to accelerate the aircraft to transition flight speeds, and the vertical force (lift) to achieve VTOL capability. This thrust vectoring is accomplished by an asymmetric extension of the rear ends of the diffusers and by use of specially designed thrust vector control jets within the ejector. The high value of thrust per unit area of ejector is achieved as a result of the effective use of injected gas at the diffuser jet in addition to the primary injection jets. This permits a high concentration of injected momentum while maintaining a high inlet area ratio and correspondingly high performance. Forces in the flight direction equivalent to 12% of the total thrust were measured with a single vector control jet in conjunction with an asymmetric extension of the rear end of the diffuser, in static tests at FDRC with a short ejector integrated into the forward end of the strake. The ejectors designed for the E205 are foldable and can be stowed completely within the strake during normal flight. Testing at high nozzle pressure ratios will be conducted at NASA Ames Research Center.

The design, based upon the use of unheated primary and diffuser gas appeared as illustrated on Figure 2. Further details are presented in Reference 11.

Ejector design considerations including performance predictions with high nozzle pressure ratio injected gas and estimated loss factors are discussed in a later section of this document.



		WING, Ref	CANARD	VERTICAL TAIL
AREA, SEMISPAN	ft²	7.682	1.539	1.907
	m²	0.714	0.143	0.177
ASPECT RATIO		3.62	2.16	1.28
TAPER RATIO		0.19	0.37	0.43
AIRFOIL	ROOT	NACA 64A204	64A005	5.3% Biconvex
	TIP		64A003	4% Biconvex

Figure 2. V/STOL Fighter (one-fifth scale model)

Ideal Ejector Performance

These programs and others in a more advanced stage of development (Reference 12 for example) have given some indication that ejectors may be satisfactory for use in thrust vectoring and augmentation at zero translational velocity or at low speeds when oriented perpendicularly to the flight path. The adverse effect of high temperature injected gas has been considered as inevitable and some systems have been developed using bleed air or fan air exhaust to minimize this problem. The real advantage achievable by thrust augmenting ejectors has awaited a more thorough analytical treatment of the problem. Previous analyses which indicated very large performance degradations due to translational motion in the thrust direction and due to the injection of heated gas were lacking in several important aspects, which have become clear to us as a result of the effort sponsored by AFFDL and some considerable in-house efforts, some of which are reported in Reference 13, "High Speed Ejectors".

The analysis of the flow through an ejector under the assumption that the fluids are incompressible, yields results which are pertinent to those systems which operate underwater (References 14 and 15 for example). However, incompressible fluid analyses cannot provide information related to the effects of high pressure and temperature injected gas, nor those phenomena which are associated with heat transfer such as thermal choking. Therefore, realistic estimates of the performance and flow characteristics within a thrust augmenting ejector required to utilize the exhaust from conventional gas turbines, ramjets or rockets as the source of power, must utilize the theory of compressible flow.

In addition to the choice between compressible and incompressible theory, the analysis of ejector flow generally utilizes a choice between constant pressure and constant area mixing. The selection of one of these two cycles permits precise expression of the momentum theorem in the mixing section in a global analysis. Of these two choices, the use of constant pressure mixing results in the simpler mathematical formulation and better pumping characteristics (Reference 1) and has been utilized extensively in jet pump ejector literature.

Unfortunately the design of a constant pressure mixing duct becomes difficult since the exact shape of the duct cannot be determined by a global analysis and would require a complex, detailed analysis of the flow throughout the mixing process. A further disadvantage to the use of constant pressure mixing is related to the obvious restriction to the variation of the static pressure during mixing. The processes are restricted to those in which the pressure after mixing is identical to the pressure of the two individual flows at the start of mixing. The potential thermodynamic and aerodynamic advantages attributable to pressure changes during mixing are obviated by the assumption of constant pressure. This is immediately evident by observation of the character of the solution to the global treatment of the mixing problem under both assumptions. Clearly the constant area mixing problem has two solutions while constant pressure mixing has only one solution, and the freedom to permit pressure variations provides the opportunity to observe many types of flow patterns not possible within the constant pressure restriction.

The assumption of constant area mixing is also restrictive, but the feasibility of using a global analysis, of writing a precise momentum equation, and the geometric simplicity has prompted its use in analytical treatments of the mixing problem. It appears possible that some special mixing duct designs would result in better performance but no such analysis has been published to date to our knowledge. Therefore all further discussion of ejector flow and performance in this report will be based upon the assumption of constant area mixing of compressible flow, and will use the symbols and station designation presented on Figure 3.

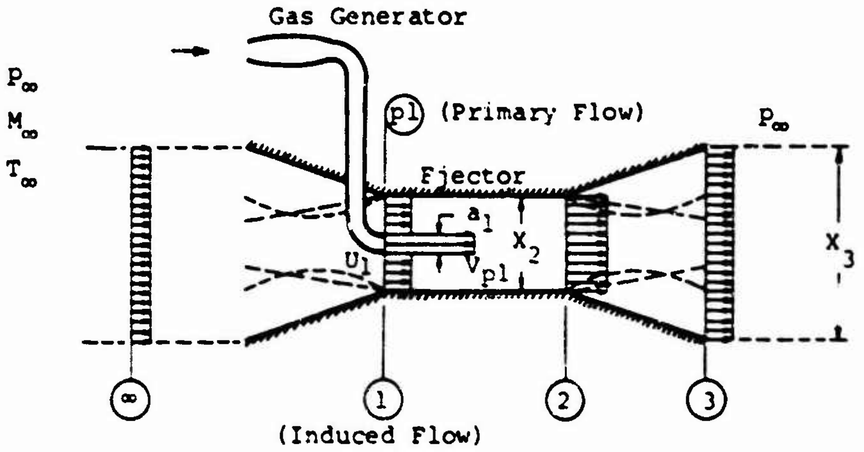


Figure 3. Ejector Configurations and Station Designation

As will be shown by means of the analyses described in the "High Speed Ejectors" report (Reference 13), high speed and hot gas injection need not have the adverse influence on ejector performance predicted by previous analyses. Although the reasons for these divergent views are complex in detail, there are several rather obvious differences in the treatment of ejector flow as presented in the "High Speed Ejector" report compared to the previous compressible flow analyses.

Most importantly, it has been observed that either of two distinct types of flow may exist after complete mixing of compressible fluid in a constant area duct. The so-called "first solution" always results in a subsonic (or sonic) flow after complete mixing, regardless of the conditions at the start of the mixing process. This solution dictates the geometric characteristics of the ejector required to return the mixed subsonic flow to ambient pressure at the outlet. The so-called "second solution" always results in a supersonic (or sonic) flow after complete mixing, regardless of the conditions at the start of the mixing process. Obviously the geometric characteristics of the ejector required to return this supersonic flow to ambient pressure will differ from those of the first solution. Since the mathematics allows only supersonic mixed flow under the second solution, it is obvious that certain conditions at the start of mixing may be inconsistent with physical reality. These conditions can be screened out by consideration of the Second Law of Thermodynamics. Although both solutions have been observed previously, to our knowledge, no use has been made of the second solution in the design of thrust augmenting ejectors to date.

Further, optimal geometries exist for all conditions examined by means of the compressible flow analysis. The optimal geometry is dependent upon the solution (first or second) utilized in the analysis, the operational and injected gas characteristics, and upon the losses within the ejector. As will be shown, the thrust augmentation achievable decreases rapidly on either side of the optimal inlet and outlet configurations. While some variations of outlet area have been examined, it is not apparent that those experiments were performed in the light of theoretical guidance nor that the geometries were even close to the optimal given by the theory.

In the following discussion, use is made of several unique parameters not generally used by theoreticians, but which we believe are important in relating theory to reality.

To evaluate the influence of any parameter on ejector performance, it is essential that the ejector size be fixed in relation to the size of its reference jet. To accomplish this it is necessary to define a reference jet as a free jet whose gas has the same stagnation properties and mass flow as those of the primary jet of the ejector. Then the relationship of the mixing section area of the ejector to that of its reference jet when expanded isentropically to ambient pressure (α_∞), defines the ejector size. When the nozzle pressure ratio is supercritical, it is sometimes convenient to relate the size of the mixing section of the ejector to the throat area of the reference jet (α_*). In either case the comparison of ejector size to that of its reference jet remains constant as the pressure at the injection plane in the ejector varies as a result of changes in the ejector geometry.

When the ejector is in motion in its thrust direction, it is necessary to compare its performance with that of its reference jet while in motion under the same conditions. Thus the variation of nozzle pressure and temperature with changes in the translational velocity must be considered in a realistic manner. This can be accomplished with reasonable realism if the nozzle pressure and temperature ratios are expressed as increments in excess of the free stream ratios. Thus in the analyses presented, the nozzle pressure and temperature ratios are expressed in terms of $\Delta P/p_\infty$ and $\Delta T/T_\infty$. Obviously data presented for a fixed flight Mach Number can be related to fixed stagnation properties as is done on the maps to be described in the following discussion, but in presenting data on ejector performance as a function of its translational velocity, it is more realistic to utilize constant values of $\Delta P/p_\infty$ and $\Delta T/T_\infty$ and permit the nozzle pressure and temperature ratios to vary with the free-stream pressure and temperature ratios. Further, the net thrust of the ejector should be compared to the net thrust of its reference jet, to provide a meaningful indication of the ability of the ejector to augment the thrust of its reference jet.

These considerations have been utilized in the preparation of the data to be discussed in the following section.

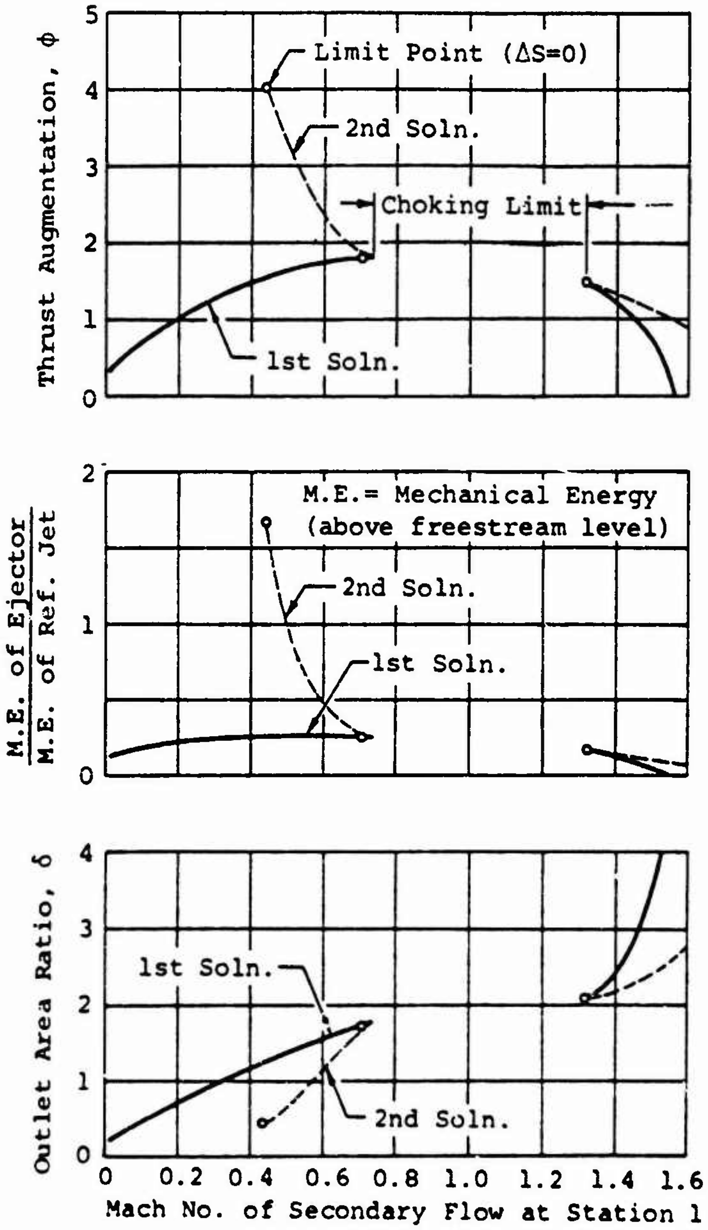
Stationary Ejector

To illustrate the importance of a proper selection of the ejector configuration, we first examine the so-called stationary case in which the ejector is at rest with respect to the undisturbed medium or oriented so that its thrust vector is normal to the flight direction. Figure 4 illustrates the influence of ejector geometry upon the performance and thermodynamics of the flow in a stationary ejector. The chart was prepared with a fixed value of α_∞ (=20) to assure consideration of an ejector whose mixing duct area has a constant relationship to the area of the reference jet when fully expanded to ambient pressure. To simplify the presentation, the thrust augmentation, ratio of mechanical energies of ejector output to reference jet output, and the outlet area ratio required to return the mixed flow to ambient pressure are plotted versus the Mach Number (M_1) of the induced flow at the start of mixing. M_1 also determines the geometry at the inlet and outlet of the ejector for any given operational and injected gas characteristics. As indicated, there are three performance points where the thrust augmentation is optimal.

Under the first solution a maximum thrust augmentation always occurs with subsonic values of M_1. In this particular case it occurs very close to the lower limit of thermal choking. This optimal point varies with operational and injected gas characteristics.

The second solution usually displays a local maximum performance point with a supersonic value of M_1, which in this case occurs at the higher limit of choking, and a limiting performance at a subsonic value of M_1, limited by the Second Law of Thermodynamics.

It is interesting to note that in this ideal limiting situation, the total entropy of the mixed flow is equal to the sum of the entropies of the flows at the start of mixing and that the mechanical energy of the ejector discharge can be larger than that of the reference jet. Thus some of the thermal energy of the reference jet is converted to mechanical energy during the mixing process. Real gas effects and wave losses obviously preclude achieving the performance predicted by the $\Delta S = 0$ point, but achievement of this second solution flow pattern would obviously result in superior performance to that achieved by the conventional first solution.



$M_\infty = 0$; $\Delta P/p_\infty = 3$; $\Delta T/T_\infty = 4$; $\alpha_\infty = 20$

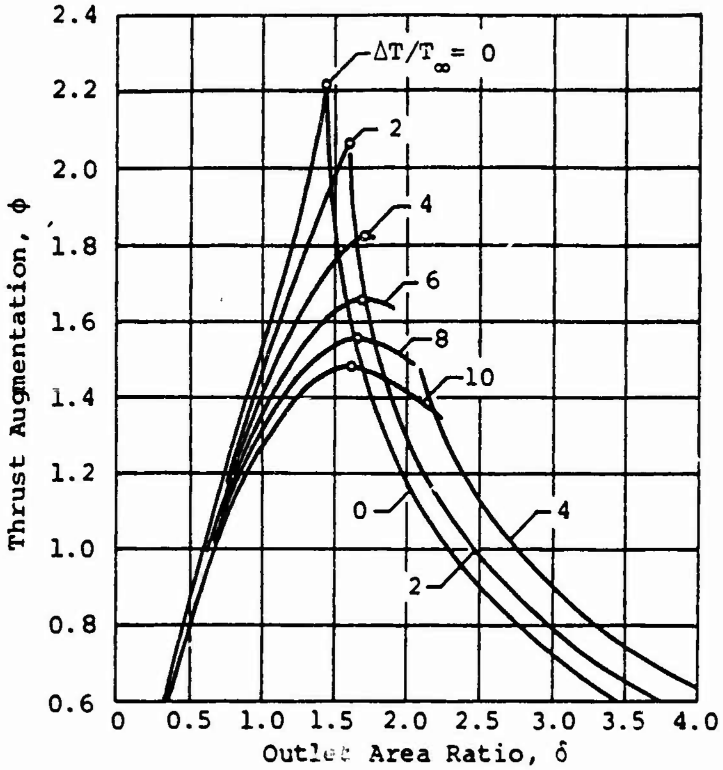
($P_{or}/p_\infty = 4$; $T_{op}/T_\infty = 5$)

Figure 4. Example of Stationary Ejector

To illustrate the influence of primary fluid stagnation temperature and outlet geometry on stationary ejector performance, the variation of thrust augmentation is plotted versus outlet area ratio, for the first solution (Figure 5) and for the second solution (Figure 6), at an arbitrarily chosen primary nozzle pressure ratio and for several different primary nozzle stagnation temperatures. On these figures, the outlet area ratio is defined as the ratio of the area at the section where the pressure returns to ambient to the area of the mixing section of the ejector. On Figure 5, the outlet in the regions of practical interest (near maximum performance) is a diverging diffuser (area ratios greater than 1.0 and all discharged flow is subsonic). Both the thrust augmentation at a fixed outlet area ratio and the maximum achievable thrust augmentation decrease with increasing primary jet stagnation temperatures when the outlet area ratio is smaller than a certain critical value. At larger values of the outlet area ratio (supersonic mixing), the thrust augmentation increases with increasing primary jet stagnation temperatures when the outlet area ratio is fixed. Maximum thrust augmentation in this region however, decreases with increasing primary jet stagnation temperatures. The natural solution of this region, (where outlet area ratios correspond to supersonic values of M_1), is presumably the second solution.

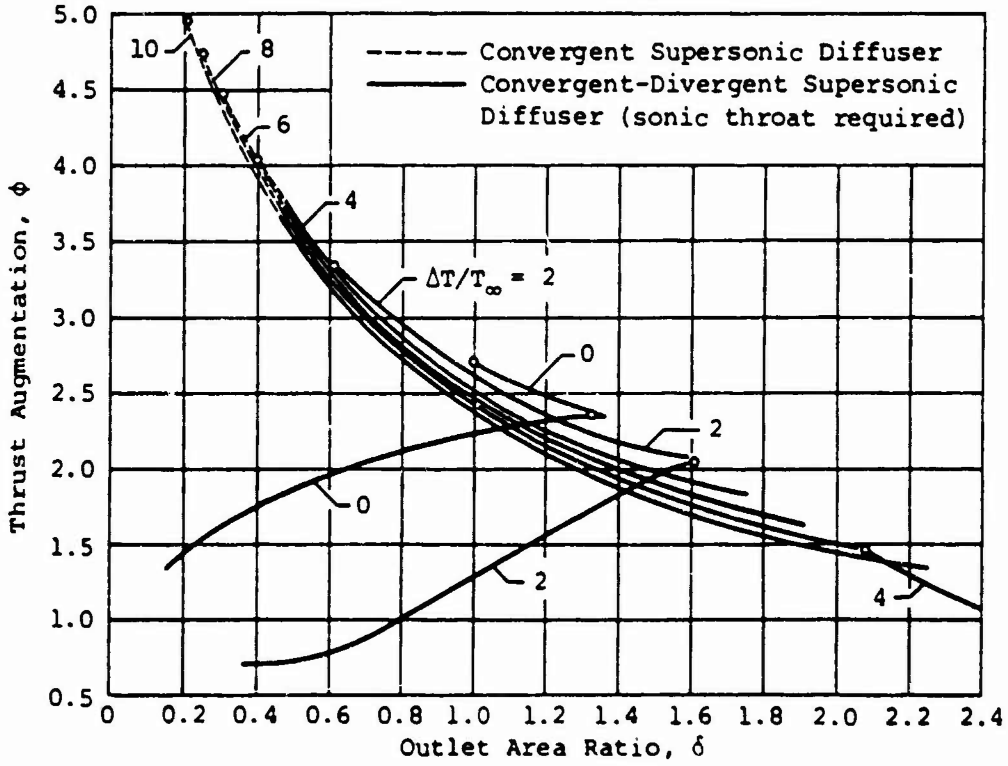
Under the second solution (Figure 6), where the flow after complete mixing is supersonic, the ideal outlet (for the cases shown) is a converging or convergent-divergent diffuser. In order to distinguish between these two different types of outlets, dashed lines are used to represent the convergent diffuser while solid lines represent the convergent-divergent diffuser (which requires a sonic throat). As in the case of the first solution, increasing primary jet stagnation temperatures have an adverse influence on performance with a given outlet area ratio. Maximum limiting performance however, improves with increasing primary jet stagnation temperatures as a result of the ability to operate at smaller outlet area ratios without violation of the Second Law of Thermodynamics.

Comparison of Figures 5 and 6 disclose the considerable performance advantage in the use of second solution design criteria, even for the stationary ejector.



$M_\infty = 0$; $\alpha_\infty = 20$; $\Delta P/p_\infty = 3$

Figure 5. Stationary Ejector Performance - First Solution



$M_\infty = 0$; $\alpha_\infty = 20$; $\Delta P/p_\infty = 3$

Figure 6. Stationary Ejector Performance - Second Solution

To provide a quick reference for determination of the influence of primary jet pressure and temperature ratios on ejector performance, maps showing ideal iso-augmentation lines with the appropriate configurations of inlet and outlet, on pressure-temperature surfaces are presented on Figures 7 to 9 for a stationary ejector with $\alpha_\infty = 20$ (a mixing duct having an area equal to 20 times that of the reference jet when fully expanded to ambient pressure).

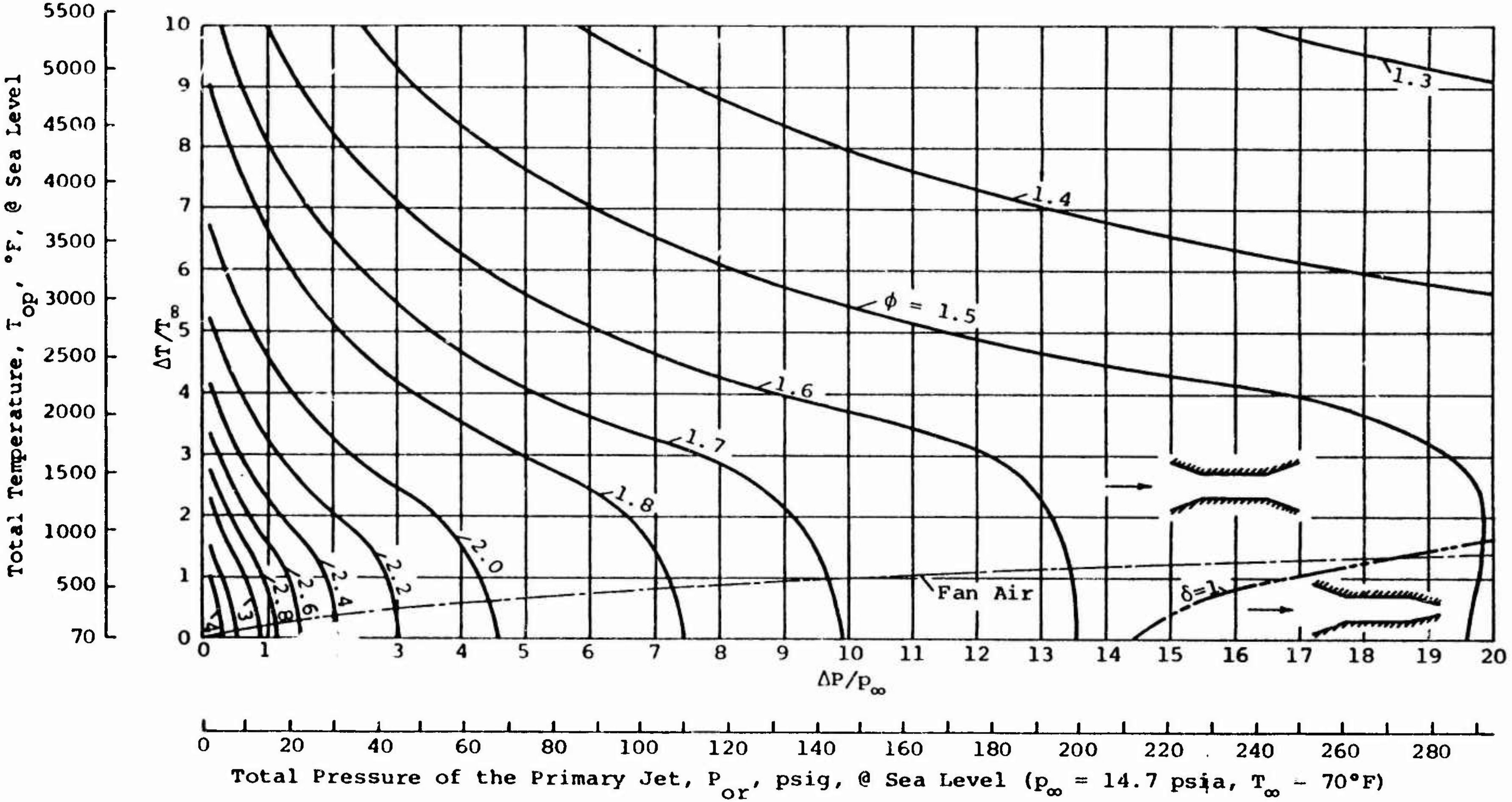
Figure 7 represents the ideal performance of an ejector at rest or whose motion is normal to the thrust direction, designed with a geometry described by the optimal condition under the first solution. To our knowledge all thrust augmenting ejectors designed to date have utilized this design criterion but little effort has been devoted to optimization. As shown on Figure 7, ejectors designed under this criterion display a performance degradation with increasing primary jet pressure and temperature ratios. These ejectors require an accelerating inlet, and a subsonic diffuser or nozzle outlet for conditions within the boundary of Figure 7. The achievement of high performance ejector designs under this criterion lies in the effective design of diffusers, the minimization of component losses and the optimization of the geometry for any given set of injected gas characteristics, as will be illustrated on Figures 16 and 17.

Figure 8 represents the same operational conditions as those of Figure 7, but assumes that design criteria are established by use of the second solution with supersonic induced flow at the start of mixing, which requires a convergent-divergent accelerating inlet. As shown, increasing primary jet pressure and temperature ratios produce performance deterioration. Excessively high temperatures result in no analytical solution.

Use of the second solution with subsonic mixing is limited to the region where the total entropy change during mixing is greater than zero. At the limit ($\Delta S = 0$), ideal ejectors display a maximum performance limited by the Second Law of Thermodynamics and at this point the performance map is as presented on Figure 9. Under this design criterion the ideal performance is very high over the entire practical range of primary pressure and temperature ratios.

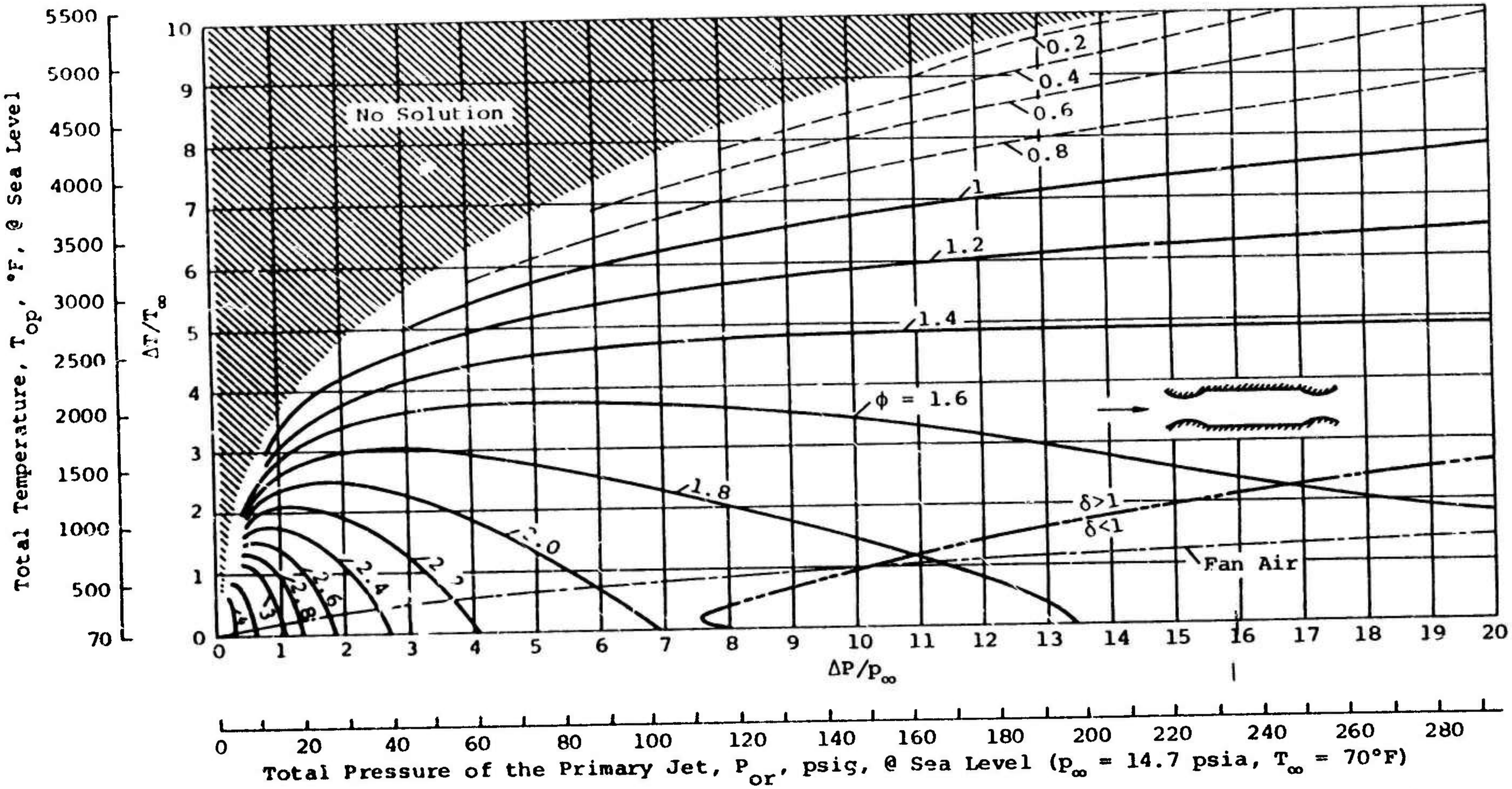
Using this design criterion, increasing primary jet stagnation temperature generally results in improved performance except at relatively low temperatures ($\Delta T/T_\infty < 1$) as illustrated. As shown, within the boundary of Figure 9, the inlet is a subsonic accelerating duct, while the outlet is either a convergent-divergent supersonic diffuser, or a convergent supersonic diffuser.

Obviously losses due to wave drag, skin friction, blockage, etc., will result in some performance degradation. These effects will be discussed in a later section of this document.



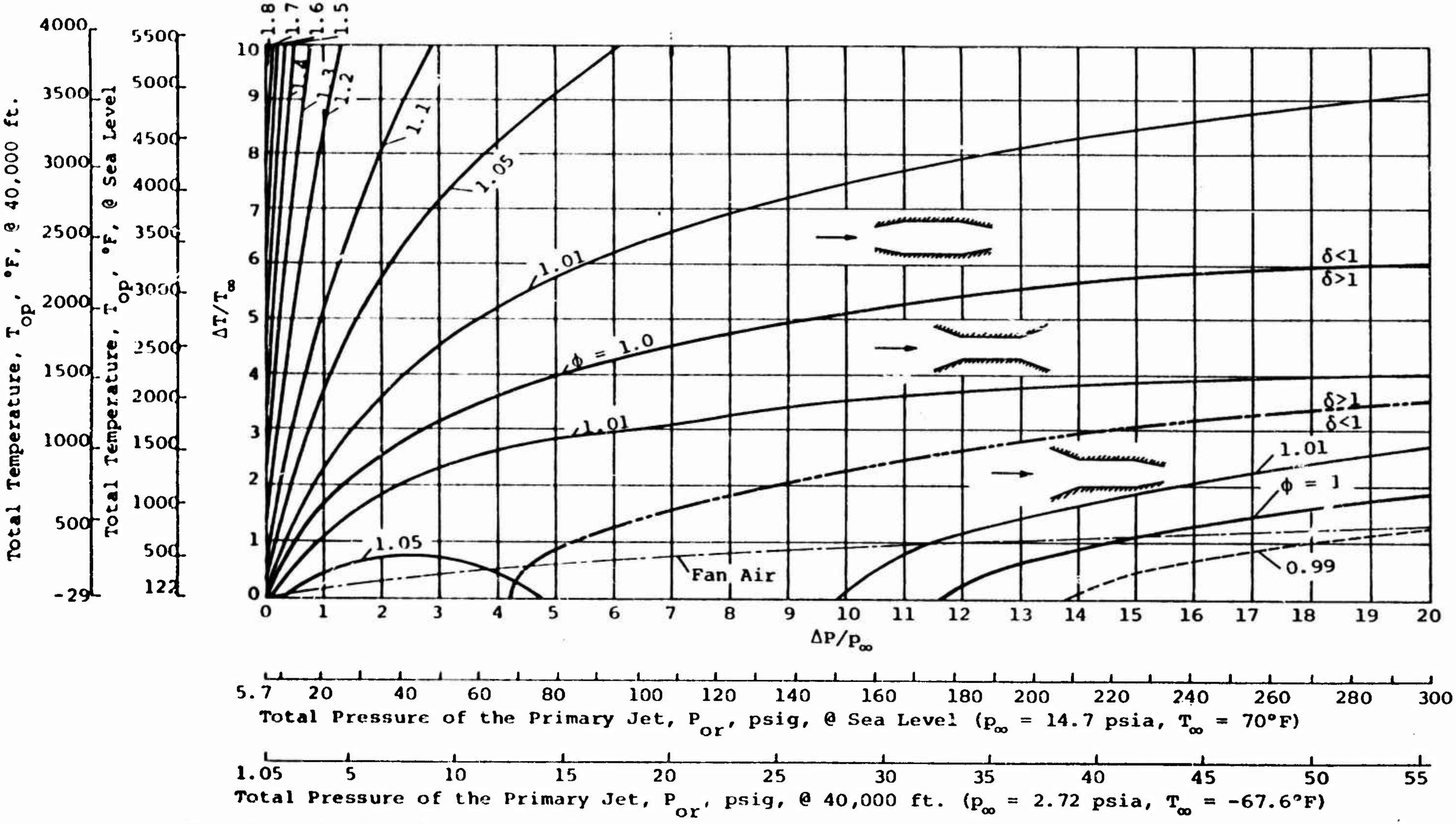
Freestream Mach Number: $M_\infty = 0$; Ejector: $\alpha_\infty = 20$, Maximum ϕ of First Solution (Subsonic Mixing)

Figure 7. Ideal Ejector Performance Map



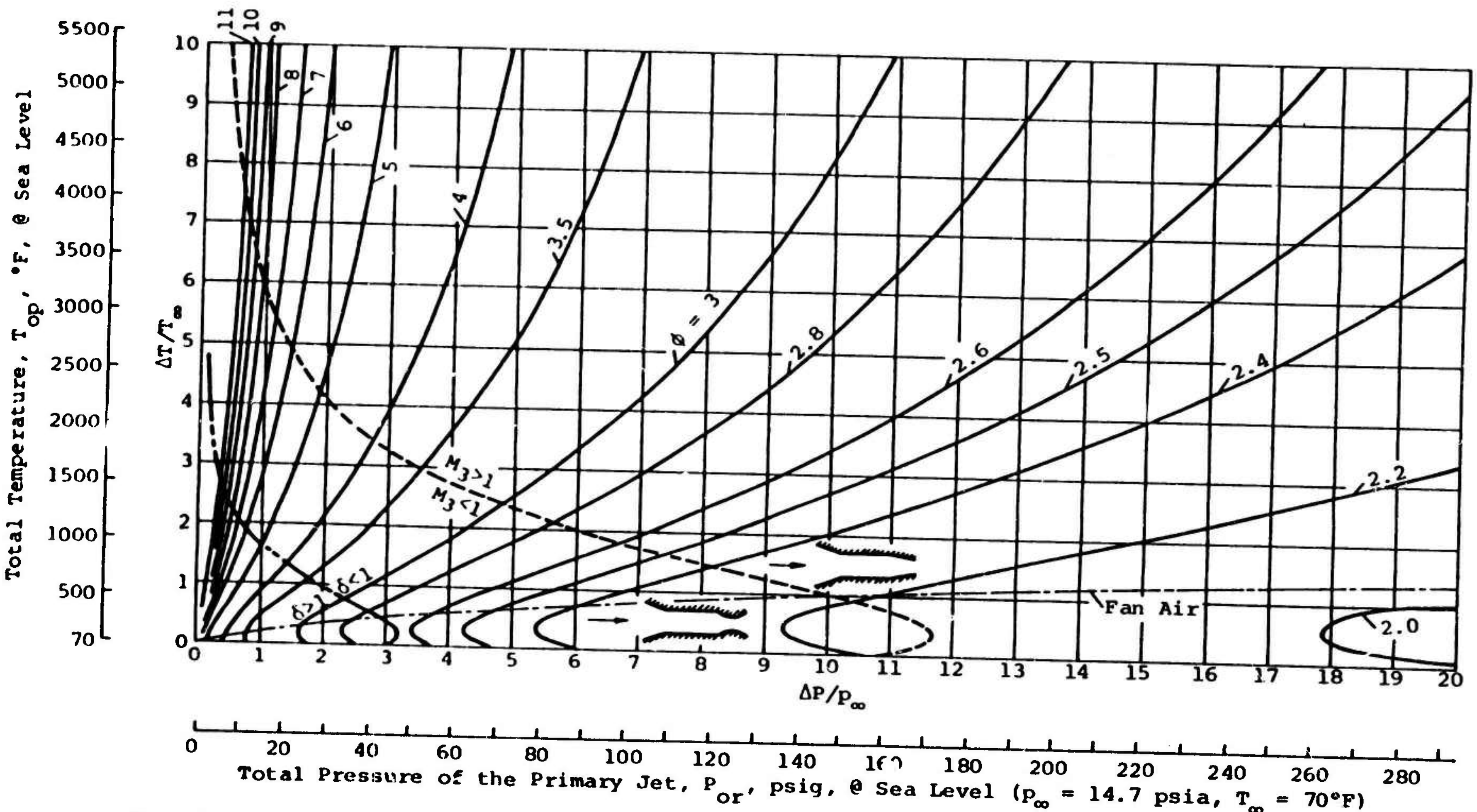
Freestream Mach Number: $M_\infty = 0$; Ejector: $\alpha_\infty = 20$, Maximum ϕ of Second Solution (Supersonic Mixing)

Figure 8. Ideal Ejector Performance Map



Freestream Mach Number: $M_\infty = 0.7$; Ejector: $\alpha_\infty = 20$, Maximum ϕ of First Solution (Subsonic Mixing)

Figure 10. Ideal Ejector Performance Map



Freestream Mach Number: $M_\infty = 0$; Ejector: $\alpha_\infty = 20$, ϕ of Second Solution with Subsonic Mixing, $\Delta S = 0$

Figure 9. Ideal Ejector Performance Map

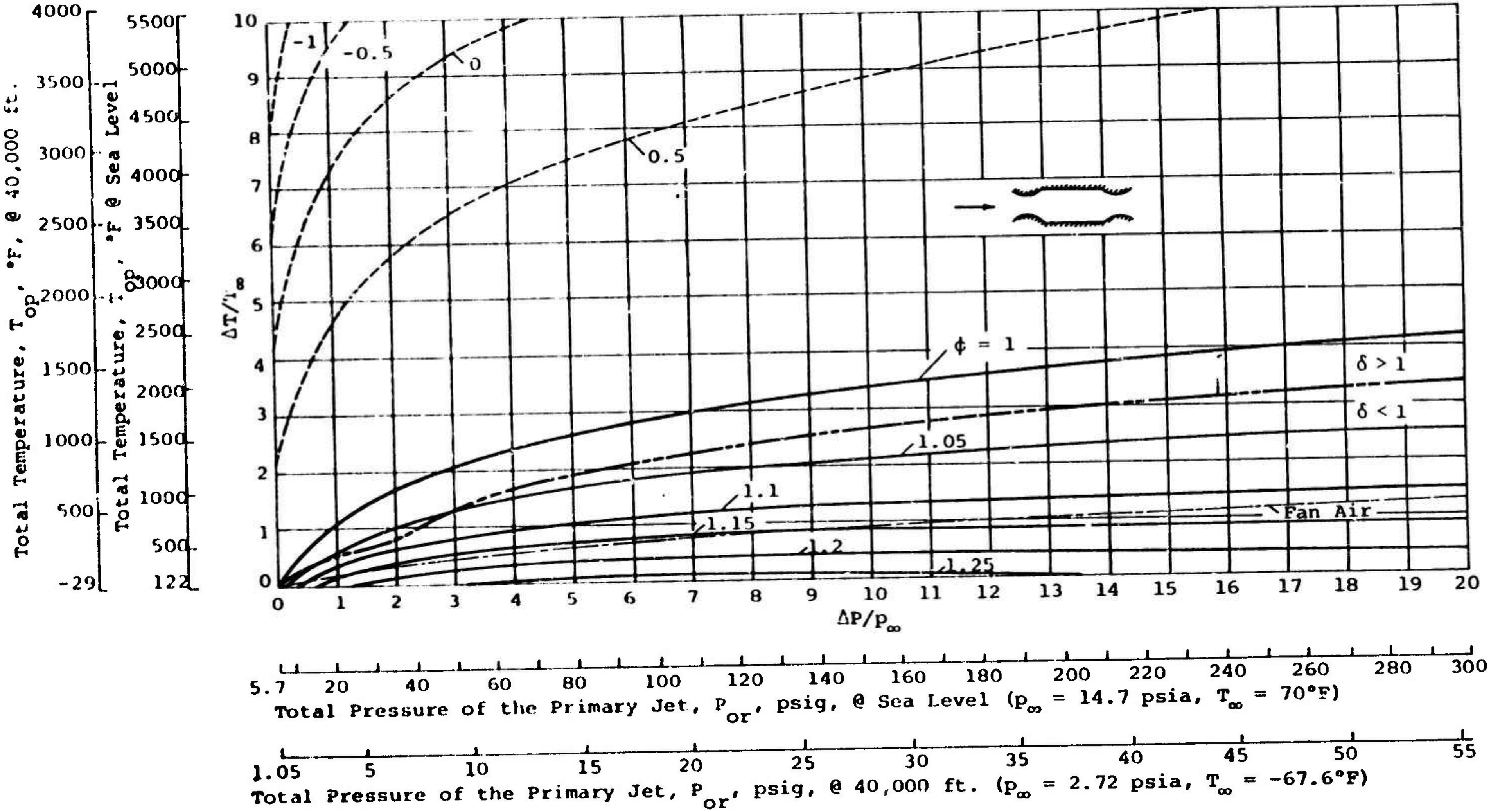
Subsonic Flight Speeds

Thrust augmenting ejectors encounter their most difficult operational conditions in the mid-subsonic range of flight speeds. At these speeds, the beneficial effects of ram compression tend to be balanced by their adverse effects.

Optimal ejector designs based upon the first solution with subsonic mixing are distinctly divided into two types separated by the upper line of $\phi = 1$ on Figure 10. The first type is sketched below the $\phi = 1$ line and as shown has similar geometry to the conventional stationary ejector design which has an accelerating inlet and operates best with relatively low temperature injected gas (like fan air for example). The second type requires a compression inlet, and operates best at relatively low nozzle pressure ratios and high temperatures, or ramjet like injected gas. The first (conventional) type ejector can not achieve adequate performance at this flight speed, as shown. The performance of the second (ramjet) type ejector becomes significant and shows good performance at this flight Mach number (0.7) as illustrated on Figure 10. This is not evident in the stationary case (Figure 7). As will be shown later, the ramjet type ejector dominates the ejector design configuration under the first solution at higher speeds.

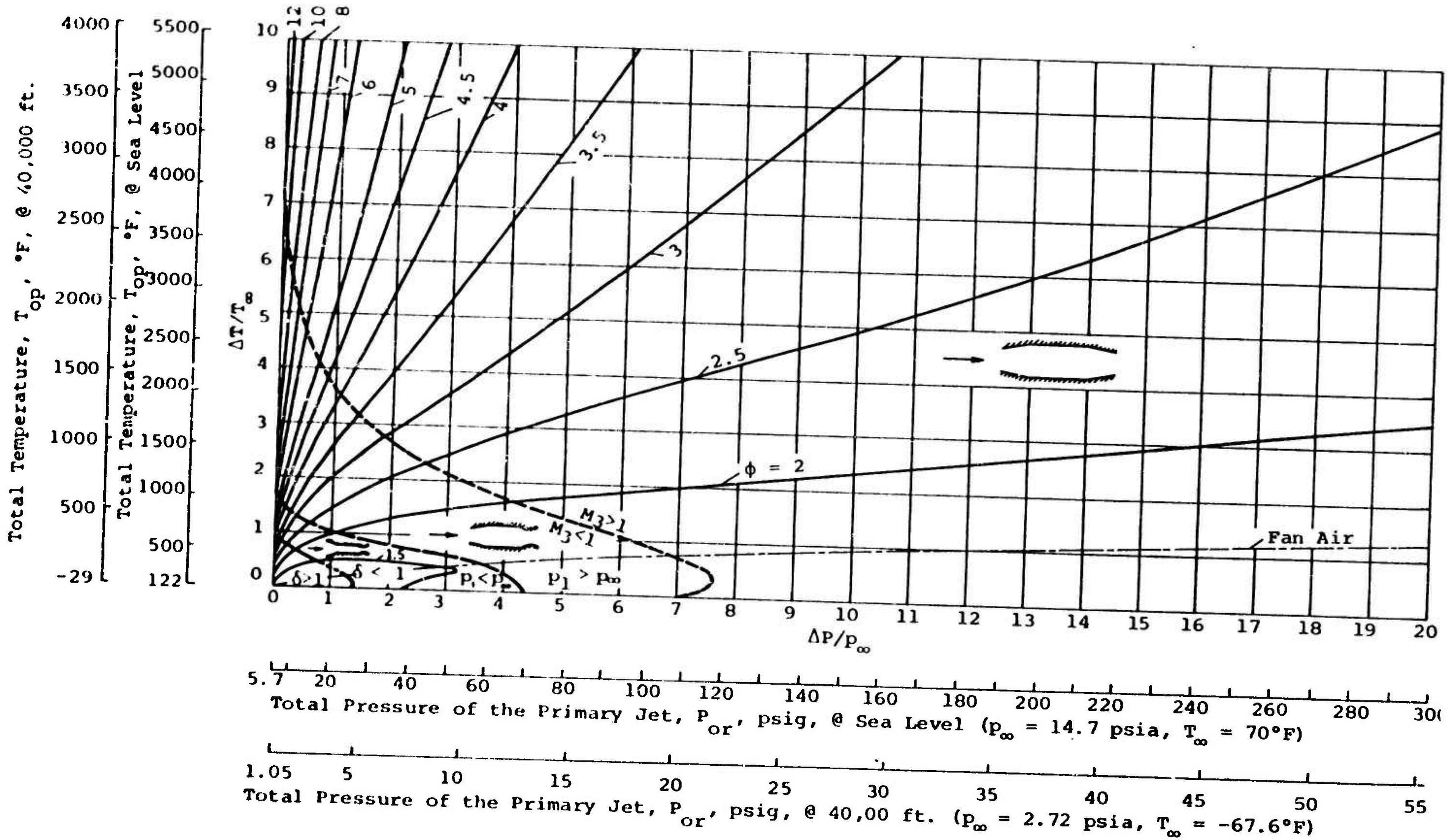
Ejectors operating at a flight Mach Number of 0.7 and designed under the second solution with supersonic mixing also display very poor optimal performance. The best performance, however, occurs at high primary nozzle pressure ratios and low primary temperature ratios as illustrated on Figure 11.

Figure 12 illustrates the tremendous advantage achievable through the use of the second solution with subsonic mixing. The ideal, limiting performance achievable by ejector designs prescribed by this type of flow can be very high over the entire practical range of primary pressure and temperature ratios. Further, the performance of these ejectors improves with increasing primary stagnation temperature (except for temperature below roughly the fan-air line), but falls off with increasing primary stagnation pressure. As indicated on Figure 12, the appropriate outlet design is a supersonic diffuser, either convergent or convergent-divergent. Therefore wave losses at the outlet become the major concern in designing this type of ejector for operation at this flight speed range.



Freestream Mach Number: $M_\infty = 0.7$; Ejector: $\alpha_\infty = 20$, Maximum ϕ of Second Solution (Supersonic Mixing)

Figure 11. Ideal Ejector Performance Map



Freestream Mach Number: $M_\infty = 0.7$; Ejector: $\alpha_\infty = 20$, ϕ of Second Solution with Subsonic Mixing, $\Delta S = 0$

Figure 12. Ideal Ejector Performance Map

Supersonic Flight Speeds

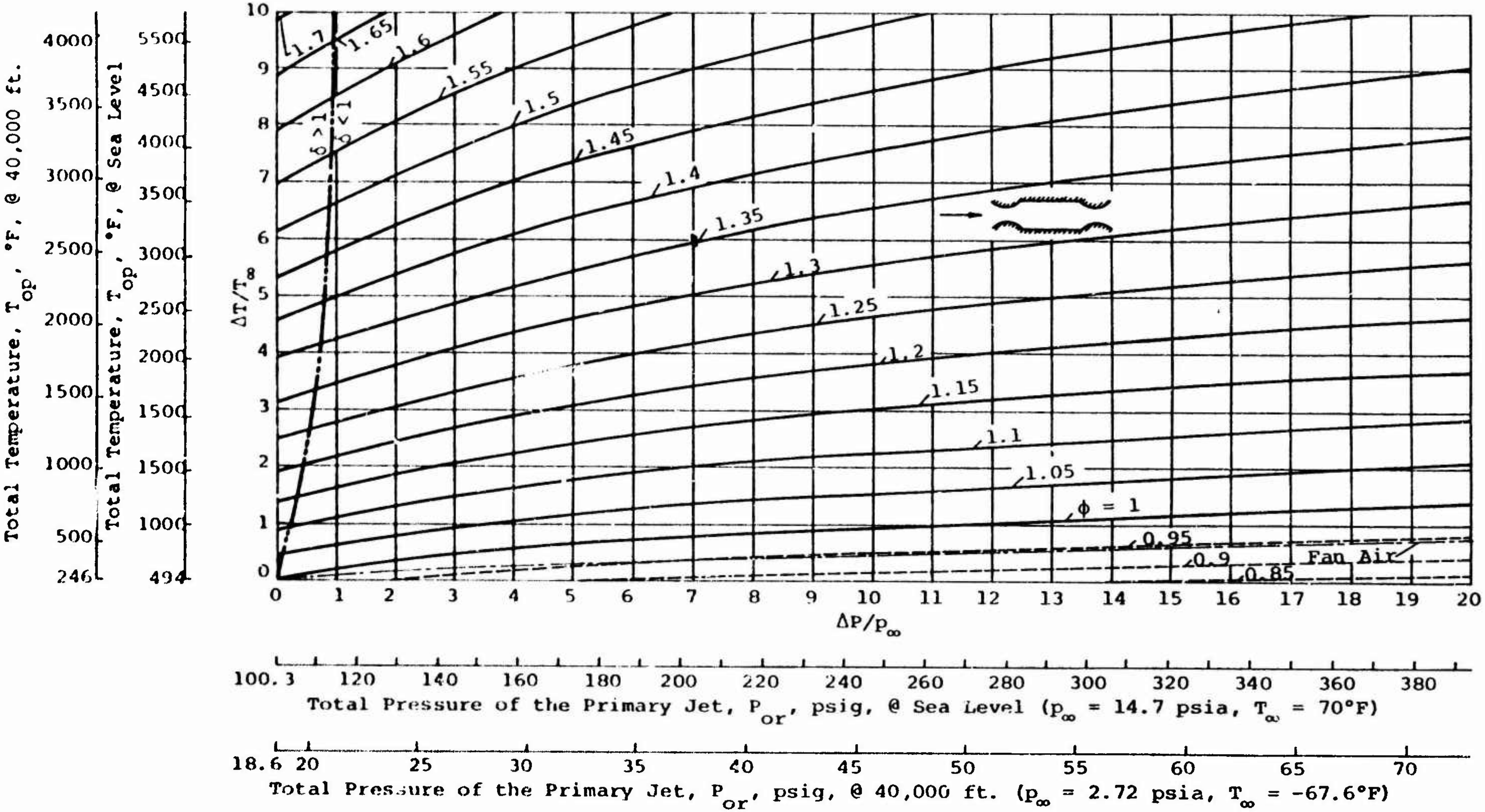
Ejectors translating at supersonic speeds can provide very large thrust augmentations provided the design criteria and injected gas characteristics are properly chosen and the configuration of the ejector is optimized.

Figure 13 illustrates the ideal performance of ejectors designed under the optimal conditions of the first solution, while translating at a Mach number of 2. As shown, better performance occurs at higher temperature and lower pressure ratio injected gas (ramjet type efflux), but the performance is a rather weak function of the nozzle pressure ratio, thus providing good performance even with turbojet or rocket type injected gas. As illustrated a supersonic convergent-divergent diffuser is required for ideal inlets and outlets over the range of specified conditions for flows resulting from this optimal design point.

Figure 14, illustrates the ideal, optimal thrust augmentation for ejectors translating at a Mach number of 2, when designed under the second solution with supersonic mixing. Better performance in this case also occurs when the injected gas has a higher stagnation temperature. The inlet is a supersonic converging diffuser, and the outlet is a diverging nozzle over most of the range of conditions illustrated.

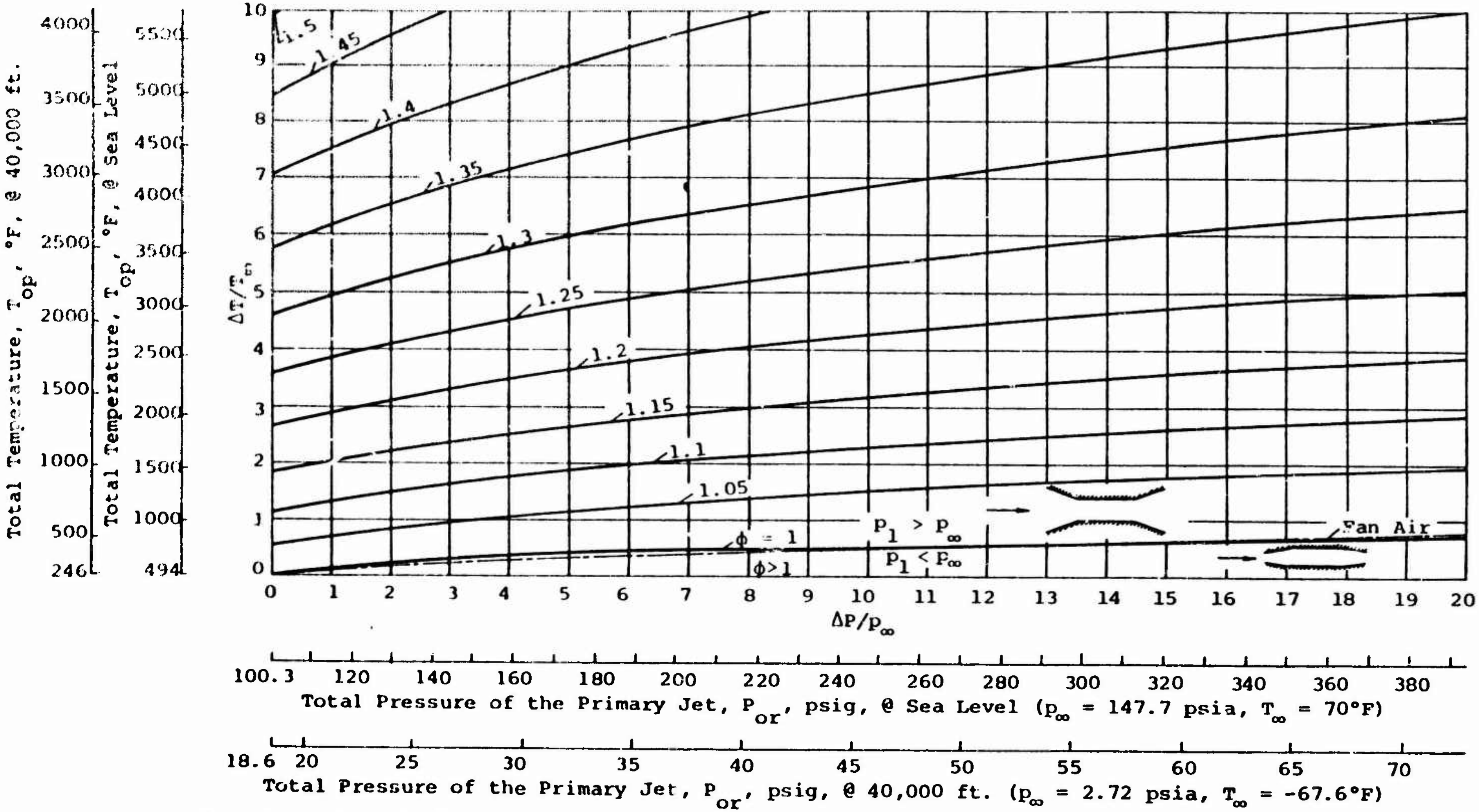
Figure 15 illustrates the ideal, limiting thrust augmentation for ejectors translating at a Mach number of 2, and designed under the second solution with subsonic mixing. This limiting performance occurs at the condition where the total entropy after mixing is equal to the sum of the entropies of the primary and induced flows at the start of mixing. As shown, the limiting performance under this condition still achieves its maximum values (for a given temperature) with ramjet type injected gas, but the performance of ejectors designed under the second solution with subsonic mixing is considerably better than that achievable by ejectors designed under either of the other conditions. To achieve this type of flow at the prescribed flight Mach number it is essential that the ideal inlet be convergent-divergent, and the outlet be a divergent nozzle as illustrated. Therefore, inlet compression loss is likely to be a major factor controlling the ejector performance.

The influence of losses in the ejector flow upon optimal design criteria and performance are illustrated in the following discussion.



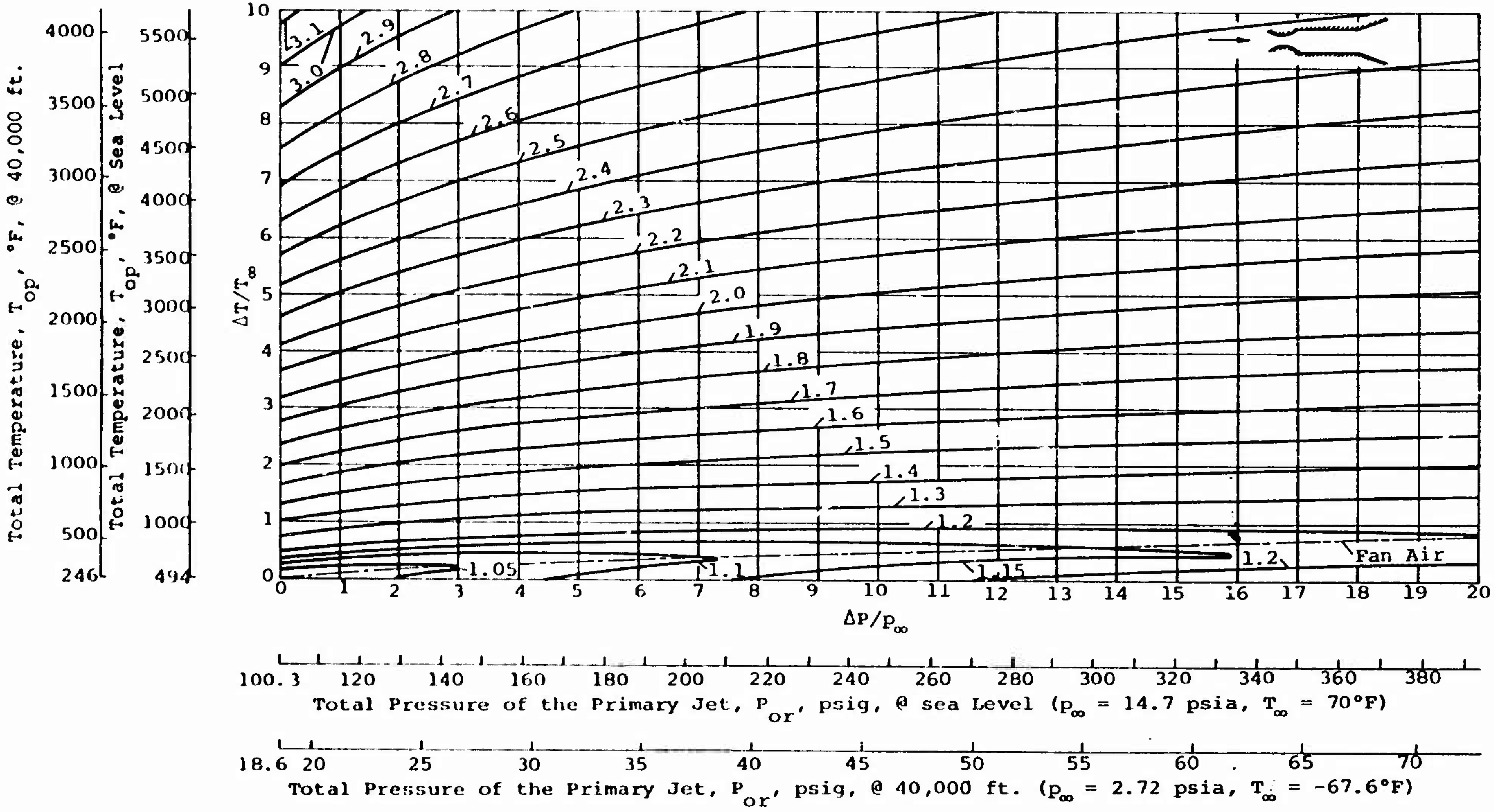
Freestream Mach Number: $M_\infty = 2$; Ejector $\alpha_\infty = 20$, Maximum ϕ of First Solution (Subsonic Mixing)

Figure 13. Ideal Ejector Performance Map



Freestream Mach Number: $M_\infty = 2$; Ejector: $\alpha_\infty = 20$, Maximum ϕ of Second Solution (Supersonic Mixing)

Figure 14. Ideal Ejector Performance Map



Freestream Mach Number: $M_\infty = 2$; Ejector: $\alpha_\infty = 20$, ϕ of Second Solution with Subsonic Mixing, $\Delta S = 0$.

Figure 15. Ideal Ejector Performance Map

Influence of Losses on Ejector Performance

The actual performance of an ejector will obviously be degraded in comparison to that calculated under the assumption of ideal flow. This degradation of performance may be attributed to skin friction, blockage, incomplete mixing and where supersonic flow is involved, to wave losses. The actual realistic performance of ejectors can only be determined by precise evaluation of the various loss factors or by experiment. Exaggerated concepts of the amount of degradation due to the losses can result from overly pessimistic estimates of some loss factors or from a failure to properly optimize the ejector geometry in view of the losses.

Since thrust augmenting ejectors operate with an overall pressure ratio of 1 (ingestion and discharge are at ambient pressure), the processes occurring within the ejector generally require compression and expansion. Constant pressure throughout the cycle always results in very poor performance. Those operational and injected gas characteristics which can result in very high ideal performance virtually always require a high degree of compression (adverse pressure gradients or shock waves) at the inlet or outlet, or both.

Obviously, an ejector at rest with respect to the undisturbed medium must have an accelerating (expansion) inlet. Thus, as is well known, high performance requires high compression (diffusion) at the outlet.

Ejectors translating at high speed (subsonic or supersonic) may have either expansion or compression inlets at their optimal performance configuration, but high performance will generally require a compression process as a part of the cycle.

It is those compressive elements of the ejector which may significantly alter the ideal flow pattern (flow separation for example) and which must be carefully designed to avoid excessive losses if high performance is to be achieved. The following discussion is intended to illustrate the influence of those major losses on ejector design and performance.

Subsonic Compression

In a conventional ejector configuration, the outlet generally consists of a subsonic diffuser. This is particularly true for ejectors designed under the first solution for operation at low subsonic speeds, with low stagnation pressure and temperature primary fluid (Figure 7 and the lower part of 10 for example).

At high subsonic speeds, subsonic compression inlets dominate the configurations which achieve optimal performance, as illustrated on Figures 10 and 12. This inlet configuration is somewhat similar to those utilized for subsonic jet engine inlets, but the details of these designs remain to be investigated.

The jet-diffuser ejector was created to overcome the subsonic compression problem involved in the conventional ejector outlet. However, other major obstacles to high performance include the primary nozzle attitude and mixing, which contribute to variations of the flow pattern. In addition, skin friction and inlet blockage can contribute to high losses. Methods utilized to evaluate and optimize these factors are described below.

Jet-Diffuser Ejector - Designed Under the First Solution

To illustrate the advantage achievable by optimal ejector design, the analysis described in Reference 13 was used to evaluate the performance and to determine the optimal geometry of the stationary jet-diffuser ejector to be integrated into the E205 (as shown on Figure 2). The influence of geometric diffuser area ratio, nozzle pressure ratio and the loss factors upon the thrust augmentation of a jet-diffuser ejector with an appropriately designed inlet are described on References 9 - 11 and illustrated on Figures 16 and 17.

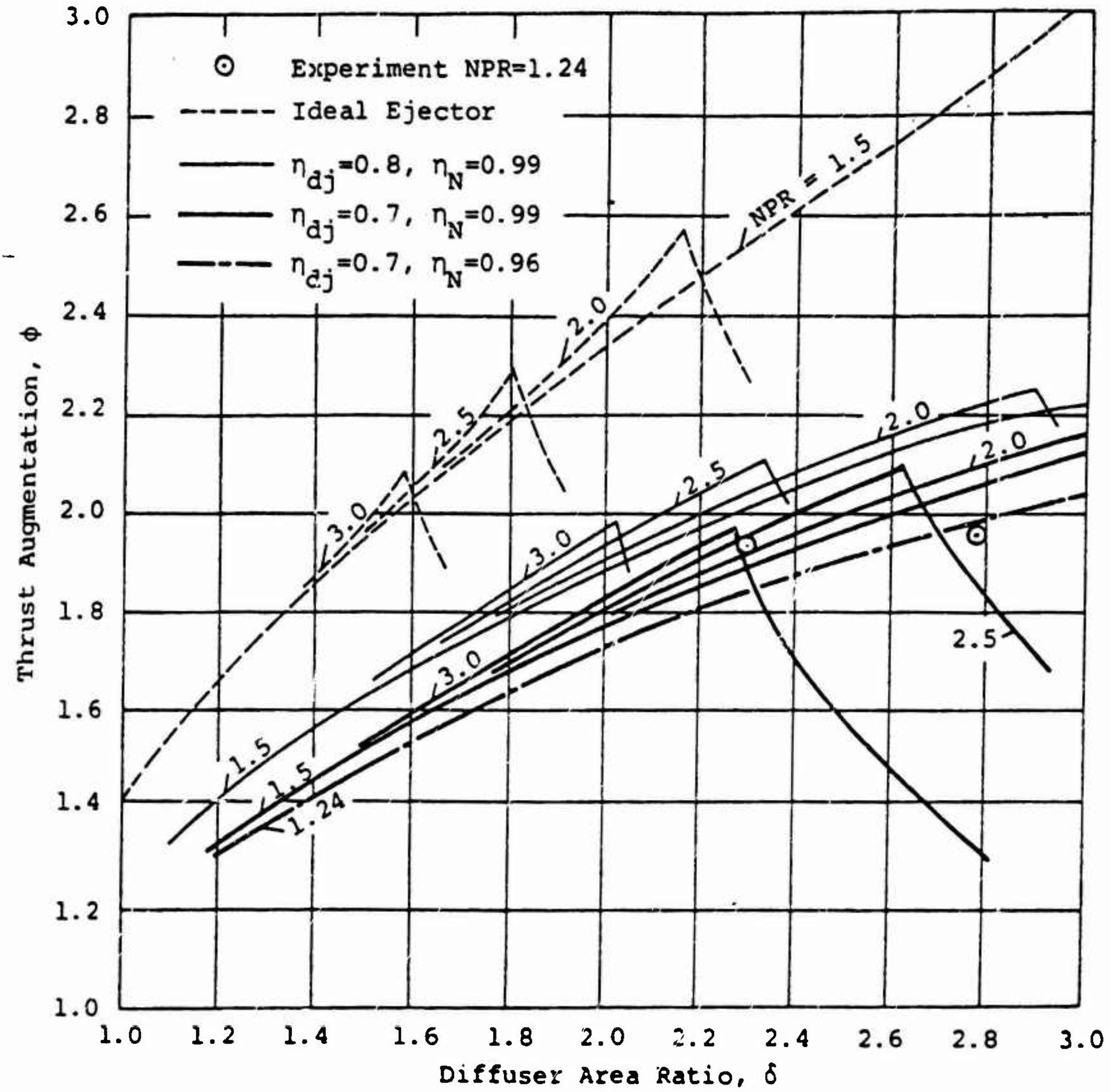
Nozzle thrust efficiency (η_N) had been evaluated experimentally at a low pressure ratio (NPR = 1.24) as reported in Reference 10. At this low pressure ratio, the nozzle thrust efficiency was determined to be 0.96, and it is estimated that at high pressure ratios, this factor will exceed 0.99 as a result of the Reynolds Number effect. The inlet drag coefficient (C_{di}) was determined by experiment and theoretical correlation to be 0.013 for a two-dimensional ejector. The increase of C_{di} due to skin friction at the ends of the ejector is a function of the throat aspect ratio of the ejector and is taken into consideration in the performance calculations used to derive Figures 16 and 17. The effect of skin friction on the diffuser jet is evaluated with the aid of conventional boundary layer theory as described in Reference 13. To include viscous effects, the influence of manufacturing and flow non-uniformities, two and three-dimensional effects and finite longitudinal dimensions, a factor (η_{dj}) called jet-diffuser efficiency was used to represent the ratio of the effective to the geometric area ratio of the solid portion of the diffuser as described in Reference 13.

Figure 16 illustrates the existence of an optimal diffuser area ratio for any given nozzle pressure ratio. The magnitude of this optimal diffuser area ratio and the corresponding thrust augmentation achievable with this optimal design depend upon the other geometric ejector factors and the loss factors. Thus, as shown on Figure 16, an increase of the diffuser area ratio can compensate somewhat for the performance degradation due to increased losses. Conversely, diffuser area ratios in excess of the optimal values can result in large performance losses. The lowest dashed curve on Figure 16 is drawn to indicate the correlation between analysis and experiment for the test conditions utilized in the experiments. The measured thrust augmentation of 1.95 achieved during the testing is very close to the theoretical curve resulting from the use of the factors derived for the ejector having a diffuser area ratio of 2.78.

As shown on Figure 16, testing of this same ejector at high pressure ratios (greater than about 2.0 to 2.5) would result in operation beyond the optimal point with drastic degradation of performance. For example, at a nozzle pressure ratio of 3.0, the thrust augmentation would be reduced from its optimal value of 1.95 to about 1.32, if the diffuser area ratio remained at 2.78. To provide optimal performance at a nozzle pressure ratio of 3.0, the solid diffuser area ratio must be reduced to about 2.3 if the losses at this pressure ratio are as assumed. Experiments were conducted with the diffuser cut down to an area ratio of 2.3 and, as illustrated, the measured thrust augmentation was 1.93 at the nozzle pressure ratio of 1.24. This experimental point lies above the theoretical curve indicating an improvement of the jet-diffuser efficiency (η_{dj}), due to the decreased diffuser area ratio.

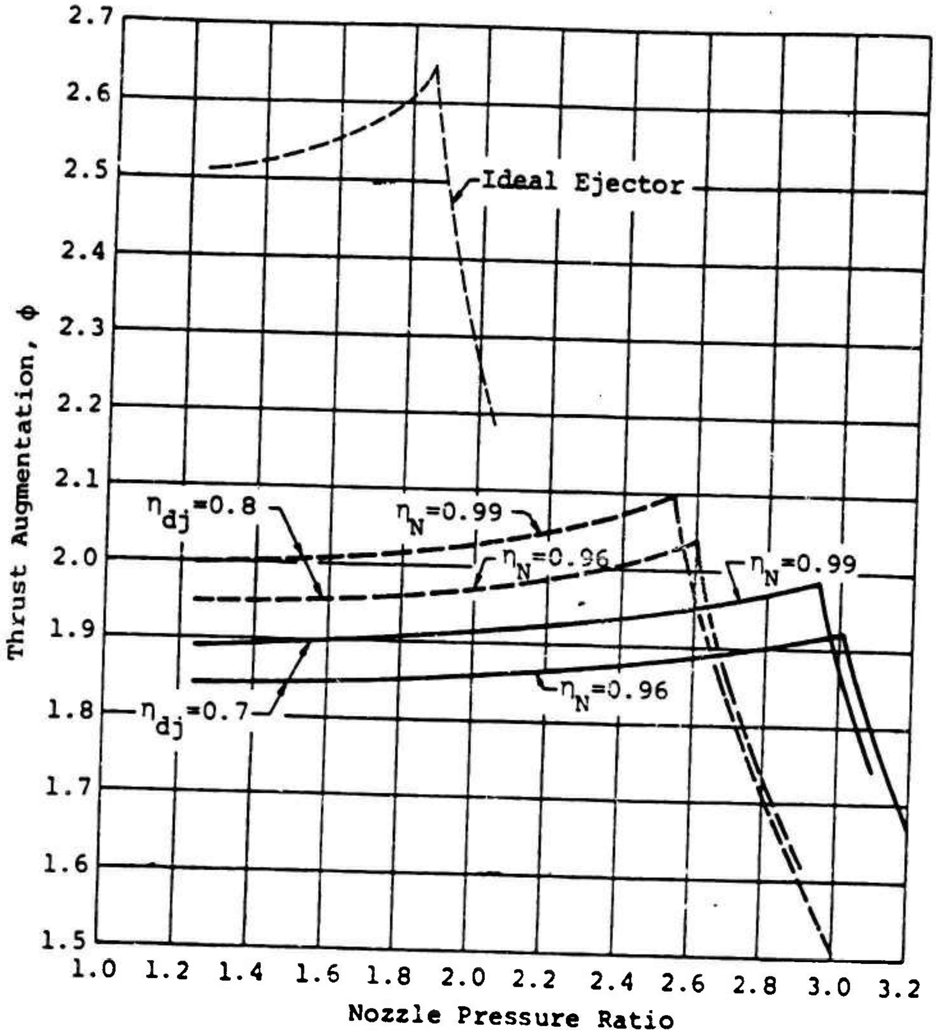
Figure 17 illustrates the variation of the thrust augmentation of the jet diffuser ejector as a function of nozzle pressure ratio for an ejector with a fixed diffuser area ratio of 2.3. As illustrated, a change of the jet-diffuser efficiency from 0.7 to 0.8 results in a reduction of the cut-off nozzle pressure ratio from 2.95 to 2.55. Thus if the jet-diffuser efficiency is increased as a result of the reduction of the area ratio, testing at a nozzle pressure ratio of 3.0 would result in very poor performance, since it would exceed the cut-off point shown on Figure 17. In that case, it would be desirable to reduce the nozzle pressure ratio to about 2.5 or to further reduce the diffuser area ratio to about 2.0, or increase the stagnation temperature of the primary jet.

A carefully planned experiment for correlation with this theory would be of great value in the design of thrust augmenting ejectors.



Air Source: Fan Air (isentropically compressed @ sea level)
Ejector Throat: 10.16 cm wide, 38.1 cm long (4 in wide, 15 in. long)
Inlet Area Ratio: 32.14 (=throat area/primary nozzle area)
Diffuser Jet/Primary Jet Mass Flow Ratio: 0.7

Figure 16. Stationary Jet-Diffuser Ejector Performance, as a Function of Diffuser Area Ratio



Air Source: Fan Air (isentropically compressed, @ sea level)
Ejector Throat: 10.16 cm wide, 38.1 cm long
(4 in. wide, 15 in. long)
Inlet Area Ratio: 32.14 (=throat area/primary nozzle area)
Diffuser Jet/Primary Jet Mass Flow Ratio: 0.7
Diffuser Area Ratio: 2.3

Figure 17. Stationary Jet-Diffuser Ejector Performance, as a Function of Nozzle Pressure Ratio

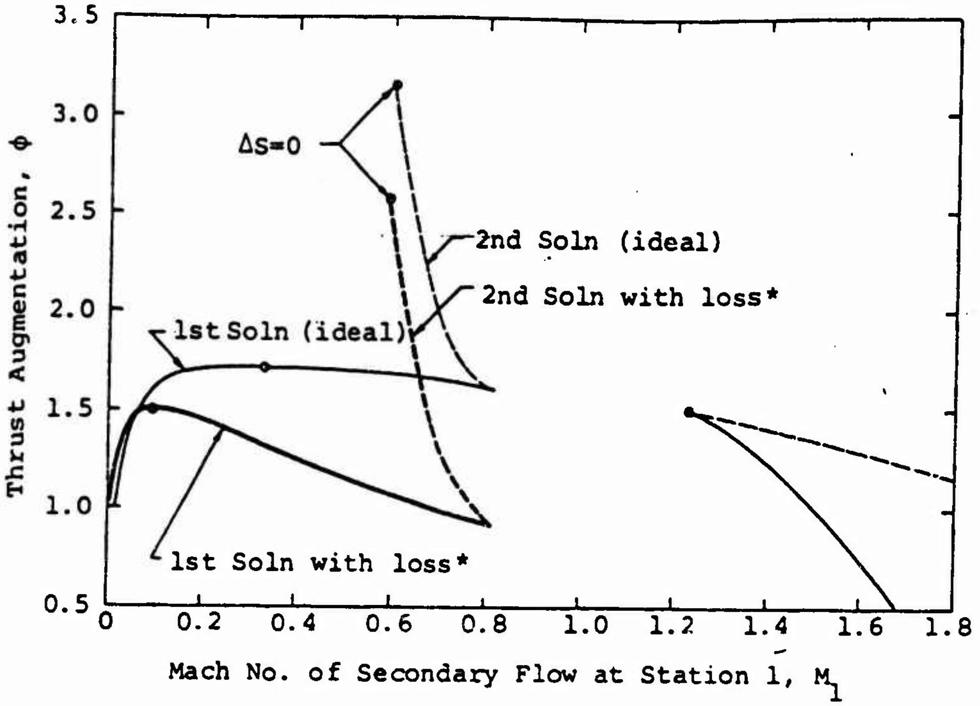
Inlet Wave Losses

Ejectors translating at supersonic speeds generally have an expansion outlet (either subsonic to supersonic or supersonic to supersonic, as shown on Figures 13 to 15) or a very weak supersonic compression outlet at low supersonic flight speeds. Therefore compression losses at the outlet are not a concern to the ejector design. The ejector may have a subsonic or a supersonic Mach No. (M_1) at the start of mixing. With supersonic mixing (M_1 greater than 1.0), the inlet for optimal designs is usually decelerating, and requires some weak compression of the supersonic flow. Since high compression can be avoided in this case, the performance degradation compared to the ideal should be small and can be evaluated. With subsonic mixing (M_1 less than 1), ejectors generally perform better than with supersonic mixing, but the inlet wave loss is also significant.

Figure 18 illustrates the change in the optimal ejector configuration and the performance degradation of supersonic ejectors, resulting from inlet losses. The losses were evaluated with the use of the standard engine inlet compression loss specification as required by MIL-E-5007D, in an ejector translating at a Mach number of 2. As indicated, the inlet compression loss results in a performance reduction and a configuration change. The optimal geometry is modified by the losses for designs under the first solution. Consideration of performance in the light of known losses requires a smaller value of M_1 than in the ideal case, and if properly optimized, the performance degradation can be small. Under the second solution, with subsonic M_1, the design configuration change is small but the thrust augmentation is degraded from a value of 3.13 to 2.56 due to the inlet losses.

Outlet Wave Losses

The performance achievable by ejectors designed under the second solution with subsonic mixing has been shown to be considerably better than that achieved by designs under the other optimal conditions over the entire range of flight conditions encountered by modern aircraft. This second solution with subsonic mixing design criterion is particularly important for flight from the mid-subsonic to transonic speed range, since other optimal conditions generally can not achieve the desired performance, with efficient gas generators. The actual achievement of the flows required to obtain this high performance involves the design of outlets capable of accepting supersonic flows at some arbitrary pressure and returning them to ambient pressure with minimal or acceptable wave losses.



Condition: (Ram-jet) $M_\infty = 2$; $\Delta P/p_\infty = 0$; $\Delta T/T_\infty = 10$; $\alpha_\infty = 20$

*loss: MIL-E-5007D Inlet Recovery Factor
for $1 < M_\infty < 5$ is,

$$\eta_i = \frac{P_{o1}}{P_{o\infty}} = 1-0.075\ (M_\infty - 1)^{1.35}$$

Figure 18. Supersonic Ejector with Inlet Compression Loss

In an attempt to demonstrate the feasibility of achieving the flow and performance attributable to second solution-subsonic mixing ejectors, FDRC with support from AFOSR and AFFDL, has initiated studies of the outlet design required by such ejectors. The study included an investigation of the starting problem for such supersonic flows and the losses and realistic performance of fixed geometry outlets capable of "swallowing" the starting shock wave and of avoiding excessive outlet losses. As a continuing part of this study, the use of simple, adjustable outlets have also been investigated.

Fixed Geometry Outlets

The starting problem can be avoided if the ideal isentropic outlet has a minimum area larger than that required for accomodating the mass flow when a normal shock wave is present in the mixing section, similar to the supersonic wind tunnel design discussed in Reference 16. In other words, the starting problem disappears if the mixed flow has a static pressure high enough to permit isentropic return to ambient pressure without excessive supersonic compression. Investigation to date showed that avoiding the starting problem is possible only at high flight speed (especially supersonic) and at high primary stagnation pressures and temperatures. These characteristics represent some realistic, in flight conditions and are encouraging from the point of view of the feasibility of designing operational systems which are quite simple. However, at supersonic speeds, the inlet compression loss becomes dominant, as discussed earlier. It has been observed that the utilization of an exit area large enough to accomodate the mass flow when a normal shock wave exists in the mixing section, can result in one of four possible outlet flows, described schematically on Figure 19.

When the steady state flow after mixing is supersonic and has a sufficiently high pressure to be returned isentropically to ambient pressure with very little or no supersonic compression, a shockless outlet can be utilized. In this case no starting problem exists. The outlet design for these conditions can appear as illustrated on Figures 19a and 19b.

In Figure 19a, the isentropic outlet design for starting as well as cruise operation is a divergent nozzle, which represents the case in which the supersonic flow at the end of mixing has a pressure in excess of ambient and must be expanded to return to ambient pressure.

Figure 19b illustrates the case in which the supersonic flow after mixing has a pressure less than ambient and isentropic compression to ambient pressure results in a smaller, but still supersonic Mach number, and with a minimum area larger than that required for accomodation of the subsonic mass flow behind a normal shock wave in the mixing section.

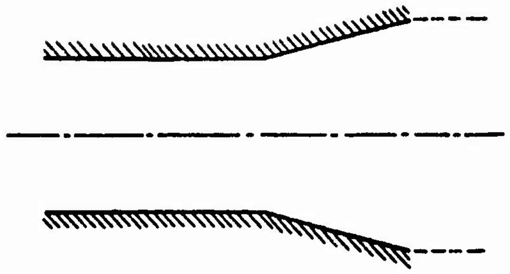
When the flow after mixing is supersonic and has properties such that its isentropic return to ambient pressure requires high compression, and results in low supersonic or subsonic exit velocities, it is impossible to avoid outlet shock waves if the starting problem is considered. The outlet flow pattern and schematic shapes for these situations are represented on Figures 19c and 19d.

Figure 19c illustrates the situation in which the ideal steady state operating outlet has a minimum area (either a "sonic throat" or the outlet opening) which is smaller than the minimum area required for starting (swallowing the starting shock wave). If the properties of the flow after mixing are such that a normal shock wave at the minimum starting area will result in exit pressure greater than ambient, and if the ejector outlet has an opening corresponding to the minimum starting area, the final compression of the flow to return to ambient pressure will be accomplished by a system of oblique waves. This situation appears to dominate ejectors which are translating at low subsonic to transonic speeds.

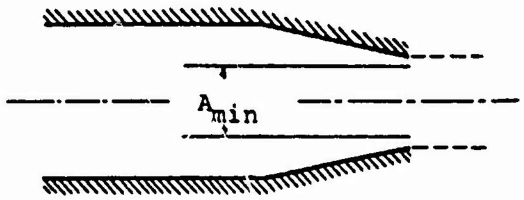
Figure 19d illustrates a situation in which the ideal steady state operating outlet has a minimum area which is smaller than that required for starting and in which a normal shock wave at the minimum starting area will result in an exit pressure which is smaller than ambient. This situation can result in either a stronger shock wave to satisfy the exit pressure requirement or, more desirably, a weak normal shock wave at the minimum area followed by a subsonic diffuser. The application for this type of outlet appears to be in the low subsonic flight speed regime, or low primary stagnation pressure and temperature gas at higher subsonic speeds.

The performance of ejectors suitable for laboratory study, using cold air supplies and translating at a Mach number of 0.65 and utilizing fixed geometry outlets is shown as a function of the nozzle pressure ratio on Figure 20. As indicated this type of outlet design for a second solution ejector with subsonic mixing is capable of performance which is considerably better than that obtained from ideal optimal first solution or second solution with supersonic mixing designs.

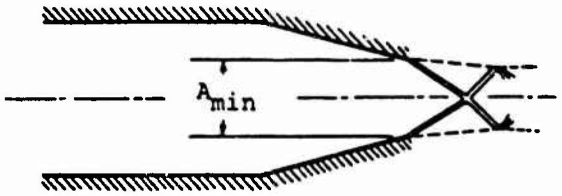
Note that α_* is 25 for the considerations presented on Figure 20. This means that the mixing duct area is fixed at 25 times the throat area (for supercritical pressure ratios) or 25 times the jet area when fully expanded to ambient pressure (for subcritical pressure ratios). Also, the entrainment ratio (induced mass flow rate/primary mass flow rate) for the conditions shown on Figure 20, decrease rapidly from about 21 at $\Delta P/p_\infty = 0.2$ to about 1 at $\Delta P/p_\infty = 20$ under the second solution at the limiting design point, $\Delta S = 0$.



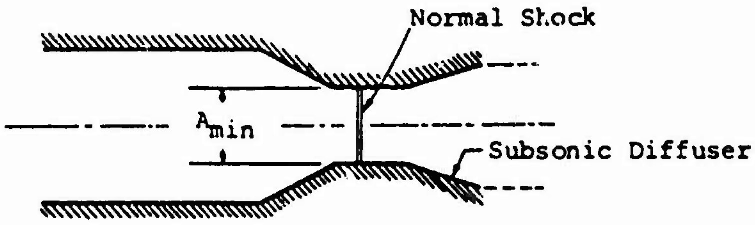
a) Shock-Free Supersonic Nozzle



b) Shock-Free Supersonic Diffuser

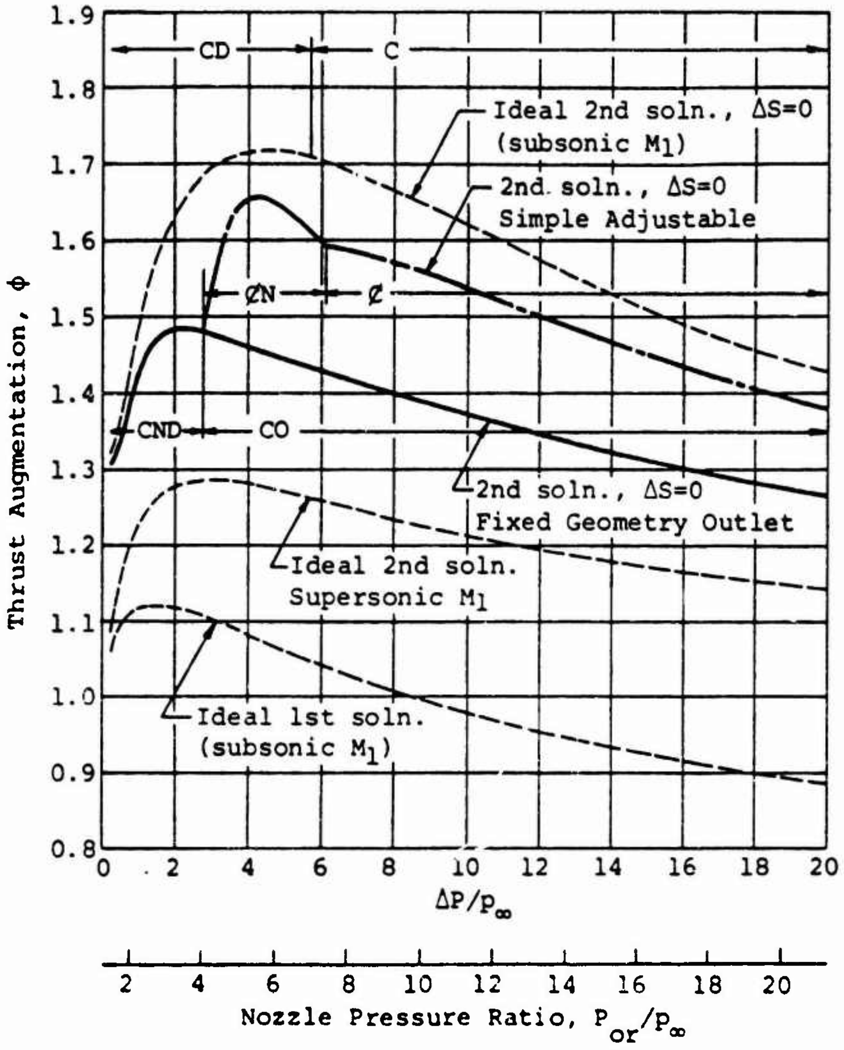


c) Supersonic Diffuser Followed by
Systems of Oblique Waves



d) Supersonic Diffuser, Normal Shock Wave
Followed by a Subsonic Diffuser

Figure 19. Fixed Geometry Outlets For Thrusting Ejectors Designed Under Second Solution



Configuration Notes:

C = Convergent supersonic diffuser (isentropic)

₵ = Convergent supersonic diffuser with flat wall and two oblique shock waves

D = Divergent subsonic diffuser (isentropic)

N = Normal shock wave, either at minimum starting section or at exit

O = Oblique wave systems required at the exit for final compression

Figure 20. Influence of Outlet Wave Losses on Ejector Performance
$M_\infty = 0.65$; $\alpha_* = 25$; $T_{op} = T_\infty$

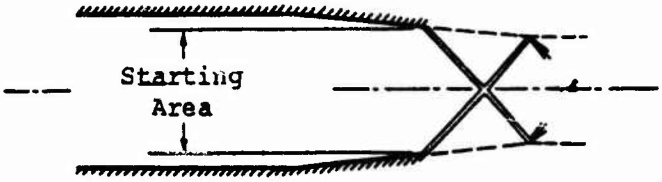
Simple Adjustable Outlet

Those conditions which require outlet designs in which the ideal minimum area cannot accomodate the mass flow under the starting condition are of two types illustrated on Figures 19c and 19d. Figure 19d, represents a configuration which requires a subsonic diffuser downstream of the minimum starting area. This would require a complex mechanism, similar to an adjustable second throat utilized in supersonic wind tunnels, for achieving an efficient outlet capable of swallowing the starting shock wave and providing an optimal outlet during cruise operation. Fixed geometry outlets corresponding to the starting condition are probably the most practical design for these conditions.

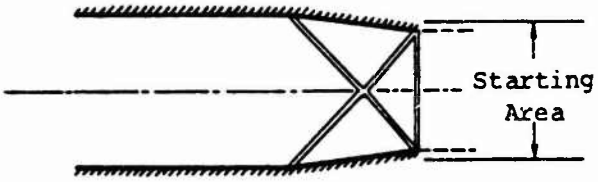
Figure 19c represents a more universal wave pattern which can provide acceptable but still quite degraded performance compared to the isentropic case as illustrated on Figure 20. By designing a simple adjustable outlet, the cruise performance of the ejector will be almost equivalent to that of the ejector with an isentropic outlet.

The simple adjustable outlet consists of a flat surface (in a two-dimensional ejector) on either side of the outlet, capable of very small rotation only, as illustrated on Figure 21. In the starting configuration these surfaces are adjusted to provide the required minimum starting area (Figure 19c or 21a). The surfaces can then be rotated to reduce the outlet area. The reduction of outlet area results in a reduction of the Mach number and an increase of the static pressure at the exit section. Obviously, the Mach number inside the exit section is still supersonic. When the increase of the static pressure is sufficient to compress the mixed flow to ambient pressure, the external starting oblique wave system (Figure 21a) will be eliminated during the outlet area adjustment, and the wave pattern associated with the cruise configuration will appear as shown on Figure 21c, which has a supersonic exit flow. When the increase of static pressure is not sufficient to return the mixed flow to ambient pressure, the external starting oblique shock wave system will require larger wave angles, due to the decreasing Mach number and finally form a normal shock wave at the exit section (Figure 21b), during the outlet area adjustment. Since the final compression of the mixed flow is accomplished by a normal shock wave, the discharged flow is subsonic.

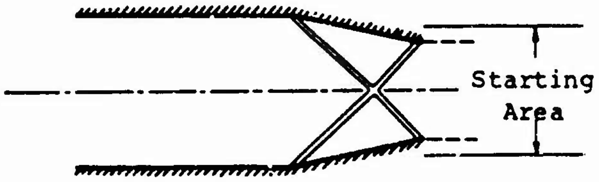
The performance improvement resulting from the adjustment of the outlet is shown on Figure 20. This analysis for the simple adjustable outlet utilized the concept of two internal oblique shock waves as shown on Figure 21, for realistic evaluation of the wave loss. These wave losses have been shown to have very little effect upon the ejector performance both during starting or with fixed geometry configurations (Figure 19) and therefore the internal waves (if any) in the fixed geometry configurations have been neglected.



a) Starting Configuration



b) Cruise Configuration with a
Normal Shock at the Exit



c) Cruise Configuration without a
Normal Shock at the Exit

Figure 21. Simple Adjustable Outlets

Conclusions

The process of mixing of compressible gases represents one of the most outstanding examples of the erroneous conclusions which can be drawn as a result of the use of incompressible flow theory where the fluids are actually compressible. Limitations implicit in the incompressible flow theory result in a failure to display the reality of a "second solution" which represents configurations having the best ideal performance over the entire ranges of operational and injected gas characteristics. Incompressible flow theory also fails to describe limitations due to thermal effects and choking.

The analysis of ejector flows based upon the use of compressible flow theory provides insights into the influence of motion in the thrust direction and thermal effects due to the injection of hot primary gas, which are of great value in the design of thrust augmenting ejectors operating in and with compressible fluids.

As shown by the compressible flow analysis, a properly designed ejector can derive beneficial performance from the utilization of the thermal energy content of its primary, injected fluid. Further, the performance of thrust augmenting ejectors need not deteriorate as rapidly due to motion in the thrust direction as is indicated by incompressible flow theory, provided the variation of stagnation characteristics of the injected and ingested gas are properly treated with changes of velocity.

The choice of ejector geometry required to achieve optimal performance is also essential to the design of high performance ejectors. The variation of optimal geometry with loss factors, in addition to the operational and injected gas characteristics must be considered in the final selection of ejector geometry.

Properly designed thrust augmenting ejectors can achieve high performance over the entire range of flight conditions, and can achieve large savings of energy (fuel consumption) in most aircraft applications.

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