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DISTRIBUTION STATEMENT Approved for Public Release Distribution is unlimited

1	Attorney Docket No. 78009
2	
3	MULTI-STAGE MAXIMUM LIKELIHOOD TARGET ESTIMATOR
4	
5	STATEMENT OF GOVERNMENT INTEREST
6	The invention described herein may be manufactured and used
7	by or for the Government of the United States of America for
8	Governmental purposes without the payment of any royalties
9	thereon or therefor.
10	
11	BACKGROUND OF THE INVENTION
12	(1) Field of the Invention
13	The present invention relates generally to the field of
14	radar and sonar systems. In particular, the invention employs an
15	algorithm in a process for enhanced target detection and
16	tracking.
17	(2) Description of the Prior Art
18	State of the art combat systems rely heavily on target
19	motion analysis (TMA) subcomponents. A target motion analysis
20	subcomponent estimates the current position and velocity
21	components of a contact. Estimates from target motion analysis
22	are important to combat personnel because the estimate allows
23	the personnel to predict the location of the hostile contact
24	some time in the future. Precise determination of the future

position of the contact is required for accurate targeting of
 weapons systems as well as for defensive maneuvering and evasion
 of the contact by friendly units.

In both radar and sonar detection systems, an antenna array receives a reflected signal. Preliminary processing then occurs and the locations of contacts are generated. An example of this type of processing is disclosed in Chang et al., Active Sonar Range-Beam Partitioner, U.S. Patent 5,737,249 (Filed 7 April 1998).

10 The next stage in processing is to determine range and 11 bearing estimates for each target. Prior attempts have led to 12 two distinct approaches for these determinations. The first 13 approach, (sequential algorithms) uses an averaged measurement 14 to reflect historic information and combines this average in a 15 weighted manner with the most current measurement. This approach yields minimal computational needs due to the small size of the 16 17 input dataset. Sequential algorithms also can respond quickly to 18 targets that have rapidly varying direction of movement. However, the condensation of all historic measurements into a 19 20 single set of input numbers results in a great loss in the granularity of the data. Sequential algorithms have not been 21 22 able to utilize the complete historic dataset to dynamically 23 recompute the output range and bearing as a cache set of input values is received. 24

1 Batch processing algorithms have developed to meet this 2 precise need. However, batch processing algorithms have also 3 been plaqued with a plethora of problems. First, computational requirements have consistently been exceptionally high. As a 4 5 result, algorithm designers have been limited in the amount of 6 processing steps which could be performed while still providing 7 real time output. In some circumstances, computational needs 8 have been so high as to require limiting the number of 9 individual historic input measurements which are processed. As 10 such, all viable prior attempts have used a single stage 11 algorithm for processing.

12 The first type of algorithm often used is grid searching. 13 The grid search technique divides the target space into a number 14 of cells. Contact range and bearing are computed by detecting 15 movement between cells. In order for this technique to be 16 successful, the resolution of the target grid must be very fine. 17 This fine resolution has resulted in extreme computational power 18 requirements.

19 The second type of algorithm is a stand-alone endpoint 20 coordinate maximum likelihood estimation (MLE). In maximum 21 likelihood estimation, an iterative least-squares technique is 22 used to determine contact range and bearing. However,

this approach has been subject to over-sensitivity, especially
 in cases where iterations on the quadratic solution lead to a
 divergence rather than a convergence.

4

5

SUMMARY OF THE INVENTION

Accordingly, it is an object of the present invention to
provide a software algorithm which provides range and bearing
estimates for target acquisition systems.

9 It is a further object of the present invention to minimize 10 computational requirements while processing substantial historic 11 data.

It is a still further object of the present invention to maintain a high granularity or resolution in the target field. It is a still further object of the present invention to prevent divergence of the least-squares solution and of the target falses which result from such divergence.

17 In accordance with these and other objects, the invention 18 is a process using a multi-stage algorithm for estimating the 19 current position and velocity components of contacts. The 20 algorithm comprises four major stages. In the first stage, pre-21 processing aimed at elimination of angle errors associated with 22 the time measurements is developed for use in later stages. In 23 the second stage, a coarse grid search, in endpoint coordinates, 24 is performed to yield a refined range estimate at each of the

1 time measurements. In the third stage, an endpoint Gauss-Newton 2 type maximum likelihood estimation (MLE) solution is performed 3 to yield an accurate range estimate. Finally, in the fourth 4 stage, the computed range and bearing values are refined more 5 precisely through a Cartesian coordinate MLE.

6 The four-stage process or method provides the advantage of 7 allowing each stage of the algorithm to work with well-defined 8 input data. Additionally, this method allows the overall 9 algorithm to perform computationally heavy operations over a 10 smaller data space.

Also, the initial stage of the operation is held to a 11 12 coarse estimation requiring little processing power. In this 13 way, the present invention is able to handle large amounts of 14 historic target information without sacrificing resolution in 15 the target space. Furthermore, the procedure of using 16 preprocessing and early estimation stages before the least 17 squares operations in the MLE stages, steps the algorithm from 18 selecting iteration points at local minimums rather than true 19 minimums. This procedure prevents divergence in the solution 20 and prevents the resulting false radar and sonar targets.

BRIEF DESCRIPTION OF THE DRAWINGS

1

2	The foregoing objects and other advantages of the present
3	invention will be more fully understood from the following
4	detailed description and reference to the appended drawings
5	wherein:
6	FIG. 1 is a block diagram depicting the multi-stage maximum
7	likelihood estimation (MLE) method;
8	FIG. 2 is a geographic plot of MLE endpoints; and
9	FIG. 3 is a three-dimensional representation of the sonar
10	beam and bottom reflected beam of a towed array sensor.
11	
12	DESCRIPTION OF THE PREFERRED EMBODIMENTS
13	The multistage maximum likelihood estimator (MLE) processes
14	sonar data and computes a target solution (range, bearing,
15	course, speed) and a localization ellipse by processing data in
16	several stages. The algorithm processes azimuthal bearing
17	measurements, direct path or bottom-bounce conical angle
18	measurements, horizontal range, direct path or bottom bounce
19	frequency measurements from multiple trackers and sonar arrays.
20	Frequency data from a maximum of 2 trackers may be processed.
21	The algorithm constraints include a non-maneuvering target at a
22	known depth, a flat ocean bottom, and an isovelocity environment
23	(straight-line sound propagation). The propagation path is
24	constrained to be either direct path or bottom bounce-on ray

б

reversal and the measurement noise is assumed to be
 uncorrelated. When measurement data has been partitioned into
 segments, propagation path hypothesis testing is performed.

Referring now to FIG. 1, the overall process 10 is depicted
showing the four major stages of the present invention, a
endpoint angle smoothing stage 12, a coarse endpoint coordinate
grid search stage 22, an endpoint Gauss-Newton type MLE 32, and
a fourth stage, the Cartesian coordinate MLE stage 42.

9 In the first stage 12, the algorithm calculates angle 10 smoothing on the angle measurements at the endpoints of the data 11 window in order to reduce angle errors associated with the tie 12 down times (depicted in FIG. 2 as time line 1 (t_1) and time line 13 2 (t_2)) used by the endpoint coarse grid search and endpoint 14 maximum likelihood estimator.

15 In the second stage 22 of FIG. 1, a coarse grid search in 16 endpoint coordinates is performed to obtain a reasonable initial 17 stage estimate of target range at the two times lines. Referring again to FIG. 2, the target position at time line 1 18 19 (t_1) and time line 2 (t_2) is constrained to lie on either the 20 azimuthal bearing lines or conical angle hyperbolas for bottom 21 bounce propagation or conical angle hyperbolic asymptotes for 22 direct path propagation, thereby producing the constrained track 23 of a target 26. The actual track 28 is depicted showing the 24 convergence of the solution.

1 These target restraints may be better visualized by reference to FIG. 3. In FIG. 3, ownship 62 is submerged at a 2 submarine depth plane 68 with a representation of the sonar-3 4 emitted, cone-shaped beam 64. The cone-shaped beam 64 either 5 directly impinges a target or can be reflected off the ocean bottom 72. As shown, the bottom reflection 66 produces a 6 hyperbola. As a result, the reflected beam 70 is a conical 7 angle hyperbola. 8

9 In the third stage 32, an endpoint Gauss-Newton type MLE
10 estimates target range at the two times lines along with a
11 target base frequency for a maximum of two frequency trackers.
12 Again, the target position at time line 1 and time line 2 is
13 constrained to lie on either the azimuthal bearing lines,
14 conical angle hyperbolas or conical angle hyperbolic asymptotes.

In the fourth stage 42, the solution is further refined using the Cartesian coordinate MLE, which also provides errors bounds on various target parameters. The Cartesian coordinate MLE is also Gauss-Newton type MLE that estimates target x, yposition and velocity using the same assumptions made by the endpoint MLE.

21

22 Endpoint Angle Smoothing

23 The first stage, the endpoint angle smoothing stage24 receives input data from the target tracker, in this example, a

sonar sensor, and provides preliminary data for follow-on
 stages. The algorithm performs angle smoothing on the angle
 measurements at the endpoints of the data window in order to
 reduce angle errors associated with the tie-down times (referred
 to as time line 1 and time line 2).

6 Because the coarse grid search constrains its target 7 solution to lie on the azimuthal bearing lines or conical angle 8 hyperbolae (bottom bounce) or conical angle hyperbolic 9 asymptotes (direct path) at time line 1 and time line 2, 10 significantly noisy measurements at either timeline may result 11 in a significantly biased target solution. In order to avoid 12 biased solutions due to endpoint constraints, the coarse grid 13 search constrains the target track to lie on the smoothed (vice measured) bearing lines or conical angle hyperbolae/asymptotes. 14 15 The angle measurements from the tracker or trackers associated 16 with time line 1 and time line 2 are smoothed by fitting 17 measurement data collected within a specified time window of 18 either time line 1 or time line 2 with a quadratic model using 19 standard (normal equation) least-squares theory. Sophisticated 20 orthogonalization techniques are simply not necessary in this 21 application.

Assuming a quadratic model, the angle measurements from the tracker associated with time line 1 that are within 120 seconds of time line 1 (a_1, a_2, \ldots, a_m) can be described as

a_1		1	$\Delta t_{\rm i}$	$\frac{\Delta t_1^2}{2}$	
<i>a</i> ₂		1	Δt_2	$\frac{\Delta t_2^2}{2}$	$\begin{bmatrix} a_0 \end{bmatrix}$
•	=	•	٠	•	a_0
•		•	٠	•	$\left\lfloor a_{0}^{*}\right\rfloor$
_a _m _		•	• Δt_m	$\frac{\Delta t_m^2}{2}$	

- 2 or
- 3

1

z=Hx

4 where Δt_i is the time of the *i*th measurement of time line 1 5 a_i is the *i*th angle measurement $(-\pi \le a_i < +\pi)$

 a_{0} is the smoothed angle at time line 1

7 a_0 is the angle rate at time line 1

8 a''_0 is the angle acceleration at time line 1

9 The curve fit coefficients (x) can be computed using a10 standard unweighted normal equation approach as

11

 $\boldsymbol{x} = \left[\boldsymbol{H}^{T}\boldsymbol{H}\right]^{-1}\boldsymbol{H}^{T}\boldsymbol{z} \tag{3}$

(1)

(2)

12 where the matrix inverse is performed using a standard Gaussian 13 elimination method. In order to tie down to the smoothed angle 14 at time line 1, the smoothed angle estimate a_0 can be substituted 15 for the measured angle at time line 1.

In similar fashion, a smoothed angle estimate at time line
2 can be generated using tracker data associated with the time
line 2 tracker that is within 120 seconds of time line 2, and

this smoothed angle can also be substituted for the measured
 angle at time line 2.

If the root mean square (RMS) error of the curve fit at either time line exceeds 3⁰, then the smoothed angle estimates shall be discarded.

6

7 Coarse Grid Search

8 The coarse grid search can process frequency data for up to 9 two separate frequency trackers. For improved clarity, only a 10 single frequency tracker is described.

11 1. Where at least three frequency measurements are available 12 for a given frequency tracker, frequency data from that tracker 13 is processed and the estimated base frequency for that tracker 14 (*Fb*) is set to the most recent frequency measurement.

15 2. Set the minimum and maximum range at t1 with respect to the 16 sensor associated with time line 1 (Rl_{min} , Rl_{max}) and the minimum 17 and maximum range at t2 with respect to the sensor associated 18 with time line 2 ($R2_{min}$, $R2_{max}$) as follows:

a. If the measurement at time line 1 is a bearing,
set the minimum range at *t1* with respect to the sensor
associated with time line 1 (*R1_{min}*) to the minimum
range constraint which is defaulted to 100.
b. If the measurement at time line 1 is a conical
angle, compute the minimum range at *t1* with respect to

1		the sensor associated with time line 1 ($R1_{min}$). If
2		Rl_{min} is less than the minimum range constraint, set
3		$R1_{min}$ to the minimum range constraint which is
4		defaulted to 100. The minimum range with respect to
5		the sensor is computed as follows:
6		
7		i. Compute the plane depth (Rz) associated with
8	· ·	a measurement as follows:
9		
10		1.) If the propagation path is direct (zero
11		ray reversals), then the image plane depth
12		is computed as follows:
13		
13 14		$Rz = Zt - Zs \tag{4}$
		Rz = Zt - Zs (4) where Zt is the assumed target depth and
14		
14 15		where Zt is the assumed target depth and
14 15 16		where Zt is the assumed target depth and
14 15 16 17		where Zt is the assumed target depth and Zs is the sensor depth
14 15 16 17 18		where Zt is the assumed target depth and Zs is the sensor depth 2.) If the propagation path is bottom bounce
14 15 16 17 18 19		<pre>where Zt is the assumed target depth and Zs is the sensor depth 2.) If the propagation path is bottom bounce (one ray reversal), then the image plane</pre>
14 15 16 17 18 19 20		<pre>where Zt is the assumed target depth and Zs is the sensor depth 2.) If the propagation path is bottom bounce (one ray reversal), then the image plane</pre>
14 15 16 17 18 19 20 21		<pre>where Zt is the assumed target depth and Zs is the sensor depth 2.) If the propagation path is bottom bounce (one ray reversal), then the image plane depth is computed as follows:</pre>

1 ii. Compute the maximum D/E angle with respect to 2 the sensor (θ max). If the measured conical angle (βm) is between 0 and $\pi/2$ inclusive, 3 $\theta_{\rm max} = \beta_m - C_s$ 4 (6)where $C_{\rm s}$ is the sensor cant angle. If the 5 measured conical angle is less than π , 6 $\theta_{\rm max} = \pi - \beta_m + C_s$ 7 (7)iii. The minimum range with respect to the sensor 8 9 (R_{min}) can then be computed as $R_{\min} = \frac{R_z}{\tan \theta}$ 10 (8)11 where R_z is the image plane depth. 12 13 Set the maximum range t1 with respect to the с. 14 sensor associated with time line $1(RI_{max})$ to the maximum 15 range constraint which is defaulted to 200000. 16 If the measurement at time line 2 is a bearing, d. set the minimum range at t2 with respect to the sensor 17 18 associated with time line 2 $(R2_{min})$ to the minimum range 19 constraint which is defaulted to 100. 20 If the measurement at time line 2 is a conical e. angle, compute the minimum range at t2 with respect to 21 22 the sensor associated with time line 2 $(R2_{min})$. If

R2min is less than the minimum range constraint, set 1 2 $R2_{min}$ to the minimum constraint which is defaulted to The minimum range with respect to the sensor is 100. 3 computed as for equations (4) thru (8). 4 Set the maximum range at t2 with respect to the 5 f. sensor associated with time line $2(R2_{max})$ to the 6 maximum range constraint which is defaulted to 200000. 7 8 3. Compute three values of range at t1 with respect to the 9 sensor associated with time line 1 $(Rl_i, j=1,...,3)$ and three 10 11 values of range at t2 with respect to the sensor associated with 12 time line 2 ($R2_k$, $k=1,\ldots,3$) as follows: $R1_{i} = R1_{\min} + 5000j$ 13 (9) $R2_{k} = R2_{min} + 5000k$ 14 (10)15 If $Rl_j > Rl_{max}$, set Rl_j to Rl_{max} . If $R2_k > R2_{max}$, set $R2_k$ to $R2_{max}$. 16 17 If frequency data is being processed from a particular 4. tracker, then five base frequency estimates $(Fb_1, 1=1, \ldots, 5)$ are 18 19 computed as follows: $Fb_l = Fr_{avg} + 0.005(l-1)$ 20 (11)21 where Fr_{avg} is averaged measured frequency measurement between t122 and t2.

1	5. For each combination of $R1_j, R2_k, Fb_1$ compute the Endpoint
2	coordinate performance index (PI _{jk1})as follows:
3	
4	a. Compute Endpoint Parameters as follows:
5	
6	i. If the measurement at time line 1 is a
7	bearing, set true bearing at <i>t1</i> with respect to
8	the sensor associated with time line 1 $(B1)$ to
9	the bearing estimate at time line 1.
10	ii. If the measurement at time line 1 is a
11	conical angle,
12	1.) Compute the target image depth at $t1$
13	with respect to the sensor associated with
14	time line 1 (Rzl) as described in for
15	equations (4) and (5).
16	2.) Compute the maximum depression/elevation
17	(D/E) angle at $t1$ with respect to the sensor
18	associated with time line 1 ($ heta l_{max}$) as
19	described for equations (6) thru (8).
20	3.) Compute the slant range at $t1$ with
21	respect to the sensor associated with time
22	line 1 (<i>Rs1</i>):
23	

1	$Rs1 = \sqrt{R1^2 + Rz1^2} $ (12)
2	4.) Compute the D/E angle at $t1$ with respect
3	to the sensor associated with time line 1
4	$(\theta 1):$
5	
6	$\theta 1 = \sin^{-1} \left(\frac{Rz1}{Rs1} \right) \tag{13}$
7	5.) If $\theta 1 > \theta 1_{max}$, the D/E angle is invalid and
8	processing shall terminate.
9	6.) Compute the cosine of relative bearing
10	at t1 with respect to the sensor associated
11	with time line 1 (<i>cBr1</i>) as follows:
12	$cBr1 = \frac{\cos\beta 1 + \sin Cs 1 \sin \theta 1}{\cos Cs 1 \cos \theta 1} $ (14)
13	
14	where <i>Cs1</i> is the cant angle at <i>t1</i> of the sensor
15	associated with time line 1
16	eta1 is the conical angle estimate at time line
17	1.
18	
19	7.) Insure that -0.99999< <i>cBr1</i> <0.99999.
20	8.) Compute the relative bearing at <i>t1</i> with
21	respect to the sensor associated with time
22	line 1 (Br1) as follows:

1	$Br\mathbf{l} = \cos^{-1} cBr\mathbf{l} \tag{15}$
2	
3	9.) If the port/starboard assumption for
4	time line 1 indicates port, set $Br1=2\pi$ -Br1.
5	10.) Compute the true bearing at $t1$ with
6	respect to the sensor associated with time
7	line 1 (B1) as follows:
8	
9	$B1 = Br1 + Hs1 \tag{16}$
10	where <i>Hsl</i> is the heading at <i>tl</i> of the sensor
11	associated with time line 1.
12	iii. If the measurement at time line 2 is a
13	bearing, set true bearing at $t2$ with respect to
14	the sensor associated with time line 2 $(B2)$ to
15	the bearing estimate at time line 2.
16	iv. If the measurement at time line 2 is a
17	conical angle,
18	1.) Compute the target image depth at $t2$
19	with respect to the sensor associated with
20	time line 2 ($Rz2$) as described for equations
21	(4) and (5).
22	2.) Compute the maximum D/E angle at $t2$ with
23	respect to the sensor associated with time

1		line 2 ($\theta 2_{max}$) as described for equations (6)
2		thru (8).
3		3.) Compute the slant range at $t2$ with
4		respect to the sensor associated with time
5		line 2 (<i>Rs2</i>):
6		$Rs2 = \sqrt{R2^2 + Rz2^2} $ (17)
7		4.) Compute the D/E angle at $t2$ with respect
8		to the sensor associated with time line 2
9		(<i>θ</i> 2):
10		$\theta 2 = \sin^{-1} \left(\frac{Rz2}{Rs2} \right) \tag{18}$
11		5.) If $\theta_{2} > \theta_{2_{max}}$, the D/E angle is invalid and
12		processing shall terminate.
13		6.) Compute the cosine of relative bearing
14		at $t2$ with respect to the sensor associated
15		with time line 2 (<i>cBr2</i>) as follows:
16		$cBr2 = \frac{\cos\beta 2 + \sin Cs2\sin\theta 2}{\cos Cs2\cos\theta 2} $ (19)
17		where $Cs2$ is the cant angle at $t2$ of the
18		sensor associated with time line 2
19		eta 2 is the conical angle estimate at time line
20		2.
	· · · · · · · · · · · · · · · · · · ·	

1	7.) Insure that -0.99999< <i>cBr2</i> <0.99999.
2	8.) Compute the relative bearing at $t2$ with
3	respect to the sensor associated with time
4	line 2 (Br2) as follows:
5	$Br2 = \cos^{-1} cBr2 \tag{20}$
6	9.) If the port/starboard assumption for
7	time line 2 indicates port set $Br2=2\pi-Br2$.
8	10.) Compute the true bearing at $t2$ with
9	respect to the sensor associated with time
10	line 2 (B2) as follows:
11	$B2 = Br2 + Hs2 \tag{21}$
12	where $Hs2$ is the heading at $t2$ of the sensor
13	associated with time line 2.
14	b. For each measurement in the batch:
15	i. Compute the x-component of range t_i with
16	respect to the sensor assoicated with the ith
17	measurement (Rx_i) and the y-component of range at
18	t_i with respect to the sensor associated with the
19	ith measurement (Ry _i):
20	$T1_{i} = \frac{t_{i} - t1}{t2 - t1} \tag{22}$
21	$T2_{i} = 1 - T1_{1} \tag{23}$
22	$Rx_{i} = T2_{i}R1_{j}\sin B1 + T1_{i}R2_{k}\sin B2 + T1_{i}(Xs2 - Xs1) - (Xs_{i} - Xs1) $ (24)

1	$Ry_{i} = T2_{i}R1_{j}\cos B1 + T1_{i}R2_{k}\cos B2 + T1_{i}(Ys2 - Ys1) - (Ys_{i} - Ys1) $ (25)
2	where Xs_i is the x-coordinate of the position at t_i of
3	the sensor associated with the <i>i</i> th measurement
4	Ys_i is the y-coordinate of the position at t_i of the
5	sensor associate with the <i>i</i> th measurement
6	t_i the time of the <i>i</i> th measurement
7	ii. If the <i>i</i> th measurement is a bearing, the
8	following shall be performed:
9	1.) Compute the true bearing at t_i with
10	respect to the sensor associated with the
11	ith measurement (B_i) :
12	$B_i = \tan^{-1} \left(\frac{Rx_i}{Ry_i} \right) $ (26)
13	2.) Compute the bearing residual ($RESB_i$) such
14	that $-\pi \leq RESB_i \leq \pi$:
15	$RESB_i = Bm_i - B_i \tag{27}$
16	where Bm_{i} is the measured bearing at t_{i}
17	
18	3.) Compute the normalized bearing residual
19	$\left(\overline{RESB_{i}}\right)$:
20	$\overline{RESB_i} = \frac{RESB_i}{\sigma B_i} $ (28)

1	where σ_{B_i} is the standard deviation of the
2	measured bearing at t_i
3	iii. If the <i>i</i> th measurement is a conical angle,
4	the following shall be performed:
5	1.) Compute the target image depth at t_i with
6	respect to the sensor associated with ith
7	measurement (Rz_i) as decribed for equations
8	(4) and (5).
9	2.) Compute the maximum D/E angle at t_i with
10	respect to the sensor associated with the
11	ith measurement ($ heta_{mexi}$) as described for
12	equations (6) thru (8).
13	3.) Compute the slant range at t_i with
14	respect to the sensor associated with the
15	ith measurement (Rs _i):
16	$Rs_{i} = \sqrt{Rx_{i}^{2} + Ry_{i}^{2} + Rz_{i}^{2}} $ (29)
17	4.) Compute the D/E angle at t_i with respect
18	to the sensor associated with the i th
19	measurement (θ_i) :
20	$\theta_{i} = \sin^{-1} \left(\frac{Rz_{i}}{Rs_{i}} \right) $ (30)
21	5.) If $ heta_{i} < heta_{maxi}$, the D/E angle is valid and
22	the following shall be performed:

1		<u>a</u> Compute the x-component of range at t_i
2		with respect to the sensor associated
3		with the <i>i</i> th measurement (xta_i), the y-
4		component of range at t_i with respect to
5		the sensor associated with the <i>i</i> th
6		measurement (yta_i) and the z-component
7	·	of range at t_i with respect to the
8		sensor associated with the ith
9		measurement rotated to the axis of the
10		array (<i>zta_i</i>):
11		
12		$xta_i = Rx_i \cos Hs_i - Ry_i \sin Hs_i $ (31)
13		$yta_i = (Rx_i \sin Hs_i + Ry_i \cos Hs_i) \cos Cs_i - Rz_i \sin Cs_i (32)$
14		$zta_i = (Rx_i \sin Hs_i + Ry_i \cos Hs_i) \sin Cs_i + Rz_i \cos Cs_i (33)$
15		where CS_i is the cant angle at t_i of the
16		sensor associated with the <i>i</i> th
17		measurement
18		Hs_i is the heading at t_i of the sensor
19		associated with the <i>i</i> th measurement
20		
21		<u>b</u> Compute the conical angle at ti with
22		respect to the sensor associated with
23		the ith measurement (eta_i):

·].	
2	$if yta_i \neq 0$
3	$\beta_i = \tan^{-1} \left(\frac{\sqrt{xta_i^2 + zta_i^2}}{yta_i} \right) $ (34)
4	otherwise
5	$\beta_i = \frac{\pi}{2} \tag{35}$
6	<u>c</u> . Compute the conical angle residual
7	$(RES\beta_i)$ such that $-\pi \leq RES\beta_i \leq \pi$:
8	
9	$RES\beta_i = \beta m_i - \beta_1 \tag{36}$
10	where $eta m_{\pm}$ is the measured conical angle
11	at t_i .
12	d. Compute the normalized conical
13	angle residual $(\overline{RES\beta_i})$:
14	$\overline{RES\beta_i} = \frac{RES\beta_i}{\sigma\beta_i} $ (37)
15	where σeta_i is the standard deviation of
16	the measured conical angle at t_i .
17	iv. If the <i>i</i> th measurement is a horizontal range:
18	1.) Compute the range at t_i with respect to
19	the sensor associated with the <i>i</i> th
20	measurement (R_i) :

$$R_i = \sqrt{Rx_i^2 + Ry_i^2} \tag{38}$$

2.) Compute the range residual $(RESR_i)$:

$$RESR_i = Rm_i - R_i \tag{39}$$

where Rm_i is the measured range at t_i . 3.) Compute the normalized range residual $(\overline{RESR_i})$:

$$\overline{RESR_i} = \frac{RESR_i}{\sigma R_i} \tag{40}$$

where σR_i is the standard deviation of the measured range at t_i .

respect to the sensor associated with the

v. If the *i*th measurement is a frequency and frequency data are being processed:

12 1.) Compute the target image at t_i with 13 respect to the sensor associated with the 14 ith measurement (Rz_i) as described for 15 equations (4) and (5).

162.) Compute the maximum D/E angle at t_i with17respect to the sensor associated with the18ith measurement (θ_{maxi}) as described for19equations (6) thru (8).203.) Compute the slant range at t_i with

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ith measurement (Rs_i) :

$$Rs_{i} = \sqrt{Rx_{i}^{2} + Ry_{i}^{2} + Rz_{i}^{2}}$$
(41)

4.) Compute the D/E angle at t_i with respect to the sensor associated with the *i*th measurement (θ_i) :

$$\theta_i = \sin^{-1} \left(\frac{R z_i}{R s_i} \right) \tag{42}$$

5.) If $\theta_i < \theta_{maxi}$, the D/E angle is valid and the following shall be performed:

8 <u>a</u> Compute the x-component of target
9 velocity (Vxt) and the y-component of
10 target velocity (Vyt):

$$V_{xt} = \frac{R2_k \sin B2 + Xs2 - R1_j \sin B1 - Xs1}{t2 - t1}$$
(43)

$$Vyt = \frac{R2_k \cos B2 + Ys2 - R1_j \cos B1 - Ys2}{t2 - t1}$$
(44)

<u>b</u>. Compute the frequency at t_i with respect to the sensor associated with the *ith* measurement (F_i) :

$$F_{i} = Fb \frac{cRs_{i} + Vxs_{i}Rx_{i} + Vys_{i}Ry_{i}}{cRs_{i} + VxtRx_{i} + VytRy_{i}}$$
(45)

where c is the average speed of sound. <u>c</u>. Compute the frequency residual (RESF_i):

1
$$RESF_i = Fm_i - F_i$$
(46)2where Fm_i is the measured frequency at3 t_i .4d. Compute the normalized frequency5residual $(\overline{RESF_i})$:6 $\overline{RESF_i} = \frac{RESF_i}{\sigma F_i}$ 7where σF_i is the standard deviation of8the measured frequency at t_i .9.10c. If a range constraint is being imposed, then the11following computations shall be performed:12.13i. Compute the target range (R) as follows:14.15 $R = \sqrt{Rx_i^2 + Ry_i^2}$ 16.17.18 $RESR = Rc - R$ 19where Rc is the assumed target range.20.111. Compute the normalized range residual21. (\overline{RESR}) :

1	$\overline{RESR} = \frac{RESR}{\sigma R}$	(50
2	where σR is the standard deviation of the as	sumed
3	target range.	
4	d. If a speed constraint is being imposed, then	the
5	following computations shall be performed:	
6		
7	i. Compute the x-component of target veloci	ty
8	(Vxt) and the y-component of target velocity	
9	(<i>Vyt</i>):	• .
10		
11	$Vxt = \frac{R2_k \sin B2 + Xs2 - R1_j \sin B1 - Xs1}{t2 - t1}$	(51
12	$Vyt = \frac{R2_k \cos B2 + Ys2 - Rl_j \cos Bl - Ys1}{t2 - t!}$	(52
13	ii. Compute the target speed (V):	
14		·
15	$V = \sqrt{Vxt^2 + Vyt^2}$	(52

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iii. Compute the speed residual (RESV):

RESV = Vc - V(54)

(50)

(51)

(52)

(52)

where Vc is the assumed target speed.

Compute the normalized speed residual (\overline{RESV}) : iv.

> $\overline{RESV} = \frac{RESV}{\sigma V}$ (55)

1	where σV is the standard deviation of the assumed
2	target speed.
3	
4	e. Compute the Endpoint coordinate performance index
5	(PI_{jkl}) as the square root of the mean of the squared
6	normalized residuals, which include measurements as
. 7	well as constraints.
8	6. Select the value of $R1_j$, $R2_k$ and Fb_l associated with the
9	smallest PI _{jk1} .
10	Then assuming zero mean unit variance measurements, $R^{-1} = I$,
11	The non-linear, least-squares algorithm, which employs
12	Householder transformations, applies to both the third and
13	fourth stages of the target estimator, the endpoint MLE and the
14	Cartesian coordinate MLE. The sequence of operations are:
15	
16	Initialization
17	$x_1 = x_0$
18	
19	1=1, NITER Gauss-Newton iterations
20	i=1, m measurement loop m=# of
21	measurements
22	$H = \partial h(x_{1-1}) / \partial x \qquad \qquad \text{Jacobian matrix m x ns}$
23	ns=# of state variables

1	$Z=z-h(x_{1-1})$	residual vector m x 1
2	$H = [H \mid z]$	augmented Jacobian m x (ns+1)
3	A=QH Householder Transfe	ormation
4		U upper triangular ns x ns
5	$= \begin{bmatrix} U & Y \\ 0 & d \end{bmatrix}$	Y is normalized residual ns x 1
6	$P=U^{-1}U^{-t}$	state covariance matrix ns x ns
7	$\Delta x = U^{-1}Y$	correction vector ns x 1
8	$PI=1/2[z-h(x_{1-1})]R^{-1}[z-h(x_{1-1})]$	initial performance index (scalar)
9	$X_1 = X_{1-1} + \alpha \Delta X$	state update
10		α =stepsize via line search
11	PI'=1/2[z-h(x ₁)] $R^{-1}[z-h(x_1)]$	updated performance index
12	$\Delta PI = (PI - PI') / PI'$	change in performance index
13	if Δ PI <threshold, exit="" loop<="" td=""><td>convergence test</td></threshold,>	convergence test
14	•	
15	Endpoint Coordinate MLE	
16	The endpoint coordinate	MLE can process frequency data for
17	up to two separate frequency	trackers. For improved clarity,
18	only a single frequency trac	ker is described.
19		
20	1. Initialize the following	Endpoint coordinate MLE solution
21	parameters to zero:	
22	Roc (range with respect	to own ship at current time)
23	Boc (bearing with respe	ct to own ship at current time)

1 Ct (target course)

2 Vt (target speed)

3 Fb (target base frequency)

5 2. Initialize the number of Gauss-Newton iterations to zero.
6 A maximum of twenty-five Gauss-Newton iterations shall be
7 performed as described in paragraphs 15a through 15r.

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9 3. Determine the number of state variables as follows:
10 If a least three frequency measurements are available, then
11 frequency data will be processed, target base frequency shall be
12 estimated and the number of states (*ns*) shall be set to three.
13 Otherwise, the number of state variables shall be two, frequency
14 data shall not be processed and target base frequency shall not
15 be estimated.

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17 4. Initialize values for range at *t1* with respect to the sensor 18 associated with time line 1 (*R1*) and range at *t2* with respect to 19 the sensor associated with time line 2 (*R2*) using the outputs 20 from the coarse grid search.

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$$R1 = R1_{init} \tag{56}$$

(57)

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where Rl_{init} and $R2_{init}$ are output by the grid search algorithm t1 is the time line 1 time

 $R2 = R2_{init}$

1	t2 is the time line 2 time
2	
3	5. If frequency is being processed, initialize the base
4	frequency state (Fb) with the base frequency output by the
5	coarse gird search algorithm.
6	
7	6. Compute the Endpoint coordinate performance index (PI) based
8	on the initial states as follows:
9	a. First compute endpoint parameters:
10	i. If the measurement at time line 1 is a
11	bearing, set true bearing at <i>t1</i> with respect to
12	the sensor associated with time line 1 ($B1$) to
13	the bearing estimate at time line 1.
14	ii. If the measurement at time line 1 is a
15	conical angle,
16	1.) Compute the target image depth at t1
17	with respect to the sensor associated with
18	time line 1 $(Rz1)$ as described for equations
19	(4) and (5).
20	2.) Compute the maximum
21	depression/elevation (D/E) angle at $t1$ with
22	respect to the sensor associated with time
23	line 1 ($ heta$ 1 max) as described for equations
24	(6) thru (8).

3.) Compute the slant range at *t1* with respect to the sensor associated with time line 1 (*Rs1*):

$$Rs1 = \sqrt{R1^2 + Rz1^2} \tag{58}$$

4.) Compute the D/E angle at t1 with respect to the sensor associated with time line 1 (θ 1):

$$91 = \sin^{-1}\left(\frac{Rz1}{Rs1}\right) \tag{59}$$

5.) If $\theta 1 > \theta 1_{max}$, the D/E angle is invalid and processing shall terminate.

6.) Compute the cosine of relative bearing at *t1* with respect to the sensor associated with time 1 (*cBr1*) as follows:

$$cBr1 = \frac{\cos\beta 1 + \sin Csl\sin\theta 1}{\cos Csl\cos\theta 1}$$
(60)

where Cs1 is the cant angle at t1 of the sensor associated with time line 1 β 1 is the conical angle estimate at time line 1

7.) Insure that -0.99999<*cBr1*<0.99999.
8.) Compute the relative bearing at *t1* with respect to the sensor associated with time line 1 (*Br1*) as follows:

1	$Br1 = \cos^{-1} cBr1 \tag{61}$
2	9.) If the port/starboard assumption for
3	time line 1 indicates port, set $Br1=2\pi$ -Br1.
4	10.) Compute the tear bearing at $t1$ with
5	respect to the sensor associated with time
6	line 1 (<i>B1</i>) as follows:
7	$B1 = Br1 + Hs1 \tag{62}$
8	where <i>Hsl</i> is the heading at <i>tl</i> of the sensor
9	associated with time line 1.
10	iii. If the measurement at time line 2 is a
11	bearing, set true bearing at $t2$ with respect to
12	, the sensor associated with time line 2 $(B2)$ to
13	the bearing estimate at time line 2.
14	iv. If the measurement at time line 2 is a
15	conical angle,
16	1.) Compute the target image depth at t2
17	with respect to the sensor associated with
18	time line 2 (Rz2) as described for equations
19	(4) and (5).
20	2.) Compute the maximum D/E angle at $t2$
21	with respect to the sensor associated with
22	time line 2 ($ heta 2_{max}$) as described for equations
23	(6) thru (8).

3.) Compute the slant range at t2 with respect to the sensor associated with time line 2 (Rs2):

$$Rs2 = \sqrt{R2^2 + Rz2^2}$$
(63)

4.) Compute the D/E angle at t2 with respect to the sensor associated with time line 2 ($\theta 2$):

$$\theta 2 = \sin^{-1} \left(\frac{Rz2}{Rs2} \right) \tag{64}$$

5.) If $\theta_{2} > \theta_{2_{max}}$, the D/E angle is invalid and processing shall terminate.

11 6.) Compute the cosine of relative bearing
12 at t2 with respect to the sensor associated
13 with time line 2 (cBr2) as follows:

$$CBr2 = \frac{\cos\beta 2 + \sin Cs2 \sin \theta 2}{\cos Cs \cos \theta 2}$$
(65)

where Cs2 is the cant angle at t2 of the sensor associated with time line 2 and $\beta2$ is the conical angle estimate at time line 2 7.) Insure that -0.99999<cBr2<0.99999. 8.) Compute the relative bearing at t2 with respect to the sensor associated with time line 2 (Br2) as follows:

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$$Br2 = \cos^{-1} cBr2$$

(66)

1	9.) If the port/starboard assumption for	
2	time line 1 indicates port, set $Br2=2\pi$ -Br2.	
- 3		
2	10.) Compute the true bearing at t2 with	
4	respect to the sensor associated with time	
5	line 2 (B2) as follows:	
6	$B2 = Br2 + Hs2 \tag{67}$	
7	where Hs2 is the heading at t2 of the sensor	
8	associated with time line 2	
9	b. Second, for each measurement in the batch:	
10	i. Compute the x-component of range at ti with	
11	respect to the sensor associated with the ith	
12	measurement (Rx_i) and the y-component of range at	
13	ti with respect to the sensor associated with the	
14	ith measurement (Ryi):	
15	$TI_{i} = \frac{t_{i} - t1}{t2 - t1} $ (68)	
16	$T2_i = 1 - T1_i$ (69)	
17	$Rx_{i} = T2_{i}R1 \sin B1 + T1_{i}R2 \sin B2 + T1_{i}(Xs2 - Xs1) - (Xs_{i} - Xs1) $ (70)	
18	$Ry_{i} = T2_{i}R1 \cos B1 + T1_{i}R2 \cos B2 + T1_{i}(Ys2 - Ys1) - (Ys_{i} - Ys1) $ (71)	
19	where Xs_i is the x-coordinate of the position at	
20	t_i of the sensor associated with the <i>i</i> th	
21	measurement	
22	Ys_i is the y-coordinate of the position at t_i of	
23	the sensor associated with the <i>i</i> th measurement	
1	t_i is the time of the <i>i</i> th measurement	
----	--	
2	ii. If the ith measurement is a bearing, the	
3	following shall be performed:	
4	1.) Compute the true bearing at t_i with	
5	respect to the sensor associated with the	
6	ith measurement (B _i):	
7	$B_{i} = \tan^{-2} \left(\frac{Rx_{i}}{Ry_{i}} \right) $ (72)	
8	2.) Compute the bearing residual (RESb _i)	
9	such that	
10	$-\pi \leq RESb_i \leq \pi$:	
11	$RESD_i = Bm_i - B_i \tag{73}$	
12	where Bm_i is the measured bearing at t_i	
13	3.) Compute the normalized bearing residual	
14	$\left(\overline{RESb_i}\right)$:	
15	$\overline{RESb_i} = \frac{RESb_i}{\sigma b} $ (74)	
16	where σb_i is the standard deviation of the	
17	measured bearing at t_i .	
18	· iii. If the <i>i</i> th measurement is a conical angle,	
19	the following shall be performed:	
20	1.) Compute the target image depth at t_i	
21	with respect to the sensor associated with	

the *i*th measurement (Rz_i) as described for equations (4) and (5).

2.) Compute the maximum D/E angle at t_i with respect to the sensor associated with the ith measurement (θ_{maxi}) as described for equations (6) thru (8).

3.) Compute the slant range at t_i with respect to the sensor associated with the ith measurement (Rs_i) .

$$Rs_{i} = \sqrt{Rx_{i}^{2} + Ry_{i}^{2} + Rz_{i}^{2}}$$
(75)

4.) Compute the D/E angle at t_i with respect to the sensor associated with the *i*th measurement (θ_i) :

$$\theta_i = \sin^{-1} \left(\frac{Rz_i}{Rs_i} \right) \tag{76}$$

5.) If $\theta_i < \theta_{maxi}$, the D/E angle is valid and the following shall be performed:

<u>a</u>. Compute the x-component of range at t_i with respect to the sensor associated with the *i*th measurement (xta_i) , the y-component of range at t_i with respect to the sensor associated with the *i*th measurement (yta_i) and the

1	z -component of range at t_i with respect
2	to the sensor associated with the <i>i</i> th
3	. measurement (zta_i) rotated to the axis
4	of the array:
5	$xta_i = Rx_i \cos Hs_i - Ry_i \sin Hs_j \tag{77}$
6	$yta_i = (Rx_i \sin Hs_i + Ry_i \cos Hs_i) \cos Cs_i - Rz_i \sin Cs_i$ (78)
7	$zta_i = (Rx_i \sin Hs_i + Ry_i \cos Hs_i) \sin Cs_i + Rz_i \cos Cs_i$ (79)
8	where Cs_i is the cant angle at t_i of the
9	sensor associated with the <i>i</i> th
10	measurement and Hs_i is the heading at t_i
11	of the sensor associated with the i th
12	measurement
13	<u>b</u> . Compute the conical angle at t_i with
14	respect to the sensor associated with
15	the ith measurement $(m{eta}_i)$:
16	If $yta_i \neq 0$:
17	$\beta_i = \tan^{-1} \left(\frac{\sqrt{xta_i^2 + zta_i^2}}{yta_i} \right) $ (80)
18	otherwise:
19	$\beta_i = \frac{\pi}{2} \tag{81}$
20	\underline{c} . Compute the conical angle residual
21	(RES β_i) such that $-\pi \leq \text{RES}\beta_i \leq \pi$:

1		$RES\beta_i = \beta m_i - \beta_i$	(82)
2		where $eta m_i$ is the measured conical	angle
3		at t_i	
4		\underline{d} . Compute the normalized conic	al
5		angle residual $\left(\overline{RES\beta_i}\right)$:	
6		$RES\beta_i = \frac{RES\beta_i}{\sigma\beta_i}$	(83)
7	· .	where $\sigma \! eta_{_{i}}$ is the standard deviati	on of
8		the measured conical angle at t_i	· ·
9		iv. If the <i>i</i> th measurement is a horizonta	la
10		range:	
11		1.) Compute the range residual $(RESr_i)$):
12		$RESr_i = Rm_i - R_i$,
13		where Rm_i is the measured range at t_i .	
14		2.) Compute the normalized range res	idual
15		$\left(\overline{RESr_i}\right)$:	
16		$\overline{RESr_i} = \frac{RESr_i}{\sigma r_i}$	(84)
17		where $\sigma r_{ m i}$ is the standard deviation of	f the
18		measured range at t_i .	

1 If the ith measurement is a frequency and v. 2 frequency data are being processed, then the 3 following shall be performed: 4 Compute the target image depth at t_i 1.) 5 with respect to the sensor associated with . 6 the ith measurement (Rz_i) . 7 2.) Compute the maximum D/E angle at t_i 8 with respect to the sensor associated with 9 the *i*th measurement (θ_{maxi}) . 10 3.) Compute the slant range at t_i with 11 respect to the sensor associated with the 12 ith measurement (Rs_i) : $Rs_i = \sqrt{Rx_i^2 + Ry_i^2 + Rz_i^2}$ 13 (85)14 4.) Compute the D/E angle at t_i with respect 15 to the sensor associated with the ith 16 measurement (θ_i) : $\theta_i = \sin^{-1}\left(\frac{Rz_i}{Rs_i}\right)$ 17 (86) 5.) If $heta_i$ < $heta_{ extsf{max}\,i}$, the D/E angle is valid and 18 19 the following shall be performed: 20 a. Compute the x-component of target 21 velocity (Vxt) and the y-component of 22 target velocity (Vyt):

1i. Compute the range residual (RESr):2
$$RESr = Rc - R$$
(92)3where Rc is the assumed target range.4ii. Compute the normalized range residual $(RESr)$:5 $\overline{RESr} = \frac{RESr}{\sigma R}$ (93)6where σR is the assumed target range standard deviation.78d. If a speed constraint is being imposed, then the9following processing shall be performed:10i. Compute the x-component of target velocity11(Vxt) and the y-component of target velocity12(Vyt):13 $Vxt = \frac{R2 \sin B2 + Xs2 - RI \sin BI - XsI}{t2 - tI}$ (94)14 $Vyt = \frac{R2 \cos B2 + Ys2 - RI \cos BI - YsI}{t2 - tI}$ (95)15ii. Compute the target speed (V):(96)16 $V = \sqrt{Vxt^2 + Vyt^2}$ (96)17iii. Compute the speed residual (RESv):18 $RESV = Vc - V$ (97)19where V is the assumed target speed

Compute the normalized speed residual 1 iv. RESv): 2 $\overline{RESV} = \frac{RESV}{\pi V}$ 3 (98) 4 where σV is the assumed target speed standard 5 deviation. 6 7 e.) Compute the Endpoint coordinate performance index 8 as the square root of the means of the squared 9 normalized residuals. 10 11 7. Set the minimum and maximum range at t1 with respect to the 12 sensor associated with time line 1 $(R1_{min}, R1_{max})$ and the minimum and maximum range at t2 with respect to the sensor associated 13 14 with time line 2 $(R2_{min}, R2_{max})$ as follows: 15 16 If the measurement at time line 1 is a bearing, a. 17 set the minimum range at t1 with respect to the sensor 18 associated with time line 1 (RI_{min}) to the minimum 19 range constraint which is defaulted to 100: 20 21 b. If the measurement at time line 1 is a conical 22 angle, compute the minimum range at t1 with respect to

1 the sensor associated with time line $1(R1_{min})$. If $R1_{min}$ 2 is less than the minimum range constraint, set R1min to 3 the minimum range constraint which is defaulted to 4 100. The minimum range with respect to the sensor is 5 computed as described in equations (4) thru (8). 6 7 c. Set the maximum range at t1 with respect to the 8 sensor associated with time line 1 $(R1_{max})$ to the 9 maximum range constraint with is defaulted to 200000. 10 11 d. If the measurement at time line 2 is a bearing; 12 set the minimum range at t2 with respect to the sensor 13 associated with time line $2(R2_{min})$ to the minimum range 14 constraint which is defaulted to 100. 15 16 If the measurement at time line 2 is a conical e. 17 angle, compute the minimum range at t2 with respect to 18 the sensor associated with time line $2(R2_{min})$. If $R2_{min}$ 19 is less than the minimum range constraint, set $R2_{min}$ to 20 the minimum range constraint which is defaulted to 21 100. The minimum range with respect to the sensor is 22 computed as described in equations (4) thru (8). 23

1		f. Set the maximum range at $t2$ with respect to the
2		sensor associated with time line $2(R2_{max})$ to the
3]	maximum range constraint which is defaulted to 200000.
4		
5	8. Compute	e the endpoint parameters as follows:
б	, i	a. If the measurement at time line 1 is a bearing,
7	2	set true bearing at <i>t1</i> with respect to the sensor
8	ä	associated with time line 1 ($B1$) to the smoothed
9	}	bearing estimate at time line 1 output by the endpoint
10	5	smoother algorithm.
11		
12	ł	b. If the measurement at time line 1 is a conical
13	ā	angle,
14		i. Compute the target image depth at t1 with
15		respect to the sensor associated with time line 1
16		(Rz1) as described for equations (4) and (5).
17		ii. Compute the maximum depression/elevation
18		(D/E) angle at $t1$ with respect to the sensor
19		associated with time line 1($ heta 1_{max}$) as described for
20		· equations (6) thru (8).
21		iii. Compute the slant range at $\tau 1$ with respect
22		to the sensor associated with time line 1(Rs1):
23		$Rs1 = \sqrt{R1^2 + Rz1^2} \tag{99}$

iv. Compute the D/E angle t1 with respect to the sensor associated with time line $1(\theta 1)$:

$$\theta 1 = \sin^{-1} \left(\frac{Rz1}{Rs1} \right) \tag{100}$$

v. If $\theta > \theta 1_{max}$, the D/E angle is invalid and processing shall terminate.

vi. Compute the cosine of relative bearing at *t1* with respect to the sensor associated with time line 1(*cBr1*) as follows:

$$cBr1 = \frac{\cos\beta 1 + \sin Cs1\sin\theta 1}{\cos Cs1\cos\theta 1}$$
(101)

where Cs1 is the cant angle at t1 of the sensor associated with time line 1

 β 1 is the smoothed conical angle estimate at time line 1 output by the endpoint smoother algorithm. vii. Insure that -0.99999<*cBr1*<0.99999.

viii. Compute the relative bearing at *t1* with respect to the sensor associated with time line 1 (*Br1*) as follows:

$$Br1 = \cos^{-1} cBr1 \tag{102}$$

ix. If the port/starboard assumption for time line 1 indicates port, set $Br1=2\pi$ -Br1.

1	x. Compute the true bearing at t1 with respect
2	to the sensor associated with time line $1(B1)$ as
3	follows:
4	$B1 = Br1 + Hs1 \tag{103}$
5	where <i>Hs1</i> is the heading at <i>t1</i> of the sensor
6	associated with time line 1
7	
8	c. If the measurement at time line 2 is a bearing,
9	set true bearing at $t2$ with respect to the sensor
10	associated with time line $2(B2)$ to the smoothed
11	bearing estimate at time line 2 output by the endpoint
12	smoother algorithm.
13	
14	d. If the measurement at time line 2 is a conical
15	angle,
16	i. Compute the target image depth at $t2$ with
17	respect to the sensor associated with time line 2
18	(Rz2) as described for equations (4) and (5).
19	ii. Compute the maximum D/E angle at $t2$ with
20	respect to the sensor associated with time line 2
21	$(heta 2_{max})$ as described for equations (6) thru (8).
22	iii. Compute the slant range at t2 with respect
23	to the sensor associated with time line 2 ($Rs2$):

1
$$Rs2 = \sqrt{R2^2 + Rz2^2}$$
(104)
2 iv. Compute the D/E angle at t2 with respect to
3 the sensor associated with time line 2 ($\theta 2$):
4
$$\theta 2 = \sin^{-1}\left(\frac{Rz2}{Rs2}\right)$$
(105)
5 v. If $\theta 2 > \theta 2_{sex}$, the D/E angle is invalid and
6 processing shall terminate.
7 vi. Compute the cosine of relative bearing at t2
8 with respect to the sensor associated with time
9 line 2 ($cBr2$) as follows:
10 $cBr2 = \frac{\cos \beta 2 + \sin Cs2 \sin \theta 2}{\cos Cs2 \cos \theta 2}$ (106)
11 where $Cs2$ is the cant angle at t2 of the sensor
12 associated with time line 2.
13 $\beta 2$ is the smoothed conical angle estimate at time
14 line 2 output by the endpoint smoother algorithm.
15
16 vii. Insure that -0.99999 $< cBr2 < 0.99999$.
17
18 viii. Compute the relative bearing at t2 with
19 respect to the sensor associated with time line 2
20 (Br2) as follows:
21 $Br2 = \cos^{-1} cBr2$ (107)

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1ix. If the port/starboard assumption for time2line 2 indicates port, set
$$Br2=2\pi-Br2$$
.3x. Compute the true bearing at t2 with respect4to the sensor associated with time line 2 (B2) as5follows:6 $B2=Br2+Hs2$ (108)7where $Hs2$ is the heading at t2 of the sensor8associated with time line 2.99109. Compute the initial x-component of target velocity (Vxt) and11initial y-component of target velocity (Vyt):12 $Vxt = \frac{R2\sin B2 + Xs2 - R lish B1 - Xs1}{l2 - l1}$ (109)13 $Vyt = \frac{R2\cos B2 + Ys2 - R los B1 - Ys1}{l2 - l1}$ (110)14where $Xs2$ is the x-coordinate of the position at t2 of the15sensor associated with time line 216Ys2 is the y-coordinate of the position at t2 of the sensor17associated with time line 218Xs1 is the x-coordinate of the position at t1 of the sensor19associated with time line 120Ys1 is the y-coordinate of the position at t1 of the sensor21associated with time line 120Ys1 is the y-coordinate of the position at t1 of the sensor

10. Compute the initial target course (Ct) and speed (Vt)1 2 estimates: $Ct = \tan^{-1} \left(\frac{Vxt}{Vyt} \right)$ $Vt = \sqrt{Vxt^{2} + Vyt^{2}}$ 3 (111)(112)4 5 11. Compute initial x-coordinate of target position at tc (Xtc) 6 and initial y-coordinate of target position at tc (Ytc): 7 $Xtc = R2\sin B2 + Xs2 + Vxt(t2 - tc)$ 8 (113) $Ytc = R2\cos B2 + Ys2 + Vyt(t2 - tc)$ 9 (114)10 where tc is current time 11 12 12. Compute initial x-component of range at tc with respect to 13 own ship (Rxoc) and initial y-component of range at tc with 14 respect to own ship (Ryoc): 15 Rxoc = Xtc - Xoc(115)Ryoc = Ytc - Yoc16 (116)17 where Xoc is the x-coordinate of own ship position at tc 18 Yoc is the y-coordinate of own ship position at tc 19 20 13. Compute initial range at tc with respect to own ship (Roc) 21 and true bearing at tc with respect to own ship (Boc): $Roc = \sqrt{Rxoc^2 + Ryoc^2}$ 22 (117)

$$Boc = \tan^{-1}\left(\frac{Rxoc}{Rvoc}\right)$$

(118)

If a range constraint is being imposed, limit the initial 2 14. 3 range at tc with respect to own ship to the maximum target range constraint. If a speed constraint is being imposed, limit the 4 5 initial target speed estimate (Vt) to the maximum target speed constraint. 6 7 8 15. Gauss-Newton iterations shall be performed as described in 9 paragraphs a through r below, until the algorithm converges as 10 described in paragraph r or until twenty-five iterations have 11 been performed. 12 If the measurement at time line 1 is a conical а. 13 angle, compute endpoint parameters at the time of the 14 measurement at time line 1: 15 Limit the range at t1 with respect to the i. 16 sensor associated with time line 1 (R1) to a 17 minimum of $R1_{min}$ +0.1. 18 19 Compute the target image depth at t1 with ii.

21 (*Rz1*) as described for equations (4) and (5).

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respect to the sensor associated with time line

1		iii. Compute the maximum depression/elevation	
2		(D/E) angle at $t1$ with respect to the sensor	
3		associated with time line 1 ($ heta 1_{max}$) as described	d
4		for equations (6) thru (8).	
5			
б	• •	iv. Compute the slant range at <i>t1</i> with respec	t
7		to the sensor associated with time line 1 (Rs1):
8		$Rs1 = \sqrt{R1^2 + Rz1^2} $	119)
9			
10		v. Compute the D/E angle at $t1$ with respect t	0
11		the sensor associated with time line 1 ($ heta$ 1):	
12	•	$\theta l = \sin^{-l} \left(\frac{Rzl}{Rsl} \right) \tag{6}$	120)
13		vi. If $\theta 1 < \theta 1_{max}$, perform the following:	
14		1.) Compute the cosine of relative bearin	g
15		at <i>t1</i> with respect to the sensor associat	ed
16	•	with time line 1 (<i>cBr1</i>) as follows:	
17		$cBr1 = \frac{\cos\beta 1 + \sin Cs 1\sin\theta 1}{\cos Cs 1\cos\theta 1} $	121)
18			
19		2.) Insure that -0.999999< <i>cBr1</i> <0.999999.	
20		3.) Compute the relative bearing at $t1$ w	ith
21		respect to the sensor associated with tim	e
22		line (Br1) as follows:	

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$$Brl = \cos^{-1} cBrl$$
(122)24.) If the port/starboard assumption for3time line 1 indicates port, set $Brl=2\pi$ - Brl .45.) Compute the true bearing at tl with5respect to the sensor associated with time6line 1 (Bl) as follows:7 $Bl = Brl + Hsl$ (123)86.) Compute the slant range at tl respect9to the sensor associated with time line 110 $(Rs1)$ as follows:11 $Rsl = \sqrt{Rl^2 + Rzl^2}$ (124)12vii. If $\theta l > \theta l_{max}$, terminate all processing.13b. If the measurement at time line 2 is a conical14angle, compute endpoint parameters at the time of the15measurement at time line 2:16i. Limit the range at $t2$ with respect to the17sensor associated with time line $2(R2)$ to a18minimum of $R_{2min} + 0.1$.19ii. Compute the target image depth at $t2$ with20($Rz2$) as described for equations (4) and (5).

1	iii. Compute the maximum D/E angle at $t2$ with
2	respect to the sensor associated with time line 2
3	$(heta 2_{max})$ as described for equations (6) thru (8).
4	iv. Compute the slant range at $t2$ with respect
5	to the sensor associated with time line 2 ($Rs2$):
6	$Rs2 = \sqrt{R2^2 + Rz2^2} $ (125)
7	v. Compute the D/E angle at $t2$ with respect to
8	the sensor associated with time line 2 $(heta 2)$:
9	$\theta 2 = \sin^{-1} \left(\frac{Rz2}{Rs2} \right) \tag{126}$
10	vi. If $\theta 2 < \theta 2_{max}$, perform the following:
11	1.) Compute the cosine of relative bearing
12	at t2 with respect to the sensor associated
13	with time line 2 (<i>cBr2</i>) as follows:
14	$cBr2 = \frac{\cos\beta 2 + \sin Cs2\sin\theta 2}{\cos Cs2\cos\theta 2} $ (127)
15	2.) Insure that -0.99999<< <i>cBr2</i> <0.99999.
16	3.) Compute the relative bearing at $t2$ with
17	respect to the sensor associated with time
18	line 2 (Br2) as follows:
19	$Br2 = \cos^{-1} cBr2 \tag{128}$
20	4.) If the port/starboard assumption for
21	time line 2 indicates port, set $Br2=2\pi-Br2$.

1	5.) Compute the true bearing at $t2$ with
2	respect to the sensor associated with time
3	line 2 (B2) as follows:
4	$B2 = Br2 + Hs2 \tag{129}$
5	6.) Compute the slant range at $t2$ respect
6	to the sensor associated with time line 2
7	(<i>Rs2</i>) as follows:
8	$Rs2 = \sqrt{R2^2 + Rz2^2} $ (130)
9	vii. If $\theta_{2} > \theta_{2_{max}}$, terminate all processing.
10	
11	c. For each measurement in the batch:
12	i. Compute the x-component of range at t_i with
13	respect to the sensor associated with the <i>i</i> th
14	measurement (Rx_i) and the y-component of range at
15	t_i with respect to the sensor associated with the
16	ith measurement (Ry_i) :
17	$TI_{i} = \frac{t_{i} - t1}{t2 - t1} $ (131)
18	$T2_i = 1 - T1_i$ (132)
19	$Rx_{i} = T2_{i}R1 \sin B1 + T1_{i}R2 \sin B2 + T1_{i}(Xs_{i}^{2} - Xs_{i}) - (Xs_{i} - Xs_{i}) (133)$
20	$Ry_{i} = T2_{i}R1\cos B1 + T1_{i}R2\cos B2 + T1_{i}(Ys2 - Ys1) - (Ys_{i} - Ys1) $ (134)
21	where Xs_i is the x-coordinate of the position at
22	t_i of the sensor associated with the <i>i</i> th
23	measurement

 Ys_i is the y-coordinate of the position at t_i of the sensor associated with the *i*th measurement t_i is the time of the *i*th measurement ii. Compute the range at t_i with respect to the sensor associated with the *i*th measurement (R_i) and bearing at t_i with respect to the sensor associated with the *i*th measurement (B_i) :

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$$R_{i} = \sqrt{Rx_{i}^{2} + Ry_{i}^{2}}$$
(135)

$$B_i = \tan^{-1} \left(\frac{Rx_i}{Ry_i} \right) \tag{136}$$

iii. Compute the target image depth at t_i with respect to the sensor associated with the *i*th measurement (Rz_i) and D/E angle at t_i with respect to the sensor associated with the *i*th measurement (θ_i) as described for equations (75) and (76). iv. If the measurement at time line 1 is a bearing, the following shall be performed: 1.) Compute the partial derivative of the *x*-component of target range at t_i with respect to the sensor associated with the *i*th measurement with respect to range at *t*1 with respect to the sensor associated with time line 1 $\left(\frac{\partial Rx_i}{\partial R1}\right)$ and the partial derivative

of the y-component of target range at t_i with respect to range at t1 with respect to the

sensor associated with line 1 $\left(\frac{\partial Ry_i}{\partial RI}\right)$:

$$\frac{\partial Rx_i}{\partial R1} = T2_i \sin B1 \tag{137}$$

$$\frac{\partial RY_i}{\partial R1} = T2_i \cos B1 \tag{138}$$

2.) Compute the partial derivative of target horizontal range at t_i with respect to the sensor associated with the *i*th measurement with respect to range at *t1* with respect to the sensor associated with time

line 1
$$\left(\frac{\partial R_i}{\partial R_i}\right)$$
:

 $\frac{\partial R_{i}}{\partial R1} = \frac{Rx_{i}}{\frac{\partial Rx_{i}}{\partial R1}} + \frac{Ry_{i}}{\frac{\partial Ry_{i}}{\partial R1}}$ (139)

3.) Compute the partial derivative of the bearing at t_i with respect to the sensor associated with the *i*th measurement with respect to range at t1 with respect to the

sensor associated with time line 1 $\left(\frac{\partial B_i}{\partial R_i}\right)$:

$$\frac{\partial B_i}{\partial R_i} = \frac{T 2_i \sin \left(B_i - B_i\right)}{R_i} \tag{140}$$

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4.) Compute the partial derivative of the sine of true bearing at t1 with respect to the sensor associated with time line 1 with respect to range at t1 with respect to the sensor associated with time line 1 $\left(\frac{\partial SB1}{\partial R1}\right)$ and the partial derivative of the cosine of true bearing at t1 with respect to the sensor associated with time line 1 with respect to range at t1 with respect to the sensor associated with time line 1 with respect to the sensor associated with time line 1 with respect to the sensor associated with time line 1 with respect to the sensor $(\partial CB1)$

associated with time line 1 $\left(\frac{\partial CBI}{\partial RI}\right)$:

$$\frac{\partial sB1}{\partial R1} = 0 \tag{141}$$

$$\frac{\partial CB1}{\partial R1} = 0 \tag{142}$$

If the measurement at time line 1 is a v. conical angle, the following shall be performed: 1.) Compute the partial derivative of the sine of true bearing at t1 with respect to the sensor associated with time line 1 with respect to range at t1 with respect to the sensor associated with time line 1 $\left(\frac{\partial sB1}{\partial P1}\right)$ and the partial derivative of the cosine of true

bearing at *t1* with respect to the sensor associated with time line 1 with respect to range at *t1* with respect to the sensor

associated with time line 1 $\left(\frac{\partial CB1}{\partial R1}\right)$:

$$TMP1 = \frac{-Rz1}{R1^2 \cos Cs1 \left(\frac{Rz1 \cos \beta 1}{Rs1} + \sin Cs1\right)}$$
(143)

$$TMP2 = -\left(\frac{\cos Br1}{\sin Br1}\right)TMP1 \tag{144}$$

$$\frac{\partial cB1}{\partial R1} = TMP1\cos Hs1 - TMP\sin Hs1$$
(145)

$$\frac{\partial sB1}{\partial R1} = -\left(\frac{\cos B1}{\sin B1}\right)\frac{\partial cB1}{\partial R1}$$
(146)

2.) Compute the partial derivative of the x-component of range at t_i with respect to the sensor associated with the *i*th measurement with respect to range at *t1* with respect to the sensor associated with time

line 1
$$\left(\frac{\partial Rx_i}{\partial R1}\right)$$
 and the partial derivative of
the y-component of range at t_i with respect
to the sensor associated with the *i*th
measurement with respect to range at *t1* with

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respect to the sensor associated with time

 $\frac{\partial Rx_i}{\partial RI} = T2_i \left(RI \frac{\partial SBI}{\partial RI} + \sin BI \right)$

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 $\frac{\partial RY_i}{\partial P^1} = T2_i \left(R1 \frac{\partial CB1}{\partial P^1} + \cos B1 \right)$ (148)

(147)

3.) Compute the partial derivative of horizontal range at t_i with respect to the sensor associated with the *i*th measurement with respect to the range at t1 with respect to the sensor associated with time line 1

 $\left(\frac{\partial R_i}{\partial R_1}\right)$:

line 1 $\left(\frac{\partial Ry_i}{\partial P^1}\right)$:

 $\frac{\partial R_i}{\partial P_1} = \frac{Rx_i}{\frac{\partial Rx_i}{\partial R_1}} + Ry_i \frac{\partial Ry_i}{\partial R_1}}{P_i}$ (149)

4.) Compute the partial derivative of true bearing at t_i with respect to the sensor associated with the *i*th measurement with respect to range at *t1* with respect to the

sensor associated with time line 1 $\left(\frac{\partial B_i}{\partial RI}\right)$:

$$\frac{\partial B_i}{\partial R_1} = \frac{Ry_i}{\frac{\partial Rx_i}{\partial R_1}} - \frac{Rx_i}{\frac{\partial Ry_i}{\partial R_1}}$$
(150)

1 If the measurement at time line 2 is a vi. 2 bearing, the following shall be performed: 3 Compute the partial derivative of the 1.) 4 x-component of target range at t_i with 5 respect to the sensor associated with the 6 ith measurement with respect to range at t27 with respect to the sensor associated with time line 2 $\left(\frac{\partial Rx_i}{\partial R^2}\right)$ and the partial derivative 8 9 of the y-component of target range at t_i with 10 respect to the sensor associated with the 11 ith measurement with respect to range at t2 12 with respect to the sensor associated with time line 2 $\left(\frac{\partial Ry_i}{\partial R^2}\right)$: 13 $\frac{\partial Rx_i}{\partial R^2} = Tl_i \sin B2$ 14 (151) $\frac{\partial R y_i}{\partial P^2} = Tl_i \cos B2$ 15 (152)16 2.) Compute the partial derivative of 17 target horizontal range at t_i with respect to 18 the sensor associated with the ith

measurement with respect to range at t2 with

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respect to the sensor associated with time

line 2 $\left(\frac{\partial R_i}{\partial R^2}\right)$:

$$\frac{\partial R_{i}}{\partial R^{2}} = \frac{Rx_{i}}{\frac{\partial Rx_{i}}{\partial R^{2}}} + Ry_{i}\frac{\partial Ry_{i}}{\partial R^{2}}}{R_{i}}$$
(153)

3.) Compute the partial derivative of the bearing at t_i with respect to the sensor associated with the *i*th measurement with respect to range at t_2 with respect to the

sensor associated with time line 2 $\left(\frac{\partial B_i}{\partial R^2}\right)$:

$$\frac{\partial B_i}{\partial R^2} = \frac{Tl_i \sin (B^2 - B_i)}{R_i}$$
(154)

4.) Compute the partial derivative of the sine of true bearing at t2 with respect to the sensor associated with time line 2 with respect to range at t2 with respect to the

sensor associated with time line 2
$$\left(\frac{\partial sB2}{\partial R2}\right)$$

and the partial derivative of the cosine of true bearing at t2 with respect to the sensor associated with time line 2 with respect to range at t2 with respect to the

sensor associated with time line 2 $\left(\frac{\partial cB2}{\partial R2}\right)$:

$$\frac{\partial sB2}{\partial R2} = 0 \tag{155}$$

$$\frac{\partial cB2}{\partial R2} = 0 \tag{156}$$

vii. If the measurement at time line 2 is a conical angle, the following shall be performed: 1.) Compute the partial derivative of the sine of true bearing at t2 with respect to the sensor associated with time line 2 with respect to range at t2 with respect to the sensor associated with time line 2 $\left(\frac{\partial SB2}{\partial R2}\right)$ and the partial derivative of the cosine of true bearing at t2 with respect to the sensor associated with time line 2 with respect to the sensor associated with respect to the sensor associated with time line 2 with respect to the sensor associated with time line 2 with respect to the sensor associated with time line 2 with respect to the sensor associated with time line 2 with respect to the sensor associated with time line 2 with respect to the sensor associated with time line 2 with respect to the sensor associated with time line 2 $\left(\frac{\partial CB2}{\partial R^2}\right)$:

$$TMP1 = \frac{-Rz2}{R2^2 \cos Cs2 \left(\frac{Rz2 \cos \beta 2}{Rs2} + \sin Cs2\right)}$$
(157)

$$TMP2 = -\left(\frac{\cos Br2}{\sin Br2}\right)TMP1 \tag{158}$$

$$\frac{\partial cB2}{\partial R2} = TMP1 \cos Hs2 - TMP2 \sin Hs2$$
(159)

$$\frac{\partial sB2}{\partial R2} = -\left(\frac{\cos B2}{\sin B2}\right)\frac{\partial cB2}{\partial R2}$$
(160)

2.) Compute the partial derivative of the x-component of range at t_i with respect to the sensor associated with the *i*th measurement with respect to range at t2 with respect to the sensor associated with time line 2 $\left(\frac{\partial Rx_i}{\partial R2}\right)$ and the partial derivative of the y-component of range at t_i with respect to the sensor associated with the *i*th measurement with respect to range at t2 with respect to the sensor associated with time

line 2
$$\left(\frac{\partial Ry_i}{\partial R^2}\right)$$
:

$$\frac{\partial Rx_i}{\partial R2} = TI_i \left(R2 \frac{\partial sB2}{\partial R2} + \sin B2 \right)$$
(161)

$$\frac{\partial Ry_i}{\partial R^2} = T I_i \left(R^2 \frac{\partial c B^2}{\partial R^2} + \cos B^2 \right)$$
(162)

3.) Compute the partial derivative of horizontal range at t_i with respect to the sensor associated with the *i*th measurement with respect to the range at t_2 with respect to the sensor associated with time line 2

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 $\left(\frac{\partial R_i}{\partial R_2}\right)$:

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$$\frac{\partial R_i}{\partial R^2} = \frac{Rx_i \frac{\partial Rx_i}{\partial R^2} + Ry_i \frac{\partial Ry_i}{\partial R^2}}{R_i}$$
(163)

4.) Compute the partial derivative of true bearing at t_i with respect to the sensor associated with the *i*th measurement with respect to range at t2 with respect to the sensor associated with time line $2\left(\frac{\partial B_i}{\partial R^2}\right)$:

$$\frac{\partial B_i}{\partial R^2} = \frac{Ry_i \frac{\partial Rx_i}{\partial R^2} - Rx_i \frac{\partial Ry_i}{\partial R^2}}{R_i^2}$$
(164)

viii. If the *i*th measurement is a bearing, then the following shall be performed:

1.) Compute the bearing residual $(RESb_i)$ such that $-\pi \leq RESb_i \leq \pi$:

$$RESb_i = Bm_i - B_i \tag{165}$$

(166)

where Bm_i is the measured bearing at t_i 2.) Compute the normalized bearing residual

 $\left(RESb_{i} \right)$ and normalized partial derivatives

 $\overline{RESb_i} = \frac{RESb_i}{\sigma b_i}$

 $rac{\partial B_i}{\partial R1}$,

 $\left| \frac{\partial B_i}{\partial R^2} \right|$:

$$\frac{\overline{\partial B_i}}{\partial R1} = \frac{\frac{\partial B_i}{\partial R1}}{\sigma b_i}$$
(167)

$$\frac{\overline{\partial B_i}}{\partial R^2} = \frac{\frac{\partial B_i}{\partial R^2}}{\sigma b_i}$$
(168)

where σb_i is the standard deviation of the bearing measurement

3.) If frequency data are not being
processed, set the next row of the augmented
Jacobian H Matrix as follows:

$$\left[\frac{\partial B_i}{\partial R1} \frac{\partial B_i}{\partial R2} \quad \overline{RESb_i}\right]$$
(169)

If frequency data are being processed, set the next row of the augmented Jacobian H matrix as follows:

$$\left[\frac{\overline{\partial B_i}}{\partial R^2} \frac{\overline{\partial B_i}}{\partial R^2} O \overline{RESb_i}\right]$$
(170)

ix. If the *i*th measurement is a conical angle and the D/E-mark indicates a valid D/E:
1.) Compute the true bearing at t_i with respect to the sensor associated with the *i*th measurement (B_i):

$$B_i = \tan^{-2} \left(\frac{R x_i}{R y_i} \right)$$

(171)

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2.) Compute the slant range at t_i with respect to the sensor associated with the ith measurement (Rs_i) :

$$Rs_{i} = \sqrt{Rx_{i}^{2} + Ry_{i}^{2} + Rz_{i}^{2}}$$
(172)

3.) If the measurement at time line 1 is a conical angle:

<u>a</u> Compute the partial derivative of slant range at *t1* with respect to the sensor associated with time line 1 with respect to range at *t1* with respect to the sensor associated with time line 1

 $\left(\frac{\partial Rs1}{\partial R1}\right)$:

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 $\frac{\partial Rs1}{\partial R1} = \frac{R1}{Rs1} \tag{173}$

<u>b</u> Compute the partial derivative of cosine of relative bearing at *t1* with respect to the sensor associated with time line 1 with respect to range at *t1* with respect to the sensor associated

with time line $\left(\frac{\partial cBr1}{\partial R1}\right)$ and the partial

derivative of sine of relative bearing at *t1* with respect to the sensor associated with time line 1 with

respect to range at *t1* with respect to the sensor associated with time line 1

 $\left(\frac{\partial sBr1}{\partial R1}\right)$:

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$\frac{\partial CBr}{\partial Rl}$	$\frac{1}{Rl^2} = \frac{\cos\beta l \left(Rl \frac{\partial Rsl}{\partial Rl} - Rsl \right) - Rzl \sin Csl}{Rl^2 \cos Csl}$	(174)
	$\frac{\partial sBr1}{\partial R1} = -\frac{\cos Br1}{\sin Br1} \frac{\partial cBr1}{\partial R1}$	(175)

4.) If the measurement at time line 2 is a conical angle:

<u>a</u> Compute the partial derivative of slant range at t2 with respect to the sensor associated with time line 2 with respect to range at t2 with respect to the sensor associated with time line 2

 $\left(\frac{\partial Rs2}{\partial R2}\right):$

 $\frac{\partial Rs2}{\partial R^2} = \frac{R2}{Rc^2}$

<u>b</u> Compute the partial derivative of cosine of relative bearing at t2 with respect to the sensor associated with time line 2 with respect to range at t2with respect to the sensor associated

(176)

with time line 2 $\left(\frac{\partial cBr2}{\partial R2}\right)$ and the

partial derivative of sine of relative bearing at t2 with respect to the sensor associated with time line 2 with respect to range at t2 with respect to

1		the sensor associated with time line 2
2		$\left(\frac{\partial sBr2}{\partial R2}\right)$:
		$\partial R s^2$ $\partial R s^2$ $\partial R s^2$
3		$\frac{\partial cBr^2}{\partial R^2} = \frac{\cos\beta r^2 \left(R^2 \frac{\partial Rs^2}{\partial R^2} - Rs^2\right) - Rz 2\sin Cs^2}{R^2 \cos Cs^2} $ (177)
4		$\frac{\partial sBr2}{\partial R2} = -\frac{\cos Br2}{\sin Br2} \frac{\partial cBr2}{\partial R2} $ (178)
5		5.) Compute the partial derivative of slant
6		range at t_i with respect to the sensor
7		associated with the <i>i</i> th measurement with
8		respect to the x-component of range at t_i
9		with respect to the sensor associated with
10		the i th measurement $\left(rac{\partial { m Rs}_i}{\partial { m Rx}_i} ight)$ and the partial
11		derivative of slant range at t_i with respect
12		to the sensor associated with the <i>i</i> th
13	•	measurement with respect to the y-component
14		of range at t_i with respect to the sensor
15		associated with the <i>i</i> th measurement $\left(\frac{\partial Rs_i}{\partial Ry_i}\right)$:
16		$\frac{\partial Rs_i}{\partial Rx_i} = \frac{Rx_i}{Rs_i} $ (179)
17		$\frac{\partial Rs_i}{\partial Ry_i} = \frac{Ry_i}{Rs_i} $ (180)
18		6.) Compute the partial derivative of
19		relative at t_i with respect to the sensor
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20		associated with the <i>i</i> th measurement with

sensor associated with time line 1 $\left(\frac{\partial Br_i}{\partial RI}\right)$ and

the partial derivative of relative bearing at t_i with respect to the sensor associated with the *i*th measurement with respect to range at t2 with respect to the sensor

associated with time line 2 $\left(\frac{\partial Br_i}{\partial R^2}\right)$:

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$$\frac{\partial Br_{i}}{\partial R1} = \frac{Ry_{i} \frac{\partial Rx_{i}}{\partial R1} - Rx_{i} \frac{\partial Ry_{i}}{\partial R1}}{R_{i}^{2}}$$
(181)

$$\frac{\partial B_{I_{i}}}{\partial R^{2}} = \frac{R_{Y_{i}}}{R_{i}} \frac{\partial R_{X_{i}}}{\partial R^{2}} - R_{X_{i}} \frac{\partial R_{Y_{i}}}{\partial R^{2}}$$
(182)

7.) Compute the partial derivative of D/E at t_i with respect to the sensor associated with the *i*th measurement with respect to range at *t1* with respect to the sensor associated with time line 1 $\left(\frac{\partial \theta_i}{\partial RI}\right)$ and the partial derivative of D/E angle at t_i with respect to the sensor associated with the *i*th measurement with respect to range at *t2* with respect to the sensor associated with

time line 2 $\left(\frac{\partial \theta_i}{\partial R^2}\right)$:

$$\frac{\partial \theta_i}{\partial R1} = \frac{-Rz_i}{Rs_i^2} \frac{\frac{\partial R_i}{\partial R1}}{Rs_i^2}$$
(183)

$$\frac{\partial \theta_i}{\partial R^2} = \frac{-Rz_i \frac{\partial R_i}{\partial R^2}}{Rs_i^2}$$
(184)

8.) Compute the sine and cosine of D/E angle at t_i with respect to the sensor associated with the *i*th measurement:

$$\sin\theta_i = \frac{Rz_i}{Rs_i} \tag{185}$$

$$\cos\theta_i = \frac{R_i}{Rs_i} \tag{186}$$

9.) Compute the relative bearing at t_i with respect to the sensor associated with the *i*th measurement (Br_i) :

$$Br_i = B_i - Hs_i \tag{187}$$

10.) Compute the conical angle at t_i from a horizontal array with respect to the sensor associated with the *i*th measurement (βh_i) :

$$\beta h_i = \cos^{-1} (\cos \theta_i \cos Br_i) \tag{188}$$

11.) Is $\sin \beta h_i \neq 0$, compute the partial derivative of the conical angle at t_i from a horizontal array with respect to the sensor associated with the *i*th measurement with respect to range at *t1* with respect to the
sensor associated with time line 1 $\left(\frac{\partial eta h_i}{\partial R1}\right)$ and

the partial derivative of the conical angle at t_i from a horizontal array with respect to the sensor associated with the *i*th measurement with respect to range at t2 with respect to the sensor associated with time

line 2
$$\left(\frac{\partial\beta h_i}{\partial R^2}\right)$$
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$$\frac{\partial \beta h_i}{\partial Rl} = \frac{\cos Br_i \sin \theta_i \frac{\partial \theta_i}{\partial Rl} + \sin Br_i \cos \theta_i \frac{\partial Br_i}{\partial Rl}}{\sin \beta h_i}$$
(189)

$$\frac{\partial \beta h_{i}}{\partial R^{2}} = \frac{\cos Br_{i} \sin \theta_{i} \frac{\partial \theta_{i}}{\partial R^{2}} + \sin Br_{i} \cos \theta_{i} \frac{\partial B r_{i}}{\partial R 2}}{\sin \beta h_{i}}$$
(190)

12.) Compute the x-component of range at t_i with respect to the sensor associated with the *i*th measurement at the time of the *i*th measurement (xta_i) , the y-component of range at t_i with respect to the sensor associated with the *i*th measurement at the time of the *i*th measurement (yta_i) and the z-component of range at t_i with respect to the sensor associated with the *i*th measurement at the time of the *i*th measurement (zta_i) rotated to the axis of the array:

$$xta_i = Rx_i \cos Hs_i - Ry_i \sin Hs_i \qquad (191)$$

$$vta_i = (Rx_i \sin Hs_i + Ry_i \cos Hs_i) \cos Cs_i - Rz_i \sin Cs_i \qquad (192)$$

 $zta_{i} = (Rx_{i} \sin Hs_{i} + Ry_{i} \cos Hs_{i}) \sin Cs_{i} + Rz_{i} \cos Cs_{i}$ (193) 13.) Compute the conical angle at t_{i} with respect to the sensor associated with the ith measurement (β_{i}) :

If $yta_i \neq 0$,

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$$\beta_{i} = \tan^{-1} \left(\frac{\sqrt{xta_{i}^{2} + zta_{i}^{2}}}{yta_{i}} \right)$$
(194)

otherwise,

$$\beta_i = \frac{\pi}{2} \tag{195}$$

14.) If $sin\beta_i \neq 0$, compute the partial derivative of the conical angle at t_i with respect to the sensor associated with the ith measurement with respect to range at t1with respect to the sensor associated with time line $1\left(\frac{\partial\beta_i}{\partial R1}\right)$ and the partial derivative of the conical angle at t_i with respect to range at t2 with respect to the sensor associated with time line $2\left(\frac{\partial\beta_i}{\partial R2}\right)$:

$$\frac{\partial \beta_{i}}{\partial R1} = \frac{\cos C s_{i} \sin \beta h_{i} \frac{\partial \beta h_{i}}{\partial R1} + \sin C s_{i} \cos \theta_{i} \frac{\partial \theta_{i}}{\partial R1}}{\sin \beta_{i}}$$
(196)

$$\frac{\partial \beta_{i}}{\partial R^{2}} = \frac{\cos C s_{i} \sin \beta h_{i} \frac{\partial \beta h_{i}}{\partial R^{2}} + \sin C s_{i} \cos \theta_{i} \frac{\partial \theta_{i}}{\partial R^{2}}}{\sin \beta_{i}}$$
(197)

15.) Compute the conical angle residual

 $(RES\beta_i)$ such that $-\pi \leq RES\beta_i \leq \pi$:

 $RES\beta_{i} = \beta m_{i} - \beta_{i}$ (198)

where βm_i is the measured conical angle at $t_i.$

16.) Compute the normalized conical angle residual $(\overline{RES\beta_i})$ and normalized partial

derivatives
$$\left(\frac{\partial \beta_i}{\partial R1}, \frac{\partial \beta_i}{\partial R2} \right)$$
:

 $\overline{RES\beta_i} = \frac{RES\beta_i}{\sigma\beta_i}$ (199)

$$\frac{\partial \beta_{i}}{\partial R_{1}} = \frac{\frac{\partial \beta_{i}}{\partial R_{1}}}{\sigma \beta_{i}}$$
(200)

$$\frac{\overline{\partial \beta_i}}{\partial R^2} = \frac{\frac{\partial \beta_i}{\partial R^2}}{\sigma \beta_i}$$
(201)

where σeta_i is the standard deviation of the conical angle measurement.

17.) If frequency data are not being processed, set the next row of the augmented Jacobian *H* matrix to:

$$\begin{bmatrix} \frac{\partial \beta_i}{\partial R1} & \frac{\partial \beta_i}{\partial R2} \\ \hline RES\beta_i \end{bmatrix}$$

If frequency data are being processed, set the next row of the augmented Jacobian H matrix to:

$$\left[\frac{\partial \beta_{i}}{\partial R1} \frac{\partial \beta_{i}}{\partial R2} 0 \frac{\partial \beta_{i}}{RES\beta_{i}}\right]$$
(203)

x. If the *i*th measurement is a horizontal range:
1.) Compute the range residual (*RESr_i*):

$$RESr_i = Rm_i - R_i \tag{204}$$

where Rm_i is the measured horizontal range at t_i .

2.) Compute the normalized range residual $(\overline{RESr_i})$ and normalized partial derivatives

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$$\overline{RESr_i} = \frac{RESr_i}{\sigma r_i}$$

 $\frac{\partial R_i}{\partial R1}$, $\frac{\partial R_i}{\partial R2}$

∂R, ∂R1 ∂R1 σr_{\cdot}

(206)

(205)

(202)

$$\frac{\partial R_i}{\partial R_2} = \frac{\frac{\partial R_i}{\partial R_2}}{\sigma r_i}$$
(207)

(208)

where $\sigma r_{\rm i}$ is the standard deviation of the range measurement.

3.) If frequency data are not being processed, set the next row of the augmented Jacobian H matrix to:

$$\left[\frac{\partial R_i}{\partial R_i} \frac{\partial R_i}{\partial R_2} \frac{\partial R_i}{RESr_i}\right]$$

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If frequency data are being processed, set the next row of the augmented Jacobian H matrix to:

$$\frac{\overline{\partial R_i}}{\partial R1} \frac{\overline{\partial R_i}}{\partial R2} O \overline{RESr_i}$$
(209

xi. If frequency data are being processed and the D/E mark associated with the *i*th measurement indicates a valid D/E, then the following shall be performed:

16 1.) Compute the x-component of target 17 velocity (Vxt) and the y-component of target 18 velocity (Vyt):

$$Vxt = \frac{R2\sin B2 + Xs2 - R1\sin B1 - Xs1}{t2 - t1}$$
(210)

 $Vyt = \frac{R2\cos B2 + Ys2 - R1\cos B1 - Ys1}{t2 - t1}$ (211)

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2.) Compute the x-component of target velocity at t_i relative to the sensor associated with the *i*th measurement (Vx_i) and the y-component of target velocity at t_i relative to the sensor associated with the *i*th measurement (Vy_i)

 $Vx_i = Vxt - Vxs_i \tag{212}$

$$Vy_{t} = Vyt - Vys_{t} \tag{213}$$

where Vxs_i is the x-component of velocity of the sensor associated with the *i*th measurement

 Vys_i is the y-component of velocity of the sensor associated with the *i*th measurement 3.) Compute the slant range at t_i with respect to the sensor associated with the *i*th measurement (Rs_i)

$$Rs_i \sqrt{R_i^2 + Rz_i^2} \tag{214}$$

4.) Compute the partial derivative of frequency at t_i with respect to the sensor associated with the *i*th measurement with respect to the *x*-component of range at t_i with respect to the sensor associated with

the *i*th measurement $\left(\frac{\partial f_i}{\partial Rx_i}\right)$ and the partial

derivative of frequency at t_i with respect to the sensor associated with the *i*th measurement with respect to the y-component of range at t_i with respect to the sensor

associated with the *ith* measurement $\left(\frac{\partial f_i}{\partial R_{\mathcal{V}_i}}\right)$:

$$\frac{\partial f_i}{\partial Rx_i} = \frac{Fb}{\left(cRs_i + Vxs_iRx_i + Vys_iRy_i + Rx_iVx_i + Ry_iVy_i\right)^2} \bullet \left(\left(Rx_iVx_i + Ry_iVy_i\right)\left(\frac{c}{Rs_i}Rx_i + Vxs_i\right) - \left(cRs_i + Vxs_iRx_i + Vys_iRy_i\right)Vx_i\right)\right)$$
(215)

$$\frac{\partial f_i}{\partial Ry_i} = \frac{Fb}{\left(cRs_i + Vxs_iRx_i + Vys_iRy_i + Rx_iVx_i + Ry_iVy_i\right)^2} \bullet \left(\left(Rx_iVx_i + Ry_iVy_i\right)\left(\frac{c}{Rs_i}Ry_i + Vys_i\right) - \left(cRs_i + Vxs_iRx_i + Vys_iRy_i\right)Vy_i\right)\right)$$
(216)

5.) Compute the partial derivative of frequency at t_i with respect to the sensor associated with the ith measurement with respect to the x-component of target relative velocity at t_i with respect to the sensor associated with the *i*th measurement

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 $\left(\frac{\partial f_i}{\partial V x}\right)$ and the partial derivative of

frequency at t_i with respect to the sensor associated with the ith measurement with

respect to the y-component of target relative velocity at t_i with respect to the sensor associated with the *i*th measurement

 $\left(\frac{\partial f_i}{\partial V v_i}\right)$:

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 $\frac{\partial f_i}{\partial Vx_i} = \frac{Fb}{\left(cRs_i + Vxs_iRx_i + Vys_iRy_i + Rx_iVx_i + Ry_iVy_i\right)^2} \bullet (217)$ $\left(\left(cRs_i + Vxs_iRx_i + Vys_iRy_i\right)Rx_i\right)$

$$\frac{\partial f_i}{\partial Vy_i} = \frac{Fb}{\left(cRs_i + Vxs_iRx_i + Vys_iRy_i + Rx_iVx_i + Ry_iVy_i\right)^2} \bullet (218)$$
$$\left(\left(cRs_i + Vxs_iRx_i + Vys_iRy_i\right)Ry_i\right)$$

6.) Compute the partial derivative of the x-component of target relative velocity at t_i with respect to the sensor associated with the *i*th measurement with respect to range at *t1* with respect to the sensor associated

with the measurement at time line 1 $\left(\frac{\partial V x_i}{\partial R l} \right)$

and the partial derivative cf the ycomponent of target relative velocity at t_i with respect to the sensor associated with the *i*th measurement with respect to range at *t1* with respect to the sensor associated

with time line 1 $\left(\frac{\partial Vy_i}{\partial R1}\right)$:

$$\frac{\partial Vx_i}{\partial R1} = \frac{R1}{\frac{\partial \sin B1}{\partial R1}} + \sin B1}{t2 - t1}$$
(219)

$$\frac{\partial V_{Y_i}}{\partial R_1} = \frac{R1}{\frac{\partial \cos B1}{\partial R_1}} + \cos B1}{t2 - t1}$$
(220)

7.) Compute the partial derivative of the x-component of target relative velocity at t_i with respect to the sensor associated with the *i*th measurement with respect to range at t_2 with respect to the sensor associated with time line 2 at $t_2 - \left(\frac{\partial Vx_i}{\partial R_2}\right)$ and the partial derivative of the y-component of target relative velocity at t_i with respect to the sensor associated with the *i*th measurement with respect to range at t_2 with respect to the sensor associated with time line 2

 $\left(\frac{\partial V y_i}{\partial R^2}\right):$

$$\frac{\partial Vx_i}{\partial R^2} = -\frac{R2}{\frac{\partial sB^2}{\partial R^2}} + \sin B^2}{t^2 - t^1}$$
(221)

$$\frac{\partial V_{y_i}}{\partial R^2} = -\frac{R2}{\frac{\partial cB^2}{\partial R^2}} + \cos B^2}{t^2 - t^1}$$
(222)

1 8.) Compute the partial derivative of 2 frequency at t_i with respect to the sensor 3 associated with the ith measurement with 4 respect to range at t1 with respect to the sensor associated with time line 1 $\left(\frac{\partial f_i}{\partial P^{\dagger}}\right)$, 5 6 the partial derivative of frequency at t_i 7 with respect to the sensor associated with 8 the *i*th measurement with respect to range at 9 t2 with respect to the sensor associated with time line 2 $\left(\frac{\partial f_i}{\partial P^2}\right)$ and the partial 10 derivative of frequency at t_i with respect to 11 12 the sensor associated with the ith 13 measurement with respect to base frequency $\left(\frac{\partial f_i}{\partial F_h}\right)$: 14 15 $\frac{\partial f_i}{\partial R1} = \frac{\partial f_i}{\partial Rx} \frac{\partial Rx_i}{\partial R1} + \frac{\partial f_i}{\partial Ry} \frac{\partial Ry_i}{\partial R1} + \frac{\partial f_i}{\partial Vx} \frac{\partial Vx_i}{\partial R1} + \frac{\partial f_i}{\partial Vy} \frac{\partial Vy_i}{\partial R1} (223)$ 16 $\frac{\partial f_i}{\partial R^2} = \frac{\partial f_i}{\partial Rx_i} \frac{\partial Rx_i}{\partial R^2} + \frac{\partial f_i}{\partial Ry_i} \frac{\partial Ry_i}{\partial R^2} + \frac{\partial f_i}{\partial Vx_i} \frac{\partial Vx_i}{\partial R^2} + \frac{\partial f_i}{\partial Vy_i} \frac{\partial Vy_i}{\partial R^2} (224)$ 17 $\frac{\partial f_i}{\partial Fb} = \frac{cRs_i + Vxs_iRx_i + Vys_iRy_i}{(cRs_i + Vxs_iRx_i + Vys_iRy_i + Rx_iVx_i + Ry_iVy_i)}$ (225) 18

9.) Compute the estimated frequency at t_i with respect to the sensor associated with the *i*th measurement at the time of the *i*th measurement:

$$f_{i} = Fb \frac{cRs_{i} + Vxs_{i}Rx_{i} + Vys_{i}Ry_{i}}{cRs_{i} + VxtRx_{i} + VytRy_{i}}$$
(226)

10.) Compute the frequency residual $(RESf_i)$:

$$RESf_i = fm_i - f_i \tag{227}$$

where fm_i is the measured frequency at t_i . 11.) Compute the normalized frequency residual $(\overline{RESf_i})$ and normalized partial

derivatives
$$\left(\frac{\overline{\partial f_i}}{\partial R1}, \frac{\overline{\partial f_i}}{\partial R2}, \frac{\overline{\partial f_i}}{\partial Fb}\right)$$
:

$$\overline{RESf_i} = \frac{RESf_i}{\sigma f_i}$$
(228)

$$\frac{\overline{\partial f_i}}{\partial R1} = \frac{\frac{\partial f_i}{\partial R1}}{\sigma f_i}$$
(229)

$$\frac{\partial f_i}{\partial R^2} = \frac{\frac{\partial f_i}{\partial R^2}}{\sigma f_i}$$
(230)

$$\frac{\overline{\partial f_i}}{\partial Fb} = \frac{\frac{\partial f_i}{\partial Fb}}{\sigma f_i}$$
(231)

where σf_i is the standard deviation of the frequency measurement.

12.) Set the next row of the augmented 1 2 Jacobian H matrix to: $\begin{bmatrix} \frac{\partial f_i}{\partial R1} & \frac{\partial f_i}{\partial R2} & \frac{\partial f_i}{\partial Fb} \\ \hline RESF_i \end{bmatrix}$ 3 (232)4 If a range constraint is being imposed, then the d. 5 following processing shall be performed: 6 i. Compute the range residual (RESR): RESR = Rc - R7 (233)8 where Rc is the assumed target range. ii. Compute the normalized range residual (RESR): 9 $\overline{RESR} = \frac{RESR}{\sigma R}$ 10 (234)11 where σR is the assumed target range standard 12 deviation. 13 iii. If frequency data are not being processed, 14 set the next row of the augmented Jacobian H 15 Matrix to: $\begin{bmatrix} \overline{\partial R:} & \overline{\partial R:} \\ \overline{\partial RI} & \overline{\partial R2} & \overline{RESR} \end{bmatrix}$ 16 (235)If frequency data are being processed, set the 17 18 next row of the augmented Jacobian H matrix to: $\begin{bmatrix} \frac{\partial R:}{\partial R!} & \frac{\partial R:}{\partial R2} & 0 \end{bmatrix} \xrightarrow{RESR}$ 19 (236)

1	e. If a speed constraint is being imposed, then the
2	following processing shall be performed:
3	i. Compute the x-component of target velocity
4	(Vxt) and the y-component of target velocity
5	(<i>Vyt</i>):
6	$Vxt = \frac{R2\sin B2 + Xs2 - R1\sin B1 - Xs1}{t2 - t1} $ (237)
7	$Vyt = \frac{R2\cos B2 + Ys2 - R1\cos B1 - Ys1}{t2 - t1} $ (238)
8	ii. Compute the partial derivative of the x -
9	component of target velocity with respect to
10	range at t1 with respect to the sensor associated
11	with the measurement at time line 1 $\left(rac{\partial Vxt}{\partial R1} ight)$ and
12	the partial derivative of the y-component of
13	target velocity with respect to range t1 with
14	respect to the sensor associated with time line 1
15	$\left(\frac{\partial V_{yt}}{\partial T_{1}}\right)$:

 $\left(\frac{\partial Vyt}{\partial Rl}\right)$:

$$\frac{\partial Vxt}{\partial R1} = -\frac{R1}{\frac{\partial sB1}{\partial R1}} + \sin B1}{t2 - t1}$$
(239)

$$\frac{\partial V_{xt}}{\partial R1} = -\frac{R1}{\frac{\partial SB1}{\partial R1}} + \cos B1}{t2 - t1}$$
(240)

iii. Compute the partial derivative of the xcomponent of target velocity with respect to

range at t2 with respect to the sensor associated

with time line 2
$$\left(rac{\partial Vxt}{\partial R2}
ight)$$
 and the partial

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derivative of the y-component of target velocity with respect to range at t2 with respect to the sensor associated with time line 2 $\left(\frac{\partial Vyt}{\partial R2}\right)$:

$$\frac{\partial V_{X}t}{\partial R^{2}} = -\frac{R^{2} \frac{\partial sB^{2}}{\partial R^{2}} + \sin B^{2}}{t^{2} - t^{1}}$$
(241)

$$\frac{\partial Vyt}{\partial R^2} = -\frac{R^2 \frac{\partial cB^2}{\partial R^2} + \cos B^2}{t^2 - t^1}$$
(242)

iv. Compute the partial derivative of target speed with respect to range at *t1* with respect to

the sensor associated with time line 1 $\left(rac{\partial v}{\partial R I}
ight)$ and

the partial derivative of target speed with respect to range at t2 with respect to the sensor

associated with time line 2
$$\left(\frac{\partial v}{\partial R^2}\right)$$
:

$$\frac{\partial v}{\partial R1} = \frac{Vxt}{\frac{\partial Vxt}{\partial R1}} + Vyt}{\frac{\partial Vyt}{\partial R1}}$$
(243)

$$\frac{\partial v}{\partial R^2} = \frac{Vxt \frac{\partial Vxt}{\partial R^2} + Vyt \frac{\partial Vyt}{\partial R^2}}{\sqrt{Vxt^2 + Vyt^2}}$$
(244)

v. Compute the estimated target speed:

 $V = \sqrt{Vxt^2 + Vyt^2} \tag{245}$

vi. Compute the speed residual (RESv):

$$RESv = Vc - V \tag{246}$$

where Vc is the assumed target speed.

vii. Compute the normalized speed residual (\overline{RESv})

and normalized partial derivatives $\begin{pmatrix} -\frac{\partial v}{\partial r}, \frac{\partial v}{\partial R^2} \\ \frac{\partial v}{\partial R^2}, \frac{\partial v}{\partial R^2} \end{pmatrix}$

 $\overline{RESv} = \frac{RESv}{\sigma v}$ (247)

$$\frac{\overline{\partial v}}{\partial R_{1}} = \frac{\frac{\partial v}{\partial R_{1}}}{\sigma v}$$
(248)

$$\frac{\overline{\partial v}}{\partial R^2} = \frac{\frac{\partial v}{\partial R^2}}{\sigma v}$$
(249)

where σv is the standard deviation of the assumed target speed.

viii. If frequency data are not being processed, set the next row of the augmented Jacobian H matrix to:

$$\begin{bmatrix} \frac{\partial v}{\partial R1} & \frac{\partial v}{\partial R2} & \frac{\partial v}{RESv} \end{bmatrix}$$

If frequency data are being processed, set the next row of the augmented Jacobian H matrix to:

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$$\begin{bmatrix} \overline{\partial v} \\ \overline{\partial Rl} & \overline{\partial v} \\ \overline{\partial Rl} & \overline{\partial r} \\ 0 & \overline{RESv} \end{bmatrix}$$
(251)
f. Reorder the rows of the H matrix such that a zero-
valued partial derivative does not appear along the
diagonal.
g. Perform the Householder transformation on the m-
by-ns+1 matrix H:
i. Compute values of s, μ , and β as:
 $s = -sgn(H(1,1)) \left(\sum_{i=1}^{m} [H(i,1)^2] \right)$ (252)
 $u(1) = H(1,1) - s$ (253)
 $u(1) = H(1,1) - s$ (253)
 $u(i) = H(i,1) \ i = 2, \dots, m$ (254)
 $\beta = \frac{1}{su(1)}$ (255)
ii. For j=2, ..., ns+1, evaluate the following
equations (apply Householder transformation to
the successive columns of H):
 $\gamma = \beta \sum_{n=1}^{m} u(i)H(i,j)$ (256)

.

 $H (i, j) = H (i, j) + \gamma \iota (i) i = 1, \dots, m$

(257)

1	h. Extract the upper triangular matrix R from the
2	upper left hand corner of the transformed matrix H:
3	R(i,i) = H(i,i)i = 1,,ns (258)
4	i. Compute R^{-1} by back substitution:
5	i. Compute R^{-1} (1,1) as follows:
6	$R^{-1}(1, 1) = \frac{1}{R(1, 1)} $ (259)
7	ii. For $j=2, \ldots, ns$ perform the following:
8	$U(j,j) = \frac{1}{R(j,j)}$ (260)
9	$U(k, j) = -\left(\sum_{\ell=k}^{j-1} U(k, \ell) R(\ell, j)\right) U(j, j) , k = 1,, j - 1 $ (261)
10	
11	j. Set the Y vector to the last column of the
12	transformed matrix H:
13	Y(i) = H(i, n+1)i = 1,,ns (262)
14	k. Compute the gain vector (G):
15	$G = R^{-1}Y \tag{263}$
16	
17	1. Determine if the gain vector is near zero. If
18	both $ G(1) $ and $ G(2) $ are less than or equal to 0.1, then
19	the algorithm has converged and Gauss-Newton
20	iterations shall terminate, and processing shall
21	resume as described in paragraph g.
	88

1 Otherwise, processing shall continue as described 2 below. 3 4 Limit the range changes such that the updated m. 5 range estimates will be within bounds as follows: If |G(1)| > 10000 or |G(2)| > 10000, perform the 6 i. 7 following calculations up to twenty times which 8 divide $\Delta R1$ and $\Delta R2$ by 2 until the updated range 9 estimates will be within bounds: $\alpha = 1$ (*Initialization*) 10 (264)11 $Rl_{remp} = Rl + \alpha G(l)$ (265) $R2_{temp} = R2 + \alpha G(2)$ 12 (266) 13 If $Rl_{min} < Rl_{temp} < Rl_{max}$ and $R2_{min} < R2_{temp} < R2_{max}$, update 14 gain vector as follows: $G = \alpha G$ 15 (267)16 and continue as described in paragraph 16n below. Otherwise, divide α by 2 and repeat the process. 17 If |G(1)| < 10000 or |G(2)| < 10000, perform the 18 ii. 19 following calculations up to twenty times which 20 decreases $\Delta R1$ and $\Delta R2$ by 5% until the updated 21 range estimates will be within bounds: ii = 0(*Initialization*) 22 (268)

1	$\alpha = \frac{100 - 5ii + 5}{100} \tag{269}$
2	$R1_{temp} = R1 + \alpha G(1) \tag{270}$
3	$R2_{lemp} = R2 + \alpha G(2) \tag{271}$
4	If $R1_{min}$ < $R1_{temp}$ < $R1_{max}$ and $R2_{min}$ < $R2_{temp}$ < $R2_{max}$, update
5	gain vector as follows:
6	$G = \alpha G \tag{272}$
7	and continue as described in paragraph 16 below.
8	Otherwise, increase ii by 1 and repeat the
9	process.
10	
11	n. Compute the stepsize (s) via the quadratic fit
12	type line search as follows:
13	i. This following procedure provides a method
14	for selecting the stepsize a_t in the modified
15	Gauss-Newton iterative formula
16	$x_{l+1} = x_l + a_l \Delta x_l $ (273)
17	where Δx_i is the correction vector. Actually,
18	because it is not normalized, the correction Δx_t
19	also contributes to the size of the step. It is
20	convenient to redefine equation (273) as
21	$x_{l+1} = x_l + a_j \Delta x_l $ (274)

1 where a_j denotes the *j*th value of the step size at the 1th Gauss-Newton iteration. 2 3 Once Δx_i is found from the Gauss-Newton ii. equations, the performance index PI_{t} is a function 4 5 only of a_i, $PI_1(a_i) = PI_1(x_1 + a_i \Delta x_1)$ 6 (275)7 and this is minimized by a judicious selection of 8 a_j . Here, a_j is defined by the minimum of a · 9 quadratic polynomial which passes through three 10 data points $(a_j, PI(a_j), j=1, 2, 3)$. For equally 11 spaced values of a_j , the step size occurring at 12 the minimum of this quadratic is given by $a_{m} = \frac{(a_{2} + a_{3})PI_{\ell}(a_{1}) - 2(a_{1} + a_{3})PI_{\ell}(a_{2}) + (a_{2} + a_{1})PI_{\ell}(a_{3})}{2PI_{\ell}(a_{1}) - 4PI_{\ell}(a_{2}) + 2PI_{\ell}(a_{2})}$ 13 (276)14 where $a_3 > a_2 \ge 0$. 15 iii. The first of these data points is readily available, namely, $a_1 = (0, PI_1(x_1))$; and if 16 17 $PI_t(1) < PI_t(0)$, (277)18 then $a_2=1$ gives the second data point and $a_3=2a_2$ gives the third. However, if equation (302) is 19 20 . not satisfied, the length of the interval is reduced by selecting $a_2 = \frac{1}{2}$ and $a_3 = 2a_2 = 1$, provided 21

1	$PI_{\ell}\left(\frac{1}{2}\right) < PI_{\ell}(0) . \tag{278}$
2	iv. If this is successful, the next selection is
· 3	$a_2 = \frac{1}{4}$ and $a_3 = 2a_2 = \frac{1}{2}$, and subsequent selections
4	are given by repeatedly dividing a_2 by 2. This
5	continues until $PI_{\ell}(a_2) < PI_{\ell}(a_1)$ or a threshold is
6	crossed which causes termination of the line
7	search. After a_m is found, then
8	$PI_{\ell}(a_m)$, $PI_{\ell}(a_1)$ and $PI_{\ell}(a_3)$ are compared to determine
9	which of these is the smallest. This is
10	necessary because the quadratic polynomial may
11	not always provide a good fit to the cost
12	function and $PI_t(a_2)$ or $PI_t(a_3)$ may be smaller than
13	$PI_{l}(a_{m})$.
14	
15	o. Update the states using the selected stepsize:
16	i. Update the range states:
17	$Rl_{new} = Rl_{old} + sG(1) \tag{279}$
18	$R2_{new} = R2_{old} + sG(2)$ (280)
19	and insure $Rl_{min}+0.1 < Rl_{max}-0.1$ and
20	$R2_{min}+0.1 < R2_{new} < R2_{max}-0.1.$
21	ii. If frequency data is being processed update
22	the frequency state:

1
$$Fb_{uvr} = Fb_{ud} + sG(3)$$
(281)
2 and insure $1 < Fb_{nev}$.
3
4 p. Compute the new performance index (PI_{nev}) based on
5 the updated states $(RI_{nev}, R2_{nev}, Fb_{nev})$.
6 q. Compute range, bearing, course and speed at tc :
7 i. Compute the x-component of target velocity
8 (Vxt) and the y-component of target velocity
9 (Vyt):
10 $Vxt = \frac{R2 \sin B2 + Xs2 - RI \sin BI - XsI}{t2 - tI}$ (282)
11 $Vyt = \frac{R2 \cos B2 + Ys2 - RI \cos BI - YsI}{t2 - tI}$ (283)
12 ii. Compute target course (Ct) and target speed
13 (Vt):
14 $Ct = \tan^{-1}\left(\frac{Vxt}{Vyt}\right)$ (284)
15 $Vt = \sqrt{Vxt^2 + Vyt^2}$ (285)
16 iii. Compute x-component of target position at
17 $tc (Xtc)$ and y-component of target position at
18 (Ytc):
19 $Xtc = R2 \sin B2 + Xs2 + Vxt(t2 - tc)$ (286)
20 $Ytc = R2 \cos B2 + Ys2 + Vxt(t2 - tc)$ (287)

1	iv. Compute x -component of range at tc with		
2	respect to own ship (Rxoc) and y-component of		
3	range at tc with respect to own ship (Ryoc):		
4	Rxoc = Xtc - Xoc (289)		
5	Ryoc = Ytc - Yoc (290)		
6	v. Compute range at <i>tc</i> with respect to own ship		
7	(Roc) and true bearing at tc with respect to own		
8	ship (Boc):		
9	$Roc = \sqrt{Rxoc^2 + Ryoc^2} $ (291)		
10	$Boc = \tan^{-1} \left(\frac{Rxoc}{Ryoc} \right) $ (292)		
11	vi. Limit the range at <i>tc</i> with respect to own ship to		
12	the maximum target range constraint.		
13			
14	Propagation path hypothesis testing can be performed by the		
15	endpoint MLE algorithm on up to a maximum of four data segments		
16	which may be from different sonar arrays, and the endpoint MLE		
17	algorithm is capable of processing an additional six azimuthal		
18	bearings only or azimuthal bearing/horizontal range segments		
19	(from any array) which may be direct path only. Each segment		
20	which contains either conical angle or frequency measurements is		

21 tested to determine whether the best propagation path is a 22 direct path or is a bottom bounce single ray reversal path. 23 Propagation path testing is performed by alternating the

propagation path for each segment to be tested from a direct 1 path to a bottom bounce path, running the endpoint MLE algorithm 2 for each propagation path combination and each appropriate 3 port/starboard combination and by saving the four best solution 4 based on the performance index, along with the associated 5 6 port/starboard indicators at the time lines and propagation 7 paths for each segment. Thus, if there are four conical angle only segments and six azimuthal bearing segments, then the .8 9 endpoint MLE may be invoked up to sixty-four times if testing 10 all possible port/starboard combinations. If the selected time 11 lines are associated with conical angle measurement and bearing measurements are available close in time to the conical angle 12 13 measurements which can remove all port/starboard ambiguity, then 14 the endpoint MLE will tie down to the bearing measurements and port/starboard hypothesis testing will not be performed. 15

Once the endpoint MLE has computed the four best solutions, the best solution is used to initialize the Cartesian coordinate MLE which will refine the solution using the optimal propagation path combinations. The Cartesian coordinate MLE shall be allowed to change the port/starboard designations if a particular part/starboard combination has been specified.

22

23 Cartesian Coordinate MLE

24 1. Initialize the number of Gauss-Newton iterations to zero.

1 2 2. Determine the number of state variables as follows: 3 If at least three frequency measurements are available, 4 then frequency data will be processed, target base 5 frequency shall be estimated and the number of state variables (ns) shall be set to five. Otherwise the number 6 7 of state variables shall be four, frequency data shall not 8 be processed and target base frequency shall not be 9 estimated. 10 Initialize values for x-coordinate of target position at tm 3. 11 (Xtm), y-component of target position at tm (Ytm), x-component 12 of target velocity (Vxt) and y-component of target velocity 13 (Vyt) using the outputs from the Endpoint MLE as follows: (293)14 Xtm = Roc sin Boc + Xoc - Vxt(tc - tm)15 (294) $Ytm = Roc \cos Boc + Yoc - Vyt(tc - tm)$ 16 (295)Vxt = Vt sin Ct17 (296) $Vyt = Vt \cos Ct$ 18 where Roc is the range at to with respect to own ship 19 Boc is the true bearing at tc with respect to own ship 20 Ct is the target course 21 Vt is the target speed 22 *Xoc* is the x-coordinate of own ship position at tc 23 . Yoc is the y-coordinate of own ship position at tc 24 tm is the time of the most recent measurement

1	tc is current time
2	If frequency data are being processed, initialize the base
3	frequency (Fb) to the base frequency output by the endpoint MLE.
4	
5	4. Compute the Cartesian coordinate performance index (PI)
6	based on the initial states as follows:
7	a. First, for each measurement in the batch:
8	i. Compute the x-component of range at t_i with
9	respect to the sensor associated with the ith
10	measurement (Rx_i) and the y-component of range at
11	t_i with respect to the sensor associated with the
12	ith measurement (Ry _i):
13	
14	$Rx_i = Xtm - Vxt(tm - t_i) - Xo_i $ (297)
15	$Ry_i = Ytm - Vyt(tm - t_i) - Yo_i $ (298)
16	where Xo_i is x-coordinate of own ship position at
17	ti
18	Yo _i is y-coordinate of own ship position at t_i
19	t; is the time of the ith measurement
20	tm is the time of the latest measurement
21	
22	ii. Compute the range at t_i with respect to the
23	sensor associated with the <i>i</i> th measurement (R_i) ;
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1	$R_{i} = \sqrt{Rx_{i}^{2} + Ry_{i}^{2}} $ (299)
2	iii. Compute the target image depth at t_i with
3	respect to the sensor associated with the <i>i</i> th
4	measurement (Rz_i) and D/E angle at t_i with respect
5	to the sensor associated with the <i>i</i> th measurement
6	$(heta_i)$.
7	iv. If the <i>i</i> th measurement is an azimuthal
8	bearing:
9	1.) Compute the true bearing at t_i with
10	respect to the sensor associated with the
11	ith measurement (B _i):
12	
13	$B_i = \tan^{-2} \left(\frac{R x_i}{R y_i} \right) \tag{300}$
14	
15	2.) Compute the bearing residual (RESb _i) such
16	that $-\pi \leq RESb_i \leq \pi$:
17	$RESb_i = Bm_i - B_i \tag{301}$
18	
19	where Bm_i is the <i>i</i> th measured bearing
20	3.) Compute the normalized bearing residual
21	$\left(\overline{RESb_i}\right)$:

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1	$\overline{RESb_i} = \frac{RESb_i}{\sigma b_i} $ (302)
2	where σb_i is the measured bearing standard
3	deviation
4	
5	v. If the <i>i</i> th measurement is a conical angle:
6	1.) Compute the target image depth at t_i
7	with respect to the sensor associated with
8	the ith measurement (Rz_i) and D/E angle at t_i
9	with respect to the sensor associated with
10	the ith measurement ($ heta_i$).
11	2.) If the D/E angle associated with the
12	conical angle measurement is valid:
13	<u>a</u> . Compute the true bearing at t_i with
14	respect to the sensor associated with
15	the <i>i</i> th measurement (B_i) :
16	$B_{i} = \tan^{-i} \left(\frac{Rx_{i}}{Ry_{i}} \right) \tag{303}$
17	
18	<u>b</u> . Compute the slant range at t_i with
19	respect to the sensor associated with
20	the <i>i</i> th measurement (Rs_i) :
21	$Rs_{i} = \sqrt{Rx_{i}^{2} + Ry_{i}^{2} + Rz_{i}^{2}} $ (304)

1	<u>c</u> . Compute the conical angle at t_i with
2	respect to the sensor associated with
3	the <i>i</i> th measurement (eta_i) :
4	$\beta_{i} = \cos^{-1} \left(\frac{\left(\cos Cs_{i} \left(Rx_{i} \sin Hs_{i} + Ry_{i} \cos Hs_{i} \right) - \sin Cs_{i} Rz_{i} \right)}{Rs_{i}} \right) (305)$
5	where Cs_i is the sensor cant angle at
6	the ith measurement
7	Hs_i is the sensor heading at the ith
8	measurement
9	
10	<u>d</u> . Compute the conical angle
11	$(RES\beta_i)$ such that $-\pi \leq RES\beta_i \leq \pi$:
12	$RES\beta_{i} = \beta m_{i} - \beta_{i} \qquad (306)$
13	where eta_{m_i} is the i th measured conical
14	angle
15	e. Compute the normalized conical angle
16	residual $\left(\overline{RES\beta_i}\right)$:
17	$\overline{RESb_i} = \frac{RESb_i}{\sigma b_i} $ (307)
18	where $\sigma \beta_i$ is the measured conical angle
19	standard deviation.
20	vi. If the <i>i</i> th measurement is a range:
21	1.) Compute the range residual (RESr _i):

$$RESr_{i} = Rm_{i} - R_{i}$$
(308)

$$RESr_{i} = Rm_{i} - R_{i}$$
(308)
where Rm_{i} is the ith measured range

$$2.) Compute the normalized range residual
$$\left(\frac{RESr_{i}}{r}\right):$$

$$RESr_{i} = \frac{RESr_{i}}{\sigma r_{i}}$$
(309)
where σr_{i} is the measured range standard
deviation
where σr_{i} is the measured range standard
vii. If frequency data are being processed and
the ith measurement is a frequency:
1.) Compute the x-component of target
relative velocity at t_{i} with respect to the
sensor associated with the ith measurement
(Vx_{i}) and the y-component of target relative
velocity at t_{i} with respect to the sensor
associated with the ith measurement (Vy_{i}):

$$Vx_{i} = Vxt - Vxs_{i}$$
(310)$$

18 $Vy_i = Vyt - Vys_i$ (311) 19 where Vxs_i is the x-component of sensor 20 velocity at t_i

1 Vys_i is the y-component of sensor 2 velocity at t_i 3 2.) Compute the target image depth at t_i with 4 5 respect to the sensor associated with the *i*th measurement (Rz_i) and D/E angle at t_i with respect 6 to the sensor associated with the ith measurement 7 8 (θ_i) 9 10 If the D/E angle associated with the 3.) 11 frequency is valid, compute the slant range at t_i 12 with respect to the sensor associated with the 13 ith measurement (Rs_i): $Rs_i = \sqrt{R_i^2 + Rz_i^2}$ 14 (312)15 Compute the estimated frequency at t_i with 4.) 16 respect to the sensor associated with the ith 17 measurement: $f_{i} = Fb \frac{cRs_{i} + Vxs_{i}Rx_{i} + Vys_{i}Ry_{i}}{cRs_{i} + VxtRx_{i} + VytRy_{i}}$ 18 (313) 19 5.) Compute the frequency residual $(RESf_i)$ $RESf_i = fm_i - f_i$ 20 (314) 21 where fm_i is the *i*th measured frequency

1 6.) Compute the normalized frequency residual $\left(\overline{RESf_i}\right)$: 2 $\overline{RESf_i} = \frac{RESf_i}{\sigma f_i}$ 3 (315)where σf_i is the measured frequency standard 4 5 deviation. 6 7 If a range constraint is being imposed, then the b. 8 following computations shall be performed: 9 i. Compute the range residual (RESr): 10 RESr = Rc - R(316)11 12 where Rc is the assumed target range ii. Compute the normalized speed residual (RESr): 13 14 $\overline{RESr} = \frac{RESr}{\sigma^R}$ 15 (317)16 17 where σR is the assumed target range standard 18 deviation. 19 20 с. If a speed constraint is being imposed, then the 21 following computations shall be performed.

3

4

i. Compute the estimated target speed:

$$V = \sqrt{Vxt^2 + Vyt^2}$$
(318)

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$$RESV = VC - V \tag{319}$$

(320)

where Vc is the assumed target speed

iii. Compute the normalized speed residual

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11

$$\overline{RESV} = \frac{RESV}{\sigma V}$$

12 where σV is the assumed target speed standard 13 deviation.

RESv

14 d. Compute the performance index as one half of the15 sum of the squared normalized residuals.

16

17 5. Gauss-Newton iterations shall be performed as described in 18 paragraphs a through n below, until the algorithm converges as 19 described in paragraph n or until twenty-five iterations have 20 been performed.

21

a. For each measurement in the batch:

1		i. Compute the x-component of range at t_i with
2		respect to the sensor associated with the <i>i</i> th
3		measurement (Rx_i) and the y-component of range at
4		t_i with respect to the sensor associated with the
5		ith measurement (Ry _i):
6	·	$Rx_i = Xtm - Vxt(tm - t_i) - Xo_i \qquad (321)$
7		$Ry_i = Ytm - Vyt(tm - t_i) - Yo_i $ (322)
8		where Xo_i is x-coordinate of own ship position at
9		ti
10	• • •	Yo _i is y-coordinate of own ship position at t_i
11		t_i is the time of the <i>i</i> th measurement
12	•	ii. Compute the range at t_i with respect to the
13		sensor associated with the <i>i</i> th measurement (R_i) :
14		$R_i = \sqrt{Rx_i^2 + Ry_i^2} \tag{323}$
15		iii. Compute the target image depth at t_i with
16	· .	respect to the sensor associated with the <i>i</i> th
17		measurement (Rz_i) and D/E angle at t_i with respect
18		to the sensor associated with the <i>i</i> th measurement
19		(θ_{I})
. 20		iv. If the <i>i</i> th measurement is an azimuthal
21		bearing:
22		1.) Compute the partial derivative of true
23		bearing at t_i with respect to the sensor

associated with the <i>i</i> th measurement wit	h
respect to the x-coordinate of target	
position at $t_m \left(\frac{\partial B_i}{\partial X t m} \right)$, the partial derive	ative
of true bearing at t_i with respect to the	ne
sensor associated with the <i>i</i> th measurem	ent ,
with respect to the y-coordinate of tar	get
position at $t_{\pi}\left(\frac{\partial B_{i}}{\partial Ytm}\right)$, the partial derive	vative
of true bearing at t_i with respect to the	ne
sensor associated with the <i>i</i> th measurem	ent
with respect to the x-component of targ	et
velocity $\left(\frac{\partial B_i}{\partial Vxt}\right)$, and the partial derivation	cive
of true bearing at t_i with respect to the	ne
sensor associated with the <i>i</i> th measurem	ent
with respect to the y-component of targ	et
velocity $\left(\frac{\partial B_i}{\partial Yxt}\right)$:	
$\frac{\partial B_i}{\partial X tm} = \frac{R Y_i}{R_i^2}$	(324)
$\frac{\partial B_i}{\partial Ytm} = -\frac{Rx_i}{R_i^2}$	(325)
AR AR	• .

 $\frac{\partial B_i}{\partial Vxt} = -(t_m - t_i) \frac{\partial B_i}{\partial Xtm}$ (326)

1	$\frac{\partial B_{i}}{\partial Vyt} = -(t_{m} - t_{i}) \frac{\partial B_{i}}{\partial Ytm} $ (327)	
2	2.) Compute the bearing residual (RESb _i)	
3	such that $-\pi \leq \text{RESb}_1 \leq \pi$:	
4	$RESb_i = Bm_i - B_i \tag{328}$	
5	where Bm_i is the <i>i</i> th measured bearing	
6	3.) Compute the normalized bearing residual	
7	$\left(\overline{\textit{RESb}_{i}} ight)$ and normalized partial derivative	
8	$\left(\frac{\overline{\partial B_{i}}}{\partial Xtm}, \frac{\overline{\partial B_{i}}}{\partial Ytm}, \frac{\overline{\partial B_{i}}}{\partial Vxt}, \frac{\overline{\partial B_{i}}}{\partial Vyt}\right):$	
9	$\frac{1}{RESb_{i}} = \frac{RESb_{i}}{\sigma b_{i}} $ (329)	
10	$\frac{\overline{\partial B_i}}{\partial Xtm} = \frac{\frac{\partial B_i}{\partial Xtm}}{\sigma b_i} $ (330)	
11	$\frac{\overline{\partial B_i}}{\partial Y tm} = \frac{\frac{\partial B_i}{\partial Y tm}}{\sigma b_i} $ (331)	
12	$\frac{\partial B_i}{\partial Vxt} = \frac{\frac{\partial B_i}{\partial Vxt}}{\sigma b_i} $ (332)	
13	$\frac{\overline{\partial B_i}}{\partial Vyt} = \frac{\frac{\partial B_i}{\partial Vyt}}{\sigma b_i} $ (333)	
14	where σb_i is the measured bearing standard	
15	deviation	
4.) If frequency data are not being processed, then set the next row of the augmented Jacobian matrix H to:

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$$\begin{bmatrix} \overline{\frac{\partial B_i}{\partial Xtm}} & \overline{\frac{\partial B_i}{\partial Ytm}} & \overline{\frac{\partial B_i}{\partial Vxt}} & \overline{\frac{\partial B_i}{\partial Vyt}} & \overline{RESB_i} \end{bmatrix}$$
(334)

If frequency data are being processed, then set the next row of the augmented Jacobian matrix H to:

$$\begin{bmatrix} \overline{\partial B_i} & \overline{\partial B_i} \\ \overline{\partial Xtm} & \overline{\partial Ytm} & \overline{\partial Vxt} & \overline{\partial Vyt} & 0 & \overline{RESB_i} \end{bmatrix}$$
(335)

If the *i*th measurement is a conical angle: 1.) Compute the target image depth at t_i with respect to the sensor associated with the *i*th measurement (Rz_i) and D/E angle at t_i with respect to the sensor associated with the *i*th measurement (θ_i).

2.) If the D/E angle associated with the conical angle measurement is valid:

<u>a</u> Compute the true bearing at t_i with respect to the sensor associated with the *i*th measurement (B_i) :

$$B_i = \tan^{-1} \left(\frac{Rx_i}{Ry_i} \right) \tag{336}$$

 $Rs_{i} = \sqrt{Rx_{i}^{2}}$ c Comp

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<u>b</u> Compute the slant range at t_i with respect to the sensor associated with the *i*th measurement (Rs_i):

$$Rs_{i} = \sqrt{Rx_{i}^{2} + Ry_{i}^{2} + Rz_{i}^{2}}$$
(337)

<u>c</u> Compute the conical angle at t_i with respect to the sensor associated with the *i*th measurement (β_i) :

$$\beta_{i} = \cos^{-1} \left(\frac{(\cos Cs_{i}(Rx_{i} \sin Hs_{i} + Ry_{i} \cos Hs_{i}) - \sin Cs_{i}Rz_{i})}{Rs_{i}} \right)$$
(338)

where Cs_i is the sensor cant angle at the *i*th measurement and Hs_i is the sensor heading at the *i*th measurement <u>d</u> Compute the partial derivative of conical angle at t_i with respect to the sensor associated with the *i*th measurement with respect to the *x*coordinate of target position at t_m

 $\left(\frac{\partial \beta_i}{\partial Xtm}\right)$, the partial derivative of conical angle at t_i with respect to the sensor associated with the *i*th measurement with respect to the *y*-

coordinate of target position at t_m 1 $\left(\frac{\partial \beta}{\partial v_{tm}}\right)$, the partial derivative of 2 conical angle at t_i with respect to the 3 4 sensor associated with the *i*th 5 measurement with respect to the xcomponent of target velocity $\left(\frac{\partial \beta_i}{\partial V \times t}\right)$, 6 and the partial derivative of conical 7 angle at t_i with respect to the sensor 8 associated with the *i*th measurement 9 with respect to the y-component of 10 11

target velocity
$$\left(\frac{\partial \beta_i}{\partial Vyt}\right)$$
:

$$12 \quad \frac{\partial \beta_i}{\partial Xtm} = -\frac{(\cos Cs_i)(Ry_i^2 + Rz_i^2)\sin Hs_i - Rx_i Ry_i \cos Hs_i) + Rx_i Rz_i \sin Cs_i)}{\sin \beta_i R_i^3}$$
(339)

15

$$14 \quad \frac{\partial \beta_i}{\partial Ytm} = -\frac{(\cos Cs_i (Rx_i^2 + Rz_i^2)\cos Hs_i - Rx_i Ry_i \sin Hs_i) + Ry_i Rz_i \sin Cs_i)}{\sin \beta_i R_i^3}$$
(340)

$$\frac{\partial \beta_i}{\partial V \times t} = -(t_m - t_i) \frac{\partial \beta_i}{\partial X t m}$$
(341)

16
$$\frac{\partial \beta_i}{\partial V_Y t} = -\langle t_m - t_i \rangle \frac{\partial \beta_i}{\partial Y t m}$$
(342)

e Compute the conical angle (RES β_i)

such that $-\pi \leq \text{RES}\beta_i \leq \pi$:

$$RES\beta_i = \beta m_i - \beta_I \tag{343}$$

where βm_i is the *i*th measured conical angle.

<u>f</u> Compute the normalized conical angle residual $(\overline{RES\beta})$ and normalized partial

derivatives
$$\left(\frac{\partial \beta_i}{\partial Xtm}, \frac{\partial \beta_i}{\partial Ytm}, \frac{\partial \beta_i}{\partial Vxt}, \frac{\partial \beta_i}{\partial Vyt}\right)$$

$$\frac{1}{RES\beta_{i}} = \frac{RES\beta_{i}}{\sigma\beta_{i}}$$
(344)

$$\frac{\partial \beta_{i}}{\partial Xtm} = \frac{\partial \beta_{i}}{\partial Xtm}$$
(345)

$$\frac{\partial \beta_{i}}{\partial Ytm} = \frac{\partial \beta_{i}}{\partial Ytm}$$
(346)

$$\frac{\overline{\partial \beta_{i}}}{\partial Vxt} = \frac{\frac{\partial \beta_{i}}{\partial Vxt}}{\sigma \beta_{i}}$$
(347)

$$\frac{\overline{\partial \beta_i}}{\partial Vyt} = \frac{\frac{\partial \beta_i}{\partial Vyt}}{\sigma \beta_i}$$
(348)

where $\sigma eta_{ extsf{I}}$ is the measured conical angle standard deviation

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1 2 If frequency data are not being q 3 processed, set the next row of the augmented Jacobian matrix H to: 4 $\left| \begin{array}{c} \partial \boldsymbol{\beta}_{i} & \partial \boldsymbol{\beta}_{i} & \partial \boldsymbol{\beta}_{i} \\ \overline{\partial Xtm} & \overline{\partial Ytm} & \overline{\partial Vxt} & \overline{\partial Vyt} \end{array} \right| RES \boldsymbol{\beta}_{i}$ 5 (349)If frequency data are being processed, 6 7 then set the next row of the augmented Jacobian matrix H to: 8 9 $\left| \frac{\partial \beta_{i}}{\partial X tm} \frac{\partial \beta_{i}}{\partial Y tm} \frac{\partial \beta_{i}}{\partial V xt} \frac{\partial \beta_{i}}{\partial V yt} \frac{\partial \beta_{i}}{\partial V yt} \right| 0 \overline{RES\beta_{i}}$ (350)10 11 If the ith measurement is a range: vi. 1.) Compute the partial derivative of range 12 13 at t_i with respect to the sensor associated 14 with the ith measurement with respect to the x-coordinate of target position at $t_m \left(\frac{\partial R_i}{\partial X_{tm}}\right)$, 15 16 the partial derivative of range at t_i with 17 respect to the sensor associated with the ith measurement with respect to the y-18 coordinate of target position at $t_m \left(\frac{\partial R_i}{\partial Y_{im}}\right)$, 19 the partial derivative of range at t_i with 20

respect to the sensor associated with the ith measurement with respect to the x-

component of target velocity $\left(\frac{\partial R_i}{\partial Vxt}\right)$, and the partial derivative of range at t_i with

respect to the sensor associated with the ith measurement with respect to the y-

component of target velocity $\left(\frac{\partial R_{i}}{\partial Vyt}\right)$:

$$\frac{\partial R_i}{\partial Xtm} = \frac{RR_i}{R_i}$$
(351)

$$\frac{\partial R_i}{\partial Y tm} = \frac{R Y_i}{R_i}$$
(352)

$$\frac{\partial R_i}{\partial Vxt} = -(t_m - t_i) \frac{\partial R_i}{\partial Xtm}$$
(353)

$$\frac{\partial R_{i}}{\partial Vyt} = -(t_{m} - t_{i}) \frac{\partial R_{i}}{\partial Ytm}$$
(354)

2.) Compute the range residual $(RESr_i)$:

 $RESr_i = Rm_i - R_i \tag{355}$

where Rm_i is the *i*th measured range

3.) Compute the normalized range residual $\overline{(RESr_i)}$ and normalized partial derivative

$$\left(\frac{\partial R}{\partial Xtm}, \frac{\partial R}{\partial Ytm}, \frac{\partial R}{\partial Vxt}, \frac{\partial R}{\partial Vyt}\right):$$

$$\frac{1}{RESr_i} = \frac{RESr_i}{\sigma r_i}$$
(356)

$$\frac{\partial R_i}{\partial Xtm} = \frac{\partial Xtm}{\sigma r_i}$$
(357)

б

$$\frac{\overline{\partial R_i}}{\partial Y tm} = \frac{\frac{\partial R_j}{\partial Y tm}}{\sigma r_i}$$
(358)

$$\frac{\partial R_{i}}{\partial Vxt} = \frac{\partial R_{i}}{\partial Vxt}$$
(359)

$$\frac{\partial R_{i}}{\partial V v t} = \frac{\partial R_{i}}{\sigma r}$$
(360)

where $\sigma r_{\rm i}$ is the measured range standard deviation.

4.) If frequency data are not being processed, then set the next row of the augmented Jacobian matrix H to:

$$\begin{bmatrix} \overline{\partial R_i} & \overline{\partial R_i} & \overline{\partial R_i} & \overline{\partial R_i} & \overline{\partial R_i} \\ \overline{\partial Xtm} & \overline{\partial Ytm} & \overline{\partial Vxt} & \overline{\partial Vyt} & \overline{RESr_i} \end{bmatrix}$$
(361)

If frequency data are being processed, then set the next row of the augmented Jacobian matrix *H* to:

$$\frac{\partial R_{i}}{\partial Xtm} \frac{\partial R_{i}}{\partial Ytm} \frac{\partial R_{i}}{\partial Vxt} \frac{\partial R_{i}}{\partial Vyt} \frac{\partial R_{i}}{\partial F_{i}} \frac{\partial R_{i}}{\partial F_{i}}$$
(362)

• *	
1	vii. If frequency data are being processed and
2	the <i>i</i> th measurement is a frequency:
3	1.) Compute the x-component of target
4	relative velocity at t_i with respect to the
5	sensor associated with the <i>i</i> th measurement
б	(Vx_i) and the y-component of target relative
7	velocity at t_i with respect to the sensor
8	associated with the <i>i</i> th measurement (Vy_i) :
9	$Vx_i = Vxt - Vxs_i \tag{363}$
10	$V_{Y_i} = V_Y t - V_Y s_i \tag{364}$
11	where Vxs_i is the x-component of sensor
12	velocity at t_i and Vys_i is the y-component of
13	sensor velocity at t_i .
14	
15	2.) Compute the target image depth at t_i
16	with respect to the sensor associated with
17	the ith measurement (Rz_i) and D/E angle at t_i
18	with respect to the sensor associated with
19	the ith measurement ($ heta_i$).
20	
21	3.) If the D/E angle associated with the
22	frequency is valid, compute the slant range
23	at t_i with respect to the sensor associated
24	with the <i>i</i> th measurement (Rs_i) :

1
$$R_{S_{i}}\sqrt{R_{i}^{2} + R_{Z_{i}}^{2}}$$
 (365)
2
3 4.) Compute the partial derivative of
4 frequency at t_{i} with respect to the sensor
5 associated with the ith measurement with
6 respect to the x-coordinate of target
7 position at $t_{x}\left(\frac{\partial f_{i}}{\partial Xm}\right)$, the partial derivative
8 of frequency at t_{i} with respect to the sensor
9 associated with the ith measurement with
10 respect to the y-coordinate of target
11 position at $t_{x}\left(\frac{\partial f_{i}}{\partial IXm}\right)$, the partial derivative
12 of frequency at t_{i} with respect to the sensor
13 associated with the ith measurement with
14 respect to the x-component of target
15 velocity $\left(\frac{\partial f_{i}}{\partial IXn}\right)$, the partial derivative of
16 frequency at t_{i} with respect to the sensor
17 associated with the ith measurement with
18 respect to the y-component of target
19 velocity $\left(\frac{\partial f_{i}}{\partial Iyq}\right)$ and the partial derivative of
20 frequency at t_{i} with respect to the sensor

æ.,

associated with the ith measurement with

	7.) Compute the normalized frequency	
	residual $\left(\overline{\textit{RESF}_{i}} ight)$ and normalized partial	
·	derivatives $\left(\frac{\partial f_{i}}{\partial Xtm}, \frac{\partial f_{i}}{\partial Ytm}, \frac{\partial f_{i}}{\partial Vxt}, \frac{\partial f_{i}}{\partial Vyt}, \frac{\partial f_{i}}{\partial Fb}\right)$:
	$\frac{1}{RESf_i} = \frac{RESf_i}{\sigma f_i}$	(373)
	$\frac{\partial f_i}{\partial X tm} = \frac{\frac{\partial f_i}{\partial X tm}}{\sigma f_i}$	(374)
н -	$\frac{\partial f_i}{\partial Y tm} = \frac{\partial f_i}{\partial Y tm}$	(375)
	$\frac{\overline{\partial f_i}}{\partial V \times t} = \frac{\frac{\partial f_i}{\partial V \times t}}{\sigma f_i}$	(376)
	$\frac{\partial f_{i}}{\partial Vyt} = \frac{\partial f_{i}}{\partial Vyt}$	(377)
	$\frac{\partial \bar{f}_i}{\partial Fb} = \frac{\partial \bar{f}_i}{\partial Fb}$	(378)
	where σf_i is the measured frequency star	ndard
	deviation.	

where fm_i is the *i*th measured frequency

13 8.) Set the next row of the augmented14 Jacobian matrix H to:

 $\left[\frac{\partial f_{i}}{\partial Xtm} \frac{\partial f_{i}}{\partial Ytm} \frac{\partial f_{i}}{\partial Vxt} \frac{\partial f_{i}}{\partial Vyt} \frac{\partial f_{i}}{\partial Fb} \frac{\partial f_{i}}{RESf_{i}}\right]$

19

2 If a range constraint is being imposed, the b. following computations shall be performed: 3 4 Compute the partial derivative of range at t_i i. 5 with respect to the sensor associated with the 6 ith measurement with respect to the x-coordinate of target position at $t_m \left(\frac{\partial R_i}{\partial Xtm}\right)$, the partial 7 derivative of range at t_i with respect to the 8 9 sensor associated with the ith measurement with 10 respect to the y-coordinate of target position at $t_m \left(\frac{\partial R_i}{\partial Ytm}\right)$, the partial derivative of range at t_i 11 12 with respect to the sensor associated with the 13 ith measurement with respect to the x-component of target velocity $\left(\frac{\partial R_i}{\partial Vxt}\right)$, and the partial 14 15 derivative of range at t_i with respect to the 16 sensor associated with the ith measurement with 17 respect to the y-component of target velocity $\left(\frac{\partial R_i}{\partial V_{Vt}}\right)$: 18

 $\frac{\partial R_i}{\partial X tm} = \frac{R x_i}{R_i}$

(380)

(379)

$$\frac{\partial R_{i}}{\partial Ytm} = \frac{R_{Y_{i}}}{R_{i}}$$
(381)

$$\frac{\partial R_{i}}{\partial Vxt} = -(t_{n} - t_{i}) \frac{\partial R_{i}}{\partial Xtm}$$
(382)

$$\frac{\partial R_{i}}{\partial Vyt} = -(t_{n} - t_{i}) \frac{\partial R_{i}}{\partial Ytm}$$
(383)

$$\frac{\partial R_{i}}{\partial Vyt} = -(t_{n} - t_{i}) \frac{\partial R_{i}}{\partial Ytm}$$
(383)

$$\frac{\partial R_{i}}{\partial Yyt} = RC - R$$
(384)

$$\frac{\partial R_{i}}{\partial tm} = RC - R$$
(384)

$$\frac{\partial R_{i}}{\partial tm} = RC - R$$
(384)

$$\frac{\partial R_{i}}{\partial tm} = \frac{\partial R_{i}}{\partial tm}$$
(385)

$$\frac{\partial R_{i}}{\partial Xtm} = \frac{\partial R_{i}}{\partial t}$$
(385)

$$\frac{\partial R_{i}}{\partial Xtm} = \frac{\partial R_{i}}{\partial R}$$
(385)

$$\frac{\partial R_{i}}{\partial Xtm} = \frac{\partial R_{i}}{\partial R}$$
(386)

$$\frac{\partial R_{i}}{\partial Ytm} = \frac{\partial R_{i}}{\partial R}$$
(386)

$$\frac{\partial R_{i}}{\partial Ytm} = \frac{\partial R_{i}}{\partial R}$$
(386)

$$\frac{\partial R_{i}}{\partial Ytm} = \frac{\partial R_{i}}{\partial R}$$
(386)

$$\frac{\partial R_{i}}{\partial Yyt} = \frac{\partial R_{i}}{\partial R}$$
(386)

$$\frac{\partial R_{i}}{\partial Yyt} = \frac{\partial R_{i}}{\partial R}$$
(386)

where $\sigma_{\mathbf{R}}$ is the measured range standard deviation.

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iv. If frequency data are not being processed, then set the next row of the augmented Jacobian matrix H to:

 $\left[\frac{\partial \mathcal{R}_{i}}{\partial Xtm} \frac{\partial \mathcal{R}_{i}}{\partial Ytm} \frac{\partial \mathcal{R}_{i}}{\partial Vxt} \frac{\partial \mathcal{R}_{j}}{\partial Vyt} \frac{\partial \mathcal{R}_{j}}{\partial Vyt}\right]$ (390)

If frequency data are being processed, then set the next row of the augmented Jacobian matrix *H* to:

$$\begin{bmatrix} \frac{\partial R_i}{\partial Xtm} & \frac{\partial R_i}{\partial Ytm} & \frac{\partial R_i}{\partial Vxt} & \frac{\partial R_i}{\partial Vyt} & 0 & \overline{RESr_i} \end{bmatrix}$$
(391)

c. If a speed constraint is being imposed, the following computations shall be performed:

i. Compute the estimated target speed:

$$V = \sqrt{V_X t^2 + V_Y t^2}$$
(392)

15 ii. Compute the partial derivative of target 16 speed with respect to the x-coordinate of target 17 position at $t_m \left(\frac{\partial V}{\partial X tm}\right)$, the partial derivative of 18 target speed with respect to the y-coordinate of 19 target position at $r_m \left(\frac{\partial V}{\partial Y tm}\right)$, the partial

derivative of target speed with respect to the x-

component of target velocity $\left(\frac{\partial V}{\partial Vxt}\right)$ and the

partial derivative of target speed with respect to the y-component of target velocity $\left(\frac{\partial V}{\partial Vvt}\right)$:

 $\frac{\partial V}{\partial Xtm} = 0 \tag{393}$

$$\frac{\partial V}{\partial Ytm} = 0 \tag{394}$$

$$\frac{\partial V}{\partial V x t} = \frac{V x t}{V} \tag{395}$$

$$\frac{\partial V}{\partial Vyt} = \frac{Vyt}{V} \tag{396}$$

iii. Compute the speed residual (RESv):

$$RESv = Vc - V \tag{397}$$

where Vc is the assumed target speed

iv. Compute the normalized speed residual $\left(RESv \right)$

and normalized partial derivatives

14
$$\left(\frac{\partial V}{\partial Xtm}, \frac{\partial V}{\partial Ytm}, \frac{\partial V}{\partial Vxt}, \frac{\partial V}{\partial Vyt}\right)$$
:

$$RESv = \frac{RESv}{\sigma R}$$
(398)

$$\frac{\partial V}{\partial Xtm} = \frac{\partial Xtm}{\sigma V}$$
(399)

$$\frac{\partial V}{\partial Ytm} = \frac{\partial Ytm}{\sigma V}$$
(400)

$$\frac{\partial V}{\partial V x t} = \frac{\partial V}{\sigma V}$$
(401)

$$\frac{-}{\frac{\partial V}{\partial V_{yt}}} = \frac{\frac{\partial V}{\partial V_{yt}}}{\sigma V}$$
(402)

(403)

where $\sigma {\tt V}$ is the assumed target speed standard deviation.

v. If frequency data are not being processed, then set the next row of the augmented Jacobian matrix H to:

If frequency data are being processed, then set the next row of the augmented Jacobian matrix H to:

 $\begin{array}{cccc} \partial V & \partial V & \partial V & \partial V \\ \partial X tm & \partial Y tm & \partial Vxt & \partial Vyt \end{array}$

 $\begin{bmatrix} \frac{\partial V}{\partial X tm} & \frac{\partial V}{\partial V tm} & \frac{\partial V}{\partial V t} & \frac{\partial V}{\partial V y t} & 0 & RESv \end{bmatrix}$ (404)

d. Reorder the rows of the matrix *H* such that a zero valued partial derivative does not appear along the diagonal.

1		e. Perform the Householder transformation on the m \mathbf{x}
2		n+1 matrix H.
3		
4		f. Extract the upper triangular matrix R from the
5		upper left hand corner of the transformed matrix H.
6		
7		g. Compute R^{-1} by back-substitution.
8		
9		h. Extract the Y vector from the upper right hand
10		corner of the transformed matrix H.
11	•	
12		i. Compute the gain vector (G):
13		$G = R^{-1}Y \tag{405}$
14		
15		j. Determine if the gain is near zero. If both $ G(i) $
16		and $ G(2) $ are less than 0.1 and $ G(3) $ and $ G(4) $ are less
17		than 0.01, then the algorithm has converged, Gauss
18		Newton iterations shall terminate, and processing
19		shall be performed as described in paragraph 6.
20		Otherwise, processing shall continue as described
21		below.
22		

121. Update the states using the optimal stepsize(s):34i. Update the position and velocity states:56
$$Xtm = Xtm + sG(1)$$
 (406)7 $Ytm = Ytm + sG(2)$ (407)8 $Vxt = Vxt + sG(3)$ (408)9 $Vyt = Vyt + sG(4)$ (409)10ii. If frequency data are being processed,11update the frequency state:121314iii Compute range with respect to own ship at t_m 15(Rom) and target speed (Vt) as161718 $Vt = \sqrt{Vxt^2 + Vyt^2}$ (411)18 $Vt = \sqrt{Vxt^2 + Vyt^2}$ (412)192010i.nsure $R_{min} + 0.1 < Rom < R_{max}$ and $V_{min} +$ 20i.v. Insure $R_{min} + 0.1 < Rom < R_{max}$ and $V_{min} +$ 21 $0.1 < Vt < V_{max}$. If either Rm or Vt is out of bounds,22limit the appropriate parameter and recompute23 Xtm, Ytm, Vxt and Vyt .

1 2 Compute the new performance index (PInew) based on m. 3 the updated states (Xtm, Ytm, Vxt, Vyt, Fb) 4 5 Compute range, bearing, course and speed at n. 6 current time: 7 8 i. Compute target course (Ct) and target speed 9 (Vt): 10 $Ct = \tan^{-1} \left(\frac{Vxt}{Vyt} \right)$ 11 (413) $Vt = \sqrt{Vxt^2 + Vyt^2}$ 12 (414)13 14 Compute x-coordinate of target position at ii. 15 tc(Xtc) and y-coordinate of target position at 16 tc(Ytc): 17 18 Xtc = Xtm + Vxt(tc - tm)(415)19 Ytc = Ytm + Vyt(tc - tm)(416)20 21 iii. Compute x-component of range at tc(Rxc) and 22 y-component of range at tc with respect to own 23 ship(Ryc):

$$Rxc = Xtc - Xoc \tag{417}$$

 $Ryc = Ytc - Yoc \tag{418}$

where Xoc is the x-coordinate of own ship position at tc and Yoc is the y-coordinate of own ship position at tc.

iv. Compute range at tc with respect to own ship
(Rc) and true bearing at tc with respect to own
ship (Bc):

$$Rc = \sqrt{R \times c^2 + R y c^2}$$
(419)

$$Bc = \tan^{-1} \left(\frac{Rxc}{Ryc} \right) \tag{420}$$

v. Limit range at to with respect to own ship to a maximum of the target maximum range.

18 vi. Limit target speed to a maximum of the19 target maximum speed.

o. Determine if the change in the performance index
is negligible. If so, processing shall terminate,
otherwise, Gauss-Newton iterations shall continue.

1 2 i. Compute change in the performance index (ΔPI) : 3 1.) If $PI_{old} > 0$, 4 $\Delta PI = \frac{\left| PI_{new} - PI_{old} \right|}{PI_{old}}$ (421)5 2.) If *PI*_{old}=0, 6 $\Delta PI = 0$ 7 (422)8 9 ii. If $\Delta PI \leq 0.00001$ and $PI_{new} \leq threshold_{cc}$, stop iterating. 10 11 Compute the ns by ns Cartesian coordinate covariance matrix: 12 6. $P = R^{-1}R^{-T}$ (423)13 14 Extrapolate the covariance matrix forward to current time: 15 7. If frequency data are not being processed, the 16 a. transition matrix Φ shall be defined as follows: 17 $\phi = \begin{bmatrix} 1 & 0 & tc - tm & 0 \\ 0 & 1 & 0 & tc - tm \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ (424)18

$$\sigma_{R} = \sqrt{\frac{P_{11}Rxc^{2} + 2P_{12}RxcRyc + P_{22}Rxc^{2}}{Rc^{4}}}$$

$$\sigma_{C} = \sqrt{\frac{P_{11}Ryc^{2} - 2P_{12}RxcRyc + P_{22}Rxc^{2}}{Rc^{4}}}$$

$$(423)$$

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$$\sigma_{s} = \sqrt{\frac{P_{33}Vxt^{2} + 2P_{34}Vxt * Vyt + P_{44}Vyt^{2}}{Vt^{2}}}$$
(430)

15 If frequency data are being processed.

7

$$\sigma_F = \sqrt{P_{55}}$$

9. Compute major and minor localization ellipse axis length
(X_{maj}, X_{min}) and orientation of major axis from North (ORIEN):

$$\lambda_{maj} = \frac{P_{11} + P_{22} + \sqrt{(P_{11} - P_{22})^2 + 4P_{12}^2}}{2}$$
(432)

(431)

$$\lambda_{\min} = \frac{P_{11} + P_{22} - \sqrt{(P_{11} - P_{22})^2 + 4P_{12}^2}}{2}$$
(433)

$$X_{maj} = 2.1459 \sqrt{\lambda_{maj}} \tag{434}$$

$$X_{\min} = 2.1459 \sqrt{\lambda_{\min}}$$
 (435)

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$$ORIEN = \tan^{-1} \left[\frac{P_{12}}{\lambda_{maj} - P_{11}} \right]$$
 (436)

11 10. Outputting to a display computer.

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13 It will be understood that many additional changes in the 14 details, materials, steps and arrangement of parts, which have 15 been herein described and illustrated in order to explain the 16 nature of the invention, may be made by those skilled in the art 17 within the principle and scope of the invention as expressed in 18 the appended claims.

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Attorney Docket No. 78009

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MULTI-STAGE MAXIMUM LIKELIHOOD TARGET ESTIMATOR

ABSTRACT OF THE DISCLOSURE

6 A multi-stage maximum likelihood target estimator for use 7 with radar and sonar systems is provided. The estimator is a 8 software implemented algorithm having four computational stages. 9 The first stage provides angle smoothing for data endpoints 10 thereby reducing angle errors associated with tie-down times. 11 The second stage performs a coarse grid search to obtain the initial approximate target state to be used as a starting point 12 13 for stages 3 and 4. The third stage is an endpoint Gauss-Newton 14 type maximum likelihood target estimate which determines target range along two time lines. The final refinement of the target 15 state is obtained by the fourth stage which is a Cartesian 16 17 coordinate maximum likelihood target estimate. The four-stage processing allows the use of target historic data while reducing 18 19 processing time and computation power requirement.



FIG. 1



