Dielectric Properties of Snow

W.H. Stiles and F.T. Ulaby

Remote Sensing Laboratoiy
University of Kansas Center for Research, Inc.
Lawrence, Kansas 66045

ABSTRACT

This paper is a review of the dielectric properties of snow in the radio frequency range from 100 kHz to 35 GHz. Applicable dielectric mixing formulas are discussed and compared to available experimental data. Below 1 GHz, the dielectric behavior of snow is well understood and has led to development of sensors to measure the liquid water content of snow, while above 1 GHz, the scarcity of experimental data and the discrepancies between the few available measurements, particularly when liquid water is present, are responsible for the rather sketchy understanding of the dielectric behavior of snow in the microwave region.

INTRODUCTION

As the importance of snow as a renewable resource becomes increasingly recognized, methods of assessing snow properties in a timely fashion over large areas become indispensable. It is clear that, for both timely observations or large areal coverage, the physical handling of the snow must be minimized. Remote sensing techniques satisfy the above requirement; however, interpretation of the remotely sensed data is not always straightforward. These data are usually in the form of processed quantities which are determined by the parameters of the measurement system and the characteristics of the observed scene (in this case, snow). In the radio region of the spectrum, the interaction of electromagnetic energy with snow is governed by its geometrical and electrical (dielectric) properties; therefore, a more complete description of the dielectric properties of snow should lead to a better understanding of remote sensing observations of snow.

Several review articles of the electrical properties of ice and snow have been published over the last 20 years [1-4]. The most recent of these reviews, by Glen and Paren [3], stressed the electrical behavior in the spectral range that is particularly suited to glacial sounding (DC to 100 kHz). This paper presents a review of the dielectric properties of snow, with special emphasis on its behavior at the radio frequencies above 100 kHz. Included in this review are: 1) a discussion of mixing formulas used to characterize the dielectric properties of a snow mixture in terms of its constituent parts—air, ice and liquid water (if present), and 2) a summary of experimental data reported in the literature.

PROPERTIES OF CONSTITUENTS

The electromagnetic propagation properties of a material are defined in terms of its magnetic permeability, μ, and its relative complex dielectric constant, ϵ. For most naturally occurring materials, including snow, $\mu \cong \mu_0$, the magnetic permeability of free space. Hence, the propagation is governed solely by ϵ, which has the complex form:

$$\epsilon = \epsilon' - j\epsilon'' \tag{1}$$

Since snow is a heterogeneous mixture of air, ice, and, under certain conditions, liquid water, the constituent dielectric properties must be examined first in order to understand the dielectric properties of the mixture. For practical purposes, the characteristics of air are indistinguishable from those of free space. Water and ice, however, exhibit dielectric behavior which can be described, at least to the first order, by the Debye equation:

$$\epsilon = \epsilon_\infty + \frac{\epsilon_0 - \epsilon_\infty}{1 + j2\pi f\tau} \tag{2}$$

or in terms of its real and imaginary parts:

$$\epsilon' = \epsilon_\infty + \frac{\epsilon_0 - \epsilon_\infty}{1 + (2\pi f\tau)^2} \tag{2a}$$

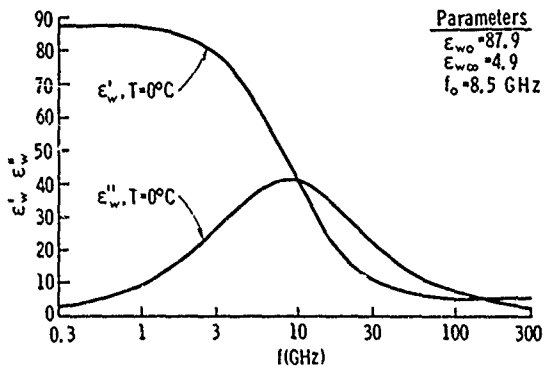$$\epsilon'' = \frac{2\pi f\tau(\epsilon_0 - \epsilon_\infty)}{1 + (2\pi f\tau)^2} \tag{2b}$$

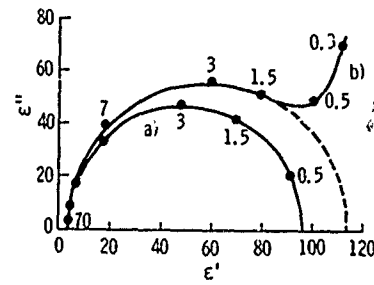Figure 1. Complex dielectric constant of water at 0°C from the Debye equation.

Figure 2. Cole-Cole diagram of ε" and ε' for ice samples at −10.8°C in a) pure ice and b) ice with impurities. Points represent measurements at the indicated frequencies in kHz. (From Evans [1].)

where $\epsilon_0 = \lim_{f \to 0} \epsilon$ or the static dielectric constant, dimensionless

$\epsilon_\infty = \lim_{f \to \infty} \epsilon$ or the optical limit of dielectric constant, dimensionless

τ = relaxation time of the material (s)

f = electromagnetic frequency (Hz).

This equation describes the contribution to the polarizability of a polar molecule (H_2O) from the permanent dipole moment of that molecule. An applied electric field tends to align the dipole against the thermal forces which induce disorde, or "relaxation" and are a function of the temperature and viscosity of the dielectric material. The relaxation time as derived by Debye for spherical polar molecules in a viscous medium is a good approximation for water [5]:

$$\tau = \frac{4\pi \eta a^3}{kT} \qquad (3)$$

where η is the viscosity, a is the molecular radius, k is Boltzmann's constant and T is the temperature.

Empirical expressions based on experimental results have been developed for the constants in equation 2 for water as functions of physical temperature [5–8] and are reviewed in [9]. Figure 1 illustrates the behavior of ϵ_ω' and ϵ_ω'' of water at 0°C as a function of frequency. The peak in ϵ_ω'' at 8.5 GHz results from the Debye dispersion being centered in the microwave region. The frequency at which the peak occurs is termed the relaxation frequency f_0 where $f_0 = (2\pi\tau)^{-1}$.

For the frequency interval of emphasis, 100 kHz to 35 GHz, the dispersion causes the dielectric properties of water to vary significantly. Below approximately 1 GHz, low frequency approximations to equations 2a and 2b yield $\epsilon_\omega' \cong \epsilon_{0\omega}$ and $\epsilon_\omega'' \cong 2\pi f T(\epsilon_{0\omega} - \epsilon_{\infty\omega})$, while

above 1 GHz the exact form must be used. For liquids or solids, interactions between molecules and viscosity variations tend to violate some of the assumptions of the Debye equation giving rise to a non-unique relaxation time. The Cole-Cole [10] modification to the Debye form (eq 2), i.e. raising the $j2\pi f\tau$ term to the power $(1 - \alpha)$, allows this equation to describe the dispersion for a distribution of relaxation times. For water, α has been determined to be less than 0.03 [8], indicating that the Debye form for a single relaxation time is adequate, but for ice and snow this is not always the case.

The dielectric properties of ice, at least over a large frequency interval, can also be described by the Debye equation of the same form as for water, equation 3. While the relaxation frequency of water lies in the microwave region, for ice (as a result of the higher viscosity compared to that of water) the relaxation frequency is reduced to the kilohertz region and as in the case for water, it is temperature-dependent. Auty and Cole [11] measured the dielectric coefficients for pure ice and found that $\epsilon_{i\infty} = 3.10$, that τ_i had a minimum value at 0° corresponding to $f_0 = 7.5$ kHz, and that ϵ_{i0} had a minimum value of 91.5 at −1 °C.

A convenient form for examining the spectral behavior of a material describable by the Debye form is the Cole-Cole diagram which is a plot of the imaginary part ϵ'' versus the real part ϵ'. Curves defined by equation 2 are semicircles in this representation. Figure 2 from Auty and Cole's data [11] illustrates measurements of pure ice and ice with impurities along with their respective Debye curves. Each symbol corresponds to a measurement at frequencies denoted in kHz. Two observations are apparent from this figure: 1) the DC conductivity of sample (b) causes the behavior to deviate from the Debye form below 1.5 kHz and 2) since the optical limit is the left-most intersection of the semicircle with the ϵ'-axis, this representa-

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Table 1. Measured values of the real part of the relative dielectric constant of pure or freshwater ice, ϵ_i' (from Ulaby et al. [9]).

Frequency (GHz)	Temperature range (°C)	ϵ_i'	Reference
0.15 to 2.5	−1 to −60	2.90 to 2.95	Westphal (in [1])
9.375	0 to −18	3.15	Cumming [14]
10	−12	3.17	Von Hippel [20]
10	0 to −35	3.14	Vant et al. [16]
10	−1 to −49	3.17	Lamb [17]
24	0 to −185	3.18	Lamb and Turney [18]
26.4 to 40.0	0 to −35	2.92	Vant et al. [16]
94.5	−28	3.08	Perry and Straiton [15]
1000	−173	3.20	Bertie et al. [21]
99	—	3.17	Blue [19]

tion is not very illuminating for the high frequency response because of the compression of the variation in ϵ ".

It should be noted that in the frequency range below 10 kHz, several loss mechanisms are present [3, 12, 13], primarily due to impurities, structural effects, and molecular or ionic defects. A good overview of these dispersions is given in the review by Glen and Faren [3] and in a paper by Paren and Glen [12]. These dispersions may be describable by the Debye form but do not occur at the same relaxation frequencies or have the same strengths. In the high-frequency limit, however, these values will each approach their optical limits and consequently will have very little contribution above 100 kHz. Therefore, the macroscopic dielectric behavior can be described by effective quantities that include contributions from all of the dispersion mechanisms.

For frequencies much greater than the relaxation frequency in ice, $f/f_0 = 2\pi f \tau_i \gg 1$, the following high frequency approximations can be made to the rationalized form of the Debye equation (eq 2a and 2b):

$$\epsilon_i' = \epsilon_{i\infty} \tag{4a}$$

$$\epsilon_i'' = \frac{\epsilon_{i0} - \epsilon_{i\infty}}{2\pi f \tau_i} = (\quad - \epsilon_{i\infty}) \frac{f_{i0}}{f} \tag{4b}$$

From measurements for which these approximations are valid, between 10 MHz and 1000 GHz, the optical limit $\epsilon_{i\infty}$ has been found to be insensitive to temperature (below 0 °C) and independent of frequency. Values from several experimental investigations [1, 14–21] for $\epsilon_{i\infty}$ are given in Table 1 and excellent agreement generally is observed; hence, the value of $\epsilon_{i\infty} = 3.15$ from Cumming [14] will be used as being representative. While the real part is well-behaved, the imaginary part is dependent on both frequency and temperature as expected from equations 3 and 4b. Figure 3 is a survey of the available data on ϵ_i'' as a function of fre-

quency for pure or freshwater ice at two temperatures. For this frequency interval, equation 4b indicates that ϵ_i'' should decrease as f^{-1}; therefore on a log-log scale, the behavior should be linear with a negative slope. The experimental data given in Figure 3 show, however, that the Debye form is inapplicable above 1 GHz, indicating that the single resonance assumption is not valid. Evans [1] attributed the observed spectral behavior of ϵ_i'' to contributions by infrared dispersions.

Since the theoretical expressions for ϵ_i'' are not adequate in the microwave frequency range, experimental results must be relied upon even though considerable variability exists in the data above 10 GHz. This variation is at least in part a consequence of the problems associated with making accurate measurements of the loss factor of a very low-loss material, which ice is.
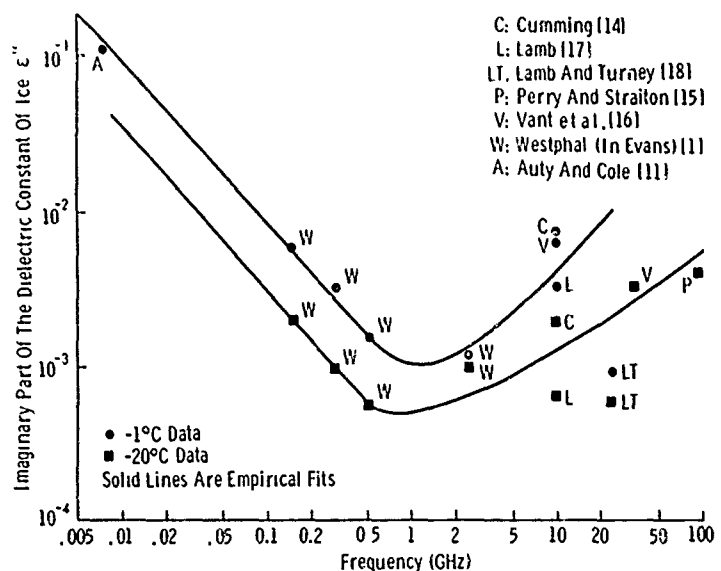

C: Cumming [14]
L: Lamb [17]
LT: Lamb And Turney [18]
P: Perry And Straiton [15]
V: Vant et al. [16]
W: Westphal (In Evans) [1]
A: Auty And Cole [11]

● −1°C Data
■ −20°C Data
Solid Lines Are Empirical Fits

Figure 3. Imaginary part of the relative dielectric constant of pure and fresh water ice. (From Ulaby et al. [9].)

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Table 2. Mixing formulae for snow.

Formula Number	Mixing Formula	Comments	Inclusion Shape	Reference
1	$$\frac{c_{ds} - 1}{3c_{ds}} = V_i \frac{c_i - 1}{c_i + 2c_{ds}}$$	Simplification of 6 for spherical with $V_w = 0$ $c_{ws} = c_{ds}$	spherical	Cumming [14] Glen and Paren [3] (Bottcher)
2	$$\frac{c_{ds} - 1}{c_{ds} + F} = V_i \frac{c_i - 1}{c_i + F}$$	Dry snow only	F = 0 vertical to F = ∞ planar	Evans (Weiner for dry snow) [1]
3	$$c_{ws} = \frac{c_w V_w U + c_{ds}(1-V_w)}{V_w U + (1-V_w)}$$ where $U = \frac{c_{ds} + F}{c_w + F}$	Wet snow	F = 0 vertical to F = ∞ planar	Edgerton et al. [22] (Weiner)
4	$$c_{ds}^{1/3} - 1 = V_i(c_i^{1/3} - 1)$$	Dry snow	Spherical	Glen and Paren [3] (Looyenga)
5	$$c_{ws} = c_{ds} + \frac{V_w}{3}(c_w - c_{ds}) \sum_{j=1}^{3} \frac{1}{1 + (\frac{c_w}{c_n} - 1)A_{wj}}$$	c_n - local dielectric constant accounts for particle interaction	Ellipsoidal	Sweeney and Colbeck [23] (de Loor)
6	$$c_{ws} = 1 + \frac{c_{ws}}{3} V_i(c_i - 1) \sum_{j=1}^{3} \frac{1}{c_{ws} + (c_i - c_{ws})A_{ij}}$$ $$+ \frac{c_{ws}}{3} V_w(c_w - 1) \sum_{j=1}^{3} \frac{1}{c_{ws} + (c_w - c_{ws})A_{wj}}$$	Different shape factors for water and ice	Ellipsoidal	Colbeck [24] (Polder and van Santen)
7	$$c_{ws} = c_{ds} + \frac{V_w}{3}(c_w - c_{ds}) c_{ws} \sum_{j=1}^{3} \frac{1}{c_{ws} + (c_w - c_{ws})A_{wj}}$$	Simplification of 5 with $c_n = c_{ws}$	Ellipsoidal	Ambach and Denoth [25] (Polder and van Santen)
8	$$c_{ds} = 1 + \frac{3V_i(c_i - 1)}{(2 + c_i) - V_i(c_i - 1)}$$	Simplification of 9	Spherical	Tinga et al. [26]
9	$$c_{ws} = 1 + \left\{ 3\left[\left(\frac{r_w}{r_a}\right)^3 (c_w - 1)(2c_w - c_i) - \left(\frac{r_i}{r_a}\right)^3 (c_w - c_i)(2c_w + 1)\right] \right.$$ $$\times \left[(2 + c_w)(2c_w + c_i) - 2\left(\frac{r_i}{r_w}\right)^3 (c_w - 1)(c_w - c_i) \right.$$ $$\left.\left. - \left(\frac{r_w}{r_a}\right)^3 (c_w - 1)(2c_w + c_i) + \left(\frac{r_i}{r_a}\right)^3 (c_w - c_i)(2c_w + 1) \right]^{-1} \right\}$$		Spherical of ice coated with water	Tiuri and Schultz [27] (Tinga et al.)

MIXING FORMULAS

The dielectric properties of a heterogeneous mixture such as snow must be described by mixing formulas that incorporate the dielectric properties and proportions of the component materials. In the rigorous formulations, structure will also be a factor. Snow can be treated as either a two-component or three-component mixture consisting of ice, air and liquid water. The two component formulation is usually applied to dry snow (air and ice) and sometimes to wet snow through an additional application of the formula for dry snow and water. In the three-component formulations, air, ice and water are each accounted for as individual components.

Both theoretical and purely empirical mixing formulas have been reported in the literature. In this section, the discussion will be limited to theoretical approaches,

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while discussion of the regression models will be presented in the section on dielectric properties. The developments of mixing formulas have usually been restricted to modeling one component as regularly shaped inclusions uniformly distributed within a host or background material. The mathematical difficulties of handling inclusion shapes and inter-inclusion interactions have required the choice of ellipsoidal or spherical particles. This limitation poses no real problem for snow that has been wet since the particle shapes tend toward spherical anyway; however, very fresh snow is not usually even close to spherical. Applicability for fresh snow therefore may be suspect but also may be difficult to examine because handling requirements and the action of metamorphism immediately start modifying its structure towards more stable thermodynamic shapes.

Mixing formulas which have been applied to snow are given in Table 2. With the exceptions of Formulas 2 and 3, which are variations on the formulation due to Weiner [1, 22] and which attempt to use a single factor to describe a three-dimensional shape (and are therefore inherently unsound), all the other formulas can be related to one another.

Formulas 5, 6 and 7 are descriptive of ellipsoidal inclusions with axes a, b and c and associated depolarization factors A_{kj}, where k denotes the inclusion type, either ice or water, and j denotes the axis. Usually two of the axes are assumed equal, thus since the sum of the A_j must equal one, only one integration is needed. Formula 5 is the most rigorous in that ϵ_n (the effective dielectric constant in the immediate region of the particle) can be different from either of the constituents or the mixture average. This formulation is the most general; however, it is also the most complicated to apply. Formula 7 makes the approximation that $\epsilon_n = \epsilon_{ws}$, the dielectric constant of the mixture, and appears to provide an adequate fit to the available data. Both of these formulations assume that the dry snow dielectric properties are already known, while Formula 6 treats the three-component case. In applying Formula 6 to measured dielectric data, Colbeck [24] found that the ellipsoidal ice particles may be approximated as spherical in shape without incurring significant error in the results, but for the water inclusions the shape factor was found to be very important.

Formula 1, which applies to dry snow only, is a special case of Formula 5; if the particles are spherical (i.e., $A_1 = A_2 = A_3 = \frac{1}{3}$), Formula 5 reduces to Formula 1 upon replacing ϵ_{ws} and ϵ_{ds} with ϵ_{ds} and ϵ_i, respectively, and setting $\epsilon_n = \epsilon_{ds}$. Similarly, Formula 4 can be derived from Formula 5 for dry snow conditions.

The expressions given by Formula 9 are a special case of a more general formulation derived by Tinga et al. [26] for ellipsoids of dielectric ϵ_1 covered by a layer of material of dielectric ϵ_2 contained in a host material of dielectric ϵ_3. As given in Table 2, Formula 9 is for spherical ice particles covered by a uniform layer of water. Formula 8 is the same as Formula 9 with no water present (i.e., dry snow). While Colbeck [24] states that the water distribution in the snowpack is not in spherical layers around the ice particles, Formula 9 has been found to give acceptable results when compared to experimental data, as discussed in a later section.

DIELECTRIC PROPERTIES OF SNOW

The dielectric properties of dry and wet snow are discussed in separate sections because the addition of liquid water to dry snow significantly alters its properties.

Dry snow

Because the relaxation frequency of ice is in the kilohertz range, dry snow is not expected to exhibit a dispersion behavior above 100 kHz. Although surface effects and other processes may cause additional low-frequency dispersions [5], the mixture with air, which can be considered to have $f_0 = \infty$, makes the relaxation frequency of snow always higher than that of ice. The relaxation frequency for natural dry snow remains below approximately 100 kHz. Dielectric measurements, conducted for densities ranging between 0.08 and 0.41 g cm^{-3} and for temperature between $-10\,°C$ and $0\,°C$, show that the relaxation frequency of dry snow, f_{ds}, is in the 5- to 60-kHz range [1, 12, 28].

Ambach and Denoth [25] measured the frequency dependence of ϵ'_{ds} and the loss tangent, $\tan \delta_{ds}$, for fine-grained (<0.25-mm radius) and coarse-grained (>0.25-mm radius) dry snow. The loss tangent is defined as the ratio of the imaginary part to the real part of the dielectric constant. Their results, given in Figure 4, indicate different relaxation spectra for the two samples; the coarse-grained sample exhibits a single Debye-type relaxation, while the fine-grained sample exhibits a wide relaxation spectrum indicative of a range of relaxation frequencies. This result does not imply that the size is the effect; the depolarization factors of the ice particles were very much different from the two cases: fine-grained $A \leq 0.03$ and coarse-grained $A \geq 0.20$. Above 10 MHz, the real part is independent of frequency and the effect of grain shape on the relaxation spectrum in the high-frequency limit is negligible.

For frequencies above 1 MHz the dominant physical parameter that determines the dielectric constant of dry snow is its density. An alternate and useful expression for density is the volume fraction of ice V_i, which is related to the ice density ϱ_i and the snow density ϱ_s through:
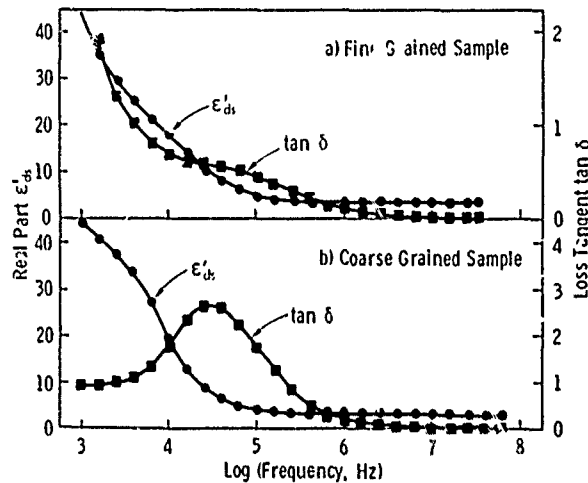
Figure 4. Dependence of the real part of the complex dielectric constant and loss tangent of dry snow with frequency for a) fine-grained and b) coarse-grained samples. (From Ambach and Denoth [25].)

Figure 5. Real part of the dielectric constant of dry snow as a function of snow density ϱ_s along with empirical fits.

$$V_i = \varrho_s / \varrho_i. \qquad (5)$$

Figure 5, from Cumming [14], depicts the dependence of ϵ'_{ds} on ϱ_s at 9.375 GHz, along with curves defined by mixing Formula 4 (in Table 2) and several empirical expressions [3, 25, 29]. Although this data set was measured at 9.375 GHz, it is applicable above 1 MHz where ϵ'_{ds} is frequency-independent. Cumming [14] also found ϵ'_{ds} to be independent of temperature as expected from ice and air properties. The empirical fits [3, 25, 29] are approximately what one would get from a volumetrically weighted average of the dielectric constants of the constituents. Any of the mixing formulas applicable to dry snow will also provide a good fit to these data. One useful simple formulation for the real part results from Formula 4 with $\epsilon_i = 3.15$

$$\epsilon'_{ds} = (1 + 0.508 \, \varrho_s)^3. \qquad (6)$$

While ϵ'_{ds} is independent of both temperature and frequency, ϵ''_{ds} is sensitive to temperature as shown in Figure 6 and to frequency by inference from Figure 3 for ice. Although spectral measurements of dielectric properties have been made up to around 100 MHz, in the microwave region single-frequency measurement programs have predominated and precise determination of the spectral dependence is not available. As a result, one must rely on mixing formulas, which often lead to
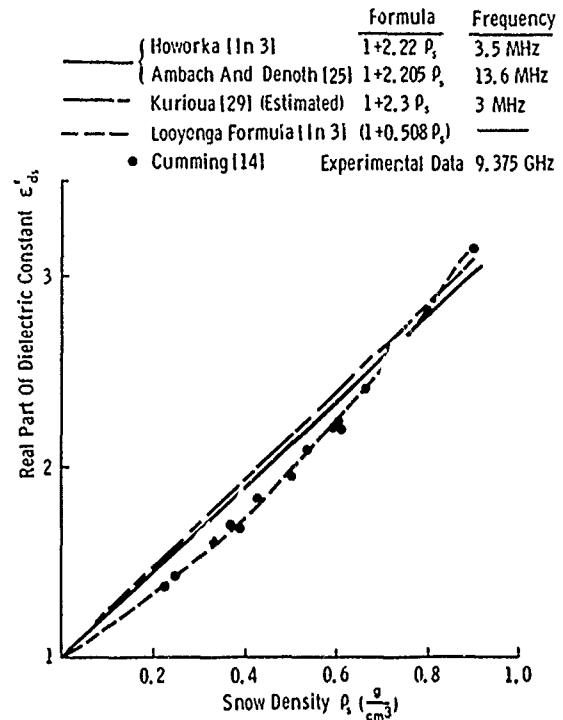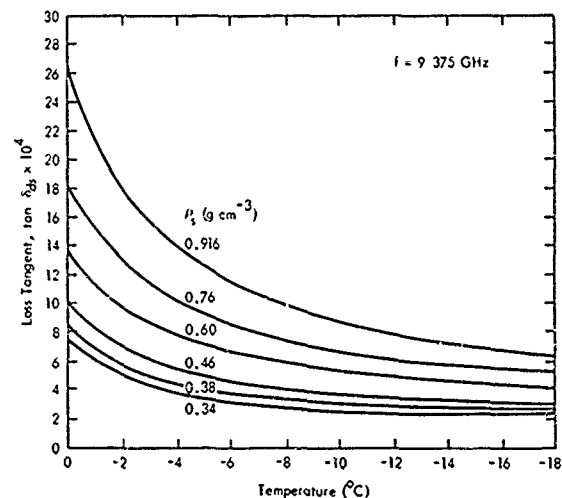

Figure 6. The loss tangent of dry snow as a function of temperature at a frequency of 9.375 GHz. (From Cumming [14].)

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inconsistencies. For example, Royer [2] found that if Formula 2 of Table 1 is used, different values of F are required to fit the real and imaginary parts of ϵ_{ds} to experimental data suggesting that a better model is needed and therefore Formula 2 should not be used.

An expression derived from Formula 8 (of Table 1) by Ulaby et al. [9] is given by

$$\epsilon_{ds}'' = \frac{0.34 V_i \epsilon_i''}{(1 - 0.417 V_i)^2} \qquad (7)$$

and provides a fit close to Cumming's [14] data.

For dry snow, if one avoids the low-frequency dispersion region, the real part of ϵ_{ds} is well-known and easily determined from density and the mixing formulas; for the imaginary part, however, the fits are approximate due to the limited accuracy of the available data on the imaginary part of the dielectric constant of ice.

Wet snow

As 0 °C is approached, liquid water appears in snow and causes a significant change in the dielectric properties of the mixture because both the real and imaginary parts of the dielectric constant of water are much greater than those of ice.

10-MHz to 1-GHz region

For the frequency interval between approximately 10 MHz and 1 GHz, propagation loss through dry or wet snow is very small. Consequently, most of the research activities conducted in this frequency range have concentrated on the dependence of ϵ_{ws}' on V_w, the volume fraction of water. The use of mixing formulas is facilitated by the fact that for $f > 10$ MHz, $\epsilon_i' \approx 3.15$ and, as mentioned earlier, it is frequency- and temperature-independent, and for $f < 1$ GHz, $\epsilon_w' \approx \epsilon_{wo}'$, the static

dielectric constant of water, which, of course, is frequency-independent and weakly dependent on temperature. Hence, the dielectric constant of the ice-air-water mixture is also frequency-independent and approximately temperature-independent.

The empirical expressions given in Figure 5 for dry snow have been extended to wet snow by adding a term that accounts for the water volume fraction, V_w,

$$\epsilon_{ws}' = 1 + a\varrho_s + bV_w \qquad (8)$$

where the constants (a, b) were found experimentally to have the values (2.22, 23.6) of Howorka [in 3] and similar values (2.205, 21.3) were determined by Ambach and Denoth [25].

The above expression provides a useful approximation of the relationship between ϵ_{ws}' and V_w, although a more exact representation using Formula 7 of Table 1 allows for modeling the water particles as ellipsoids. With the assumption that the depolarization factors A_1 and A_2 are equal, implying that $A_3 = 1 - 2A_1$, Ambach and Denoth [25] employed experimental data to compute A_1, by fitting the data to Formula 7. Their results are shown in Figure 7. It is observed that A_1 is a function of V_w, however, $A_1 \approx 0.05$ seems to be a good average depolarization factor.

In a more detailed modeling of liquid distribution, Colbeck [24] suggested that water actually appears in two basic shapes (fillets and veins) because of the geometry of wet snow ice-grains. All wet snow tends to form multiple crystal clusters and the fillets and veins of water form in the air spaces between three or two grains, respectively.

The practical application of dielectric measurements in this frequency range is in the measurement of liquid water, which will be covered in a later section.
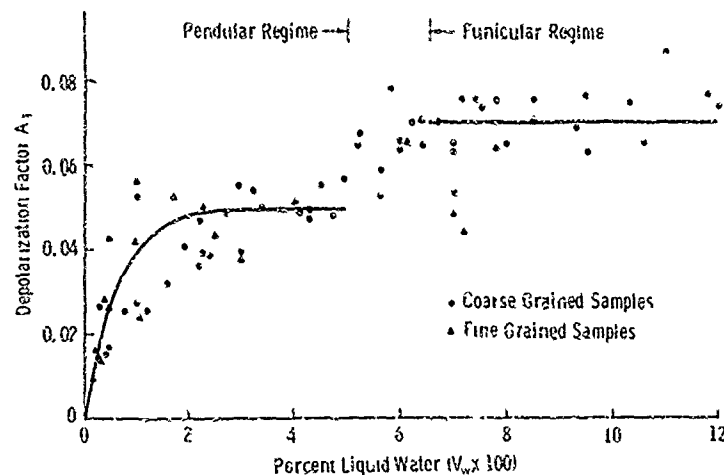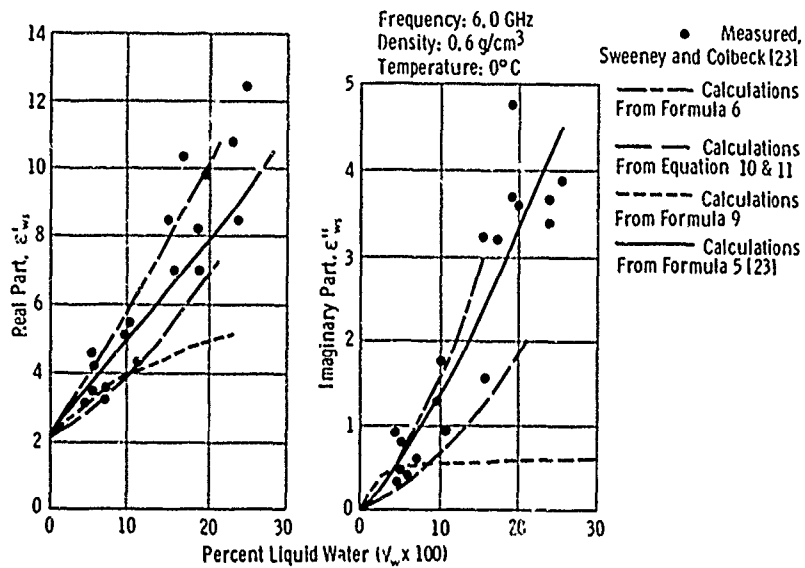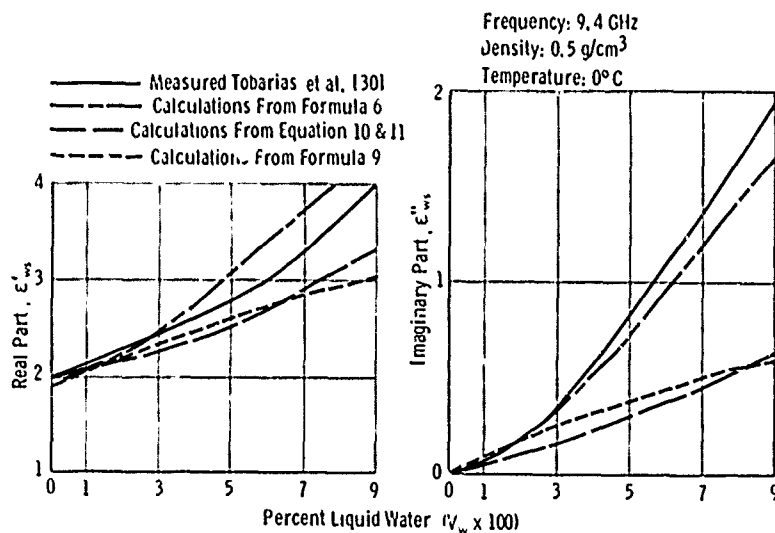

Figure 7. Depolarization factor A_{w1} for mixing Formula 7. (From Ambach and Denoth [25].)

Figure 8. Real and imaginary parts of the relative dielectric constant of snow as a function of liquid water content at 6 GHz and 9.4 GHz.

Above 1 GHz

In the microwave region, both real and imaginary parts of ϵ_{ws} are frequency-dependent, therefore frequency extrapolation is questionable. Four sets of data have been reported in the literature for ϵ_{ws} as a function of snow liquid water content V_w. Cumming [14] reported the variation in loss-tangent in response to V_w; however, his data covered a narrow range extending between $V_w = 0$ and 0.006. Figure 8 presents dielectric measurements given by Sweeney and Colbeck [23] at 6 GHz and Tobarias et al. [30] at 9.4 GHz, together with

the results reported by Linlor [31] for both frequencies. Also shown are curves computed using the mixing formulas indicated on the figure. The curves due to Linlor [31] were computed from expressions he had developed on the basis of experimental data acquired over the 4 to 12-GHz region. These expressions are:

$$\epsilon'_{ws} = 1 + 2\varrho_s + bV_w^{3/2} \tag{9}$$

$$\epsilon''_{ws} = \frac{1.0994\alpha}{f} \sqrt{\epsilon'_{ws}} \tag{10}$$

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where

$$b = 5.87 \times 10^{-2} - 3.10 \times 10^{-4} (f - 4)^2$$

$$\alpha = V_w [0.045(f - 4) + 0.066(1 + a)] dB/cm \qquad (11)$$

with f in GHz and with a representing the average grain diameter in mm. The ranges of validity are stated to be: 0.1 mm $\leq a \leq$ 1.0 mm and $0 \leq V_w \leq 0.12$ [31].

The experimental data in Figure 8 show significant differences at both frequencies, especially for the imaginary part. In addition, the curves generated from mixing Formulas 5, 6 and 9 also show some discrepancies. The behavior at 6 GHz shows that Formulas 5 and 6 give similar results as expected because of their common basis. Formula 6 was applied using spherical ice particles ($A_{11} = \frac{1}{3}$), ellipsoidal water particles ($A_{11} = 0.04$, an average from Figure 7), and the constituent dielectric properties. The results from this formula fall near an upper bound on the data, indicating that at the higher values of V_w a different value of A is needed. Formula 9 shows good agreement with ϵ' data from Linlor [31] up to about $V_w = 0.1$; however, the curvature does not follow the correct trend and therefore should only be used with great caution. Its prediction for ϵ''_{ws} is even further in error.

At 9.4 GHz, again the data from Linlor [31] are much lower than the other data. Formula 9 gives a better fit to Linlor's data than at 6 GHz, while Formula 6 fits the data of Tobarias et al. [30] fairly well with the same depolarization factors used at 6 GHz.

The differences in dielectric constants for these sets of data suggest that the dielectric behavior of wet snow in the microwave region is influenced by some parameter(s) besides snow density. Dielectric mixing theory and observations of Ambach and Denoth [36] indicate that snow grain size or structure should not affect the properties. It is the author's belief that grain structure does indirectly influence the dielectric properties. The method of preparing the samples of reference [23] was to flood the snow sample and then drain it. Colbeck [37] states that this leads to multi-grain clusters of three and more particles and the liquid inclusions exist at grain boundaries. This results in ellipsoidal liquid inclusions which may explain the better fits obtained by Formulas 5 and 6. Colbeck [37] also states that in seasonal snowpack two-grain clusters are more probable, indicating that the water will take on a ring-like shape. The data obtained by Linlor [31] were for snow that had been subjected to "vigorous" manual mixing, which probably resulted in single ice grains with a coating of water which lends itself to a good fit by Formula 9. It should be noted, however, that this is not an equilibrium state for water distribution [37] and although it may be valid for freshly wetted snow, it should be considered at most a lower limit for ϵ'_{ws} and ϵ''_{ws}.

Current understanding of the microwave dielectric behavior of snow may be summarized as follows:

1. For dry snow, ϵ'_{ds} is independent of frequency, approximately temperature-independent, and its variation with snow density is fairly well established (Figure 5) and may be approximated by Equation 7 or Formula 6.

2. For dry snow, ϵ''_{ds} varies with temperature, frequency and density. Very few experimental data are available, particularly above 10 GHz.

3. For wet snow, the presence of water adds another dimension to the complexity of the dielectric behavior of snow, and yet available data on ϵ'_{ws} and ϵ''_{ws} are even fewer than those reported for dry snow, thereby resulting in a rather sketchy understanding of the variation of ϵ_{ws} with frequency, and water content. However, Formulas 5 and 6 appear to provide the best fit to non-mixed snow.

Clearly, a vigorous experimental program designed to establish the dielectric behavior of snow is called for, particularly when free water is present.

PENETRATION DEPTH

The penetration depth in a medium is defined as the depth at which the average power of a wave traveling downward in the medium is equal to $1/e$ of the power at a point just beneath the surface of the medium. For a medium with uniform extinction coefficient x_e, the penetration depth δ_p is given by

$$\delta_p = 1/x_e.$$

In the general case, x_e consists of an absorption coefficient x_a and a scattering coefficient x_s,

$$x_e = x_a + x_s.$$

As a first order estimate of x_e, x_s usually is ignored because its computation involves the dielectric properties of the ice particles and their size and shape distributions relative to the wavelength. For wet snow, setting $x_e \cong x_a$ is a fair approximation because $x_s << x_a$ under most snow conditions in the microwave region. The same approximation holds true for dry snow at frequencies in the lower part of the microwave spectrum (<5 GHz), but for frequencies above 10 GHz, x_s may be comparable in magnitude to x_a for some dry snow conditions (large particle sizes). Thus, when the penetration depth is calculated from $\delta_p = 1/x_a$, the values then obtained should be regarded as upper limits since they do not account for scattering losses in the medium.
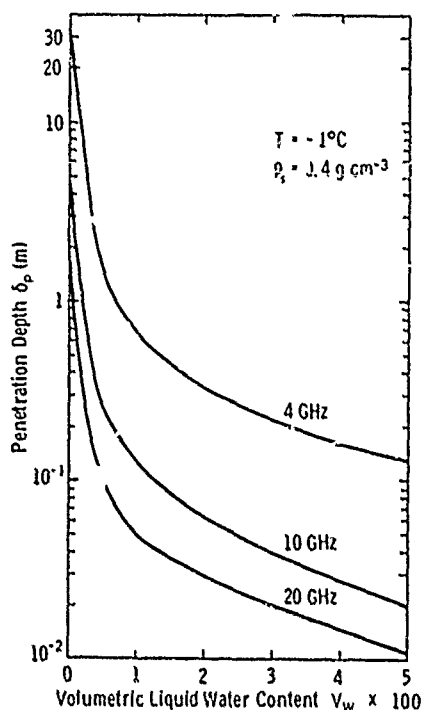
99

Figure 9. Computed values of penetration depth in snow as a function of V_w for three frequencies.

The absorption coefficient is defined as:

$$x_a = \frac{4\pi}{\lambda_0} \left[\frac{\epsilon'}{2} \left\{ \left[1 + \left(\frac{\epsilon''}{\epsilon''} \right)^2 \right]^{1/2} - 1 \right\} \right]^{1/2}$$

where λ_0 is the wavelength in free space. If $(\epsilon''/\epsilon') << 1$, the above expression reduces to

$$x_a \cong (2\pi/\lambda_0)(\epsilon''/\sqrt{\epsilon'})$$

which is applicable to dry snow throughout the microwave region and to wet snow for $V_w < 2\%$.

Figure 9 shows computed values of δ_p plotted as a function of V_w for three microwave frequencies. The values corresponding to dry snow conditions ($V_w = 0$) were computed using $\epsilon'_{ds} = 1.74$ (from equation 6) and ϵ''_{ds} from equation 7 with $\varrho_s = 0.4$ g cm^{-3} and ϵ''_i as given in Figure 3. For $V_w > 0$, equations 9–11 due to Linlor [31] were used for computing ϵ'_{ws} and ϵ''_{ws}. These equations were used at all three frequencies even though they are based on measurements over the 4- to 12-GHz region only. Furthermore, as noted earlier, because Linlor's measurements are representative of freshly mixed samples of wet snow which exhibit lower values of ϵ''_{ds} than snow under natural conditions, the values of δ_p shown in Figure 9 are larger than would be expected for natural snow. That is, these curves should be regarded as estimates of the upper limit of δ_p.

Perhaps the most significant information one can deduce from Figure 9 is that the penetration depth can decrease by more than an order of magnitude as V_w increases from 0 to 0.5%, followed by a much slower rate of decrease as a function of increasing V_w.

ENGINEERING APPLICATIONS

Two of the uses of dielectric properties of snow are: 1) development of sensors to measure snow liquid water content and 2) interpretation and modeling of microwave remote sensing data. For liquid water determination, frequencies below 100 MHz have been found useful, while for remote sensing observations only frequencies above 1 GHz are usually employed.

Knowledge of the liquid water content of snow is of interest in many areas of snow research, including snow strength, snow melt, rate of metamorphism, water flow in snow, avalanche prediction and as a ground-truth factor for remote sensing observations at the higher frequencies. Ambach and Denoth [25] have worked for several years to develop a dielectric measuring device that can be used to ascertain the liquid water content of snow. This indirect method has numerous advantages over traditional methods such as calorimetric and centrifugal separation, among which are rapidity of measurement and reduction in the required ancillary equipment (dry ice, silicone, etc.). Also, when evaluated against the freezing calorimeter as a reference, an absolute accuracy of ±0.5% for V_w was observed [25]. This accuracy is comparable to that of a field calorimeter and, although contact with the snow is required for measurement, the time savings represented by this in-situ method, as compared with calorimetric methods, are very important in field operations.

This technique is based upon the substantial difference in the magnitude of the dielectric properties of water and dry snow as discussed in the preceding section. Ambach and Denoth's device consists of a bridge circuit that measures the capacitance of a probe filled with a snow sample. The change in capacitance due to the presence of the snow sample is used to compute ϵ' of the snow sample. Using equation 10, and either measuring or estimating the snow density ϱ_s, the snow liquid water content V_w is obtained. In one configuration, the probe consists of five parallel plates with a 3-cm spacing and an effective volume of approximately 1000 cm. This volume was found to be near the lower practical limit based on constraints of minimal compaction of the snow sample and negligible fringing fields.

In many cases liquid water determination in thinner layers is desired; near the surface of a snowpack, the liquid water content can have a large gradient with

depth. Planar comb-shaped capacitors have been developed [25] that, through control of widths and spacings, have varying effective measurement depths. Liquid water contents in layer thicknesses as small as millimeters can conceptually be obtained. Hence, the potential exists for measuring a very detailed liquid water profile of the snowpack near-surface layer which would be very useful in the interpretation of microwave remote sensing observations.

Microwave radiometers and radars have shown promise for mapping snow depth, or water equivalent, over large areas for hydrologic applications [32-34]. With the ice crystals in snow being comparable in size to the wavelength in the microwave region, particularly in the millimeter wavelength range, propagation in snow is characterized by both absorption and scattering. Hence the radar backscatter and radiometric emission from snow are governed by the mean dielectric constant of the snow volume as well as the spatial variability of the dielectric constant within the volume. Dry snow (and wet snow at low microwave frequencies in the neighborhood of 1 GHz) is a relatively low-loss medium, which means that the observed backscatter and emission from a snowpack usually consist of contributions from the entire pack as well as from the ground below. Thus detailed knowledge of the snow structure and of the dielectric depth profile is the key to successful modeling of the microwave interaction with snow and to accurate interpretation of microwave observations. Due to the lack of experimentally established models that can adequately describe the dielectric behavior of snow as a function of frequency and liquid water content, most research activities related to microwave remote sensing of snow have been limited to "first-order" investigations.

In these first-order investigations, the mean dielectric constant usually is assumed to be uniform with depth and the snow crystals are assumed spherical in shape and uniform in size. These assumptions are used in theoretical and empirical models developed to interpret experimental results. Such investigations have established the overall behavior of backscattering and emission from snow, but in order to use remotely sensed data to predict snow parameters—such as water equivalent and liquid water content—and to assess the state of the underlying ground medium (frozen or thawed), it will be necessary to use multi-frequency and multi-temporal observations in a prediction algorithm that incorporates in greater detail the role of frequency and dielectric profile shape than is currently possible. Thus, until further research of the microwave dielectric properties of snow is conducted, development of remote sensing applications of snow can only proceed at a slow pace.

CONCLUDING REMARKS

The electrical properties of snow have been evaluated as functions of the following physical properties: snow density, temperature, ice particle shape, liquid water content, and the shape of the liquid water inclusions. Analyses of the dielectric dependence on the above parameters indicates that the liquid water content exercises the single most important influence, and consequently the discussions were divided into wet and dry snow sections. Additionally, two spectral regions were examined in terms of potential applications.

In the lower of the two regions, 100 kHz to 1 GHz, dielectric measurements have been used to measure the liquid water content of snow. By further restricting this range to above 10 MHz, the dependence of ϵ_{ws} on ice particle shape becomes negligible. Hence, for a specific snow sample, only snow density ϱ_s must be measured for a calculation of liquid water content. Although not available commercially, operational sensors have been built, are in use for measuring liquid water in the snow, and hopefully will provide a good alternative to traditional techniques.

In the spectral region applicable to remote sensing, 1 GHz to 35 GHz, the general electrical characteristics of snow are known; however additional measurements are needed, especially for the imaginary part of the dielectric constant above 10 GHz. Application of this information is not direct as in the liquid water measurement, but it is needed for refinement of the models used to describe the electromagnetic wave interaction with snow. The sensitivity of microwave sensors to changes in the dielectric properties of snow (due to changes in liquid water content) may be illustrated by the following example. Corresponding to a change in liquid water content of the snow surface layer from $V_w = 0$ (dry snow) to $V_w = 2\%$, the radar backscattering coefficient has been observed to change by a factor of 30 (15 dB) at 35 GHz and the radiometric temperature increased by 120 K [35]. These results indicate that dry-wet snow determination is a relatively simple task, but quantifying the wetness of measuring the water equivalent of the snowpack is not as straightforward. To realize these latter objectives with a usable degree of prediction accuracy, improved knowledge of the microwave dielectric properties of snow is needed.

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NOMENCLATURE

A_{kj} = depolarization factor

k = index where i = ice and w = water

$$A_j = \tfrac{1}{2}\, abc \int_0^{\infty} \frac{ds}{(a^2+S)^{1/2}(b^2+S)^{1/2}(c^2+S)^{1/2}(j^2+S)}$$

where a, b and c are ellipsoid axes and S is distance along the respective axis

a = molecular radius, constant

F = form factor

f = frequency

f_0 = relaxation frequency

k = Boltzmann's constant

r_k = radius of i = ice, a = air, w = water, where

$$r_w = r_i\left(1 + \frac{0.916\,W_w}{100 - W_w}\right)^{1/3}$$

$$r_a = r_i\left(\frac{0.0916}{\varrho_s}\right)^{1/3}\left(1 + \frac{W_w}{100 - W_w}\right)^{1/3}$$

T = temperature

V_i = volumetric fraction of i = ice, w = water

W_v = percent liquid water by weight

α = attenuation coefficient

δ_p = penetration depth

ϵ_k = dielectric constant of i = ice, w = water, ds = dry snow, ws = wet snow, 0 = static value, ∞ = optical limit

ϵ' = real part of ϵ

ϵ'' = imaginary part of ϵ

η = viscosity

x_e = extinction coefficient

x_a = absorption coefficient

x_s = scattering coefficient

λ_0 = wavelength in free space

ϱ_k = density of s = snow, i = ice

τ = relaxation time